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Pedagogical design capacity for teaching elementary mathematics: A cross-case analysis of four teachers

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Pedagogical design capacity for teaching elementary mathematics: A cross-case analysis of four teachers

by

Tonia J. Land

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Education

Program of Study Committee:
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    Alejandro Andreotti
    Heather Bolles
    Ann Thompson

Iowa State University
Ames, Iowa
2011

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CHAPTER 1. ASKING A QUESTION

Introduction

When asked to generate mathematical expressions that equaled 75 and included a fraction, a second grader, Kasey, generated the following list:

- $25 \times 3 = 75$
- $24 \frac{1}{3} \times 3 = 75$
- $\frac{1}{2} \times 150 = 75$
- $\frac{1}{4} \times 300 = 75$
- $1 \times 75 = 75$

It seems likely that Kasey used the first equation and her knowledge of $\frac{1}{3} \times 3 = 1$ to generate the second equation, but her error in the second equation suggests that she is still grappling with how to accurately use that knowledge. Given that Kasey is a second grader, it is impressive that she has some knowledge of multiplying fractions. She does, however, need some guidance in how to apply that knowledge when multiplying mixed numbers. In the latter three equations, it seems that Kasey used halving and doubling of factors to generate equal expressions. Another student, Malcolm, in the same class generated these expressions:

- $74 \frac{1}{4} + \frac{1}{4} = 75$
- $37 \frac{1}{2} + 37 \frac{1}{2} = 75$
- $60 \frac{1}{2} + 14 \frac{1}{2} = 75$

It seems likely that Malcolm used his knowledge of $\frac{1}{2} + \frac{1}{2} = 1$ and decomposing numbers to generate expressions that equaled 75. The first sentence contained an error, but it seems likely that it was a recording error as Malcolm’s latter two sentences are correct.

Debbie and Jayda, students in a fifth grade classroom, used the relationship
between $\frac{1}{2}$ and $\frac{1}{4}$ to find $\frac{1}{4}$ of 60 baseball cards:

Debbie’s strategy for finding $\frac{1}{4}$ of 60 baseball cards

$60 - 30 = 30$
$30 - 15 = 15$

Jayda’s strategy for finding $\frac{1}{4}$ of 60 baseball cards

$60 \div 2 = 30$
$30 \div 2 = 15$

Using subtraction and division respectively, Debbie and Jayda halved 60 then halved 30 providing evidence that the two girls understood that $\frac{1}{4}$ is half of $\frac{1}{2}$. Adam, a classmate of Debbie and Jayda, found $\frac{1}{4}$ of 44 baseball cards using this strategy:

Adam’s strategy for finding $\frac{1}{4}$ of 44 baseball cards - Adam said, “I put ten in each circle because I knew that would make 40. There was four left over, so I put one more in each circle.”

![Diagram of baseball cards]

$44 \div 4 = 11$

From his written work and explanation, it is evident that Adam knew that finding $\frac{1}{4}$ of a group of objects involved dividing those objects into four equal groups. He also used his knowledge of base-ten to decompose 40 into four groups of ten, and then he distributed the remaining four baseball cards. Furthermore, Adam was able to represent his work in two ways – a drawing and an equation.

The National Council of Teacher of Mathematics (NCTM) wrote about their vision for school mathematics:

Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find
methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers (NCTM, 2000, p. 2).

The student vignettes given above reflect NCTM’s vision in that the students approached the instructional tasks from different perspectives and represented the mathematics in different ways. To generate expressions that equaled 75, Kasey used halving and doubling of factors while Malcolm used his knowledge of \( \frac{1}{2} + \frac{1}{2} = 1 \) and decomposing numbers. Each was able to use his/her own knowledge to generate expressions equaling 75. Additionally, the vignettes illustrate that the students were flexible and resourceful problem solvers. To find a fraction of a set, Jayda and Debbie used the relationship between \( \frac{1}{4} \) and \( \frac{1}{2} \), while Adam used his knowledge of base-ten.

Also stated in NCTM’s vision is that mathematical tasks should be carefully chosen by teachers. Instructional tasks that engage students in cognitively demanding ways, however, are more difficult for teachers to facilitate (Stein, Grover, & Henningsen, 1996). The tasks are more difficult, because teachers tend to be unfamiliar with the non-procedural ways of solving mathematics problems and the many possible solution strategies put demands on teachers’ capacities (Ball & Cohen, 1996). Children are problem solvers – they can construct viable solutions to problems without formal instruction on specific algorithms (Carpenter, Fennema & Franke, 1996). Given that we know this about children, the challenge lies in designing instruction that provides students with opportunities to construct viable solutions and not simply mimic solution paths provided by textbooks and teachers. That challenge is the motivation behind this dissertation study. From the above vignettes, we know NCTM’s vision is possible, but
the question I wanted to investigate was: “How do teachers design a context where NCTM’s vision is put into practice?” Pedagogical design capacity (PDC) is a construct that helps answer that question.

This dissertation is structured around three stand-alone articles (Chapters 3, 4, and 5). Chapter 1 provides an introduction to those three chapters by first describing the PDC construct, curricular resources available to the study participants, and an overall methodology. The methodology used in Chapters 3-5 is described in more detail in each of those respective chapters. I conclude Chapter 1 with an overview of Chapters 2-6.

**Pedagogical Design Capacity**

PDC stems from the idea that teaching is a design activity (Brown & Edelson, 2003). “Teachers must perceive and interpret existing resources, evaluate the constraints of the classroom setting, balance tradeoffs, and devise strategies – all in pursuit of their instructional goals. These are all characteristics of design” (Brown & Edelson, 2003, p. 1). Brown and Edelson (Brown & Edelson, 2003; Brown, 2009) introduced the PDC construct as a way to understand how teachers perceive and mobilize existing resources to design instruction. *Perceive* indicates the ability to recognize, or notice, potential resources and *mobilize* highlights the importance of teachers’ abilities to act on or with those resources (Remillard, 2005).

Viewing teaching as a design activity is a relatively new concept, but it is compatible with a range of cognitive theories that accentuate the interactions an individual has with the tools he/she uses to accomplish particular goals (Brown, 2009).
Artifacts are human-created tools. “A key feature of artifacts is that they assist people in achieving goals they could not accomplish on their own” (Brown, 2009, p. 19).

Curriculum materials, Brown contends, are artifacts. Drawing on the work of several researchers, Brown compiled a list of ways teachers interact with curriculum materials: 1) they select, 2) they interpret, 3) they reconcile their perceptions of the intended goal with their own goals, 4) they make accommodations for their students, and 5) they depart from the plan by adding to, modifying, or omitting altogether (Brown, 2009).

PDC describes the capacity of a teacher to mobilize his/her existing resources in order to achieve instructional goals. Teachers have access to different tools or resources (e.g. curriculum materials, professional development) depending on their setting and they use those resources differently depending on their experience, goals, and abilities. For instance, two teachers who have seemingly similar knowledge and skills can produce different enacted curricula, “because they possess very different capacities to create deliberate, productive designs” (Brown, 2009, p. 29).

Similarly, Remillard (2005) writes that teachers and their curriculum materials have a participatory relationship - each is a significant and active contributor to the planned curriculum. Brown and Edelson (2003) describe three patterns of curriculum use as offloading, adapting, and improvising. Offloading refers to adhering closely to curriculum materials. Adapting refers to the practice of using curriculum materials, but also contributing one’s own design elements to instruction. When a teacher improvises, he/she “pursues instructional paths of their own design” (Brown & Edelson, p. 7, 2003).

Brown and Edelson contend that the PDC construct has implications for teacher
preparation and professional development, noting that, “Teachers require support in exploring which resources to use and how to use them” (Brown & Edelson, 2003, p. 6). Many teachers, however, are required to use a particular curriculum series (Archer, 2005). Therefore, much of the work of teacher educators and school district leaders lies in how to support teachers in using the curriculum series well. One way to understand how to support teachers in their use of Standards-based curriculum materials is to study how teachers already use them in productive ways. Stein and Kaufman (2010) conducted one such study and found that rather than the education, experience, and knowledge a teacher brings to a classroom; teachers tended to have higher quality lessons, (measured by maintaining high levels of cognitive demand, attending to student thinking and vesting intellectual authority in mathematical reasoning) when they “talked about or reviewed big mathematical ideas that students were supposed to be learning” (Stein & Kaufman, 2010, p. 681). In other words, how teachers mobilized the curriculum materials shaped the quality of the lessons more than individual capacity (Stein & Kaufman, 2010). This dissertation study also investigated the ways in which teachers mobilized a Standards-based curriculum series, Investigations (TERC, 2008), in addition to another curricular resource – Cognitively Guided Instruction (CGI).

An introduction to these two curricular resources (Investigations and CGI) is provided in the next section.

**Available Curricular Resources**

All study participants had access to Investigations (TERC, 2008) - an innovative Standards-based elementary mathematics curriculum – and CGI professional

Study participants also had participated in CGI professional development - an approach to instruction that focuses primarily on children’s mathematical thinking (Carpenter et al., 1999). One of many studies focusing on the effectiveness of CGI examined changes in the beliefs and instructional practices of 21 teachers who participated in a CGI teacher development program over the course of four years (Fennema et al., 1996). This study provided strong evidence to indicate that understanding children’s thinking is a powerful tool in changing instruction (Fennema et al., 1996). Over the course of the study, 17 of the 21 teachers came to believe more strongly that children could solve problems they had not previously been taught procedures for, which changed the teachers’ perceptions of their own role in the
classroom. Specifically, teachers came to believe that their role was to provide opportunities for children to solve problems and report on solution strategies rather than telling children how to think.

Additionally, Franke and colleagues (2001) found that professional development in CGI is sustainable when they interviewed 22 teachers four years after participating in a CGI professional development program. All teachers continued to use children’s thinking in their practice and ten continued to be engaged in generative growth. The teacher’s role in a CGI classroom is to:

- continually upgrade their understanding of how each child thinks, select activities that will engage all the children in problem solving and enable their mathematical knowledge to grow, and create a learning environment where all children are able to communicate about their thinking and feel good about themselves in relation to mathematics” (Carpenter et al., 1999, p. 101).

Due to grounding my work in PDC, I am most interested in the aspect of the teacher’s role that entails selecting “activities that will engage all children in problem solving.” To engage all students in problem solving and design situations where all students are growing mathematically is a tremendous task for a teacher. More information is needed as to how teachers are designing instruction where all students are working productively. We know that CGI is an effective, sustainable, and generative teaching practice that has positive effects on student learning. However, we do not yet know much about how CGI teachers mobilize their CGI knowledge.

**Methodology**

**Participants**
Four expert teachers were involved with this study: Kathy, Olivia, Nancy, and Violet\(^1\). I identified them as expert teachers, because they continually engaged their students in cognitively demanding tasks. They were able to mobilize their existing resources to craft instructional contexts that engage students in doing mathematics. I provide a thorough description of each of these teachers in Chapter Two.

**Data Sources**

Kathy, Olivia, and Violet were each observed on six occasions. After each classroom observation, I watched the videotape and developed a video-stimulated recall interview for each of the six observations. Nancy was observed on eleven occasions. The video-stimulated recall interview was conducted for three of her observations. These interviews were designed to gain knowledge of possible key dimensions of PDC. For each segment of classroom observations (opening routine, launch of activity, student exploration, and strategy sharing), I would ask each teacher to state and explain her intended goal, provide details in how that segment was designed, and describe what she noticed about the student learning. For instance, the following five questions were asked of Violet for one part of one observation:

1. Can you tell me about the game - Roll Around the Clock? How does the game fit in with the goals for the unit and year? Did only some students play? Why?
2. Tell me about the beginning problem. Why did you develop the beginning problem? How does the problem tie into the curriculum and your goals for the year?
3. What do you notice about Kael’s strategy? Where do you want him to progress to from here?
4. What do you notice about Leslie’s strategy?
5. What is your goal with your conversation with Liev?

\(^1\) All names are pseudonyms.
After each interview was complete, I transcribed all interview data.

**Data Analysis**

In previous work (Land & Drake, 2010), I drew on the work of Remillard (2005) and Davis et al. (2007) to generate a list an initial list of possible key dimensions of PDC to investigate the PDC of one teacher. During that study, my co-author and I generated a second list of possible key dimensions of PDC through a process of emergent coding. We used this list as a coding scheme to analyze interview data:

- Knowledge of Curricular Resource
- View/Perception of Curricular Resource
- Mobilization of Curricular Resource
- Knowledge/Perception of Students
- Mobilization of Student Resources
- Beliefs
- Goals
- Tolerance for Discomfort
- Previous Teaching Experiences
- Subject Matter Knowledge

Once final codes and definitions were established, I recoded all interview data. A second coder coded 34% of the data (14 of 41 pages of interview transcripts). The two coders were in disagreement on 10 segments of data comprising 13% of the 14 pages of transcripts. Of those disagreements, we reached 90% consensus. The remaining segment pertained to Nancy using the CGI framework to gain knowledge of her students. The segment remained under dispute, because one coder felt it should be coded as mobilization of curricular resources while the other thought it should be coded as knowledge of students. It was agreed that the segment could be double-coded. Moving forward, I decided not to double-code data, and that codes entailing how teachers perceived or mobilized a particular resource took precedence over other
codes.

For this dissertation study, I coded all transcripts using the above list of possible key dimensions as a coding scheme. I added one additional code – progressions. I made this decision based on the fact that the teachers continually talked about progressions in their interviews and wanted to capture what was seemingly an important part of their teaching practices. For Chapter Three, I pulled interview data coded as goals, progressions, mobilization of resources, and knowledge of students to build representations of four progression types. To come to understand teachers’ knowledge of students and how the teachers were mobilizing student resources, I analyzed data coded as such for the results described in Chapter Four. Many times teachers posed contextualized word problems. The problems and the rationales were analyzed to generate a list describing the ways in which the teachers were mobilizing number choices. Those mobilizations of number choices are described in Chapter Five.

**My Interest in this Work**

I graduated with a degree in elementary education in 1995. After graduation, I spent nine years teaching in the Aldine and Cy-Fair Independent Schools Districts in Houston, Texas. My positions included teaching math, science, and technology. Math, of course, was my favorite. Looking back on my math instruction, I would say that I had a traditional teaching practice, but I looked for alternate ways of teaching. One particular day stands out for me. As per the textbook, I was to teach the procedure for converting mixed numbers to improper fractions and vice versa. I thought at the time that there must be a better way and was not even sure if my students understood conceptually
mixed numbers and improper fractions. If my students did not understand mixed numbers and improper fractions, why was I teaching procedures to convert between the two? Therefore, instead of teaching those procedures, I asked students to draw representations of the mixed numbers/improper fractions (e.g. 3 ½, 4 3/4), then to represent the drawing with two numbers (e.g., 3 ½ and 7/2). Finally, I asked students to look for patterns in the two numbers and develop rules for shifting back and forth between the two. I am not saying that this was a great teaching moment, but it was the beginning of a shift in philosophy. I no longer wanted to teach procedures, but I struggled with how to alter my teaching practice.

My philosophy shift developed further when I started teaching the elementary mathematics methods course at Iowa State University in 2006. To prepare for that teaching endeavor, I observed my advisor for a semester. It was in her classroom that I first learned about CGI. I studied the CGI book, watched the videos, and observed my advisor facilitating instruction. I became convinced that teaching through problem solving was a much better form of instruction. Additionally, children’s alternate solution strategies were intriguing to me. I came to believe that the teacher’s role is to figure out how students are thinking about the mathematics rather than having students figure out how the teacher is thinking about mathematics.

While conducting research around teachers’ use of Standards-based curriculum materials, I became acquainted with Investigations (TERC, 2008) and other Standards-based curriculum series. I became attracted to Investigations (TERC, 2008), because of its focus on teaching through problem solving and conceptual understanding. I found
the teacher’s guide to have valuable information for teachers – specific learning goals, examples of student work, professional development within the materials, differentiation options, and example teacher dialogue. Unlike other Standards-based series, I felt Investigations (TERC, 2008) aligned with the CGI philosophy.

I became interested in the work around PDC after reading two book chapters – one authored by Brown (2009) detailing the construct of PDC and another by Stein and Kim (2009) around the four levels of cognitive demand (memorization, procedures without connections, procedures with connections, and doing mathematics) for mathematical tasks. A quote from the Brown chapter really resonated with me. Brown was describing a teacher who was able to mobilize a curriculum series in productive ways, and stated, “She possesses a skill.” In the following chapter, Stein and Kim (2009) talked about the challenge for teachers to implement cognitively demanding tasks. It became clear to me at that time that the skills required of a teacher to implement memorization tasks are quite different than the skills required to implement tasks categorized as doing mathematics.

The force behind my interest is the design and assessment of the elementary mathematics methods course. I want to help PSTs use their available supports in productive ways. That is, I want to help PSTs implement instruction where students are engaged in doing mathematics. I believe that coming to possess the skills of successfully implementing “doing mathematics” tasks happens on a trajectory, and we can start PSTs on that trajectory during the elementary mathematics methods course. Then, when PSTs become practicing teachers, they will continue on that trajectory as they
learn from their generative teaching practices. Defining the end of the trajectory entails studying teachers who are already there. Figure 1 represents a curriculum user trajectory.

![Figure 1: Curriculum User Trajectory](image)

Figure 1: Curriculum User Trajectory

I am not saying that teachers are ever finished with learning, but instead, they become expert curriculum users - teachers who are able to perceive the intended meaning of, mobilize the potential of, and continually develop their pedagogy through curricular resources.

I have this conceptualization of what a classroom facilitated by an expert teacher looks like – one where students are engaged in interesting and cognitively demanding mathematics like the examples given above. The central question that I wanted to answer was, “What is the PDC of teachers who successfully facilitate classrooms where students are doing mathematics?” What resources do teachers with a high degree of PDC perceive as valuable to their instruction? How do they use those resources? How much of their teaching practice is of their own design and why? I was able to find some answers to these questions, which I describe in the remaining chapters.

**Overview of Chapters**

Chapter Two provides a detailed description for each of the four teachers – Kathy, Nancy, Olivia, and Violet. I structure each description around the teachers’
primary and secondary resources. The four teachers all had access to *Investigations* (TERC, 2008) and CGI professional development, but used those resources differently. For instance, Violet mobilized *Investigations* (TERC, 2008) as her primary resource, but Olivia did not use *Investigations* (TERC, 2008) at all due to her perceptions of the curriculum series. My teacher descriptions also provide insight into other factors that shaped each of the teachers’ practices. For instance, during her early career experiences, Kathy collaborated with professors from State University on the Developmental Activities Program. Olivia had an interesting collaboration with the other two teachers on the second grade team. I mention these factors, because in the context of understanding each teacher’s PDC, they have substantial bearing.

Chapter Three describes how the teachers mobilized four different types of progressions: unit, series of instructional activities, number choices, and student solutions. Different curricular resources provided support for one or more progression types. For instance, *Investigations* (TERC, 2008) provided support for Violet for unit and series of instructional activities progressions, but not for number choice and student solution progressions. She was supported, however, by her CGI knowledge for these two progression types. These results add to the existing research in mathematics education around the notion of mobilizing hypothetical learning trajectories to provide rationales for designing instruction (e.g. Clements & Sarama, 2004; Fuson, Carroll, & Drueck, 2000; Simon, 1995).

In Chapter Four, I report findings on what kind of knowledge the teachers had of students as well as how the four teachers mobilized student resources to design
instruction. Grounding that study in PDC and other studies focusing on teachers
learning how to use children’s mathematical thinking for instructional decisions, I
found that teachers could detail strategies and were able to distinguish between details
that speak to a child’s conceptual understanding and details that speak to other
mathematical practices. Furthermore, the teachers possessed individual knowledge of
students including knowledge of the strategies students tended to use as well as
individuals’ dispositions, and mobilized individual knowledge of students to make
instructional decisions. Those instructional decisions are based on moving students
along a student solution progression. Student resources were also mobilized to
introduce instructional topics, to develop PDC, and to take on roles traditionally
reserved for teachers.

Chapter Five describes how the four teachers mobilized number choices – one
type of progression identified in Chapter Three. Choosing number choices in problem
posing is a knowledge base that has received little, if any, attention. By analyzing
problems the four teachers posed, I found that the teachers mobilized number choices
in seven different ways: to address mathematical content, to encourage a particular
strategy, to provide differentiation, to develop relational thinking, to respond to
children’s mathematical thinking, for assessment, and to provide an entry point. The
teachers mobilized number choices in these ways to move students along the four
different types of progressions.

Chapter Six is a conclusion chapter. In that chapter, I return to the notion of PDC
and discuss how my research contributes to the PDC construct for teaching elementary
mathematics. I then make connections between this dissertation research and another research interest – teacher learning about and from Standards-based curriculum materials. I end the chapter with a discussion about how this research has affected my teaching and possible future research endeavors. Future research endeavors include investigating expert elementary teachers’ subject matter knowledge, accessibility of expert teachers’ practices to pre-service teachers, and designing and assessing the elementary mathematics methods course.
CHAPTER 2. THE TEACHERS

Introduction

In this chapter, I provide an extensive description of each of the four teachers involved with this study: Kathy, Nancy, Olivia, and Violet. I start each description with some background information, and then move into describing the teachers’ primary and secondary resources. They each had access to two supportive resources – Investigations (TERC, 2008) and Cognitively Guided Instruction (CGI) professional development. Investigations (TERC, 2008) is an innovative reform-based elementary mathematics curriculum. Stein and Kaufman (2010) describe Investigations (TERC, 2008) as a high-demand/high-support curriculum series, which means that it provides cognitively demanding tasks for students and numerous supports for teachers to implement those tasks. CGI professional development is an approach to instruction that focuses on children’s mathematical thinking. Much research has been generated around the effectiveness of CGI (e.g., Carpenter, Fennema, & Franke, 1996; Carpenter et al., 1989). Given that the four teachers had access to these two resources, they were highly supported both in terms of curriculum materials and in professional development. Even with access to these same supportive resources, however, each teacher is unique in how she perceived and mobilized those resources, which is evident in the following descriptions.

Nancy

Nancy had been teaching for 16 years at the time of the study. She started her teaching career at an elementary school in which she was provided with a traditional
textbook to support her mathematics instruction. After two years there, Nancy took a position at a school in her present district, Lakefront, where she stayed for three years.

In 1998, Nancy transferred within the Lakefront district to her current school, Washington Elementary, where she taught a 2nd and 3rd grade multi-age classroom. At Washington Elementary, Nancy had access to *Investigations* (TERC, 2008). She used *Investigations* (TERC, 2008) almost exclusively until 2004 when she began participating in CGI professional development.

**Primary Resources**

Nancy used two primary curricular resources: *Investigations* (TERC, 2008) and CGI professional development. During each observation, Nancy either facilitated a lesson from *Investigations* (TERC, 2008) or posed a problem informed by the CGI problem-type framework (Carpenter et al., 1999). CGI professional development also supported Nancy in her questioning of students and facilitation of whole-group discussions during her facilitation of *Investigations* (TERC, 2008) lessons and problem posing. Because Nancy’s use of these two resources was so connected, I contend that both are Nancy’s primary resources. I discuss Nancy’s mobilization of these resources individually.

**Investigations.**

Nancy began using *Investigations* (TERC, 2008) when she started teaching at Washington Elementary. Nancy perceived *Investigations* (TERC, 2008) as an excellent resource comprised of open-ended math tasks that afford multiple access points in her multi-age classroom and integrated problem solving. Furthermore, Nancy thought the
curriculum guide was teacher-friendly, provided valuable information for the teaching
of mathematics, offered great discussion questions that probed students’ thinking, and
helped her make an easy transition to CGI due to its focus on children’s solutions

One, the way it was set up in the book. It required me to do a lot of reading in
order to teach the lesson, which was good because like, it had sections for the
teacher that then told me information about what kids might do. What I would
see and just some of the mathematical why this is happening and that piece of it
helped me. And then, it was pretty open-ended, so kids had different ways to
solve it.

Above, Nancy talked about how she mobilized *Investigations* (TERC, 2008) - by
thoroughly reading the teacher instructions when planning instruction. The section that
provides information on how students might engage with the task was particularly
useful.

Nancy also used the examples of students’ solutions provided in the curriculum
materials to help her facilitate classroom discussions. In one instance, students were
asked to make as many possible Hexagon Cookies (TERC, 2008) as they could think of
by placing smaller pattern blocks on the larger yellow hexagon – the cookie. Although
there was no pattern block that represented ¼, the curriculum materials mentioned
that a student may bring it up, and if a student does not, suggested a way for the teacher
to bring it up. Therefore, Nancy was ready to facilitate a discussion around ¼ when a
student did happen to create a cookie using ¼. Nancy talked about that experience:

I would have never thought about talking about fourths with this lesson. And
that it brought that up. And then, my kids had brought it up without me even
talking about it... So, I just think it does a lot of that foreseeing of what might
happen. I think the more that we can foresee that, the better we are in the
classroom on the spot.
In other words, Nancy did not believe that she would have been prepared to appropriately address a student’s solution that included ¼. By seeing that possible solution and suggestions for how to address that solution, reading the curriculum materials prepared Nancy to facilitate an important discussion. Nancy talked further about her experience using *Investigations* (TERC, 2008):

> ... Or when a child brings up something maybe I didn’t really understand it fully, you know, right away exactly what their thinking was, but talking to them, or questioning them then did allow me to understand what they were doing or to think about the questions. The questions in the teacher’s guide or whatever were very helpful to me to even what type of questions even to ask. Um, because I think I started more with the how did you get that? How did you think about that? Where before it was more - what was your answer? And that’s changed a lot. I mean that whole *Investigations* changed that piece for me. I became more concerned about the child’s processing, rather than them being right or wrong.

Another feature that was particularly useful to Nancy were the discussion questions. They helped Nancy formulate what questions to ask students to understand their thinking processes. The questions changed Nancy’s focus from determining if students were right or wrong to determining what children’s processes were instead.

**CGI Professional Development.**

Several years after Nancy began using *Investigations* (TERC, 2008), she started participating in CGI professional development (Carpenter et al., 1999). Nancy attributed her experiences with *Investigations* (TERC, 2008) for her easy transition into using CGI frameworks and principles:

> It was not a big aha to me that kids aren’t going to solve problems the same way. It was not a big aha to me that there are different types of problems. Like, I had seen join-change unknown problems in Investigations... The framework piece just gave me organization to what I had experienced.
Nancy’s experiences with CGI built on her knowledge about problems and children’s solution strategies by giving her a framework for each. Below, Nancy talked about how she mobilized CGI strategies:

I think that CGI has helped me. It has helped me to listen to kids better - to take the time to listen to kids. It has also given me kind of some understanding, just reinforcing things that I knew is right. Not all kids are [going to] know every multiplication fact when they leave third grade, and that’s OK. And it’s really keyed me in on those types of things is that there’s a reason I don’t give timed tests and here’s why. You know, it’s given me that background like it’s not good for kids because. And I feel comfortable talking to parents about that when I’m questioned. I mean I feel like I have an understanding. I knew it wasn’t right. Before and I didn’t do it, but I could never always justify why it’s not good. Now, I feel like I know the why.

Along with CGI professional development prompting Nancy to listen to kids better, there is also evidence in this quote that her experience with CGI provided her with explanations as to why she had certain beliefs about mathematics. Before CGI, Nancy thought that giving timed tests was not a good approach to mathematics instruction, but she did not have the information she needed to support that belief. CGI gave her that information and helped her justify her beliefs. Furthermore, Nancy could communicate and justify her approach to mathematics instruction to parents.

Nancy further mobilized CGI strategies by making adaptations to Investigations (TERC, 2008) lessons, creating series of true/false and open-number sentences to introduce or reinforce mathematical concepts in opening tasks, guiding lesson planning, and focusing on children’s mathematical thinking. To illustrate the kinds of adaptations Nancy made to Investigations (TERC, 2008), I describe one observed lesson. The written lesson asked students to represent several numbers (46, 56, 66, and 86) with multiple representations – manipulatives (strips of 10 and single stickers), number
symbols, and equations (e.g. $40 + 6 = 46$). Then, students were to solve six contextualized word problems. Instead of having students work all six problems, Nancy posed just one problem, but provided multiple number choices – a typical differentiation strategy drawn from CGI. Then, Nancy asked students to share their solutions and facilitated a discussion around the various solutions and made connections between solutions. The discussion was not suggested in the curriculum, but a discussion around student solutions is a common routine in Nancy’s classroom due to her experiences with CGI. In summary, Nancy adapted *Investigations* (TERC, 2008) by focusing on one problem per lesson based on her students’ needs, offering alternate number choices also based on her students’ needs, and facilitating an in-depth discussion of solution strategies.

The second mobilization of CGI strategies was Nancy’s creation of number sentence sequences. Nancy mobilized the true/false and open-number sentences she learned about in CGI professional development by using them as opening activities in her daily instruction on days she was not facilitating an opening activity from *Investigations* (TERC, 2008). Often times, Nancy developed a sequence of true/false sentences to address the same concept that was addressed in the main lesson. For example, during a unit on fractions, Nancy asked her students if $\frac{1}{2} > \frac{1}{4}$ was true or false and facilitated a discussion around how to prove if that statement was true or false. Then, based on the children’s thinking that was shared, Nancy posed subsequent inequalities or equations. For instance, one student asked if $\frac{1}{2}$ was equal to $\frac{3}{6}$. Nancy posed that equation, then asked students to justify their answer.
The third way in which Nancy mobilized her CGI training was to guide lesson planning. For much of her lesson planning, Nancy mobilized the CGI problem-type framework (See Carpenter et al., 1999) to guide the order in which she posed problems. Nancy laid out how she mobilized that framework in the excerpt below:

I usually start with like join-result, separate-result unknown, part-part-whole, and then the join-change unknown is one problem though I give early in the year because it tells me, um, like if they can keep track of that second set of numbers.... So, I kind of give that as an indicator problem. But then moving on later to multiplication and then measurement division before partitive division. And then last is like start unknowns, or compare referent unknown. Those are harder.

From the above excerpt, it is evident that Nancy has a clear plan for her problem posing using the CGI framework. How long Nancy spent on each problem type was determined by the needs of her students. Some years, Nancy needed to spend little time on a certain problem type and in other years, she needed to spend more time depending on how long it took for students to understand and master a particular concept. When using *Investigations* (TERC, 2008), Nancy would choose the problems that her students needed or would alter the problem type.

As mentioned in an earlier excerpt, Nancy felt that her experiences in CGI prompted her to listen to kids better, and caused her to focus her questioning techniques on students' problem-solving processes. Nancy used these questioning techniques during student exploration time and facilitation of whole-group discussions.

During a number of the day task, Amber generated the following series of equations:

\[
\begin{align*}
100 - 75 &= 25 \\
101 - 74 &= 25 \\
102 - 73 &= 25 \\
103 - 72 &= 25
\end{align*}
\]
Even though this student had a misconception, Nancy perceived Amber’s thinking as an opportunity to discuss an important relationship – when the minuend is increased by one, you also have to increase the subtrahend by one – and facilitated a whole-group discussion around it. Below are several lines from the transcript:

Nancy: Can I stop you for a second? Boys and girls, I want you to think about this because I think [Amber] has a good pattern that she’s thinking about in her head. We know that numbers make a pattern sometimes…. She said 74. If she was at 100 and took away 75, she knew it was 25, but let’s think. What happens if she only takes away 74?

Students: 24, 26

Nancy: This is really that thing that we’ve been talking about is she’s trying to compensate, isn’t she? She’s trying to know if she knows this fact… So now she’s trying to think if I add one more, how many do I have to take away? … She wants to have 25 for her answer, doesn’t she? Hmmm, so if she wants 25 for her answer…

Stacey: You need to take away 76, because you added the one, you need to add it to what you take away.

Nancy: Amber listen to what Stacey is saying. Stacey will you repeat that?

Stacey: If you added the 1 to 100, then you need to take it away.

Claude: You have to take it away. 100 + 1 = 101. 75 + 1 = 76.

Nancy: Ok, so if I’m using this as my base, what should it be? If I’m 1 here, then I have to add 1 to what I take away?

Stacey: Yeah

Claude: It’s sort of just like 101 take away 1 and then...

Nancy: Let’s see if that worked out. Amber, do you see why this one doesn’t work out? Does anyone have another way that you could prove to Amber that this isn’t right? (Gestures to 101 - 74 = 25.) That that didn’t work? How else could you prove it?

Nancy’s questioning technique allowed students to work through Amber’s misconception. First, Nancy asked, “if I add one more, how many do I have to take away?” This question prompted other students (Stacey and Claude) to contribute their thinking to the conversation. Then, Nancy voiced Stacey and Claude’s thoughts, asked Stacey to repeat her thinking, asked Amber if she understood, and finally asked
students to find another way to prove the that 101 – 24 does not equal 25. Nancy is using her students’ thinking and her questioning technique to help Amber correct her misconception instead of explaining Amber’s mistake herself.

**Conclusion**

Nancy began her reform-oriented teaching practice when she gained access to *Investigations* (TERC, 1998, 2008). She described her use of *Investigations* (TERC, 2008) as focusing on the features that prepared her to attend to children’s mathematical thinking – examples of student solutions and discussion questions. After using *Investigations* (TERC, 2008) for several years, Nancy participated in CGI professional development, which provided her with frameworks for problem-types and children’s strategies; and supported her in her adaptation of *Investigations* (TERC, 2008), creation of series of true/false and open-number sentences to introduce or reinforce mathematical concepts in opening tasks, guide lesson planning, and focus on children’s mathematical thinking.

**Violet**

At the time of the study, Violet, like Nancy, had been teaching for 16 years. She too, used a traditional textbook at the beginning of her career, but believed at the time it was not the best approach to teaching mathematics. Resources were unavailable, however, for Violet to teach mathematics in the ways she wanted to until she gained access to *Investigations* (TERC, 2008) in 1998. In 2007, Violet began participating in CGI professional development. Like Nancy, Violet mobilized *Investigations* (TERC, 2008) and CGI consistently to design instruction resulting in many similarities in their
teaching practices. Unlike Nancy, Violet considered and mobilized *Investigations* (TERC, 2008) as her primary resource. Therefore, there were distinct differences in how the two teachers designed instruction. At the time of the study, Violet taught a 4th and 5th grade multi-age classroom.

**Primary Curricular Resource**

Violet began using *Investigations* (TERC, 2008) after it was suggested to her at a conference:

> We happened to go to a Marilyn Burns conference, and one of the instructors when we broke off into groups was talking about TERC. We were talking about our frustrations and things like that. She said, “You need to use this. This is a great series.” We started *Investigations* and buying it on our own... We started using it, and to be honest, I was not used to teaching that way.

Violet talked about how she began using *Investigations*:

> I came from a very traditional school, which I didn’t agree with, but I didn’t know any better. And so, I literally would read exactly what they would say - teacher says this. And that’s how I started. And after a while, you set that aside and you get what they’re trying to do, but I started using that and found that the kids were really thinking about what they were doing and using strategies that I hadn’t even thought about before in my whole life. And as a teacher, I was like why shouldn’t they subtract this way? This makes perfect sense, so I had these aha moments like this child, and I’m thinking of one in particular. She subtracted from left to right. She just came up with it... It was one of those aha moments where this is how it should be. There is not one way. Then, I just kind of bought into *Investigations*, because they took concepts at such a deep level. It was so hands-on. They allowed kids to share the way they were thinking about math.

Because Violet had no experiences with teaching mathematics in a reform-oriented manner, she followed the book closely. Violet stated, “I literally would read exactly what they would say.” After a while, however, Violet understood the intentions of the curriculum designers and no longer needed to follow the guide so closely. Additionally, Violet began to have “aha moments” when students would solve problems in alternate,
nontraditional ways prompting her to learn mathematics differently. The alternate algorithms made sense to Violet. Violet “bought into” *Investigations* (TERC, 2008) because it explored concepts at a deep level, was hands-on, and allowed students to share their thinking. Violet spoke further about why she bought into *Investigations* (TERC, 2008):

Well, it [*Investigations*] ties in perfectly with this whole philosophy here at our school... When I looked at our district objectives at that time, it was just listed as math objectives. There were four pages for fourth grade and six for fifth. How do I teach without teaching it procedurally? And having kids understand and being able to apply... We do projects here too. How can we have them apply what they’re learning to real-life situations and projects? It was really great, rich, lessons. They were meaningful. The kids were excelling. It just worked here.

Violet found that her district gave no support for teaching the required objectives. The objectives were simply listed and gave no guidance in how to teach them in a way that students would understand conceptually and be able to apply them. *Investigations* (TERC, 2008) also fit into the philosophy of Violet’s school, which entailed project-based instruction. The lessons were rich and meaningful, and most importantly, the students “were excelling.”

Violet provided compelling reasons as to why she used *Investigations* (TERC, 2008). She obviously perceived *Investigations* (TERC, 2008) as an important resource in her design of classroom instruction. More important, however, is how Violet used *Investigations* (TERC, 2008). Violet spoke about her use:

I usually use the *Investigations* series to kind of guide my instruction, but I also pull into it... Sometimes, I'll start with a CGI problem that is usually tied into the Investigations lesson. For yesterday, for example, I knew my focus was going to be adding fractions. I usually start by writing a problem just to kind of see the kinds of strategies they’re going to use. Then, from there, I make some decisions... What do they need? ... In general, I use that book.
What is most notable about Violet’s use of *Investigations* (TERC, 2008) is how she mobilized the curriculum’s “Math Focus Points” to “guide” her instruction. The focus points are a list of targeted learning goals for each unit or part of unit. For instance, Violet facilitated a 4th grade fraction unit titled, “Fraction Cards and Decimal Squares” that entailed the following focus points for “understanding the meaning of fractions and decimal fractions” (TERC-b, 2008, p. 10):

- Finding fractional parts of a rectangular area
- Finding fractional parts of a group
- Interpreting the meaning of the numerator and the denominator of a fraction
- Writing, reading, and applying fractional notation
- Representing fractions greater than 1
- Identifying everyday uses of fractions and decimals
- Reading and writing tenths and hundredths
- Representing tenths and hundredths as parts of an area

There are also focus points for “comparing the values of fractions and decimal fractions” as well as “using representations to add rational numbers” (TERC-b, 2008, p. 11-12). Below, Violet talked about how she mobilized the focus points:

**Tonia:** So, how did you have a sense of those goals or concepts within fractions?

**Violet:** Those are within the TERC. I will use that to guide instruction... To start with, it’s started with just representing halves, fourths, eighths. And then it moved to, um, fractions of a group... Then, I had pulled in other resources as needed. That’s how I use TERC. Just the basic concepts, I follow that.

**Tonia:** The math focus points?

**Violet:** Yeah, exactly... Like within fractions of a group, I wanted them to see, do they see it as division? And giving lots of experiences to see how that is really dividing. Using whatever strategy they use to divide. So, that was kind of my focus for that particular group. Then, it moves into adding fractions and then ordering fractions. So, those are like the focus points and I follow those. And then pull in additional resources. Sometimes I use TERC exactly the way it’s set up. Sometimes I don’t, but it all depends on what I see the kids doing in class.

**Tonia:** So if there’s like 15 focus points and you see on the first problem that your kids have mastered the first four, then do you move on?
**Violet:** I skip. Absolutely, there’s a group of kids here that when I meet with them on the floor, I’ve jumped to the fifth grade book and I go ok, we’re working on addition of fractions, let’s go on to that focus in this book.

Violet used the focus points to determine the topics of instruction. If students had mastered certain focus points, then she skipped those focus points. If students needed to spend more time on a focus point than *Investigations* (TERC, 2008) allowed, Violet would pull in different resources to provide students with more experiences with that particular concept. Violet also discussed using the 5th grade edition of *Investigations* (TERC, 2008) with a small group of students to work on adding fractions, because she determined that they needed more challenging tasks for that particular focus point.

Violet’s mobilization of the focus points also occurred during large group discussions. On several occasions, I observed Violet facilitating mini-lessons on mathematical concepts that came up when students were sharing their solution methods. One day, Violet posed the Baseball Card Problem:

**Baseball Card Problem**
Dustin has ____ baseball cards. He gives \(\frac{1}{4}\) to his friend. How many baseball cards did he give to his friend?

8  24  44  60  100  144

One student, Liev, used the following strategy:

**Liev’s strategy for finding \(\frac{1}{4}\) of 8 baseball cards**

\[
\begin{align*}
\frac{1}{2} & \quad \frac{1}{2} \\
\frac{1}{2} & \quad \frac{1}{2}
\end{align*}
\]

\(\frac{1}{2} + \frac{1}{2} = 1\)

1 + 1 = 2
After Liev explained his strategy, Violet took that opportunity to ask students if $4/2$ was equal to 2 and facilitated a discussion around that question. I asked her about that episode:

Violet: So, he was adding up those halves to four halves, and I wanted him to know... I don't know if they've seen those improper fractions before. That's new to them. So four halves, is that the same as two? How do you know that? So they can see two representations of the same number. Four halves is the same.

Tonia: So, are improper fractions a focal point for Investigations?

Violet: I think they touch on it here and there, but it's one of those moments like here's a point to talk about it. I don't want to teach improper fractions, here's how we convert improper fractions to, you know, mixed numbers. Here's how we... whatever. Here's a perfect place to pull it in, because it's a natural place to discuss it.

Violet explained that large group discussion is an ideal opportunity to discuss other focus points. She was able to make the most of those opportunities based on her extensive knowledge of the focus points:

I'm going to read through all of that, and I'm going to look at their focal points, and I'm going to get those in my mind. And so when moments like these come up, you know, I can kind of jump on them. But I have so much experience with that textbook, I just kind of know them like the back of my hand.

Secondary Resource

CGI principles and strategies were incorporated in Violet's teaching practice in three primary ways – as opening tasks, to assess students, and to facilitate sharing sessions. Nancy also used CGI in these ways, but differed from Violet in that many of Nancy's main lessons entailed posing a CGI problem. Violet posed CGI problems as an opening task to a unit or lesson, but then generally facilitated a lesson from Investigations (TERC, 2008). When talking about starting a new unit, Violet stated, "I usually start by writing a problem just to kind of see the kinds of strategies they're
going to use. Then, from there, I make some decisions...” Violet would start a new unit with a CGI problem as in the Baseball Card Problem presented earlier. Other opening tasks included true/false number sentences and number of the day tasks. Each of the openings tasks reflected the concepts being explored in the main Investigations (TERC, 2008) lesson. Sometimes Violet’s true/false number sentences would focus on relational thinking (Carpenter, Franke, & Levi; 2003) as in the following series of two number sentences:

- \( \frac{1}{4} \) of 12 < \( \frac{1}{2} \) of 12
- \( \frac{1}{4} \) of 24 > \( \frac{1}{4} \) of 44

In the first number sentence, Violet hoped that students would use the relationship between \( \frac{1}{4} \) and \( \frac{1}{2} \) to determine that the number sentence was true. Relational thinking is a concept explored in CGI professional development, but identifying relationships between unit fractions where one unit fraction is a multiple of another is also a math focus point for comparing fractions as per Investigations (TERC, 2008). In the second number sentence, Violet hoped that students would notice that 24 is less than 44 and use that knowledge to determine that the number sentence is false instead of computing for both sides.

Violet stated that she sometimes begins a unit with a problem to “see the kinds of strategies they're [students] going to use.” Knowledge about students’ solution strategies from CGI allowed Violet to assess those strategies. Then, based on her assessment of students’ strategies, Violet made decisions on how to address the focus points in Investigations (TERC, 2008). In other words, the nature of students’ solutions informed Violet on how much time she needed to spend on each of the focus points.
A large part of Violet’s practice was questioning students to assess their understandings. When talking about her role as students were solving problems, Violet had this to say:

My role is just to go around and um. First of all, the purpose was to see how they would attempt... what kind of strategies they would use to attempt this problem, what misconceptions they have about these kinds of problems. So my role is just to go around talk to them, question them. I want more information from them. You just can’t see that on their paper. So, I’m going to question - why did you write that? Why did you write it that way? What were you thinking? Um, can you explain this? It’s just more for me, more information for me, um, on the way that they’re thinking.

Like Nancy, Violet’s questioning technique started to develop when she began using *Investigations* (TERC, 2008). As mentioned earlier, Violet would pose the exact questions written in the text. Violet’s questioning technique further developed after participating in CGI:

CGI has helped a lot with the questioning. It has actually a lot and how to question kids, and just looking at different strategies and how those strategies are related and how to record that. It’s helped a lot in recording.

This questioning process allowed Violet to gain substantial knowledge of her students and to design individual instruction. For instance, a few students used direct modeling for the above Baseball Card Problem. When asked about two of those students, Violet had this to say:

Response for the first direct modeler - Stacey
I want her to stop direct modeling. She’s beyond direct modeling. She doesn’t need to use cubes to solve this problem. She’s very comfortable... I think she solved the rest of them using that strategy right there (referring to video clip of Stacey using multiplication)... I know she has a really good understanding of division, and she uses multiplying to divide. And so, I know she knew that. And she doesn’t need cubes to divide. So that’s why I wanted to say, how else could you do it?
Response for the second direct modeler - Leslie
I need to work on division with her. I don’t think she’s even ready to move beyond direct modeling at this point especially with division and to see that relationship. I’m not sure she really understands the one-fourth part. That’s really abstract. Um, the sets are hard for her. So, lots of experiences with this, and lots of use with direct modeling. That’s what she needs.

Violet knew that Stacey is “beyond direct modeling” and is able to use a more sophisticated strategy, which would demonstrate a greater understanding of that particular focal point – finding parts of a set. Violet prompted Stacey to work where she is capable of by asking Stacey to solve the problem in another way. Leslie, on the other hand, needed to continue direct modeling until she had greater mastery of finding parts of a set. Violet hypothesized that Leslie would need to have several experiences with this concept before she could move beyond direct modeling.

Conclusion

Violet had a traditional teaching practice until she gained access to *Investigations* in 1999, which she has used primarily and almost exclusively since. Violet felt that *Investigations* (TERC; 1998, 2008) was responsible for her shift to a more reform-oriented teaching philosophy. At the time of the study, Violet had participated in three years of CGI professional development, which prompted her to adapt *Investigations* (TERC, 2008) lessons using her CGI knowledge. Adaptations included replacing opening routines in *Investigations* with contextualized problems or series of true/false and open number sentences, questioning students extensively, and facilitating large-group discussion around children’s solutions strategies daily.
Olivia

At the time of the study, Olivia had been teaching for 13 years. All of her teaching experience was within the Lakefront District, but at multiple grade levels. She had been teaching 2nd grade for the last eight years. During her early career experiences, Olivia had a more traditional teaching practice that focused on standard algorithms. Even though she knew that standard algorithms caused confusion for her students, Olivia did not have the support to change her practice until she participated in CGI professional development in 2004. Olivia also facilitated CGI professional development for other teachers in the district.

Primary Curricular Resource

Olivia’s primary curricular resource was her CGI professional development. She also used a few secondary resources, (e.g. Maintenance math, Kathy Richardson & Marilyn Burns books, Number of the Day) which had been part of her practice for several years, but made adaptations to these secondary resources based on her CGI knowledge. Olivia designed classroom activities by first considering the learning objectives set forth by the Lakefront District. She stated, “The geometry lessons that I’ve been planning, and this is what I would do with any topic or area of study in math, I would look at our objectives first.” After choosing the learning objective, Olivia then either wrote a CGI problem or chose an activity from her secondary resources that met that particular objective, but also gave tremendous consideration to student need.

During the remainder of this section, I provide a snapshot of Olivia’s teaching practice and how her CGI knowledge has shaped it in two primary ways – to make
adaptations to her secondary resources and in her problem posing. Of course, Olivia used her CGI knowledge in other ways, like assessing students and facilitating sharing sessions. However, I have already talked about those aspects of a CGI teaching practice when describing Nancy and Violet’s teaching practices. Olivia’s teaching practice was unique to the other two teachers due to her use of secondary resources and how she made adaptations to them; and also in how she posed problems due to her collaboration with her colleagues, her integration of children’s literature, and her nonuse of *Investigations* (TERC, 2008).

**Series of five equations.**

Olivia began each day by posing a series of five open-number sentences like the ones posed below:

1. $25 + \Box = 36$
2. $48 = \Box + 37$
3. $19 \div 2 = \Box$
4. $\Box = 9 \div 2$
5. $14 \div 2 = \Box$

The purpose for these equations was for reinforcement or review. Olivia stated, “usually things we’ve already done before, or what we’re working on. And like today, they were a review.” Olivia described how this teaching practice came about:

A long time ago when I first started, our district had like what they called a maintenance math in a packet. It might have been from [Harry Whitehead], that guy from [State University]. That professor from [State University], he had a maintenance math thing. Some teachers would have the kids turn them in everyday. It was just kind of a review, but when we started taking CGI training, it was like, why do they have to be those questions? Why can’t we do what’s right with our kids? So, that’s kind of why I still do it. I like that part, but I wanted it to be adapted towards... I just make it every morning or before I leave at night.
[Harry Whitehead Maintenance Math] was a program that provided students with daily review in the form of a workbook. Olivia liked that the equations from Harry Whitehead provided a review, but since attending CGI, she found that the equations in the workbook did not meet her students’ needs nor fit into what her students were exploring in class. Therefore, she continued the practice of daily review, but adapted the equations to better fit her students. Olivia described her rationale for the above equations:

**Olivia:** Oh, missing addends there, but I wanted to see if they would count up by ten, then one more on that one.

**Tonia:** Which one? The first one?

**Olivia:** Yep, I think it’s 25 + 11 = 36, yeah, plus blank. The second one, the same thing, but I just changed the two around. I wanted to see if they were going to count up by ten.... Then, of course, I did that, because we were doing fractions.

**Tonia:** 19 divided by 2, ok

**Olivia:** And 9 divided by 2, and 14 divided by 2.... It had to have been at the beginning [of fractions] if this is the first videotape.

**Tonia:** This is the very first time I came.

**Olivia:** We divided things in two first - divided things in half. Cause we thought two was easy - dividing things into two groups. It’s interesting how much thought you put into something when you think about it. There’s a reason why you’re doing this stuff, or there should be.

As the above transcript illustrates, Olivia wrote her equations based on specific learning objectives. In this case, her objectives were counting up by ten, then adding one more, and dividing objects in half. Olivia also varied the position of the equal sign based on students’ tendencies to develop misconceptions about the equal sign, which she learned about in CGI professional development.

**Number of the day.**

In two of the six observations, Olivia facilitated a Number of the Day (NOTD) activity before her main lesson, and I know from interview data that it is an activity that
she used often. Like the Harry Whitehead Maintenance Math, this is a teaching strategy Olivia has used for years, but the nature of the strategy has changed since her CGI training. Before CGI, Olivia would ask for students’ expressions, record them on the board, and then move on:

When I first did number of the day, ok, what was your number sentence? Oh great, and I would write it up there. Then I’d go to the next kid (laughter). And it wasn't meaningful. I mean it was an attempt to be. But now, it’s... When I call on kids, you talk about what you did, and you’re making connections between the ones that they’re doing. You get mini-lessons out of what the kids are doing there. Like the parenthesis was a huge thing that came out of number of the day. Now, they all want to use it.

Due to CGI, Olivia’s facilitation of NOTD has become more “meaningful.” In the above excerpt, Olivia indicated that she now makes connections between students’ expressions and facilitates mini-lessons when appropriate. The transcript below illustrates one of these mini-lessons when NOTD was ½:

Student: \( \frac{3}{4} \)
Natalie: \( \frac{3}{4} \)? Ok.
Student: minus \( \frac{1}{4} \) equals \( \frac{1}{2} \)
Natalie: How did you know that?
Student: I just kind of knew like... I drew a box in four.
Natalie: (Draws a rectangle split into four parts.) Like this?
Student: And then 3 (inaudible)
Natalie: (Shades in three of the boxes) Ok
Student: I knew \( \frac{1}{2} \) was two of the boxes, so I just took \( \frac{1}{4} \) of it away.
Natalie: So, you had 3 of the fourths, and you took one of the fourths away, and you have \( \frac{1}{2} \) left over. Could we write that a different way where our answer is something different...?
Student: 2/4
Natalie: Aahhh, is \( \frac{1}{2} \) the same as 2/4?
Student: Yes
Natalie: Is it? It is. So, I’m going to write this one next, Allie. \( \frac{3}{4} - \frac{1}{4} \) equals... Did you say 2/4 Karen?
Karen: Yeah
One student shared her expression of $\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$. Olivia asked this student to share how she knew this expression to be true. During the student’s explanation, Olivia drew a model to help illustrate the explanation. Then, she asked students for an additional representation intending for students to generate an equivalent fraction for $\frac{1}{2}$.

Also before her CGI training, Natalie would choose random numbers; not necessarily numbers that had connections to concepts she and her students were exploring in class. The two times I observed NOTD were during a fraction unit. The above transcript comes from the second time Olivia posed $\frac{1}{2}$ as the NOTD. The first time she posed $\frac{1}{2}$, students were unable to generate many expressions. After having more experiences with the number $\frac{1}{2}$, Olivia decided to pose $\frac{1}{2}$ again. The second time, students were able to generate many more expressions. The other number for NOTD was 75 and students had to include at least one fraction in their expressions. Olivia chose 75 because her students had considerable experiences with 75 providing them with an entry point:

I thought they know a lot about 75. They know about 10s and 5. They know about 5s, 10s, and 1s. But they also know about 25s and 50s. I just thought they knew a lot about that number as a whole, that I didn't think it would be hard for them to add a fraction in there... And you know, the relationship with 75 and 100.

**Main Lesson.**

The main lesson each day usually consists of a contextualized problem due to Olivia’s CGI knowledge:

We write a lot of our own story problems. We don't usually get them from anywhere. We’ll write our own. We try to write them around, like if we’re studying... like I was studying bats at the beginning of the year. I know you asked about that. So, I would try to write a lot of problems that had to do with bats, or
we studied penguins. If those two weren’t studying it, it might just be a random one, or I'll just change it to fit my topic - like the weather or tornadoes.

Olivia and the other two second-grade teachers at her school wrote their own story problems. To choose the context of the problem, Olivia looked to topics of study in other subject areas – science and reading. Relating story problems to her topic of study was a big part of Olivia’s teaching practice. Below is an excerpt where Olivia talked about writing story problems around Tacky the Penguin books and spiders:

I remember doing them [part-part-whole problems] specifically just because we read a lot of Tacky books. And so, we talk about fish. He makes fish snow cones.... He makes these treats for the emperor to come - emperor penguin. So he makes fish snow cones and some other. And then it was like how many total treats? But there were this many total treats, how many whatever did he make? .... We found this many tarantulas, or so and so found this many tarantulas, so and so found this many brown recluse spiders. We found this many total spiders. How many brown-recluse spiders did we find?

In addition to the context, Olivia and her colleagues also considered what type of problem to pose based on their learning goals and the number choices:

We'll decide what kind of problem we'll want to write. At first, it's a little bit more of the easier problem types and then we'll work on more difficult. But we also plan our number choices. That's where the three of us sit down and we'll say what kind of problem do we want to write. You know, what's our goal. We'll decide the problem type. Then decide our number choices.

Below Olivia talked further about her and her colleague’s rationales for their number choices:

We look more at what kind of strategies we want the kids to use. Then our number choices would tell us if they are going to count up by tens. That's what we want them to do. Are they going to do that? Or do numbers close to 100. So, do they know like 64 + 36? Are they using that knowledge to get to 100? Or are they counting by 10s to get to 100?
When planning problems, Olivia considered units of study to generate a context for the problems and made connections across subject areas. After deciding the mathematical goal, Olivia and her colleagues decided which problem type that would address that particular goal, and then chose the number choices. Number choices were chosen based on specific learning goals and strategies they (Olivia and her colleagues) wanted to see their students using. Much consideration was given to which number choices would generate the type of strategies Olivia and her colleagues were hoping for.

In one observation, Olivia posed the Tornado Problem. It is a part-part-whole problem. Olivia liked to pose these types of problems, because they “tell you a lot about your kids, because you can't directly model that problem. I think it really pushes them to think outside the box - how am I going to solve this problem?” In posing part-part-whole problems, Olivia wanted to push her students to use more sophisticated strategies than direct modeling. In the Tornado Problem, Olivia provided several number choices to meet the wide-range of students in her classroom.

**Tornado Problem**

Last year the National Weather Service recorded ___ tornadoes in the United States. They recorded some tornadoes in other parts of the world. They record a total of ___ tornadoes. How many of the tornadoes were in other parts of the world?

(18, 28) (26, 48) (22, 75) (39, 81) (83, 150) (77, 168) (95, 194)
(101, 283) (156, 381) (274, 475)

**Natalie’s Rationale for the Tornado Problem**

Well, I think this was by 10, wasn’t it, the first one, the very first one? And 26 and 48, well, I was going to see if they would add 20 to start off with. Cause they’d get really close, 46, then 2 more... Yeah, and here (39, 81) if they would know their doubles - 40 + 40. So, the close numbers here. And this one (83, 150), just to see if they know 17 and then 50. Cause 100, 150, but just putting those two together, which is going over the 100. I thought [it] would be so hard for these kids, and it’s so easy. Cause they know this chunk and this chunk, and they put it together.
And even the kids that know the 83 + 17 is 100. Where some kids are still doing... They might do 83 + 10 is 93, then plus 7. And then their 50, or are they going to increment all the way up to 150. Or are they going to go 100 and that 50 on ... I’m not sure the rhyme or reason for that one [77, 168]. It might have just been a number over 100 again and a number under a hundred just trying to get a difference. And this one [95, 194] was to see plus 100 would be..., but it’s one less. Seeing if they could look at it without doing any calculating on their paper. That one [101, 283], cause I wanted to do something out of the 100s, but this is just over. So, they know 101 to 200 is 99. That’s huge if they can look at that and say that’s 99 plus their 83. What’s this one? 156 + 81... I’m hoping they’ll add up in big increments of 100. Plus 100, or they might go 44 gets me to 200 and then 81, so looking at that. And this one again [274, 475], 74 and 75 are just off by one to see what they would do with that.

One can see that the number choices increase in size and complexity. In the first number choice, the difference between the two numbers is ten. Differences between the numbers increase in size (with the exception of (39, 81)) until the last number choice, which had a difference of 201. The number choices increased in complexity in that students would have to use more sophisticated solution strategies to solve, which Olivia intended and hoped for. Olivia hoped that students will add on 10 for (18, 28); add on 20 then 2 more for (26, 48); use their knowledge of doubles and compensating for (39, 81); compensate for (95, 194), (101, 283) and (274, 475); and increment for (83, 150) and (156, 381). If the strategies that Olivia hoped for happened to be used by one or more of her students, she would make sure that strategy was shared (not taught by her) in a whole-group facilitation of strategies.

Collaboration

When Olivia talked about her planning process, she often referred to her colleagues - the two other second grade teachers at her school. The three of them worked in close collaboration with each other.
We collaborate, because we choose to. It is not mandatory. We meet every week once a week for sure. And like I said, we’ll meet just out in the hallway after this [a lesson]. We’ll talk about what our kids did and get ideas from each other. What’s really nice is [Jack] is really good. He’s a math major. So, he has that math mind that I don’t come by naturally. But [Leticia] and I are really good about making sure our problems fit what we’re talking about and making sure they are... You know, if we’re talking about this in our science, animals, and we tie in our problems...We balance each other out really well.

The three teachers chose to work with each other and found particular roles in their collaboration. As indicated in an above excerpt, they chose the learning goals together. Then, Olivia and Leticia’s primary role was to provide the context for the problem, while Jack’s primary role was to generate the number choices.

Jack lent his mathematical knowledge to his colleagues in other instances as well. In the next excerpt, Olivia talked about a time where she was confronted with a student question that she could not answer immediately. The question was, “Is 28 divided by 2 the same as 28 divided by ½?” Olivia described her thoughts:

The sad thing is, I was trying to find the best way to explain to the kids why they were not the same. The only thing I could think of was the “inverse and multiply” method which is what I memorized from my early days. Thus, it was a good reason to tell the kids that it is better to understand what they are doing then to simply memorize procedures.

When the question was posed, Olivia asked her students to go home, think about it, and maybe ask their parents. Then, Olivia asked Jack to help her:

I talked with one of my colleagues and he [Jack] had a good way to explain it to the kids, by giving ½ of a cookie to each kid. Then counting the number of kids that were given ½ pieces. Then we discovered that we could also think about it like cutting the 28 cupcakes in half and counting our halves. Needless to say, it was a great discussion and the kids actually got it!! Once I wrote the number sentence to go with the numbers, they were seeing that they were simply doubling the first number. They asked what would happen if we divided by fourths. Then we did it by thirds. I cannot believe what kids are capable of doing at this age and what I learn from them every day!
Several times during our interview process, Olivia talked about her limited mathematical knowledge as she did in the above excerpt when she said that she does not come by math naturally. When confronted with the question about if dividing by two is the same as dividing by half, Olivia did not know how to explain that concept. If Olivia only had access to her own mathematical knowledge, she could have only resorted to a memorized procedure to explain that dividing by two is not the same as dividing by half, which would have been inappropriate for second graders.

Olivia relied on Jack for mathematical content knowledge, but was also learning mathematical knowledge from Jack. For instance, in her rationale for the Tornado problem, it is evident that Olivia understood the mathematics behind the number choices. That is, she can look at the number choices and identify which strategy the three of them were aiming for. Furthermore, Olivia now understands the mathematics of dividing a number by $\frac{1}{2}$. She could present a model of that concept (giving students $\frac{1}{2}$ a cookie and then counting how many students) and knew that when you divide a whole number by $\frac{1}{2}$, you are doubling the whole number. Olivia was also able to transfer her knowledge of dividing by $\frac{1}{2}$ to dividing by $\frac{1}{4}$ and $\frac{1}{3}$ when her students asked about those fractions.

Another notable aspect of the dividing by $\frac{1}{2}$ lesson is that a student question became the lesson objective for the next day. Therefore, Olivia sometimes used student questions to determine which mathematical concepts were explored in class. In a geometry unit, students developed conjectures about the relationship between a polygon’s number of sides and the number of lines of symmetry. Some students had
noticed that equilateral triangles had three lines of symmetry and squares had four, and then thought the number of sides in a polygon and the number of lines of symmetry would be the same. A rectangle had two lines of symmetry because it had two pairs of equal sides. Their theory, however, was challenged when confronted with a pentagon with two right angles. Students attempted to fold the pentagon five different ways, but were unsuccessful. Then, Olivia asked if the pentagon had equal sides, which it did not. The question, “Does a pentagon with equal sides have five lines of symmetry” became the mathematical concept explored the next day. Olivia gave every student a cutout of a regular pentagon and let him or her explore that question.

Since CGI, Olivia found basing instruction off her students was easier than teaching out of textbook. She stated:

It’s [instruction] not because well the book said to. That’s why I think using CGI is so much easier than teaching out of a textbook. I just think that it’s easier, because you’re really looking at your kids and what they know going that way.

CGI provided Olivia with a continuum of strategies that students progress through as they solve problems. Olivia analyzed where her students were at on that continuum then designed instruction to progress them through that continuum like in the Tornado Problem. Like other CGI teachers, Olivia used students’ sharing of solutions to help other students progress through that continuum. She stated, “then, from the kids sharing, the other kids see it [solution method].”

Olivia’s Nonuse of Investigations

My study participants were chosen because of their CGI knowledge and their access to Investigations (TERC, 2008). Because Nancy, Violet and myself were so
attracted to this particular curriculum series, I assumed all my study participants would be as well, but this was not the case with Olivia. I knew that Olivia primarily used her CGI knowledge to design instruction, but in pre-observation discussions, I had asked her if she would be teaching from *Investigations* (TERC, 2008), as I wanted to understand how she would perceive and mobilize it. Olivia agreed that she would look at *Investigations* (TERC, 2008) in preparation to teach fractions and geometry, but she did not end up using it at all. As part of my interview process, I asked her to explain why:

I think I could use a lot of the things in *Investigations* if I took the time to look at it... I think there are different things that I could pull out. It’s just the time commitment. Do I want to? Cause I already had some ideas. Yeah, pulling out different things, and would that fit your objective better than what I already have?

Olivia already had an established practice that she felt meets her objectives, so pulling lessons from *Investigations* (TERC, 2008) would entail a time commitment that she was not willing to do. Below, Olivia talked further about why she did not use *Investigations* (TERC, 2008):

**Olivia:** I’m not saying you would dumb it [instruction] down any, but I just feel more, I don’t know, trapped.

**Tonia:** When you use a textbook?

**Olivia:** Yeah, I feel trapped or something. Like I have to do what it says.

**Tonia:** What do you think that [feeling trapped] comes from?

**Olivia:** Oh, the experience, because when I did use it, that was how it was used. That’s how it was when I first started teaching, which is sad. I feel like if I... I don’t know if it was my team that I worked on, because that’s how they did it. I just did it, because I was new and just thought... Not that that was my teaching style anyway. When you’re new, you feel like you need to grasp at things. What do I need to do just to learn this curriculum? But I think once you learn it, then you feel like you can kind of go out on your own. It’s kind of hard when everybody you teach with just teaches like that, and you’re new. And I felt like that’s how it was used. And a lot of people who are traditional do it that way.
That’s trapping to me... It’s more freeing now that I’ve taken CGI classes. I feel more like you don’t have to. I think it comes with experience too. Like knowing what kids can do, but having the mindset that kids are capable of a lot more than what you think they can do, but also knowing your curriculum well enough to know what your objectives are and where the kids need to be by the end of school year and feeling comfortable with that too I think.

In the first sentence in the above transcript, Olivia mentioned feeling trapped when using a curriculum. Feeling trapped first stemmed from teaching from a traditional curriculum when Olivia began her teaching career. She felt like she had to do what the textbook said and not make adaptations. Olivia’s colleagues at the time also taught in that manner, thus they did not provide support for other options. Since participating in CGI professional development, Olivia does not feel trapped any more– she does not have to stick to a curriculum. Furthermore, Olivia felt she could take her students further, because they are capable of doing a lot more than the learning objectives in a textbook.

I also asked Olivia to examine the fraction unit in *Investigations* (TERC, 2008) and explain her nonuse. The unit began by asking students to discuss what they know about $\frac{1}{2}$. Olivia explained her thoughts:

I’m looking at this and it talks about asking kids what they know about a half already. I just like... I don’t know why, but I thought I don’t really like that. I like how we didn’t ever say the word half. We didn’t talk about what a half was. We just talked about dividing something in two, and what do you call that? I liked hearing what the kids call those. They didn’t know what they were called. They didn’t know how to write a half. That told me a lot from the get go. You knew the kids who knew it, but you knew the kids who had no clue.

When Olivia started her own fraction unit, she gave a problem where students would have to divide something in two. In doing that, Olivia felt she gained important knowledge about her students that she would not have if she would have started the
unit with a discussion about halves. Below, Olivia talked about her students work on the problem she posed:

It was what they were calling the pieces. A lot of them would call that extra piece or the half, they would count it as one... They were dividing 15 by 2. So, they would have 7 wholes and then that 1. You could see their work. You could see the short stick. They said it was 8. It’s not that they didn’t know what to do. They understood that. They didn’t know what to call that piece. That’s why I like that, because it’s not the process that they don’t understand. They know that’s a piece. They don’t know what it’s called. I like being able to say, that’s just a real thing that happened and you can talk about it. It’s your teaching point. A lot of kids didn’t know what that was called; so then you talk about does anyone know what that’s called? Well, that’s called a half. Does anyone know how you write it? Yep, this is how you write it - one over two. Why do you call it one over two? It just seemed more... Here, you just say what do you know about a half, which is interesting. But a lot of kids know what half a cookie means... So, it just made me... I didn't want to do it. I didn't want to do it, because I saw this right from the beginning. I was like... I just liked how we did it, it really shows you where the misconceptions are right from the beginning.

By posing a problem, Olivia discovered that her students understood the process of dividing objects in half; it was that students did not know what to call the “short stick.” (Students would distribute tallies when solving measurement division problems.) Some students would call the short stick (tally) one instead of a half. Olivia was able to make several teaching points stemming from the students’ drawing of the “short stick” – what its called, how to write it, why it’s written one over two. If Olivia had implemented the Investigations (TERC, 2008) curriculum, those opportunities would have been missed. Furthermore, in her fraction problems, Olivia discovered where student misconceptions laid and was able to address them.

In another lesson, students were asked to fold pieces of paper to represent fractional parts. Olivia discussed why she did not want to implement that lesson:
Oh, it was the folding paper. I didn't want to do that either, because that's always how I taught it before. So, I didn't want to do the folding paper. I don't know. Maybe it would have been helpful. It was certain things that I'd had always done before. I just didn't want to do it... Maybe it brought back bad memories.

**Conclusion**

Olivia primarily mobilized her CGI knowledge to design instruction, which entailed generating contextualized problems with her colleagues and adapting secondary resources. Standards set forth by the Lakefront School District determined Olivia's goals for instruction. Olivia's collaboration with her colleagues provided her access to important content knowledge. *Investigations* (TERC, 2008) was not a resource used by Olivia, because she felt trapped when using it, as it reminded her of how she used to teach traditionally. Furthermore, Olivia felt that many teaching points would have been missed in her teaching of fractions if she had used lessons in *Investigations* (TERC, 2008).

**Kathy**

Kathy had been teaching in the Lakefront District since 1985 at multiple primary grade levels. She was unique from the other three teachers in that she had different early career experiences. The other three teachers all talked about how they taught in a traditional manner, because they “didn’t know any better.” Kathy, on the other hand, talked about using the Developmental Activities Program (DAP):

> Which is DAP, which was with a couple of professors from [State University]. When I first started teaching, I worked with them a lot. It's very Piagetian. So, um, from the very beginning, this very strong belief in knowing the progression that children learn things. And so, I feel like I have really tried to develop strong foundation in understanding how kids develop. Not only in place value, but how they develop in their number sense. How do they develop in their spatial relations?
Also different from the other teachers was that Kathy is and always has been confident in her mathematical ability. She grew up in a small town where she and her family grew and sold vegetables. Through that experience, Kathy learned to measure, count, sort, weigh, add, subtract, multiply, divide and use fractions in a real world context at an early age. Kathy also had been participating in CGI professional development since 2004 and facilitated sessions for other teachers.

**Primary Resource**

When talking about DAP, Kathy mentioned that she has always had a “strong belief in knowing the progression that children learn things.” This notion is what guided Kathy’s design of instruction. Therefore, I claim that Kathy’s knowledge of “progressions” was her primary resource:

> What I do is look at the progression that my kids go through and then I look at which activities are going to fit, um, into where that progression would be…. So like if I am knowing that my next thing is decomposing numbers and getting kids to decompose numbers, I probably have at my hand many different activities at hand that are ready that all of us kind of have.

In her mind, Kathy had a progression of mathematical concepts that her “kids go through.” One such concept was decomposing numbers. Once Kathy had determined which concept needed to be explored, she designed lessons using several secondary sources - CGI problems and other resources (e.g. teacher resource books authored by Kathy Richardson and Marilyn Burns; NCTM’s Navigation Series) to move students along the progression. How much time Kathy allotted for a concept depended on her students:
Then, I just look to see how quickly they're moving. Do I need to make things more difficult, more easy, more experiences, less experiences? So, I'll have a bank of things together, and sometimes I'll skip and sometimes I don't.

Through formal and informal assessments, Kathy determined how quickly, or not, students were mastering concepts. Students' mastery of concepts determined if Kathy needed to spend more or less time on a particular concept. Additionally, through her design of instruction, Kathy planted seeds for concepts or skills further down the progression to ensure instruction was successful:

... And if you wait until they have this skill before you plant the seed for the next skill one, you're going to be a big trouble. And those kids can't progress through. I kind of always look at it as a range, and then I look at my kids if I'm going to go in this direction and I'm not going in this direction.

I asked Kathy where the big ideas come from. In other words, how did she know which mathematical concepts to address with her instruction? She claimed the big ideas were something that she just knew, but could acquire from the Iowa Core:

The big ideas come from... I could refer to the Iowa Core Curriculum. A lot of those are embedded in my mind now just because I've been in this grade level for a while. If I wasn't; I'd probably go back and check the Iowa Core. I would probably check my grade-level expectations. I always look above those grade-level expectations too.

Several times during our interview process, Kathy spoke about how the lesson being discussed fit into the progression or how her number choices for a CGI problem illustrated a progression. For instance, Kathy adapted The Star Lesson from a Marilyn Burns resource book that involved having students draw as many stars, then tallies, as they could in a minute and count. Below Kathy talked about why this lesson was appropriate in February of that year:
We had done a lot of counting with groupings. Some children had some work with counting collections with counting by 2s and 5s and 10s, and had come to the conclusion that you can count by different structures and conserve the number and keep the number the same. I wanted to start giving kids much more of an experience with tens and ones as an opportunity to break numbers apart... With this class, this is the time of the year it took place in.

Before The Star Lesson, Kathy’s students had been counting by 2s, 5s, and 10s in counting collection activities. Through those activities, students developed the idea that “you can count by different structures and conserve the number” – one concept on Kathy’s progression. After students had developed understanding of that concept, students should move into experiences involving decomposing numbers into tens and ones. The Star Lesson provided one such experience. Another notable aspect of the above excerpt is that Kathy talked about that February was the “time of year it took place in” suggesting that in other years, students had come to understand number conservation at other times of the year depending on how quickly they moved along the progression.

Consider Kathy’s rationale for the Stegosaurus Problem given in January:

**Stegosaurus Problem**
Stegosaurus has two rows of plates on his back. Each row has 8 plates. How many plates are there altogether?
8   11   24   42   55

**Kathy’s Rationale for the Stegosaurus Problem**
You can kind of see the progression. So, the 8 and 8 is a known fact, which always just great for the kids to practice, which is one that is over 10.... 10 and 10 is pretty easy. Are they going to use what they know about 10 and 10 to solve 11 and 11? That one is just 20, 20, 40, 4, 4, 8 is pretty easy... Well, the next one is going over the hundred. Somewhat pretty easy, because 50 and 50, a lot of kids know that’s a 100... 5 and 5 is 10, and so it’s just having them go over a hundred... So, it’s going over, but it’s easy. If you had to consider an easy one, that would pretty much be an easy one. And actually 52 would probably be a little bit more complex, because they would be more apt to write 104 more
incorrectly than they would 110. Because they would write 104, they'll write 100 - 4, 1, 0, 0, 4. So kids who don't know, but 110, they will know enough not to do that. They don't know enough not to do that, which is kind of interesting I always find when kids go to write numbers.

In the “progression” of Kathy's number choices, several mathematical concepts were addressed. Kathy first mentioned that the number choice of 8 is a fact “over 10.” Of course, there are also facts with sums less than 10 and facts with sums more than 10. Kathy did not include a fact with a sum less than 10, because her students had mastered that concept. Next, Kathy talked about two-digit numbers (11, 24, and 42) with sums less than 100 and how her students would solve them. Next, is a number choice (55) with a sum over 100, but is “pretty easy” for her students to solve. Finally, Kathy spoke about a number choice (52) that would be harder for her students to solve, because her students could possibly write 104 incorrectly. Key defining points in this progression are numbers with sums below and over 10 and 100.

As with all the teachers in this study, Kathy facilitated CGI professional development for other teachers in Lakefront. That work involved helping teachers understand and develop progressions, which is something Kathy claimed teachers really struggle with:

You really have to, I think, give kids those opportunities. And I think that is what's hardest for teachers is knowing that progression. What is that piece, and how do you develop that? And I think teachers who are newer to using CGI or this kind of philosophy it’s frustrating, because it doesn't come as naturally to them. They really have to fight to figure it out, but you have to go through that as a teacher.

To help teachers with this process, Kathy spoke about how her and her fellow facilitators “plant that seed” with teachers:
So what we’re trying to do in our training is to really plant that seed with teachers that your numbers are very important. There is a progression. Not just in todays, but what that next day’s is going to look like. And what the day before looked like... You’re going to repeat some of those ideas, but you know, if they’re real proficient right here, the majority of your kids, how are you going to move them?

**Secondary Resources**

Kathy used a variety of secondary resources (e.g. CGI, NCTM’s Navigation Series, Kathy Richardson and Marilyn Burns resource books, and *Investigations* (TERC, 2008)). Activities were selected from these secondary resources based on the current big idea. Due to the access of CGI and *Investigations* (TERC, 2008) by all four teachers, I was most interested in these two curricular resources. Kathy mobilized CGI much like the other three teachers – opening tasks, problem posing, assessing students, and facilitating sharing sessions, but she also talked about using CGI to get to know her students’ personalities and help them overcome their lack of confidence:

**Tonia:** Tell me about Lynn. What do you think?

**Kathy:** She’s got a perky personality. She’s really got a lot of math ability. She’s an interesting one, because sometimes she’ll make errors when she really doesn’t need to be making errors just because she wants it for attention purposes. She’s really pretty good on a lot of things, but it’s interesting because when I have people in the room and things like that, sometimes she will fake having issues so someone will sit down and listen to her and go through it. As you talk with her and work with her more, you’ll see that she has some good understandings and that going on. She’s very easily swayed to using somebody’s strategy if she perceives that that person is getting attention... The nice thing about CGI is that you know your kids’ personalities. Everything is about math, but sometimes it’s about making that child have a more confident personality and being comfortable with what their thoughts are, and sharing what their thoughts are. Even though I’m talking about things that are not mathematical, CGI can be a venue to help kids get over those kinds of things.

Through her questioning, Kathy learned several things about Lynn - she had a lot of math ability, she made errors in an attempt to receive attention, and she was easily
swayed to use others’ strategies. Because Kathy had been mobilizing CGI strategies and practices, she contended Lynn has become a more confident mathematician.

With regards to *Investigations* (TERC, 2008), Kathy did not use it very often and when she did use it, she used it much like she used other resource books, in that she chose an activity that fit where her students were on a progression. Kathy felt that most of the lessons were “too easy” for her students, thus the lessons were more appropriate for her students more towards the beginning of the school year. Kathy did, however, facilitate one lesson, with considerable adaptations, from *Investigations* (TERC, 2008) about patterns when I was observing. When deciding to use one lesson in the pattern unit, Kathy first noted that her students had already met the objectives in the first two unit lessons through other instructional activities. The third lesson of the *Investigations* unit (TERC, 2008) met her students’ needs because it addressed identifying the unit pattern, extending patterns, and reversing patterns.

**Conclusion**

Kathy began her teaching career working with nearby university professors implementing a program that she described as “very Piagetian.” Because of this experience, Kathy felt that she always had “this very strong belief in knowing the progression that children learn things. And so, I feel like I have really tried to develop strong foundation in understanding how kids develop.” Kathy used her knowledge of “progressions” to guide her instruction. She would choose instructional activities that would address the point of the progression that her students were currently working at. At the time of the study, Kathy had participated in five years of CGI professional
development and facilitated CGI professional development for other teachers in the
district. Kathy had access to *Investigations* (TERC, 2008), but did not mobilize the first
grade edition often due to already having a bank of high-quality instructional activities
and because the lessons were often “too easy.”
CHAPTER 3. PEDAGOGICAL DESIGN CAPACITY FOR TEACHING ELEMENTARY MATHEMATICS: MOBILIZING PROGRESSIONS

A paper to be submitted for publication.

Tonia J. Land

Abstract

In this study, I examined how curricular resources supported three expert teachers in their conception and mobilization of hypothetical learning progressions and trajectories. Results indicate that the teachers mobilized four different types of progressions - unit of study, series of instructional activities, number choices, and student solutions – and that the progressions were imbedded within each other. This study has implications for defining key dimensions of PDC and supporting teachers at each of the four progression levels with curricular resources.

Introduction

Pedagogical design capacity (PDC) is a teacher’s “ability to perceive and mobilize existing resources in order to craft instructional contexts” (Brown & Edelson, 2003, p.6; Brown, 2009, p. 29). The PDC construct is in its infancy. That is, the key dimensions of PDC have not been identified; and ways to measure teachers’ PDC have not yet been developed (Brown, 2009). Understanding how teachers perceive and mobilize Standards-based curriculum materials in productive ways is of particular importance due to the prevalence and frequent mandate of Standards-based materials in elementary mathematics classrooms. Furthermore, recent research from Stein and Kaufman (2010) suggests that how a teacher mobilizes a curriculum series is more important than the knowledge, education, and experiences a teacher has. Thus,
supporting teachers in their mobilization of curriculum materials is imperative.

My work has entailed investigating the ways in which expert teachers with access to the Standards-based elementary mathematical curriculum, *Investigations* (TERC, 2008), and Cognitively Guided Instruction (CGI) professional development (Carpenter, Fennema, Franke, & Levi, 1999) perceived and mobilized these two curricular resources to design instruction and develop their PDC (see Land & Drake, 2010; Land, Chapter 4; Land, Chapter 5). In this study, I specifically examined how curricular resources supported three expert teachers in their conception and mobilization of hypothetical learning progressions and trajectories.

I first ground this study in three constructs – pedagogical design capacity, the participatory relationship between a teacher and his/her curriculum materials, and hypothetical learning trajectories and progressions. Data collection and analysis entailed building representations of four different progression types: unit of study, series of instructional activities, number choices, and student solutions. I end this paper with implications about supporting in-service and pre-service teachers in their own conceptualization and mobilization of the four progression types.

**Theoretical Framework**

The theoretical framework guiding this study is grounded in the notion of PDC (Brown & Edelson, 2003; Brown, 2009) and the participatory relationship between a teacher and his/her curricular resources (Remillard, 2005). Additionally, this study is informed by the literature pertaining to hypothetical learning trajectories and progressions.
PDC stems from the notion that teaching is a design activity (Brown & Edelson, 2003). “Teachers must perceive and interpret existing resources, evaluate the constraints of the classroom setting, balance tradeoffs, and devise strategies – all in pursuit of their instructional goals. These are all characteristics of design” (Brown & Edelson, 2003, p. 1). PDC takes into account the use of tools. Teachers have access to different tools or resources (e.g. curriculum materials, professional development) depending on their setting and they use those resources differently depending on their experience, goals, and abilities.

Similarly, Remillard writes that teachers and the curriculum materials that they use have a participatory relationship (2005). Each is a significant and active contributor to the planned curriculum. Ways to describe this relationship have been generated. For example, Brown and Edelson (2003), describe three patterns of curriculum use as offloading, adapting, and improvising. Offloading refers to adhering closely to curriculum materials. Adapting refers to the practice of using curriculum materials, but also contributing one’s own design elements to instruction. When a teacher improvises, he/she “pursues instructional paths of their own design” (Brown & Edelson, p. 7, 2003). Sherin and Drake (2009) found that teachers’ patterns of curriculum use could be explained by determining how teachers read, evaluate, and adapt curriculum materials before, during, and after instruction.

Brown and Edelson contend that the PDC construct has implications for teacher preparation and professional development. “Teachers require support in exploring which resources to use and how to use them” (Brown & Edelson, 2003, p. 6). Many
teachers, however, are required to use a particular curriculum series (Archer, 2005). One way to understand how to support teachers in their use of Standards-based curriculum materials is to study how teachers already use them in productive ways. Stein and Kaufman (2010) conducted one such study and found that rather than the education, experience, and knowledge a teacher brings to a classroom; teachers tended to have higher quality lessons, (measured by maintaining high levels of cognitive demand, attending to student thinking and vesting intellectual authority in mathematical reasoning) when they “talked about or reviewed big mathematical ideas that students were supposed to be learning” (Stein & Kaufman, 2010, p. 681). In other words, teachers who mobilized curriculum materials in particular ways had high-quality lessons even if they had limited individual capacity.

**Hypothetical Learning Progressions and Trajectories**

Building on the work of both mathematics and science educators (Baroody, Cibulskis, Lai, & Li, 2004; Clements & Sarama, 2004; Simon, 1995), Stevens, Shin, and Krajcik (2009), a group of science educators, developed a taxonomy of working definitions for terms associated with learning progressions. A learning progression:

- Organizes the content of the discipline and describes a potential route towards more sophisticated knowledge.
- Explicitly specifies the connections between ideas students need to build an integrated knowledge framework.
- Links the content to appropriate phenomena or models that students should be able to explain.
- Provides potential instructional strategies and learning tasks to help students move from one level to the next. (Stevens et al., 2009, p. 2).

A hypothetical learning progression (HLP) is a learning progression that includes the current learning research (Stevens, Shin, & Krajcik, 2009). A hypothetical learning
trajectory is different from an HLP in that it addresses a specific learning goal, specifies the process that will help students meet the learning goal, and includes potential student ideas and difficulties (Simon, 1995; Stevens et al., 2009). HLTs are subsets of HLPs. One or more HLTs “will describe how students can move from one point in the HLP to another” (Stevens et al., 2009, p. 2).

Simon (1995) points out that the definition of an HLT is not meant to imply that teachers only pursue one goal or HLT at a time. “Rather, it is meant to underscore the importance of having a goal and rationale for teaching decisions and the hypothetical nature of such thinking” (Simon, 1995, p. 136). Since Simon introduced HLTs, curriculum developers have used it as a foundation to develop innovative mathematics curricula (Clements & Sarama, 2004). However, the developers and authors have conceptualized and applied HLTs in different ways. Clements and Sarama conceptualized learning trajectories as:

Descriptions of children’s thinking and learning in a specific mathematical domain and a related, conjectured route through a set of instructional tasks designed to engender those mental process or actions hypothesized to move children through a developmental progression of levels of thinking, created with the intent of supporting children’s achievement of specific goals in that mathematical domain (2004, p. 83).

In this study, I report on the ways in which three teachers conceptualized progressions and used them in their instruction. The teachers did not differentiate between progressions and trajectories. Thus, they labeled any particular sequence of instruction as a progression. Because the teachers conceptualized the progressions described in this paper, they are practitioner-based and are different than learning progressions described by researchers. Hiebert, Gallimore, and Stigler (2002) point out
several features of practitioner knowledge that make it valuable to teaching and learning. First, practitioner knowledge is linked with practice in that it responds “to specific problems of practice” (Hiebert et al., 2002, p. 6). Additional features of practitioner knowledge include that it is detailed, concrete, specific, and integrated (Hiebert et al., 2002). I found that the progressions conceptualized by the teachers had the features of practitioner knowledge as they were directly related to each teacher’s context. For instance, a progression mobilized by Violet may or may not be useful to the other teachers depending on their own teaching contexts.

**Methodology**

**Participants**

Kathy, Olivia, and Violet\(^2\) are three teachers employed at two different schools in the Lakefront School District. All three teachers have access to the *Investigations* (TERC, 2008) curriculum series and have participated extensively in CGI professional development (Carpenter et al., 1999). Kathy teaches 1\(^{st}\) grade; Olivia teaches 2\(^{nd}\) grade; and Violet is in a 4\(^{th}\)/5\(^{th}\)-grade multi-age classroom. Although these teachers have access to these two major curricular resources, they differ in when each resource became available to them and how they perceived and mobilized the two. Next, I provide a brief description of each teacher.

**Kathy.**

Kathy began her teaching career working with nearby university professors implementing a program that she described as “very Piagetian.” Because of this

\(^2\) All names are pseudonyms.
experience, Kathy felt that she has always had “this very strong belief in knowing the progression that children learn things. And so, I feel like I have really tried to develop strong foundation in understanding how kids develop.” Kathy used her knowledge of “progressions” to guide her instruction. She would choose instructional activities that would address the point of the progression(s) that her students were currently working at. At the time of the study, Kathy had participated in five years of CGI professional development and facilitated CGI professional development for other teachers in the district. Kathy had access to *Investigations* (TERC, 2008), but did not mobilize the first grade edition often due to already having a bank of high-quality instructional activities and because the lessons were often “too easy.”

**Olivia.**

Olivia had a “traditional” teaching practice until she participated in CGI five years ago. She primarily mobilized her CGI knowledge to design instruction, which entailed generating contextualized problems and adapting other secondary resources that had been a part of her practice for several years (e.g. number of the day tasks; Kathy Richardson and Marilyn Burns resource books). Olivia gained access to *Investigations* (TERC, 2008) the same year this study took place. However, Olivia did not perceive *Investigations* (TERC, 2008) as a valuable resource. She felt like she already has a successful teaching practice due to her CGI knowledge, and thus, did not need to mobilize another primary resource. I had asked Olivia to consider using *Investigations* (TERC, 2008) for upcoming fraction and geometry units. Olivia did read through the units and examined the lessons, but she did not use them due to feeling “trapped.”
Furthermore, certain aspects of the lessons (e.g., paper folding) reminded her of how she used to teach traditionally also contributing to her nonuse of *Investigations*.

**Violet.**

Violet also had a traditional teaching practice until she gained access to *Investigations* in 1999, which she has used primarily and almost exclusively since. Violet felt that *Investigations* (TERC; 1998, 2008) was responsible for her shift to a more reform-oriented teaching philosophy. At the time of the study, Violet had participated in three years of CGI professional development, which prompted her to adapt *Investigations* (TERC, 2008) lessons using her CGI knowledge. Adaptations included replacing opening routines in *Investigations* (TERC, 2008) with contextualized problems or series of true/false and open number sentences, questioning students extensively, and facilitating large-group discussion around children’s solutions strategies daily.

**Data Sources**

Kathy, Olivia, and Violet were each observed on six occasions. After each classroom observation, I watched the videotape and developed a video-stimulated recall interview for each of the six observations. These interviews were designed to gain knowledge of possible key dimensions of PDC. For each segment of classroom observations (opening routine, launch of activity, student exploration, and strategy sharing), I would ask each teacher to state and explain her intended goal, provide details in how that segment was designed, and describe what she noticed about the student learning. For instance, the following five questions were asked of Violet for one
part of one observation:

1. Can you tell me about the game, Roll Around the Clock? How does the game fit in with the goals for the unit and year? Did only some students play? Why?
2. Tell me about the beginning problem. Why did you develop the beginning problem? How does the problem tie into the curriculum and your goals for the year?
3. What do you notice about Kael's strategy? Where do you want him to progress to from here?
4. What do you notice about Leslie's strategy?
5. What is your goal with your conversation with Liev?

Early in the interview process, the teachers talked about “progressions.” In those instances, I asked them to describe specifically what they meant. I also began to ask more explicitly about “progressions” to gain a better understanding of what constituted a “progression” as per the teachers. For example, I would ask, “why is this instructional activity appropriate for this time of year?” and “what are you going to do next and why?” I became more interested in the teachers’ discourse surrounding progressions, because the progressions seemed to drive the teachers’ instructional decisions. In fact, in Kathy’s case, progressions were her primary resource (See Chapter One). Because progressions seemed to be such a distinguishing feature of the teachers’ practices and because the teachers’ progressions seemed similar to the HLPs and HLTs discussed in the research literature, I decided to investigate “progressions” further. Thus, my research question was:

**Research Question**

- How were Kathy, Olivia, and Violet conceptualizing and mobilizing progressions?

**Data Analysis**

In previous work (Land & Drake, 2010), I drew on the work of Remillard (2005)
and Davis et al. (2007) to generate a list an initial list of possible key dimensions of PDC to investigate the PDC of one teacher. During that study, my co-author and I generated a second list of possible key dimensions of PDC through a process of emergent coding. We used this list as a coding scheme to analyze interview data:

- Knowledge of Curricular Resource
- View/Perception of Curricular Resource
- Mobilization of Curricular Resource
- Knowledge/Perception of Students
- Mobilization of Student Resources
- Beliefs
- Goals
- Tolerance for Discomfort
- Previous Teaching Experiences
- Subject Matter Knowledge

Once final codes and definitions were established, I recoded all interview data. A second coder coded 34% of the data (14 of 41 pages of interview transcripts). The two coders were in disagreement on 10 segments of data comprising 13% of the 14 pages of transcripts. Of those disagreements, we reached 90% consensus. The remaining segment pertained to Nancy using the CGI framework to gain knowledge of her students. The segment remained under dispute, because one coder felt it should be coded as mobilization of curricular resources while the other thought it should be coded as knowledge of students. It was agreed that the segment could be double-coded. Moving forward, I decided not to double-code data, and that codes entailing how teachers perceived or mobilized a particular resource took precedence over other codes.

After transcribing all interview data for this current study, I coded all transcripts using the above list of possible key dimensions as a coding scheme. Additionally, I
coded data as “progression” whenever teachers were talking explicitly about a progression. In these instances, I did not create different codes for HLPs and HLTs, because I could not always distinguish between the two. I justified this decision in three ways. First, I was keeping true to the language used by the teachers. Even though the progressions were clearly of different types based on the different types of learning goals, the teachers were not using different terms to distinguish between them. Second, many in the mathematics education field have used the construct of “hypothetical learning trajectories” as a foundation for their curriculum development and research, but have interpreted and applied that construct in different ways (Clements & Sarama, 2004). Therefore, there are not clear distinctions between HLPs and HLTs in the mathematics education field. Third, my purpose is not to distinguish between HLPs and HLTs. Instead, I am reporting on the ways in which the three teachers were conceptualizing and mobilizing progressions. Additionally, I was investigating the curricular resources that supported teachers in their conceptualization and mobilization of progressions.

After all transcripts were coded, I examined the data coded as progressions. From the interview data coded as progressions, there were clearly two types of progressions: number choice and student solutions. Whenever the teachers posed a problem, they would mobilize a multiple number choice structure as in the Pennies Problem below:
**Pennies Problem**

Lenore has ____ pennies and Max has ____ pennies. How many pennies do they have together?

(6, 30)    (40, 20)    (10, 68)    (45, 13)

Several times, the teachers described their number choices as a progression. Thus, I identified these types of progressions as number choice progressions.

Second, the teachers would describe a progression of student solutions from least sophisticated to most sophisticated. As per the teachers’ recollections, a common activity in CGI professional development was to order student work. A group of students would be given a similar problem and then the students’ work would be ordered from least to most sophisticated. For example, a counting strategy would be considered more sophisticated than a direct modeling strategy. Any time a teacher talked about a progression of student work, I identified that segment of transcript as a student solution progression. Fuson, Carroll, and Drueck (2000) had a similar finding when they reported that teachers using the *Everyday Mathematics* curriculum “articulated their vision of the curriculum as consisting of a progression or range of solution methods through which they helped all children move” (p. 292). Once I identified a student solution progression, I went back to the original transcript to pull related interview data to build the student solution progressions. An example of a student solution progression is given in the results section.

Each of these progression types (number choice and student solution) did not fit the definition of HLPs and HLTs from Stevens and her colleagues (2007). That is, number choice and student solution progressions did not provide organization to the
big mathematical ideas as in HLPs, nor did they provide the means (instructional activities) for students to reach a particular learning goal as in HLTs. Number choice progressions are imbedded within a larger instructional activity (e.g., contextualized problem). Student solution progressions describe the efficiency and sophistication of students’ strategies, which indicates a level of student understanding.

Therefore, I looked to other interview data to look for instances of HLPs and HLTs. I organized the transcript data into segments by teacher and instructional activity. Sometimes there was more than one instructional activity in each observation. For each of these segments, I pulled interview data coded as goals and mobilization of resource, and looked for organizations of mathematical content and specific learning goals along with a means (instructional activities) for students to reach that goal. Data coded as goals identified the learning goal for each instructional activity. Data coded as mobilization of resources identified the resource and how the resource was being used to reach the learning goal.

From that process, I was able to identify two additional progression types: unit of study and series of instructional activities. Unit of study progressions do not address specific learning goals. Instead, they address something broader such as a sequence of mathematical concepts that are closely related. Like HLPs, they provide an organization of and sequence to the big mathematical ideas. A series of instructional activities was a set of tasks that addressed one or more mathematical concepts. One could consider this progression type to be similar to an HLT, because the progression of activities addressed a specific learning goal and specified the means (instructional activities) that
would potentially help students to meet that goal. I describe all four progression types in more detail in the results section.

**Results**

Because I was only in the three teachers’ classrooms on six occasions, I could capture only segments of progressions. There may be points on the progressions before, between, and after the points I have been able to define. In the remainder of this section, I present examples for each of the four types of progressions: unit of study, series of instructional activities, number choice, and student solutions. I chose the most developed progressions to present as examples. For each example, I first present data from which each progression was built, then a visual representation.

**Unit of Study**

During the interview process, Violet spoke extensively about her use of *Investigations* (TERC, 2008). When teaching a fraction unit, I asked Violet to talk about how she had a sense of fraction concepts:

**Interview Excerpt**

*Tonia:* So, how did you have a sense of those goals or concepts within fractions?

*Violet:* Those are within the TERC. I will use that to guide instruction... To start with, it’s started with just representing halves, fourths, eighths. And then it moved to, um, fractions of a group... Then, I had pulled in other resources as needed. That’s how I use TERC. Just the basic concepts, I follow that.

*Tonia:* The math focus points?

*Violet:* Yeah, exactly... Like within fractions of a group, I wanted them to see, do they see it as division? And giving lots of experiences to see how that is really dividing. Using whatever strategy they use to divide. So, that was kind of my focus for that particular group. Then, it moves into adding fractions and then ordering fractions. So, those are like the focus points and I follow those. And then pull in additional resources. Sometimes I use TERC exactly the way it’s set up. Sometimes I don’t, but it all depends on what I see the kids doing in class.

In the above interview excerpt, Violet stated that she followed those concepts. First, the
The unit started with “representing halves, fourths, eighths.” Then, the unit moved to “fractions of a group.” Violet intended to provide several experiences to address this progression point using what strategies students felt comfortable with. From there, the unit moved into “adding fractions and then ordering fractions.” The unit of study progression is represented below in Figure 1 as recalled by Violet:

![Figure 1: Unit of Study Progression](image)

I first observed Violet facilitating instruction for finding fractions of a group. She posed the Baseball Card Problem:

**Baseball Card Problem:**
Leonard has ____ baseball cards. He gives \( \frac{1}{4} \) to his friend. How many baseball cards did he give to his friend?

8 24 44 60 100 144

Violet started that particular progression point with finding \( \frac{1}{4} \) of a group, because \( \frac{1}{2} \) would have been “too easy.” The following day, Violet posed the following two inequalities and asked her students to state if they were true or false:

- \( \frac{1}{4} \) of 12 < \( \frac{1}{2} \) of 12
- \( \frac{1}{4} \) of 24 > \( \frac{1}{4} \) of 44

Again, Violet’s focus was finding a fractional part of a group, but she wanted students to use the relationship between \( \frac{1}{2} \) and \( \frac{1}{4} \) as part of their solution method. A few days later, I saw that Violet had moved on to the next progression point, adding fractions, when I observed a lesson where students were generating equations with fractions that
equaled one (e.g., \( \frac{1}{2} + \frac{1}{2} = 1 \), \( \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1 \)). Following a lesson in *Investigations* (TERC, 2008), students were given 4 x 6 grids to help them generate the equations and represent their work.

In the *Investigations* unit (TERC, 2008), there are several more focus points listed, but what is notable about both Violet’s recall of the progression and the math focus points is that they both depict a “path” that provides organization to several fraction concepts. Violet defined the first point as “representing halves, fourths, and eighths.” The authors of *Investigations* (TERC, 2008) defined the same point as “finding fractional parts of an area.” What is interesting about that is that Violet actually described a second progression that is imbedded with the first. The first lesson in the 4th grade fraction unit (TERC, 2008) provides a series of instructional activities addressing the first unit of study progression point starting with halves, and then moving into fourths and eighths. A second example of a series of instructional activities progression is given next.

**Series of Instructional Activities - Measurement Division Problems**

In this second example, Olivia designed a series of measurement division problems to meet the district’s objective of “identifying parts of a whole to model halves, thirds, and fourths.” For each of the measurement division problems, Olivia provided a progression of number choices. Therefore, there is a second progression, number choice, imbedded in the series of instructional activities. First, Olivia posed problems that involved dividing things up. Olivia stated:

*We started out dividing just like jellybeans into Easter baskets. So, we did a lot of measurement division problems first to get them [students] thinking about*
dividing things up. Then, we started with the halves and dividing something in two.

Although I do not have an example of one of these problems, I know from the interview data that all the number choices involved dividing things up “equally,” meaning there were no “leftovers.” I used this data to define my first point – dividing objects into groups with no leftovers. Olivia talked about this same point in her statement below, but also mentioned even numbers:

Equal numbers first just to see what kids are going to do. Are they going to direct model? Which a lot of them do at first. Then even numbers, they pick up on that really quickly. Even numbers - they each have a partner. They think back to what they learned in kindergarten, or first grade even.

Because Olivia mentioned “even numbers – they each have a partner,” I understood this to mean that she posed problems involving dividing an even numbers of objects between two people defining the second progression point. Additional data to support defining this progression point is when Olivia next posed problems that involved “halves and dividing something in two” as in the Sharing Cookies Problem below:

**Sharing Cookies Problem**
Trisha and Allie are sharing _____ chocolate chip cookies. If they are shared equally, how many will each of them get?

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In the Sharing Cookies Problem, students are dividing even and odd number of objects between two people. From this, I defined the third point as dividing an odd number of objects between two people. After dividing objects between two people, Olivia posed problems where a set of objects needed to be divided among four people.
She stated, “Our district objective for second grade is halves, thirds, and fourths. As a team, we feel that going from halves to fourths is a more natural move then halves to thirds. So, that’s usually our next move.” Below is the problem that was posed immediately after the Sharing Cookies Problem:

**Sharing Brownies Problem**
Trisha, Allie, Lance, and Kathy are sharing brownies. If they are sharing ____ brownie equally, how many will each person get?

4  5  8  9  16  17  20
32  33  44  45  48  49  50

In the Sharing Brownies Problem, Olivia provided a number that is divisible by four first followed by the next whole number, except in the last number choice of 50. With these number choices, correct solutions entailed no brownie needing to be divided in parts, or one brownie needing to be divided up giving each person a certain amount of whole brownies and \( \frac{1}{4} \) of another. From these data, I defined the fourth and fifth progression points as dividing a multiple of four objects between four people and dividing objects between four people where a whole number plus \( \frac{1}{4} \) would be given to each person.

The next problem posed by Olivia again involved dividing objects between four people, but the number choices were different. Olivia stated, “I didn’t, obviously, want very many of them to come out equal.”

**Miniature Candy Bar Problem**
There are ____ miniature candy bars. Dustin, Jose, Sam, and Joe are going to share the candy bars. If they split up the candy bars equally, how many will each of them get?

11  17  22  35  48
65  83  75  99  104
In most of these number choices, they “don’t come out equal.” I know from our full interview that Olivia meant that every person would not be given a certain amount of whole miniature candy bars or a certain amount of whole miniature candy bars and \(\frac{1}{4}\) of another as was the case in the Sharing Brownies Problem. For example, when 11 candy bars are shared among four people, each person would get 2 \(\frac{3}{4}\) candy bars. This data caused me to define the final progression point as dividing objects among four people where each person would be given a whole number plus one or more fourths.

Olivia’s progression for students meeting the district objective of learning about “halves, thirds, and fourths” is represented below in Figure 2:

![Figure 2: Series of Instructional Activities](image)

I have added arrows connecting the first progression point, dividing objects with no leftovers, to the second and fourth points. This is to denote that the second (dividing an even number of objects between two people) and fourth (dividing a multiple of four objects between four people) points are actually more specified aspects of the first point – dividing objects with no leftovers. I point this out because it is an interesting strategy to scaffold learning. Olivia had students working back and forth between two progression points with her number choices in the Sharing Cookies and Sharing Dividing objects with no leftovers

Dividing objects among four people where a whole number plus \(\frac{1}{4}\) would be given to each person

Dividing objects among four people where each person would be given a whole number plus one or more fourths

Dividing objects among four people where a whole number plus \(\frac{1}{4}\) would be given to each person

Dividing objects among four people where each person would be given a whole number plus one or more fourths

Dividing objects among four people where a whole number plus \(\frac{1}{4}\) would be given to each person

Dividing objects among four people where each person would be given a whole number plus one or more fourths

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Brownies Problems, which provided students with a transition or entry into a more complex progression point.

**Number Choice Progression - Pennies Problem**

In one observation, Kathy posed the Pennies Problem to help address the goal of base-ten understanding. Below is the Pennies Problem and Kathy’s rationale:

**Pennies Problem**
Lenore has ____ pennies and Max has ____ pennies. How many pennies do they have together?

(6, 30)  (40, 20)  (10, 68)  (45, 13)

**Kathy's Rationale for the Pennies Problem**
[Base-ten] and wanting kids to start looking at tens and ones and being able to add them easily - like the 6 and 30. Obviously, I'm putting the 6 first instead of the 30 first, because I want them to see that 6 and 30 is the same as 30 and 6, which is pretty attainable for them...If I was looking at 6 and 30, you wouldn't want them to hold 6 and count on 30... The 10 and 68, there's 2 ways that my kids could look at this. Most of them will be looking at 60 and 8, then adding the 10, which would be 70 + 8. And some kids, just knowing that when you add 10 to number like 68, it's 78. That's a little bit more complex for the kids. So, wanting them to think about that strategy. Then 45 and 13 is really more of having to decompose two numbers. You want them to look at it as 40 and 10 and 5 and 3.

Presenting a progression of number choices is an almost daily teaching strategy in all the teachers’ classrooms, which provides differentiation for students. In this particular problem, I was able to define several points on a number choice progression. First, Kathy talked about a single-digit number plus a decade number defining one point on the progression. From her statement, “Obviously, I’m putting the 6 first instead of the 30 first,” I inferred that a preceding point would entail presenting a decade number first and a single-digit number second. Thus, I defined that notion as my first point. Kathy did not discuss the second number choice of two decade numbers, but she did
include number choices of that type in her problem defining the third progression point – adding two decade numbers. The third number choice entailed 10 plus a two-digit non-decade number. The final number choice involved adding two two-digit non-decade numbers, which Kathy stated involves “having to decompose two numbers.”

Figure 3 is a visual representation of Kathy’s progression:

![Number Choice Progression Diagram]

*Figure 3: Number Choice Progression*

At the number choice level, Kathy talked about how she moved students to increasing more complex number choices:

If I’m thinking about what I would plan for the next day or if I would for the next couple of weeks, I would look at this [student work] and kind of see where my kids are. If I’m noticing on the first three choices... If most of them are fairly successful with that, then I’m going to take that number choice out... If I look at a number choice and I see a lot of my kids having to direct model it, I want them to move them. Then, I might have 2 or 3 number choices that are just kind of tone in or hone in on that specific piece.

If students become successful with particular types of number choices (e.g. decade number plus single-digit number), Kathy would take that number choice out, which allowed her to add another more complex number choice. This could entail, for instance, sums over 100. Kathy also mentioned that if several students were direct modeling, she would pose two or three number choices of the same type. In other
words, Kathy assessed students’ solution strategies to decide what type of number choices to offer.

**Student Solution Progression – Parts of a Set**

As mentioned earlier, Violet posed the Baseball Card Problem in one observation:

**Baseball Card Problem:**
Leonard has ____ baseball cards. He gives ¼ to his friend. How many baseball cards did he give to his friend?

8  24  44  60  100  144

**Violet’s Rationale for the Baseball Card Problem**
So, I changed the numbers, but kept the fraction the same, and I picked a fourth because I knew it was a friendly fraction that they all would be able to have access to… A half would be too easy… I wanted to see if they could do some relational thinking with using a half and then breaking that in half and see what kind of relationship they had with that… Fractions of a set is much more challenging than fractions of an area. So, um, this was our first attempt at this, so I needed it accessible to most kids.

From the above rationale, I defined one point on a student solution progression as using relational thinking, because of Violet’s statement, “I wanted to see if they could do some relational thinking…” Once students solved the problem, Violet asked a few students to present their solution strategies. As per her CGI professional development experience, Violet had students share their solutions in order of sophistication:

I usually start with the least sophisticated to most sophisticated… I want to be able to have the kids see the progression. Like, let’s say for the division, we see some kids using the blocks. This is a strategy, and I want them to understand that every strategy is perfectly fine… From there, I might be doing tally marks. You know, I don’t need to use cubes. That’s a great way to record it without cubes. And then, from those tally marks those kids might make that next jump to oh; I don’t need to do one at a time. I can do a friendly number like groups of ten, and we have that conversation…
In the above statement, I defined the following sequential points – direct modeling (using the blocks), use of tally marks, use of numbers (friendly number like groups of 10). In the case of the Baseball Card Problem, Violet had students present solutions in the following order:

**Liev's strategy for finding \( \frac{1}{4} \) of 8 baseball cards**

\[
\begin{array}{c}
\text{\( \frac{1}{2} \)} \\
\text{\( \frac{1}{2} \)} \\
\text{\( \frac{1}{2} \)} \\
\text{\( \frac{1}{2} \)}
\end{array}
\]

\[
\frac{1}{2} + \frac{1}{2} = 1 \\
1 + 1 = 2
\]

**Noah’s strategy for finding \( \frac{1}{4} \) of 24 baseball cards** – Noah put tallies one at a time in the circles until he had drawn 24 tallies then wrote the equation below the drawing.

\[
24 \div 4 = 6
\]

**Adam’s strategy for finding \( \frac{1}{4} \) of 44 baseball cards** - Adam said, “I put 10 in each circle because I knew that would make 40. There was four left over, so I put one more in each circle.”

\[
44 \div 4 = 11
\]

**Debbie’s strategy for finding \( \frac{1}{4} \) of 60 baseball cards**

\[
60 - 30 = 30 \\
30 - 15 = 15
\]

**Jayda’s strategy for finding \( \frac{1}{4} \) of 60 baseball cards**

\[
60 \div 2 = 30 \\
30 \div 2 = 15
\]

These strategies provided me with a second piece of evidence that direct modeling, use of tally marks, use of numbers, and relational thinking are defined points on the
progression. Liev is not actually direct modeling since he is using an area model instead of a discrete model. I included his strategy here because Violet had him his share his strategy during class. I did not create two separate points for Debbie and Jayda’s strategy, because both of them used the relationship between $\frac{1}{2}$ and $\frac{1}{4}$ to solve the problem. Finally, in the interview process, I asked Violet to describe what she noticed about Adam’s strategy. She stated:

Well, he’s different than Liev in that he’s dividing. He’s breaking it up into four groups, and he has a more efficient strategy. He’s not doing one tally at a time. And that he’s started doing groups of, you know, tens, which I’m happy with. The next step I would take with him would be to take those tens and actually write an equation to show. So, if he had four groups of ten, I would next record that as four groups of ten.

From her above statement, I defined an additional point as writing an equation as a multiple representation. Adam has a productive solution, but now Violet wants him to represent that solution with an equation. Using all the above data, I built the following representation of a student solutions progression:

![Figure 4: Student Solution Progression](image)

Violet posed the Baseball Card Problem while facilitating the fraction unit from *Investigations* (TERC, 2008). The problem was not a problem given in the materials, but one that Violet created to address the focus point “parts of a group.” Therefore, this progression representing student solutions is imbedded in the larger unit of study.
progression. Similarly, each progression is imbedded in another progression if conceptualized as such by the teacher. I will provide a description of the imbedded progressions in the next section.

**Imbedded Progressions – Fraction Unit**

Earlier, I presented a unit of study progression (Figure One) for a fraction unit that Violet facilitated from *Investigations* (TERC, 2008). To address one point on that progression (fraction of a group), Violet facilitated a series of instructional activities (e.g., Baseball Card Problem, true/false sentences, lessons from *Investigations* (TERC, 2008)). There was a progression to that series of activities. Violet alluded to that progression when she talked about representing halves, fourths, and eighths. She also alluded to that progression in her rationale for the Baseball Card Problem; she stated, “A half would be too easy.” That is, Violet could facilitate an activity around finding ½ of a group, but that particular activity would be too easy for her students. Therefore, she needed to hit the next point on the progression – finding ¼ of a group. One of the instructional activities was the Baseball Card Problem in which there was a number choice progression given. Finally, there was a progression of student solutions from least to most sophisticated. Figure 5 represents the imbedded progressions:
Figure 5: Imbedded Progressions

While designing instructional activities, Kathy, Olivia, and Violet mobilized their knowledge of each progression type. Therefore, their instructional goals entailed moving students along each of these progressions. As indicated in Figure Five, each type of progression is connected to the others. For instance, the types of strategies students are using and the number choices students are able to solve successfully indicate how sophisticated their knowledge is of fractions of a group. Figure Five also illustrates the complexity of a teaching practice that involves multiple progressions.

Discussion

Given the results of this study, the taxonomy of terms regarding learning progressions I borrowed from science education does not adequately depict all the types of progression conceptualized and mobilized by Kathy, Olivia, and Violet when teaching elementary mathematics. It is important to recognize each type of progression individually, as there are implications for how teachers are prepared and supported. I grounded this study in the notion of pedagogical design capacity – how teachers
perceive and mobilize their existing resources. To end this chapter, I return to that construct and discuss resources the three teachers mobilized to conceptualize and develop progressions of each type. I start with unit of study progressions.

Unit of Study.

Kathy and Olivia mobilized their district objectives to develop organization for their unit of study progressions. Olivia stated, “The geometry lessons that I've been planning, and this is what I would do with any topic or area of study in math, I would look at our objectives first... Our [Lakefront] district objectives.” Olivia also stated that the district’s objectives were limited. That is, they did not provide much support in helping teachers conceptualize the big mathematical ideas. With regards to fractions in second grade, the learning objective was, “identify parts of a whole to model halves, thirds, and fourths,” which did not provide a way to come to know or organize the many big mathematical ideas associated with fractions (e.g., finding equal parts, dividing an area into equal parts, naming parts, and different shaped parts can have the same area). Kathy and Olivia have generated their own conceptualization and organization to the big mathematical ideas through their experiences with CGI and teaching.

CGI professional development, however, does not actually provide organization to the big mathematical ideas. CGI provides frameworks for problem types and student solutions (Carpenter et al., 1999). Organization of the big mathematical ideas is something that Kathy and Olivia, along with their colleagues, have developed on their own through their extensive knowledge of number and operations. My conjecture is that few teachers would be able to develop unit of study progressions with CGI.
Furthermore, CGI addresses number and operation. It does not provide frameworks for the other content areas – algebra, geometry, measurement, and data analysis and probability.

Violet, on the other hand, despite also working in the Lakefront School District mobilized the “math focus points” in *Investigations* (TERC, 2008), which she used as a progression. Stein and Kaufman (2010) pointed out that the *Investigations* curriculum (TERC, 2008) provides substantial support for teachers in their locating and understanding of the mathematical big ideas. Kathy and Olivia did not mobilize the math focus points from *Investigations* (TERC, 2008) due to their perceptions of the curriculum. Using the terms Brown and Edelson (2003) generated to describe teachers’ curriculum use, Violet offloaded the conceptualization of unit of study progressions onto the curriculum materials. Kathy and Olivia, however, chose to either adapt or improvise these progressions, which they were able to do because of their extensive knowledge of number and operation.

**Series of Instructional Activities.**

A series of instructional activities would address a particular point on a unit progression. The teachers would target a point on a unit progression and then design a series of instructional activities (by offloading, adapting, or improvising) that addressed that point. In several lessons, Violet facilitated instruction around finding fractions of a group. To address that unit progression point, Violet primarily pulled from *Investigations* (TERC, 2008), thus offloaded, but also pulled from other resources if her students needed additional instruction. The types of strategies used by students
determined student need. For example, if several students were direct modeling, Violet knew that she needed to spend more time on that progression point (e.g., finding a fraction of a group). Because *Investigations* (TERC, 2008) did not provide the exact number of instructional activities needed by her students, Violet would need to pull from other resources. On the other hand, if students did not need a particular lesson, Violet would omit it. For instance, Violet did not facilitate the activity for finding $\frac{1}{2}$ of a group in *Investigations* (TERC, 2008), because her students did not need it. A half would be “too easy for them.” Therefore, Violet began instruction for that unit progression point by asking students to find $\frac{1}{4}$ of a group.

Kathy and Olivia mostly mobilized CGI to develop a series of instructional activities. Once they chose a learning goal, a series of instructional activities could be developed to meet that goal. For example, Olivia developed a series of instructional activities to meet the district’s objective of “halves, thirds, and fourths.” That series of instructional activities entailed several measurement division problems with varying number choices. Olivia did not, however, mobilize CGI for a series of instructional activities for her geometry unit.

**Number Choices.**

All three teachers developed the practice of providing multiple number choices in their CGI professional development experiences. In my examination of several Standards-based curriculum materials (e.g., *Investigations*, *Everyday Mathematics*, *Math Expressions*, *Trailblazers*), multiple number choices are not systematically provided in each lesson. In some lessons, however, suggestions for alternative numbers are
provided in differentiation sections. Providing a progression of number choices in a single activity, however, would be arbitrary. A primary reason to offer multiple number choices is to provide differentiation, but curriculum developers do not know specifically what differentiation is needed. Therefore, it should be left up to the teacher to decide which number choices are appropriate for his/her teacher. Teachers, however, will need support in this task either from participation in professional development or from curriculum materials.

Kathy talked about her experiences with helping other teachers think about appropriate number choices:

And I think teachers who are newer to using CGI or this kind of philosophy it’s frustrating, because it doesn’t come as naturally to them. They really have to fight to figure it out, but you have to go through that as a teacher. The number choice progression that I do now is so much different than when I first started this.

Kathy pointed out that developing the skill of providing number choice progressions is frustrating to teachers. Providing multiple number choices is something that Kathy learned about in CGI professional development, but she has further developed through her own teaching. What is noteworthy here is that teachers have little to no support for developing progressions of number choices.

**Student Solution.**

According to the CGI research, students first direct model when exploring a new concept, then move on to more sophisticated strategies, such as some form of counting, use of derived facts, and invented algorithms (Carpenter et al., 1999). A large component of CGI professional development is ordering student work from least to
most sophisticated. All three teachers developed extensive knowledge of student solution progressions through CGI. That knowledge became more developed through their teaching experiences. Violet gained some knowledge of students’ solutions, but not progressions, through her use of *Investigations* (TERC, 2008). Some student work is provided in curriculum materials (e.g., *Investigations, Everyday Mathematics*) to help teachers anticipate students’ solutions, and some research has found that teachers’ knowledge of children’s solution strategies is linked to their use of curriculum materials (Empson & Junk, 2004). Providing a progression of student solutions, however, is another limitation of curriculum materials currently, but could be developed by curriculum designers.

In summary, with regards to the four different types of progressions, *Investigations* (TERC, 2008) can help teachers by conceptualizing and mobilizing unit of study and series of instructional activity progressions. *Investigations* (TERC, 2008) could provide support for number choice and student solution progressions, but it does not currently. CGI provides limited support for developing unit of study progressions for number and operations and provides no support for unit of study progressions in the other content areas. CGI can provide support for the other three progression types. It is clear that *Investigations* (TERC, 2008) and CGI supported teachers, but it is not clear whether other materials could support teachers in similar ways.

**Conclusion**

I chose to study Kathy, Olivia, and Violet’s teaching practices because they each had high levels of PDC. From studying their practices, I found that these expert teachers
conceptualized and mobilized four different types of progressions. In their paper about learning trajectories in mathematics education, Clements and Sarama (2004) highlighted a similar notion when they stated that “there is evidence that superior teachers use a related structure” (p. 82) to HLTs. HLTs are imperative because they provide teachers with a rationale for instructional design decisions (Simon, 1995).

More important than the evidence that expert teachers mobilize multiple types of learning goals and progressions, is what resources contributed to their knowledge of different progression types. One of the major aspects of grounding a study in PDC is that the PDC construct has implications for teacher preparation and professional development (Brown & Edelson, 2003). Therefore, the fundamental question that arises from this study is, “How do we support other teachers in their conceptualization and mobilization of progressions?” My belief is that we need to start with the teachers’ current resources. Many school districts in the United States are mandating the use of a single, often Standards-based, elementary mathematics curriculum series, which is to be the teachers’ primary resource for designing instruction. Recently, the majority of states have adopted the Common Core State Standards (CCSSO & NGA, 2010). Do these resources provide the needed support? If the resources do provide the support, will teachers use them in ways that include the mobilization of progressions?

Currently, I do not think teachers are supported in developing their ability to conceptualize and mobilize the four progression types unless they have access to both Investigations (TERC, 2008) and CGI or other similar resources. Stein and Kaufman (2010) pointed out that Investigations (TERC, 2008) supported teachers in locating the
big mathematical ideas compared to another series. I believe this finding may have something to do with how the curriculum developers unpack and present the big ideas. For instance, Violet facilitated the 4th grade unit titled, “Fraction Cards and Decimal Squares” that entailed the following focus points for “understanding the meaning of fractions and decimal fractions” (TERC-b, 2008, p. 10):

• Finding fractional parts of a rectangular area
• Finding fractional parts of a group
• Interpreting the meaning of the numerator and the denominator of a fraction
• Writing, reading, and applying fractional notation
• Representing fractions greater than 1
• Identifying everyday uses of fractions and decimals
• Reading and writing tenths and hundredths
• Representing tenths and hundredths as parts of an area

The focus points provide a concise, unpacked bulleted list of the big mathematical ideas related to understanding fractions. The list also suggests an order to which the big ideas should be explored. The unit also consisted of two similar lists for comparing and adding fractions.

The Common Core provides the following information for a similar topic to the Investigations (TERC, 2008) unit used by Violet – “extend understanding of fraction equivalence and ordering” (CCSSO & NGA, 2010, p. 30):

Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (CCSSO & NGA, 2010, p. 30).

That excerpt from the Common Core does not provide the needed support for conceptualizing a unit progression for several reasons. First, the language is more difficult to comprehend. Second, the excerpt seems to be focusing on generating
equivalent fractions only and not other big ideas like finding fractional parts and fraction notation. Third, the excerpt does not suggest an appropriate fraction model. Finally, it does not suggest an order in how topics should be introduced.

Violet was able to conceptualize and mobilize learning goals and progressions for units of study and series of instructional activities in her use of *Investigations* (TERC, 2008), because she mobilized *Investigations* (TERC, 2008) in a particular way. We know from the work of several researchers (e.g. Brown & Edelson, 2003; Remillard, 1999, 2000; Sherin & Drake, 2009), that different teachers use curriculum materials in numerous ways. In other words, each teacher has their own unique mobilization of curriculum materials, which means that providing a teacher with *Investigations* (TERC, 2008) does not guarantee that he/she will mobilize it in the same way that Violet did.

Teachers also have various orientations towards curriculum materials (Remillard & Bryans, 2004). That is, teachers perceive curriculum materials differently. That notion was evident in this study – Violet perceived *Investigations* (TERC, 2008) as a valuable resource, whereas Kathy and Olivia did not due to their other curricular knowledge. Additionally, not all Standards-based curriculum series are alike, which means that not all curriculum materials may provide the necessarily support. Stein and Kaufman (2010) found that different curriculum materials provided different levels (low or high) of support.

*Investigations* (TERC, 2008), a strong curricular support for Violet, was limited in that it did not provide efficient support for number choice progressions and student solution progressions. Violet relied on her CGI knowledge to provide that support, as
did Kathy and Olivia. CGI is one resource that provided the type of support needed. Are there other professional development opportunities that could do the same? Is there a way to provide support grounded in the Standard-based curriculum materials teachers are mandated to use? Further research will need to be conducted to find the answers to these questions.

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CHAPTER 4. PEDAGOGICAL DESIGN CAPACITY FOR TEACHING ELEMENTARY MATHEMATICS: KNOWLEDGE OF STUDENTS AND MOBILIZING STUDENT RESOURCES

A paper to be submitted for publication.

Tonia J. Land

Abstract

Several studies have found that developing an understanding of children’s mathematical thinking is a productive means for teachers to base instructional decisions and increase student achievement. Grounding this study in pedagogical design capacity (PDC) and studies that focused on teachers learning how to use children’s mathematical thinking for instructional decisions, I found that expert teachers elicited and attended to the details in children’s strategies. They distinguished between details that speak to a child’s conceptual understanding and details that speak to other mathematical practices. Furthermore, the teachers possessed general knowledge of students, general knowledge for a group, and individual knowledge of students; and mobilized that knowledge to make instructional decisions. Instructional decisions were based on moving students along a student solution progression. Student resources were also mobilized to introduce instructional topics, to carry out roles traditionally reserved for teachers, and to develop PDC. This study has implications for how to support teachers in their acquisition of knowledge of students and mobilizing student resources.

Introduction

Brown and Edelson (Brown & Edelson, 2003; Brown, 2009) introduced the PDC construct as a way to understand how teachers perceive and mobilize existing
resources to design instruction. *Perceive* indicates the ability to recognize, or notice, potential resources and *mobilize* highlights the importance of teachers’ abilities to act on or with those resources (Remillard, 2005). One potential resource that has emerged from the Cognitively Guided Instruction (CGI) research is greater knowledge about a framework for children’s mathematical thinking (Fennema et al., 1996; Fennema, Franke, Carpenter, & Carey, 1993; Franke et al., 1998). Through CGI professional development opportunities, some teachers changed their beliefs about mathematics teaching and learning in that they came to believe that instruction should be guided by knowledge about their students. Additionally, some teachers began to use children’s mathematical thinking as a basis for instructional design. In other words, teachers began to *perceive* children’s mathematical thinking as a valuable resource and *mobilized* that resource to craft instructional contexts.

Teachers acquire knowledge about their students through their teaching practice, but they do not generally use that knowledge due to it not being structured in a meaningful way (Carpenter et al., 1989, Franke et al., 1998). It is not clear, however, what kinds of knowledge teachers have of their students and when particular knowledge is useful for designing instruction. Other CGI studies (Fennema et al., 1996; Fennema et al., 1993; Franke et al., 1998) have documented that teachers who have participated in CGI use their knowledge of students to make instructional decisions. But again, it is not clear how, more specifically, teachers used their knowledge of students or other student resources when making instructional decisions. By studying four teachers (Kathy, Nancy, Olivia, and Violet) who had participated extensively in CGI
professional development, I examined what kind of knowledge these teachers had of students and how the teachers mobilized student resources.

My theoretical framework is grounded in the PDC construct and the research around teachers learning how to use children’s mathematical thinking in their instruction. Using a process of emergent coding, I was able to identify different types of knowledge teachers have of students and how they mobilized student resources to design instructional contexts. Implications include a developmental process for acquiring the different types of student knowledge and ways for teachers to gain the skills needed to elicit and mobilize student resources.

**Theoretical Framework**

PDC stems from the notion that teaching is a design activity (Brown & Edelson, 2003). “Teachers must perceive and interpret existing resources, evaluate the constraints of the classroom setting, balance tradeoffs, and devise strategies – all in pursuit of their instructional goals. These are all characteristics of design” (Brown & Edelson, 2003, p. 1). The notion that teaching is a design activity is a relatively new concept, but it is compatible with a range of cognitive theories that accentuate the interactions an individual has with the tools he/she uses to accomplish particular goals (Brown, 2009). Artifacts are human-created tools. “A key feature of artifacts is that they assist people in achieving goals they could not accomplish on their own” (Brown, 2009, p. 19).

PDC describes the capacity of a teacher to mobilize his/her existing resources in order to achieve instructional goals. Teachers have access to different tools or
resources (e.g. curriculum materials, professional development) depending on their setting and they use those resources differently depending on their experience, goals, and abilities. For instance, two teachers who have seemingly similar knowledge and skills can produce different enacted curricula, “because they possess very different capacities to create deliberate, productive designs” (Brown, 2009, p. 29). Brown talks about PDC in the context of curriculum materials:

This perspective is rooted in the notion that all teaching involves a process of design in which teachers use curriculum materials in unique ways as they craft instructional episodes. (Brown, 2009, p. 18)

This same notion (teachers use curriculum materials in unique ways) also applies to teachers’ use of children’s mathematical thinking. A major aspect of CGI is that teachers learn how to elicit student thinking. In other words, teachers learn how to question their students. Once teachers come to know and understand how their students are thinking, they can mobilize that thinking in different ways.

For example, Franke and her colleagues (1998) found that three teachers used children’s mathematical thinking in different ways after participating in CGI professional development. Ms. Nathan changed her practice by providing students with opportunities to solve a variety of problems, not telling students how to solve problems, and listening to student explanations. It was not evident for the researchers, however, if Ms. Nathan would use children’s mathematical thinking as a basis for making instructional decisions. Ms. Carroll changed in her beliefs in that she came to believe that she should build on her students’ existing mathematical knowledge, but did not change in how she used children’s mathematical thinking for instructional purposes.
The researchers contend Ms. Carroll did not possess the understandings of children’s mathematical thinking that supported her belief that she should build on children’s thinking. The third teacher, Ms. Andrew, was able to use children’s mathematical thinking as a basis for instructional decisions. “Ms. Andrew understood that she needed to build on her children’s mathematical thinking, she also realized that she needed to understand the children’s thinking in detailed and connected ways in order to accomplish this” (Franke et al, 1998, p. 78).

Using the PDC construct, I interpreted how the three teachers (Ms. Nathan, Ms. Carroll, and Ms. Andrew) differed in the ways in which they perceived and mobilized children’s mathematical thinking. From the study results, it seems that Ms. Nathan did not perceive or mobilize children’s mathematical thinking as a resource in her instruction. Ms. Carroll perceived that children’s mathematical thinking was a valuable resource, but was not able to mobilize children’s mathematical thinking, because she did not fully understand it. Ms. Andrew perceived children’s mathematical thinking as a valuable resource and was also able to mobilize it to make instructional decisions.

Fennema and her colleagues (1993) researched specifically how one teacher, Ms. J, was using children’s mathematical thinking. The researchers found that Ms. J made sure students worked at their own ability, and that her students were supportive of each other’s thinking. If a problem was too difficult, Ms. J would change the numbers or save it for another day. Ms. J used all the problem types, but varied the problem types and size of numbers based on what she knew about the children, and made extensions to multiplication, division, and fractions when she felt students were ready. Contexts of
problems were chosen based on children’s interests. Ms. J would carefully record the problem types students could solve and the number choices.

The researchers (Fennema et al., 1993) found many aspects of Ms. J’s practice that mobilized children’s mathematical thinking. However, the study brought up, for me, several additional questions: How did Ms. J come to know each student’s ability level? How did she make sure that students worked productively? What did Ms. J know about her students to cause her to vary the problem type and number choices?

In a longitudinal study about teachers learning to use children’s mathematical thinking in their instruction, Fennema and colleagues (1996) found that there were five different levels of cognitively guided beliefs. Table 1 outlines each of those levels:

<p>| Table 1: Levels of Teacher Cognitively Guided Beliefs (Fennema et al, 1996, p. 413) |</p>
<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Does not believe that children can solve problems without instruction or believe in children’s use of their own strategies.</td>
</tr>
<tr>
<td>2</td>
<td>Struggling with the beliefs that children can solve problems without instruction and should use their own strategies.</td>
</tr>
<tr>
<td>3</td>
<td>Believes that children can solve problems without instruction. Believes only in a limited way that his or her students’ thinking should be used to make instructional decisions.</td>
</tr>
<tr>
<td>4-A</td>
<td>Believes that children can solve problems without instruction in specific domains included in workshops and that he or she should use knowledge of his or her students to guide interactions with them.</td>
</tr>
<tr>
<td>4-B</td>
<td>Believes that children can solve problems without instruction across mathematics content domains and that what he or she knows about children’s thinking should inform his or her decision making, both regarding interactions with the students and curriculum design.</td>
</tr>
</tbody>
</table>

Interpreting these levels of teachers’ beliefs through the PDC construct, Level 1 and 2 teachers do not perceive children’s mathematical thinking as a resource for instruction. Level 3 teachers are beginning to recognize that children’s mathematical thinking can be a resource, while Level 4-A and 4-B perceive children’s mathematical thinking as a
resource. Again, it is not clear what knowledge teachers have of students. In that same study, researchers identified five levels in how teachers used or mobilized children's mathematical thinking:

Table 2: Levels of Cognitively Guided Instruction (Fennema et al, 1996, p. 412)

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Provides few, if any, opportunities for children to engage in problem solving or to share their thinking.</td>
</tr>
<tr>
<td>2</td>
<td>Provides limited opportunities for children to engage in problem solving or to share their thinking. Elicits or attends to children’s thinking or uses what they share in a very limited way.</td>
</tr>
<tr>
<td>3</td>
<td>Provides opportunities for children to solve problems and share their thinking. Beginning to elicit and attend to what children share but doesn’t use what is shared to make instructional decisions.</td>
</tr>
<tr>
<td>4-A</td>
<td>Provides opportunities for children to solve a variety of problems, elicits their thinking, and provides time for sharing their thinking. Instructional decisions are usually driven by general knowledge about his or her students’ thinking, but not by individual children’s thinking.</td>
</tr>
<tr>
<td>4-B</td>
<td>Provides opportunities for children to be involved in a variety of problem-solving activities. Elicits children’s thinking, attends to children sharing their thinking, and adapts instruction according to what is shared. Instruction is driven by teacher’s knowledge about individual children in the classroom.</td>
</tr>
</tbody>
</table>

There are two noteworthy distinctions between Level 4 (A and B) teachers and teachers at the other levels. First, Level 4 teachers elicit their students’ thinking, thus providing them knowledge of their students. Second, Level 4 teachers mobilize children’s mathematical thinking to help make instructional decisions. What types of instructional decisions were made, however, is not clear.

In a follow-up to the Fennema et al. (1996) study, Franke and colleagues (2001) studied how participants in CGI professional development continued to implement the principles of the program four years after it ended. Of the 22 participants, all continued some use of children’s thinking, but 10 continued to learn and engaged in generative growth. Generativity refers to the idea that an individual can continue to add to his/her
understanding. In this case, ten teachers continued to add to their understanding of children’s mathematical thinking. The researchers described how teachers could become generative in their knowledge of children’s mathematical thinking:

If teachers can learn to talk to their students about their thinking, puzzle about what the responses tell them about students’ understanding, decide how to use this knowledge in planning instruction and interacting with students, and figure out how to learn more about the students' thinking, then the teachers’ own learning can become generative. (Fennema et al., 1996, p. 656)

I conceptualize generativity as a teacher continually increasing his/her PDC. Teachers are able to continue to develop their PDC if they engage in the practices described in the above quote.

Also investigating how teachers mobilized children’s mathematical thinking, Jacobs and colleagues found that there were three different levels in how teachers used children’s understanding as a basis for responding to students (Jacobs, Lamb, & Philipp, 2010). Attending to, interpreting, and responding are three interrelated skills that comprise the construct of professional noticing of children's mathematical thinking (Jacobs et al., 2010). Participants were asked to provide details for student solutions to a particular problem (attend to), explain what they learned about the students (interpret), and pose a follow-up problem (respond). Within the broader construct of noticing, responding refers to teachers using what they have learned about children’s understandings from a specific situation when designing subsequent instruction (Jacobs et al., 2010, p. 173). The researchers identified three levels of responding: robust evidence of teachers using children’s understandings, limited evidence of teachers using children’s understandings, and lack of evidence of teachers using
children’s understandings. A response categorized as using robust evidence explicitly considered the child’s existing strategy, anticipated a possible next strategy, and provided a next problem to encourage students to use a more sophisticated strategy (Jacobs et al., 2010). This study provides us with some information about how teachers can make instructional decisions based on children’s mathematical thinking.

These studies around children’s mathematical thinking provide substantial evidence that children’s mathematical thinking can be a resource for teachers to develop instruction specific to their students and to continually develop their PDC. Before teachers can mobilize children’s mathematical thinking, however, they first need to have knowledge of their students. That is, teachers need to question and listen to their students as well as make sense of students’ solutions. To provide a framework for thinking about knowledge of students, I return to the professional noticing of children’s mathematical thinking construct.

Jacobs and colleagues (2010) identified one skill within that construct as attending to children’s strategies. Attending to children’s strategies refers to how teachers pick up on the “mathematical details in children’s strategies” (Jacobs et al., 2010, p. 172). The researchers distinguished between teachers who showed evidence of attending to children’s strategies and those who showed lack of evidence of attending to children’s strategies. Four different groups of teachers were asked to examine three examples of student work in response to the following problem: “Todd has 6 bags of M&M’s. Each bag has 43 M&M’s. How many M&M’s does Todd have?” (Jacobs et al., 2010, p. 178). Then, the four teacher groups were asked to describe in detail what they
thought each child did. Each separate description was scored either as 1 for evidence of attending to the children's or 0 for lack of evidence. Below is an example of each type of description:

**Example from teacher who showed evidence of attending to a child's strategy**

I think that Cassandra made 6 circles with the number 43 in each one. Then she combined every 2 circles by adding the 10s together and then adding the 1s together for each pair. Next, she added the 10s (80 + 80) and the 1s (6 + 6) for the first 4 circles. After adding 160 + 12 to equal 172, she needed to add 86. Knowing that 80 + 20 = 100 (a familiar #), she took 20 from the 70 to get to 100. Then she figured she needed to add the 52 left from the 172. What she forgot about was the 6 left from the 86. That’s why her answer is off by 6. (Jacobs et al., 2010, p. 183)

**Example from teacher who showed lack of evidence of attending to a child’s strategy**

Cassandra’s work is very practical and simple too, but it’s not understandable. Why did she subtract 20 and where did she get the 70 from? Her work was not very clean, and she probably lost herself with too many numbers and lots of adding. (Jacobs et al., 2010, p. 184)

The first example described in detail and captured mathematically what Cassandra did to solve the M & M problem. The second example did not capture any of the mathematical details and made assumptions (e.g., “she probably lost herself”) about Cassandra’s understanding.

Attending to children’s strategies allows teachers to gain knowledge of those strategies. In the first description of Cassandra's strategy, it is clear that the participant knew how Cassandra solved the M & M problem in detail, which leads to some knowledge of Cassandra. When I examined Cassandra’s work it led me to believe, for instance, that Cassandra can represent multiplication problems as evidenced by her drawing 6 circles with 43 in each circle. Cassandra can also decompose two-digit numbers into tens and ones. Examining Cassandra’s strategy also provides me with
some knowledge about children in general. When solving multiplication problems, a child might draw a representation of that problem and decompose two-digit numbers into tens and ones.

One of the goals for Jacobs and her colleagues was to “identify group differences among the four participant groups to capture the development of professional-noticing expertise” (Jacobs et al., 2010, p. 181). The four participant groups consisted of: prospective teachers, initial participants (K-3 teachers about to start sustained professional development focused on children’s thinking), advancing participants (2 years of same professional development), emerging teacher leaders (4 or more years of same professional development). The group differences are shown in Table 3.

Table 3: Group means (Standards Deviations) for the Attending to Children’s Strategy Skill (Jacobs et al., 2010, p. 181)

<table>
<thead>
<tr>
<th>Component skill</th>
<th>Scale</th>
<th>Prospective Teachers</th>
<th>Initial Participants</th>
<th>Advancing Participants</th>
<th>Emerging Teacher Leaders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attending to children’s strategies</td>
<td>0–1</td>
<td>0.42 (0.50)</td>
<td>0.65 (0.49)</td>
<td>0.90 (0.30)</td>
<td>0.97 (0.17)</td>
</tr>
</tbody>
</table>

The researchers contend that these findings provide evidence that “expertise in attending to children’s strategies grew with teaching experience and continued to grow with 2 years of professional development” (Jacobs et al., 2010, p. 182). In others words, attending to children’s thinking is a developmental process.

Using the characteristics of Cognitively Guide Instruction Level 4 teachers (Table 2, Fennema et al., 1996), I identified two types of knowledge of students to look for in my data analysis: general knowledge of his/her students and knowledge of individual
students. Level 4-A teachers based instruction on groups of students. On the other hand, “Level 4-B teachers had more detailed knowledge of each child’s thinking than Level 4-A teachers and seemed always to be aware of the impact that instruction would have on each individual” (Fennema et al., 1996, p. 421).

That process of questioning and listening to students allowed teachers to gain knowledge of their students, which can be used as a basis for making instructional decisions. Wanting to understand these processes more specifically led me to ask the following two questions:

**Research Questions**

- What knowledge do Level 4 teachers have of students?
- How do Level 4 teachers mobilize student resources?

**Methodology**

**Participants**

Kathy, Nancy, Olivia, and Violet\(^3\) teach at two different schools within the Lakefront School District. Kathy teaches 1st grade; Nancy is in a 2nd/3rd-grade multi-age classroom; Olivia teaches 2nd grade; and Violet is in a 4th/5th-grade multi-age classroom. At the time of this study, Kathy, Nancy, and Olivia had been participating in CGI professional development for five years and have continued to participate in the training each year. In addition, they facilitated CGI professional development for other teachers in the Lakefront School District. Violet had been participating in CGI training every year for three years. Before CGI, Nancy and Violet implemented the Standards-

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\(^3\) All names are pseudonyms.

**Data Sources**

Kathy, Olivia, and Violet were each observed on six occasions. After each classroom observation, I watched the videotape and developed a video-stimulated recall interview for each of the six observations. These interviews were designed to gain knowledge of possible key dimensions of PDC. For each segment of classroom observations (opening routine, launch of activity, student exploration, and strategy sharing), I would ask each teacher to state and explain their intended goal, provide details in how that segment was designed, and describe what they noticed about the student learning. For instance, the following five questions were asked of Violet for one part of one observation:

6. Can you tell me about the game, Roll Around the Clock? How does the game fit in with the goals for the unit and year? Did only some students play? Why?
7. Tell me about the beginning problem. Why did you develop the beginning problem? How does the problem tie into the curriculum and your goals for the year?
8. What do you notice about Kael’s strategy? Where do you want him to progress from here?
9. What do you notice about Leslie’s strategy?
10. What is your goal with your conversation with Liev?

**Data Analysis**

In previous work (Land & Drake, 2010), I drew on the work of Remillard (2005)
and Davis et al. (2007) to generate a list an initial list of possible key dimensions of PDC to investigate the PDC of one teacher. During that study, my co-author and I generated a second list of possible key dimensions of PDC through a process of emergent coding. We used this list as a coding scheme to analyze interview data:

- Knowledge of Curricular Resource
- View/Perception of Curricular Resource
- Mobilization of Curricular Resource
- Knowledge/Perception of Students
- Mobilization of Student Resources
- Beliefs
- Goals
- Tolerance for Discomfort
- Previous Teaching Experiences
- Subject Matter Knowledge

Once final codes and definitions were established, I recoded all interview data. A second coder coded 34% of the data (14 of 41 pages of interview transcripts). The two coders were in disagreement on 10 segments of data comprising 13% of the 14 pages of transcripts. Of those disagreements, we reached 90% consensus. The remaining segment pertained to Nancy using the CGI framework to gain knowledge of her students. The segment remained under dispute, because one coder felt it should be coded as mobilization of student resources while the other thought it should be coded as knowledge of students. It was agreed that the segment could be double-coded.

Moving forward, I decided not to double-code data, and that codes entailing how teachers perceived or mobilized a particular resource took precedence over other codes.

After transcribing all interview data for this study, I coded all transcripts using the above list of possible key dimensions as a coding scheme. Then, I pulled all the data
coded as knowledge of students and mobilization of student resources. Table 4 provides the definition and an example of each of these codes.

*Table 4: Knowledge of Student and Mobilization of Student Resources Codes (Land & Drake, 2010)*

<table>
<thead>
<tr>
<th>Code</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge/Perception of Students</td>
<td>For instances when the participants were talking about their students with regards to what they knew or how they learn. It was also used when participants talked about how they perceived or felt about their students.</td>
<td>“You know, they enjoy math. And they see themselves as mathematical thinkers. And, um, I think their confidence about math has improved. And all of those are positives, because if they have that attitude, that they can do it, and it's not too hard. There's not a problem I can't solve.”</td>
</tr>
<tr>
<td>Mobilization of Student Resources</td>
<td>For instances when participants used a child as a resource for meeting instructional goals and making instructional decisions. Instances where the participant talked about students sharing strategies in small and large groups were included. This code also included instances where a student's mathematical discourse would change the direction of the lesson. This code was also used to describe instances when children's mathematical thinking was used to develop PDC.</td>
<td>“And I just think their interactions with each other. I think that's been the biggest change too. Also using CGI is that my kids talk a lot during math to one another.”</td>
</tr>
</tbody>
</table>

For all text coded as knowledge of students and mobilization of student resources, I developed a set of sub-codes, using my theoretical framework as a guide, to capture more specifically what kinds of knowledge the teachers had of their students and how
they mobilized student resources. For knowledge of students, those codes consisted of detailing students’ strategies (Jacobs et al., 2010), general knowledge of students (Fennema et al., 1996), general knowledge of a group, and knowledge of individual students (Fennema et al., 1996). Table 5 provides a list of the sub-codes as well as definitions and examples for each sub-code related to knowledge of students.

<table>
<thead>
<tr>
<th>Sub-Code</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detailing Strategies</td>
<td>For instances when the teachers were describing students’ strategies.</td>
<td>“She’s counting by 1s, and the other person is counting by 10s. She didn’t actually cross out the last number. She started at 11, but she didn’t cross out the 60.”</td>
</tr>
<tr>
<td>General knowledge of students</td>
<td>For instances when the participants talked about what they knew about children in general, but the knowledge did not necessarily pertain to their current classroom.</td>
<td>“Now we know why it’s so difficult for kids when they get older, because they don’t have really good beginning experiences with fractions.”</td>
</tr>
<tr>
<td>General knowledge of a group</td>
<td>For instances when the participants talked about what they knew about children in a specific group. The groups consisted of the participants’ former or current classrooms.</td>
<td>“My kids right now, they can count by tens. I mean they’re pretty proficient.”</td>
</tr>
<tr>
<td>Knowledge of individual students</td>
<td>For instances when the participants talked about what they knew about a specific child.</td>
<td>“He’s not going to be confident that he made an error. He’s going to try and manipulate the numbers to match what he has. He would.”</td>
</tr>
</tbody>
</table>

For mobilization of student resources, the sub-codes consisted of instructional decisions (Fennema et al., 1996), instructional topics, roles traditionally reserved for the teacher, and developing PDC. Table 6 provides a list of the sub-codes as well as
definitions and examples for each sub-code related to mobilization of student resources.

Table 6: Sub-codes for Mobilization of Student Resources

<table>
<thead>
<tr>
<th>Sub-Code</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructional Decisions</td>
<td>For instances when a participant made an instructional decision based on a student’s current or former strategy.</td>
<td>“I want her to stop direct modeling. She’s beyond direct modeling. She doesn’t need to use cubes to solve this problem.”</td>
</tr>
<tr>
<td>Instructional Topics</td>
<td>For instances when a student(s) question or thinking became the topic of instruction for the entire class.</td>
<td>“Yes, they had that theory. That’s what they were wondering. They wanted to find out if the pentagon has five lines of symmetry.”</td>
</tr>
<tr>
<td>Traditional Teacher Roles</td>
<td>For instances when the participants talked about asking a student to carry out a role that is traditionally a teacher's role.</td>
<td>“Because Linda was a person who I think... I knew would be able to see if it was right or it wasn’t right. So, Linda could have been the person to question her rather than me.... Because she takes on that role of teacher.”</td>
</tr>
<tr>
<td>Develop PDC</td>
<td>For instances when the participants developed their PDC by attending to or engaging with a child's strategy or question.</td>
<td>“Watching what they do in their math thinking and how they solve problems... And, it’s interesting because I keep learning more about things. I think fractions are a new area that I’ve really started to understand more as an adult by teaching it with my kids.”</td>
</tr>
</tbody>
</table>

Results

My results are organized around the two research questions. Regarding knowledge of students, I begin by describing the teachers’ attention to children’s mathematical thinking by detailing their strategies. Then, I present results on three different kinds of knowledge the teachers had of students: general knowledge of
students, general knowledge of a group, and knowledge of individual students. I also present results on the ways in which the teachers mobilized student resources – to make instructional decisions, for instructional topics, to take on roles traditionally reserved for teachers, and to continually develop PDC.

**Detailing Children’s Strategies**

In all classroom observations, students were to use any strategy they felt comfortable with to solve a given problem. In the data collection process, I was able to videotape numerous students working. Therefore, I was able to capture several students’ strategies and the teachers working with individual students. In the video-stimulated recall process, I would ask the teachers to describe what they noticed about the children’s strategies and/or their goal for their individualized attention. In those instances, the teachers provided details about the students’ strategies as Olivia does in the excerpt below. In the excerpt, Olivia described students’ strategies for dividing various numbers by two.

**Olivia:** But a couple of kids… they looked at the tens, and then the ones, which wasn’t quite the same as the chunking. Then I had a group of kids who were doing the chunking. They’d go up by 10. They’d each get 10. Oh, they have 11 left. So then, they might do 5 and 5. So they were doing it until they got close to the number. Then, I would say that the kids were using their doubles to get close. Then, I had the kids looking at the separate places - the tens and the ones. And they were doing some into the hundreds.

**Tonia:** So, say there were 64. They were doing 30 and 30 and 2 and 2?

**Olivia:** Yes, they’d look at 60. They knew that 30 and 30 was 60, and 4 was 2 and 2. And so that’s how they got that.

**Tonia:** How would they use doubles?

**Olivia:** Let’s say for 19, today they did the 9 and 9 to get close to 19 and then there was one left over and they divided it into half. A lot of them were using their doubles facts to solve it… They were looking at what number was close to 19 - not necessarily breaking it to 10 and 9. See what I mean?
In the above excerpt, Olivia attended to the strategies of three different groups of students – those separating tens and ones, those using “chunking”, and those using doubles.

While detailing the strategies, the teachers distinguished between students’ conceptual understanding and getting the right answer. Consider the following statement from Olivia when talking about how her students solved a problem that involved dividing 15 objects between two people:

They were dividing 15 by 2. So, they would have 7 wholes and then that 1. You could see their work. You could see the short stick. They said it was 8. It’s not that they [students] didn’t know what to do. They understood that. They didn’t know what to call that piece. They know that’s a piece. They don’t know what it’s called.

Students divided the 15 objects by distributing tallies one by one or by larger numbers (e.g., 2 or 5). Once they distributed 14 objects, they would have one left over. The students would then split that one leftover in half. Because students did not know what to call that half a piece (the short stick), they deemed it as a whole, added it to 7, and called it 8. As Olivia pointed out, students had conceptual understanding. They did not get the right answer, however, because they did not know what to call the short stick.

This example highlights the importance of not only being able to detail strategies, but identifying which details provide evidence of conceptual understanding and which details are about other mathematical practices.

At the same time that Olivia was detailing her students’ strategies, she was also gaining knowledge about her students. For example, she knew that it was not the process that her students were struggling with, but what to call the pieces. My next
sections describe general knowledge of students, general knowledge about a group, and knowledge of individual students.

**General Knowledge of Students**

The Fennema et al. (1996) study identified Level 4-A teachers as ones who considered general knowledge of students as they made instructional decisions. I also found that the teachers used general knowledge about students, but that knowledge was not always specific to students in the teachers’ classrooms. For instance, Kathy spoke about students’ abilities to transfer knowledge to a different context:

> Sometimes they [students] can solve it [number choice] when it’s written as an equation, but then you can use those exact same numbers in a CGI problem, and they’ll direct model it... When you start doing both of those things, um, you can’t take it for granted that if they do it in this venue that they're going to show those same strategies when it’s in a different setting for them.

This statement was referring to students in general. Kathy is saying that every year, no matter who your students are, you cannot “take it for granted” that students can transfer what they know from one context to another. In another example, Kathy spoke about children’s difficulties with adding fractions:

> So, if I had ¾ + ¾, they can draw the model and shade in the ¾. And they can draw this model and shade in the ¾, but when you want to know how much it is together, they try to draw these arrows and lines and it’s confusing for them... That’s really hard to kids to make that leap, because they can't hold that in their heads. It’s kind of like making tens and ones. Kids learn that by doing it over and over with objects. They don't have that opportunity with fractions.

General knowledge about students is something that teachers can come to expect to possibly be the case with any classroom. Then, there is knowledge teachers can have that pertains specifically to a group.
General Knowledge of a Group

Teachers had general knowledge about students, but that knowledge applied, more specifically, to students who were in current or former classrooms. For instance, Nancy talked about her students’ abilities to split an even number:

And so, cause I think most of them could split a whole number very easily, or an even number very easily. You know, half of 10, 5. Half of 8- I think they can do that easily, but 9. How do I solve that?

Most of the students in Nancy’s classroom, at that time, could split an even number easily, but an odd number would be more difficult. In another example, Nancy spoke about strategies her students would use for subtraction:

Because none of my kids would trade. They would all use negative numbers or add up. They would not do it that way [trade].

General knowledge about a group is different than general knowledge of students, because it is specific to a particular group of students. A teacher can have a large knowledge base about students in general, but only some of that knowledge pertains to his/her current classroom. Furthermore, general knowledge of a group changes. For instance, Kathy made the following statement:

My kids right now, they can count by tens. I mean they’re pretty proficient. They’ve used the hundreds charts.

At the time of the observation, Kathy’s students could count by ten. The way Kathy talked about her students’ abilities to count by ten suggests that there was a time when her students could not count by ten.

It is not clear whether general knowledge of a group is a smaller subset of a general knowledge base about children, or if it is an accumulation of knowledge about
individual students. If general knowledge of a group is a smaller subset, then the teachers have a larger knowledge base about students, and then they choose which parts of that knowledge base pertains to her students and ignore, at the time, knowledge that does not pertain to her students. If general knowledge of a group is an accumulation of knowledge about individual students, then the teacher has knowledge about each individual student and then accumulates all those pieces to generate a sense of her students as a group.

**Knowledge about Individual Students**

Like the Level 4-B teachers in the Fennema et al. (1996), Kathy, Nancy, Olivia, and Violet had knowledge of individual students. This type of knowledge consisted of having knowledge of what type of strategies students tended and needed to use, and knowledge of students’ dispositions. In one observation, Violet posed the following problem:

**Baseball Card Problem:**
Leonard has ____ baseball cards. He gives ¼ to his friend. How many baseball cards did he give to his friend?

8  24  44  60  100  144

After choosing one or more number choices, students used a variety of strategies.

Stacey and Leslie both direct modeled the problem using cubes. When I asked her about each of these students, Violet had this to say:

**Statement about Stacey**
I want her to stop direct modeling. She’s beyond direct modeling. She doesn’t need to use cubes to solve this problem. She’s very comfortable... I think she solved the rest of them using that strategy right there (referring to video clip of Stacey using multiplication)... I know she has a really good understanding of division, and she uses multiplying to divide. And so, I know she knew that. And
she doesn’t need cubes to divide. So that’s why I wanted to say, how else could you do it?

**Statement about Leslie**
I need to work on division with her. I don’t think she’s even ready to move beyond direct modeling at this point especially with division and to see that relationship. I’m not sure she really understands the one-fourth part. That’s really abstract. Um, the sets are hard for her. So, lots of experiences with this, and lots of use with direct modeling. That’s what she needs.

Violet knew that Stacey had a “good understanding of division” demonstrated by Stacey’s use of multiplication to divide and her former nonuse of direct modeling. On the other hand, Violet felt that Leslie did not have good understanding of division. Based on her knowledge of each individual student, Violet made different instructional decisions for each even though they were both direct modeling for the Baseball Card Problem. Because Violet had individual knowledge of students, it allowed her to ensure that each child was working productively.

Knowledge of individual students allowed the teachers to make more accurate interpretations of students’ understandings. In one observation, Rebecca drew 37 circles when adding 18 and 19. Kathy offered the following interpretation of Rebecca’s strategy:

Then on this one, the 18 and 19, which is really kind of tricky, she wrote 37 circles. I think on this one, she thought of it has 20 and 17 ... You wouldn’t draw tens and ones if you were compensating. How do you do a pictorial representation of that? You can’t. So, she just drew 37 singles... So, if you didn’t know anything about her at all...you would mark this down as a pretty low-level direct modeler. She’s actually compensating. So, that’s the power of knowing the kids and talking with her.

As Kathy pointed out, if she did not know Rebecca, she would have misinterpreted Rebecca’s understanding of addition, which would consequently affect how she made
instructional decisions. Additionally, Kathy understood the difficulty of communicating the compensating strategy for Rebecca and possibly other students.

Knowledge of individual students did not always entail what students understood mathematically. Many times the teachers talked about students’ dispositions. For instance, Kathy had this to say about Lynn:

She's got a perky personality. She's really got a lot of math ability. She's an interesting one, because sometimes she'll make errors when she really doesn't need to be making errors just because she wants it for attention purposes. She's really pretty good on a lot of things, but it's interesting because when I have people in the room and things like that, sometimes she will fake having issues so someone will sit down and listen to her and go through it. As you talk with her and work with her more, you'll see that she has some good understandings and that going on. She's very easily swayed to using somebody’s strategy if she perceives that that person is getting attention... The nice thing about CGI is that you know your kids' personalities. Everything is about math, but sometimes it's about making that child have a more confident personality and being comfortable with what their thoughts are, and sharing what their thoughts are. Even though I’m talking about things that are not mathematical, CGI can be a venue to help kids get over those kinds of things.

Through her questioning, Kathy learned several things about Lynn - she had a lot of math ability, she made errors in an attempt to receive attention, and she was easily swayed to use others’ strategies. Because Kathy had been mobilizing CGI strategies and practices, she contended Lynn became a more confident mathematician.

Kathy mobilized CGI strategies to acquire knowledge of individual students, which was true for all the teachers, but Nancy and Violet also mobilized particular features of Investigations (TERC, 2008) to acquire individual knowledge of students:

**Nancy**
The questions in the teacher’s guide or whatever were very helpful to me to even what type of questions even to ask. Um, because I think I started more with the how did you get that? How did you think about that? Where before it was more - what was your answer? And that’s changed a lot. I mean that whole
*Investigations* changed that piece for me. I became more concerned about the child's processing, rather than them being right or wrong.

**Violet**
And so, I literally would read exactly what they would say - teacher says this..., but I started using that and found that the kids were really thinking about what they were doing and using strategies that I hadn't even thought about before in my whole life...

Both of the above excerpts provide evidence that the questions in the teacher’s guide afforded a means for Nancy and Violet to gain knowledge of individual students. Kathy, Nancy, Olivia, and Violet had significant knowledge of students in general and individual knowledge of students, but maybe more importantly, they were able to draw on that knowledge to design instructional contexts.

**Mobilizing Student Resources**

In other work (Land, Chapter 3), I found that Kathy, Olivia, Nancy, and Violet mobilized four different progression types - unit, series of instructional activities, number choice, and student solutions – to make instructional decisions. With regards to student solutions, the teachers made instructional decisions to move students along the progression. In other words, the teachers wanted the students to use and understand more sophisticated strategies. Below, Kathy and Olivia talk about their planning based on children's thinking:

**Kathy**
If you know the progression that students follow through, and you’ve seen that progression, it makes your planning so much more beneficial for them.

**Olivia**
It’s [instruction] not because well the book said to. That’s why I think using CGI is so much easier than teaching out of a textbook. I just think that it's easier, because you’re really looking at your kids and what they know going that way.
This section describes how the teachers mobilized knowledge of students to make instructional decisions and other ways they mobilized student resources - instructional topics, in roles traditionally reserved for teachers, and developing PDC.

**Instructional Decisions**

Instructional decisions were based on what strategies students were currently using. Consider the following statements from Violet:

> Sometimes, I'll start with a CGI problem that is usually tied into the *Investigations* lesson. For yesterday, for example, I knew my focus was going to be adding fractions. I usually start by writing a problem just to kind of see the kinds of strategies they're going to use. Then, from there, I make some decisions... What do they need?

In the above quote, Violet talked about assessing what kind of strategies students use before making instructional decisions. In other words, Violet needed to determine where students lie on a student solution progression before she can make instructional decisions about topics needing to be covered. For example, if students are using a strategy such as direct modeling, Violet will spend more time on that topic. If students are using more sophisticated strategies successfully, Violet will spend less time on that topic. This made Violet's planning, and the others, a day-to-day event, because the teachers needed to see what strategies the students were using before they could plan what was going to happen the next day. When asked what she was going to do the next day after one observation, Olivia stated:

> I think I want to see how they share first. You know to see. Everyone I've had come up to the table, I've checked their halves so far, but I haven't checked anybody with the fourths. So, I want to see what they're doing with that first.

Similarly, students' strategies determined how fast instruction moved as indicated by
Kathy:

Now, we’re more into structuring and sequencing where it fits our kids and how fast they move.... I would look at this [student work] and kind of see where my kids are at. If I’m noticing on the first three choices, which are pretty much counting by 10s, like 10 + 60. If most of them are fairly successful with that, then, I’m going to take that number out. If this 10 and 70 was really difficult for my kids, then the next time I would have multiples, like 20 and 50. I would have 40 and 80.

Instructional decisions were also made to encourage students to use particular strategies. Olivia described that process below:

We look more at what kind of strategies we want the kids to use. Then our number choices would tell us if they are going to count up by tens. That’s what we want them to do. Are they going to do that? Or do numbers close to 100. So, do they know like 64 + 36? Are they using that knowledge to get to 100? Or are they counting by 10s to get to 100?

The teachers will choose problems and number choices based on the types of strategies they want students to use. In the excerpt, Olivia talked about composing numbers to get to 100 and counting by 10s. In another observation, Olivia posed a part-part-whole problem. Olivia liked to pose these types of problems, because they “tell you a lot about your kids, because you can't directly model that problem. I think it really pushes them to think outside the box - how am I going to solve this problem?” In posing part-part-whole problems, Olivia wanted to push her students to use more sophisticated strategies than direct modeling.

In an another example, Nancy posed Hank’s Sister’s Problem:

Hank has ___ dollars in the bank. His sister Emily wants to borrow ____ dollars. How many dollars will Hank have left if he loans the money to Emily?

A (12, 9)  B (33, 22)  C (90, 61)  D (419, 321)
A (16, 13)  B (44, 25)  C (150, 53)  D (672, 480)
Nancy’s Rationale for Hank’s Sister Problem

For Hank's sister problem I chose the numbers because the day before Billy had shown his strategy and explained it to the class. Here is an example of what Billy did:

\[ 33 - 22 = \_\_\_\_ \]
\[ 32 - 22 = 10 \]
\[ 10 + 1 = 11 \]

Another student did this:

\[ 33 - 22 = \_\_\_\_ \]
\[ 33 - 23 = 10 \]
\[ 10 + 1 = 11 \]

They [students] started to show some understanding of what he had done, so I thought I would try. The numbers in the ones column are close together, in a hope that some of them would try the strategy that they were shown the day before. We've been talking about looking at the number and making the strategy that would make the most sense with those numbers, rather than always resorting to the same strategy.

Because Nancy's students were demonstrating some understanding of compensation, she posed numbers that were “close together in the ones place.” For instance, 44 is only one away from 25 in the ones place and 419 is two away from 321 in the ones place if one goes over the 20. Therefore, Nancy is designing a context where the compensation strategy is easily noticed and applied by students.

**Instructional Topics**

Students’ strategies were also a means for the teachers to address particular topics when opportunities arose. For instance, a student in Violet’s class, Liev, used the following strategy to find \( \frac{1}{4} \) of 8 baseball cards:
Liev’s strategy for finding $\frac{1}{4}$ of 8 baseball cards

\[
\begin{align*}
\frac{1}{2} + \frac{1}{2} &= 1 \\
1 + 1 &= 2
\end{align*}
\]

Liev used an area model instead of a set model to solve the above problem.

Nevertheless, after Liev explained his strategy, Violet took that opportunity to ask students if $\frac{4}{2}$ was equal to 2 and facilitated a discussion around that question. I asked her about that episode:

**Violet:** So, he was adding up those halves to four halves, and I wanted him to know... I don’t know if they’ve seen those improper fractions before. That’s new to them. So four halves, is that the same as two? How do you know that? So they can see two representations of the same number. Four halves is the same.

**Tonia:** So, are improper fractions a focal point for *Investigations*?

**Violet:** I think they touch on it here and there, but it’s one of those moments like here’s a point to talk about it. I don’t want to teach improper fractions, here’s how we convert improper fractions to, you know, mixed numbers. Here’s how we... whatever. Here’s a perfect place to pull it in, because it’s a natural place to discuss it.

Violet explained that a sharing session is an ideal opportunity to discuss additional mathematical topics.

Olivia talked about a time where she was confronted with a student question she could not answer immediately. The question was, “Is 28 divided by 2 the same as 28 divided by $\frac{1}{2}$?” Olivia described her thoughts:

The sad thing is, I was trying to find the best way to explain to the kids why they were not the same. The only thing I could think of was the “inverse and multiply” method which is what I memorized from my early days. Thus, it was a good reason to tell the kids that it is better to understand what they are doing than to simply memorize procedures.
When the question was posed, Olivia asked her students to go home, think about it, and maybe ask their parents. Then, Olivia asked her colleague to help her. Together, they designed an activity where they distributed ½ cookie to each student, then counted the number of students that were given ½ a cookie. Thus, 28 cookies were divided in half and distributed to 56 students. Olivia had this to say after instruction:

Needless to say, it was a great discussion and the kids actually got it!! Once I wrote the number sentence to go with the numbers, they were seeing that they were simply doubling the first number. They asked what would happen if we divided by fourths. Then we did it by thirds. I cannot believe what kids are capable of doing at this age and what I learn from them every day!

This example illustrates how a student question became the instructional topic for the next day, but it also illustrates an instance where Olivia mobilized student resources to develop her PDC.

**Developing PDC**

When confronted with the question about if dividing by two is the same as dividing by half, Olivia did not know how to address that concept. Instead of dismissing the question or relying on what she already knew, Olivia took the student’s question as an opportunity for learning. The only instructional activity that Olivia knew how to design to explore division of fractions was to explain the invert and multiply procedure, which would be inappropriate for second graders. By consulting her colleague, Olivia was able to design a much more appropriate activity – distributing ½ of a cookie to each second grader and counting the second graders, writing an equation, and looking for relationships between the dividend and the quotient.
Other times, students’ strategies led the teachers to acquire more mathematical knowledge. Violet talked about a time where a student subtracted left to right. She had never seen that strategy, but it made so much more sense mathematically. Nancy talked about her learning from students:

Watching what they do in their math thinking and how they solve problems... And, it’s interesting because I keep learning more about things. I think fractions are a new area that I’ve really started to understand more as an adult by teaching it with my kids.

Because the teachers facilitated instruction in a way that allowed students to develop their own solution paths, it provided them with opportunities for learning, both pedagogically and mathematically. These opportunities would not have arisen if the teachers asked students to use one particular solution method.

**Roles Traditionally Reserved for Teachers**

In traditional mathematics teaching, teachers explain concepts and procedures to students. Due to *Investigations* (TERC, 2008) and CGI, that role of explaining has shifted to the students in these teachers’ classrooms. After solving a problem, students explained their strategy to the class. Not only are students communicating their thinking, but also other students are hearing about that strategy and learning about it.

Olivia talked about the reasoning behind this process:

I would particularly want to ask if there’s a kid who is on the fence, like doing a strategy, but you want them to do a more sophisticated strategy, something I could do is really be choosy on who is going to come up and share that day... So, I’m going to bring her up and ask her a lot of questions about what she’s doing.

Essentially, Olivia is mobilizing a student resource to move another student along a progression. The teachers did not want to explain strategies to students, so they used
various techniques for students to take on that role. For instance, when pulling small
groups based on levels of student understanding, Violet would include students who
had higher levels of understanding to help others make sense of the concept:

Tonia: And you had said that day, that you kept Carl. Carl didn’t need to be in
this group, but you kept him. So, explain that.
Violet: Well, because he has a greater understanding than some of the other kids
in that group, and I needed... I just don’t always want to tell them. When it comes
from other kids, sometimes they understand it better. So, I wanted him there to
help guide that instruction. To help guide and get kids thinking in a way that I
may not have been able to explain.

The teachers felt that students learn better from each other than they do from a teacher.

Often times, more advanced topics were not introduced until a student brought it up.

Kathy stated, “I’m hoping there’s some kids in there that will give me that opportunity
to have that discussion with kids.”

Along those same lines, students would be asked to take on the teacher role to
another student in a one-on-one exchange. For example, Amber, a student in Nancy’s
classroom, had sorted geometric shapes into two groups. Afterward, Nancy asked Linda
to listen to Amber explain her thinking. Nancy spoke about that strategy:

Because Linda was a person who I think... I knew would be able to see if it was
right or it wasn’t right. So, Linda could have been the person to question her
rather than me.... Because she takes on that role of teacher.

In a final example, Violet talked about a student who needed to solve a problem in a
different way (a more sophisticated strategy).

I sent her to Jamal... and said, I want you to think about your thinking and about
how you solved it, because he did some multiplying too. How could you do 126
divided by 14 with Jamal’s strategy?

This section illustrated how the teachers mobilized student resources to make
instructional decisions, introduce instructional topics, develop PDC, and to take on roles traditionally reserved for teachers. Next, I discuss implications of these results for teacher education.

**Implications**

Results from the Jacobs et al. (2010) suggest that learning to attend to children’s solutions strategies is a developmental process, because teachers with more years of experience and participation in professional development showed more evidence of attending to the details of children’s solutions. The Cognitively Guided Instruction levels (Table 2) developed by Fennema and her colleagues (1996) suggest that becoming a cognitively guided teacher is also a developmental process. After learning to attend to children’s strategies, the most developed teachers (Level 4-A and Level 4-B) use general knowledge of students and knowledge of individual students when making instructional decisions, respectively. Using these two studies along with the results from my study, I first discuss a cyclical process for teachers to develop knowledge of students and a beginning hypothetical learning trajectory for mobilizing that knowledge to make instructional decisions.

**Cyclical Process for Acquiring Knowledge of Students**

To begin to understand children’s mathematical thinking, a teacher first must attend to it, which involves detailing students’ solutions. Detailing strategies leads teachers to gain knowledge of students in general. For instance, in the M & M problem (Jacobs et al., 2010), detailing Cassandra’s strategy could lead a teacher to conjecture that another student might draw bags of M&Ms, break apart each 43 into 40 and 3, and
then add. This kind of knowledge allows teachers to anticipate possible solution methods. After giving the M&M problem to his/her own students, a teacher would come to know how his/her own students, either in general or individually, would solve the M & M problem.

After a school year ends, knowledge of individual students becomes general knowledge of students. Thus, the process of acquiring knowledge of individual students becomes a cyclical process. As a teacher gains more experience and/or attends more frequently to students’ solutions, his/her bank of general knowledge increases. One could say that the teacher’s schema for children’s mathematics increases. Thus, making acquiring general knowledge of students specific to a group and knowledge of individual students a shorter process. As a teacher builds schemata through experiences with children’s mathematical thinking, he/she can begin to respond more efficiently and effectively to present and future experiences with children’s mathematical thinking. Figure 1 illustrates the cyclical process.

![Cyclical Process for Developing Knowledge of Students](image)

*Figure 1: Cyclical Process for Developing Knowledge of Students*
The bottom three boxes illustrate the cyclical process for teachers' acquisition of general knowledge of students described above. The top three boxes describe the means for teachers to acquire general knowledge of students. Teachers gain knowledge of students through previous teaching experiences, detailing children's strategies, and reading curriculum materials. Some *Standards*-based curriculum materials provide general knowledge of students through their examples of student solutions (Stein & Kaufman, 2010).

With regards to mobilizing student resources to guide instructional decisions, teachers cannot use general knowledge of students or knowledge of individual students until they have gained that knowledge. It is not evident from the CGI studies or this study if there is a developmental process for using general knowledge of students or individual knowledge of students to design instruction. As it stands, a beginning hypothetical learning trajectory for mobilizing children’s mathematical thinking for making instructional decisions is represented in Figure 2:

*Figure 2: Hypothetical Learning Trajectory for Mobilizing Children’s Mathematical thinking*

Are there more points along the trajectory or degrees of instruction for each point? I believe that it is probable that there are more points.
Supporting Teachers in the Developmental Process

The implication from the above cyclical process of acquiring knowledge of students is how to support teachers in building their schemata for children’s mathematics. All four teachers acquired knowledge of individual students by using CGI strategies. Nancy and Violet also used particular features (student examples and questions in teacher guide) in *Investigations* (TERC, 2008). Student examples support teachers’ general knowledge of students, whereas questions in the teacher’s guide support teachers in their acquisition of knowledge of individual students.

Others have also suggested that curriculum materials can help teachers with knowledge of students. In their recommendations for educative materials, Davis and Krajcik (2005) suggest that curriculum materials help teachers anticipate what students might do in response to instructional activities. Providing information such as this adds to a teacher’s general knowledge of students, which would consequently support them in gaining knowledge of individual students. Empson and Junk (2004) found that using *Investigations* (TERC, 2008), a *Standards*-based curriculum series, influenced 13 teachers’ knowledge of children’s mathematics. “Children’s mathematics” refers to the integrated knowledge of concepts, procedures, and mathematics with knowledge of children’s thinking. The two researchers found that the teachers developed integrated knowledge of multiplication by using *Investigations* (TERC, 1998). The teachers’ integrated knowledge of division and subtraction, however, was less developed. The authors speculate that this difference is attributed to the ways in which the three topics were addressed in the curriculum materials. The difference in the
teachers’ knowledge of multiplication and division supports the claim that curriculum materials can help teachers with knowledge of students.

Once teachers know their students, they could possibly mobilize the curriculum materials in different ways. When first using *Investigations* (TERC, 2008), Violet would read all parts of the curriculum guide including possible student solutions. At the time of the study, Violet no longer read those sections, because her students would not use those strategies. Furthermore, Violet could anticipate which strategies her students would use. Violet already had knowledge of individual students. Thus, she did not need the general knowledge of students the curriculum materials provided.

**Conclusion**

To close this chapter, I return to the PDC construct. Teachers with a high degree of PDC are able to detail students’ strategies and are able to distinguish between details that speak to a child’s conceptual understanding and details that speak to other mathematical practices. Teachers with a high degree of PDC have individual knowledge of students - including knowledge of the strategies students tended to use as well as individuals’ dispositions. Finally, teachers with a high degree of PDC mobilize individual knowledge of students to make instructional decisions. Those instructional decisions are based on moving students along a student solution progression. Student resources can also be mobilized to introduce instructional topics, develop PDC, and in roles traditionally reserved for teachers.

To develop teachers’ knowledge of students, we know of two means that can support that endeavor – CGI professional development and the mobilization of
particular features of *Investigations* (TERC, 2008). Unfortunately, not all teachers have access to these two curricular resources. Furthermore, not all teachers mobilize those resources in the same way. Therefore, if we want teachers to make instructional decisions based on general and individual knowledge of students, they need access to these two curricular resources and provided the support in how to use them well.

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CHAPTER 5. PEDAGOGICAL DESIGN CAPACITY FOR TEACHING ELEMENTARY MATHEMATICS: MOBILIZING NUMBER CHOICES

A paper to be submitted for publication.

Tonia J. Land

Abstract

Problem posing is a significant aspect of mathematics education reform (Crespo & Sinclair, 2003). Some research has been generated around the kinds of problems teachers pose, but little, if any, research has been conducted around the act of choosing numbers. By analyzing problems posed by four expert teachers, I was able to identify seven different ways in which these teachers mobilized number choices – to address mathematical content, to encourage a particular strategy, to provide differentiation, to develop relational thinking, to respond to children’s mathematical thinking, for assessment, and to provide an entry point. This study has implications for preparing pre-service teachers for productive problem posing.

Introduction

Brown and Edelson (Brown, 20002; Brown & Edelson, 2003; Brown, 2009) introduced the notion of pedagogical design capacity (PDC) as a way to understand how teachers perceive and mobilize existing resources to design instruction, which would consequently have implications for how to successfully prepare novice teachers. In other work (Land, Chapter 3), I investigated the PDC of expert elementary teachers. The major finding from that study was that the teachers mobilized structures similar to hypothetical learning trajectories (Simon, 1995) to advance student thinking. One of those structures was a number choice progression. In this study, I examined more
closely how four expert teachers (Kathy, Nancy, Olivia, and Violet) mobilized number choice progressions when designing instruction.

Understanding how expert teachers mobilize number choices when problem posing has implications for designing and assessing instruction in elementary mathematics methods courses for pre-service teachers (PSTs). Some research has been generated about the kinds of problems pre-service teachers pose (e.g., Crespo, 2003; Crespo & Sinclair, 2008), but little, if any, research has been conducted around the act of choosing numbers. My intent with this study was to develop instructional activities for PSTs around the different ways number choices can be mobilized. Additionally, I wanted to better understand how much of an expert teacher’s practice is accessible and/or appropriate for PSTs. How much of the ways in which expert teachers mobilize number choices can be noticed, understood, and applied by PSTs? A first step toward that understanding is knowing more specifically what expert teachers do.

I first frame the study in the notion of PDC and the Cognitively Guided Instruction (CGI) research. Then, I describe my data collection and analysis process, which resulted in a list of ways that Kathy, Nancy, Olivia, and Violet mobilized number choices. Several example problems and number choices are given followed by a discussion about each mobilization type. I end this paper with implications concerning the design and assessment of instruction in elementary mathematics methods courses.

**Theoretical Framework**

Pedagogical design capacity is defined as a teacher’s “ability to perceive and mobilize existing resources in order to craft instructional contexts” (Brown & Edelson,
Perceive indicates the ability to recognize, or notice, potential resources; and mobilize highlights the importance of teachers’ abilities to act on or with those resources (Remillard, 2005). The four teachers perceived number choices as a fundamental part of their teaching practices and mobilized them in powerful ways to address specific learning goals.

PDC stems from the notion that teaching is a design activity (Brown & Edelson, 2003). “Teachers must perceive and interpret existing resources, evaluate the constraints of the classroom setting, balance tradeoffs, and devise strategies – all in pursuit of their instructional goals. These are all characteristics of design” (Brown & Edelson, 2003, p. 1). PDC takes into account the use of artifacts. Teachers have access to different teaching artifacts or resources (e.g., professional development, and curriculum materials) depending on their setting, and they use those resources differently depending on their experience, goals, and abilities. Almost all elementary mathematics teachers pose problems and choose numbers, either problems they generate themselves or problems from curriculum materials.

Brown & Edelson contend that the PDC construct has implications for how teachers are prepared:

In addition to support for learning subject matter and ways of teaching the content, which many have long advocated, teachers also require support in exploring which resources to use and how to use them. This latter aspect of professional development should help teachers link their instructional goals to the specific features and affordances of curriculum materials, and should support teachers in making the necessary design modifications required to achieve this alignment. Thus, teacher preparation and professional development might explicitly target the design skills required for effective use of instructional materials (p. 6.).
Problem posing is a significant aspect of reformed-oriented mathematics instruction, and choosing numbers is a substantial part of problem posing. Even though choosing numbers is a substantial part of problem posing, no knowledge base around choosing numbers yet exists.

The CGI research provided important frameworks around the different problem types and children’s mathematical thinking. One of many studies focusing on the effectiveness of CGI examined changes in the beliefs and instructional practices of 21 teachers who participated in a CGI teacher development program over the course of four years (Fennema et al., 1996). This study provided strong evidence to indicate that understanding children’s thinking is a powerful tool in changing instruction (Fennema et al, 1996). Over the course of the study, 17 of the 21 teachers came to believe more strongly that children could solve problems they had not previously been taught procedures for, which changed the teachers’ perceptions of their own role in the classroom. Specifically, teachers came to believe that their role is to provide opportunities for children to solve problems and report on solution strategies rather than telling children how to think.

Additionally, Franke and colleagues (2001) found that professional development in CGI is sustainable when they interviewed 22 teachers four years after participating in a CGI professional development program. All teachers continued to use children’s thinking in their practice and ten continued to be engaged in generative growth. Based on these findings, and others, many mathematics teacher educators now spend a significant portion of time in elementary mathematics methods courses helping PSTs
understand the teacher’s role in classrooms based on CGI and similar approaches (Roth-McDuffie, Drake, & Herbel-Eisenmann, 2008).

The teacher’s role in a CGI classroom is to:

- Continually upgrade their understanding of how each child thinks, select activities that will engage all the children in problem solving and enable their mathematical knowledge to grow, and create a learning environment where all children are able to communicate about their thinking and feel good about themselves in relation to mathematics (Carpenter et al., 1999, p. 101).

Due to grounding my work in PDC, I am most interested in the aspect of the teacher’s role that entails selecting "activities that will engage all children in problem solving." To engage all students in problem solving and design situations where all students are growing mathematically is a tremendous task for a teacher. More information is needed as to how teachers are designing instruction where all students are working productively. We know that CGI is an effective, sustainable, and generative teaching practice that has positive effects on student learning. CGI teachers pose problems using the problem type frameworks for guidance. However, we do not yet know much about how CGI teachers mobilize the problem-type framework. When I began investigating how teachers were mobilizing the problem-type framework by interviewing teachers, I quickly noticed that the teachers mostly talked about their number choices. That is, the teachers posed different types of problems depending on their learning goal, but the number choices were actually the key factor in reaching instructional goals. Therefore, this study was designed to answer the following research question:
Research Question

• How do expert teachers mobilize number choices?

Methodology

Participants

Kathy, Nancy, Olivia, and Violet teach at two different schools within the Lakefront School District. Kathy teaches 1st grade; Nancy is in a 2nd/3rd-grade multi-age classroom; Olivia teaches 2nd grade; and Violet is in a 4th/5th-grade multi-age classroom. At the time of this study, Kathy, Nancy, and Olivia had been participating in CGI professional development for five years and have continued to participate in the training each year. In addition, they facilitated CGI professional development for other teachers in the Lakefront School District. Violet had been participating in CGI training every year for three years. Before CGI, Nancy and Violet implemented the Standards-based curriculum *Investigations* (TERC; 1998, 2008) beginning in 1998. Through that implementation, Nancy and Violet became familiar with different problem types and children’s solution strategies. Now, they use knowledge gained from CGI training to design instruction and make adaptations to *Investigations* (TERC, 2008) to better meet their students’ needs and focus on children’s mathematical thinking (To read an extensive case study about Nancy, see Land & Drake; 2010).

Data Sources

The data for this study come from a larger study investigating how teachers use Standards-based curriculum materials and key dimensions of PDC. Kathy, Olivia, and Violet were each observed on six occasions. Nancy was observed on 11 occasions. After
the observations, the four teachers completed a video-stimulated recall interview process in which they were asked a series of questions designed to understand how they perceived and mobilized their resources and capture key dimensions of their PDC.

In 17 of 29 total observations, the teachers posed a contextualized problem informed by the CGI problem-type framework (Carpenter et al., 1999). During the interview process, the teachers were asked to provide a rationale for their problems. I did not, however, conduct the interview process with Nancy for all of her observations. Therefore, there are rationales for 11 of the 17 problems. I analyzed the eleven problems and rationales to generate a list of how the four teachers mobilized number choices to meet their instructional goals.

**Data Analysis**

These data were analyzed through a process of open and emergent coding (Strauss & Corbin, 1998). Through the inductive analysis process, I identified deliberate acts of instruction. To understand this part of data analysis process, I provide an example. Below is the Fishbowl Problem posed by Nancy and her rationale for doing so.

**Fishbowl Problem.**

Sam had ___ fish bowls. He had ____ fish in each bowl. How many fish did Sam have?

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(2, 10)</td>
<td>(4, 20)</td>
<td>(3, 11)</td>
<td>(4, 12)</td>
</tr>
<tr>
<td></td>
<td>(5, 10)</td>
<td>(8, 20)</td>
<td>(6, 11)</td>
<td>(8, 12)</td>
</tr>
</tbody>
</table>

Nancy intended for her students to solve for one or more sets of numbers. For example, students choosing Set B needed to solve for both set of number choices. This multiple number choice structure is a typical differentiation strategy that each of the four
teachers use after participating in CGI professional development. Students are to choose the numbers that they feel are right for them.

Nancy's Rationale for Fishbowl Problem
I had a couple of different goals for the lesson with the goldfish bowl problem. For some of my students I wanted to see if they were able to skip count by multiples of ten. For others, that I knew could, I wanted to see if they could see any relationships between the numbers I had chosen for them to solve. I had asked them to pick a pair of numbers to solve, hoping they would see this. Also, when choosing the numbers 11 and 12, I was looking to see if any of the students used the distributive property and their knowledge of tens to help them solve the problem.

In the first sentence, Nancy indicated that she had multiple goals for this problem. First, Nancy wanted her students to master a particular solution strategy - skip counting by multiples of tens. To meet this objective, Nancy posed a multiplication problem with decade numbers. For students who already knew how to skip count by tens, Nancy wanted them to notice the relationships between a pair of number choices. Nancy asked her students to choose a pair of numbers to solve for. The number choices become increasingly more complex. That is, the first two groups of number choices involve decade numbers while the latter two are two-digit non-decade numbers.

Nancy's number choices are also in response to her students’ mathematical thinking (Jacobs, Lamb, & Philipp, 2010) as she is taking into consideration that some students have already mastered skip counting by tens and need to move on to another concept. Finally, Nancy hoped her number choices encouraged students to use the “distributive property and their knowledge of tens to help them solve the problem.” In other words, Nancy used her number choices to encourage students to use and reinforce their knowledge of a particular mathematical concept.
In this one problem, Nancy mobilized her number choices in five different ways: 1) to encourage students to use a particular strategy; 2) to develop relational thinking; 3) to provide differentiation; 4) to respond to children's mathematical thinking; and 5) to help students understand a mathematical concept.

For each problem and rationale, I made a similar mobilization list. Then, I compiled the lists and created a definition for each mobilization. In all, there were seven different mobilizations. This list is not exhaustive in demonstrating the different ways number choices could be and are mobilized by teachers. These are the types of mobilizations I found in Kathy, Nancy, Olivia, and Violet’s problem posing. Table One provides a list of mobilizations and definitions for each:

Table 1: Mobilizations with definitions

<table>
<thead>
<tr>
<th>Mobilization</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 To help students understand a particular mathematical concept</td>
<td>Number choices were posed to help students gain understandings of a particular mathematical concept as stated explicitly in the teachers' rationales.</td>
</tr>
<tr>
<td>M2 To encourage students to use a particular strategy</td>
<td>Number choices are posed to encourage a particular strategy. Thus, number choices need to make sense with the type of strategy the teacher is encouraging. Encouraging a strategy is different than teaching a strategy in that a teacher is not modeling a strategy. Instead, the teacher is designing a situation where the strategy could be easily noticed and applied.</td>
</tr>
<tr>
<td>M3 Differentiating instruction by providing multiple number choices</td>
<td>The multiple number choice structure is mobilized to provide differentiation. It is assumed that not all students will work productively with the same number choices. Therefore, multiple number choices are given. The number choices need to increase in complexity to be considered as providing differentiation. If number choices do not increase in complexity, then the number choices are providing additional practice, not differentiation.</td>
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<tr>
<td>M4 To develop relational thinking (Carpenter,</td>
<td>Number choices are chosen to encourage students to use the relationship between a set of numbers to help them understand...</td>
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<tr>
<td>M5</td>
<td>To respond to children’s mathematical thinking (Jacobs, Lamb, &amp; Philipp; 2010)</td>
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<td>M6</td>
<td>For assessment</td>
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<tr>
<td>M7</td>
<td>As an entry point</td>
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**Results**

In the remainder of this section, I provide four more example problems and the rationales for those problems given by the four teachers. I provide several examples of problems and rationales to give readers a more complete sense of the data, and so that I have discussed at least two examples for each type of mobilization. Following each problem and rationale, I provide some discussion connecting the rationales to the above mobilization list. I highlight the different mobilizations by denoting them by M1, M2, M3, and so on throughout this section.
Baseball Card Problem (M1, M3, M4, M5, M6, M7)

Leonard has ____ baseball cards. He gives \( \frac{1}{4} \) to his friend. How many baseball cards did he give to his friend?

8  24  44  60  100  144

Violet’s number choices are interesting, because they are all multiples of four, span from single-digit to three-digits, and are ones her students might be familiar with. Furthermore, 44, 100, and 144 are connected in that \( 44 + 100 = 144 \). Students can use their knowledge of finding \( \frac{1}{4} \) of 44 and 100 to find \( \frac{1}{4} \) of 144 – a more complex number choice. Providing numbers with this relationship is appropriate in Violet’s classrooms, as her students often decompose numbers when solving problems. Below is Violet’s rationale for the baseball card problem:

Violet’s Rationale for the Baseball Card Problem

So, I changed the numbers, but kept the fraction the same, and I picked a fourth because I knew it was a friendly fraction that they all would be able to have access to…. A half would be too easy… I wanted to see if they could do some relational thinking with using a half and then breaking that in half and see what kind of relationship they had with that… Fractions of a set is much more challenging than fractions of an area. So, um, this was our first attempt at this, so I needed it accessible to most kids.

In giving her students the Baseball Card Problem, Violet wanted her students to understand fractions of a set as mentioned in the second to last sentence (M1). The number choices increased in size and some complexity spanning from single-digit to triple-digit numbers providing differentiation (M3). I state that the numbers increase in size and some complexity, because 100 is a more complex number to find \( \frac{1}{4} \) of than 8, but not necessarily more complex than 60. Violet chose to use \( \frac{1}{4} \) instead of \( \frac{1}{2} \), because she knew that \( \frac{1}{2} \) would be too easy for her students. Therefore, she responded to her
students, because she used what she knew about them from previous work when deciding what fractional part to have her students explore (M5). Additionally, Violet thought that students might use relational thinking when posing \( \frac{1}{4} \) (M4). She stated, “I wanted to see if they could do some relational thinking with using a half and then breaking that in half.” I know from other interview data that Violet posed this problem to assess what her students knew about finding a fractional part of a set as they started a new unit in *Investigations* (M6), a *Standards*-based series published by TERC (2008), and that she used that knowledge to help her plan instructional activities. Because this was the first time Violet’s students found a fraction of a set, she used a number that her students were familiar with, providing them an entry point into these types of problems (M7).

**Pennies Problem – (M1, M2, M3, & M4)**

Lenore has ____ pennies and Max has ____ pennies. How many pennies do they have together?

(6, 30)    (40, 20)    (10, 68)    (45, 13)

Notice that the first number choice entailed adding a single-digit number to a decade number. Then, the number choices moved into adding two decade numbers, adding ten to a non-decade number, and finally, adding two non-decade numbers.

**Kathy’s Rationale for the Pennies Problem**

[Base-ten] and wanting kids to start looking at tens and ones and being able to add them easily - like the 6 and 30. Obviously, I’m putting the 6 first instead of the 30 first, because I want them to see that 6 and 30 is the same as 30 and 6, which is pretty attainable for them... If I were looking at 6 and 30, you wouldn’t want them to hold 6 and count on 30... The 10 and 68, there’s two ways that my kids could look at this. Most of them will be looking at 60 and 8, then adding the 10, which would be 70 + 8. And some kids, just knowing that when you add 10 to a number like 68, it’s 78. That’s a little bit more complex for the kids. So, wanting
them to think about that strategy. Then 45 and 13 is really more of having to decompose two numbers. You want them to look at it as 40 and 10 and 5 and 3.

In the Pennies Problem, Kathy wanted her students to develop some base-ten understanding (M1). Kathy encouraged relational thinking by “putting the 6 first instead of the 30” (M4). She wanted students to see that “6 and 30 is the same as 30 and 6.” Implied in the relational thinking mobilization is the notion that teachers are addressing a mathematical concept, which in this case is the commutative property. Also implied in that mobilization is that a teacher is encouraging a particular strategy. In her rationale, Kathy stated, “If I was looking at 6 and 30, you wouldn’t want them to hold 6 and count up 30.” Other strategies are encouraged with subsequent number choices (M2). For instance, Kathy wanted students when solving for 45 and 13 to break apart by place, “you want them to look at it as 40 and 10 and 5 and 3.” Kathy talked about the increasing complexity of her number choices (M3) in her rationale starting from two numbers that are easily added together (6 and 30) to “having to decompose two numbers” (45, 13).

After Kathy’s students solved the Pennies Problem and others like it, Kathy discussed what she would do next:

**After the Pennies Problem – (M5 & M7)**

When I start having my kids look at the 10 and the 68 and kind of putting that together easily, then I’ll probably start doing some join-change unknown problems. Where I have 68, I get some more, now I have 78. I might take some of these same number choices. I have 20 cookies. I get some more. Now I have 60 cookies. How many did I get? I could take some of those things after the kids are successful. After the kids are successful, I might take some of those same number choices and put it in a more complex problem.
Kathy provided an entry point into more complex problems and thus, another series of instructional activities with her number choices (M7). She used experiences with one type of problem to provide an entry point in another more complex problem type. When Kathy decides to start posing the more complex join-change-unknown problems, based on her students’ successes (M5), she will use number choices that students are familiar with to scaffold their learning.

**Hank’s Sister Problem – (M2 & M3)**

Hank has ___ dollars in the bank. His sister Emily wants to borrow ____ dollars. How many dollars will Hank have left if he loans the money to Emily?

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<td>(12, 9)</td>
<td>(33, 22)</td>
<td>(90, 61)</td>
<td>(419, 321)</td>
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<td>(16, 13)</td>
<td>(44, 25)</td>
<td>(150, 53)</td>
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Notice that these number choices spanned from smaller two-digit numbers to large three-digit numbers and that the differences between the numbers increased; and as Nancy pointed out her rationale below, the numbers are “close together in the ones place.” For instance, 44 is only one away from 25 in the ones place and 419 is two away from 321 in the ones place if one goes over the 20. These numbers made sense with Nancy’s goal of encouraging students to use compensating. See below for her rationale:

**Nancy’s Rationale for Hank’s Sister Problem**

For Hank’s Sister problem I chose the numbers because the day before Billy had shown his strategy and explained it to the class. Here is an example of what Billy did:

33 - 22 = ___
32 - 22 = 10
10 + 1 = 11

Another student did this:
33 - 22 = ___
33 - 23 = 10
10 + 1 = 11

They [students] started to show some understanding of what he had done, so I thought I would try. The numbers in the ones column are close together, in a hope that some of them would try the strategy that they were shown the day before. We’ve been talking about looking at the number and making the strategy that would make the most sense with those numbers, rather than always resorting to the same strategy.

In Hank’s Sister Problem, Nancy encouraged her students to use a strategy (compensating) that Billy had used and explained the day before (M2). From other interview data, I know that Nancy wanted her students to be flexible with their strategies, which is also evidenced in the last sentence of her rationale. Instead of using the same strategy repeatedly, Nancy wanted her students to use the strategy “that would make the most sense with those numbers.” In other words, Nancy wanted her students to analyze the numbers before choosing a strategy. The numbers increased in complexity providing differentiation (M3). I could assume that Nancy intended to address some mathematical content, but she did not explicitly state the intended content in her rationale, thus I did not include M1 as a mobilization.

**Sharing Cookies Problem (M1, M4, M7)**

Leslie and Allison are sharing _____ chocolate chip cookies. If they are shared equally, how many will each of them get?

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<td>30</td>
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<td>51</td>
<td>66</td>
<td>67</td>
<td>83</td>
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Notice that with the exception of 2 and 83, even numbers are posed followed by the next whole number (e.g., 4 and 5).
Olivia's Rationale for Sharing Cookies Problem

This is when we were doing halves... 30 divides equally. So, are they going to use the knowledge from that number previously to help them to solve the next one, or are they [students] going to start over?

In the Sharing Cookies Problem, Olivia addressed the district’s objective of “halves, thirds, and fourths” (M1). With the exception of the first number choice of two, Olivia posed even numbers followed by the next whole number in hopes that students would use the relationship between the two numbers (e.g., 30 and 31) to help them solve for the odd numbers (M4). Additionally, the even numbers provided students with an entry point into the problem (M7), as students had solved previous measurement division problems involving sharing a set of objects between two people. Natalie stated that students “pick up on” even numbers quickly, because students “think back to what they learned in kindergarten, or first grade even.”

In each problem, the teacher mobilized the number choices in multiple ways. There was not a problem that was used for just one purpose. In some cases, a specific aspect of the problem could be considered as two or more different mobilizations. For instance, in the Baseball Card Problem, \( \frac{1}{4} \) provided an entry point into the problem, encouraged relational thinking, and was in response to children’s mathematical thinking. Differentiation was counted in each of the problems, because all the problems had multiple number choices with increasing complexity. The smallest number of mobilizations characterizing a problem was two and the largest number was six.

Discussion

With the exception of differentiation, I only counted mobilizations that teachers talked explicitly about in their rationales or in other interview data. However, from
what I know of the teachers’ practices, their number choices always addressed relevant mathematics, they assessed their students daily, and they responded appropriately to children’s mathematical thinking. Therefore, I contend that a set of good number choices addresses relevant mathematical content, provides differentiation, is used for assessment purposes, and responds to children’s mathematical thinking. In some special cases, number choices can also be used to encourage particular strategies, develop relational thinking, and provide an entry point into a problem. In the remainder of this section, I talk in more detail about each mobilization and include other research related to each mobilization. This discussion provides some context for my implications section concerning the mobilizations’ accessibility to PSTs during the elementary mathematics methods course.

**To help students understand mathematical content**

Kathy, Nancy, and Olivia determined the mathematical content they addressed in the classroom by first referring to a set of standards provided by their district:

> The geometry lessons that I’ve been planning, and this is what I would do with any topic or area of study in math, I would look at our objectives first... Our [Lakefront] district objectives. (Olivia, initial interview)

The teachers thought the standards were “limited,” however, in that they did not provide specific learning objectives. Through their CGI professional development, they were able to conceptualize more specific goals. For instance, in the Fishbowl Problem, Nancy’s specific goals were skip counting by multiples of ten, doubling, and applying the distributive property. Violet, on the other hand, used the focal points in *Investigations* (TERC, 2008) to conceptualize her specific learning goals.
Hiebert and colleagues (2007) talk about the importance of specific learning goals and outline two criteria for specifying learning goals. “First, goal descriptions are more useful when they are more specific, when they include subgoals and primary or general goals. Second, goal descriptions are more useful when they use the language of the subject” (p. 51). The researchers talk about this usefulness in the context of finding evidence that students are meeting learning goals, but specifying learning goals is also necessary in other aspects of teaching.

Morris and colleagues (2009) found that PSTs did not, in general, unpack learning goals and identify sub-concepts. To provide a sense of what the researchers mean by “unpacking learning goals” and “identifying sub-concepts,” I provide an example from their work. Adding fractions is a learning goal that involves the following sub-concepts:

- A quantity is identified as the quantity “one.”
- We obtain units of size 1/n by partitioning the “one” into n equal parts.
- The numerator is the number of unit of size 1/n.
- The addends must both be expressed in terms of the same-size unit.
- The addends must be joined.
- The sum must be expressed in terms of a unit of size 1/n (Morris et al., 2010, p. 499).

The researchers did find, however, that PSTs could identify sub-concepts in supportive contexts (Morris et al., 2009). Supportive contexts consisted of tasks where PSTs were required to use a sub-concept to complete the task.

**To provide differentiation**

The teachers provided differentiation by mobilizing a multiple number choice structure. It is assumed that students will not work productively with the same number
choices. Therefore, multiple number choices are provided. Providing differentiation is not as easy as providing numbers that are simply larger. The teachers provided progressions of number choices that increased in complexity (Land, Chapter Three). To provide number choices that increase in complexity, consideration needs to be given to both the problem and the numbers.

For example, in the Pennies Problem, Kathy stated in her rationale that the beginning numbers were ones that are easily added together (6 and 30). The second number choice entailed adding two decade numbers (40, 20), the third entailed adding ten to a non-decade number (10, 68), and the last number choice entailed two non-decade numbers (45, 13) where students would “have to decompose two numbers” (45, 13). Kathy has categorized numbers and ordered them according to how easy or hard they are to add together.

To further illustrate the need that consideration needs to be given numbers when providing differentiation, I describe an additional problem posed by Olivia. After the Sharing Cookies problem (above) that entailed students dividing cookies between even and then odd numbers of people, Olivia posed a problem where students were splitting brownies between four people. Her number choice progression began with numbers that were divisible by four (e.g., 8, 16, 32, 44) resulting in a whole number, then numbers divisible by four plus one (e.g., 9, 17, 33, 45) resulting in a whole number and ¼, then numbers that resulted in a whole number plus two or three fourths (e.g., 11, 22, 35). The number progression Olivia posed illustrates the complexity involved with choosing numbers to provide differentiation.
To assess students

Kathy, Nancy, Olivia, and Violet assessed students daily. This daily assessment is important when determining what to do next (or respond) to students. In her discussion about the Pennies Problem, Kathy talked about her role when students are solving problems – “If I’m thinking about what I would plan for the next day or for the next couple of weeks, I would look at this [students’ solutions] and kind of see where my kids are at.”

Additionally, Violet would use problems to pre-assess her students as they began a new unit of instruction and to help her design instructional units. For instance, Violet posed the Baseball Card Problem in part to find out what her students understood about fractions. Particular lessons in the unit will address certain focus points. Violet analyzed the Baseball Problem to determine which focus points and thus lessons her students needed to work on.

I usually start by writing a problem just to kind of see the kinds of strategies they’re going to use. Then, from there, I make some decisions. First of all, what do kids need? In this situation, we have some different groups that I need to split and extend. So usually, I start with a problem of the day and figure that out. From there, I make some decisions about where I need to go with that. What do they need? In this case, a lot of them don’t have experiences with basic fractions and putting them together. Some of them can find equivalent fractions. So, I start there.

By “kinds of strategies,” Violet meant that she wanted to see if her students were going to direct model or use other, more sophisticated strategies. The more sophisticated the strategy, the greater understanding students have.
As an entry point

In several instances, Kathy, Nancy, Olivia, and Violet posed number choices that their students were familiar with to provide students an entry point. For instance, in the Baseball Card Problem, Violet posed \( \frac{1}{4} \) because she knew her students would have access to the problem. Olivia provided number choices that were sequential, like 30 and 31, so students could use what they knew about 30, an easier number choice, to help them solve for 31, a harder number choice. Kathy talked about giving students experiences with certain numbers in easier problem types to help give them access to problems that are more complex, which essentially is providing students an entry into another series of instructional activities.

To develop relational thinking

In four problems of the total problem set, the teachers chose certain number choices in hopes that students would see relationships between the numbers to help them solve. “Relational thinking represents a fundamental shift from an arithmetic focus (calculating answers) to an algebraic focus (examining relations)” (Jacobs et al., 2007, p. 260). Jacobs and colleagues (2007) describe relational thinking and its place in elementary mathematics:

In short, relational thinking entails an awareness of relations among numbers and the fundamental properties of number operations. Students can use relational thinking to simplify calculations, construct and learn new concepts, extend procedures to new number domains, and generally make sense of arithmetic... One fundamental goal of integrating relational thinking into the elementary curriculum is to facilitate students’ transition to the formal study of algebra in the later grades so that no distinct boundary exists between arithmetic and algebra (p. 261).
In posing number choices that have relationships with each other, the teachers were developing important algebraic concepts in their students.

In most cases, the multiple number choice structure was mobilized when attempting to encourage relational thinking. Multiple number choices allowed students to look for relationships across sets of numbers, and use their knowledge of those relationships when problem-solving. For example, Nancy tried to encourage students to use doubling to solve for the numbers 8 and 20 after solving for 4 and 20 in the Fishbowl Problem.

In the case of the Baseball Card problem, Violet tried to encourage relational thinking in a different way. She asked students to find $\frac{1}{4}$ of set of objects, in part, to encourage students to use their knowledge of the relationship between $\frac{1}{2}$ and $\frac{1}{4}$ to solve. Observation data showed that some students did, in fact, use that relationship.

Below are two students’ strategies solving for 60:

**Debbie’s strategy for finding $\frac{1}{4}$ of 60 baseball cards**

$60 - 30 = 30$

$30 - 15 = 15$

**Jayda’s strategy for finding $\frac{1}{4}$ of 60 baseball cards**

$60 \div 2 = 30$

$30 \div 2 = 15$

Also interesting about the Baseball Card Problem is that some of the number choices have a meaningful relationship – 44, 100, and 144. Students could add their answers from 44 and 100 to solve for 144.

One study, conducted by Jacobs and her colleagues, examined the effects of professional development on teachers’ understanding of relational thinking (2007). The
researchers found that teachers participating in the professional development could generate more solutions that reflected relational thinking for five problems than teachers who do not participate. However, 89% of non-participants did use relational thinking for at least one problem. Given these results, the notion of relational thinking may be easily understood and applied by PSTs.

**To encourage students to use a particular strategy**

Participation in CGI professional development provided the teachers with a continuum of strategies that students use when solving contextualized problems. When solving problems with single-digit numbers, children first direct model, then use counting strategies, then use derived facts before facts become typically known (Carpenter et al., 1999). For double-digit numbers, children will use a variety of invented algorithms based on what they know about numbers to solve problems. As with single-digit numbers, strategies for double-digit numbers vary in levels of sophistication. In CGI professional development, teachers learn about the range of strategies and how to encourage students to use more sophisticated strategies with problem posing. In the Pennies Problem, Kathy chose numbers that hopefully encouraged students to use a strategy that was at a certain level of sophistication. For instance, Kathy talked about not wanting her students to “hold 6 and count on 30.” Instead, Kathy wanted her students to count on from the larger number, which is a more sophisticated strategy than counting on from the smaller number and takes knowledge of the commutative to understand.
In Hank’s Sister Problem, Nancy also wanted her students to use a particular strategy, but in a different way. Nancy posed number choices to help students understand and possibly use a strategy used by another student. The other students started to show some understanding of the strategy. Therefore, Nancy posed a problem to further develop students’ understanding of the strategy. Her intention was that students notice and analyze numbers before choosing a strategy. The level of sophistication was not the primary concern in that case, so I argue it is a different way to encourage students to use a particular strategy.

**To respond to children’s mathematical thinking**

Jacobs and colleagues in 2010 introduced the construct of *professional noticing of children's mathematical thinking*. This construct is separated into three interrelated skills: attending to children’s strategies, interpreting children’s understandings, and deciding how to respond on the basis of children’s understandings. One way to respond to children’s mathematical thinking is to pose a next problem (Jacobs et al., 2010). Jacobs and colleagues analyzed how teachers were responding to children’s mathematical thinking by asking them to respond to an episode of children’s mathematical thinking (video-clip) by posing a next problem (2010). They found that 82% of teachers who had four or more years of professional development focused on children’s mathematical thinking considered children’s thinking when responding to them. It is not surprising, then, that our teachers considered children’s understandings when posing problems and choosing number choices due to their extensive CGI professional development experiences.
Other participants in the Jacobs et al. study included prospective teachers, practicing K-3 teachers with no professional development, and teachers with two years of professional development. Of these participants, 14% of prospective teachers, 26% of practicing K-3 teachers with no professional development, and 54% of teachers with two years of professional development considered children’s thinking when responding. Given these results, it seems like the majority of PSTs could not develop the skill of responding to children’s mathematical thinking during the methods course, because the skill takes years to develop (Jacobs et al., 2010). However, in other work, (Land, 2007; Land & Drake, 2008), I found that PSTs did develop in their responses to children’s mathematical thinking over the course of a semester. That is, PSTs did not consider children’s mathematical thinking when writing subsequent problems at the beginning of the course, but did so by the end. I attribute this difference in results due to the methods instructor’s focus on responding in the Land (2007) and Land & Drake (2008) studies.

To summarize this section, I return to the notion of PDC – how teachers perceive and mobilize existing resources. This study contributes to the understanding of how expert elementary mathematics teachers perceive and mobilize number choices to design instruction. Kathy, Nancy, Olivia, and Violet recognize that number choices are an important aspect of teaching elementary mathematics, and that number choices can be mobilized to meet various instructional objectives. They came to have this perception about number choices primarily through their experiences in CGI professional development and continued to develop their use of number choices
through their teaching practices. Due to my interest in the development of the elementary mathematics methods course, my primary question now is – “in what ways can the elementary methods course be designed to start PSTs on a trajectory to mobilize number choices in the ways expert teachers do?”

**Implications**

Good number choices first and foremost address relevant mathematics. Having PSTs develop number choices for relevant mathematics should be given appropriate attention throughout the methods course. Nonetheless, it may also be one of the more challenging mobilizations for PSTs. Morris and colleagues (2009) found that PSTs could unpack learning goals in supportive contexts, but did not spontaneously do so in unsupportive contexts, which means supportive contexts need to be designed for PSTs for them to generate number choices for specific learning goals. For example, PSTs could be given a specific learning goal and be asked to generate problems with number choices that would address the given goal.

A constraint with providing PSTs with a support is that these same supports may not be available to PSTs once they begin their own teaching practice. PSTs eventually will need to be able to generate specific learning goals on their own using curricular supports available to practicing teachers. However, few supports are available to practicing teachers for the generation of specific learning goals. Kathy, Nancy, and Olivia talked about the limitation of their district’s standards in providing specific learning goals. They relied on their CGI knowledge for this process. Many states have
adopted the Common Core (CCSSO & NGA, 2010), but it is not known yet if this document will be accessible and supportive to teachers in identifying learning goals.

Violet mobilized the *Standards*-based curriculum *Investigations* (TERC, 2008) to generate specific learning goals. Stein and Kaufman (2010), in their analysis of two *Standards*-based curriculum series, found that *Investigations* (TERC, 2008) provided more support than *Everyday Mathematics* (UCSMP, 2007) for “locating and understanding the big mathematical ideas within lessons” (Stein & Kaufman, 2010, p. 663). Therefore, only certain *Standards*-based materials may be able to provide the needed support for generating specific learning goals.

Along with addressing mathematics content, good number choices provide differentiation, are used for assessment purposes, and respond to children's mathematical thinking. Providing multiple number choices is a relatively easy differentiation strategy that is accessible to PSTs. PSTs can grasp the concept of providing number choices that increase in complexity. The challenge, however, will be in coming to know easier and harder number choices for particular problem types. In other work (Land, Chapter Three), Kathy mentioned that number choice progressions are frustrating for teachers to conceptualize. I believe that PSTs can grasp the concept of differentiation and will be able to begin to mobilize number choices in this manner, but will need to continue to develop the skill once they become practicing teachers.

That number choices can be mobilized for assessment, I conjecture, is also accessible to PSTs if they are familiar with the range of student solution strategies (Carpenter et al., 1999). PSTs can examine an example of student work and assess a general level of
sophistication. Using that assessment to respond appropriately, however, will be more for PSTs. Even with the results of my previous work (Land, 2007; Land & Drake, 2008), other research (Jacobs et al., 2010) has found that the skill of responding takes years to develop.

In some special cases, number choices can also be used to develop relational thinking, provide an entry point into a problem, and encourage particular strategies. Given the results from the Jacobs et al. study (2007) that 89% of non-participants used relational thinking at least once to solve a problem and my own experiences in teaching the methods course, I believe that PSTs could mobilize number choices to encourage relational thinking if the relationships can be easily noticed and generated – like doubling. Other relationships, like $44 + 100 = 144$ in the Baseball Card Problem, may be more difficult for PSTs to generate. I also believe that providing an entry point into a problem is accessible to PSTs. Many times smaller number choices were given to allow students to direct model the problem, which is an easier concept to understand. Providing number choices that students are familiar with to provide an entry point, as in the Pennies and Baseball Card Problems, may be more difficult, because it requires having knowledge of students. Finally, I conjecture that mobilizing number choices to encourage a particular strategy will be more difficult for PSTs. Kathy, Olivia, Nancy, and Violet could mobilize number choices in this manner because of their participation in CGI professional development, which gave them extensive knowledge of children’s solution strategies. The teachers have also had opportunities to test number choices
with students. PSTs do not have continuous access to these kinds of opportunities to try out ideas and number choices with students.

In light of the above implications, I propose that instruction in the elementary mathematics methods course regarding number choice selection should occur in a particular sequence. Figure 1 illustrates that sequence:

*Figure 1: Sequence of Instruction*

I have based this sequence based on the accessibility of number choice mobilizations to PSTs. I would like to highlight the fourth box. Asking PSTs to generate problems using curricular supports, such as the Common Core (CCSSO & NGA, 2010) and Standards-based curriculum materials needs to occur at some point in the methods course due to the availability of these curricular supports to practicing teachers. PSTs need to be able to use those resources in productive ways, without scaffolds from methods instructors, once they become practicing teachers as those scaffolds will not necessarily be available. The nature of Figure One may suggest that instruction should occur in a linear fashion, but that is not what I propose. Instead, I propose that instruction on these
mobilizations overlap in some ways. To be clear, this proposed sequence of instruction is a conjecture. Further research will need to be conducted to confirm, or refute, my claims.

An additional implication from the ways in which Kathy, Nancy, Olivia, and Violet mobilized number choices is the mathematical content knowledge needed to choose number choices that are productive. What kinds of subject matter knowledge do teachers need to have to be able to provide number choices in each of the seven mobilizations? For instance, one of Nancy’s goals in the Fishbowl Problem was for students to use the distributive property to find how many total fish were in three fishbowls containing eleven fish each. Nancy not only needed to be able to understand and apply the distributive property when choosing these numbers, but also needed to recognize the importance of the distributive property to further her students’ understandings of number and operations. Furthermore, Nancy needed to know the type of numbers that would encourage students to use the distributive property. Nancy chose 11 and 12. She thought a two-digit number with a smaller digit in the ones place would accomplish that task. This is a reasonable choice for this goal, because her students have worked extensively with tens and decomposing numbers.

In conclusion, by analyzing Kathy, Nancy, Olivia, and Violet’s problems and rationales, I was able to generate a list of seven number choice mobilizations – ways in which teachers mobilize number choices. Using existing research, I made conjectures as to which mobilizations may be accessible to PSTs. My future work will entail designing instruction around mobilizing number choices, and assessing that instruction
using the list of mobilizations (Table One) as a framework. I hope to answer the following question – which mobilizations are accessible to PSTs during the elementary mathematics methods course and with what supports?

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CHAPTER 6. A PARTIALLY ANSWERED QUESTION

Introduction

I began this study wanting to find an answer to the following question: What is the PDC of teachers who successfully facilitate classrooms where students are doing mathematics? This was a big question to which I found a partial answer. I used the work of Remillard (2005) and Davis et al. (2007) to generate a list of possible key dimensions of PDC. From that list of possible key dimensions, I found that mobilization of curricular resources and knowledge of students were fundamental aspects of an expert teacher’s PDC. Identifying those aspects of PDC in itself is not surprising nor does it really add to what we already know, as an education field, about expert teachers. My contribution is that I found productive ways in which the teachers mobilized curricular resources and gained general and individual knowledge of students. Additionally, I found key dimensions of PDC that were not in my original list: mobilization of progressions and mobilization of student resources. In the next section, I summarize my contribution to the PDC construct. Then, I make connections from my dissertation study to another research interest – teacher learning about and from Standards-based curriculum materials. Lastly, I end this chapter and dissertation discussing how this study has made positive changes to my teaching and opened up other research possibilities.

Adding to the PDC Construct

My study adds to the PDC construct for elementary mathematics in that I found expert teachers mobilize four different types of progressions: unit, series of instructional activities, number choices, and student solution. Each of these
progressions was imbedded in and informed movement along the other progression
types. Different curricular resources provided support for one or more progression
types. This dimension (mobilizing progressions) is something that I contend needs to be
included in the PDC construct. This work builds on other research around hypothetical
learning trajectories. Simon introduced hypothetical learning trajectories (HLTs) in
1995 as a model and vision for constructivist teaching. Since then, HLTs have been
interpreted in different ways (Clements & Sarama, 2004). Therefore, a need existed to
differentiate between different types of HLTs. Science educators, Stevens, Shin, and
Krajcik (2009), distinguished between learning progressions and learning trajectories,
but those distinctions do not adequately describe the progressions involved with
teaching mathematics. I am not sure that the language I used for identifying the
progressions is the language the mathematics education field as whole will want to use,
but distinctions do need to be made so that we can support teachers for each
progression type.

The Fennema et al. (1996) study established that the most cognitively guided
teachers (Level 4-B) had knowledge of individual students and used that knowledge as
a basis for instructional decisions. My study reiterated that claim, but also identified the
types of knowledge teachers had of students and a process by which teachers might
come to possess that knowledge. Of course, teachers can acquire knowledge of students
through CGI, but not all teachers have access to CGI professional development nor do all
teachers with access to CGI develop knowledge of students as evidenced in the differing
levels of CGI teachers (Fennema et al., 1996). Other ways for teachers to gain
knowledge of students is through *Standards*-based curriculum materials. Empson and Junk (2004) and Land (Chapter Four) provide evidence that curriculum materials can provide a means for teachers to gain general and individual knowledge of students.

Additionally, the Fennema et al. (1996) study found that teachers used their knowledge of students to make instructional decisions. Again, my study reiterated that claim, but investigated more specifically what making instructional decisions based on knowledge of students entailed. Furthermore, I found other ways expert teachers mobilized student resources – for instructional topics, in roles traditionally reserved for teachers, and developing PDC. Finally, I examined how the teachers mobilized number choices and found that they mobilized number choices in seven ways: to address mathematical content, to encourage a particular strategy, to provide differentiation, to develop relational thinking, to respond to children’s mathematical thinking, for assessment, and to provide an entry point.

In Table 1, I have summarized how my research contributes to the PDC construct and the curricular resource that supported the teachers for each PDC dimension:

*Table 1: Contributions to PDC Construct*

<table>
<thead>
<tr>
<th>Key Dimension</th>
<th>Teachers with a high degree of PDC…</th>
</tr>
</thead>
<tbody>
<tr>
<td>View/Perception of Curricular Resource</td>
<td>• perceive <em>Investigations</em> (TERC, 2008) as a valuable resource. This perception only pertained to Nancy and Violet. Kathy and Olivia did not perceive <em>Investigations</em> (TERC, 2008) as a valuable resource.</td>
</tr>
<tr>
<td>Mobilization of Progressions</td>
<td>• use curricular resources to conceptualize and mobilize four progression types. <em>Investigations</em> (TERC, 2008) can provide support for unit and series of instructional activity progressions. CGI can provide support for series of instructional activities,</td>
</tr>
<tr>
<td>Mobilization of Number Choices</td>
<td>• mobilize number choices to address mathematical content, to encourage a particular strategy, to provide differentiation, to develop relational thinking, to respond to children’s mathematical thinking, for assessment, and to provide an entry point. Much of this support came from CGI.</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>----------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Knowledge/Perception of Students</td>
<td>• have general knowledge of students, general knowledge of a group, and knowledge of individual students.</td>
</tr>
<tr>
<td>Mobilization of Student Resources</td>
<td>• mobilize knowledge of students to make instructional decisions and also use student resources to generate instructional topics, in roles traditionally reserved for teachers, and to develop PDC.</td>
</tr>
</tbody>
</table>

My research did not address some of the other key dimensions of PDC (e.g., beliefs, goals, subject matter knowledge). Research around some of those dimensions has already been conducted, while other dimensions still need to be investigated. A complete PDC construct for teaching elementary mathematics would entail synthesizing existing literature and generating additional research.

My research participants were chosen because I believed they all had a high degree of PDC and because they all had access to *Investigations* (TERC, 2008) and CGI. Nancy and Violet were the only two participants who mobilized both of these resources in tandem with each other to generate a successful teaching practice. By using both curricular resources, teachers are potentially supported in their conceptualization and mobilization for all four progression types in all content areas; in their mobilization of number choices; in their acquisition of knowledge of students; and in their mobilization of student resources. By providing teachers with access only to *Investigations* (TERC,


2008), teachers would not be supported in their conceptualization and mobilization for number choice and student solution progressions; teachers would not be supported in mobilizing number choices in the various ways; and they would have limited support in acquiring knowledge of students. By providing teachers with access only to CGI, teachers would not be supported in their conceptualization and mobilization of unit progressions. Additionally, they would not have the needed support for instruction in all content areas. Therefore, it seems like if we want all teachers to teach in the same ways these four expert teachers do, access to both Investigations (TERC, 2008) and CGI could potentially provide the needed support. The other possibility is to redesign curriculum materials in ways that provide support for all PDC dimensions.

**Teacher Learning about and from Standards-based Curriculum Materials**

My work on this dissertation is directly related to another research interest – teacher learning about and from Standards-based curriculum materials. Recent research suggests that *how* a teacher uses a curriculum may be more important than his or her education, experience, and knowledge (Stein & Kaufman, 2010). Rather than the education, experience, and knowledge a teacher brings to a classroom, it seems that teachers tended to have higher quality lessons, (measured by maintaining high levels of cognitive demand, attending to student thinking and vesting intellectual authority in mathematical reasoning) when they “talked about or reviewed big mathematical ideas that students were supposed to be learning” (Stein & Kaufman, 2010, p. 681). In other words, teachers who mobilized curriculum materials in particular ways had high-quality lessons even if they had limited individual capacity. Therefore, it becomes
imperative that we come to understand what curriculum materials can and cannot do to support teachers in reform efforts. My dissertation work contributes to that understanding.

I found that Standards-based curriculum materials could support teachers in conceptualizing unit progressions. That was the case with Violet. Violet perceived and mobilized the focal points in Investigations (TERC, 2008) as a unit progression. Providing support for unit progressions, however, is not something that all curriculum materials do. Stein and Kim (2010) found that teachers implementing Investigations in Number, Data, and Space (TERC, 1998) had higher-quality lessons than those implementing Everyday Mathematics. The researchers attribute this finding to that fact that Investigations (TERC, 2008) provided more support to teachers for "locating and understanding the big mathematical ideas within lessons compared to Everyday Mathematics" (Stein & Kaufman, 2010, p. 663). I am interpreting Stein and Kaufman’s discussion of the big mathematical ideas as a reference to the focal points, and believe that Investigations' explicit attention to the focal points contributes to teachers' understanding of them.

Violet also mobilized Investigations (TERC, 2008) for series of instructional activities progressions. A series of instructional activities progression generally addresses one or more focal points from the unit progression. Investigations (TERC, 2008) did not, however, always provide enough instructional activities for each focal point, or it provided an instructional activity that students did not need. In those cases, Violet had to make decisions about which instructional activities to facilitate and
sometimes turn to other resources for additional activities. Violet relied on her CGI knowledge to make those decisions. That is, she attended to and assessed students’ solution strategies to make those decisions. *Investigations* (TERC, 2008) provided assessment tools, but not guidance in how to interpret those assessments for time needing to be spent on particular topics.

With regards to number choice and student solution progressions, each of the four teachers relied on their CGI knowledge. *Investigations* (TERC, 2008) did not provide those supports. It is unclear to me if a curriculum series could provide those supports. I think providing number choice progressions for a problem would be as arbitrary as providing a single number choice. It is up to the teacher to decide what number choices are appropriate for his/her students. I do believe, however, that some type of support, possibly in the form of flow-chart, could be developed. I also believe that curriculum materials could provide support for student solution progressions, but I am unsure how that support could be provided due to the amount of room need - perhaps in the form of additional documents or online materials. However, Stein and Kim (2009) pointed out that most teachers do not consult materials outside of the teacher’s guide.

**Knowledge of Students**

*Investigations* (TERC, 2008) provided support to Nancy and Violet to gain general and individual knowledge of students. The series provided examples of student work and provided a means (guidance for questioning techniques) for teachers to gain knowledge of individual students. Again, not all *Standards*-based curriculum series

While *Investigations* (TERC, 2008) gave substantial support to teachers in this manner, my own analysis of *Investigations* (TERC, 2008) found limitations to that support. All the student work examples have correct responses and do not necessarily represent the range of students’ strategies in a particular classroom or school. Students do not always have the right answer. The materials provide no support for anticipating common mistakes and misconceptions.

Another limitation in the examples of student work is that they do not change because of the nature of the materials. However, the range of strategies exhibited by students would change over time. For example, the strategies of third graders using *Investigations* (TERC, 2008) for the first time are going to be different than third graders using *Investigations* (TERC, 2008) since kindergarten. Violet found this to be true in her curriculum use. At the time of the study, Violet no longer examined student work examples because those samples did not accurately represent what her own students would do.

**The Researcher**

My dissertation research has affected me professionally in two ways. I believe the research has prompted me to make effective changes to my teaching, and it has
provided me with several directions in which I could take future research. I conclude this chapter and dissertation with a discussion about each.

Teaching

First, I am much more attuned to the act of selecting number choices. I have come to understand that the selection of number choices deserves much consideration and can be a powerful means to advance student achievement. Now that I have come to that understanding, I take opportunities, when they arise, for my students to generate number choice progressions in class. For instance, we were reading an *Investigations* (TERC, 2008) lesson plan where the focal point was “adding multiples of ten to a whole number.” After reading the goal, I asked students to tell me what that goal would look like. Somebody offered $23 + 10 = \_\_\_\_\_$. From there, we generated number choices that were easier, about the same, and more difficult (e.g., $8 + 10$, $23 + 30$, $57 + 30$, $65 + 40$, and $52 + 120$). We also have assigned characteristics to particular number choices – single-digit number plus 10, non-decade number plus a multiple of ten, non-decade number plus a multiple of ten whose sum goes over the 100, non-decade number plus a three-digit multiple of ten. I believe that using particular language will help pre-service teachers (PSTs) gain understanding of number choice progressions.

Furthermore, I have asked students to consider why certain number choices are more complex in particular problems. For example, I have PSTs consider the number choices in the following two problems:
**Sharing Brownies Problem**
Trisha, Allie, Lance, and Kathy are sharing brownies. If they are sharing ____ brownie equally, how many will each person get?

4   5   8   9   16   17   20
32   33   44   45   48   49   50

**Sharing Miniature Candy Problem**
There are ____ miniature candy bars. Dustin, Jose, Sam, and Joe are going to share the candy bars. If they split up the candy bars equally, how many will each of them get?

11   17   22   35   48
65   83   75   99   104

Both of these problems entail dividing objects among four people, but the number choices makes solving the problem easier or more complex. My students noticed that the number choices become more complex (e.g., 5 is harder than 4 and 11 is harder than 9), which I thought was great. PSTs did not, however, articulate why the numbers were more complex. I would like PSTs to be able to verbalize that multiples of four (4n) are going to be the easiest number to solve for in that problem. Numbers that are one more than a multiple of four (4n + 1) are a little more complex, because they entail dividing up a single object. Numbers such as 11 (4n + 3) are the most complex, because they involve dividing up 3 objects. To help PSTs articulate the increasing complexity, I think I could facilitate a whole-group activity building on their observation of the number choices becoming more complex in the future.

With regards to general and individual knowledge of students, I include many opportunities for PSTs to detail students’ strategies and for them to interact with
students. Given what the research literature (Empson & Junk, 2004; Jacobs, Lamb, & Philipp, 2010; Land, Chapter Three) says about gaining knowledge of students, I believe providing many opportunities to detail strategies adds to PSTs’ general knowledge base about students, which consequently supports acquisition of individual knowledge. I do not think examining a piece of student work needs to be something that takes a long time. Sometimes, I will show a strategy as a warm-up, which we then discuss for a few minutes. When reading curriculum materials, I draw PSTs’ attention to student examples in curriculum materials. Furthermore, I have asked PSTs to complete student interviews where they ask students a series of problems, then describe the details of their solutions.

At this point, I am still struggling with how to incorporate the idea that the four different types of progressions are imbedded within each other and promote movement along each. I address the each of those progression types individually in my teaching. For instance, PSTs examine a unit progression and consider how instructional activities address particular points on the progression. I discussed how I address number choice progressions earlier. I also ask PSTs to order examples of student work from least to most sophisticated creating a student solution progression. I have not determined, however, how to design a context where the relationships between the different progressions are apparent. Furthermore, I am not certain if that idea is entirely accessible to PSTs.
Research

I believe my dissertation work could take me in a few different directions. First, I could attempt to identify what kind of subject matter knowledge teachers with high PDC for teaching elementary mathematics have. Kathy was the only teacher who believed she had a solid background in mathematics. The other three talked extensively about their lack of content knowledge until they began using Investigations (TERC, 2008) and/or CGI. By their accounts, it seems likely that Nancy, Olivia, and Violet acquired their content knowledge through attending to, interpreting, and responding to children’s mathematical thinking, and not through traditional mathematics instruction. Similarly, Empson and Junk (2004) found that teachers’ knowledge of children’s mathematics was not necessarily associated with strong subject matter knowledge. In addition, Stein and Kaufman (2010) found that how a teacher mobilized a curriculum series mattered more than his/her education, experience, and knowledge. It is obvious that elementary mathematics teachers need subject matter knowledge, but exactly what subject matter knowledge they need remains to be seen (Hill, Rowan, & Ball, 2005).

Second, I could investigate how much of an expert teachers’ practice is appropriate and accessible to PSTs. We know that PSTs cannot become expert teachers before they leave a teacher educator program. They could, however, be on a path to become an expert teacher. What tools and abilities would they need to begin that path? More specifically, I would like to design instruction around mobilizing number choices, and assess that instruction using the list of mobilizations from Chapter 5 (Table One) as a framework. I hope to answer the following question – which mobilizations are
accessible to PSTs during the elementary mathematics methods course and with what supports?

Finally, I would like to measure PSTs’ curriculum use before and after taking the elementary mathematics course grounded in the use of Standards-based curriculum series (Land, Drake, & Tyminski, in progress). We (Corey and I) have already developed a curriculum-use survey. The next steps are to refine that instrument as well as develop a scoring rubric. Included in the curriculum-use survey are the many aspects of expert teaching (e.g., knowledge of students, identification of learning goals).

I believe that the value of a research study depends on how much it helps solve problems of practice. Currently, a problem in mathematics education is determining how to teach elementary mathematics in ways that reflects NCTM’s vision widespread. We know that NCTM’s vision exists in small pockets, but the challenge lies in giving all children access to challenging and engaging mathematics. My study contributes to solving that problem in that it has identified key dimensions of expert teachers’ PDC and ways in which to develop and support those dimensions in other teachers.

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