INTRODUCTION

Ultrasonic techniques are widely employed in the nondestructive characterization of materials. For example, the use of grain backscattered ultrasonic signals for the estimation of grain size has been studied extensively [1,2,3,4]. Several techniques to process the grain backscattered signals and extract information related to grain size have been reported in [4]. In this paper, we describe a new technique to process these signals and extract features that can be used for material characterization. The technique consists of the following three steps: i) deconvolution of the backscattered signal to remove the effect of the measurement system, ii) estimation of the spectrum of the resulting reflection coefficient sequence, and iii) extraction of features from the spectrum related to the average scattered energy and the rate of change of scattered energy with frequency, both computed within the bandwidth of the ultrasonic transducer. The spectral features so extracted are influenced by the microstructural properties of a material pertaining to scattering, e.g., average grain diameter, and can be used in the characterization of these properties.

In the following, we first examine the grain scattering process in some detail and then describe the signal processing steps. Experimental results involving some pure titanium samples are next presented. The results include the effect of different spectral estimation methods and window sizes on the features.

GRAIN SCATTERING

The scattering of ultrasonic waves at the grain boundaries of a material is influenced by several factors such as grain anisotropy, grain orientation, grain geometry, average grain diameter, and frequency. The effect of grain scattering is best seen through the attenuation of an ultrasonic wave travelling through a material. The frequency-dependent attenuation coefficient $\alpha$ of a material can be expressed as [5,6].

$$\alpha = \alpha_a + \alpha_s$$

where $\alpha_a$ is the absorption coefficient and $\alpha_s$ is the scattering coefficient. Attenuation due to absorption is relatively small and is caused by the direct conversion of ultrasonic energy into heat. The absorption coefficient is essentially independent of the average grain diameter and varies linearly with frequency $f$ over a wide range as given by

$$\alpha_a = C_1 f$$

where $C_1$ is a constant.
The scattering coefficient, on the other hand, has different expressions depending on the relative values of the average grain diameter $D$ and the acoustic wavelength $\lambda$. In the Rayleigh region where $\lambda > D$, the scattering coefficient is expressed as

$$\alpha_s = C_2 D^3 f^4.$$  

(3)

In the stochastic region where $\lambda = D$, the scattering coefficient is given by

$$\alpha_s = C_3 D f^2.$$  

(4)

In the diffusion region where $\lambda < D$, it is given by

$$\alpha_s = C_4 D^{-1}.$$  

(5)

The constants $C_2$, $C_3$, and $C_4$ account for factors such as grain anisotropy, grain geometry, and grain orientation. From the above expressions for $\alpha_s$, it can be inferred that the scattered ultrasonic energy as viewed through the frequency window provided by a (broadband) transducer will have different average values and different slopes (rate of change with frequency) depending on the material microstructure. Features related to these quantities can therefore be quite useful for material characterization purposes.

**SIGNAL PROCESSING**

As pointed out earlier, our processing technique consists of three steps: i) deconvolution of the backscattered signal, ii) spectrum estimation of the reflection coefficient sequence, and iii) feature extraction from the spectrum. These steps are described below.

**Deconvolution**

The grain backscattered signal obtained from a material sample is obviously colored by the measurement system response. Deconvolution of this signal with the help of a reference pulse (representing the measurement system response) allows us to obtain a signal, viz., the reflection coefficient sequence, which is dependent only on the material microstructure. The deconvolution algorithm used in our technique is based on the Kalman filter and is described in detail in [7]. A brief description of the algorithm follows.

We first model the ultrasonic backscattered signal $z(k)$ as the outcome of convolving the incident acoustic pulse $p(k)$ with the material reflection coefficient sequence $u(k)$ and further corrupting it with some additive noise $v(k)$. Here $k$ is the sampling time index. Mathematically, this relationship is expressed as

$$z(k) = p(k) * u(k) + v(k).$$  

(6)

The convolution operation in (6) suggests that we can also regard $z(k)$ as the corrupted [by $v(k)$] output of a system with impulse response $p(k)$ and input $u(k)$. In this case, we can use state-space representation to express $z(k)$ as follows.

$$\mathbf{x}(k) = F \mathbf{x}(k) + G u(k)$$
$$z(k) = H \mathbf{x}(k) + v(k)$$  

(7)

(8)

In the above expressions, $\mathbf{x}(k)$ is the $(n \times 1)$ state vector, $F$ is the $(n \times n)$ state transition matrix, $G$ is the $(n \times 1)$ input vector relating the states to the scalar input $u(k)$, $H$ is the $(1 \times n)$ measurement vector relating the states to the scalar measurement $z(k)$, and $v(k)$ is the scalar measurement noise. The matrices $F$, $G$, and $H$ together describe the measurement system and should be chosen such that the impulse response of the system approximates the acoustic pulse $p(k)$. As is customary in standard Kalman filter formulation, we assume that the input
sequence $u(k)$ and the measurement noise sequence $v(k)$ are zero-mean, white, and are mutually uncorrelated. The variances of $u(k)$ and $v(k)$ are denoted respectively by $Q$ and $R$.

The deconvolution problem is now that of estimating the input $u(k)$ given the measurement $z(k)$, the system matrices $F$, $G$, and $H$, and estimates of $Q$ and $R$. The deconvolution proceeds in two steps. In the first step, the innovations $\tilde{z}(k)$ are computed. The set of recursive equations used for this purpose are as follows.

**Predictor**

\[
\begin{align*}
\hat{x}(klk-1) &= F \hat{x}(k-1|k-1) \\
P(klk-1) &= FP(k-1|k-1)F^\prime + QGG^\prime
\end{align*}
\]  

(9) (10)

**Innovations**

\[
\begin{align*}
\tilde{z}(klk-1) &= z(k) - H\hat{x}(klk-1) \\
\eta(k) &= HP(klk-1)H^\prime + R
\end{align*}
\]  

(11) (12)

**Corrector**

\[
\begin{align*}
K(k) &= P(klk-1)H^\prime \eta^{-1}(k) \\
\hat{x}(klk) &= \hat{x}(klk-1) + K(k)\tilde{z}(klk-1) \\
P(klk) &= [I - K(k)H^\prime]P(klk-1)
\end{align*}
\]  

(13) (14) (15)

In these equations, $\hat{x}(klk-1)$ and $\hat{x}(klk)$ denote respectively the estimate of the state vector $x(k)$ based on the measurements $z(1)$ through $z(k-1)$ and $z(1)$ through $z(k)$. The corresponding error covariance matrices are denoted respectively by $P(klk-1)$ and $P(klk)$. In addition, $\tilde{z}(klk-1)$ denotes the innovations, $\eta(k)$ denotes its variance, and $K(k)$ is called the Kalman gain vector. The recursion is started by assuming suitable values for $\hat{x}(1|0)$ and $P(1|0)$.

In the second step, the innovations $\tilde{z}(k)$ are smoothed to obtain a minimum variance estimate of $u(k)$. The equations describing a fixed-interval smoothing algorithm are as follows.

\[
\begin{align*}
\hat{u}(klL) &= QG^\prime r(kL) \\
P_u(kL) &= Q - QG^\prime S(kL)GQ
\end{align*}
\]  

(16) (17)

In these equations, $\hat{u}(klL)$ is the optimal (minimum variance) fixed-interval smoothed estimate of $u(k)$ and $P_u(kL)$ is its variance. The quantity $r(kL)$, called the residual state vector, is defined as

\[
r(kL) = P^{-1}(klk-1)[\hat{x}(klL) - \hat{x}(klk-1)].
\]  

(18)

The residual state vector and its covariance matrix $S(kL)$ can be found recursively using the following equations.

\[
\begin{align*}
r(kL) &= [I - K(k)H^\prime]F^\prime r(k+1|L) + H\eta^{-1}(k)\tilde{z}(klk-1) \\
S(kL) &= [I - K(k)H^\prime]F^\prime S(k+1|L)F[I - K(k)H^\prime] + H\eta^{-1}(k)H^\prime
\end{align*}
\]  

(19) (20)

where $k = L, L-1, \ldots, 1$, $r(L+1|L) = 0$, and $S(L+1|L) = 0$.

**Spectrum Estimation**

Once the reflection coefficient sequence $\hat{u}(k)$ has been estimated, its spectrum can be obtained using different methods [8,9]. We consider two methods here, viz., Periodogram (PER) method and Auto-Regressive (AR) method. In periodogram method, the spectrum of a given sequence of $N$ samples is computed as follows.

\[
P_{PER}(f) = \left| \sum_{k=0}^{N-1} u(k) e^{-j2\pi fk} \right|^2
\]  

(21)
Table 1. Particle sizes and average grain diameters of the titanium samples A, B, and C

<table>
<thead>
<tr>
<th>Sample</th>
<th>Particle size (µm)</th>
<th>Average grain diameter (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Average grain diameter1</td>
</tr>
<tr>
<td>A</td>
<td>150-300</td>
<td>44.94</td>
</tr>
<tr>
<td>B</td>
<td>125-150</td>
<td>39.05</td>
</tr>
<tr>
<td>C</td>
<td>106-125</td>
<td>48.85</td>
</tr>
</tbody>
</table>

1. Average maximum grain diameter measured by hand
2. Average length of a randomly drawn line that intercepts the grain boundaries and is measured by hand
3. Average length of a line which is inside the grain boundaries and is randomly drawn using Monte Carlo method by computer

The periodogram method is computationally efficient, but provides poor estimate when the data record is short, i.e., \(N\) is small. In AR spectrum estimation method, we first compute a set of predictor coefficients \(a_i, i = 1, 2, \ldots, p\) from the given data record such that the average squared prediction error \(K\) is minimized. In the method used by us (Burg’s method), both forward and backward prediction errors are considered. The spectrum is then estimated as

\[
P_{AR}(f) = \frac{K}{1 + \sum_{i=1}^{p} a_i e^{-j2\pi ft}}^2
\]

(22)

The AR spectrum estimation method generally gives a better estimate when the data record is short.

Feature Extraction

After estimating the spectrum of the reflection coefficient sequence, two features are extracted from it, viz., \(F_0\) related to the average scattered energy, and \(F_1\) related to the rate of change of scattered energy with frequency. The values of these features are obtained by least-squares fitting a portion of the spectrum within the transducer bandwidth with respectively a zeroth order and a first order polynomial.

EXPERIMENTAL RESULTS

The signal processing technique described above was implemented in software - the deconvolution step was implemented through a "C" language program and the spectrum estimation and feature extraction steps were implemented through the "MATLAB" package [10]. Backscattered signals were obtained from three pure titanium samples A, B, and C. These samples were prepared using powder metallurgy techniques starting from particles of different sizes. It is believed that the microstructures of these samples differ only in terms of their average grain diameters. The particle sizes used as well as the average grain diameters measured using three different methods are shown in Table 1. A-scan data were obtained from each titanium sample at ten different locations using a 15 MHz focussed transducer (radius: 0.25", focal length: 3.5") inside a water immersion tank. The data were then digitized at a sampling frequency of 100 MHz. In obtaining the grain backscattered data, the transducer was focussed at the center of the samples. A reference pulse was obtained by focussing the transducer at the front surface of a sample.

The processing of the backscattered signals was accomplished as follows. An 8th order ARMA (Auto-Regressive Moving Average) system model with its impulse response approximating the reference pulse was first built using a nonlinear least-squares fitting utility in the IMSL Math Library [11]. The backscattered signals were then deconvolved to obtain the corresponding reflection coefficient sequences. A band-pass filter was next used to
Figure 1. Feature vector distribution for $N = 350$ and $p = 10$

Figure 2. Feature vector distribution for $N = 128$ and $p = 10$
Table 2. Quantitative separability measure for different window sizes and different orders of AR model

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>350   256  128  64</td>
</tr>
<tr>
<td>Periodogram</td>
<td>2.93  3.73  1.69  0.75</td>
</tr>
<tr>
<td>AR (2)</td>
<td>3.25  2.34  1.47  0.51</td>
</tr>
<tr>
<td>AR (4)</td>
<td>1.20  0.98  1.98  0.59</td>
</tr>
<tr>
<td>AR (6)</td>
<td>3.72  2.07  2.69  0.83</td>
</tr>
<tr>
<td>AR (8)</td>
<td>4.65  2.92  1.76  0.50</td>
</tr>
<tr>
<td>AR (10)</td>
<td>5.15  3.81  2.21  1.07</td>
</tr>
</tbody>
</table>

remove the dc and higher frequency components from the reflection coefficient sequences. The spectra of the reflection coefficient sequences were then estimated using both periodogram and AR methods. Using the frequency range from 5 to 15 MHz, the features $F_0$ and $F_1$ were finally extracted from these spectrums. In estimating the spectrums, different window sizes ($N$) and different AR model orders ($p$) were used.

The results corresponding to an AR (10) model and window sizes of 350 and 128 are shown respectively in Figure 1 and Figure 2. The ellipses in the figures indicate contours of constant probability density functions when the feature vectors corresponding to each sample were fitted with a twodimensional gaussian distribution. The lengths of the major and minor axes of each ellipse represent twice the standard deviation ($2\sigma$) along the respective directions. It is seen that the features $F_0$ and $F_1$ can clearly distinguish between the three samples even though the order of the samples along the $F_0$ axis does not correspond to any of the orders of the average grain diameters indicated in Table 1. It is also seen that the three titanium samples are well separated in the feature space even when the window size used is relatively small. (Note: For titanium material and 100 MHz sampling frequency, a distance of 1 mm corresponds to a window size $N$ of about 32) This suggests the possibility of using the features for flaw detection as well when the microstructures of the host material and flaw are different. In order to quantify the separation between the different distributions in the feature space, a separability measure $J_4$ was used [12]. This is a measure of the ratio of the variance of the means of the different distributions and the mean of their variances. For different window sizes and different orders of the AR model, this separability measure is listed in Table 2. It is seen that the separability generally increases with the AR model order $p$ and that it decreases as the window size $N$ decreases.

CONCLUSIONS

A new technique for processing grain backscattered ultrasonic signals and extracting useful features from them has been described. The technique consists of three steps: deconvolution, spectrum estimation, and feature extraction. The features chosen are related to the average scattered energy and the rate of change of scattered energy with frequency, both computed within the transducer bandwidth. The features show good potential for material characterization and flaw detection as well. Further work needs to be performed on several material samples with different microstructures to validate the usefulness of the features.

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REFERENCES


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