ANALYSIS OF MICROSTRIP PATCH ANTENNAS FOR DIELECTRIC MEASUREMENT

H. R. Hassani and D. Mirshekar-Syahkal

Department of Electronic Systems Engineering
University of Essex, Colchester CO4 3SQ, UK

INTRODUCTION

The microstrip patch antenna has been employed by Shimin [1] to measure the dielectric constant of thin slab materials. Although the applicability of the method in [1] is not restricted by the size of the material under test, the etching of the patch and the introduction of the ground plane on the material cannot be practical in many cases. A more practical approach based on the microstrip antenna is reported in [2] for the measurement of the dielectric constant of snow. The technique employs the antenna as an applicator. The antenna is brought to the close proximity or in contact with the surface of the material in order to cause interaction between the antenna's near field and the object. The result is a change in the input impedance and the resonant frequency of the antenna. Similar to short antenna applicators [3], a patch antenna applicator can take measurements under resonant and off resonant conditions. However, at resonance, achieving accurate measurements are straight-forward. The technique can be applied to solids as well as to liquids with small or large losses. It can also be used in the thickness measurement of layered materials.

Like many other techniques, a precise determination of the complex permittivity or the thickness of a material with patch antenna applicators depends on the accuracy of the mathematical or experimental modeling used for the interpretation of measurements. In this connection, no theoretical modeling has been reported in the literature. In this paper a frequency dependent mathematical technique based on the spectral domain method is presented for accurately quantifying the behavior of a coaxial fed rectangular patch antenna under various dielectric loadings. The technique can be easily extended to stacked rectangular microstrip antenna applicators.

THEORETICAL MODELING

The mathematical modeling is presented for an ordinary rectangular microstrip antenna applicator, Fig. 1.a. The extension of the method to stacked antennas, Fig. 1.b, is straightforward following the method in [4]. Both the material under test and the substrate are lossy with loss tangents \( \tan \delta \), Fig.1.a. The patch is highly conductive with an effective conductivity \( \sigma_{\text{eff}} \) including losses due to the surface roughness. It is assumed that both the material under test and the substrate are infinite in extents with no air gaps between them. The latter calls for the patch to be infinitely thin. Furthermore, it is assumed that the substrate thickness is significantly smaller than the wavelength. This allows the excitation probe to be replaced by a short electric probe (a current filament) with current distribution

\[
i = \delta (x - x_p) \delta (y - y_p) a_z
\]

(1)
where unit current has been assumed on the feed. This current excites the patch. Since the probe is very small, direct radiation from the probe and the probe self impedance can be neglected in the modeling. In view of the assumptions made, the mathematical modeling of the structure can be carried out using the spectral domain technique. The technique is originally proposed for the analysis of microstrip transmission lines [5]. In this technique, one can start with the boundary condition for the tangential electric field at the interface where the patch lies;

\[ E_{t}^{inc} + E_{t}^{scat} = \frac{E_{t}^{T}}{Z_{s}J} \]

over the patch

outside the patch

(2)

where \( E_{t}^{inc}, E_{t}^{scat} \) and \( E_{t}^{T} \) are, respectively, the tangential components of the incident, the scattered and the total electric field at the boundary and \( J \) and \( Z_{s} \) are the current and the surface impedance of the patch. Defining the two-dimensional Fourier transform pair

\[ f = \frac{1}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{j(k_{x}x + k_{y}y)} \, dx \, dy \]

and applying it to (2) lead to a compact boundary equation in the spectral (Fourier) domain:

\[ \tilde{E}_{t}^{inc} + \tilde{E}_{t}^{scat} = Z_{s} \tilde{J} + \tilde{E}_{t}^{T} \]

(4)

Since the current on the patch gives rise to the scattered field, a relation between \( E_{t}^{scat} \) and \( J \) can be established. In the Fourier domain this relation is given by

\[ \tilde{G} \cdot \tilde{J} = \tilde{E}_{t}^{scat} \]

(5)

where \( G \) is the dyadic spectral domain Green’s function. For the problem of concern, the expressions for the elements of this function are

\[ G_{xx} = -j \frac{\sin \gamma_{1} d_{1}}{\omega \varepsilon_{0} \beta^{2}} \left( \frac{k_{x}^{2} \gamma_{1} N}{T_{m}} + \frac{k_{0}^{2} k_{x}^{2} M}{T_{e}} \right), \quad G_{yy} = -j \frac{\sin \gamma_{1} d_{1}}{\omega \varepsilon_{0} \beta^{2}} \left( \frac{k_{y}^{2} \gamma_{1} N}{T_{m}} + \frac{k_{0}^{2} k_{y}^{2} M}{T_{e}} \right) \]

\[ G_{xy} = G_{yx} = -j k_{x} k_{y} \frac{\sin \gamma_{1} d_{1}}{\omega \varepsilon_{0} \beta^{2}} \left( \frac{\gamma_{1} \gamma_{2} N}{T_{m}} + \frac{k_{0}^{2} k_{x}^{2} M}{T_{e}} \right) \]

where

\[ T_{m} = \varepsilon_{1} \gamma_{2} \cos \gamma_{1} d_{1} + \varepsilon_{2} \gamma_{1} \sin \gamma_{1} d_{1} - \varepsilon_{3} \gamma_{2} \sin \gamma_{1} d_{1} \]

\[ T_{e} = \gamma_{1} \cos \gamma_{1} d_{1} + \gamma_{2} \sin \gamma_{1} d_{1} - \gamma_{2} \sin \gamma_{1} d_{1} \]

(6)

Fig. 1 (a) Ordinary and (b) stacked rectangular patch antennas when applied to a material.
N = \gamma_2 \epsilon_2 \cos \gamma_2 d_2 + j \gamma_2 \sin \gamma_2 d_2, \quad M = \gamma_2 \cos \gamma_2 d_2 + j \gamma_3 \sin \gamma_2 d_2, \quad \gamma_i^2 = k_i^2 - \beta^2,
\beta^2 = \omega^2 \mu_0 \epsilon_0 \epsilon_1 \quad \text{and} \quad \epsilon_1 = \epsilon_{ri} (1 - j \tan \delta_i) \quad (i=1, 2, 3 \text{  layers' indices in Fig. 1. a})

To derive these elements, the standard procedure in [5] was followed. This includes solving Helmholtz's equation in the Fourier domain for scalar electric and magnetic potential functions associated with each dielectric layer in Fig. 1.a, expressing the field of each layer in terms of these potentials and finally applying the necessary boundary conditions. Substituting (5) in (4) leads to a new form for the boundary condition

\[ \vec{G} \cdot \vec{J} - Z_s \vec{J} = - \vec{E}_{inc} \cdot \vec{E}_t^T \]  

This equation can be solved by expanding the components of \( \vec{J} \) (ie: \( J_x \) and \( J_y \)) in terms of suitable basis functions:

\[ J_x = \sum_{m=1}^{M} a_m J_{x,m}, \quad J_y = \sum_{n=1}^{N} b_n J_{y,n} \]  

and then applying the Galerkin technique:

\[ \sum_{m} a_m \langle (\vec{G}_{xx} - Z_s) J_{x,m} J_{x,p}^* \rangle + \sum_{n} b_n \langle (\vec{G}_{xy} J_{y,n} J_{x,p}^* \rangle = - \langle \vec{E}_{inc} J_{x,p}^* \rangle \quad p = 1, \ldots, M \]
\[ \sum_{m} a_m \langle (\vec{G}_{xy} J_{x,m} J_{y,q}^* \rangle + \sum_{n} b_n \langle (\vec{G}_{yy} - Z_s) J_{y,q} J_{y,q}^* \rangle = - \langle \vec{E}_{inc} J_{y,q}^* \rangle \quad q = 1, \ldots, N \]

where the inner product is defined as

\[ \langle f, g \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f \cdot g \, dk_x \, dk_y \]  

In order to evaluate the inner products on the right hand side of (9), expressions for the incident field are required. Alternatively, the reciprocity theorem can be used to replace the integral over the scatterer (the patch) by an integral over the source [5,6]. Thus, for example,

\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \vec{E}_{inc} \cdot \vec{E}_{inc}^* \, dk_x \, dk_y = \frac{1}{4\pi^2} \int_{0}^{d} E_{z,1}(x_p,y_0,z) \, dz \]

where \( E_{z,1}(x_p,y_0,z) \) is the electric field along the probe due to the basis current \( J_{x,p} \). An expression for this field in terms of \( J_{x,p} \) can be obtained after finding the spectral domain Green's function (6) of the problem.

From (9), coefficients \( a_m \) and \( b_n \) are obtained. Subsequently the electric field along the probe, the voltage at the feed, and finally the input impedance can be determined. A significant computer time and memory can be saved using established well-behaved basis functions for approximating currents \( J_x \) and \( J_y \) in (8). One such set of functions is given in [6]. Several terms of those functions were used in the computation, but the results were found to be extremely close to those produced using

\[ J_x = \sin \frac{\pi}{a} (x + a/2) \]  

where \( J_y \) has been omitted. Such an observation has also been reported in [7] for the case of unloaded microstrip patch antennas. The effect can be attributed to the negligible transverse current \( J_y \) (at the frequency of concern), to the resemblance of (12) to the physical current distribution on the patch, and to the stationary property of the spectral domain technique.

In the computation of (9), integrals of form (10) are involved. To facilitate the numerical integrations, the standard procedure is to apply the coordinate transformation

\[ k_x = \beta \cos \alpha \quad k_y = \beta \sin \alpha \]  

to convert the integral from the rectangular to polar coordinates;
\[
\int_0^{2\pi} \int_{\Gamma} \mathbf{f} \cdot \mathbf{g} \, \beta \, d\alpha \, d\beta
\]

(14)

where \(\Gamma\) is the path of integration. To choose the correct path(s), the two-valued nature of \(g_i\) in (6) and the location of the singularities of the integral in the complex \(\beta\) plane must be taken into consideration. The singularities correspond to the roots of \(T_e\) and \(T_m\) in (6), representing the propagation constants of the surface waves of the structure. Details of a technique for the integration of an integral like (14) can be found in [8].

RESULTS

The accuracy of the mathematical modeling was initially verified. For this purpose, the technique was applied to the antenna reported in [9] for which limited experimental data is available on its resonant frequencies when it is covered with certain dielectric layers. Excellent agreement between the computed and the measured results was achieved.

To investigate the behavior of the rectangular patch antenna under different dielectric loadings, a typical antenna with \(a = 3.7\) cm, \(b = 4.5\) cm, \(d_1 = 0.159\) cm, \(\gamma_p = -1.5\) cm, \(\varepsilon_r = 2.5\), \(\tan \delta_1 = 0.0025\) and \(\sigma_{eff} = 10^7\) S/m was considered. Fig. 2 shows the results when the antenna is applied to lossless slabs of different dielectric constants. As can be seen in Fig. 2.a, for thin dielectrics, it is the resonant frequency which effectively changes with the dielectric constant. In this case, both the input impedance (at resonance) and the Q-factor are virtually unaltered. Results for thick dielectrics, Fig. 2.c, however, indicate that the resonant frequency and the input impedance are both sensitive to a change in the dielectric constant while the Q-factor remains virtually uninfluenced by the change. From Fig. 2, it can be generally inferred that the measurement of dielectric constants of thick materials using a patch antenna applicator should be more accurate. In Fig. 3, the input impedance of the antenna when applied to different lossy dielectric slabs of the same dielectric constant (\(\varepsilon_r = 5.4\)) are plotted. These results reveal that for thin materials, Fig. 3.a, the resonant frequency (defined as the frequency at which the real part of the input impedance is maximum) is insensitive to the loss tangent of the material, but the impedance at the resonant frequency appreciably decreases when this parameter increases. For thick low loss materials, the resonant frequency appears to be independent of the loss factor, but thick lossy materials would shift the frequency upwards. Both sets of results in Fig. 3 indicate that the impedance of the antenna is highly inductive at resonance for very lossy materials.

Since in the patch antenna method of measuring permittivity and thickness, the input impedance is a parameter used to extract information, its high sensitivity to small changes in the permittivity and thickness is very desirable. In patch antennas, the input impedance is a function of the feed position. Fig. 4 shows an example of the variation of the real part of the impedance at resonance with the feed position for the antenna (specified earlier) under four different dielectric loads. In this example the feed is along the centre line (ie: \(\gamma_p = 0\) cm) of the antenna. From Fig. 4 it is clear that the impedance of the antenna has a consistent behavior, taking its maximum value at about \(x_p = -1.5\) cm irrespective of the thickness of the material under test. At this feed position the antenna is expected to offer the highest sensitivity.

From Fig. 2, it can be concluded that the measurement accuracy of the patch antenna method depends on the accuracy with which the resonant frequency and the input impedance can be determined. For accurate measurements of the resonant frequency, high Q patch antenna applicators are preferred. It is known that the Q-factor of a patch antenna is a function of the substrate thickness. Fig. 5 shows that this fact is still valid even when the antenna is under a dielectric load. From this figure, it is evident that a higher Q can be achieved at the expense of a lower input impedance (corresponding to a lower sensitivity in the measurement of the impedance).

The modeling introduced in this paper was extended to the case of stacked microstrip antenna applicators, Fig. 1.b. Results shown in Fig. 6 are for a stacked antenna with patch sizes, substrate thicknesses and loss tangents equal to those of the ordinary microstrip.
antenna specified earlier. The effective conductivity of the patches, $\sigma_{\text{eff}} = 0.7 \times 10^7$ S/m is different. As the results indicate, the antenna resonates at two different frequencies, but it seems that only one of them is strongly affected by the introduction of the material. At this

Fig. 2 Input impedances of the antenna at different frequencies for various lossless materials; (a) $d_2 = d_1$, (b) $d_2 = 3d_1$, (c) $d_2 = 6d_1$. 
Fig. 3 Input impedances of the antenna at different frequencies for various lossy materials with \( \varepsilon_{r2} = 5.4 \); (a) \( d_2 = d_1 \), (b) \( d_2 = 3d_1 \).

Fig. 4 Variation of the real part of the input impedance of the antenna at resonant frequency against the probe position for a material with \( \varepsilon_{r2} = 5.4 \) and \( \tan \delta_2 = 0.01 \).
Fig. 5  Effects of the substrate thickness on the input impedance and the Q-factor of the antenna loaded with a material with \( d_2 = 0.159 \) cm, \( \varepsilon_r_2 = 5.4 \) and \( \tan \delta_2 = 0 \).

Fig. 6  Input impedances of the stacked microstrip antenna at different frequencies for various materials with \( \tan \delta_3 = 0.0025 \); (a) \( d_3 = 0.159 \) cm, (b) \( \varepsilon_r_3 = 2.5 \).
resonance, both the frequency and the impedance have behaviors similar to those of the ordinary microstrip antenna. Further work is required to establish whether stacked microstrip antennas offer any advantages over the ordinary patch antenna applicators.

CONCLUSIONS

A theoretical technique for an accurate analysis of the microstrip patch antenna loaded with a dielectric material was presented. The accuracy of the technique is checked against available experimental data and an agreement of better than 2% was achieved. The technique can be used to generate data for inverting impedances and resonant frequencies into permittivities or thicknesses of dielectric materials.

The behaviors of the resonant frequency and the input impedance of an ordinary microstrip antenna when applied to various dielectric materials were theoretically investigated. These parameters were found to be strongly dependent on the complex permittivity and the thickness of the material under test. From simulations, it was inferred that the measurement accuracy of the patch antenna applicators increases with the thickness of the material when the purpose is the dielectric measurement, but it decreases when the object is the thickness measurement. For thick materials the input impedance of the antenna was found to be more sensitive to changes in the material thickness than the resonant frequency. Simulations showed that the sensitivity in the impedance measurement can be maximum at a particular feed position. The effect of the substrate thickness on the Q-factor of the antenna when loaded with a material was also investigated. It was revealed that thin substrates offer high Q-factors. This fact can be utilized when an accurate measurement of the resonant frequency is required.

The analysis technique was extended and applied to a stacked microstrip antenna. The results showed that the introduction of a material to the antenna does not significantly affect the behavior of the antenna around the upper resonance, but produces measurable shifts in the resonant frequency and the input impedance at the second resonance. These shifts are similar to those observed in the case of the ordinary patch antenna.

REFERENCES


