RAYLEIGH WAVE SCATTERING FROM THREE DIMENSIONAL SURFACE SLOTS AND SEMI-CIRCULAR DEPRESSIONS

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INTRODUCTION

Rayleigh wave based NDE methods have been hampered by the lack of a theory to describe the scattering process. In previous meetings we have discussed numerical schemes which model the two dimensional problem of plane Rayleigh waves scattering from a plane defect [1], and recently we have presented preliminary details of a simple three-dimensional model [2] for plane Rayleigh wave scattering from surface features accommodated on a Cartesian grid of points. In this paper we will present an analysis of plane Rayleigh wave scattering from a three dimensional surface slot and discuss results for plane Rayleigh wave scattering from a semi-circular trench.

SCATTERING FROM SLOTS

Last year [1], we presented details of a full three dimensional dynamic model for elastic wave scattering from simple geometries that can be built up from rectangular bricks. Finite elements are used for the spatial discretization of the elastic equations of motion and an explicit centered difference scheme is used for the temporal integration. The iterative algebraic equations are solved efficiently on an Intel iPSC/860 distributed memory parallel computer.

The models use a plane Ricker pulse of Rayleigh waves for the initial conditions. The Ricker pulse has a central wavelength $\lambda_0$ with maximum amplitude which defines the spatial grid increments and the dimensions of the scattering features. Fig. 1 presents numerical snapshots of the wavefield scattered by a slot with dimensions of $2\lambda_0$ by $2\lambda_0$ by $\lambda_0$, on the free surface and on the planes which bisect the slot normal to the
Fig. 1. Numerical visualisations of plane pulsed Rayleigh waves scattered from a three dimensional surface slot; plan view and side views which bisect the slot.
free surface. The displacements are drawn as vectors from the equilibrium positions, projected onto the planes. As the Rayleigh wave pulse moves from left to right it interacts strongly with the leading face of the slot producing large scale amplifications of the displacements at the top edge. This motion is similar to that found in the plane slot geometry. Strong spherical surface wavefields are generated at the top corners and these radiate out along the edges of the slot and into the rest of the surface. As the scattering picture develops it is clear that the left hand surface corners radiate most strongly into the quadrant containing the leading face of the slot.

The left hand half-space contains three wavefields generated by the leading face of the slot. The first pulse, marked as R₁, is reflected directly from the top edge and overlaps a smaller amplitude pulse R₂ produced at the bottom edge of the leading face by the tail of the incident wavefield. A pulse travels up the leading face of the slot and is then transmitted onto the left-hand half-space. A separate pulse marked R₃ arises from the surface wavefield transmitted onto the left hand vertical surface of the slot. This high frequency pulse travels down the vertical surface, is reflected by the bottom leading edge and travels up the vertical surface before being transmitted past the leading edge back onto the left-hand free surface. A small compression wavefield marked as C is radiated out beneath the half space but the amplitude is far less than for the plane strain geometry [3,4]. The radiation pattern of the shear wavefield marked as S is similar to the equivalent plane strain geometry.

In Fig. 2 we plot the components of displacement Uₓ, Uᵧ and Uₓ for lines of points on the surface. On the line from A to A', the small surface grazing compression wave marked C and the three reflected surface wavefields are clearly identifiable. The far-field reflection seismograms are very similar to those of the equivalent plane strain slot geometry [4]. In the near-field the spherical wavefields R₄ and R₅ radiated by the top corners on the leading face in the three dimensional model contribute significant extra amplitude. The seismograms from D' to D illustrate that there is a significant coupling of the incident x-y motion into z motion along the side faces of the slot. The Uₓ pulse is carried along by the inertia of the incident wavefield and little of the energy propagates away along the z direction. The most significant pulses travelling away from the sides of the slot originate from the spherical wavefields generated by the top corners on the opposite sides of the leading face of the slot. These wavefields have the same arrival times as the wavefields diffracted from the top corners on the trailing face of the slot by the incident pulse marked as R₆ and R₇. The R₆ and R₇ wavefields overlap to contribute a single pulse in the reflection seismograms. The pulses generated at the corners on the trailing face are strongly focussed into the quadrant containing the trailing face of the slot and pulses propagate along the top edge before diffracting around the opposite top corners. The incident wavefield burrows underneath the slot and reappears as the pulse marked R₈ in the wake of the surface diffracted pulses. The transmitted wavefield bears little resemblance to that in the plane strain problem [4].
Fig. 2. Displacements at seismometer points along two directions of the slot bearing half space. The lettering refers to the geometry in Fig. 1.
In the past we have presented a plane strain mixed implicit-explicit regular-irregular grid model which allows us to model Rayleigh wave scattering from local surface features of arbitrary complexity [1]. Fig. 3 shows a numerical visualization of the Ricker pulse of Rayleigh waves scattered by a semi-circular depression of radius $d=\lambda_0$ in a half space with Poisson’s ratio=1/3. The incident wavefield gives rise to reflected ($R_r$) and transmitted ($R_t$) Rayleigh wavefields in addition to mode converting considerable amounts of energy into shear ($S$) and compression ($C$) waves.

Fig. 4 presents the Rayleigh wave reflection coefficients obtained by gating the reflected Rayleigh wavefield, Fourier transforming the resulting signal and normalizing against the amplitude of the wavelength in the incident pulse. Our results are in excellent agreement with those derived from an integral equation solution to the problem which is valid at long wavelengths [5]. For low frequencies, there is considerable interference between wavefields scattered from the top corners of the feature. The near-field amplification of the curved up-step section at long wavelengths causes resonances similar to those observed in up-step and trough geometries [4]. For shorter wavelengths the reflection coefficients for the semi-circular depression more or less monotonically approach the limiting value for an isolated 90° corner (for $\sigma=1/3$ this is about 40%). The wavefield reflected from an up-step is small for short wavelengths and causes only a minor modulation of the wavefield reflected directly from the top left hand corner.

The behaviour of the reflection coefficients can be explained in terms of scattering from an effective wedge geometry. We assume that the effective source of the reflected wavefield is a given fraction of a wavelength beneath the surface. As the wavelength of the incident wavefield is reduced, the effective source point sees a sloping surface whose angle relative to the horizontal surface changes from 180° to 90°. The reflection coefficients for isolated obtuse wedges behave in a similar manner [6] with a small maximum at an angle of about 100°. The size of the maximum varies with Poisson’s ratio, for $\sigma=1/3$ the reflection coefficient for 100° is approximately 4% greater than for 90° [7]. We see therefore that the angular variation for a wedge is more or less translated into a wavelength dependence in the semi-circular geometry. The same physical reasoning can be used to explain the wavelength dependence of the reflection coefficients in other geometries, for example, the curved 90° corner [1].

CONCLUSIONS

We have presented a qualitative description of the interaction of plane Rayleigh waves scattering from a localised three dimensional slot and we are now in the process of developing the appropriate tools to project the numerical solutions onto the fundamental eigenfunctions of the half space geometry and thus determine the scattering matrices which can be measured experimentally. Calculations for different slot geometries and different angles of incidence of the incident Rayleigh wavefield are underway and will bring us within sight of the goal of providing a fundamental description.
Fig. 3. Numerical visualisation of plane pulsed Rayleigh waves scattered from semi-circular trench.

Fig. 4. Rayleigh wave reflection coefficients for a semi-circular trench.
of the scattering problem which must underlie any quantitative NDE surface defect characterization method. The results for Rayleigh wave scattering from a semi-circular trench indicate the true complexity of the Rayleigh wave scattering problem and the development of a three dimensional general model will allow us to incorporate curvature into surface features and thus allow us to deal with localised defects of arbitrary complexity. Results for scattering from hemi-spherical depressions will be presented next year.

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REFERENCES