WIDEBAND NONUNIFORMLY EXCITED FOCUSED TRANSDUCERS - THEORY AND EXPERIMENTS

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INTRODUCTION

In an earlier study [1], we discussed the interest of wideband nonuniform transducers in nondestructive testing (NDT). Only flat transducers were studied. Since that, we have developed a new technique that combines in a single probe three different modes: a conventional uniform mode, a mode favoring the plane wave component leading to a diffractionless beam and a mode favoring the edge diffracted wave leading to a focusing effect. This original technique is fully described in [2]. Here, we study the influence of nonuniform source profile on the characteristics of the field radiated by wideband focused transducers, as compared to conventional focused transducers with uniform profile. Classically, the transient field radiated by a conventional spherically focused transducer is described as the superposition of a geometric wave (a spherical wave converging to the focal point then diverging) and of diffracted edge wave [3]. Two nonuniform profiles are studied, both theoretically and experimentally which favor either one or the other contribution.

In the first section, we will review the most important results of the theory describing the transient field radiated by focused probe. An important remark on numerical precision will be made and a solution will be shortly described. We will use this theory combined with the superposition principle to study numerically in the second section the characteristics of the field radiated by nonuniform focused transducers as compared to uniform ones. In the third section, experiments will be compared to theoretical predictions. Finally, the last section will summarize the main results of this study and discuss possible applications of nonuniform focused transducers in the context of NDT.

TRANSIENT FIELD RADIATED BY FOCUSED TRANSDUCER


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apply the Rayleigh integral to non-planar probes, it is necessary to assume that secondary
diffraction effects are negligible, which is a reasonable assumption for slightly focused
probes. In their study, the radiating surface was such that every points of the radiating sur-
face vibrate with the same amplitude simultaneously with the same time-dependency. The
focusing is often obtained by means of a lens. Points at the surface of the lens do not vibrate
simultaneously. There is an equiphase surface in front of the lens which curvature is the ac-
tual focal length of the transducer. Djelouah et al. [5] proved that the calculations using [3]
accurately predict experiments with transducers with lens. Using the impulse response ap-
proach and the superposition principle, Verhoef et al. [6] presented theoretical calculations
of the field radiated by nonuniform focused transducers. The previous references concerned
only the field radiated into a fluid medium. This is an important step since focused transducer
are always used in immersion techniques in the context of NDT. However, it is more com-
plex to calculate the field radiated through a fluid / solid interface (the actual NDT configura-
tion) and only approximate solutions can be derived (see for example [7]). In what follows,
we will only discuss the case of radiation in fluids to concentrate on the effect of nonuniform
sources compared to conventional uniform ones. We give hereafter only the results of the
transient theory of focused probe relevant to the developments made in this paper. Readers
interested in using closed form analytic solution for focused transducers are referred to [3].

**Transient field radiated along the axis of symmetry**

Starting from the time-dependent Rayleigh integral and assuming that time and space
variables are separable in the source term, i.e. \( v_n(r_R,t) = v(t) \Gamma(r_R) \), the pressure field at \((r,t)\)
radiated by a transducer of surface \( R \) is given by,

\[
p(r,t) = v(t) * \rho_0 c \frac{\partial}{\partial t} \int_R \Gamma(r_R) \frac{\delta(t-|r-r_R|/c)}{2\pi |r-r_R|} dS_R.
\]

The origin of the (cylindrical) co-ordinate system is taken at the center of the radiating sur-
f ace. Let \( d \) be the distance between the observation point \( r \) and the geometric focal point at \( z = F \), \( F \) being the curvature of the surface. If the source profile is uniform, i.e., if \( \Gamma(r_R) = 1 \),
the on-axis pressure takes the following values, depending on the position relatively to \( F \).

\[
p(z < F, r=0,t) = \rho_0 c \frac{F}{d} [v(t - t_g) - v(t - t_e)],
\]

\[
p(z > F, r=0,t) = -\rho_0 c \frac{F}{d} [v(t - t_g) - v(t - t_e)],
\]

\[
p(z = F, r=0,t) = \rho_0 c e \frac{\partial v(t - F/c)}{\partial t}.
\]

where \( e = F - (F^2 - a^2)^{1/2} \) is the thickness of the radiating surface, \( a \) being the radius of the
transducer. Contribution arising at \( t_g \) is called the geometric wave, and that arising at \( t_e \)
is called the edge wave, given by \( t_g = (F-d)/c \) if \( z<0 \) or \( (F+d)/c \) and \( t_e = [(z-e)^2+a^2]^{1/2}/c. \)

The pressure radiated on its axis by a nonuniform but axisymmetric source is obtained
by applying the superposition principle after rather simple trigonometric calculation. We have

\[
p(z, r=0,t) = \rho_0 c v(t) * \frac{F}{d} \frac{\partial}{\partial t} \left[ \Gamma \left( \frac{\sqrt{[c^2 t^2 - (F-d)^2][(F+d)^2 - c^2 t^2]}}{2d} \right) \right] \Pi(t_{min},t_{max}) .
\]

where \( \Pi(t_a,t_b) \) is a time window between \( t_a \) and \( t_b \), and \( t_{min} = t_g \) if \( z>F \) or \( t_e \) if \( z>F \) and \( t_{max} = t_e \) if \( z<F \) or \( t_g \) if \( z>F \). This can be connected to the case of flat disk transducer [1] for which
\[ p(0, r=0,t) = \rho_0 c \ \nu(t) \ast \frac{\partial}{\partial t} \left[ \Gamma \left( \sqrt{c^{-2} \tau^2 - z^2} \right) \right] \Pi(t_{\min}^{t_{\max}}). \]  

When the field-point is off-axis, the expression for the velocity potential impulse response derived in [3] is more complex than that for on-axis field-points.

**Numerical precision in the calculation of the field in the focal region**

It is interesting to dispose of exact analytic solutions as that given in [3]. What is calculated analytically has not to be computed by a numerical scheme. However, in the specific case of a focused radiator, the numerical evaluation of the analytic expression poses a problem of precision. When an impulse response is computed, one must choose a time-step at which the continuous analytic expression is evaluated. In order to have a very large bandwidth in the calculation of impulse response, a typical time step \( \Delta t \) is 10 ns (allowing to convolve impulse responses with signal including frequencies up to 50 MHz). However, in the most interesting region of the field, the focal region, the various acoustic paths from the source to the observation point have almost the same time-of-flight. Therefore, impulse responses in this region are defined over a small number of discrete instants — at the geometric focal point, the impulse response is a \( \delta \)-function, the numerical impulse response is defined at one discrete time. In the numerical convolution of impulse response with input signal, when the former is defined over a small number of point, the quantity that actually counts is its area. This area can vary discontinuously and this leads to numerical errors. Consider the example shown in Fig. 1. For two neighboring field points on axis, the velocity potential impulse response has the shape of a time-window. The impulse responses have almost the same area and one expects them to lead to a result of almost the same amplitude after convolution with the input signal. However, once impulse responses have been evaluated according to the time step of discretization, the second numerical impulse response is 1.5 times greater than the first one, which is unacceptable and cause numerical errors. To solve this problem, one possibility is to oversample impulse responses but this leads to unnecessary lengthy computations. We chose to evaluate the impulse response \( h(t) \), whenever it is necessary, by assigning at a given discrete time \( i \Delta t \) the following value to the numerical impulse response \( h_{num}(t) \),

\[
h_{num}(i \Delta t) = \frac{1}{\Delta t} \int_{i \Delta t}^{(i+1) \Delta t} h(\tau) \, d\tau.
\]

When the analytic impulse response is constant, computing its exact area is trivial. When it is not (off-axis field-points) and that such problem may arise, the computation at each time step is done using a 4-point-Gauss’ method of integration. When impulse responses are defined over a sufficiently large number of time steps, above precautions are not required.

**NUMERICAL STUDY OF THE FIELD RADIATED BY NONUNIFORM FOCUSED TRANSDUCER**

The present section deals with a numerical study of the field radiated by nonuniform

![Fig. 1 Two impulse responses of almost the same “analytic area” but which “numerically evaluated” areas differ by a factor of 1.5.](image)
transducers. The three different source profiles considered (one uniform and two nonuniform) are the same as those considered in [1], given by

\[
\begin{align*}
\Gamma_1(r_R) &= 1, & \text{for } r_R \leq a, \\
\Gamma_2(r_R) &= 1, & \text{for } r_R < a_0, \\
\Gamma_2(r_R) &= \left\{\cos[\pi (r_R-a_0)/(a-a_0)] + 1\right\}/2, & \text{for } a_0 \leq r_R \leq a. \\
\Gamma_3(r_R) &= 0, & \text{for } r_R < a_0, \\
\Gamma_3(r_R) &= \left\{1 - \cos[\pi (r_R-a_0)/(a-a_0)]\right\}/2, & \text{for } a_0 \leq r_R \leq a.
\end{align*}
\]

(8) (9) (10)

Note that \(\Gamma_2(r_R) + \Gamma_3(r_R) = \Gamma_1(r_R)\). A source profile \(\Gamma_2\) attenuates diffraction by the edge of the aperture while a profile \(\Gamma_3\) attenuates the radiation by the center of the transducer.

In the strictly numerical study that follows, we consider focused disk transducers of 19-mm-Ø and take \(a_0 = a/2\) for the nonuniform profiles. The geometric focal length \(F\) is a varying parameter in the range of 50 to 150 mm. A synthetic excitation pulse is taken for the computations with \(v = 3\) MHz, given by

\[
v(t) = \left[\sin(2\pi v t) - 1/2 \sin(4\pi v t)\right] \Pi(0, 1/v).
\]

(11)

Fig. 2 compares maps of peak-to-peak amplitudes (C-Scan) of the field radiated by the

![Fig. 2 Iso-contour maps of the field radiated by 19-mm-Ø transducers with \(F = 150\) mm vibrating with profiles \(\Gamma_1\) or \(\Gamma_2\). \(z\) varies in the range \([40, 240]\) mm and \(r\) \([0, 3.5]\) mm.](image-url)
uniform transducer with $F = 150$ mm and by a transducer with the same curvature but nonuniform profile $\Gamma_2$ favoring the geometric wave.

Fig. 3 compares maps of peak-to-peak amplitudes (C-Scan) of the field radiated by the uniform transducer with $F = 150$ mm and by a transducer with the same curvature but nonuniform profile $\Gamma_3$ favoring the diffracted edge waves.

In Figs. 2 and 3, the maximum amplitudes of both kind of transducers have been normalized. From the previous figures, one can measure the -3 dB focal length and the -3 dB diameter of the beam (in general, focused transducers are characterized by their length and diameter at -6 dB in transmit-receive mode which are comparable with the -3 dB values in transmission). One sees that the profile $\Gamma_2$ leads to a longer but wider focal zone, while the profile $\Gamma_3$ leads to a longer but thinner focal zone, as compared to that of the uniform transducer. For both nonuniform sources, the center of the focal zone is closer to the transducer than for uniform transducer.

Fig. 4 compares the variation of the -3 dB diameter, -3 dB length and maximum peak-to-peak amplitude for the three modes with varying geometric focal length. The overall tendencies appear very clearly from these figures. First, beam diameter of transducers with nonuniform profile $\Gamma_2$ is wider than that of uniform ones, while that of transducers with
nonuniform profile $\Gamma_3$ is thinner. In general, focused transducers are used to enhance radial resolution. Therefore, profile $\Gamma_3$ favoring the edge diffracted waves are more interesting than conventional uniform transducer from this point of view. It can be noticed that the difference between transducers with $\Gamma_2$ profile and uniform profile is smaller than that between uniform transducer and transducers with $\Gamma_3$ profile. Quantitatively, uniform focused transducers radiate more energy than nonuniform transducers (as it might be expected).

A very interesting property of nonuniform profile favoring the geometric wave is the slow variation of waveform within the focal zone. Fig. 5 compares waveforms at different field-points in the focal zone of a uniform focused transducer and a transducer with nonuniform profile $\Gamma_2$.

Fig. 5 Example of waveforms variation in the focal zone of transducers with uniform profile [a) and b)] and profile $\Gamma_2$ [c) and d)]. a) and c): on-axis. b) and d): 1 mm off-axis.
At ranges longer than the geometric focal length, waveforms for the two transducers look the same. In fact, waveform variations result of the interference between the geometric and the edge waves. On axis, these waves are time-separated by an interval $\Delta t = t_g - t_e$ which varies with $z$ as shown in Fig. 6. $\Delta t$ asymptotically tends to $e / c$ which is smaller than the pulse duration used to compute waveforms shown on Fig. 5. At a range of 50 or 70 mm, $\Delta t$ is larger than the pulse duration so that geometric and edge waves are time-separated. With the nonuniform profile, the edge waves are greatly attenuated at these ranges so that waveforms do not show time-separated pulses and waveforms globally show less variations.

**PRACTICAL REALIZATION OF FOCUSED NONUNIFORM TRANSDUCERS**

We realized nonuniform transducers on the base of a uniform focused transducer by a technique similar in its principle to the practical realization of flat nonuniform transducer presented in [2]. It consists in dividing the active area of the transducer into two regions such that their shape models the wanted nonuniform profiles, as shown in Fig. 7. The idea comes from Stepanishen's geometric interpretation of the Rayleigh integral [4]. At a point $r$ at instant $t$, the velocity potential impulse response is proportional to the intersection of the sphere centered at $r$ of radius $ct$ with the active area of the transducer. Therefore, partial masking of the surface is equivalent to nonuniform source profile. For the experiments shown hereafter, this shape is obtained by masking a conventional uniform transducer with an ultrasonically reflecting material serrated according to the shape shown on Fig. 7.

Fig. 8 b) compares theoretical computations of the -6 dB focal zone computed for transmit-receive mode (computed with the model for calculating echo-response from targets in fluids described in [8] fully validated in [9]) with pulse-echo experiments with echoes measured from a small disk target of diameter 1 mm. The excitation pulse used for the computations is that shown in Fig. 8 a).

![Fig. 6 Time interval between geometric and edge waves for on-axis field points.](image)

![Fig. 7 Shape of the masks (in white) and resulting nonuniform profiles.](image)
Fig. 8 a) : time dependence of the source. b) experimentally measured and calculated limits of the -6dB zone for the two nonuniform profiles compared to that of the uniform.

The diameter of the transducer used in experiments is of 54 mm and its focal length is 225 mm. Modeled and measured results compare very well. The small discrepancies may be due to the fact that the transducer used in experiments was focused by a lens. Attenuation in the lens may have create itself an apodization, not taken into account in modeled results.

CONCLUSION

The numerical study presented in this paper describes the main characteristics of the field radiated by two nonuniform focused transducers as compared to conventional uniform focused transducers. The main interest of nonuniform profile favoring the edge diffracted waves is to lead to better focusing characteristics. That of nonuniform profile favoring the geometric wave is to lead to slow waveform variations within the focal zone. However, uniform focused transducer has still the advantage of higher sensitivity. Since the two nonuniform profiles considered are complementary, an electrode design such as that described in [2] could be easily adapted to focused sources. This would allow one to combine the three different modes studied herein in one single probe and therefore to take advantage of their three different characteristics during the same NDT experiments.

REFERENCES

FIDELITY OF MICHELSON INTERFEROMETRIC AND CONICAL PIEZOELECTRIC ULTRASONIC TRANSDUCERS

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INTRODUCTION

The motivation for this research is our ongoing effort in the development of ultrasonic, waveform-based, materials characterization techniques. Having developed a three-dimensional representation of the elastodynamic Green's Function for anisotropic plates [1], we now seek to verify the applicability of this representation. With elastic properties measurement as our end goal, we also seek a transducer for measurement of theoretically predicted waveforms. Our waveforms typically exhibit a large range in both amplitude (40 to 60 dB) and frequency (20 kHz to 2 MHz). The transducer used must exhibit both high-sensitivity and high-fidelity so that multiple reflections can be detected and identified. In a companion paper we describe a transducer developed at NIST for acoustic emission (AE) studies [2]. In that paper we determine that this transducer has a displacement sensitivity of approximately $5 \times 10^{-17} \text{ m/} \sqrt{\text{Hz}}$ in the 250 kHz to 1 MHz frequency region on aluminum. This transducer appears to be a good candidate for waveform-based materials characterization. In this paper we evaluate this transducer's fidelity by comparing it with both theoretical results and measurements from a path-stabilized, Michelson interferometer. We conclude that the current transducer design does not have sufficient fidelity for waveform-based materials characterization and discuss the reasons for its shortcomings and potential solutions to these problems.

ELASTODYNAMIC GREEN'S FUNCTION FOR ANISOTROPIC PLATES

Two different elastodynamic Green's Functions were used as a baseline for this fidelity study. In addition to Tewary's new delta-function representation for anisotropic plates, Hsu's

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Fourier representation for isotropic plates was also used [3]. While the boundary conditions used in Tewary's calculations prevent the propagation of plate modes, Hsu's computer model includes such displacements. For this reason, the results of the two computations are not identical.

PIEZOELECTRIC DISPLACEMENT SENSOR

The piezoelectric sensor considered in this study is a modification of the Proctor transducer (NIST-SRM #9)[4]. Fortunko and Hamstad [5] have changed the sensor design and added an internal preamplifier. In order to make the transducer practical for field applications they have added a copper wear plate, electromagnetic shielding and a ruggedized case.

In the case of the Proctor transducer, brass is used in for the backing mass because of its similar characteristic impedance to the piezoelectric material (PZT-5A). The backing mass is also shaped such that elastic waves transmitted into it from the piezoelectric element are contained within the mass until they dissipate. The Fortunko-Hamstad transducer uses a cylindrical mass which absorbs the transmitted signal efficiently, but has a substantially different characteristic impedance than the piezoelectric. This impedance mismatch results in reflected signals within the piezoelectric element. Figure 1 shows a cross section of the Fortunko-Hamstad transducer.
MICHELSON INTERFEROMETER

A path-stabilized, polarized optics, Michelson interferometer was used as a primary reference to absolute displacement in this study. A 500 mW (nominal), polarized laser with a wavelength of 1.064 \( \mu \text{m} \) was used. The stabilization system consisted of a loop-filter, high-voltage amplifier, and a mirror mounted to a PZT-5H stack with a maximum displacement of 4 \( \mu \text{m} \) at +250 V. The loop filter and high-voltage amplifier allowed stabilization to approximately 500 Hz at amplitudes of more than 2 \( \mu \text{m} \). The 10 MHz, high frequency cut-off of the interferometer’s response is determined by the 20 MHz sampling rate of the digital storage oscilloscope (DSO) used. It’s 500 Hz, low-frequency limit is a result of the active stabilization system. The detector system used in the interferometer is a \( \pm 15 \text{ volt} \), reverse-biased, differential circuit similar to that used by Deaton [6] and others. The output of the detection circuit is amplified and recorded on the 12-bit DSO. When incident on a polished aluminum surface, the interferometer exhibits a saturation voltage of approximately 18 V, peak-to-peak. The noise-level of the interferometer is approximately 50 mV. For displacement signals on the order of a few angstroms this gives a signal-to-noise ratio (SNR) of approximately 62 which corresponds to a sensitivity of approximately \( 1 \times 10^{-15} \text{ m/}\sqrt{\text{Hz}} \). The laser wavelength used in the interferometer determines the maximum detectable displacement which, for a 1.064 \( \mu \text{m} \) wavelength is 0.169 \( \mu \text{m} \). The interferometer, therefore, has a dynamic range of approximately 92 dB in the frequency range from 500 Hz to 10 MHz. The response of the interferometer near its upper displacement limit, however, is nonlinear. The displacements of our signals were well below the nonlinear response region.

MEASUREMENT CONFIGURATION

A large aluminum plate was chosen for this study for both its isotropic elasticity and high reflectivity. The plate had a length and width of 609.9 mm and was 50.8 mm thick. The source used to generate broadband acoustic signals was either a glass capillary break or a lead break at the center of the plate. These two sources were chosen for their close approximation of a step-release and the broad-band frequency content of the resultant acoustic wave [7]. Although the capillary break produced a sharper step-function, the amplitude of the resultant wave was beyond the saturation point of the electronics used in the conical piezoelectric. For this reason data for the capillary are only presented for the interferometer for comparison with theoretical results.

For each measurement, the Fortunko-Hamstad transducer and the interferometer were placed equidistant from the source. A broadband elastic wave signal was generated and then simultaneously detected by the two sensors. The time-dependent waveform from each sensor was recorded on the DSO and stored to floppy disk.
The source-to-receiver distance was varied from 25.4 mm to 101.6 mm. In terms of the ratio of the receiver distance to plate thickness, d/t in Figure 2, the variance was from 0.5 to 2.0. The time-dependent waveforms were then compared.

RESULTS

Figure 3 compares the computed waveforms from both the Tewary and Hsu representations as well as the waveform detected by the Michelson interferometer for a d/t ratio of 0.5. All three are in agreement with the noted absence of the low-frequency, plate modes in the Tewary representation as expected from the boundary conditions mentioned previously.

The measured waveforms from the interferometric and piezoelectric sensors for a d/t ratio of 1.0 are shown in Figure 4. Compared to the interferometric response, the conical piezoelectric exhibits both a resonance with a periodicity corresponding to approximately 450 kHz, and a low-frequency cut-off that appears to correspond to a periodicity of approximately 60 kHz. These factors make it impossible to accurately identify the small-amplitude, multiple reflections of the signal seen in the interferometric waveform. From these results we concluded that, in its current form, the fidelity of the Fortunko-Hamstad transducer is not sufficient for waveform-based materials characterization.
IDENTIFICATION OF FIDELITY DEGRADATION SOURCES

Figure 5 shows waveforms recorded at a d/t ratio of 1.0 using the NIST-SRM, the NIST-SRM without its preamplifier, and the conical piezoelectric without its preamplifier. These results show that both sensors exhibit a resonance at 450 kHz, but that the notch filter contained within the NIST-SRM preamplifier significantly reduces this resonance. The Fortunko-Hamstad transducer, however, exhibits a substantially larger resonance. We attribute this to both the poorer mechanical impedance match of the PZT-5A to the granite (vs. brass used in the SRM) and the superior attachment process used to bond the piezoelectric to the backing mass in the SRM.

It is also evident from Figure 5 that the removal of the preamplifier electronics from the Fortunko-Hamstad transducer restores the low-frequency response of the sensor. An integrated preamplifier with better low-frequency response would correct this deficiency.
SUMMARY

We have found the Fortunko-Hamstad transducer, designed for AE studies, does not currently have sufficient fidelity for waveform-based materials characterization. We have further identified the reasons for this sensor’s lack of fidelity by comparing it with the NIST-SRM transducer. Work is underway to improve the Fortunko-Hamstad transducer which will result in both a high-sensitivity and a high-fidelity sensor.
Figure 5. Waveforms from: a) NIST-SRM, b) NIST-SRM without preamplifier and c) Fortunko-Hamstad transducer without preamplifier.
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