SKIN DEPTH CONSIDERATIONS IN EDDY CURRENT NDT

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INTRODUCTION

Eddy current nondestructive testing depends upon the interaction of time-varying electromagnetic fields with the material under test. The electromagnetic fields are applied to the material under test via some finite sized transducer, usually an inductive coil or set of coils. If the material is conducting the fields will penetrate the conductor, but will attenuate to negligible levels after some distance. The classical skin depth,$\delta = \sqrt{2/\omega \mu \sigma} = \sqrt{1/\pi f \mu \sigma}$, is the standard assumption for the characteristic distance of field penetration in conductors. This paper reports the results of a study which examined the decay of sinusoidal steady-state (AC) fields in conductors induced by finite sized coils. Comparisons are made among the classic Dodd and Deeds formulations [1], the 3D-axisymmetric finite element method (FEM) [2], and published results [3]. Where possible experimental observations were compared to the computed and published results.

The objectives of the study were to compare the classical skin effect (exponential decay and linear phase delay) to computed (integral solution, FEM) eddy current density distributions in conductors, to determine a “region of validity” for the classical skin effect, and to confirm the validity of the computed results by comparison to experiment.

NUMERICAL MODELING

The coil configurations most common to eddy current NDT are a coil over a conductor, Fig. 1(a), and a differential coil pair inside a conducting tube (not shown). A set of integral solutions for these two geometries exist [1] which give the magnetic vector potential,$\vec{A}$, in all regions. These integral solutions satisfy the underlying partial differential equation (PDE),

$$\nabla \times \frac{1}{\mu} \nabla \times \vec{A} = \vec{J}_s - j \omega \sigma \vec{A},$$

which is elliptic because a time dependence of the form $e^{j\omega t}$ is assumed. The integral equations were coded in FORTRAN-77 and results are given in this paper for an air-cored coil over a conducting slab. The solution for the vector potential was also effected via a well known 3D-axisymmetric finite element method (FEM) [2] which uses an energy functional minimization process. The FEM has the advantage that arbitrary axisymmetric defects or inhomogeneties can be introduced in the conducting media without added complexity, therefore, the FEM is ideal for field/defect interaction studies. Also, the FEM
STANDARD MODEL:

$$R_s = 2.667 \text{ mm}$$
$$r_1 = \frac{1}{3}R_s$$
$$r_2 = \frac{5}{3}R_s$$
$$L_2-L_1 = \frac{2}{3}R_s$$
$$\sigma_1 = 1351 \text{ (S/mm) (stainless steel)}$$
$$\mu_1 = \mu_0$$
$$\mu = \mu_0 \text{ (medium above conductor)}$$
$$\text{turns} = 160$$

Figure 1. Air-cored coil over a planar conductor, (a) geometry, (b) standard model

gives a full field solution; that is the vector potential is known everywhere in the FE mesh region at program completion.

DISCUSSION

As was noted in Mottl [3], a dimensionless parameter appears in the integral solutions for a coil over a conductor when all physical dimensions are normalized by the coil mean radius, $R_s$. The parameter given by the expression $R_s^2 \omega \mu \sigma$ can also be written in terms of the classical skin depth:

$$\frac{R_s}{\delta_s} = R_s \sqrt{\pi f \mu \sigma}.$$  \hspace{1cm} (2)

If $R_s/\delta_s$ and the ratio of permeabilities, $\mu_a/\mu_b$, of the various media are constant then all solutions have similarity. In other words, if the mean radius is increased by two and the conductivity reduced by four $R_s/\delta_s$ remains constant and both solutions are similar in form and differ only by a coordinate scale change. The ratio of the permeabilities must also be considered since their ratio occurs in one of the boundary conditions, namely
\[
\frac{1}{\mu_a} \nabla \times \vec{A}_a \bigg|_{\text{tangent}} = \frac{1}{\mu_b} \nabla \times \vec{A}_b \bigg|_{\text{tangent}},
\]

therefore a change in the permeability ratios anywhere in the solution region destroys the similarity of solutions even if \( R_s/\delta_s \) is held constant.

The advantage of this parameter is that any number of physical coils can be modeled on the computer if the relative dimensions of coil and model are the same. This feature saves one from having to alter the FE code significantly for each physical coil.

**RESULTS**

The eddy current density magnitude on the surface of the conductor from the coil center to \( R = 5 \) is plotted in Fig. 2(a) (normalized radius, \( R = \rho/R_s \), no coil lift-off). The peak current density occurs at approximately \( R = 1 \) or directly beneath the mean radius of the coil. Note that in this figure a standard model (see Fig. 1(b)) has been plotted (solid line) and, for comparison, various combinations have been plotted such that \( R_s/\delta_s \) and \( \mu_a/\mu_b \) remain constant. All solutions show similarity when plotted against normalized coordinates except for the case where the permeability ratio is different even with \( R_s/\delta_s \) remaining constant. In Fig. 2(b) the eddy current density is again plotted versus normalized radius, but \( R_s/\delta_s \) is varied (\( \mu_a/\mu_b \) = constant). Also plotted in Fig. 2(b) is the eddy current density computed by Mottl [3] which is significantly different from the integral solution and FEM results obtained in this study. The integral solutions and the FEM results computed here are indistinguishable.

The eddy current density magnitude versus normalized depth into the conducting slab under the mean radius of the coil (\( R = 1 \)) is plotted in Fig. 3 for two values of \( R_s/\delta_s \) together with the classical exponential decay. In both cases the magnitude decays faster than the classical case. Mottl defines a true depth of penetration, \( \delta_t \), to be the depth at which the computed magnitude of the eddy current density has decayed to \( e^{-1} \) or 0.3679 of its value at the surface. For \( R_s/\delta_s \) = 0.1, 1.0 \( \delta_t/\delta_s \approx 0.07, 0.6 \), respectively, implying that subsurface defect detection would be affected.

To understand the variation of the true depth of penetration the ratios \( \delta_t/\delta_s \) and \( \delta_t/R_s \) are plotted versus \( R_s/\delta_s \) in Fig. 4(a). This analysis, which is due to Mottl, shows that for \( R_s/\delta_s \) varying from 0.1 to 100.0 the true depth of penetration only becomes ninety percent of the classical skin depth for \( R_s/\delta_s \geq 4 \). Below \( R_s/\delta_s = 4 \) the true depth of penetration becomes progressively smaller relative to \( \delta_s \) which shows the classical skin depth is a poor approximation for the characteristic penetration depth. It is interesting to note that the ratio \( \delta_t/R_s \) becomes essentially constant for \( R_s/\delta_s \leq 0.5 \). Why does the coil mean radius affect the field penetration? Plotted in Fig. 4(b) is the phase angle of the eddy current density. The term \( \beta_t(\delta_t) \) is defined to be the true phase angle at the true depth of penetration; \( \beta_t(\delta_s) \) is the true phase angle at the classical skin depth; \( \beta_t(\delta_s) \) is the classical phase angle at the classical skin depth (always one radian). The true phase at the true depth of penetration has the same characteristic variation with respect to \( R_s/\delta_s \) as does the true depth of penetration. In both plots the FEM and integral solutions computed here are effectively equivalent except for values \( R_s/\delta_s \geq 30 \) since the number of finite elements per true skin depth becomes relatively small. In other words, the discretization is becoming too coarse for the given field variation. Notice that results obtained in this study compare well with those of Mottl.

Finally, a coil with relative dimensions the same as the standard model was constructed and its impedance measured for various coil lift-off values above an effective infinite slab of aluminum (\( \sigma_{Al} \approx 26 \, MS/m \)). The reactance and resistance are plotted in Fig. 5 along with integral solution results. The experimental and analytical results generally show good correspondence except when the coil nears its self-resonant frequency. The variation in \( R_s/\delta_s \) was achieved by a variation in excitation frequency and the impedance measurement was made with a Hewlett-Packard 4194A impedance and gain/phase analyzer. The effect
Figure 2. Induced eddy current density magnitude in the radial direction, (a) variable material parameters, (b) variable $R_s/\delta_s$.
Figure 3. Induced eddy current density magnitude in the axial direction compared with the classical solution, (a) \( R_s/\delta_s = 0.1 \), (b) \( R_s/\delta_s = 1.0 \)
Figure 4. Normalized true depth of penetration (DoP) versus $R_s/\delta_s$ for $R = 1$, (a) DoP, (b) eddy current phase angle.
Figure 5. Normalized impedance for aluminum versus coil lift-off and $R_s/\delta_s$ for $R = 1$, (a) reactance, (b) resistance
of the coil self-resonance (parallel resonance) causes the effective series reactance and resistance to become very large as resonance is approached. Since coils are usually employed in eddy current NDT at frequencies below their self-resonance, it is evident that the integral solutions and especially the FEM are good models for the coils in this region.

CONCLUSION

From the results presented it is clear that use of the classical skin depth, $\delta_s$, as an estimate for the characteristic penetration depth of eddy currents in conductors due to finite size coils is inadequate. The implications for eddy current NDT are obvious since field penetration depth dictates ones ability to detect subsurface defects. The classical skin effect approximation cannot be applied in general to situations in which the source is finite and the induced field is localized and inhomogeneous. This study has shown that the induced eddy current density decays faster than the classical exponential in the region near the coil where most defect detection occurs, that is, excluding remote field eddy current NDE. The classical exponential approximation can model the field distribution for certain values of the parameter $R_s/\delta_s$, but for the coils used in this study the region of validity was beyond the useful inspection frequency range ($R_s/\delta_s \geq 10$). Since induced eddy current decay is faster than predicted by the classical “plane wave” solution for moderate and low values of $R_s/\delta_s$, defects of a given size cannot be detected below a distance probably much more than three classical skin depths ($3\delta_s$). A more conservative but realistic estimate for defect detection would be one classical skin depth, but even this becomes poor at low measurement frequencies.

The measurements performed do, in general, confirm the validity and usefulness of the integral solutions and finite element method for frequencies below the self-resonant frequency of the coil, but only the FEM has the flexibility to investigate defect/field interactions in the presence of material inhomogeneities and nonlinearities.

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REFERENCES

