A NOVEL TECHNIQUE FOR MICROWAVE THICKNESS MEASUREMENT OF DIELECTRIC SLABS
USING AN OPEN-ENDED RECTANGULAR WAVEGUIDE

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INTRODUCTION

Media consisting of a dielectric slab backed by a conducting plate are used in many facets of industry. Monitoring the thickness or permittivity uniformity of such slabs is of great importance. Ability of microwaves to penetrate inside dielectrics and its sensitivity to material inhomogeneities makes them suitable for this type measurements [1,2].

Information extracted in terms of reflection coefficient and phase is commonly used in microwave nondestructive testing of materials. Here, a thickness measurement technique using an open-ended rectangular waveguide placed on top of the dielectric slab backed by a conducting plate is presented. Although the theoretical derivation deals with admittance of the guide, results can be easily transformed to the desired reflection coefficient information. The admittance expression is then used in two root finding codes in an iterative manner to extract information about the thickness and permittivity of the material. Some preliminary measurement results are provided to verify the validity of the theoretical analysis.

WAVEGUIDE ADMITTANCE

Figure 1a shows the geometry of a flange mounted rectangular waveguide radiating into an infinite half-space. The half-space is assumed to be isotropic dielectric material with arbitrary permittivity characteristics and has free-space permeability.

Considering the dominant mode incident on the aperture, the terminating admittance of the waveguide can be written as [1]

\[
Y = G + jB = \frac{\int_{x=-a/2}^{a/2} \int_{y=-b/2}^{b/2} [\bar{E}(x,y,0) \times \bar{W}(x,y) \cdot \hat{z}] \, dx \, dy}{\left[ \int_{x=-a/2}^{a/2} \int_{y=-b/2}^{b/2} [\bar{E}(x,y,0) \times \bar{\varepsilon}_r(x,y) \cdot \hat{z}] \, dx \, dy \right]^2}
\]

(1)
where

\[ \vec{W}(x,y) = \vec{H}(x,y,0) + \sum_{n=1}^{\infty} Y_n \vec{h}_n(x,y) \int \vec{E}(\eta,\xi,0) \cdot \vec{e}_n(\eta,\xi) \, d\eta \, d\xi \]  \tag{2} \]

The admittance expression is constructed using transverse vector mode functions and their orthogonal properties \[3\]. \( \vec{e}_n \) and \( \vec{h}_n \) are the \( n \)th vector mode functions and \( Y_n \) is the characteristic admittance of the waveguide for the \( n \)th mode. It has been shown that Equation 1 is stationary with respect to variations of the E-field about its exact value. Thus, an approximation for the E-field would result in a good estimate for the admittance \[4\].

CONSTRUCTION OF SOLUTION

Figure 1b shows the cross-section of the geometry under consideration. The conductor backed slab has finite thickness in \( z \)-direction and extends to infinity in other directions. By evoking a vector potential formulation the fields in region 1 (i.e. dielectric slab) can be constructed in the following manner. Fields in this region satisfy the wave equation:

\[ \nabla^2 \vec{\psi} + k^2 \vec{\psi} = 0 \]  \tag{3} \]

where

\[ \vec{\psi} = \phi \hat{x} + \psi \hat{y} \]  \tag{4} \]

A possible set of solutions for the field components can be expanded in a Fourier integral form as

\[ E_{x1}(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ -jk_{z1} I_{\phi} e^{-jk_{z1} z} + jk_{z1} R_{\phi} e^{jk_{z1} z} \right] e^{-jk_x x} e^{-jk_y y} \, dk_x \, dk_y \]  \tag{5} \]
By using Fourier properties of the above expressions and applying the appropriate boundary conditions at \( z=d \), the unknown coefficients \( I_\Phi, R_\Phi, I_\Psi, R_\Psi \) can be found. By substituting these into Equations 5 through 8, the complete set of solutions for the field components are constructed. Upon normalization with respect to the free-space wavenumber and applying a polar coordinate transformation the admittance expression and its related parameters are presented as:

\[
E_{y1}(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ jkz_1 I_\Phi e^{-jkz_1 z} - jkz_1 R_\Phi e^{jkz_1 z} \right] e^{-jkx_1 x - jky_1 y} \, dx_1 \, dy_1 
\]

\[
H_{x1}(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \left( k_1^2 - k_2^2 \right) \frac{k_x k_y}{j\omega\mu_0} \left[ I_\Phi e^{-jkz_1 z} + R_\Phi e^{jkz_1 z} \right] - \frac{k_x k_y}{j\omega\mu_0} \left[ I_\Psi e^{-jkz_1 z} + R_\Psi e^{jkz_1 z} \right] \right] \times e^{-jkx_1 x - jky_1 y} \, dx_1 \, dy_1 
\]

\[
H_{y1}(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \left( k_1^2 - k_2^2 \right) \frac{k_x k_y}{j\omega\mu_0} \left[ I_\Psi e^{-jkz_1 z} + R_\Psi e^{jkz_1 z} \right] - \frac{k_x k_y}{j\omega\mu_0} \left[ I_\Phi e^{-jkz_1 z} + R_\Phi e^{jkz_1 z} \right] \right] \times e^{-jkx_1 x - jky_1 y} \, dx_1 \, dy_1 
\]

It should be mentioned that for lossless dielectric medium the slab supports surface waves. For such cases singularities occur in integration of Equation 13. One can overcome this problem by integrating over a contour around the singular points [5]. This problem has not been dealt with in this work since only generally lossy dielectric materials are of interest.

THEORETICAL RESULTS

To gain a better insight into the nature of the problem at hand some tests were performed based on the admittance formulation presented earlier. Figures 2a and 2b show variations of \( G \) and \( B \) for two types of dielectric materials versus thickness at a frequency of 10 GHz. The thick line represents a low loss dielectric and the other a dielectric of higher loss. The permittivity values chosen are generally the range...
values documented for these two types of rubber materials at this frequency [6]. As expected, for both G and B the material with larger dielectric constant and loss tangent undergoes more pronounced variations for small thicknesses and displays faster convergence.

These tests are provided to stress some critical points that must be taken into account in this type measurements. To increase the sensitivity for thickness measurements, the electrical length of the test subject should not be very close to an infinite half space. This fact is clear from both graphs showing larger variations occur at smaller thickness. On the contrary, a better estimation of permittivity can be achieved for a thicker sample. Since permittivity is constant for a uniform sample, variations of these parameters would have less effect on the measurement of permittivity.

Fig. 2a. Variations of G vs. thickness for two types of dielectric samples at 10 GHz.

Fig. 2b. Variations of susceptance vs. thickness for two types of Dielectric samples at 10 GHz.
To calculate the thickness and permittivity characteristics, conductance G and susceptance B of the complex admittance expression were treated separately in two root finding codes. Starting with an initial lower and upper bound, G and B were iterated alternatively to come up with a close estimation of either thickness or dielectric characteristics of the dielectric sample.

Figure 3 shows a simplified block diagram of the code implemented to calculate the dielectric characteristics of a slab of known thickness. A similar procedure was followed for thickness calculations also. Figures 4a and 4b display variations of G and B as a function of $\varepsilon'$ and $\varepsilon''$ for a 1 cm thick dielectric sample at 10 GHz. Each graph represents a line of constant $\varepsilon'$ value. Such graphs, although not critical, are a great tool in providing useful information about the initial bounds of data needed in the root finding codes.

<table>
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<tr>
<th>Thickness (meas.)</th>
<th>G (meas.)</th>
<th>B (meas.)</th>
<th>Thickness (est.)</th>
<th>% Error</th>
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**PRELIMINARY MEASUREMENT RESULTS**

To verify the validity of the theoretical formulation presented earlier some preliminary measurements were performed at a frequency of 10 GHz. In this experiment, three slab-like samples (carbon black composite rubber) with roughly known permittivity characteristics and different thicknesses were chosen. It should be noted that this type material is elastic in nature and its thickness varies slightly upon inserting any pressure. Furthermore, these samples are extremely difficult to be made in sheets of uniform thickness and their measured thickness over an area the size of standard x-band waveguide aperture (i.e. approximately 1x2.3 cm) varied as much as one to two percent.

Once relatively accurate measurements of the sample thicknesses were conducted, their reflection properties were measured by placing them in front of an open-ended rectangular waveguide and a conducting slab in back. By feeding the information from this measurement into the permittivity calculation code, a mean value of for the relative complex dielectric characteristic was estimated. This value is close to some reported range of values for such materials [7]. Using this estimate, the second code was used to estimate the thicknesses of the three samples. The results of this experiment are shown in Table 1. For all three samples the estimated thickness values were within two percent of their measured values.
Fig. 3. Simplified block diagram of the code for finding the dielectric characteristics.

Fig 4a. Variation of $G$ vs. $\varepsilon'_r$ and $\varepsilon''_r$ for a 1 cm thick slab at 10 GHz.
CONCLUSIONS

A technique has been discussed for microwave thickness and permittivity measurement (once either one is known) of generally lossy dielectric slabs backed by a conducting plate. A theoretical model was developed for the admittance calculation of an open-ended rectangular waveguide radiating into a lossy dielectric medium with a conductor backing. The integral expression for the admittance experiences singularities for lossless dielectric materials, although this problem can be taken care of by appropriate integration over a contour around the singular points. This case was ignored due to treatment of generally lossy dielectrics. Some important issues were pointed out in application of such measurements of both thickness and permittivity. Limited measurement results prove this method to be a reliable technique for this type of microwave nondestructive measurements.

REFERENCES