THE MODELLING AND EXPERIMENTAL STUDY OF THE PIEZOFILM
TRANSDUCER WAVEFORMS IN CYLINDRICAL GEOMETRY

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INTRODUCTION

Components with curved surfaces are very common in nondestructive
evaluation. In order to detect flaws in curved components using the contact mode,
transducers must be made to conform to the curvature of the component. The flexible
polymer piezofilm transducers have a distinct advantage in such application. When
applied on samples of a rod geometry, a film transducer assumes the shape of a part
of a cylindrical shell. When a signal is generated by such a source, the received echo
waveform will be significantly different from that of the initial excitation due to the
effects of focusing and diffraction. This phenomenon was encountered when pulse-
echo measurement was made on rod samples using PVDF polymer film transducers. A
film transducers was wrapped around the rod surface covering approximately one
sixth of the circumference. The first reflected echo was almost out of phase with the
initial input signal and the second echo was out of phase with the first one. This
phenomenon cannot be explained by ray approximation. An effort was therefore
made to build a model to explain the pressure wave behavior in cylindrical field and
to predict the received echo waveform. The efforts spent so far are concentrated on
the longitudinal wave. A film transducer, conforming to the sample curvature and
launching waves at normal incidence everywhere, can be considered as a
longitudinal pressure wave source.

To treat the diffraction effect, the model is based on the impulse response of the
source. This method was originally used to calculate the wave profile from flat piston
sources [1], and was then used to evaluate the field distribution of curved sources,
such as spherically focusing lenses [2,3], and axially symmetric ring-shaped
cylindrical sources [4]. The impulse response in cylindrical geometry, which is
usually calculated numerically, will be derived analytically under appropriate
simplification.

Since the film transducer is polarized as a longitudinal transducer, the
disturbance of the shear wave and the boundary effect can be neglected. In this
approximation, the scalar potential method is applicable and the acoustic velocity \( v \)
(the time derivative of the displacement) can be expressed in terms of the velocity
potential \( \phi \):

\[
\mathbf{v} = \frac{\partial \mathbf{u}}{\partial t} = \nabla \phi.
\]  \hspace{1cm} (1)

The pressure can also be expressed in terms of the potential:
where \( \rho \) is the density of the medium. Therefore, the field characteristics can be determined once the velocity potential in the field is known.

**IMPULSE RESPONSE APPROACH**

The Rayleigh surface integral\([2-4]\)

\[
\phi(r, t) = \frac{1}{2\pi} \iint_{S_0} \frac{V_0(t - \frac{r'}{c})}{r'} \, dS_0
\]

(3)

gives the potential function of the field generated by a distributed source on an infinite baffle, as shown in Fig. 1.

The Rayleigh integral is over the entire area of the source; each active element on the source acts as a point source. The subscript 0 denotes the quantities related to the source, \( r' \) is the distance from a source point to the field point \( M(x,y,z) \), \( c \) is the speed of wave propagation, \( V_0(t) \) is the velocity excitation of the source and \( t \) is the time. If we define the impulse response at field point \( M \) as

\[
h(r, t) = \iiint_{S_0} \frac{\delta \left( t - \frac{r'}{c} \right)}{r'} \, dS_0,
\]

(4)

then the potential function and the pressure can be written as:

\[
\phi(r, t) = V_0(t) \ast h(r, t),
\]

(5)

\[
p(r, t) = -\rho \frac{\partial \phi}{\partial t} = -\rho \left( \frac{\partial V_0(t)}{\partial t} \right) \ast h(r, t).
\]

(6)

Fig. 1 Field Generated by a Finite Source Area
Therefore, the convolution between the initial source excitation and the impulse response will give rise to the potential function and the pressure in the field. To calculate the impulse response of Eq.(4), a two-dimensional surface integral must be performed in the source region. For cylindrically shaped sources, elliptic integrals will be involved and numerical method must be used to solve this problem [4].

The transducer we modeled is part of a cylindrical shell located on the surface of a cylindrical rod, Fig. 2 shows the top view of the transducer around a cylindrical rod (the cylinder doesn't appear in this figure). Similar to the approach for calculating the impulse response of a spherical lens [2,3], the field space in Fig. 2 is divided into three parts, marked as I, II and III, by the two normal rays at the edges of the source. In Fig. 2, L_0 is the length of the transducer along the cylindrical wall, a is the distance from the field point M to the cylindrical focusing axis O, R is the radius of curvature, \( \theta \) is the angle on the axis formed by the field point and the source element dL_0, and \( r_0, r_1 \) and \( r_2 \) represent the normal, the shortest and the longest edge paths from the source L_0 to the field point M, respectively.

To evaluate the waveform change due to the effect of the curvature of the transducer, each element of the transducer will be treated as an infinite line source parallel to the axis of the cylinder, i.e. the z direction in Fig. 2. The surface integration over the source area is then transformed into a one-dimensional integral along the curved source L_0. In this approximation, field variations along the z direction, which are not associated with the curvature effect, are ignored.

The wave generated by a line source propagates only in the radial direction. For a harmonic wave, the solution of the wave equation is a Hankel function \( H_0(kr') \) [5], and in the far field, where \( kr'>>1 \), the potential and the pressure are inversely proportional to the square root of radial distance \( r' \). Such relation is also valid for a pulse wave since the acoustic intensity, or the average power flux, is given by \( I=P_t/2\pi r' \) for unit length of a line source, where \( P_t \) is the total power from this source unit. In the far-field zone, \( p \) and \( \phi \) can be considered proportional to the square root of the intensity[6], that is, proportional to \( (1/r')^{1/2} \) (This is different from a point source for which \( p\propto 1/r' \) applies.). Noticing that the source only radiates into a half-space, the superposition of the potential functions due to each line element of the transducer results in the total potential function at the field position M(x,y). This total potential function, in analogy with the Rayleigh integral, is written as:
\[ \phi(x, y, t) = \frac{1}{\pi} \int_{L_0} \frac{V_0(t - \frac{r^{'}}{c})}{\sqrt{r'}} \, dL', \tag{7} \]

where \( L_0 \) is the source region in Fig. 2. Therefore, in the far field, the field impulse response of Eq.(4) becomes

\[ h(x, y, t) = \frac{1}{\pi} \int_{L_0} \frac{\delta(t - \frac{r^{'}}{c})}{\sqrt{r'}} \, dL'. \tag{8} \]

If we denote \( t_0 = \frac{r_0}{c}, t_1 = \frac{r_1}{c}, \) and \( t_2 = \frac{r_2}{c} \) as the wave travelling time corresponding to the normal and the two edge paths, respectively, the impulse response in different regions of the field can be calculated from Eq.(8). The result is summarized in Table 1, in which

\[ \xi_0 = \frac{(a^2 + R^2 - \frac{c^2t^2}{2})}{2aR}. \]

Since there is no normal path in field region III, \( t_0 \) has no meaning in that region.

The waveform generated by the transducer in any field point \( M(x, y) \) can then be determined by the convolution of \( h(x, y, t) \) and the initial input excitation by the source.

<table>
<thead>
<tr>
<th>h(x,y,t)</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>t &lt; t_0</td>
<td>t_0 &lt; t</td>
<td>t &lt; t_1</td>
</tr>
<tr>
<td>\frac{2c}{a\pi \sqrt{\frac{1}{1 - \xi_0^2}}}</td>
<td>t_0 &lt; t &lt; t_1</td>
<td>t_2 &lt; t &lt; t_0</td>
<td>*</td>
</tr>
<tr>
<td>\frac{c}{a\pi \sqrt{\frac{1}{1 - \xi_0^2}}}</td>
<td>t_1 &lt; t &lt; t_2</td>
<td>t_1 &lt; t &lt; t_2</td>
<td>t_1 &lt; t &lt; t_2</td>
</tr>
<tr>
<td>0</td>
<td>t_2 &lt; t</td>
<td>t &lt; t_1</td>
<td>t_2 &lt; t</td>
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</table>

* \( t_0 \) does not apply to region III.

**MODELING OF THROUGH TRANSMISSION AND PULSE-ECHO MEASUREMENTS**

For a through transmission configuration in Fig. 3, two piezofilm transducers, each of length \( L_0 \) are used, one as a transmitter and the other as a receiver. The transmitter and the receiver are placed symmetrically on the rod. The impulse response of this system could generally be calculated by performing the \( h(x, y, t) \) integration over the entire receiver, which must be carried out numerically. While for this specific geometry, the impulse response can be analytically derived by involving a weighting function into Eq.(8). For a \( \delta \)-function input, since the shortest and the longest paths are the edge paths \( r_1 \) and \( r_2 = 2R \), only the signal from the edge of the transmitter can be received at the time \( t_1 = \frac{r_1}{c} \). For a wave path \( r \) \((r_1 < r < r_2)\), the traveling time is \( t = \frac{r}{c} \), but waves generated over a finite section of the transmitter, shown as a dark region in Fig. 3, will reach a corresponding region on the receiver, all with the same travel time. Since the transducer has two symmetric edges, a weighting function \( g(r) \) equal to twice the length of the dark region should be introduced when calculating the system impulse response. Thus \( h(t) \) is written as:
Fig. 3 Cross Section of Through-transmission Configuration
\( h(t) = \frac{1}{\pi} \int_{L_0} \frac{\delta(t) - \frac{r}{c}}{\sqrt{r}} g(r) \mathrm{d}L_0 . \) (9)

The weighting function is \( g(r) = g(\alpha) = 2R(\alpha - \alpha_1) \), where \( \alpha \) is the angle subtended by path \( r \) at the center of the rod, and \( \alpha_1 = (\pi R - L_0)/R \) is the angle subtended by the edge path \( r_1 \) at the center, as shown in Fig. 3. Substituting \( r = 2R \sin(\alpha/2) \) into Eq.(9) and noticing that

\[ \delta[t - \frac{2R \sin(\alpha/2)}{c}] = \frac{1}{R} \frac{\delta(\alpha - \alpha_0)}{\cos(\alpha_0)} , \]

where \( \alpha_0 = 2 \arcsin(\alpha t/2R) \), the impulse response can be derived as:

\[ h(t) = \begin{cases} \frac{c \sqrt{2R} (\alpha_0 - \alpha_1)}{\pi \cos(\alpha_0/2) \sqrt{\sin(\alpha_0/2)}} & \frac{r_1}{c} \leq t \leq \frac{2R}{c} \\ 0 & \text{otherwise} \end{cases} \] (10)

Based on the Eq.(5) or Eq.(6) and ignoring attenuation in the material, the electrical signal \( s_r(t) \) produced by the receiving transducer for the through-transmitted ultrasonic signal can be written as[2]:

\[ s_r(t) = e_0(t) * i_t(t) * i_r(t) * h(t) = s_i(t) * h(t) , \] (11)

where \( e_0(t) \) is the electrical excitation on the transmitting transducer, \( i_t(t) \) and \( i_r(t) \) are the acoustoelectric transfer functions of the transmitter and the receiver, respectively, and \( h(t) \) is the impulse response of this system. The \( s_i(t) = e_0(t) * i_t(t) * i_r(t) \) represents the initial signal when the impulse response is unity. According to the reciprocity principle, the same \( h(t) \) holds true for the wave propagation in the inverse direction. Therefore, if we assume a unity reflection coefficient for the back wall of the cylinder in a pulse-echo measurement, the signal waveform of the round-trip echo \( s_{pe}(t) \) will be given by:

\[ s_{pe}(t) = e_0(t) * i_t(t) * i_r(t) * h(t) * h(t) = s_u(t) * h(t) . \] (12)

**COMPARISON OF THE MODEL PREDICTIONS AND THE EXPERIMENTS**

To examine the transmitted and reflected waveforms predicted by Eq.(11) and Eq.(12), respectively, experiments were conducted by applying film transducers to the surface of a 2" dia. aluminum rod. A Panametrics 5052PR pulser/receiver was used as a pulse generator, Pennwalt SDT1-028K and LDT1-028K PVDF film transducers, generating broad band and narrow band ultrasonic pulses, respectively, were used as the transmitter and receiver. The central frequencies of both the broad band and the narrow band pulse signals were about 7 MHz, which ensured that the back wall of the rod was located in the far-field, hence satisfying the assumption made in this model.
The dimensions of the SDT1-028K and LDT1-028K transducers were all 1.14" X 0.55". The electrical signal waveforms were acquired on a LeCroy 9400 digital oscilloscope. In the actual measurement, since the real initial signal emitted by the transmitting transducer was difficult to acquire, we used the first round-trip echo as the initial signal. Based on the first round trip echo being the initial signal, the
through-transmission signal is then the echo that traversed the diameter three times and the pulse-echo signal is the second round trip echo. The measured and predicted results for the through-transmission and pulse-echo signals are shown in Fig. 5 and Fig. 6, respectively.

A comparison of the waveforms in Fig. 4 with the measured and predicted results in Fig. 6 shows that the echo signal has a polarity opposite that of the input signal. This phase reversal phenomenon is unique in rod geometry and is caused by the focusing effect of the curved transducer. When a pulse-echo experiment was conducted on a flat surface aluminum block using the same transducer, there was no phase reversal between the input and the echo signals.

CONCLUSIONS

A model based on the system impulse response function has been used in predicting the waveform changes of ultrasonic echoes propagating radially in a rod. An analytic expression was derived for the impulse response function. Using the analytic result, through-transmission and pulse-echo waveforms were predicted using a digitized first round trip echo as the initial signal. PVDF piezofilm transducers were used for the transmission and detection of the waves. The experimental results and model results were in good agreement. The impulse response approach was reasonably successful in predicting the waveform but suffered from singularity difficulty in treating the signal amplitude.

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REFERENCES