INTERACTION OF ULTRASONIC WAVES
WITH SIMULATED AND REAL FATIGUE CRACKS

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INTRODUCTION

It is now well accepted that the partial contact of fracture surfaces can have significant effects on the ultrasonic response of fatigue cracks. The authors and colleagues\textsuperscript{1-4} have developed an approximate model for this effect in which the array of contacts is replaced by an equivalent distributed spring with stiffness per unit area, $\kappa$. A result of this model, the frequency dependent transmission and reflection coefficients, has been verified by comparison to exact solutions for special cases.\textsuperscript{5,6} Of particular note is the comparison to the transmission and reflection at a periodic array of strip contacts, as analyzed by Angel and Achenbach\textsuperscript{7}, which is in good agreement with that of the spring model when the wavelength is large with respect to the contact spacing. Comparison to static elasticity solutions allows $\kappa$ to be determined for a variety of interesting interfacial topographies.\textsuperscript{5,6}

Using a slightly different approach, Haines\textsuperscript{8} has derived expressions for the reflection and transmission coefficients for partially contacting surfaces characterized by the mean radius of the area of each contact, $r$, and the surface flow stress of the material, $\sigma_m$. Haines' model predicts the same frequency dependence of the transmission and reflection coefficients as the spring model. Comparing these two models\textsuperscript{2} yields that $\kappa$ has a value of

$$\kappa = kE \left( \frac{N\sigma_0}{\pi\sigma_m} \right)^{1/2}$$

(1)

where $k$ is a constant of order two, $E$ is Young's modulus, $N$ is the density of contacts per unit area, $\sigma_0$ is the average compressive contact stress at the interface and $\sigma_m$ is the surface yield stress. Haines'\textsuperscript{8} model has been found to be in semi-quantitative agreement with measurements by Woolridge.\textsuperscript{9,10}

Buck et al.\textsuperscript{11} have experimentally examined the effect of various types of interfaces on the transmission coefficient of a 10 MHz longitudinal wave incident at 45° to the interface and scattered at various
angles. They found a similarity between the angular dependence for random and periodic interfaces and a marked difference between those and the angular dependence of the solid reference block, thus showing that the nature of a real interface affects the transmissivity in a fashion that may allow deduction of the interface roughness.

Thompson et al.\textsuperscript{2,3} have experimentally investigated the behavior of the transmission and mode conversion of a longitudinal wave normally incident on both real and ideal (saw slot) cracks. The transmission behavior was then compared to the predictions of the distributed spring model for each crack type and good agreement was noted. The real fatigue crack showed an apparent lengthening at higher frequencies. It was suggested that the closure region at the tip of the crack most significantly affected the transmission of the higher frequencies due to decreased transmission in the partially closed region. This observation is consistent with the spring model for the interface and appears to provide a way to estimate the spatial extent of the closure region. The 45° diffracted shear waves for the fatigue crack were observed to be quite different from those of the ideal crack. The changes were attributed to the more gradual change from perfect contact to no contact in the fatigue crack. No theoretical predictions for this effect were presented.

This paper reports recent ultrasonic studies of both simulated and real fatigue crack scattering and compares the results obtained to the predictions of the distributed spring model. In addition, further progress on development of the model to more closely resemble the discreteness of contact in an actual crack has been accomplished. It is suggested that the discreteness of the contacts must be introduced to quantitatively describe the tip diffraction results, and that this opens the possibility of experimentally determining the number of contacts per unit area, N. Use of the value of N so determined, and the $\kappa$ value as determined from through transmission experiments, in equation (1) should then allow the contact stress, $\sigma^0$, to be determined.

SIMULATED CRACK FREQUENCY RESPONSE

The periodic grating and reference blocks prepared from 1100-HO aluminum by Buck et al.\textsuperscript{11} were also used for this work. The periodic grating was designed to have a contact width, W, of 40 $\mu$m and a periodicity, S, of 120 $\mu$m with a step height of 20 $\mu$m. Baik and Thompson\textsuperscript{5,6} have shown that, for an interface of this type, the distributed spring constant, $\kappa$, can be calculated according to the relation

$$k = \frac{E}{S(1-v^2)} \left[ 4 \pi \ln \left[ \sec \frac{\pi (1-w}{S} \right] \right]^{-1}$$

where E is the Young's modulus and v is Poisson's ratio. The calculation for the block used here yields a $\kappa$ equal to 7.5 x $10^8$ MN/m$^3$.

Figure 1 shows the results of measurements of the interface reflection and transmission coefficients as a function of frequency for various orientations of the block. Included are results for both normal and 45° incidence with respect to the interface. In all cases, 0.75 inch (1.9 cm) diameter, 10 MHz nominal center frequency transducers with a 4 inch (10 cm) focal length in water were used. The
transducers were positioned such that their focal planes coincided with the center of the sample. For each case shown, sample and reference (similar sample with no interface) waveforms were taken and then converted to the frequency domain using a fast Fourier transform technique. Shown in the figure are the results obtained by deconvolving the sample waveform by the reference waveform. The abscissa is in absolute units, so that a value of 1.0 corresponds to complete transmission or reflection.

In all cases, the frequency dependence of the deconvolution results had the correct qualitative features, but was much smaller than expected for the value of $\kappa$ calculated using Eq. 2. The approximate frequency response for a $\kappa$ of that magnitude would be as shown by the dashed lines in Figure 1. The actual behavior seen would require a value for $\kappa$ of approximately an order of magnitude lower than that predicted, suggesting that a significant change in either the contact width, $W$, or periodicity, $S$, had occurred. Examination of the contacting faces after disassembly of the block showed that many of the ridges on the grating half of the block did not exhibit the plastic deformation expected if good acoustic contact had been made. The actual contact was not truly periodic and the "effective" periodicity was much larger than the 120 $\mu$m predicted, thus yielding an actual $\kappa$ smaller than was originally assumed. Thus, although it was not possible to quantitatively test the model predictions, the model did yield a correct prediction of the unexpected nonideality of the sample structure.
In addition, experimental data were taken from this sample for the non-specular orientations as shown in Figure 2. As presently formulated, the current models make no predictions for these cases. It can be speculated, however, that if the contact between the halves of the block had been as originally intended, the response at these non-specular directions may have been even more pronounced. In any event, the acoustic response in these directions was quite large and easily observed.

REAL CRACK FREQUENCY RESPONSE

The distributed spring model\(^4\) yields a prediction of the normalized ultrasonic signal \(T^N\), i.e. the observed signal at any position along the crack normalized by the longitudinal signal \((\theta = 180^\circ)\) in an uncracked region, of the form,

\[
T^N = C \int_{-\infty}^{\infty} dx \frac{1}{1 + j\alpha^2} e^{-\left(x-x_1\right)^2/w^2} e^{jk(x-x_1)\sin \theta}
\]

where \(C\) is a normalizing constant involving the wave velocities, beam amplitude, beam widths and an angular term, \(w\) is a beam width parameter, \(k\) is the wave vector, \(x_1\) is the position of the beam center and \(\theta\) is the angle of the receiver with respect to the transmitter axis. For the forward transmission experiments, \(\theta = 180^\circ\) and for the tip diffracted experiments, \(\theta = 225^\circ\). The bracketed factor in the equation represents the interface transmissivity and the other factors describe the beam magnitude and phase overlaps. The factor \(\alpha\) in the bracketed term is

\[
\alpha = \pi \rho v f / \kappa(x)
\]

Figure 2. Transmission coefficient versus frequency for some non-specular transducer orientations.
where \( p \) is the material density, \( v \) the transmitted wave velocity, \( f \) is the frequency and \( \kappa(x) \) is the distributed spring constant. For the experimental fatigue crack results to be presented, \( \kappa(x) \) was modeled by a continuous exponential function of the form

\[
\kappa(x) = e^{A-Bx}
\]  

(5)

where \( A \) and \( B \) are constants derived by fitting the model predictions to the fatigue crack transmission versus position and frequency data. This function describes the transition from perfect contact, \( \kappa = \infty \), to no contact, \( \kappa = 0 \), in an approximate fashion.

The predictions of this model and experimental data for the case of through transmission of longitudinal waves for both the ideal and real crack have been presented elsewhere\(^2,3\) and will not be discussed further. In general the agreement was excellent. Here, the behavior of the tip diffracted waves at \( 45^\circ \) (\( \phi = 225^\circ \)), both for the longitudinal and shear modes, will be considered.

Figure 3 shows a comparison of model and experiment for a longitudinal tip diffracted wave from an ideal crack at several frequencies. One sees that the model describes both the peak amplitude and peak width of the response reasonably well. Figure 4 is a similar plot of the longitudinal tip diffracted wave data for a real fatigue crack. Both the predicted and experimental data in Figure 4 show a drop in amplitude as compared to the corresponding frequency for the ideal crack. Again, this change is thought to occur due to the region of closure at the tip of the real crack which makes the transition from the fully closed (uncracked) to the fully open condition more gradual.

Figure 3. Longitudinal tip diffracted response for an ideal (fully open) crack from experiment (left) and distributed spring model (right).
Figure 4. Longitudinal tip diffracted response for a real crack from experiment (left) and model (right).

and thereby weakens the strength of the tip diffracted signals. However, there are major differences between theory and experiment for the fatigue crack. The peak amplitudes and widths observed for the experimental data are much less dependent on frequency and larger in amplitude than the corresponding predictions from the model.

The shear wave tip diffracted results for the ideal crack are shown in Figure 5 for both theory and experiment. As in the longitudinal case, the results compare very favorably, both in peak amplitude and peak width. Comparison with Figure 6 (the corresponding data from a real crack) shows considerable changes. The frequency dependence of the signals from the real crack is much greater than those from the ideal crack for both the model and experiment. In addition, the peak amplitude of the experimental data has decreased significantly from the corresponding peak for the ideal crack. The frequency dependence of the theoretical predictions is quite pronounced, and at the higher frequencies, the theory is several orders of magnitude too small. It would appear, therefore, that the closure region plays an even more significant role for tip diffracted shear waves than for longitudinal waves, either tip diffracted or through transmission. In addition, it is evident that refinement of the continuous spring model is necessary to properly describe the observations.

EXTENSION OF MODEL TO DISCRETE CONTACTS

The distributed spring model, as initially derived, used an interface stiffness, \( \kappa \), that was a continuous function of position. Motivated by the failure to quantitatively predict the tip diffracted signal magnitudes, the model has since been extended to approximately include the effects due to discrete contact points of varying width. In equation (3), the interface transmissivity was given by the term
1/(1+j\alpha). In our first attempt to assess the significance of discrete contacts, the following assumptions were made regarding the form of the dynamic crack opening displacement (COD). (1) The spatial average of the COD is the same as that predicted by the quasi-static spring model. (2) The local COD assumes the value of zero over an effective contact of radius d and is a constant elsewhere. (3) In evaluating the ensuing scattering expressions, integrals over the small circular contact areas (with dimensions much less than a wavelength) may be approximated by the value of the integrand times the area. When the approximate COD is substituted into the representation integral for ultrasonic scattering, the result is equivalent to that which would be obtained when the interface transmissivity is replaced by the factor

\[
\frac{1}{1+j\alpha} + \left(\frac{1}{1+j\alpha}\right) \left[ 1 - \frac{j8\pi f d v (1-\nu^2)}{E(1-N_d^2)} + \frac{1}{N^{1/2}} \sum \frac{j8(1-\nu^2)\pi f d \delta(x-x_i)}{E(1-N_d^2)} \right] \quad (5)
\]

where N is the contact density (assumed to be a constant over the closed region), d is the diameter of an individual contact and the sum is to be evaluated over the x-coordinate of the crack.

If one views \( \kappa(x) \) as being known independently, e.g. from measurements of forward transmission of longitudinal waves, the d and N are related according to

\[
d(x) = 8\kappa(x)/N \pi E. \quad (6)
\]

Figure 5. Shear tip diffracted response for an ideal (fully open) crack from experiment (left) and model (right).
Figure 6. Shear tip diffracted response from a real crack from experiment (left) and model (right).
Thus, in this model, two parameters are needed to fully define the scattering, \( \kappa(x) \) and \( N \). The previously reported longitudinal forward transmission measurements agreed well with the continuous spring model.\(^2,3\) Hence those measurements can be considered as an experimental determination of \( \kappa(x) \). If the tip diffraction measurements are sensitive to \( N \), they would provide an experimental means of determining this independent parameter of the partially contacting closure zone. Knowledge of both \( \kappa(x) \) and \( N \) would allow the contact stresses to be calculated.\(^2,12\)

To test this possibility, the discretized model was used to calculate the 45° tip diffracted shear wave signal, as shown in Figure 7. As can be seen, a change in the contact density has a marked effect on the amplitude of the peak for the 4MHz tip diffracted wave. At the lowest density considered, \( N^{1/2} = 250 \) contacts/cm, the theoretical predictions have increased by three orders of magnitude from the continuum limit and are approaching the level observed experimentally. In contrast, the corresponding results for longitudinal through transmission showed an indistinguishable change. It is concluded that it is essential to include discreteness of contacts in a description of tip diffracted waves and that the above scenario for directly measuring contact stress appears to hold considerable promise.

**STRESS HISTORY OF CRACK PROPAGATION**

A second compact tension specimen of 7075-T651 aluminum containing a crack, grown in two stages, was also examined in through transmission. The specimen was precracked to a length of approximately 0.65 cm. At that point, the load cycle was removed and the specimen was allowed to age for an undetermined time. The crack was then extended an additional 1 cm. The through transmission data from this specimen is shown in Figure 8. In addition to the normal closure region at the tip of the crack, a second peak in the transmission coefficient data at the position of the earlier fatigue interruption can be seen. The peak becomes narrower with increasing frequency because of the smaller beam width, as at the end of the crack. In addition, its peak value decreases at high frequencies due to the lower value of the interface transmissivity. The appearance of what is possibly a double peak in the 12 MHz data is not well understood at this time.

Our current speculation is that the secondary closure peak in this data occurs in the region of the tip of the original precrack and is due to a stress overload condition that was placed on the crack when it was extended. A crack was grown in a third specimen of 7075-T651 in such a way that a similar overload was placed on the sample during crack initiation. This third specimen also exhibits an additional closure region similar to that shown in Figure 8, except that in this case, the closure occurs at the root of the notch. From these data, it appears that through ultrasonic measurements at least a portion of the stress history of the crack propagation can be sensed.

**SUMMARY**

The use of acoustic transmission measurements has made possible the detection of regions of closure in a fatigue crack. From the models of Haines\(^8\) and the discretized distributed spring model of
Figure 7. Effect of changes in density of contacts on shear wave tip diffracted response.

Figure 8. Through transmission response for fatigue crack showing partial closure at intermediate position.
Thompson et al.\textsuperscript{4}, an evaluation of the contact stress present across the crack faces may be possible through comparison of experimental data and model predictions to determine the density of asperity contacts. In addition, determination of the position and extent of the regions of closure may help to outline the fatigue history of propagation of a crack.

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**REFERENCES**

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