Capacitive sensors for measuring complex permittivity of planar and cylindrical structures

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Capacitive sensors for measuring complex permittivity of planar and cylindrical structures

by

Tianming Chen

A dissertation submitted to the graduate faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY

Major: Electrical Engineering

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Iowa State University
Ames, Iowa
2012

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To my father Baorong Chen

who encouraged me pursuing a Ph.D. when I was ten,

my mother Yun Xiong

who is always beautiful and perfect in my heart,

and my lovely wife Mengmeng

who makes me aware life is not all about programming.
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Figure 9.10  As for Figure 9.7 but for hydrolytically exposed wires.
Figure 9.11  Inferred complex permittivity, a) real part and b) imaginary part, of the hydrolytically exposed wires in comparison with that of polyimide HN film (23). Note the very different scales on the vertical axes of Figure 9.11 b), presented this way for clarity.
ABSTRACT

With the increasing use of low-conductivity structural and functional materials, there has been a greater need for the efficient and reliable nondestructive evaluation (NDE) of these materials. One approach to evaluate low-conductivity structural and functional materials is to characterize the material dielectric property. In this thesis, capacitive sensors are developed for measuring complex permittivity of planar and cylindrical materials. For each sensor configuration, models are developed to allow for inverse determination of material permittivity from measured capacitance, therefore realizing quantitative characterization of material dielectric properties.

In the first half of the thesis, coplanar concentric capacitive sensors are developed to meet the need of detecting water or excessive inhomogeneities caused by repairs in aircraft radome structures. Another important motivation is the absolute dielectric property characterization of laminar structures. Three coplanar sensor configurations are designed: the simple two-electrode concentric configuration, the interdigital spiral and the interdigital concentric configurations. Corresponding numerical models are developed to predict the sensor capacitance for given test-piece structures. The validity of the models is verified by comparing numerical predictions and measurement results. The advantage and disadvantage of each sensor configuration is discussed. For the two-electrode concentric configuration, a prototype handheld probe is also fabricated, and has detected successfully 1 cc of low contrast liquid in a simulated radome structure.

Curved patch capacitive sensors, presented in the second half of the thesis, are developed with the motivation of accurate and convenient permittivity measurement of cylindrical structures. It is demonstrated that the permittivity of homogeneous dielectric rods is inferred easily from measured sensor capacitance, based on analytical and numerical models developed here. Another practical application of the curved patch capacitive sensors is the quantitative evaluation of aircraft wiring insulation condition. In this work, wires are modeled as cylindrical
dielectrics with a conductive core. A numerical relationship between the complex permittivity of the insulation and the sensor capacitance and dissipation factor is established. A prototype probe, developed based on this model, has distinguished successfully degraded wires from the control ones. The feasibility of utilizing the presented capacitive approach for quantitative evaluation of aircraft wiring insulation condition is demonstrated.

Although the development of the capacitive sensors in this thesis is motivated by aerospace engineering related applications, results presented in this work have the potential to be applied to other engineering fields. Potential sensor applications and recommended future research are suggested at the end of the thesis.
CHAPTER 1. GENERAL INTRODUCTION

1.1 Introduction

The work presented in this thesis falls under the research area of nondestructive evaluation (NDE). More specifically, the capacitive sensors developed in this thesis are suitable for the NDE of dielectric structures.

1.1.1 Background

The term NDE may sound unfamiliar but the use of NDE can be easily found in our everyday lives. Before getting into the technical definition of NDE, let us look at an example that is familiar to all of us: we can tell if a wall is thin or thick simply by knocking on it and listening to the hollow response. This simple example illustrates perfectly how NDE works. In this case, we send out a certain form of energy (by knocking), the energy interacts with the material under test and something happens (the wall vibrates), and we interpret the energy that comes back to us (the sound we hear) to obtain the desired information (whether the wall is thin or thick).

An accurate definition of NDE is given in (1): NDE has been defined as comprising those test methods used to examine an object, material or system without impairing its future usefulness.

Modern NDE emerged in the early 20th century to meet people’s needs of producing “flawless” components and preventing failures. Up until this day, NDE has evolved into an interdisciplinary area that encompasses aerospace engineering, electrical engineering, mechanical engineering, materials science and many other scientific areas. Some of its popular applications include quality control during manufacturing and diagnosis during maintenance and service. Details on different types of NDE methods and their applications can be found in the texts (2).
1.1.2 NDE of Metallic Materials

The use of metallic materials can be traced back to several hundred years ago. Not surprisingly, NDE methods for metallic materials are well developed compared. For example, to detect surface or near surface defects in metallic materials, electromagnetic methods, such as eddy current testing and magnetic testing, can be applied. On the other hand, ultrasonics and radiography have been developed and utilized to detect volumetric flaws in metallic materials. Introductions to these methods are presented in (3).

1.1.3 NDE of Low-conductivity Materials

NDE of low-conductivity structural and functional materials is the focus of this thesis. Recent years have witnessed an increasing use of low-conductivity structural and functional materials. The extensive use of polymer-matrix composite materials is one of the many examples. As pointed out in (4), “The word composite in the term composite material signifies that two or more materials are combined on a macroscopic scale to form a useful third material. The advantage of composite materials is that, if well designed, they usually exhibit the best qualities of their components or constituents and often some qualities that neither constituent possesses”. Major engineered advantages of composite materials include being resistant to fatigue damage and harsh environments, repairable, low weight while sometimes being more stiff than metallic materials. Due to these promising properties, composite materials have been used increasingly in the aviation industries (e.g., the Boeing 787 Dreamliner) and the renewable energy industries (e.g., the manufacturing of wind turbine blades). In addition, Kevlar body-armor and ceramic-matrix-composites for thermal stability in hot engine environments are examples of some of the recently developed applications of low-conductivity structural and functional materials.

Also increased is the need for accurate NDE of these low-conductivity structural and functional materials. However, NDE methods for low-conductivity structural and functional materials are not as well developed as those for metallic materials. New approaches are yet to be developed to assess the quality of these materials during both manufacturing and maintenance.
One approach to evaluate low-conductivity structural and functional materials is to characterize their dielectric properties. This is analogous to using conductivity as a parametric indicator when it comes to the NDE of metallic materials. The dielectric properties of materials include the dielectric strength and the dielectric constant. Dielectric strength is the voltage a material can withstand before electrical breakdown happens. Dielectric constant is a measure of the material’s capability of storing electric energy. Most of the time, materials dielectric constant is a complex number, and a function of frequency and temperature. It is therefore not hard to understand why dielectric constant is also referred to as complex permittivity. The work presented in this thesis focuses on the characterization of complex permittivity of planar and cylindrical structures using capacitive sensors. Other approaches that can be used for the characterization of materials complex permittivity include microwave techniques and resonant testing. One advantage of utilizing capacitive methods lies in the fact that it does not require the use of expensive equipment or complicated operation procedures to achieve good inspection results.

1.2 Thesis Organization

This thesis can be divided into two parts based on the configurations of the developed capacitive sensors: the coplanar capacitive sensors are discussed in Chapters 2 through 5 and the curved patch capacitive sensors in Chapters 6 through 9. The coplanar capacitive sensors described in Chapters 2 to 5 are developed to meet the need of measuring the permittivity of laminar structures through one-sided access. Chapter 2 describes the development and modeling of a concentric coplanar capacitive sensor, and the experimental verification of the numerical model. Chapter 3 presents another approach to model the concentric capacitive sensors: the spectral domain approach as opposed to the spatial domain approach used in Chapter 2. Comparisons between these two methods are presented. In Chapter 4, a handheld capacitive probe has been developed based on the physical model described in Chapters 2 and 3. This prototype probe can be applied for practical inspection of laminar structures. For example, the probe has successfully detected 1 cc of low contrast liquid injected into a sandwich structure such as used in an aircraft radome structure. The
sensors developed in Chapters 2 through 4 are relatively simple in configuration, and have the advantage of being straightforward to model. However, the output capacitance is relatively low - typically a few pF. In order to enhance the signal strength and the signal-to-noise ratio, capacitive sensors having the spiral and the concentric interdigital electrode configurations are developed in Chapter 5, and compared with the simple two-electrode concentric configurations. One common feature of the capacitive sensors in Chapters 2 to 5 is the rotational symmetry of the sensor structure. As a result, the sensor capacitance is immune to the relative orientation of the sensor and the test-piece. This feature makes these coplanar capacitive sensors very suitable for many practical inspections.

Chapters 6 to 9 present curved patch capacitive sensors that measure the permittivity of cylindrical structures. The materials under test in Chapters 6 and 7 are homogeneous cylindrical dielectric rods. In Chapter 6, a numerical model that relates the permittivity of the dielectric rod under test to the measurable sensor capacitance has been developed and verified experimentally. As a step further, a 2D analytical solution, calculating the capacitance of a curved patched capacitor that conforms to the curvature of the same test-piece structure, has been derived in Chapter 7. A practical measurement setup based on this 2D solution has also been developed and tested. The significance of the work in Chapters 6 and 7 is that it greatly facilitates the process of permittivity measurement of dielectric rods. Currently, the most common approach is to cut a slice from the end of the rod, and measure the permittivity of the slice using a parallel plate capacitor. This process is destructive, not to mention the permittivity of the slice might be changed due to strain induced during cutting. In contrast, the measurement approaches developed in Chapters 6 and 7 are nondestructive. The test-piece permittivity can be inferred easily, based on the models, from the measured capacitance.

Research work presented in Chapters 8 and 9 is motivated by the need for quantitative evaluation of insulation condition of electrical wires. The insulation complex permittivity changes as a result of degradation. The curved patch capacitors, therefore, are employed to characterize the complex permittivity of the wiring insulation. In Chapter 8, wires are modeled as circular dielectric cylinders with a conductive core. A numerical model has been developed to relate the test-piece complex permittivity to the measurable sensor capacitance and dissipation.
factor. The validity of this model is verified through benchmark experiments on large-scale test-pieces. Based on this model, a prototype capacitive probe has been developed in Chapter 9 to evaluate the insulation condition of aircraft wires. It is demonstrated that it is feasible to use the proposed capacitive approach in the evaluation of wiring insulation condition. The capacitance technique developed in Chapters 8 and 9 is a localized measurement method. It is complementary to large-scale inspection techniques and has the potential to be built into smart embedded wiring test systems of the future.

1.3 Literature Review

Considering that in-depth literature reviews have been included in Chapters 2 through 9, a detailed survey of the literature will not be repeated here. I refer the reader to each chapter for reviews on different topics. In Chapter 2, major methods for dielectric property measurements are listed. These methods range from low frequency capacitive methods to high frequency microwave techniques. Since capacitive sensing is the focus of this work, a thorough literature review on capacitive methods is performed. In the area of materials characterization, capacitive methods based on numerical models are detailed in Chapters 2, 5, 6 and 8, whereas those based on analytical models are presented in Chapter 6. Apart from materials characterization, other applications of capacitive methods, such as pressure sensing and displacement sensing, are listed in Chapter 7. The latter part of the thesis covers the inspection of electrical wires. Different wiring inspection methods are presented in Chapter 9. Some of those methods are designed to inspect the condition of the wiring conductor while others are developed for the inspection of insulation condition.

1.4 Generic Research Approach

The development of capacitive sensors in this work follows the same generic research approach. First, the problems to solve are analyzed to determine the appropriate NDE technique and sensor configuration to use. For the problems studied in this thesis, capacitive methods are selected as the appropriate techniques. Second, numerical or analytical models are devel-
oped for the inverse determination of test-piece permittivity from measured sensor capacitance. Benchmark experiments are then performed to verify the validity of the models. For both the coplanar and the curved patch capacitive sensors developed here, prototype probes are developed for practical inspections. Uncertainty studies are also performed throughout the whole process to help better understand the measurement system.
1.5 References


CHAPTER 2. ANALYSIS OF A CONCENTRIC COPLANAR
CAPACITIVE SENSOR FOR NONDESTRUCTIVE EVALUATION OF
MULTI-LAYERED DIELECTRIC STRUCTURES

A paper published in the *IEEE Transactions on Dielectrics and Electrical Insulation*
Tianming Chen and Nicola Bowler

2.1 Abstract

A concentric coplanar capacitive sensor is analyzed for the quantitative characterization of material properties for multi-layered dielectrics. The sensor output signal, transcapacitance $C_T$, is related to the thickness and dielectric constant of each layer of the material under test. Electrostatic Green's functions due to point charges over different dielectric structures are derived utilizing the Hankel transform given the cylindrical symmetry of the proposed sensor. Numerical implementations based on the Green's functions are presented. The sensor electrodes are divided into a number of circular filaments, and the sensor surface charge distribution is then calculated using the method of moments (MoM). From the sensor surface charge, $C_T$ is calculated. Numerical calculations on sensor optimization are conducted and show that normalized $C_T$ as a function of sensor configuration is determined solely by its own relative dimensions, regardless of the overall dimensions of the sensor. In addition, calculations indicate how the sensor can be optimized for sensitivity to changes in core permittivity of a three-layer test-piece such as an aircraft radome. Benchmark experiment results are provided for one, two-, and three-layer test-pieces and very good agreement with calculated $C_T$ is observed. The sensor is also applied to water ingression measurements in a sandwich structure resembling the aircraft radome, in which the water-injected area can be successfully detected from the sensor.
output signal.

2.2 Introduction

Dielectric materials play an extensive role in both industrial applications and scientific research areas. In the modern integrated circuit industry, as electrical components are miniaturized, there are palpable needs for dielectric measurements of low-loss thin materials. The use of fine-line signal conductors requires thinner, possibly laminated, low-dielectric constant printed-wiring board materials. On the other hand, compact antenna arrays require high-dielectric constant substrates to obtain phase shifts. Moreover, lightweight structural composites in air- and space-craft, Kevlar body-armor and ceramic-matrix-composites for thermal stability in hot engine environments are examples of some of the recently developed applications of low-conductivity materials. As a result of these increased applications of dielectrics, the quantitative dielectric property characterization of these dielectric materials becomes markedly important for the process control in manufacturing, optimization of electrical apparatus design and performance, and system monitoring and diagnostics.

A number of high frequency nondestructive evaluation (NDE) techniques have been developed for dielectric measurements with their own specific applications (1). Transmission-line techniques are capable of measuring material permittivity by an open-circuit termination. The material properties of the test-piece can be interpreted from the reflection coefficient of the system. Open resonators have also been used in measuring low-loss materials in the millimeter wavelength range (2) and a certain open resonator system for measuring anisotropic thin films has been developed and is able to obtain the material tensor permittivity values (3). Measurements using surface electromagnetic waves are quite applicable for low-loss dielectric thin films and layered substrates, since they possess a high quality factor and are therefore sensitive to loss (4). Evanescent-field dielectrometry has been utilized in diagnosing and monitoring fresco degradations resulting from moisture and soluble salts (5). Besides, broadband dielectric measurements (0.01 to 3 GHz) on the effects of exposure of thick film adhesive-bonded structures to moisture have been reported (6), where the data obtained are complemented by mechanical testing and failure analysis of the bond structure measured as a function of the ex-
posure time. However, the focus of this paper is on describing electrostatic and low frequency NDE techniques for dielectric measurements.

One important and practical field of material dielectric property characterization is dielectrometry, which derives the complex permittivity of a test-piece from the measured sensor capacitance. Interdigital dielectrometry sensors, with increased effective length and output capacitance between the electrodes because of their interdigital structure, have been used for dielectrometry measurements for a long time. An excellent review paper on interdigital sensors and transducers is (7), in which the physical principles, sensor design and fabrication, and relevant applications of interdigital sensors are discussed in detail. These interdigital dielectrometry sensors have been applied in many fields such as material property monitoring, humidity and moisture sensing, electrical insulation properties sensing, monitoring of curing processes, chemical sensing, biosensing, and so on. For example, using a secant method root-searching routine for parameter estimation, interdigital electrode dielectrometry has been made capable of measuring the continuum parameters of heterogeneous media (8), which include material thickness, material permittivity with thickness known, and material surface conductivity with thickness known. The optimization of multi-wavelength interdigital dielectrometry instrumentation and algorithms has also been described in (9). Through variation of geometrical design, materials, manufacturing processes, electronic circuitry, and considerations of accumulated effects of non-ideal geometry of experimental setups, improvement of sensor performance can be achieved. Additionally, design principles for multichannel fringing electric field sensors, especially detailed analysis on how the sensor geometry affects the sensor performance and tradeoffs among different design objectives, have been carried out (10) providing insight into design of capacitive sensors in general.

Apart from using interdigital dielectrometry sensors, other sensor configurations have been used to characterize defects, moisture content, temperature, aging status, delamination, and other inhomogeneities in dielectric materials. For example, rectangular capacitive array sensors have been used for the detection of surface and subsurface features in dielectrics and surface features in conductive materials (11). Cylindrical geometry quasistatic dielectrometry sensors with signal interpretation based on semi-analytical models have also been developed in recent
years to measure the permittivity of a dielectric plate (12). For water intrusion detection in composite structures, rectangular coplanar capacitance sensors with high sensitivity have been developed (13) on the basis that the presence of defects, such as water, leads to changes of dielectric characteristics in the structure, resulting in variations in the sensor measured capacitance. Using a similar principle, rectangular coplanar capacitance sensors have been applied for damage detection in laminated composite plates (14). Also, the influence of electrode configurations on a differential capacitive rain sensor, which consists of a sensitive capacitor whose capacitance changes in the presence of water and an insensitive reference capacitor, have been investigated in (15). Moreover, these capacitance techniques have even been employed for the continuous monitoring of the thickness of biofilms and tissue cultures (16).

Electrical capacitance tomography (ECT) is another capacitance measurement technique that is used to image cross-sections of industrial processes containing dielectric materials (17). The principle is that through image reconstruction for ECT, the test-piece permittivity distribution and therefore the material distribution over its cross-section can be determined. Over the past decades, research progress on both the hardware design (18; 19) and sensor configuration optimization (20) of ECT systems has been made successfully.

In this paper, a concentric coplanar capacitive sensor is developed with the motivation of detecting water or excessive inhomogeneities caused by repairs in modern radome structures. The proposed sensor, having the advantage of rotational symmetry, consists of a charged central disc and a coplanar outer annular ring that exhibit a strong measurable transcapacitance $C_T$. The output signal depends on the material and structural properties of the test-piece with which the sensor is in surface contact. An electrostatic Green’s function for a three-layered dielectric structure in free space is derived in cylindrical coordinates through the Hankel transform method. This derived Green’s function may then be simplified, providing results for many other cases such as a half-space dielectric, a layered half-space dielectric, and one- and two-layered dielectrics in free space. Numerical implementations based on these Green’s functions are described, in which the surface charge distribution on the sensor electrodes is calculated through the method of moments (MoM). From the surface charge, $C_T$ is calculated. To verify the validity of the numerical calculation, benchmark experiments are conducted for one-, two-,
Figure 2.1 Concentric coplanar capacitive sensor. The radius of the central disc and the width of the outer ring are denoted $s$ and $t$, respectively. The gap between them is $g$, and $D$ is the sensor diameter.

and three-layer dielectric test-pieces in free space, respectively. Very good agreement is observed between the calculated and measured transcapacitance. Furthermore, water ingestion measurements in a sandwich structure are carried out and demonstrate the feasibility of using the capacitive sensor to detect water intrusion and inhomogeneities in radome structures.

2.3 Green’s Functions for Multilayered Dielectrics

The configuration of the proposed sensor is shown in Figure 2.1. This coplanar sensor consists of two concentric electrodes: the inner disc and the outer annular ring.

Electrostatic Green’s functions due to a point charge over different test-piece structures are derived first. These Green’s functions are then utilized in later MoM calculations of the sensor transcapacitance $C_T$. Because of the cylindrical symmetry of the designed sensor, the electrostatic Green’s functions are derived in cylindrical coordinates through the Hankel transform method. Additionally, the test-pieces in our theoretical analyses are assumed to be infinite in the horizontal directions and the sensor electrodes are assumed to be infinitesimally thin.

Assume there is a point charge placed at the origin in free space. The resulting electrostatic potential $\Psi$, related to the electric field $\mathbf{E} = -\nabla \Psi$, satisfies the Laplace equation and can be expressed in cylindrical coordinates as

$$
\left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial z^2} \right) \Psi(\rho, z) = 0, \quad r \neq 0
$$

(2.1)

where $\Psi(\rho, z)$ is independent of azimuthal angle $\phi$. Next, the Hankel transform $\tilde{f}(\kappa)$ of zero-
order of a function $f(\rho)$ is given by

$$\tilde{f}(\kappa) = \int_0^\infty f(\rho)J_0(\kappa\rho)\rho\,d\rho$$

(2.2)

where $J_0(z)$ is the Bessel function of the first kind and the inverse transform is of the same form. Apply the zero-order Hankel transform to (2.1), making use of the following identity

$$\int_0^\infty \left[ \left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} \right) f(\rho) \right] J_0(\kappa\rho)\rho\,d\rho = -\kappa^2 \tilde{f}(\kappa),$$

(2.3)

where $f(\rho)$ is assumed to be such that the terms $\rho J_0(\kappa\rho)\partial f(\rho)/\partial \rho$ and $\rho f(\rho)\partial J_0(\kappa\rho)/\partial \rho$ vanish at both limits. The spatial domain Laplace equation (2.1) is then transformed into a one-dimensional Helmholtz equation in the transformed domain:

$$\left( \frac{\partial^2}{\partial z^2} - \kappa^2 \right) \tilde{\Psi}(\kappa, z) = 0,$$

(2.4)

where for $\kappa$ the root with positive real part is taken. In this paper, the Green’s functions are first derived in the transformed domain and then transformed back to the spatial domain through the inverse Hankel transform.

2.3.1 Point Charge on Top of a Four-layer Dielectric

One potential application of the capacitive sensor discussed in this paper is dielectric property characterization of three-layer modern aircraft radome structures, using the knowledge of sensor geometry and the output transcapacitance $C_T$. In order to set up the governing equations in the MoM calculations for the in-contact characterization of layered dielectric structures, the potential due to a point charge in the plane $z = 0$ is derived. Without loss of generality, a four-layer half-space dielectric configuration shown in Figure 2.2 is used in the following theoretical derivation. One can easily obtain the solution for the three-layer radome structure by replacing $\epsilon_4$ by $\epsilon_0$, the permittivity of free space.

In Figure 2.2, a point charge is placed on top of a four-layer half-space dielectric. The electrostatic potential $\Psi$ satisfies the Laplace equation in each homogeneous medium. After applying the zero-order Hankel transform mentioned above, the resulting one-dimensional Helmholtz equations in the transformed domain can be expressed as

$$\left( \frac{\partial^2}{\partial z^2} - \kappa^2 \right) \tilde{\Psi}_0(\kappa, z) = \frac{1}{2\pi} \delta(z), \quad z \geq 0$$

(2.5)
Figure 2.2 Point charge on top of a four-layer dielectric.

\[
\left( \frac{\partial^2}{\partial z^2} - \kappa^2 \right) \tilde{\Psi}_i(\kappa, z) = 0, \quad -h_i \leq z < -h_{i-1},
\]

where \( i = 1, 2, 3, 4 \), and \( h_0 = 0 \) while \( h_4 \to -\infty \). The subscripts 0, 1, , 4 denote the free space above the dielectric and each homogeneous layer of the dielectric, respectively. From (2.5) and (2.6), general solutions for the potentials in each region can be expressed as

\[
\tilde{\Psi}_i(\kappa, z) = A_i(\kappa)e^{-\kappa z} + B_i(\kappa)e^{\kappa z}, \quad -h_i \leq z < -h_{i-1},
\]

where \( B_0(\kappa) = A_4(\kappa) = 0 \) due to the fact that the potential at infinity vanishes.

The interface conditions on the electric fields are

\[
\hat{z} \times (E_0 - E_1) = 0, \quad \hat{z} \cdot (D_0 - D_1) = \rho_s (2.8)
\]

\[
\hat{z} \times (E_i - E_{i+1}) = 0, \quad \hat{z} \cdot (D_i - D_{i+1}) = 0 (2.9)
\]

where \( i = 1, 2, 3 \), and \( \rho_s \) is the free surface charge density in the plane \( z = 0 \). Applying the Hankel transform to the interface conditions for \( E \) and \( D \), the corresponding boundary conditions for the potentials in the transformed domain are expressed:

\[
\tilde{\Psi}_0(\kappa, 0) = \tilde{\Psi}_1(\kappa, 0) \quad (2.10)
\]

\[
-\epsilon_0 \frac{\partial \tilde{\Psi}_0(\kappa, 0)}{\partial z} + \epsilon_1 \frac{\partial \tilde{\Psi}_1(\kappa, 0)}{\partial z} = \frac{1}{2\pi} (2.11)
\]

\[
\tilde{\Psi}_i(\kappa, -h_i) = \tilde{\Psi}_{i+1}(\kappa, -h_i) \quad (2.12)
\]
\[
\epsilon_i \frac{\partial \tilde{\Psi}_i(z, -h_i)}{\partial z} = \epsilon_{i+1} \frac{\partial \tilde{\Psi}_{i+1}(z, -h_i)}{\partial z}
\]

(2.13)

where \( i = 1, 2, 3 \). A little more explanation is made here about the \( 1/2\pi \) term on the right-hand side of (2.11). In cylindrical coordinates, the Dirac delta-function can be expressed for points on the \( z \) axis as

\[
\delta(r - r') = \frac{1}{2\pi \rho} \delta(\rho) \delta(z - z').
\]

(2.14)

Therefore, the surface charge density in the plane \( z = 0 \) is \( \rho_s = \delta(\rho)/2\pi \rho \), with its Hankel transform being \( 1/2\pi \). Applying the Hankel transform to the boundary condition (2.8), one can easily get the result shown in (2.11).

Substitute (2.7) into (2.10) and (2.13) to express the coefficient \( A_1(\kappa) \) as

\[
A_1(\kappa) = -\frac{1}{2\pi \kappa(\epsilon_0 + \epsilon_1)} \times \left[ \delta e^{-2\kappa h_3} + \gamma e^{-2\kappa h_2} + \beta e^{-2\kappa h_1} + \beta \gamma \delta e^{-2\kappa(h_1 + h_3 - h_2)} \right]
\]

\[
\times \sum_{s=0}^{\infty} (-1)^s \frac{\beta + \alpha e^{-2\kappa T_1}}{\beta + \alpha e^{-2\kappa T_1} + 1} \frac{\gamma + \delta e^{-2\kappa T_3}}{\gamma + \delta e^{-2\kappa T_3} + 1} e^{-2\kappa s T_2}
\]

(2.15)

where \( \alpha = (\epsilon_1 - \epsilon_0)/(\epsilon_1 + \epsilon_0), \beta = (\epsilon_2 - \epsilon_1)/(\epsilon_2 + \epsilon_1), \gamma = (\epsilon_3 - \epsilon_2)/(\epsilon_3 + \epsilon_2), \delta = (\epsilon_4 - \epsilon_3)/(\epsilon_4 + \epsilon_3) \).

Besides, \( T_1 = h_1, T_2 = h_2 - h_1, \) and \( T_3 = h_3 - h_2 \). In order to get the spatial domain solution, (2.15) can be expanded into the form of series summations, which facilitates application of the inverse Hankel transform. For those terms inside the summation of \( A_1(\kappa) \), we have

\[
(\beta + \alpha e^{-2\kappa T_1})^n = \sum_{r=0}^{n} \frac{n!}{r!(n-r)!} \beta^{n-r} \alpha^r e^{-2r\kappa T_1}
\]

(2.16)

\[
\frac{1}{(1 + \alpha e^{-2\kappa T_1})^{n+1}} = \sum_{s=0}^{\infty} (-1)^s \frac{(n+s)!}{n!s!} \beta^{n-s} \alpha^s e^{-2s\kappa T_1}
\]

(2.17)

and similarly for terms \( (\gamma + \delta e^{-2\kappa T_3})^n \) and \( (1 + \gamma e^{-2\kappa T_3})^{-(n+1)} \). Combining equations (2.16) and (2.17) gives

\[
\frac{(\beta + \alpha e^{-2\kappa T_1})^n}{(1 + \alpha e^{-2\kappa T_1})^{n+1}} = \sum_{s=0}^{\infty} \sum_{r=0}^{n} (-1)^s \frac{(n+s)!}{r!(n-r)!s!} \alpha^{r+s} \beta^{n-s} e^{-2(r+s)\kappa T_1}.
\]

(2.18)

Adopting \( m = r + s \), (2.18) is then written in the following form

\[
\frac{(\beta + \alpha e^{-2\kappa T_1})^n}{(1 + \alpha e^{-2\kappa T_1})^{n+1}} = \sum_{m=0}^{\infty} K_{mn}(\alpha, \beta) e^{-2\kappa m T_1}
\]

(2.19)
where
\[ K_{mn}(\alpha, \beta) = \sum_{r=0}^{\min(m,n)} (-1)^{m-r} \times \frac{(m + n - r)!}{r!(m-r)!(n-r)!} \alpha^m \beta^{m+n-2r}. \] (2.20)

Similarly
\[ \frac{(\gamma + \delta e^{-2\kappa T_3})^n}{(1 + \gamma e^{-2\kappa T_3})^{n+1}} = \sum_{l=0}^{\infty} K_{ln}(\delta, \gamma) e^{-2\kappa T_3}. \] (2.21)

Finally, the series summation form for \( A_1(\kappa) \) in the transformed domain is written as
\[ A_1(\kappa) = -\frac{1}{2\pi \kappa (\epsilon_0 + \epsilon_1)} \times \left[ \delta e^{-2h_3} + \gamma e^{-2h_2} + \beta e^{-2h_1} + \beta\gamma e^{-2(h_1 + h_3 - h_2)} \right] \] \[ \times \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} (-1)^n K_{mn}(\alpha, \beta) K_{ln}(\delta, \gamma) e^{-2\kappa mT_1} e^{-2\kappa lT_3} \] (2.22)

and it is found from the boundary conditions that
\[ A_0(\kappa) = \frac{1}{2\pi \kappa (\epsilon_0 + \epsilon_1)} + (1 + \alpha) A_1(\kappa) \] (2.23)

Applying the inverse Hankel transform to (2.23), the potential in the \( z = 0 \) plane due to a point charge at the origin is expressed as
\[ \Psi(\rho, 0) = \frac{1}{2\pi (\epsilon_0 + \epsilon_1) \rho} + (1 + \alpha) \int_{0}^{\infty} A_1(\kappa) J_0(\kappa \rho) d\kappa. \] (2.24)

The integral in (2.24) can be evaluated by applying the following Hankel transform pair to each of its power series terms, given in (2.22),
\[ \int_{0}^{\infty} e^{-\kappa z} J_0(\kappa \rho) d\kappa = \frac{1}{\sqrt{\rho^2 + z^2}} \] (2.25)

Equation (2.24) is finally expressed in real-space form as
\[ \Psi(\rho, 0) = \frac{1}{2\pi (\epsilon_0 + \epsilon_1) \rho} - \frac{1 + \alpha}{2\pi (\epsilon_0 + \epsilon_1)} (G_1 + G_2 + G_3 + G_4) \] (2.26)

where
\[ G_1 = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} (-1)^n K_{mn}(\alpha, \beta) K_{ln}(\delta, \gamma) \frac{\delta}{\sqrt{\rho^2 + [2(mT_1 + nT_2 + lT_3 + h_3)]^2}} \] (2.27)
\[ G_2 = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} (-1)^n K_{mn}(\alpha, \beta) K_{ln}(\delta, \gamma) \frac{\gamma}{\sqrt{\rho^2 + [2(mT_1 + nT_2 + lT_3 + h_2)]^2}} \] (2.28)
\[ G_3 = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} (-1)^n K_{mn}(\alpha, \beta) K_{ln}(\delta, \gamma) \frac{\beta}{\sqrt{\rho^2 + [2(mT_1 + nT_2 + lT_3 + h_1)]^2}} \]  

(2.29) \[
G_4 = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} (-1)^n K_{mn}(\alpha, \beta) K_{ln}(\delta, \gamma) \frac{\beta \gamma \delta}{\sqrt{\rho^2 + [2(mT_1 + nT_2 + lT_3 + h_1 + h_3 - h_2)]^2}} \]  

(2.30)

(2.26) to (2.30) together give the surface potential \( \Psi_0(\rho, 0) \) due to a point charge at the surface of a four-layer half-space dielectric in the spatial domain. The potential throughout the entire domain can be derived from the above equations but only \( \Psi_0(\rho, 0) \) is needed here for later MoM calculations because the sensor is in contact with the test-piece surface. By substituting \( \epsilon_0 \) for \( \epsilon_0 \) in the above relations, the potential due to a point charge on top of a three-layer dielectric in free space can be retrieved. Numerical results based on this potential are compared with corresponding experimental results in Section 2.5.1.

### 2.3.2 Point Charge on Top of a Two-layer Dielectric in Free Space

The surface potential for the case of a point charge on top of a two-layer dielectric can be simplified from (2.26) by assuming that \( \epsilon_1 = \epsilon_2 \) and \( \epsilon_4 = \epsilon_0 \). We are interested in this case for the purpose of benchmark testing described in Section 2.5. As a result, \( \beta \) becomes zero and \( G_3 = G_4 = 0 \). On the other hand, \( K_{mn}(\alpha, \beta) \) has a non-zero value, \( K_{mn}(\alpha) = \alpha^n \), only when \( m = n = r \). This is because when \( m \neq n \), the term \( m + n - 2r \) is constantly greater than zero and thus \( \beta^{m+n-2r} = 0 \). Hence, the corresponding potential is simplified as

\[
\Psi(\rho, 0) = \frac{1}{2\pi(\epsilon_0 + \epsilon_1)} + \frac{1 + \alpha}{2\pi(\epsilon_0 + \epsilon_1)} (G_1 + G_2) \]  

(2.31)

where

\[
G_1 = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (-1)^n \alpha^n K_{mn}(\delta, \gamma) \frac{\delta}{\sqrt{\rho^2 + \{2[(n + 1)T_1 + (m + 1)T_2]\]^2}} \]  

(2.32)

\[
G_2 = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (-1)^n \alpha^n K_{mn}(\delta, \gamma) \frac{\gamma}{\sqrt{\rho^2 + \{2[(n + 1)T_1 + mT_2]\]^2}} \]  

(2.33)

Here, \( T_1 \) and \( T_2 \) represent the thickness of the top and bottom homogeneous layers, respectively.
2.3.3 Point Charge on Top of a Two-layer Half-space Dielectric

The above derived potential due to a point charge over the surface of a four-layer half-space dielectric can also be reduced to the case of a point charge on top of a coated half-space dielectric. This case can be furthermore reduced to the solutions of a point charge on top of a one-layer dielectric slab in free space and a point charge on top of a homogeneous half-space dielectric. These simplified results are identical to those presented in (21) and (22). In addition, calculation results based on the potential due to a point charge on top of a one-layer dielectric in free space are used in the benchmark comparison in Section 2.5.

Assuming that $\epsilon_1 = \epsilon_2 = \epsilon_3 \neq \epsilon_0$, the structure in Figure 2.2 is simplified into the case of a half-space dielectric with a single surface layer as shown in Figure 2.3. The top layer has dielectric constant $\epsilon_1$ and thickness $h$. The bottom layer is the half-space dielectric with dielectric constant $\epsilon_2$. In this case, $\beta = \gamma = 0$. $K_{mn}(\alpha, \beta)$ only has non-zero value when $m = n = r$ and $K_{mn}(\alpha) = \alpha^n$. Similarly, $K_{ln}(\delta, \gamma)$ only has non-zero value when $l = n = t$ and $K_{ln}(\delta) = \delta^n$. (2.26) is simplified to

$$\Psi(\rho, 0) = \frac{1}{2\pi(\epsilon_0 + \epsilon_1)} \left\{ \frac{1}{\rho} - (1 + \alpha) \sum_{n=0}^{\infty} (-1)^n \frac{(\alpha)^n (\delta)^{n+1}}{\sqrt{\rho^2 + [2(n+1)h]^2}} \right\}, \quad (2.34)$$

where $\alpha = (\epsilon_1 - \epsilon_0)/(\epsilon_1 + \epsilon_0)$ and $\delta = (\epsilon_2 - \epsilon_1)/(\epsilon_2 + \epsilon_1)$. To compare the derived result with that in the literature, rewrite (2.34) in terms of coefficients $\alpha = (\epsilon_1 - \epsilon_0)/(\epsilon_1 + \epsilon_0)$ and
\[ \beta = (\epsilon_1 - \epsilon_2)/(\epsilon_1 + \epsilon_2), \]

which gives

\[ \Psi(\rho, 0) = \frac{1}{2\pi(\epsilon_0 + \epsilon_1)} \left\{ \frac{1}{\rho} + (1 + \alpha) \sum_{n=0}^{\infty} (\alpha \beta)^n \frac{\beta}{\sqrt{\rho^2 + [2(n+1)h]^2}} \right\}. \]  

(2.35) is identical with the result presented in (21), where the Green’s function is derived using a double Fourier transform in Cartesian coordinates. A special case is that in which the half-space dielectric is replaced by free space and the test-piece in contact with the sensor is then a homogeneous plate. The corresponding potential is expressed in (2.36) by replacing \( \epsilon_2 \) with \( \epsilon_0 \) in equation (2.35):

\[ \Psi(\rho, 0) = \frac{1}{2\pi(\epsilon_0 + \epsilon_1)} \left\{ \frac{1}{\rho} + (1 + \alpha) \sum_{n=0}^{\infty} (\alpha)^{2n+1} \frac{1}{\sqrt{\rho^2 + [2(n+1)h]^2}} \right\}. \]  

(2.36) can be simplified further by choosing \( \epsilon_1 = \epsilon_0 \). The series summation terms in equation (2.36) all vanish because \( \alpha = 0 \) in this case. This simplified result is identical to that presented in (22), in which the result is derived in the spatial domain directly.

### 2.4 Numerical Implementation

#### 2.4.1 Calculation Method

In order to calculate the sensor transcapacitance, \( C_T \), the method of moments (MoM) (23) is utilized in the numerical calculations. In the following calculation examples, all the sensors share the configuration shown in Figure 2.1, where the central disc is charged to the potential \( V_1 = 1 \) V and potential of the outer ring is kept at \( V_2 = 0 \) V.

The electrostatic potentials due to a point source, derived above, serve as the Green’s functions in the MoM simulations. As shown in Figure 2.4, the concentric electrodes of the sensor are divided into \( N \) circular filaments each with width \( \Delta \) and a surface charge density that is constant with respect to variation in \( \rho \). For the test-piece structure shown in Figure 2.2, the potential at a given observation point \( (\rho, \phi, 0) \) due to a source point \( (\rho', \phi', 0) \) can be expressed as follows, by slightly modifying equations (2.26) to (2.30):

\[ \Psi(\rho, 0) = \frac{1}{2\pi(\epsilon_0 + \epsilon_1)} \left[ \frac{1}{| \mathbf{r} - \mathbf{r}' |} - (1 + \alpha)(G_1 + G_2 + G_3 + G_4) \right] \]  

(2.37)
Figure 2.4 Concentric sensor is divided into \( N \) circular filaments, each with a constant surface charge density that is constant with respect to variation in \( \rho \).

where

\[
|\mathbf{r} - \mathbf{r}'| = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi - \phi')}.
\]  

(2.38)

\[
G_1(|\mathbf{r} - \mathbf{r}'|) = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} (-1)^n \frac{K_{mn}(\alpha, \beta) K_{ln}(\delta, \gamma) \delta}{\sqrt{|\mathbf{r} - \mathbf{r}'|^2 + [2(mT_1 + nT_2 + lT_3 + h_3)]^2}}
\]  

(2.39)

and \( G_2(|\mathbf{r} - \mathbf{r}'|) \), \( G_3(|\mathbf{r} - \mathbf{r}'|) \), and \( G_4(|\mathbf{r} - \mathbf{r}'|) \) can be modified similarly. For other test-piece configurations, the appropriate Green’s function should be used. Moreover, the potential at such an observation point due to points on a charged sensor shown in Figure 2.1 can be derived by integrating (2.37) over the sensor electrode surface:

\[
\Psi(\rho, 0 | \rho', 0) = \frac{1}{2\pi(\epsilon_0 + \epsilon_1)} \int_{\text{disc+ring}} \frac{K(\rho, 0 | \rho', 0) \sigma(\rho') \rho' d\rho'}{\sqrt{|\mathbf{r} - \mathbf{r}'|^2 + [2(mT_1 + nT_2 + lT_3 + h_3)]^2}}.
\]  

(2.40)

where \( \sigma(\rho') \) is the sensor surface charge density and

\[
K(\rho, 0 | \rho', 0) = \int_0^{2\pi} \left[ \frac{1}{|\mathbf{r} - \mathbf{r}'|} - (1 + \alpha) \sum_{i=1}^{4} G_i \right] d\phi'.
\]  

(2.41)

One thing to notice is that because of the cylindrical symmetry of the sensor structure, the resulting potential in space is independent of the azimuthal angle \( \phi \). Therefore, the problem of calculating the sensor surface charge distribution, which is determined by the potential distribution, is reduced to the \( \rho \)-direction only. For observation points on the sensor electrodes, the boundary conditions for the potential can be expressed as

\[
\Psi_i(\rho, z = 0) = \frac{1}{2\pi(\epsilon_0 + \epsilon_1)} \int_{\text{disc+ring}} K(\rho, 0 | \rho', 0) \sigma(\rho') \rho' d\rho' = V_m,
\]  

(2.42)
where points on the central disc are denoted by \( m = 1 \) while those on the outer ring are denoted by \( m = 2 \). In order to solve for the sensor surface charge distribution \( \sigma(\rho') \) using MoM calculations, the following expansion is used:

\[
\sigma(\rho') = \sum_{j=1}^{N} \sigma_j b_j(\rho')
\]

(2.43)

where \( b_j(\rho') \) is the basis function and \( \sigma_j \) is the unknown coefficient. Here, we choose \( b_j(\rho') \) as the following function for filaments on the inner disc

\[
b_j(\rho') = \begin{cases} 
\frac{1}{\sqrt{s^2-(\rho')^2}}, & (j-1)\Delta < \rho' < j\Delta \\
0, & \text{elsewhere}
\end{cases}
\]

(2.44)

where \( s \) is the radius of the inner disc. For the filaments on the outer annular ring, \( b_j(\rho') \) is chosen as

\[
b_j(\rho') = \begin{cases} 
\frac{1}{\sqrt{(s+g)^2-(\rho')^2}} \times \frac{1}{\sqrt{(D/2)^2-(\rho')^2}}, & (j-1)\Delta < \rho' < j\Delta \\
0, & \text{elsewhere}
\end{cases}
\]

(2.45)

where \( g \) is the gap between the two sensor electrodes and \( D \) is the diameter of the sensor. This form of basis function has the advantage of modeling the edge effect of the charge distribution discussed later. To resolve the \( N \) unknown \( \sigma_j \) coefficients, it is then required that the boundary conditions for \( V_m \) in (2.42) are satisfied for each circular filament on the sensor surface. To evaluate (2.42) in \( N \) different filaments, weighting (or testing) functions \( w_i(\rho) \) are needed. Here, we choose the weighting and basis functions to be the same, known as Galerkin’s method. For filaments on the inner disc,

\[
w_i(\rho) = \begin{cases} 
\frac{1}{\sqrt{s^2-(\rho)^2}}, & (i-1)\Delta < \rho < i\Delta \\
0, & \text{elsewhere}
\end{cases}
\]

(2.46)

while the weighting function for filaments on the outer annular ring

\[
w_i(\rho) = \begin{cases} 
\frac{1}{\sqrt{(s+g)^2-(\rho)^2}} \times \frac{1}{\sqrt{(D/2)^2-(\rho)^2}}, & (i-1)\Delta < \rho < i\Delta \\
0, & \text{elsewhere}
\end{cases}
\]

(2.47)
where \( i = 1, 2, \ldots, N \). Discretizing the integral equation using weighting functions in each of the \( N \) filaments, (2.42) turns into the following matrix equation:

\[
\begin{pmatrix}
G_{11} & G_{12} & \cdots & G_{1N} \\
G_{21} & G_{22} & \cdots & G_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
G_{N1} & G_{N2} & \cdots & G_{NN}
\end{pmatrix}
\times
\begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\vdots \\
\sigma_N
\end{pmatrix}
= \mathbf{V},
\tag{2.48}
\]

where

\[
G_{ij} = \int_{(i-1)\Delta}^{i\Delta} w_i(\rho) \left[ \int_{(j-1)\Delta}^{j\Delta} K(\rho, 0|\rho', 0)b_j(\rho')\rho' \, d\rho' \right] \rho \, d\rho.
\tag{2.49}
\]

For the \( \mathbf{V} \) matrix, if the element is located on the central electrode, its value is \( V_1 = 1 \) V; while the values for those elements located on the outer ring are \( V_2 = 0 \).

From (2.48), the sensor surface charge distribution \( \sigma(\rho') \) can be calculated. Once \( \sigma(\rho') \) is known, one can integrate over the electrode surfaces and find the total charge on both inner and outer electrodes. The sensor output signal, which is the transcapacitance \( C_T \) between those two electrodes, can be ultimately calculated through

\[
C_T = \frac{Q_{\text{outer}}}{V_{\text{inner}}} \bigg|_{V_{\text{inner}} = 0}
\tag{2.50}
\]

where \( Q_{\text{outer}} \) is the total charge on the outer electrode, while \( V_{\text{inner}} \) and \( V_{\text{outer}} \) respectively represent the voltage on the inner and outer electrodes. Choosing this convention leads to \( C_T < 0 \), whereas \( |C_T| \) is compared with experiment in the following.

2.4.2 Example Calculations

Figure 2.5 shows an example of the sensor surface charge distribution, where the sensor is placed above a half-space dielectric with relative permittivity \( \epsilon_r = 8 \). The sensor configuration is \( s = t = 10 \) mm and \( g = 1 \) mm. Due to the edge effect, the surface charge density at the edge of the inner charged electrode is singular. This positive charge distribution results in a negative surface charge distribution on the outer electrode. The surface charge density on the inner edge of the outer electrode tends to infinity much faster than that on the outer edge, because of its smaller radius and stronger interaction with the inner electrode. It is worth
Figure 2.5 Calculated surface charge distribution for the sensor shown in Figure 2.1 in contact with a half-space dielectric. Sensor configuration: $s = t = 10$ mm, $g = 1$ mm, $V_{\text{inner}} = 1$ V, and $V_{\text{outer}} = 0$ V. The test-piece has relative dielectric constant $\epsilon_r = 8$.

mentioning that when one applies a different combination of potentials on the inner and outer electrodes, the sensor surface charge distribution changes correspondingly. However, the sensor transcapacitance $|C_T|$, which is the intrinsic property of the sensor and only determined by its own structure, is unchanged. The sensor transcapacitance, $|C_T| = 5.398$ pF for this case, is calculated through (2.50).

Numerical calculations based on the same test-piece have been carried out to investigate the optimal sensor configuration giving the maximum output signal $|C_T|$. The sensor output signal as a function of $s$ and $g$ is plotted in Figure 2.6. In the calculation, the sensor outer radius $D/2 = s + g + t$ is fixed and all the curves in Figure 2.6 are normalized with respect to their own maximum values. As can be seen from the figure, for any given $g$, the sensor output signal increases to a maximum value and then decreases as $s$ increases. This is because as $s$ increases, the width of the outer electrode $t$ decreases, resulting in stronger edge effects on its surface charge distribution. These stronger edge effects result in more charges accumulated on the outer ring, and therefore the sensor output signal is increased according to (2.50). In this regime, the surface charge density is the dominant factor determining the total surface charge $Q_{\text{outer}}$. However, as $s$ increases and passes a certain value, the sensor output signal starts to decrease. This is due to the fact that the diminishing surface area of the outer electrode
becomes dominant in determining the total surface charge $Q_{\text{outer}}$. As a result, we observe an optimal sensor configuration for a given $g$ that gives the maximum $|C_T|$. It is also verified in our calculations that the shape of all the curves in Figure 2.6 do not depend on the actual size of the sensor and the applied electrode voltage, but only on the relative values of $s$, $g$, and $D$. Similarly, as $g$ increases, the interaction between the inner and outer electrodes is decreased, and the surface charge density at their neighboring edges diminishes accordingly. Because of the decreased edge effect and surface charge density, the outer electrode needs more surface area to achieve its maximum $Q_{\text{outer}}$, which is directly proportional to $|C_T|$. This is why as $g/D$ increases, the $s/D$ value that yields the maximum $|C_T|$ decreases in Figure 2.6. As one can imagine, the absolute magnitude of $|C_T|$ also becomes smaller for larger $g$ and fixed $s$ and $D$ values, due to the same reasoning mentioned above. Consequently, in order to achieve the maximum $|C_T|$, it is desirable to maintain high $s/D$ and low $g/D$ ratios. Nevertheless, it is worth mentioning that the sensitive area of the sensor closely corresponds to the location of the gap between its two electrodes, and there will be an insensitive zone at the center of those sensors with relatively large $s$ values.

Another example, addressing sensor sensitivity to changes in core permittivity of a three-layer structure, is presented here. We are interested in this problem because one potential application of the sensor is detection of ingressed water or inhomogeneities in the core of an
aircraft radome structure, which is typically a three-layer sandwich structure. In the numerical
calculation, the infinite series summations in (2.27) to (2.30) are truncated to \( N = 10 \) terms each. The difference between \( N = 10 \) and \( N = 100 \) terms is only 0.008\% while the latter is extremely time-consuming. The sensor configuration is \( s = t = 10 \) mm and \( g = 0.5 \) mm. The test-piece is shown in Figure 2.2, where \( T1 = T3 = 2.4 \) mm, \( T2 = 3 \) mm, and medium 4 is replaced by free space. The relative permittivity of the top and bottom layers, \( \epsilon_1 \) and \( \epsilon_3 \), is chosen to be the same. These parameters are also adopted in later benchmark experiments described in Section 2.5. Figure 2.7 shows how the normalized sensor output signal \( |C_T| \) changes as a function of \( \epsilon_1 \) and \( \epsilon_3 \) and of the core relative permittivity \( \epsilon_2 \). In Figure 2.7, \( \epsilon_1 = \epsilon_2 = \epsilon_3 = 1 \) gives the limiting case of the sensor in free space; \( \epsilon_1 = \epsilon_2 = \epsilon_3 \neq 1 \) gives the case of the sensor on top of a one-layer test-piece in free space; and \( \epsilon_1 = \epsilon_3 = 1 \neq \epsilon_2 \) gives the case of lift-off measurement of a one-layer test-piece in free space. It is seen from Figure 2.7 that the slope of the curve representing the normalized \( |C_T| \) as a result of changing \( \epsilon_1 = \epsilon_3 \) when \( \epsilon_2 = 10 \) is much greater than that obtained as a result of changing \( \epsilon_2 \) when \( \epsilon_1 = \epsilon_3 = 10 \) as expected due to the shielding effect of the top layer. In addition, high \( \epsilon_1 = \epsilon_3 \) values give less sensitivity to \( \epsilon_2 \) changes. This can be made more explicit by defining the percentage difference in the sensor output signal as follows:

\[
\text{difference} = P = \frac{|C_T|_{\epsilon_2 + \Delta\epsilon_2} - |C_T|_{\epsilon_2}}{|C_T|_{\epsilon_2}} \times 100\% \tag{2.51}
\]

When \( \epsilon_1 = \epsilon_3 = 3, \epsilon_2 = 2, \) and \( \Delta\epsilon_2 = 1 \), for example, then \( P \) is 3.66\%. However, for the same \( \epsilon_2 \) and \( \Delta\epsilon_2 \), when \( \epsilon_1 = \epsilon_3 = 10, \) \( P \) is only 2.99\%. This percentage change in \( |C_T| \) is expected to be even smaller when \( \epsilon_1 \) becomes larger, which is reasonable because higher density electric fields are confined in the high \( \epsilon_1 \) material. To improve sensor sensitivity to the permittivity change in the core-layer then, one can increase the gap \( g \) between the electrodes to some extent. For example, when \( g = 1 \) mm rather than 0.5 mm as in the calculations of Figure 2.7, and keeping all the other parameters the same, \( P \) is 3.62\% when \( \epsilon_1 = \epsilon_3 = 3 \) and 4.42\% when \( \epsilon_1 = \epsilon_3 = 10 \). However, the magnitude of the sensor output signal is decreased as \( g \) increases. Therefore, a trade-off between high sensor sensitivity and strong output signal is needed when
Figure 2.7 Calculated sensor output signal $|C_T|$ changes as a function of $\epsilon_{r_1} = \epsilon_{r_3}$ and the core-layer relative permittivity $\epsilon_{r_2}$. $|C_T|$ is normalized by its own maximum value for this calculation, which is 4.66 pF. Sensor configuration is as for Figure 2.5.

determining the optimal sensor configuration for measurements detecting permittivity change in the core layer.

2.5 Experiments

2.5.1 Benchmark Experiments

In order to verify the validity of the theory developed above, benchmark experiments were carried out for one-, two-, and three-layer dielectric test-pieces in free space, respectively. An Agilent E4980A precision LCR meter (20 Hz to 2 MHz) was utilized for the capacitance measurements. The operating frequency of the LCR meter was set to be 1 MHz. This particular frequency ensured that the measurement error of the LCR meter was less than 0.3% for a 1 pF capacitance, while at the same time giving a good approximation for the electrostatic case in the numerical model. A Novocontrol Alpha Dielectric Spectrometer was used to independently measure the dielectric constants of the samples used in the benchmark experiments. In the Novocontrol measurements, two 40-mm-diameter electrodes were used and the edge effect compensation was turned on, due to the fact that the thicknesses of the test-pieces were relatively large compared to the test fixtures electrode diameter. In addition, the test-piece thicknesses
were measured by a digital thickness indicator with accuracy ± 1 µm. These independently-measured test-piece thickness and dielectric constant values were used as the inputs of the calculation model.

Seven copper sensors of the configuration shown in Figure 2.1 were fabricated by photolithography. Four sensors have \( g = 0.5 \) mm and three have \( g = 1 \) mm, with different \( s = t \) values. These sensors were deposited onto a 25-µm-thick Kapton film to support the copper. By comparing the calculation result of a capacitive sensor \((s = t = 10 \text{ mm} \text{ and } g = 0.5 \text{ mm})\) on top of a half-space dielectric \((\epsilon_r = 8)\) and that of the same sensor on top of a 25-µm-thick Kapton\(^\circledR\) film over the same half-space, it was estimated that the presence of the Kapton\(^\circledR\) film influences the measurement signal by less than 0.5%. For each of the following benchmark measurements, the test-piece was supported by three acrylic stands 50 cm above a wood-top working table to approximate the infinite test-piece in free space assumption in the calculation model. Tape was used to attach each sensor tightly against the test-piece to ensure minimum air gap between the sensor and the test-piece, due to the fact that the presence of an air gap can affect measurement results significantly. The tape was attached on the edges of the Kapton\(^\circledR\) film, far away from the sensor outer electrode. \(|C_T|\) was measured by placing the probe of the Agilent probe test fixture 16095A across the two sensor electrodes. This probe test fixture was connected to the LCR meter and the capacitance values were read from the LCR screen.

To verify the results for the case of the capacitive sensor on top of a one-layer dielectric test-piece in free space, a glass plate with dimensions 305 × 305 mm\(^2\) and thickness 3.02 ± 0.01 mm was used. The test-piece dielectric constant was independently measured as 5.62 ± 0.05. Figure 2.8 gives the comparison between the numerical and experimental results. Experimental data show excellent agreement with the numerical results, to within 4%. Ten measurements were made for each sensor and the results were averaged. The maximum standard deviation in the measurements was found to be 2%. As can be seen, \(|C_T|\) increases as \( s \) increases and decreases as \( g \) increases. Meanwhile, sensors with smaller \( s \) values show relatively greater standard deviation in the measured data. This is reasonable because when the scale of the sensor becomes smaller, the output capacitance is consequently smaller, and the noise from the surroundings in the measurement environment can have a relatively greater impact on the
Figure 2.8 Measured and calculated $|C_T|$ for various sensor configurations (see Figure 2.1) in contact with a glass plate with $\epsilon_r = 5.62$ and thickness 3.02 mm.

measurement results.

The case of the capacitive sensor on top of a two-layer dielectric test-piece in free space was verified by placing a $305 \times 305$ mm$^2$ acrylic plate with thickness $2.39 \pm 0.02$ mm on top of the glass plate mentioned above. The independently measured acrylic dielectric constant was $2.85 \pm 0.05$ in this case. Plastic clamps were used to make sure there was as little air gap as possible between these two plates. Figure 2.9 gives the comparison between the numerical and experimental results. Again, very good agreement between experimental and theoretical results is observed. The maximum difference between the theory and experiment is less than 3% and the maximum standard deviation is 1% in these measurements. Similarly, Figure 2.10 shows the comparison results for the case of the capacitive sensor on top of a three-layered acrylic-glass-acrylic structure. The top and bottom acrylic plates share the same parameters and the glass plate sandwiched in the middle is the same as that used previously. It is seen from Figure 2.10 that, even for this more complex test-piece, very good agreement between theoretical predictions and experimental results is obtained. In this case, the maximum difference between the theory and experiment is 3% and the maximum standard deviation is 1%.

In conclusion, benchmark experiments show very good agreement with theoretical predictions. The output signal for the three-layer acrylic-glass-acrylic structure is slightly greater
Figure 2.9  Measured and calculated $|C_T|$ for various sensor configurations (see Figure 2.1) in contact with an acrylic plate, $\epsilon_r = 2.85$ and thickness 2.39 mm, on top of a glass plate with parameters as for Figure 2.8.

than that of the two-layer acrylic-glass structure but smaller than that of the one-layer glass plate. Because glass has a higher permittivity than acrylic, the sensor output signal of the one-layer glass plate is greater than that of the two-layer acrylic-glass structure. For the three-layer acrylic-glass-acrylic structure, the electric fields are mostly shielded by the glass plate. Therefore, adding an acrylic plate beneath the glass plate does not result in a significant change in the sensor output signal.

2.5.2 Detection of a Localized Anomaly in a Three-layer Structure

Water intrusion has been a persistent problem for composite structures on aircraft. The freezing and thawing of intruded water in radomes and honeycomb sandwich flight controls can lead to disbond and structural failures. For this reason, water ingression experiments based on a sandwich structure were conducted to demonstrate the sensors capability of detecting water intrusion in radome structures. The sandwich panel used in the following water ingression tests, shown in Figure 2.11, has a paper and resin honeycomb core covered with fiberglass skins and closely resembles a real radome structure. Table 2.1 gives the detailed properties of the sandwich panel.
Figure 2.10  Measured and calculated $|C_T|$ for various sensor configurations (see Figure 2.1) in contact with a three-layer acrylic-glass-acrylic structure. Layer parameters are as for Figures 2.8 and 2.9.

Figure 2.11  Sensor on top of a 1 cc water-injected glassfiber-honeycomb-glassfiber sandwich panel. Subfigure: photograph of the sandwich panel whose properties are given in Table 2.1.

Figure 2.11 shows the configuration for the coplanar capacitive sensor inspecting for 1 cc of injected water (4 honeycomb cells). The sensor scans from right to left on the test-piece surface, and the sensor output signal is read from the LCR meter screen. The solid line in Figure 2.12 shows the sensor output signal for the configuration shown in Figure 2.11. It is seen from the solid curve in Figure 2.12 that there are two peaks and a valley between them in the output signal. This phenomenon arises from the fact that the most sensitive region of the sensor is at the gap between its two electrodes. As the sensor scans over the water, the left gap of the sensor meets the water-injected area first. This results in a peak in the sensor output signal. As the sensor continues to move to the left and reaches the place where it is
Table 2.1  Properties of the Glassfiber-Honeycomb-Glassfiber Sandwich Panel (Figure 2.11), Supplied by the Composites Store, Inc.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core thickness</td>
<td>7.62 mm</td>
</tr>
<tr>
<td>Skin thickness</td>
<td>0.254 mm</td>
</tr>
<tr>
<td>Cell volume</td>
<td>0.25 cc</td>
</tr>
<tr>
<td>Surface area of cell</td>
<td>22 mm²</td>
</tr>
<tr>
<td>Panel length and width</td>
<td>298.45 mm</td>
</tr>
</tbody>
</table>

Figure 2.12  Measured $|C_T|$ for 1 cc of water injected into the glassfiber-honeycomb-glassfiber sandwich panel, Figure 2.11. Sensor configuration is as for Figure 2.5.

centered over the water-injected area, there is a decrease in the sensor output signal, due to the fact that the water is off the sensors most sensitive region. However, as the sensor continues moving, its right gap then meets the water-injected area. As a result, there is another peak in the sensor output signal. When the sensor moves away from the water-injected area, its output signal returns to the baseline signal for the unflawed panel.

In contrast, the dashed line in Figure 2.12 shows the sensor output signal for 5 cc of injected water. In this case there is only one peak in the sensor output signal, and the magnitude of the peak is approximately double that measured for 1 cc of injected water. This is due to the fact that the water-injected area in this case is larger than in the previous case (20 honeycomb cells). As the sensor scans from the right to the left, its left gap reaches the water-injected
area first. Correspondingly, there is an increase in the output signal. As the sensor keeps moving, its left gap still lies over the water-injected area, while its right gap starts to come into the water-injected area as well. This leads to the maximum sensor output signal shown in Figure 2.12. However, as the sensor continues moving, its left gap leaves the water-injected area first and the sensor output signal starts to decrease. When both gaps move out of the water-injected region, the sensor output signal returns to the baseline signal for the unflawed panel.

2.6 Conclusion

Electrostatic Green’s functions for a point charge on top of multi-layered dielectric structures have been derived using the Hankel transform given the cylindrical symmetry of the concentric sensor. Sensor output signal, transcapacitance $|C_T|$, has been calculated through the method of moments and corresponding benchmark experiments have been carried out. Very good agreement (within 4%) between theory and experiment on one-, two-, and three-layer dielectric test-pieces in free space has been observed. This suggests that experimental measurements can be interpreted by the theoretical model in order to determine permittivity of individual layers in multi-layered structures.

Additionally, the capability of the proposed sensor for detecting water intrusion in radome structures has been demonstrated by experiments in which 1 cc and 5 cc of water injected into the core of a glassfiber-honeycomb-glassfiber sandwich structure were clearly detected. In the future, a hand-held capacitive NDE system based on the proposed sensor will be developed and eventually differential probes for optimal defect detection in low-conductivity materials will be investigated.

2.7 Acknowledgments

The authors thank J. R. Bowler and J. M. Song for valuable discussions. This work was supported by the Air Force Research Laboratory under contract FA8650-04-C-5228 and by NASA under cooperative agreement NNX07AU54A at Iowa State University’s Center for NDE.
2.8 References


CHAPTER 3. ANALYSIS OF A CONCENTRIC COPLANAR CAPACITIVE SENSOR USING A SPECTRAL DOMAIN APPROACH

A paper published in the *Review of Progress in Quantitative Nondestructive Evaluation*
Tianming Chen, Jiming Song, John R. Bowler, and Nicola Bowler

3.1 Abstract

Previously, concentric coplanar capacitive sensors have been developed to quantitatively characterize the permittivity or thickness of one layer in multi-layered dielectrics. Electrostatic Green’s functions due to a point source at the surface of one- to three-layered test-pieces were first derived in the spectral domain, under the Hankel transform. Green’s functions in the spatial domain were then obtained by using the appropriate inverse transform. Utilizing the spatial domain Green’s functions, the sensor surface charge density was calculated using the method of moments and the sensor capacitance was calculated from its surface charge. In the current work, the spectral domain Green’s functions are used to derive directly the integral equation for the sensor surface charge density in the spectral domain, using Parseval’s theorem. Then the integral equation is discretized to form matrix equations using the method of moments. It is shown that the spatial domain approach is more computationally efficient, whereas the Green’s function derivation and numerical implementation are easier for the spectral domain approach.

3.2 Introduction

The efficient and reliable characterization of material properties of dielectric is of increasing importance in research because of the changing needs of industry. For example, there is greater
use of composite materials in new aircraft, such as the Boeing 787, because of the weight saving achieved. Correspondingly, many electromagnetic techniques, both high frequency and low frequency methods, have been developed over the years to meet the increasing need for the nondestructive evaluation (NDE) of dielectric and low-conductivity materials. For instance, dielectric resonators have been developed for precise measurements of complex permittivity and the thermal effects on permittivity for isotropic dielectric materials (1). Interdigital dielectrometry sensors have been used for applications such as humidity and moisture sensing, electrical insulation properties sensing, monitoring of curing process, chemical sensing, and so on (2). Cylindrical geometry quasistatic dielectrometry sensors have been developed for quantitative capacitance measurements of multi-layered dielectrics (3), while rectangular coplanar capacitive sensors have been applied for water intrusion detection in composite structures as well as damage detection in laminated composite plates (4; 5).

In our previous work, concentric coplanar capacitive sensors have been developed to characterize quantitatively the permittivity or thickness of one layer in multi-layered dielectrics, using a spatial domain approach in the theoretical analysis (6). Electrostatic Green’s functions due to a point source at the surface of one- to three-layered test-pieces were first derived in the spectral domain, under the Hankel transform. Green’s functions in the spatial domain were then obtained by using the appropriate inverse transform. Utilizing the spatial domain Green’s functions, the sensor surface charge density was calculated using the method of moments (MoM) and the sensor capacitance was calculated from its surface charge. In the current work, a spectral domain approach is applied. Spectral domain approaches have been widely used in calculating the dispersion characteristics of microstrip lines (7) and open and shielded microstrips (8; 9) over the decades. In this paper, the spectral domain Green’s function is used to derive the integral equation for the sensor surface charge density in the spectral domain, using Parseval’s theorem. Then the integral equation is discretized to form matrix equations using the MoM. It is shown that the spatial domain approach is more computationally efficient for both one- and three-layered structures in free space, while the Green’s function derivation and numerical implementation for the spectral domain approach are more straightforward.
3.3 Spectral Domain Green’s function for Multilayered Dielectrics

The configuration of the concentric coplanar capacitive sensor is shown in Figure 3.1. The capacitive sensor consists of two concentric electrodes: the inner disc and the outer annular ring.

In order to model the in-contact characterization of layered dielectric structures, the Green’s function due to a charged sensor over a five-layer half-space dielectric is derived. The Green’s function is then utilized in later MoM calculations of the sensor capacitance $C$. Besides, the test-pieces in our theoretical analysis are assumed to be infinite in the horizontal directions and the sensor electrodes are assumed to be infinitesimally thin.

A charged concentric sensor placed on top of a five-layer half-space dielectric is shown in Figure 3.2. The electrostatic potential $\Psi$, related to the electric field $\mathbf{E} = -\nabla \Psi$, satisfies the Laplace equation in each homogeneous medium, and can be expressed in cylindrical coordinates as

$$\left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial z^2} \right) \Psi_i(\rho, z) = 0, \quad i = 0, 1, \cdots, 5,$$

where $\Psi_i(\rho, z)$ is the potential in medium $i$ and is independent of azimuthal angle $\phi$. The
Hankel transform $\tilde{f}(\kappa)$ of zero-order of a function $f(\rho)$ is given by

$$
\tilde{f}(\kappa) = \int_0^\infty f(\rho) J_0(\kappa \rho) \rho \, d\rho,
$$

with the inverse being of the same form. Apply the zero-order Hankel transform to (3.1), making use of the following identity (10)

$$
\int_0^\infty \left[ \frac{d^2}{d\rho^2} + \frac{1}{\rho} \frac{d}{d\rho} \right] f(\rho) J_0(\kappa \rho) \rho \, d\rho = -\kappa^2 \tilde{f}(\kappa),
$$

where $f(\rho)$ is assumed to be such that the terms $\rho J_0(\kappa \rho) df(\rho)/d\rho$ and $\rho f(\rho) dJ_0(\kappa \rho)/d\rho$ vanish at both limits. The spatial domain Laplace equation (3.1) is then transformed into a one-dimensional Helmholtz equation in the transformed domain:

$$
\left( \frac{d^2}{dz^2} - \kappa^2 \right) \tilde{\Psi}_i(\kappa, z) = 0, \quad i = 0, 1, \cdots, 5,
$$

where for $\kappa$ the root with positive real part is taken. From (3.4), general solutions for the potentials in each layer can be expressed as

$$
\tilde{\Psi}_i(\kappa, z) = A_i(\kappa) e^{-\kappa z} + B_i(\kappa) e^{\kappa z}, \quad -h_i \leq z < -h_{i-1},
$$

where $i = 0, 1, \cdots, 5$, $h_{-1} \to \infty$, $h_0 = 0$, and $h_5 \to -\infty$. Note that $B_0(\kappa) = A_5(\kappa) = 0$ due to the fact that the potential at infinity vanishes. The interface conditions on the electric fields are

$$
\hat{z} \times (\mathbf{E}_0 - \mathbf{E}_1) = 0, \quad \hat{z} \cdot (\mathbf{D}_0 - \mathbf{D}_1) = \sigma_s(\rho)
$$

Figure 3.2 Concentric capacitive sensor on top of a five-layer dielectric.
\[ \hat{z} \times (E_i - E_{i+1}) = 0, \quad \hat{z} \cdot (D_i - D_{i+1}) = 0 \quad (3.7) \]

where \( i = 1, 2, 3, 4 \) and \( \sigma_s(\rho) \) is the free surface charge density on the sensor surface and is only a function of \( \rho \). Applying the Hankel transform to the interface conditions for \( E \) and \( D \), the corresponding boundary conditions for the potentials in the spectral domain are expressed:

\[ \tilde{\Psi}_0(\kappa, 0) = \tilde{\Psi}_1(\kappa, 0), \quad (3.8) \]

\[ \epsilon_1 \frac{d\tilde{\Psi}_1(\kappa, 0)}{dz} = \epsilon_0 \frac{d\tilde{\Psi}_0(\kappa, 0)}{dz} + \tilde{\sigma}_s(\kappa). \quad (3.9) \]

\[ \tilde{\Psi}_i(\kappa, -h_i) = \tilde{\Psi}_{i+1}(\kappa, -h_i), \quad (3.10) \]

\[ \epsilon_i \frac{d\tilde{\Psi}_{i+1}(\kappa, -h_i)}{dz} = \epsilon_{i+1} \frac{d\tilde{\Psi}_{i+1}(\kappa, -h_i)}{dz}, \quad (3.11) \]

where \( i = 1, 2, 3, 4 \) and \( \tilde{\sigma}_s(\kappa) \) is the Hankel transform of the spatial domain surface charge density \( \sigma_s(\rho) \) at \( z = 0 \):

\[ \tilde{\sigma}_s(\kappa) = \int_0^{\infty} \sigma_s(\rho) J_0(\kappa \rho) \rho d\rho. \quad (3.12) \]

Substitute (3.5) into (3.8) to (3.11) to express the coefficient \( A_0(\kappa) \) as

\[ A_0(\kappa) = \frac{\tilde{\sigma}_s(\kappa) F(\kappa)}{(\epsilon_0 + \epsilon_1) \kappa D(\kappa)}, \quad (3.13) \]

where

\[ F(\kappa) = 1 - \alpha_2 e^{-2\kappa h_1} - \alpha_3 e^{-2\kappa h_2} - \alpha_4 e^{-2\kappa h_3} - \alpha_5 e^{-2\kappa h_4} + \alpha_2 \alpha_3 e^{-2\kappa d_2} + \alpha_3 \alpha_4 e^{-2\kappa d_3} + \alpha_4 \alpha_5 e^{-2\kappa d_4} - \alpha_2 \alpha_3 \alpha_4 e^{-2\kappa (d_1 + d_2)} - \alpha_2 \alpha_3 \alpha_5 e^{-2\kappa (d_1 + d_3)} + \alpha_2 \alpha_3 \alpha_4 \alpha_5 e^{-2\kappa (d_2 + d_3)} - \alpha_2 \alpha_3 \alpha_4 \alpha_5 e^{-2\kappa (d_1 + d_2 + d_3)} + \alpha_2 \alpha_5 e^{-2\kappa (d_2 + d_3 + d_4)}, \quad (3.14) \]

\[ D(\kappa) = 1 + \alpha_1 \alpha_2 e^{-2\kappa h_1} + \alpha_1 \alpha_3 e^{-2\kappa h_2} + \alpha_1 \alpha_4 e^{-2\kappa h_3} + \alpha_1 \alpha_5 e^{-2\kappa h_4} + \alpha_2 \alpha_3 e^{-2\kappa d_2} + \alpha_3 \alpha_4 e^{-2\kappa d_3} + \alpha_4 \alpha_5 e^{-2\kappa d_4} + \alpha_2 \alpha_4 e^{-2\kappa (d_2 + d_3)} + \alpha_3 \alpha_5 e^{-2\kappa (d_3 + d_4)} + \alpha_1 \alpha_2 \alpha_4 e^{-2\kappa (d_1 + d_3)} + \alpha_1 \alpha_2 \alpha_5 e^{-2\kappa (d_1 + d_4)} + \alpha_2 \alpha_3 \alpha_4 e^{-2\kappa (d_1 + d_2)} + \alpha_2 \alpha_3 \alpha_5 e^{-2\kappa (d_1 + d_3)} + \alpha_1 \alpha_3 \alpha_4 e^{-2\kappa (d_1 + d_2 + d_3)} \quad (3.15) \]
\[ \alpha_i = (\varepsilon_i - \varepsilon_{i-1})/(\varepsilon_i + \varepsilon_{i-1}) \quad i = 1, 2, \ldots, 5, \] and \( d_1 \) through \( d_4 \) correspond to the thickness of layer 1 to layer 4, respectively. Substitute (3.13) into (3.5), the potential in the plane \( z = 0 \) is expressed as

\[ \tilde{\Psi}_0(\kappa, 0) = \frac{\tilde{\sigma}_s(\kappa)}{(\varepsilon_0 + \varepsilon_1)\kappa} \frac{F(\kappa)}{D(\kappa)}. \quad (3.16) \]

Now, rather than performing the inverse Hankel transform of the Green’s function in the spatial domain approach (6), the spectral domain Green’s function is used to calculate directly the sensor surface charge density \( \sigma_s(\rho) \) and eventually the sensor capacitance \( C \).

### 3.3.1 Numerical Implementation

The method of moments (MoM) (11) is utilized in the numerical calculations to calculate the sensor capacitance, \( C \). In the following calculation examples, all the sensors share the configuration shown in Figure 3.1, where the central disc is charged to the potential \( V_1 = 1 \) V and potential of the outer ring is kept at \( V_2 = 0 \) V.

As shown in Figure 3.1, the inner disc (outer annular ring) of the concentric sensor are divided into \( N \) (\( M \)) circular filaments each with width \( \Delta_1 \) (\( \Delta_2 \)) and a surface charge density that is constant with respect to variation in \( \rho \). In order to solve for the sensor surface charge distribution \( \sigma_s(\rho) \) using MoM calculations, the following expansion for the inner disc is used

\[ \sigma_s(\rho) = \sum_{n=1}^{N} \sigma_n b_n(\rho), \quad (3.17) \]

where \( \sigma_n \) is the unknown coefficient and \( b_n(\rho) \) is the pulse basis function:

\[ b_n(\rho) = \begin{cases} 1 & (n - 1)\Delta_1 < \rho < n\Delta_1 \\ 0 & \text{elsewhere} \end{cases} \quad (3.18) \]

The Hankel transform of the spatial domain surface charge density \( \sigma_s(\rho) \) at \( z = 0 \) is expressed as

\[ \tilde{\sigma}_s(\kappa) = \sum_{n=1}^{N} \sigma_n \tilde{b}_n(\kappa), \quad (3.19) \]

where

\[ \tilde{b}_n(\kappa) = \int_{0}^{\infty} b_n(\rho) J_0(\kappa \rho) \rho \, d\rho = \frac{1}{\kappa} \left[ n\Delta_1 J_1(n\Delta_1 \kappa) - (n - 1)\Delta_1 J_1((n - 1)\Delta_1 \kappa) \right]. \quad (3.20) \]
One can expand the surface charge density on the outer annular ring similarly, and (3.16) is written as

$$\tilde{\Psi}_0(\kappa, 0) = \frac{\sum_{n=1}^{L} \sigma_n \tilde{b}_n(\kappa) F(\kappa)}{(\epsilon_0 + \epsilon_1) \kappa} D(\kappa),$$

(3.21)

where $L = M + N$. Multiply both sides of (3.21) by $\tilde{b}_m(\kappa) \kappa$ and integrate with respect to $\kappa$ from 0 to $\infty$, (3.21) is expressed as the following integral form

$$\frac{1}{\epsilon_0 + \epsilon_1} \int_{0}^{\infty} \frac{F(\kappa)}{D(\kappa)} \sum_{n=1}^{L} \sigma_n \tilde{b}_n(\kappa) \tilde{b}_m(\kappa) d\kappa = \int_{0}^{\infty} \tilde{\Psi}_0(\kappa, 0) \tilde{b}_m(\kappa) \kappa d\kappa,$$

(3.22)

and can be further discretized into the matrix equation below

$$\begin{pmatrix} G_{11} & G_{12} & \cdots & G_{1L} \\ G_{21} & G_{22} & \cdots & G_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ G_{L1} & G_{L2} & \cdots & G_{LL} \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_L \end{pmatrix} = \begin{pmatrix} \tilde{v}_1 \\ \tilde{v}_2 \\ \vdots \\ \tilde{v}_L \end{pmatrix},$$

(3.23)

where

$$G_{mn} = \frac{1}{\epsilon_0 + \epsilon_1} \int_{0}^{\infty} \frac{F(\kappa)}{D(\kappa)} \tilde{b}_m(\kappa) \tilde{b}_n(\kappa) d\kappa.$$

(3.24)

On the other hand, from Parseval’s theorem, we have

$$\int_{0}^{\infty} \tilde{\Psi}(\kappa, z) \tilde{b}_m(\kappa) \kappa d\kappa = \int_{0}^{\infty} \Psi(\rho, z) b_m(\rho) \rho d\rho$$

(3.25)

and the right hand side of (3.23) is expressed as

$$\tilde{v}_m = \int_{0}^{\infty} \tilde{\Psi}_0(\kappa, 0) \tilde{b}_m(\kappa) \kappa d\kappa = \int_{0}^{\infty} \Psi(\rho, 0) b_m(\rho) \rho d\rho.$$

(3.26)

A closed-form expression can be obtained depending on the constant potential $\Psi_0(\rho, 0)$ on the sensor surface. From (3.23), the sensor surface charge distribution can be calculated. Once $\sigma_s(\rho)$ is known, one can integrate over the electrode surfaces and find the total charge on both inner and outer electrodes. The sensor output signal, which is the capacitance $C$ between those two electrodes, can be ultimately calculated through

$$C = \frac{Q}{V},$$

(3.27)

where $Q$ is the total charge on each electrode, while $V$ represents the potential difference between the inner and outer electrodes. It is worth pointing out that there is no singularity
problems in evaluating the MoM matrix elements in the spectral domain approach. However, in the spatial domain approach, care must be taken to deal with the singularity problems in the MoM matrix when the source point and the observation point are at the same location.

### 3.4 Spectral versus Spatial Domain Approaches

The spatial domain Green's function for a source point on top of a four-layer half-space dielectric is derived in (6). The Green’s function is in the form of zero to infinity series summations, and the number of summations is proportional to the number of layers present in the test-piece. In the MoM calculations, the matrix elements are formed by integrating the Green’s function along radial and azimuthal directions, where closed form expressions are available for integration along the azimuthal direction. Experimental verification of the numerical model based on the spatial domain Green’s function has been presented in (6). Very good agreement (to within 4%) between theory and experiment has been observed.

For the purpose of comparing the computational efficiency of the spatial and spectral domain approaches, the case of the concentric sensor on top of a one-layer dielectric slab in free space is considered first. The Green’s functions can be obtained by adopting $\varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \varepsilon_5 = \varepsilon_0$ in Figure 3.2. In the following numerical calculations, the zero to infinity summation in evaluating the spatial domain Green’s function is truncated to $N_0$ terms, where $N_0$ is chosen to achieve accuracy of four significant figures in the final calculated $C$, and the zero to infinity integral in (3.24) for the spectral domain approach is truncated to the region from 0 to $T$ in a similar manner. The sensors in the following numerical comparisons share the same configuration: $s = 10$ mm, $g = 0.5$ mm, and $t = 10$ mm. In the numerical calculations, the sensor is divided into $N = M = 10$ circular filaments on the disc and the outer ring, respectively (see Figure 3.1). The computer used in the calculations is MacBook Pro with Intel Core 2 Duo 2.26 GHz and 2 GB memory.

Table 3.1 shows the comparison results for the case of the concentric sensor on top of a one-layer dielectric slab with $\varepsilon_{r1} = 100$ in free space. Different slab thicknesses $h_1$ are considered. As one can see, the spatial domain approach is more efficient in calculating one-layered test-pieces in free space, especially as $h_1$ increases. Calculations for the dielectric slab with $\varepsilon_{r1} = 2$ were
Table 3.1 Sensor on top of a one-layer dielectric slab in free space. The relative permittivity of the slab is \(\epsilon_r = 100\).

<table>
<thead>
<tr>
<th>(h_1) (mm)</th>
<th>(C) (pF)</th>
<th>(N_0) in the spatial domain approach</th>
<th>(T) in the spatial domain approach (m(^{-1}))</th>
<th>Spectral domain approach CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>6.612</td>
<td>107</td>
<td>72.4</td>
<td>310</td>
</tr>
<tr>
<td>0.56</td>
<td>25.01</td>
<td>49</td>
<td>34.0</td>
<td>490</td>
</tr>
<tr>
<td>3.16</td>
<td>55.10</td>
<td>15</td>
<td>20.3</td>
<td>360</td>
</tr>
<tr>
<td>17.78</td>
<td>71.36</td>
<td>4</td>
<td>8.5</td>
<td>480</td>
</tr>
<tr>
<td>100</td>
<td>71.70</td>
<td>1</td>
<td>5.2</td>
<td>310</td>
</tr>
</tbody>
</table>

also performed. By comparing the calculation results corresponding to \(\epsilon_r = 100\) and \(\epsilon_r = 2\), it is found that for any fixed \(h_1\), the truncation range and CPU time reduce as \(\epsilon_r\) decreases, for both approaches. However, such changes are more dramatic for the spatial domain approach. For example, when \(\epsilon_r = 2\) and \(h_1 = 0.1\) mm, \(N_0 = 3\) and the CPU time spent is 7.5 s for the spatial domain approach, while for the spectral domain approach \(T = 220\) m\(^{-1}\) and the CPU time spent is 55.5 s.

In order to compare the efficiency of these two approaches in calculating \(C\) for multi-layered dielectrics, the case of a concentric capacitive sensor on top of a three-layer dielectric test-piece in free space is considered, in which media 0, 2, 4, and 5 in Figure 3.2 are replaced by free space while media 1 and 3 share a relative permittivity of \(\epsilon_r\). The thickness of layer 1, 2, and 3 is 0.34 mm. Table 3.2 shows the comparison results between these two approaches, with different permittivity contrasts between neighboring layers. It is still found that the spatial domain approach is more computationally effective than the spectral domain approach, in dealing with multi-layered structures.

### 3.5 Conclusion

The computational efficiency of a spectral domain approach is compared with that of a spatial domain approach for the numerical calculation of capacitance of a coplanar concentric sensor in contact with layered test-pieces. The spatial domain approach is found more efficient in dealing with both one- and three-layered dielectric structures, due to the fact that integration
Table 3.2  Sensor on top of a three-layer dielectric in free space, with different permittivity contrasts between neighboring layers. When $\epsilon_r = 40$, four significant figure accuracy in $C$ is not achieved.

<table>
<thead>
<tr>
<th>$h_1$ (mm)</th>
<th>$C$ (pF)</th>
<th>$N_0$ in the spatial domain approach</th>
<th>Spatial domain CPU time (s)</th>
<th>$T$ in the spatial domain approach (m$^{-1}$)</th>
<th>Spectral domain CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.688</td>
<td>2</td>
<td>9.0</td>
<td>220</td>
<td>72.5</td>
</tr>
<tr>
<td>10</td>
<td>3.370</td>
<td>12</td>
<td>40.4</td>
<td>320</td>
<td>99.9</td>
</tr>
<tr>
<td>20</td>
<td>5.288</td>
<td>20</td>
<td>72.7</td>
<td>360</td>
<td>116.8</td>
</tr>
<tr>
<td>30</td>
<td>7.140</td>
<td>24</td>
<td>91.4</td>
<td>380</td>
<td>127.6</td>
</tr>
</tbody>
</table>

of the Green’s function along the azimuthal direction has a closed form expression. Such efficiency is at the cost of performing analytical inverse Hankel transformation in the Green’s function derivation and at the cost of dealing with singularities in evaluating MoM matrix elements. The spectral domain approach, however, is less complex in both the theoretical derivation and the numerical implementation.

3.6 Acknowledgments

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3.7 References


CHAPTER 4.  A ROTATIONALLY-ININVARIANT CAPACITIVE PROBE FOR MATERIALS EVALUATION

A paper published in the *Materials Evaluation*

Tianming Chen and Nicola Bowler

4.1 Abstract

Motivated by potential applications such as defect detection in aircraft radome sandwich structures, a rotationally-invariant capacitive probe with concentric coplanar electrodes has been designed based on a theoretical model published previously. Two versions of the probe, with different target sensing penetration depths, have been constructed in such a way that removal of parasitic capacitances can be achieved. Capacitance measurements for the probes in surface contact with laminar structures show agreement with theoretical predictions to within 10%. Important factors governing the penetration depth of concentric capacitive sensors have been investigated numerically, followed by experimental investigation on stepped samples, whose results are found to be in accordance with numerical predictions. The effect of probe lift-off on inferred permittivity of dielectric slabs was also investigated. As lift-off increases, uncertainty in inferred permittivity increases dramatically. On the other hand, the capacitive probes show good sensitivity to low-contrast inhomogeneities embedded in laminar structures; 1 cc of a low contrast liquid injected into the core of a glassfiber-honeycomb-glassfiber sandwich structure has been successfully detected.
4.2 Introduction

There are numerous applications in which the use of non-traditional low-conductivity materials is increasing rapidly. Lightweight structural composites in air- and space-craft, Kevlar body-armor and ceramic-matrix-composites for thermal stability in hot engine environments are examples of these. In cases where the integrity of the material is important, there is a need for effective nondestructive evaluation (NDE) of these materials. Capacitive NDE offers a favorable solution to the dilemma of achieving high performance at relatively low cost for defect detection in, and characterization of, low-conductivity materials. Capacitive NDE works by measuring the capacitance, $C$, of a pair of electrodes in close proximity to a low-conductivity test-piece. Interdigital coplanar capacitive sensors are capable of quantitative characterization of test-piece material properties and are widely used (1). Rectangular capacitive array sensors that detect surface and subsurface features of dielectric materials have also been reported (2). Rectangular coplanar capacitance sensors have been developed to detect water intrusion in composite materials (3) and for damage detection in laminated composite plates (4). Another potential application of capacitive NDE is in the assessment of the integrity of wiring insulation. For example, curved patch electrodes that conform to the outer surface of electrical cables have been developed and modeled in order to evaluate the wire insulation permittivity from measured capacitance values (5). In other work (6), linear relationships between the capacitance of open-circuited wires (parallel insulated round wires, twisted-pairs, and coaxial cables) and their length have been proposed and enable the determination of cable length from measured capacitance values.

Finite element method simulation packages have often been employed to calculate interdigital sensor capacitance and have the advantage of being able to deal with test-pieces of arbitrary shape. One effective semi-analytical approach, called the continuum model, has been developed to relate the sensor capacitance to the complex permittivity and thickness of each layer in multi-layered configurations (7). In the continuum model, iterative relationships between complex surface capacitance densities of neighboring layers are derived, in which the complex surface capacitance density relates the electric flux density at a planar surface to the potential
at that surface. Conformal mapping techniques have also been applied to obtain closed-form solutions for the capacitance of interdigital sensors in surface contact with multi-layered dielectric structures (8). Adopting a similar analytical approach as in the continuum model, circular electroquasistatic dielectrometry sensors have been developed for evaluation of multi-layered dielectrics (9). In work closely related to this work, concentric coplanar capacitive sensors have been developed for quantitative characterization of the permittivity or thickness of individual layers in multi-layered dielectrics, using either a spatial domain approach (10) or a spectral domain approach (11) in the theoretical analysis. Analysis for the hand-held capacitive probe presented in this paper is based on the spectral domain approach.

In this study, a rotationally-invariant hand-held capacitive probe with concentric coplanar electrodes has been designed and built, Figure 4.1, motivated by defect detection in aircraft radome sandwich structures. Typical NDE radome inspection techniques include X-ray, ultrasound (12), millimeter wave/microwave (13), shearography, terahertz imaging (14), and thermography. Comparisons of the different methods for radome inspection are available in (15). It was found that X-ray and millimeter wave techniques had high detection sensitivity in this specific application, while ultrasound and shearography methods did not. The advantage of utilizing capacitive NDE for radome inspection lies in the fact that this method does not require the use of expensive equipment or complicated operation procedures to achieve good inspection results. Two versions of the probe, with different target penetration depths, have been tested. The sensors have the same outer diameter (25.4 mm) but different gap width between the inner and outer electrodes. The probes were designed with the aid of a theoretical model in which the capacitance is related to the electrode dimensions and the thickness and permittivity of each layer in a multi-layered dielectric test-piece. Experimental measurements of $C$ with the probes in surface contact with one- and multi-layered dielectric test-pieces have been carried out and measured capacitance agrees with theoretical predictions to within 10%. This work builds on a previous benchmark study in which un-encased electrodes were tested on similar multi-layered test-pieces in order to validate the theoretical model. In the previous study, measured and calculated $C$ were found to agree to within 4%. Further, the important parameters governing the penetration depth of this concentric capacitive sensor have been studied theoretically and
experimentally by measurements on stepped Delrin® and HDPE slabs. Lift-off studies, both numerical and experimental, were carried out to investigate how lift-off affects measured $C$ and the accuracy of the test-piece material properties when determined inversely from measured $C$. It is demonstrated that these hand-held probes are capable of detecting small embedded inhomogeneities in laminar structures, e.g., 1 cc of a low permittivity (low contrast) injected fluid in a glassfiber-honeycomb-glassfiber sandwich panel that gives rise to $\Delta C \sim 0.02 \text{ pF}$ is clearly detected. These capacitive probes are especially promising for discontinuity detection in sandwich structures. Instrument uncertainty in the measured capacitances in this paper is 0.3%.

4.3 Theoretical Background

Figure 4.1 a) depicts a concentric capacitive sensor in surface contact with a five-layer dielectric half-space. The capacitive sensor consists of an inner disc, radius $s$, and an outer annular ring, width $t$. The gap between these two electrodes is denoted $g$. A numerical model, described elsewhere (11), provides a quantitative relationship between the sensor output signal, which can be measured, and the permittivity and thickness of each layer in the dielectric. In the theoretical modeling, a spectral domain Green’s function is derived for the configuration in
Figure 4.1 a). This Green’s function is used to obtain an integral equation for the sensor surface charge density in the spectral domain, using Parseval’s theorem. This integral equation is then discretized to form matrix equations using the method of moments, from which the sensor surface charge density is calculated. The total charge \( Q \) on each sensor electrode is obtained by integrating the calculated surface charge density over electrode surface and the sensor output capacitance is then computed from \( C = Q/V \), where \( V \) is the potential difference between the two electrodes. The concentric sensor is considered to be infinitesimally thin in the numerical calculations while the test-piece is assumed to be laterally infinite. These assumptions are reasonable for electrodes that are relatively thin compared with the thickness of individual layers in the test-piece, and if the sensor is placed sufficiently far from the edges of the test-piece so that edge effects are negligible. For details of the calculation, see (11).

4.4 Probe Assembly

Two sets of concentric electrodes with different target penetration depths were fabricated by selectively etching a 18-\( \mu \)m-thick copper cladding (14 mL standard) off a 25.4-\( \mu \)m-thick Kapton\textsuperscript{®} film by photolithography (American Standard Circuits, Inc). Both sets of electrodes have fixed outer diameter 25.4 mm (1 inch), which was selected as a workable dimension for a hand-held probe, but have different gaps and other dimensions as listed in Table 4.1. The characteristic capacitance listed in Table 4.1 is the calculated free-space capacitance for each sensor. The gap between the two electrodes and the width of the outer electrode are relatively small values and strongly affect the sensor output capacitance. In order to measure these values very accurately, a Nikon EPIPHOT 200 microscope was used that is capable of achieving precision of \( \pm 5 \) \( \mu \)m for good calibration and 50X magnification. The sensor inner electrode radius was measured using the traveling microscope method with accuracy \( \pm 0.01 \) mm, due to its relative large dimension. It was found that the fabricated dimensions are the same as the nominal values under such measurement accuracy.

Figure 4.2 a) shows the assembled capacitive probe, Figure 4.2 b) shows the concentric electrodes, and Figure 4.3 a) shows the components used to assemble the probe. They consist of the following: a Rogers RO4003\textsuperscript{®} dielectric sensor substrate with thickness 0.31 \( \pm 0.01 \)
etching a 18-µm-thick copper cladding (14 mL standard) off a 25.4-µm-thick Kapton® film by photolithography (American Standard Circuits, Inc). Both sets of electrodes have fixed outer diameter 25.4 mm (1 inch), which was selected as a workable dimension for a hand-held probe, but have different gaps and other dimensions as listed in Table 1. The characteristic capacitance listed in Table 1 is the calculated free-space capacitance for each sensor. The gap between the two electrodes and the width of the outer electrode are relatively small values and strongly affect the sensor output capacitance. In order to measure these values very accurately, a Nikon EPIPHOT 200 microscope was used that is capable of achieving precision of ± 5 µm for good calibration and 50X magnification. The sensor inner electrode radius was measured using the “traveling microscope” method with accuracy ± 0.01 mm, due to its relative large dimension. It was found that the fabricated dimensions are the same as the nominal values under such measurement accuracy.

Figure 2 a) shows the assembled capacitive probe, Figure 2 b) shows the concentric electrodes, and Figure 3 a) shows the components used to assemble the probe. They consist of the following: a Rogers RO4003® dielectric sensor substrate with thickness 0.31 ± 0.01 mm, on which concentric electrodes are supported; pins soldered to the electrodes; a BNC-to-receptacle adaptor that connects the pins to the BNC connector of an Agilent probe 16095A; and an Agilent LCR meter E4980A that displays the measured capacitance. The entire sensor structure is enclosed in a two-part acrylic tube. Assembled parts A and B are shown in Figure 2 a) with part B shown in detail in Figure 2 b). The acrylic tube was divided into two to facilitate calibration of the probe, i.e., removal of effects of the probe structure on measured capacitance. The two parts, which can be easily attached or detached, were connected together using plastic countersunk screws.

### Table 4.1 Dimensions and calculated free-space capacitance for sensors A and B.

<table>
<thead>
<tr>
<th>Inner electrode radius $s$ (mm)</th>
<th>Outer electrode width $t$ (mm)</th>
<th>Gap between the electrodes $g$ (mm)</th>
<th>Characteristic capacitance (pF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor A</td>
<td>10.67 ± 0.01</td>
<td>0.518 ± 0.009</td>
<td>1.40</td>
</tr>
<tr>
<td>Sensor B</td>
<td>9.66 ± 0.01</td>
<td>1.51 ± 0.01</td>
<td>0.99</td>
</tr>
</tbody>
</table>

mm, on which concentric electrodes are supported; pins soldered to the electrodes; a BNC-to-receptacle adaptor that connects the pins to the BNC connector of an Agilent probe 16095A; and an Agilent LCR meter E4980A that displays the measured capacitance. The entire sensor structure is enclosed in a two-part acrylic tube. Assembled parts A and B are shown in Figure 4.2 a) with part B shown in detail in Figure 4.2 b). The acrylic tube was divided into two to facilitate calibration of the probe, i.e., removal of effects of the probe structure on measured capacitance. The two parts, which can be easily attached or detached, were connected together using plastic countersunk screws.

### 4.5 Calibration Procedures

An effective calibration procedure removes the effect on the measured capacitance of all influences apart from the desired transcapacitance of the sensor. By comparing the probe measurement setup in Figure 4.3 a) and the model used in numerical calculations (Figure 4.1 a)), it can be seen that parasitic capacitances that affect measurement results include: A) that
from the cable connecting the LCR meter to the BNC connector on the Agilent probe, B) that from the BNC-to-receptacle adaptor, C) that from the two receptacles in which the two soldered pins on the sensor are inserted, and D) that from the two pins themselves. The goal is to calibrate the whole system and take into account all the parasitic capacitances up to plane A shown in Figure 4.3.

Steps taken to remove the influences of parasitic capacitances are as follows. In the LCR meter measurement setup, the cable length option was set as 1 m. This setting automatically accounts for the parasitic capacitance A) due to the cable. In order to take into account parasitic capacitances from B) to D), open and short calibration steps are needed. Because the two pins D) are soldered to the electrodes, as shown in Figure 4.3 a), two identical pins were inserted into the receptacle ends during calibration, as indicated in Figure 4.3 b). Open and short calibrations were then performed on plane A according to the procedures provided in the LCR meter manual. All parasitic capacitances up to plane A in Figure 4.3 are accounted for after calibration. However, part B of the acrylic tube is not considered in this process, and will be accounted for by introducing an effective permittivity for the sensor substrate, as discussed below.
4.6 Probe Characterization

In previous work (10), benchmark experiments measuring the transcapacitance of two concentric electrodes in contact with various large test-pieces showed agreement between experiment and theory of better than 4%. Similar experiments are performed here to assess the level of agreement between theory and experiment for the hand-held probes, which is expected to be poorer due to the hardware associated with the hand-held probe that is not modeled explicitly.

In order to account for effects from part B of the acrylic tube, an effective permittivity for layer 1, Figure 4.1 a), was introduced. This effective permittivity was determined by placing the assembled probe in free space and measuring its capacitance. This measurement is considered in the numerical modeling as the case of a concentric capacitive sensor in surface contact with a one-layer dielectric (the sensor substrate) in free space. By assigning the thickness of layer 1 to be that of the sensor substrate and then varying its permittivity, a calculated probe output capacitance that agrees with the measured value to three significant figures was obtained. This permittivity value was subsequently assigned to be the effective permittivity of the sensor substrate with geometry shown in Figure 4.1 b). The effective sensor substrate permittivity for sensor A was determined to be 3.47 while that for sensor B was determined to be 3.31, at 1 MHz and room temperature. The effective permittivity values for both sensor configurations are greater than the substrate permittivity itself, 3.01 ± 0.05, due to the existence of the acrylic tube part B (which has a relative permittivity of around 2.8). The effective permittivity of the sensor substrate for sensor A is greater than that for sensor B, because of the fact that sensor A has higher output capacitance values and influences from part B results in larger absolute changes in the capacitance for sensor A. Consequently, its effective substrate permittivity, which is inversely determined based on the output capacitance, is larger. These fitted sensor substrate effective permittivity values were subsequently used as inputs in the numerical model for the calculation of probe capacitances.
4.7 Experiments on Laminar Structures

Measurements reported in this paper were performed at room temperature. The LCR meter operating frequency was set at 1 MHz so that the measurement error from the LCR meter was less than 0.3% for a 1 pF capacitance. At the same time, 1 MHz is low enough to be a good approximation for the electrostatic assumption made in the numerical model. Samples used in the benchmark experiments are one-, two- and three-layer test-pieces formed by combinations of acrylic and glass plates with lateral dimensions 30 cm by 30 cm. A digital thickness indicator with ± 1 µm accuracy was used to measure the plate thicknesses. The acrylic plates were 2.39 ± 0.02 mm thick and the glass plate was 3.02 ± 0.01 mm thick. A Novocontrol Alpha Dielectric Spectrometer was used to provide an independent value of the dielectric constants of the samples at 1 MHz, as inputs to the model. The dielectric constant of the glass sheet was measured as 5.62 ± 0.05 and that of acrylic was 2.85 ± 0.05.

For all the measurements reported in this paper, the test-piece was supported 10 cm above a wood-top working table to approximate the free space assumption in the calculation model. The two probes with parameters listed in Table 4.1 were tested on five different laminar structures. The hand-held probes were pressed tightly against the test-piece surface to eliminate any air gap between the sensor substrate and the test-piece. As can be seen from Table 4.2, experimental results agree with calculated results to within an average of 7% for sensor A and 9% for sensor B. Notice that absolute differences in measured and calculated capacitance values for sensors A and B are similar in magnitude, and the greater relative differences observed for sensor B are due to the fact that its capacitance values are smaller.

The agreement between theory and experiment of within 10%, shown in Table 4.2, indicates that the structure of the probe give rise to some loss of quantitative accuracy, compared to the 4% agreement obtained in previous benchmark experiments for un-encased electrodes (10). Further, the calibration process here is not perfect. For example, the electrical contact condition between the receptacles and the two soldered pins is not identical to that between the receptacles and the calibration pins. In addition, the soldered joints on the electrodes are not accounted for in the calibration process.
Table 4.2 Measured and calculated capacitance of hand-held probes in surface contact with various test-pieces. Relative differences are compared to the calculated capacitances. Uncertainty in measured $C$ is 0.3%.

<table>
<thead>
<tr>
<th></th>
<th>Calculated $C$ (pF)</th>
<th>Measured $C$ (pF)</th>
<th>Relative difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-layer acrylic</td>
<td>2.75</td>
<td>1.83</td>
<td>2.58 ± 0.01</td>
</tr>
<tr>
<td>One-layer glass</td>
<td>3.57</td>
<td>2.53</td>
<td>3.26 ± 0.01</td>
</tr>
<tr>
<td>Two layer acrylic over glass</td>
<td>2.93</td>
<td>2.03</td>
<td>2.73 ± 0.01</td>
</tr>
<tr>
<td>Two layer glass over acrylic</td>
<td>3.70</td>
<td>2.63</td>
<td>3.42 ± 0.01</td>
</tr>
<tr>
<td>Three layer acrylic-glass-acrylic</td>
<td>2.93</td>
<td>2.03</td>
<td>2.73 ± 0.01</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 4.8 Penetration Depth of Concentric Capacitive Sensors

In capacitive NDE, the penetration depth can be defined in terms of the sensor output capacitance (4) and (16). Consider a concentric capacitive sensor in surface contact with a one-layer dielectric slab with permittivity in free space (Figure 4.4 a)). The penetration depth of a concentric coplanar capacitive sensor is here defined by identifying the one-layer test-piece thickness $T$ for which the capacitance is 10% smaller than its value when in contact with a similar but infinitely thick test-piece. When this condition is satisfied, the sensor penetration depth value is equal to the test-piece thickness $T$ and is dependent on the permittivity of the test-piece.

In other works, $D_3$ is defined as the penetration depth of capacitive sensors. Here we choose $D_{10}$ because the absolute difference in capacitance will be less than 0.1 pF if the capacitance is less than 3 pF and is adopted, and such small changes in capacitance are hard to measure especially when noise is present.

Figure 4.4 b) shows the calculated sensor output capacitance as a function of the one-layer test-piece thickness and permittivity for sensors A and B and test-pieces with $\epsilon_r = 2$ and 5. It can be seen that, for a given test-piece permittivity, the sensor capacitance increases as the test-piece thickness increases and asymptotically approaches a constant value as the thickness
Figure 4.4 Penetration depth of concentric capacitive sensors: (a) cross-section view of a concentric capacitive sensor in surface contact with a one-layer dielectric in free space; (b) calculated sensor output capacitance as a function of test-piece permittivity and thickness, see (a); (c) calculated difference between the capacitance in (b) and that of a similar but infinitely thick test-piece, equation (1).

becomes large. Further insight about the sensor penetration depth is provided in Figure 4.4 c), in which the vertical axis is defined as

\[ \text{Difference\%} = \frac{|C - C_\infty|}{C_\infty} \times 100 \]  

(4.1)

\( C \) is sensor capacitance for a particular test-piece slab and \( C_\infty \) is that as the slab thickness tends to infinity. Notice that the sensor output capacitance approaches at different rates depending on sensor configuration and test-piece permittivity. For a given test-piece permittivity, sensor B always has larger penetration depth than sensor A, because of its wider inter-electrode spacing. This agrees with our intuition. It is also shown that, for a given sensor configuration, the sensor penetration depth is larger for test-pieces with higher \( \epsilon_r \) values. Detailed explanation of this point is given in the Appendix A.

Figures 4.5 a) and 4.5 b) show measured capacitance as a function of test-piece thickness, for both stepped Delrin® (\( \epsilon_r = 3.82 \)) and stepped HDPE (\( \epsilon_r = 2.65 \)) slabs. Figures 4.5
c) and 4.5 d) show relative differences between the capacitance measured values and that
on the 18-mm-thick test-piece, which approximates a half-space. It can be seen that $D_{10}$ of
both sensors is greater for the Delrin® slab than that for the HDPE slab in accordance with
the predictions of Figure 4.4 b), and both sensors sensitivity to test-piece thickness starts to
decline as $T$ increases. For a given test-piece, for sensor B is greater than for sensor A again
in accordance with predictions of Figure 4.4 c). Additionally, good agreement (to within an
average of 10%) between measured capacitances and numerical predictions is observed for test-
pieces with permittivities and thicknesses in the range 2.65 to 3.82 and 1.50 mm to 3.12 mm,
respectively.

In summary, for a given sensor configuration, sensor penetration depth increases as test-
piece permittivity increases. For a given test-piece material, sensors with wider inter-electrode
spacing have higher penetration depths but smaller output capacitances. Therefore, a trade-off
exists between sensor output signal and penetration depth.

4.9 Capacitance as a Function of Probe Lift-off

How do lift-off variations affect the measured probe capacitance and the accuracy of test-
piece permittivity values that may be derived from those measurements? The experimental
arrangement for measuring $C$ as a function of lift-off from the test-piece is shown in Figure 4.2
a). The test-piece was adjusted to be horizontal using a level. The lift-off between the hand-
held probe and the test-piece was precisely controlled by pressing the probe tightly against
the test-piece with fixed-thickness plastic shims acting as spacers in between. These plastic
shims were then removed carefully, without moving the test-piece or the hand-held probe. This
procedure helps to ensure that the plane of the electrodes and the test-piece surface are in
parallel, avoiding probe tilt. The thickness of the plastic shims was measured using a digital
indicator and the resulting value considered to be the probe lift-off value.

The capacitance of the hand-held probes as a function of probe lift-off was measured, and
compared with numerical predictions. In the numerical calculations, the probe substrate was
again assigned the effective value derived from measurement of the free space probe capacitance,
and layer 2 in Figure 4.1 a) was assumed to be air with thickness equal to the lift-off value.
Figure 4.5 Measured capacitance of hand-held probes as a function of test-piece thickness: (a) stepped Delrin\textsuperscript{®} slab $\epsilon_r = 3.82$; (b) stepped HDPE slab $\epsilon_r = 2.65$; (c) and (d) difference calculated according to equation (1) for Delrin\textsuperscript{®} and HDPE respectively, assuming an 18-mm-thick test-piece to be an approximate half-space. Uncertainty in measured $C$ is 0.3%.

The average difference between measured and calculated values was 7%. The difference $\Delta C = |C_{\text{lift-off}} - C_{\text{air}}|$ is plotted in Figures 4.6 a) and 4.6 b) for measurements on PMMA and glass slabs respectively, whose parameters are described earlier in the section discussing experiments on laminar structures. $C_{\text{lift-off}}$ corresponds to the capacitance when there is a certain lift-off value and $C_{\text{air}}$ is the probe capacitance when sensor lift-off value tends to infinity, i.e., probe in free space. Figures 4.6 a) and 4.6 b) provide useful information on how much variation to expect in capacitance when the probe scans a rough test-pieces surface.

Test-piece permittivity values can be determined inversely from measured capacitance val-
Figure 4.6 Measured and calculated differences in capacitance of hand-held probes as a function of lift-off: (a) sensor A; (b) sensor B; (c) and (d) permittivity determined for PMMA and glass, respectively. Uncertainty in measured $C$ is 0.3%.

The measured capacitances agree with the calculated ones the best when lift-off is large; since these situations are closest to the calibration environment of the probes. Figures 4.6 c) and 4.6 d) show the inversely determined permittivity values for the one-layered PMMA and glass, respectively. It is seen that when lift-off values are relatively small, the hand-held probes can characterize the test-piece material property fairly well. However, large lift-off values can result in inaccuracy in the inversely determined material permittivity information, even if the relative differences between the measured and calculated capacitances are small. This is due to the fact that the hand-held probes are most sensitive to the region near the sensor substrate. When the lift-off is large a slight difference in measured capacitance can result in a large difference in the inversely determined test-piece permittivity.
Table 4.3  Measured and calculated capacitance of hand-held probes in surface contact with various test-pieces. Relative differences are compared to the calculated capacitances. Uncertainty in measured \( C \) is 0.3%.

<table>
<thead>
<tr>
<th>Hole diameter (mm)</th>
<th>Measured capacitance on air filled holes (pF)</th>
<th>Relative difference for air filled holes (%)</th>
<th>Measured capacitance on wax filled holes (pF)</th>
<th>Relative difference for wax filled holes (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sensor A</td>
<td>Sensor B</td>
<td>Sensor A</td>
<td>Sensor B</td>
</tr>
<tr>
<td>2.5</td>
<td>2.87</td>
<td>1.91</td>
<td>0.3</td>
<td>1.0</td>
</tr>
<tr>
<td>5.0</td>
<td>2.85</td>
<td>1.89</td>
<td>1.0</td>
<td>2.1</td>
</tr>
<tr>
<td>7.5</td>
<td>2.83</td>
<td>1.86</td>
<td>1.7</td>
<td>3.6</td>
</tr>
<tr>
<td>10.0</td>
<td>2.79</td>
<td>1.82</td>
<td>3.1</td>
<td>5.7</td>
</tr>
</tbody>
</table>

4.10 Detection of Embedded Inhomogeneities in Sandwich Structures

In some structures, such as radomes, it is important that the electrical properties do not vary in an uncontrolled way. Imperfect repairs or damage followed by ingress of water can give rise to inhomogeneities with electrical properties that contrast with their surroundings. Here we investigate the ability of the capacitive probes to resolve inhomogeneities of various size and permittivity embedded in a Delrin\textsuperscript{R} plate and in the core of a glassfiber-honeycomb-glassfiber sandwich structure.

Two rows of holes of different diameters, 2.5, 5.0, 7.5, and 10 mm, were drilled in a 3.17-mm-thick Delrin\textsuperscript{R} plate with permittivity \( \epsilon_r = 4.14 \). One row of holes was left empty while the other was filled with Paraffin wax (\( \epsilon_r = 2.1 \)) to form zones with different permittivity contrasts. Table 4.3 lists the measured capacitances when the contrast zones are positioned directly beneath the sensor gaps. It is seen that both probes were able to detect the air-filled holes of all sizes, whereas both failed to detect the 2.5-mm-diameter wax filled holes, due to the lower permittivity contrast between Delrin and wax (around 2) compared with that between Delrin\textsuperscript{R} and air (around 4.1). On the other hand, because of its deeper penetration depth, sensor B was found more capable of detecting embedded zones than sensor A; see relative differences in Table 4.3.

In order to investigate the effectiveness of the hand-held probes in detecting inhomogeneities
in sandwich structures, different amounts (1, 3, and 5 cc) of water and olive oil ($\epsilon_r \approx 78$ and $\epsilon_r \approx 3$ at 1 MHz and room temperature, respectively) were injected into the honeycomb core of a glassfiber-honeycomb-glassfiber structure (Figure 4.7). Olive oil is used as easily available material to simulate the presence of, for example, excess resin ($\epsilon_r \approx 3$) in the honeycomb or an imperfect repair. Parameters of the sandwich panel are listed in Table 4.4. 1 cc of injected liquid corresponds to 4 honeycomb cells with total surface area of 88 mm$^2$, compared to the surface area of sensors A and B which is of 507 mm$^2$. Figures 4.8 a) to 4.8 d) show the capacitance measured as the hand-held probes scan a line directly over the cells containing the contrast agent. The measured probe signal strength is related to both the inhomogeneity permittivity and size. In particular, for the cases in which the injected liquid areas are smaller than the inner disc of the concentric sensor, two peaks in the output signal are observed for each measurement, due to the sensor gaps on each side of the sensor responding to the inhomogeneity separately. On the other hand, when the injected liquid area is greater than the inner electrode size, a single peak in the measurement signal is observed due to both sides of the sensor being excited simultaneously. As can be seen from Figure 4.8, the approximate size of the inhomogeneity can be inferred from the shape of the measured signal and permittivity information can be extracted from the signal magnitude.

In summary, the outstanding ability of the probes to detect low contrast zones smaller than the sensors themselves has been demonstrated. For example, both sensors detected successfully 1 cc of olive oil ($\epsilon_r \approx 3$) filling 4 cells in the honeycomb core of a laminar structure, indicating their potential application in defect detection in aircraft radome sandwich structures.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core thickness</td>
<td>7.62 mm</td>
</tr>
<tr>
<td>Skin thickness</td>
<td>0.254 mm</td>
</tr>
<tr>
<td>Cell volume</td>
<td>0.25 cc</td>
</tr>
<tr>
<td>Surface area of cell</td>
<td>22 mm$^2$</td>
</tr>
<tr>
<td>Panel length and width</td>
<td>298.45 mm</td>
</tr>
</tbody>
</table>
4.11 Conclusion

Two hand-held capacitive probes with different target penetration depths have been built and tested. Following a calibration procedure that accounts for stray capacitances and the presence of the probe casing, which is not accounted for explicitly in the accompanying model, agreement to within 10% between measured and calculated capacitances has been demonstrated for experiments on laminar structures. The penetration depth of concentric capacitive sensors has been defined and studied both numerically and experimentally. For a given electrode configuration, the sensor penetration depth increases as test-piece permittivity increases. For a given test-piece, sensors with wider electrode spacing have larger penetration depths but lower capacitance values. The hand-held probes sensitivity to lift-off variations has been assessed numerically and experimentally. In order to acquire inversely determined material permittivities close to the actual values, small lift-off values are desirable because such measurement setups give rise to the best signal strength. This suggests that, if the probe is to be used for quantitative permittivity measurement, then calibration on a known test sample may be preferable to calibration in air. The outstanding capability of the hand-held sensors in detecting relatively small contrast zones in one-layered and multi-layered structures has been demonstrated experimentally, e.g., 1 cc olive oil injection in glassfiber sandwich panel was successfully detected.

The hand-held probes discussed here were built using readily available materials and com-
Figure 4.8 Capacitance measured as hand-held probes scan over glassfiber-honeycomb-glassfiber sandwich panels containing injected dielectric contrast agents water and olive oil: (a) sensor A and injected water; (b) sensor A and injected olive oil; (c) sensor B and injected water; (d) sensor B and injected olive oil. Uncertainty in measured $C$ is 0.3%.

ponents. In the future, some refinements can be made to the probe assembly in order to improve the agreement between measurement results and numerical calculations. For example, the probe test fixture and the BNC to receptacle adaptor can be replaced by a combined lead and sensor, thereby reducing parasitic capacitance. Additionally, the lead and sensor can be enclosed in a more compact rigid case that has fewer effects on the sensor signal.
4.12 Acknowledgments

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4.13 References


CHAPTER 5. INTERDIGITAL SPIRAL AND CONCENTRIC CAPACITIVE SENSORS FOR MATERIALS EVALUATION

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5.1 Abstract

This paper describes the development of two circular coplanar interdigital sensors with i) a spiral interdigital configuration and ii) a concentric interdigital configuration for the nondestructive evaluation of multilayered dielectric structures. A numerical model accounting for sensor geometry, test-piece geometry and real permittivity, and metal electrode thickness has been developed to calculate the capacitance of the sensors when in contact with a planar test-piece comprising up to four layers. The validity of the numerical model has been demonstrated by good agreement (to within 5%) between numerical predictions and benchmark experiment results. Compared with a two-electrode coplanar capacitive sensor developed previously, the interdigital configurations were found to have higher signal strength and signal-to-noise ratio, better accuracy in materials characterization, and higher sensitivity in detecting surface flaws when prior knowledge of approximate flaw size is available. On the other hand, the two electrode sensor shows a deeper penetration depth, less susceptibility to lift-off variations and better sensitivity in detecting internal flaws in sandwich structures.

5.2 Introduction

Interdigital sensors are one kind of one-sided capacitive sensors that characterize materials dielectric and/or geometrical properties through impedance measurements. One obvious ad-
vantage of the interdigital sensors, compared to other coplanar capacitive sensors (1), is that the sensor signal strength is greatly increased. This is achieved by enhancing the interaction between the oppositely-charged electrodes. An historical perspective of interdigital sensors, modeling and fabrication techniques and their applications are presented in the review paper (2).

The most commonly used interdigital sensors are periodic structures consisting of parallel microstrips (2). Applications of these sensors range from moisture content sensing to food inspection. For applications in moisture content estimation, test-pieces of different moisture contents possess different dielectric properties, which give rise to different measured sensor responses. Such responses can be capacitance, resistance, or both. In (3), interdigital sensors have been applied to measure moisture concentration in paper pulp with high sensitivity and repeatability. Measurements of water content in the human skin using interdigital sensors is another example based on the same principle (4). Interdigital sensors have also been used as humidity sensors (5). In this application, the sensor substrate is selected in such a way that their materials properties change as they uptake water vapor from the surrounding environment. Such changes result in variations in the measured sensor impedance, which signals changes in the ambient humidity. Interdigital sensors have also been used for chemical sensing purposes. Examples include detection of nitrates and contamination in natural water sources (6), and determination of hydrogen gas concentration (7). Food inspection using interdigital sensors is another recent rising application. The feasibility of utilizing interdigital sensors for the estimation of fat in meat has been demonstrated in (8). A micro-impedance biosensor, having the interdigital electrode configuration, has been successfully utilized for bacteria detection by immersing the surface-insulated biosensor in the testing solution (9). Another interdigital sensor based sensing system that is capable of detecting marine toxins in mussel meat has been reported in (10). Cylindrical interdigital sensors that conforms to the curvature of cylindrical test-pieces have been designed and modeled in (11). The electrodes of these sensors have to be either periodic along the $\phi$- or $z$-directions. These two cylindrical sensor configurations have been applied for dielectrometry measurements of moisture diffusion and temperature dynamics in oil impregnated paper insulated electric power cables (11). In addition to the planar, rect-
angular and cylindrical electrode configurations, quarter-circular interdigital sensors have been presented in (12), to improve the sensors’ ability to detect flaws of various orientations with respect to the electrode axis.

Also independent of orientation, simple two-electrode concentric capacitive sensors, Figure 5.1 a), have been developed previously to meet the need for quantitative nondestructive evaluation of low-conductivity structures (13). Numerical modeling of the simple concentric capacitors has been reported in (13) (based on a spatial domain approach) and (14) (based on a spectral domain approach). The validity of both models was verified by benchmark experiments in which very good agreement (to within 4%) between the numerical predictions and experimental results was obtained. The numerical models allow for inverse determination of dielectric constant or thickness of one layer in a multilayered dielectric structure, based on measured capacitance. A handheld capacitive probe has been developed based on the simple two-electrode sensor configuration and applied for characterization of multilayered structures (15). The capacitive probe was found capable of detecting inhomogeneities of low permittivity contrast (approximate permittivity ratio of 3) with the surroundings in multilayered materials.

One obvious advantage of the simple two-electrode sensor configuration, shown in Figure 5.1 a), is the ability to model it accurately, thereby achieving accurate quantitative information in materials characterization. On the other hand, the regions of the sensor that contribute most to the sensor capacitance are the outer edge of the inner electrode and the inner edge of the outer electrode, where the surface charge density is greatest (13). This results in a low capacitance of the sensor of around 2 pF for a sensor with 25 mm diameter. The fact that these interaction areas are only a small fraction of the total sensor surface area, however, suggests that the sensor output capacitance can be increased for a sensor of a given size by interdigitating the electrodes. For this reason, in order to achieve higher signal strength and to improve signal-to-noise ratio while still retaining rotational symmetry, interdigital spiral and concentric interdigital sensors with increased interaction area to sensor surface area have been conceived and are shown schematically in Figures 5.2 and 5.3. This paper is concerned with the design, fabrication and testing of these sensors and is organized as follows. In Section 3, a numerical model that characterizes the interdigital spiral and concentric sensors is described. Numerical
comparisons of the performances between the interdigital configurations and the two-electrode configuration are made in Section 4, in terms of sensor sensitivity in materials characterization, penetration depth and susceptibility to lift-off variations. Experimental verification of the numerical modeling has been performed and very good agreement between numerical predictions and measurement results is observed (to within 3.4% on average), Section 5. Experimental comparisons between the interdigital and the simple two-electrode configurations also show that the two-electrode configuration is better at detecting buried inhomogeneities in sandwich structures, whereas the interdigital configurations exhibit superior performance in detecting surface flaws.

5.3 Sensor Configuration

Figure 5.2 shows the interdigital spiral sensor configuration. This configuration is referred to as the ‘spiral sensor’ in the rest of this paper. The two sensor electrodes can have different numbers of turns, $N_1$ and $N_2$, where $|N_1 - N_2| \leq 1$, and the inter-electrode spacing is fixed. Note that each turn of the spiral is formed by two semi-circles each with a fixed radius, see the subfigure in Figure 5.2. The two electrodes are connected to two patch contacts via two leads. The capacitance between the two coplanar spiral electrodes can be measured by placing a probe of an impedance measurement instrument across the two contacts.

Figure 5.3 shows the configuration of an interdigital concentric sensor. This configuration is referred to as the ‘concentric sensor’ in the following. The two oppositely charged electrodes of the sensor, each consist of a straight track and a number of circular fingers connected to the track. The number of fingers, $N_1$ or $N_2$, on each electrode may be different depending on the sensor configuration and again, are connected to two square contacts to facilitate capacitance measurements.

As can be seen from Figures 5.2 and 5.3, each electrode (finger) of the spiral and concentric sensors interacts with its neighboring electrodes (fingers) on both sides, except the innermost and outermost electrodes (fingers). Compared to the two-electrode configuration shown in Figure 5.1 b), the interaction area to sensor surface area ratio of the circular interdigital sensors has been increased substantially.
Figure 5.1  a) Simple two-electrode concentric capacitive sensor. The radius of the inner radius is denoted $s_0$, the width of the outer electrode $t_0$, and the gap between the two electrodes $g_0$. b) Numerical modeling of the circular interdigital sensors with diameter $D$. The width of each annular ring is denoted $w$, the gap between neighboring rings $g$, and the radius of the inner most ring $s$.

5.4 Modeling

Numerical modeling of the spiral and concentric sensors is composed of two steps that depend on the geometry of the electrodes. A first order approximation using the concentric ring model shown in Figure 5.1 b) is the same for both sensors. A second order correction to account for the ways in which the spiral and the concentric configurations differ from concentric rings is then applied.

5.4.1 First Order Approximation: the Concentric Annular Ring Model

In Figure 5.1 a), both of the circular interdigital sensors are modeled as a number of concentric annular rings: $N_1$ annular rings of the same color in Figure 5.1 a) are charged to the same potential to form one electrode, while the other $N_2$ rings form the other oppositely charged electrode. This concentric annular ring configuration is a reasonable first order approximation of the spiral and the concentric sensors because of the following facts. For the spiral configuration, the sensor is actually formed by two groups of concentric semi-annular rings of different $s$ ($s_1$
Figure 5.2  Schematic diagram of an interdigital spiral capacitive sensor. Subfigure: each turn of the spiral is comprised of two semi-circles.

Figure 5.3  Schematic diagram of an interdigital concentric sensor.

and $s_2$) but identical $w$ and $g$ (see Figures 5.2 and 5.1 a)). It has been demonstrated in (13) that the capacitance $C_{con}$ of concentric sensors is a linear function of $s$ for fixed $g$ and $w$, i.e., $C_{con} = ks$ where $k$ is the slope determined solely by the test-piece material property. Therefore, the capacitance of the spiral sensor can be expressed as

$$C_{spiral} = \frac{1}{2}C_{con1} + \frac{1}{2}C_{con2} = \frac{1}{2}ks_1 + \frac{1}{2}ks_2 = k\frac{s_1 + s_2}{2}. \tag{5.1}$$

In other words, one spiral loop formed from two consecutive half circles can be modeled equivalently as $s = (s_i + s_{i+1})/2$. As for the concentric configuration, capacitance resulting from the discontinuity of the circular fingers and the existence of the two straight leads is relatively small compared to the capacitance resulting from the rest of the sensor, which means that each interdigital loop can be modeled by one full circular loop, as a first approximation.
Numerical modeling of the interdigital sensors is performed in the electroquasistatic regime, i.e., the wavelength is much greater than the dimension of the problem of interest. Figure 5.4 a) shows the sensor and test-piece configuration used in the modeling. The infinitesimally thin electrodes are sandwiched by two layered-half-space dielectrics. The dielectric materials are assumed to be homogeneous and infinite in the horizontal directions. Media 1 and 4 in Figure 5.4 a) are infinitely thick while the thicknesses of media 2 and 3 are $T_2$ and $T_3$, respectively. The modeling procedures for the circular interdigital sensors are the same as those adopted in (13). A numerical method (the method of moments) is used to calculate the sensor capacitance. As a first step, the Green’s function due to a point charge has to be derived. This Green’s function will be employed to set up integral equations that eventually will be solved for the sensor capacitance. Figure 5.4 b) shows a point charge placed at the origin and sandwiched by the two layered-half-space dielectrics. The resulting electrostatic potential $\Psi_i$, at an observation point $(\rho, \phi, z)$, satisfies the Laplace equation in cylindrical coordinates:

$$
\left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial z^2} \right) \Psi_i(\rho, z) = \frac{1}{2\pi \rho} \delta(\rho) \delta(z) \quad i = 1, 2, 3, 4, \quad (5.2)
$$
where $i$ corresponds to media 1 through 4 and $\delta(x)$ is the Dirac delta function. $\Psi(\rho, z)$ is independent of the azimuthal angle $\phi$ due to the cylindrical symmetry of this problem. (2.1) is a partial differential equation. In order to reduce it to an ordinary differential equation and facilitate the process of solving for $\Psi(\rho, z)$, the Hankel transform $\tilde{f}(\kappa)$ of zero-order of a function $f(\rho)$

$$\tilde{f}(\kappa) = \int_0^\infty f(\rho) J_0(\kappa \rho) \rho \, d\rho$$

is applied to (5.2), where $J_0(\kappa \rho)$ is the Bessel function of the first kind of order zero. Making use of the following identity (16)

$$\int_0^\infty \left[ \left( \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} \right) f(\rho) \right] J_0(\kappa \rho) \rho \, d\rho = -\kappa^2 \tilde{f}(\kappa),$$

where $f(\rho)$ is assumed to be such that the terms $\rho J_0(\kappa \rho) \partial f(\rho)/\partial \rho$ and $\rho f(\rho) \partial J_0(\kappa \rho)/\partial \rho$ vanish at both limits, the spatial domain Laplace equation (5.2) is then transformed into a one-dimensional Helmholtz equation in the transformed domain:

$$\left( \frac{\partial^2}{\partial z^2} - \kappa^2 \right) \tilde{\Psi}_i(\kappa, z) = \frac{1}{2\pi} \delta(z) \quad i = 1, 2, 3, 4,$$

where for $\kappa$ the root with positive real part is taken. The Green's function $\tilde{\Psi}_i(\kappa, z)$ in the transformed domain will be derived first, and will be transformed back to the spatial domain through the inverse Hankel transform to find the expression for $\Psi_i(\rho, z)$.

The general solutions for the potentials in (5.5) in each medium can be expressed as

$$\tilde{\Psi}_i(\kappa, z) = A_i(\kappa) e^{-\kappa z} + B_i(\kappa) e^{\kappa z}, \quad z_i < z \leq z_{i-1},$$

where $z_0 \to \infty$, $z_1 = T_2$, $z_2 = 0$, $z_3 = -T_3$ and $z_4 \to -\infty$. The coefficients $B_1(\kappa) = A_4(\kappa) = 0$ due to the fact that the potential at infinity vanishes. To solve for the remaining six coefficients, the interface conditions for the electric fields

$$\hat{z} \cdot (\mathbf{E}_i - \mathbf{E}_{i+1}) = 0 \quad \text{and} \quad \hat{z} \cdot (\mathbf{D}_i - \mathbf{D}_{i+1}) = 0, \quad i = 1, 3$$

$$\hat{z} \cdot (\mathbf{E}_2 - \mathbf{E}_3) = 0 \quad \text{and} \quad \hat{z} \cdot (\mathbf{D}_2 - \mathbf{D}_3) = \rho_s$$

are introduced, where $\rho_s = \delta(\rho)/2\pi \rho$ is the surface charge density in the plane $z = 0$. The electric field $\mathbf{E}$ and the potential $\Psi$ are related by $\mathbf{E} = -\nabla \Psi$. The interface conditions (5.7)
and (5.8) can be further expressed in terms of the potential in the transformed domain by applying the Hankel transform

\[ \tilde{\Psi}_i(\kappa, z_i) = \tilde{\Psi}_{i+1}(\kappa, z_i), \quad \epsilon_i \frac{\partial \tilde{\Psi}_i(\kappa, z_i)}{\partial z} = \epsilon_{i+1} \frac{\partial \tilde{\Psi}_{i+1}(\kappa, z_i)}{\partial z}, \]

(5.9)

\[ \tilde{\Psi}_2(\kappa, 0) = \tilde{\Psi}_3(\kappa, 0), \quad - \epsilon_2 \frac{\partial \tilde{\Psi}_2(\kappa, 0)}{\partial z} + \epsilon_3 \frac{\partial \tilde{\Psi}_3(\kappa, 0)}{\partial z} = \frac{1}{2\pi}, \]

(5.10)

where \( i = 1, 3 \) and \( \epsilon_i \) represents the relative permittivity of each layer. The potential distribution in the transformed domain can therefore be solved by substituting (5.6) into (5.9) and (5.10). It is worth pointing out that only \( \tilde{\Psi}_3(\kappa, 0) \) will be needed in the following to obtain the sensor capacitance, because the electrodes lie in the plane \( z = 0 \). The potential in the spatial domain \( \Psi_3(\rho, 0) \) is obtained by applying the inverse Hankel transform to \( \tilde{\Psi}_3(\kappa, 0) \). To express the potential \( \Psi_3(\rho, \phi, 0|\rho', \phi', 0) \) at an observation point \( r' = (\rho, \phi, 0) \) due to a point charge at \( r = (\rho', \phi', 0) \), simply replace \( \rho \) by \( |r - r'| \) in \( \Psi_3(\rho, 0) \):

\[
\Psi_3(\rho, \phi, 0|\rho', \phi', 0) = \frac{1}{2\pi(\epsilon_2 + \epsilon_3)} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (-1)^n \alpha^n K_{mn}(\gamma, \beta) \times \left\{ \frac{1}{\sqrt{|r - r'|^2 + 4(nh_1 + mh_2)^2}} - \frac{\gamma}{\alpha} \sqrt{|r - r'|^2 + 4[nh_1 + (m + 1)h_2]^2} + \frac{\alpha \gamma}{\alpha} \sqrt{|r - r'|^2 + 4[(n + 1)h_1 + mh_2]^2} \right\}
\]

(5.11)

where

\[ |r - r'|^2 = \rho^2 + \rho'^2 - 2\rho \rho' \cos(\phi - \phi') \]

(5.12)

\[
K_{mn}(\gamma, \beta) = \sum_{r=0}^{\min(m, n)} (-1)^{m-r} \frac{(m + n - r)!}{r!(m - r)!(n - r)!} \gamma^m \beta^{m+n-2r}
\]

(5.13)

\[ \alpha = (\epsilon_2 - \epsilon_1)/(\epsilon_2 + \epsilon_1), \quad \beta = (\epsilon_3 - \epsilon_3)/(\epsilon_3 + \epsilon_2) \quad \text{and} \quad \gamma = (\epsilon_4 - \epsilon_3)/(\epsilon_4 + \epsilon_3). \]

This completes the derivation of the potential Green’s function. Steps to calculate the sensor capacitance using the Green’s function are the same as in (13) and will only be summarized here: i) the Green’s function (5.11) is utilized to set up an integral equation that relates the prescribed potentials on the interdigital electrodes to the unknown surface charge distribution on the electrodes; ii) the integral equation in i) is discretized into a matrix equation, from which the
unknown surface charge distribution is solved; iii) after finding the surface charge distribution, the total charge \( Q \) on each electrode is obtained. The capacitance of the interdigital sensor is computed from \( C = Q/V \), where \( V \) is the potential difference between the two electrodes.

5.4.2 Second Order Approximation: End Corrections and Compensation for Finite Electrode Thickness

To improve the first order approximation in the modeling of the sensor configuration, shown in Figure 5.1 b), end corrections have been made to both the spiral and the concentric sensors. This second order approximation takes into account the difference between the actual sensor configurations and the concentric annular ring model. The end correction for the spiral configuration is more straightforward: the total capacitance \( C_0 \) is obtained by adding the capacitance due to the two square contacts \( C_{\text{contacts}} \) to \( C_a \) as

\[
C_0 = C_a + C_{\text{contacts}},
\]

where \( C_{\text{contacts}} \) is computed using the method of moments in the same manner as described in Section 5.4.1.

As shown in Figure 5.5, the capacitance for the concentric sensor in Figure 5.3, denoted \( C_0 \), is computed as

\[
C_0 = C_a - C_b + C_c,
\]
where $C_a$ is calculated from Figure 5.1 a), $C_b$ and $C_c$ are the capacitance due to parts b) and c) in Figure 5.5, respectively. $C_b$ is the capacitance ‘lost’ when parts of the rings are removed to insert the parallel tracks, whereas $C_c$ adds the capacitance of those tracks and surrounding structure. The parallel rectangular electrode structure b) is used to approximate the structure within the dashed box in a), a reasonable approximation considering the length $l$ is small compared to the circumference of the circles. $C_b$ is calculated using the analytical model described in (18), whereas $C_c$ is computed using the method of moments in the same manner as described in Section 5.4.1.

For cases when the thickness of the electrodes are not negligible compared to their width, stray capacitance resulting from fringing fields will contribute to the total sensor capacitance. To account for the existence of this stray capacitance, compensation for the finite electrode thickness are made in the model: instead of using the actual electrode width $w$ for the circular interdigital sensors (Figure 5.1 a)), an effective electrode width $w + 2\Delta$ is adopted (17), with

$$
\Delta = \frac{t}{2\pi \epsilon_e} \left[ 1 + \ln \left( \frac{8\pi w}{t} \right) \right],
$$

(5.16)

where $t$ is the thickness of the rings and $\epsilon_e$ is the average permittivity of the two layers in contact with the rings (media 2 and 3 in Fig. 5.4 a)). The effective gap between neighboring rings now becomes $g - 2\Delta$, while the total dimension of the interdigital sensors is unchanged. This approximation was proven to work well in many cases and has been adopted in modeling coplanar capacitive sensors composed of parallel microstrips (17). The effectiveness of using (5.16) to compensate for the finite electrode thickness of circular interdigital sensors is further demonstrated by benchmark experiments presented in Section 5.6.1.

### 5.5 Numerical Examples

The purpose of numerical calculations presented in this section are to study the effect of sensor geometry on the performance of circular interdigital sensors. Comparisons have also been made between the circular interdigital sensors and the simple two electrode sensors, in terms of sensor sensitivity, penetration depth and susceptibility to lift-off effects. The interdigital
sensors studied in this section have equal number of oppositely charged annular rings, i.e., \( N_1 = N_2 = N \) in Figure 5.1 a). In this section, \( C_a \) alone is computed and second order contributions to \( C \) are neglected.

5.5.1 Sensor Sensitivity and Accuracy in Materials Characterization

Figure 5.6 shows the capacitance of circular interdigital sensors as a function of the substrate relative permittivity and the sensor geometry. The sensors are in surface contact with a one-layer dielectric substrate in free space \( (\epsilon_1 = \epsilon_2 = \epsilon_4 = \epsilon_0 \) in Figure 5.4 a)). The capacitance of the interdigital sensors is also compared to that of a simple two-electrode sensor. All the sensors in Figure 5.6 have a fixed diameter of 25.4 mm except for the top line. It is found that the sensor capacitance, \( C \), of all configurations is a linear function of the substrate permittivity. The sensor sensitivity \( k \), defined as the slope of each line, is also observed to be greater for the interdigital sensors than for the two-electrode configuration. On the one hand, for fixed \( g \) and \( w \), the sensitivity of the interdigital sensors increases as the number of annular rings \( N \) increases. On the other hand, for interdigital sensors with fixed sensor diameter, \( k \) increases as \( w \) and \( g \) decrease (smaller \( w \) and/or \( g \) means larger \( N \) for fixed \( D \)). The influence of \( g \) is found to be more significant than that of \( w \). This is because smaller \( g \) allows for more interaction between the neighboring oppositely charged electrodes, and therefore improves the sensor sensitivity.

The sensor sensitivity discussed in Figure 5.6 plays an important role in inferring test-piece permittivity from measured capacitance. As can be seen from Figure 5.6, uncertainty in the measured capacitance \( \Delta C \) is related to the uncertainty of inferred test-piece permittivity \( \Delta \epsilon_r \) as: \( \Delta C = k \Delta \epsilon_r \). This relationship shows that, for sensors with sensitivity greater than 1, the uncertainty in the inferred test-piece permittivity will be smaller than the uncertainty in the measured capacitance, and vice versa. For instance, the sensitivity of the top line in Figure 5.6 \( k = 2.31 \), whereas \( k = 0.47 \) for the bottom line (two-electrode configuration). Assume \( \Delta C = \pm 0.01 \) pF in the capacitance measurements, the uncertainties in the inferred permittivity for the top and bottom lines are \( \Delta \epsilon_r = \pm 0.004 \) and \( \pm 0.02 \), respectively. In this comparison, the \( \Delta \epsilon_r \) when using the interdigital sensor is only 1/5 of that when using the two-electrode sensor.
Figure 5.6  Sensor capacitance as a function of substrate permittivity $\varepsilon_3$ in Figure 5.4 a). All the sensor configurations have fixed starting radius $s = 1.02$ mm and diameter $D = 25.4$ mm, except for the one corresponding to the top line, for which $D = 41.66$ mm (see Figure 5.1 a)). The dimensions for the two-electrode configuration are $s_0 = 10.67$ mm, $g_0 = 0.51$ mm and $t_0 = 1.52$ mm (see Figure 5.1 a)).

Figure 5.6 shows that the interdigital configuration provides larger sensor sensitivity and better accuracy in materials permittivity characterization.

5.5.2 Penetration depth

Figure 5.7 shows the penetration depth of circular interdigital sensors as a function of test-piece permittivity and sensor geometry. The sensors are in surface contact with a one-layer dielectric slab as for Figure 5.6. The sensor penetration depth $T_{10}$ is defined by identifying the one-layer test-piece thickness $T$ for which the capacitance is 10% smaller than its value when in contact with a similar but infinitely thick test-piece (15). The vertical axes of Figures 5.7 and 5.8 are defined as

$$\text{Difference\%} = \frac{|C - C_\infty|}{C_\infty} \times 100\%, \quad (5.17)$$

where $C$ is the sensor capacitance for a particular test-piece slab and $C_\infty$ is that as the slab thickness tends to infinity. Hence $T_{10}$ is defined $T_{10} = T$ at Difference\% = 10%. It is worth pointing out that one can select other values for Difference\% besides 10%. A rule to select the Difference\% is to ensure the absolute difference in capacitance is large enough to be measured by the instrument. As can be seen from Figure 5.7, the sensor penetration depth increases as the
test-piece permittivity increases, for given electrode configurations. Mathematical explanations of this behavior can be found in (15). The penetration depth of interdigital sensors is also found to increase as $g$ increases. This is because larger inter-electrode spacing allows for more fields to penetrate into the dielectric materials. Changes in the electrode width $w$, however, have less impact on the sensor penetration depth. Note that it is found numerically that the sensor penetration depth is independent of the number of electrodes $N$.

A similar relationship between the penetration depth and test-piece permittivity is observed for the two-electrode configuration, Figure 5.8. In addition, the sensor penetration depth increases as $g_0$ increases, but is insensitive to changes in $s_0$ and $t_0$. It can be seen from Figures 5.7 and 5.8 that $T_{10}$ of the two-electrode configurations are greater than that of the interdigital configurations, for equivalent inter-electrode spacing dimensions $g$. Specifically, $T_{10}$ is greater than $g_0$ for the two-electrode configuration, but smaller than $g$ for the interdigital design. Comparisons between Figures 5.7 and 5.8 demonstrate that the two-electrode configuration is more capable of detecting subsurface flaws.
5.5.3 Susceptibility to Lift-off Variations

Figure 5.9 shows the comparison of the sensors’ susceptibility to lift-off variations. In the calculations, \( \epsilon_2 = \epsilon_4 = 1 \) and \( \epsilon_3 = 3.34 \). \( T_3 = 0.31 \text{ mm} \) and \( T_2 \) varies as lift-off. The dimension of the simple two-electrode sensor is as for Figure 5.8. The vertical axis of Figure 5.9 is defined as the relative change in sensor capacitance with respect to \( C_0 \), in which \( C_0 \) is the capacitance when the lift-off is zero. The interdigital configuration with \( w = g = 0.51 \text{ mm} \) has the same diameter, 25.4 mm, as the two-electrode configuration. It is observed that the relative change in \( C \) for the interdigital configurations shown in Figure 5.9 is at least twice that for the two-electrode configuration for any particular value of lift-off. The number of electrodes/rings \( N \) for the interdigital sensors has a negligible effect on the sensors’ susceptibility to lift-off variations. Comparisons in Figure 5.9 demonstrate that the simple two-electrode configuration has the advantage of being less susceptible to lift-off variations than the interdigital sensors. This is an important feature during practical inspections. Figure 5.9 also shows how much variations to expect in capacitance when the sensors scan over a rough test-piece surface.
Figure 5.9 Comparison of sensors’ susceptibility to lift-off variations. $C$ is the sensor capacitance at a certain lift-off, and $C_0$ is the capacitance when lift-off is zero. The parameters of the simple two-electrode sensor are as for Figure 5.8. Solid line: two-electrode configuration. Others: circular interdigital configuration. $s = 1.02$ mm for all interdigital configurations.

5.6 Experiment

Spiral and concentric sensors of different dimensions have been fabricated using photolithography (14 mL standard copper deposition). The electrode thickness is 18 µm under this copper deposition standard. Thickness compensations and end corrections, as described in Section 4. B, are included in the numerical modeling for the interdigital sensors in this section. The fabrication process is as described in (13). The sensors are deposited on a one-layer 0.31-mm-thick RO4003® substrate with measured permittivity $\epsilon_r = 3.34 \pm 0.05$ at 1 MHz and room temperature. The fabrication tolerance of the photolithography process is $\pm 13$ µm. The substrate thickness was measured using a digital caliper with uncertainty of $\pm 0.01$ mm. The permittivity of the substrate was measured using a Novocontrol Alpha Dielectric Spectrometer.

Table 5.1 shows the dimensions of the fabricated spiral and concentric sensors. For the spiral sensors, the nominal values of the electrode width and the inter-electrode spacing are 100 µm. The inner radius of the innermost spiral is 100 µm. The two square contacts have area 1 mm$^2$ and are 1 mm apart. The separation between the outermost spiral and the two contacts is also 1 mm. For the concentric sensor configuration, the width of each circular finger and the spacing between neighboring fingers are both 100 µm. Similarly, the straight tracks
Table 5.1  Dimensions of the fabricated spiral/concentric sensors. *For the large concentric configuration, the two electrodes have $N_1 = 31$ and $N_2 = 30$ fingers, respectively.

<table>
<thead>
<tr>
<th>Size of spiral (concentric) sensors</th>
<th>Number of turns (fingers) on each electrode</th>
<th>Spiral sensor diameter $D$ (mm) ± 0.01</th>
<th>Concentric sensor diameter $D$ (mm) ± 0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>12</td>
<td>9.9</td>
<td>10.0</td>
</tr>
<tr>
<td>Medium</td>
<td>18</td>
<td>14.7</td>
<td>14.8</td>
</tr>
<tr>
<td>Large</td>
<td>$31^*$</td>
<td>25.1</td>
<td>24.8</td>
</tr>
</tbody>
</table>

are $100 \ \mu m$ wide and $100 \ \mu m$ apart. The inner radius of the innermost finger, and the gap between the end of the circular fingers and the oppositely charged lead are also $100 \ \mu m$. The size of the square contacts is as for the spiral sensors. Separations between the two square contacts are $100 \ \mu m$ and $1 \ mm$ for the concentric and spiral configurations, respectively. A Nikon EPIPHOT 200 microscope was used to verify the dimensions of the fabricated sensors, for which the measurement precision is $\pm 3 \ \mu m$ with good calibration and $100X$ magnification. It was found that variation in the fabricated sensor dimensions is within $\pm 15 \ \mu m$ of the nominal values.

5.6.1 Benchmark Experiments

Benchmark experiments were carried out to verify the effectiveness of the modeling process. An Agilent E4980A precision LCR meter was connected to an Agilent probe test fixture 16095A to measure the sensor capacitance, by placing the two pins of the probe across the square contacts of the interdigital sensors. Capacitance measurements were performed at 1 MHz and room temperature. The measurement frequency was selected to achieve the best measurement accuracy, while at the same time satisfying the electroquasistatic assumption made in the modeling process. The LCR meter measurement accuracy is between $0.1\%$ and $0.3\%$ for a capacitance between 2 and 105 pF under the above measurement conditions. The measurement procedures adopted in this paper are the same as those in (13), where a detailed description of the measurement setup can be found.

Table 5.2 shows the comparison between the calculated and measured capacitance for isolated interdigital sensors (no test-piece). The sensors were modeled by assigning $\epsilon_1 = \epsilon_2 = \epsilon_4 =$
$\epsilon_0, \epsilon_3 = 3.34$ and $T_3 = 0.31$ mm. Measurement results and numerical predictions agree to within an average of 3.3%. This demonstrates that the modeling process in Section 4 can characterize effectively the behavior of the spiral and the concentric sensors. The measured and calculated capacitance of a two-electrode sensor is also included in Table II. This sensor was fabricated using the same photolithography process, although in this case the electrodes were deposited on a 25.4-µm-thick Kapton® film and the sensor was then placed in tight surface contact with the RO4003® substrate and the capacitance measured. The parameters of the two-electrode sensor are as for Figure 5.8. The Kapton® film is effectively negligible because of the fact that it exhibits no unusual dielectric properties ($\epsilon_r = 3.4$ at 1 MHz and room temperature) and its thickness is much smaller than the test-pieces studied here.

The large spiral and concentric sensors mentioned in Table 5.2 are similar in size to the two-electrode sensor, while the measured capacitances are 27 times higher. On the other hand, even though the diameters of the small- and medium-sized spiral/concentric sensors are smaller than that of the two-electrode sensor, the measured capacitances are still much higher. Note that the variation in measured capacitance, or the noise, in Table II is between ± 0.01 and ± 0.03 pF for all the sensor configurations. If one defines the signal-to-noise ratio (SNR) for the capacitive sensors as the ratio of measured capacitance to its variations, the SNR of the large spiral and concentric sensors is approximately 2850 and 1825, respectively. In contrast, the SNR of the two-electrode sensor is approximately 200. The SNR of the large spiral/concentric sensors is 9 to 14 times that of the two-electrode configuration. The interdigital sensor design obviously gives rise to not only enhanced signal strength but also enhanced signal-to-noise ratio.

### 5.6.2 Inverse Determination of Test-piece Permittivity Based on Measured Capacitance

The test-pieces used in this subsection were an Acrylic slab ($\epsilon_r = 2.85 \pm 0.05$, 30 cm × 30 cm and thickness 2.39 ± 0.02 mm) and a glass sheet ($\epsilon_r = 5.62 \pm 0.05$, 30 cm × 30 cm and thickness 3.02 ± 0.01 mm). The sample permittivities were measured using a Novocontrol Alpha Dielectric Spectrometer at 1 MHz and room temperature. Each test-piece was placed in contact with the electrodes in such a way that the electrodes were sandwiched between their
Table 5.2  Comparison between measured and calculated capacitance for the isolated interdigital/two-electrode sensors. S: small, M: medium and L: large. The parameters of the two-electrode sensor are as for Figure 5.8.

<table>
<thead>
<tr>
<th>Sensor configuration</th>
<th>Measured $C$ (pF)</th>
<th>Calculated $C$ (pF)</th>
<th>Relative difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spiral S</td>
<td>8.86 ± 0.01</td>
<td>8.65</td>
<td>-2.4</td>
</tr>
<tr>
<td>Spiral M</td>
<td>19.40 ± 0.02</td>
<td>18.88</td>
<td>-2.7</td>
</tr>
<tr>
<td>Spiral L</td>
<td>56.94 ± 0.02</td>
<td>54.65</td>
<td>-4.0</td>
</tr>
<tr>
<td>Concentric S</td>
<td>9.07 ± 0.01</td>
<td>8.63</td>
<td>-4.9</td>
</tr>
<tr>
<td>Concentric M</td>
<td>19.22 ± 0.02</td>
<td>18.83</td>
<td>-2.0</td>
</tr>
<tr>
<td>Concentric L</td>
<td>54.74 ± 0.03</td>
<td>52.78</td>
<td>-3.6</td>
</tr>
<tr>
<td>Two-electrode</td>
<td>2.02 ± 0.01</td>
<td>2.08</td>
<td>-2.8</td>
</tr>
</tbody>
</table>

substrate and the test-piece. In the measurements, the square contacts were not covered by the test-piece in order to allow electrical contact with the probes to be made. In the corrections for the square contacts, only the sensor substrate is considered. Pressure was applied to the test-piece and the sensor substrate to minimize the air-gap between the test-piece and the sensor electrodes. It was found that measurement results and numerical predictions agreed to within 2.5% for the Acrylic sample and 5.5% for glass. The capacitance of the two-electrode sensor in Table 5.2 was also measured for comparison. Its measured and calculated capacitance agreed to within 2.0%.

Table 5.3 shows the test-piece permittivity determined inversely from measured sensor capacitance, by utilizing the numerical model. No significant difference is observed in terms of the accuracy of inferred $\epsilon_r$, between using the spiral/concentric sensors and the two-electrode sensor. This finding looks contradictory to what is shown in Figure 5.6 at a glance, i.e., the sensitivity $k$ of the interdigital configurations is higher and should therefore give rise to more accurate inferred test-piece permittivity, but can be explained as follows. $\Delta C$ is assumed to be identical for all sensor configurations in Figure 5.6. Here, however, the percentage difference between the measured and numerically-predicted capacitance was similar for the interdigital configurations and the two-electrode one (2.0% to 5.5%). The absolute difference in capacitance $\Delta C$ is much higher for the interdigital sensors simply because of their higher capacitance. The difference between the inferred and independently-measured test-piece permittivity is de-
Table 5.3  Comparison of inversely determined test-piece permittivity using different sensors. S: small, M: medium and L: large (Table 5.1). The independently measured test-piece relative permittivities are $\epsilon_r = 2.85 \pm 0.05$ for Acrylic and $\epsilon_r = 5.62 \pm 0.05$ for glass. The relative difference is calculated with respect to the independently measured values.

<table>
<thead>
<tr>
<th>Sensor configuration</th>
<th>Acrylic: inferred $\epsilon_r$</th>
<th>Acrylic: rel. diff. in $\epsilon_r$</th>
<th>Glass: inferred $\epsilon_r$</th>
<th>Glass: rel. diff. in $\epsilon_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spiral S</td>
<td>2.82</td>
<td>-1.1%</td>
<td>5.87</td>
<td>4.5%</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.01$</td>
<td>$\pm 0.01$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spiral M</td>
<td>2.71</td>
<td>-4.9%</td>
<td>5.29</td>
<td>5.9%</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.01$</td>
<td>$\pm 0.01$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spiral L</td>
<td>2.706</td>
<td>-4.9%</td>
<td>5.159</td>
<td>-8.2%</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.004$</td>
<td>$\pm 0.004$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concentric S</td>
<td>2.70</td>
<td>-5.3%</td>
<td>5.85</td>
<td>4.1%</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.01$</td>
<td>$\pm 0.01$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concentric M</td>
<td>2.86</td>
<td>0.4%</td>
<td>5.34</td>
<td>-5.0%</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.01$</td>
<td>$\pm 0.01$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concentric L</td>
<td>2.747</td>
<td>-3.5%</td>
<td>5.108</td>
<td>-9.1%</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.003$</td>
<td>$\pm 0.003$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two-electrode</td>
<td>2.77</td>
<td>-2.8%</td>
<td>5.42</td>
<td>-3.6%</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.02$</td>
<td>$\pm 0.02$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

termined from $\Delta \epsilon_r = \Delta C/k$, Figure 5.6. Therefore, although $k$ is greater for the interdigital configurations, $\Delta C$ is also greater, and the $\Delta \epsilon_r$ is not necessarily smaller. It is observed for these cases that variations in the inferred test-piece permittivity $\epsilon_r$ for the spiral/concentric configurations are approximately half of those of the two-electrode configuration.

5.6.3 Detection of Surface Flaws in a One-layered Dielectric

The capability of the different sensors in detecting surface flaws is compared here. The one-layered test-piece was a 30 cm $\times$ 30 cm $\times$ 3.17 mm Delrin$^\text{®}$ plate ($\epsilon_r = 4.14$). Two rows of holes of different diameters (2.5, 5.0, 7.5 and 10.0 mm) were drilled in the plate to form contrast zones of different dimensions. One row was left empty while the other was filled with Paraffin wax ($\epsilon_r = 2.1$) to form zones of different permittivity contrasts with the surroundings. The interdigital sensors (Table 5.1) and the two-electrode sensor (Table 5.2) were used to inspect for the contrast zones by recording the capacitance $C$ when the contrast zones were positioned directly beneath the sensor gaps. $C$ was then compared to the sensor capacitance
Figure 5.10  Detection of air- and wax-filled holes in a 3.17-mm-thick Delrin® plate. Hashed bars: wax-filled holes. Solid bars: air-filled holes. Horizontal axis: diameters of different holes.

$C_0$ when the sensor was positioned a homogeneous part of the plate. The percentage change in capacitance $(C_0 - C)/C_0 \times 100\%$ is plotted in Figure 5.10 for flaws of different diameters and permittivity contrasts. Note that the interdigital sensors were sandwiched between the sample and their substrate. If one places the substrate directly on the test-piece, the interdigital sensors can barely sense the Delrin® plate due to their limited penetration depth. As in the above measurements, pressure was applied to the sample and the substrate to minimize the effects of air gaps.

Since the measurement results of the spiral and the concentric configurations are similar, only the results of the spiral configurations are shown in Figure 5.10. The percentage change in capacitance, for all sensor configurations, is observed to be greater for the air-filled holes than wax-filled ones. On the other hand, for a given contrast zone, $(C_0 - C)/C_0$ varies for sensors of different configurations. The detecting capability of the two-electrode sensor is poorer than that of the small and medium spiral configurations, but better than that of the large spiral one. This indicates that an optimal sensor configuration exists for surface flaws of given dimensions. For example, the small spiral configuration is found to have the best detection capability in this comparison (50% higher than for the two-electrode sensor). This is because its diameter (9.9 mm) most closely matches that of the contrast zones. In addition, more of the interactive area of the small spiral sensor is above the contrast zones compared to the two-electrode case,
making it more sensitive to changes in the test-piece materials property.

The two-electrode sensor is found to be more capable of detecting contrast zones than the large spiral configuration, although they have similar outer diameters. The ratio between the area of the contrast zones and the interactive area of the sensor is lower for the large spiral configuration. This means the capacitance contribution from the contrast zones will be smaller, and the sensor are therefore less likely to detect them.

As a summary, when prior knowledge of the size of surface flaws is available, utilizing circular interdigital sensors of similar size to the flaw can significantly increase the detectability. When the size of the contrast zones is much smaller than the area of the sensor, however, the two-electrode configuration will be more likely to detect such inhomogeneities.

5.6.4 Detection of Internal Inhomogeneities in a Sandwich Structure

In order to compare the capability of the capacitive sensors in detecting inhomogeneities in sandwich structures, different amounts (0.25, 1, and 3 cc) of water ($\epsilon_r \approx 78$ at 1 MHz and room temperature) were injected into the honeycomb core of a glassfiber-honeycomb-glassfiber structure. The glass fiber layer is 0.254 mm thick, and the thickness of the honeycomb cores is 7.62 mm. Other parameters of the sandwich panel can be found in (13). 0.25 cc of injected liquid corresponds to 1 honeycomb cell with total surface area of 22 mm$^2$. The experimental setup and sensors are as described in Section 6. C. The concentric and the spiral sensors were found to have similar performance. For brevity, results for only the spiral sensors are shown here. The surface areas of the sensors are: two-electrode 506 mm$^2$, small spiral: 79 mm$^2$, medium spiral: 170 mm$^2$ and large spiral: 495 mm$^2$.

Figures 5.11 and 5.12 show the measured capacitance as the sensors scan a line directly over the cells containing the contrast agent. The corresponding relative increase in capacitance is also indicated. The signal strength of a given sensor is observed to be related to the size of the inhomogeneity. This agrees with our intuition. On the other hand, the shapes of the curves are different for the spiral and the two-electrode configurations. This is explained as follows. The most sensitive regions of the capacitive sensors are the gaps between oppositely charged electrodes. When the injected water is directly beneath the sensor gaps, the sensor capacitance
Figure 5.11  Capacitance measured as the small spiral and the two-electrode sensors scan over
glassfiber-honeycomb-glassfiber sandwich panels containing various volumes of
injected water. Uncertainty in the measured capacitance is ± 0.02 and ± 0.03 pF
for the two-electrode and the small spiral configurations, respectively. Percentage
increase in C is relative to the capacitance measured on a region remote from the
injected water.

will increase due to the higher permittivity of water compared to the surroundings. For the
two-electrode configuration, the size of the area of the injected water is smaller than the surface
area of the sensor. When the sensor scans over the cells containing water, the sensor gaps on
each side of the sensor respond to the inhomogeneity separately. As a result, two peaks are
observed in the output signal. On the contrary, only one peak is observed in the output signal
of the small spiral sensor, independent of the relative size of the water injection area. This is
because the sensitive areas of the interdigital sensors are distributed more uniformly over the
sensor surface and are smaller than the area of the injected water. When the sensor scans over
the cells containing water, therefore, the capacitance starts to increase, and reaches the peak
when the overlap between the sensor and the area of injected water achieves its maximum.

The two-electrode sensor is found to have the highest sensitivity in detecting embedded
inhomogeneities, Figures 5.11 and 5.12. The confined sensitive area and deeper penetration
depth make the two-electrode configuration more capable of detecting inhomogeneities in the
sandwich structure studied here. The medium spiral sensor is observed to have the highest
Figure 5.12 Capacitance measured as the medium and large spiral sensors scan over glass-fiber-honeycomb-glassfiber sandwich panels containing injected water. Uncertainty in the measured capacitance is ± 0.04 and ± 0.07 pF for the medium and large spiral configurations, respectively. The percentage increase is defined as for Figure 5.11.

sensitivity among the spiral configurations, due to the fact that the its surface area is the closest to the areas of the inhomogeneities. However, due to the limited penetration depth, fewer electric fields of the interdigital configurations penetrate through the 0.254-mm-thick glassfiber surface, compared to the two-electrode configuration.

5.7 Conclusion

Spiral interdigital sensors and concentric interdigital sensors have been developed, to improve the output capacitance and signal-to-noise ratio when compared with a previously developed two-electrode coplanar concentric design. A numerical model has been developed to describe the behavior of the interdigital sensors. Experimental verifications showed that the numerical predictions and measurement results agreed to within 5%. Through numerical and experimental comparisons, the two-electrode configuration was found to possess advantages such as deeper penetration depth, better immunity to lift-off variations and higher sensitivity in detecting internal inhomogeneities in sandwich structures. The interdigital configurations were found to be able to achieve higher output signal strength, better signal-to-noise ratio,
better accuracy in materials characterization and higher sensitivity in detecting surface defects when prior knowledge of approximate flaw size is available.

5.8 Acknowledgments

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5.9 References


CHAPTER 6. ANALYSIS OF ARC-ELECTRODE CAPACITIVE SENSORS FOR CHARACTERIZATION OF DIELECTRIC CYLINDRICAL RODS

A paper published in the IEEE Transactions on Instrumentation and Measurement
Tianming Chen, Nicola Bowler, and John R. Bowler

6.1 Abstract

An arc-electrode capacitive sensor has been developed for the quantitative characterization of permittivity of cylindrical dielectric rods. The material property of the cylindrical test-piece can be inversely determined from the sensor output capacitance based on a theoretical model. For the modeling process, the electrostatic Green’s function due to a point source exterior to a dielectric rod is derived. The sensor output capacitance is calculated numerically using the method of moments (MoM), in which the integral equation is set up based on the electrostatic Green’s function. Numerical calculations on sensor configuration optimization are performed. Calculations also demonstrate the quantitative relationship between the sensor output capacitance and the test-piece dielectric and structural properties. Capacitance measurements on different dielectric rods with different sensor configurations have been performed to verify the validity of the numerical model. Very good agreement (to within 3%) between theoretical calculations and measurement results is observed.

6.2 Introduction

Increasing demands for dielectric measurements have been observed over the past decade, with new applications of advanced composites in modern aircraft, automobiles, and ship-
building. Specifically, dielectric measurements are important for the characterization of thin films, substrates, circuit boards, printed-wiring boards, bulk materials, powders, liquids and semisolids.

Capacitance methods, because of their simplicity, relatively low cost, and high accuracy, have been applied to characterize the dielectric properties of many different materials. Over the past 100 years, closed-form solutions for capacitances due to various canonically shaped electrodes have been found, by mapping out the electrostatic field in the vicinity of the conductors. Canonical electrode shapes are those formed from surfaces easy to describe in standard coordinates, including strips, circular discs, annular rings, cylindrical arcs, spherical caps, etc.

It is convenient to solve capacitance problems associated with electrified strips using Cartesian coordinates. The capacitances for two parallel and coplanar infinite strips, as well as charged thin-strip quadrupoles, have been solved using the triple integral equations. In addition, the potential associated with a physically more realistic strip of finite length, the potential due to polygonal plates, and the potential due to a charged elliptical plate have been derived using dual integral equations in Cartesian coordinates.

Using cylindrical coordinates, the solution to dual integral equations has been applied to obtain the surface charge distribution of a charged disc in free space, and also to obtain the potential due to a circular disc placed between two parallel earthed planes. The solution of the Fredholm equation has been applied to solve for the capacitance of an electrified disc situated inside an earthed coaxial infinitely long hollow cylinder. Also, the field due to two equal coplanar electrified discs has been solved by the method of Kobayashi potentials, while the capacitance between two identical, parallel and coaxial discs has been obtained by solving Love's integral equation. These solutions are available in the classic book written by TIMSneddon. Furthermore, an axisymmetric problem of several charged coaxial discs has been considered by the dual integral equation method, and the solution has been obtained for the case when the distance between neighboring discs is large compared to their radii. The potential of a system of \( N \) charged, arbitrarily located, circular discs has also been considered in. Aside from classic problems associated with discs, Cooke's solution of a set of triple integral equations has been applied to solve for the field due to a charged annular ring of finite width in free space...
(1), while the solution to the integro-series equations has been applied to obtain the total charge for a capacitor that consists of a coupled disc and spherical cap (2).

Another set of canonical capacitance problems discussed in the literature are infinitely long axially slotted open cylinders. In (2), the capacitance generated by a pair of charged symmetrically or asymmetrically placed circular arcs has been calculated in circular cylindrical coordinates, whereas the capacitance due to a pair of charged symmetrically placed elliptic arcs has been solved in elliptic cylinder coordinates (2).

The canonical capacitance problems, mentioned above, are all discussed in free space situations, and need further modifications to be applicable for characterization of material dielectric properties. Other semi-analytical and numerical capacitive solutions have been developed over the past decade to keep pace with new applications of dielectric materials. For example, interdigital dielectrometry has been applied for material dielectric property characterization as one of the most commonly used capacitance sensors. An excellent review paper on interdigital sensors is (4), in which sensor modeling, fabrication, measurement setup, and applications are discussed in detail. In addition to a widely-used effective semi-analytical approach, called the continuum model (5), conformal mapping techniques have also been applied to obtain closed-form solutions for the capacitance of interdigital sensors in surface contact with multi-layered dielectric structures (6). Examples of practical applications of interdigital sensors include estimation of meat fat content (7) and insulation damage detection in power system cables (8). Multichannel fringing electric field sensors, designed by finite-element (FE) method calculations for sensor modeling, optimization and performance evaluation, have been used for material property measurements (9). Cylindrical geometry electroquasistatic dielectrometry sensors have been developed using semi-analytical models to quantitatively relate the dielectric properties of multi-layered test-pieces to sensor output transcapacitance (10). Concentric coplanar capacitive sensors for nondestructive evaluation of multi-layered dielectric structures have been developed in (11), and can be applied to detect water ingestion and inhomogeneities in aircraft radome structures. In addition, rectangular coplanar capacitance sensors have been developed to detect water intrusion in composite materials (12) and for damage detection in laminated composite plates (13). Rectangular capacitive array sensors that detect surface and
subsurface features of dielectric materials have been developed in (14). In (15), approaches of determining the length of open-circuited aircraft wires through capacitance measurements have been presented.

In this paper, a model-based capacitive method is developed for the quantitative dielectric property characterization of circular cylindrical dielectric rods. The work is motivated by testing of cylindrical components such as wiring insulation or polymeric tubing, and will be developed to deal with those cases in future. The capacitance sensor consists of two arc-shaped patch electrodes that are located exterior to and coaxially with the cylindrical test-piece. These two sensor electrodes exhibit a measurable capacitance whose value depends on both the dielectric and geometrical properties of the dielectric rod. The arc-electrode configuration offers a nondestructive and convenient way of determining the dielectric constant of cylindrical test-pieces, compared to cutting a slice from the test-piece for a conventional parallel-plate capacitance measurement. A numerical method, the method of moments (MoM), is employed in the numerical calculations. First, the Green’s function for a point source over the surface of a dielectric rod is derived in cylindrical coordinates, in the form of modified Bessel functions of the first and second kinds of integer order \( n \): \( I_n(z) \) and \( K_n(z) \). This Green’s function then serves as the integration kernel in MoM calculations, from which the sensor surface charge distribution is obtained. Once the sensor surface charge distribution is known, one can easily calculate the sensor output capacitance \( C \) through \( C = \frac{Q}{V} \), where \( Q \) is the total charge on one electrode and \( V \) is the potential difference between the two sensor electrodes. Validation of numerical results by benchmark experiments has been performed, and very good agreement (to within 3%) between theoretical calculations and measurements is observed. The advantage of the arc-electrode capacitive sensor reported in this paper, compared to existing planar capacitive sensors, is that the arc electrodes conform to the surface of a cylindrical test-piece and concentrate the electric field in the material under test. In addition, the physics-based model developed in this paper allows inverse determination of test-piece permittivity from measured arc-electrode capacitance.
Figure 6.1  Arc-electrode capacitive sensor. The radii of the sensor electrodes and the cylindrical dielectric rod are denoted $\rho_0$ and $a$, respectively. The arc-angle of each sensor electrode is $\phi_0$ (rad). The length of each electrode in the vertical direction is $l$ and the width in the horizontal direction is $w = \phi_0 \times \rho_0$.

6.3 Modeling

6.3.1 Sensor Configuration

Figure 6.1 shows the configuration of the arc-electrode capacitive sensor. The capacitive sensor consists of two identical arc-electrodes coaxial with a cylindrical dielectric rod, and exhibits a measurable capacitance $C$ that is quantitatively related to the permittivity and diameter of the material under test. In the theoretical modeling, the cylindrical dielectric rod is assumed to be infinitely long while the arc-electrodes are infinitesimally thin. The more general case in which the electrodes and the test-piece have different radii, as shown in Figure 6.1, is considered theoretically. However, in order to achieve maximum output capacitance, it is more desirable to have the sensor electrodes in tight contact with the cylindrical test-piece in measurements. One practical approach to achieving tight surface contact between the electrode and the test-piece is to deposit the sensor electrodes on a compressible dielectric material used as the sensor substrate, and press the substrate against the test-piece to conform the arc electrodes to the test-piece surface. This approach will be attempted in a future version of the sensor.
### 6.3.2 Derivation of Green’s Function in Cylindrical Coordinates

The electrostatic Green’s function due to a point charge outside of an infinitely long dielectric rod is derived in cylindrical coordinates, to form the integral equations later used in MoM calculations. Figure 6.2 shows a point charge placed at \((\rho', \phi', z')\) exterior to a cylindrical dielectric rod of radius \(a\) and dielectric constant \(\varepsilon_2\). Without loss of generality, the dielectric constant for the medium exterior to the dielectric rod is assumed to be \(\varepsilon_1\). The resulting potential \(\Psi(\rho, \phi, z)\) due to such a point charge satisfies the Laplace equation in each homogeneous region:

\[
\left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right] \Psi(\rho, \phi, z) = -\frac{1}{\rho} \delta(\rho - \rho')\delta(\phi - \phi')\delta(z - z'),
\]

(6.1)

and is subject to the interface conditions at the surface defined by \(\rho = a\):

\[
\Psi^{(1)} = \Psi^{(2)} \quad \text{and} \quad \varepsilon_1 \frac{\partial}{\partial \rho} \Psi^{(1)} = \varepsilon_2 \frac{\partial}{\partial \rho} \Psi^{(2)},
\]

(6.2)

where superscripts \((1)\) and \((2)\) correspond to the regions defined by \(\rho > a\) and \(0 < \rho < a\), respectively. To find a suitable solution, one starts with the fundamental solution of the Laplace equation in free space:

\[
G_0(\mathbf{r} | \mathbf{r'}) = \frac{1}{4\pi |\mathbf{r} - \mathbf{r'}|},
\]

(6.3)
where \( \mathbf{r} \) corresponds to the observation point at \((\rho, \phi, z)\) and \( \mathbf{r}' \) corresponds to the source point at \((\rho', \phi', z')\). Using the integral (16)

\[
\frac{1}{r} = \frac{2}{\pi} \int_0^\infty K_0(\kappa \rho) \cos(\kappa z) \, d\kappa,
\]

(6.4)

where \( r = \sqrt{\rho^2 + z^2} \) and \( K_0(\kappa \rho) \) is the modified Bessel function of the second kind of order zero, one can rewrite the fundamental solution (6.3) in cylindrical coordinates, which is convenient considering the cylindrical boundary surfaces of the problem. By using the addition theorem (16)

\[
K_0(\kappa \chi) = I_0(\kappa \rho_<)K_0(\kappa \rho_>) + 2 \sum_{t=1}^\infty \cos[t(\phi - \phi')] \int_0^\infty \tilde{G}_t(\rho, \rho', \kappa) \cos[\kappa(z - z')] \, d\kappa,
\]

(6.5)

where \( \chi = \sqrt{\rho^2 + \rho'^2 - 2\rho \rho' \cos(\phi - \phi')} \), \( \rho_< \) is the lesser of \( \rho \) and \( \rho' \), and \( \rho_> \) is the greater, (6.4) is transformed to an arbitrary coordinate system. The potential at an observation point \((\rho, \phi, z)\) due to a source point at \((\rho', \phi', z')\) in free space is expressed as

\[
G_0(\mathbf{r}|\mathbf{r}') = \frac{1}{2\pi^2} \times \left\{ \int_0^\infty \tilde{G}_0(\rho, \rho', \kappa) \cos[\kappa(z - z')] \, d\kappa + 2 \sum_{t=1}^\infty \cos[t(\phi - \phi')] \int_0^\infty \tilde{G}_t(\rho, \rho', \kappa) \cos[\kappa(z - z')] \, d\kappa \right\},
\]

(6.6)

where

\[
\tilde{G}_t(\rho, \rho', \kappa) = I_t(\kappa \rho_<)K_t(\kappa \rho_>) \quad t = 0, 1, 2, \cdots,
\]

(6.7)

and \( I_t(\kappa \rho_<) \) is the modified Bessel function of the first kind of order \( t \). It is noted that the difference between the Green’s function due to a point charge in free space and that due to a point charge exterior to an infinitely long dielectric rod lies in the interface conditions at a surface \( \rho = \text{constant} \). To find the Green’s function due to a point charge outside of a dielectric rod, one needs only to modify the integral kernel \( \tilde{G}_t(\rho, \rho', \kappa) \) in (6.6) so that the interface conditions in (6.2) are satisfied:

\[
\tilde{G}_t^{(1)}(\rho, \rho', \kappa) = \tilde{G}_t^{(2)}(\rho, \rho', \kappa),
\]

(6.8)

and

\[
\epsilon_1 \frac{\partial}{\partial \rho} \tilde{G}_t^{(1)}(\rho, \rho', \kappa) = \epsilon_2 \frac{\partial}{\partial \rho} \tilde{G}_t^{(2)}(\rho, \rho', \kappa).
\]

(6.9)
To create a kernel that satisfies (6.8) and (6.9), define
\[
\tilde{G}_t^{(1)}(\rho, \rho', \kappa) = I_t(\kappa \rho) K_t(\kappa \rho') + A(\kappa) K_t(\kappa \rho) K_t(\kappa \rho'),
\]
(6.10)
and
\[
\tilde{G}_t^{(2)}(\rho, \rho', \kappa) = B(\kappa) I_t(\kappa \rho) K_t(\kappa \rho').
\]
(6.11)
The first term in (6.10) represents the primary field due to the point source, while the second term represents the reflected field with a reflection coefficient \(A(\kappa)\). In the region \(0 < \rho < a\) there exists only the transmitted field and \(B(\kappa)\) in (6.11) is the transmission coefficient. Substitute (6.10) and (6.11) into (6.8) and (6.9) to find that the coefficient \(A(\kappa)\) is expressed as
\[
A(\kappa) = -\frac{(\epsilon_2 - \epsilon_1) I_t(\kappa a) I_t'(\kappa a)}{\epsilon_2 I_t'(\kappa a) K_t(\kappa a) - \epsilon_1 I_t(\kappa a) K_t'(\kappa a)},
\]
(6.12)
where \(I_t'(z_0) = dI_t(z)/dz|_{z=z_0}\) and similarly for \(K_t'(z_0)\). Now the potential \(G^{(1)}(\mathbf{r}|\mathbf{r}')\) at an observation point exterior to the dielectric cylinder due to a point charge also outside of the cylinder is expressed as
\[
G^{(1)}(\mathbf{r}|\mathbf{r}') = \frac{1}{2\pi^2} \times \left\{ \int_0^\infty \tilde{G}_t^{(1)}(\rho, \rho', \kappa) \cos[\kappa(\mathbf{z} - \mathbf{z}')] \, d\kappa \right. \\
+ \left. 2 \sum_{t=1}^{\infty} \cos[t(\phi - \phi')] \int_0^\infty \tilde{G}_t^{(1)}(\rho, \rho', \kappa) \cos[\kappa(\mathbf{z} - \mathbf{z}')] \, d\kappa \right\}.
\]
(6.13)
The derived Green’s function, dependent on the permittivity and radius of the dielectric rod under test, is used later to calculate the capacitance of the arc-electrode sensor.

6.3.3 Note on the Choice of the Bessel Function Kernel

Instead of using the identity in (6.4), one can also express \(1/r\) in terms of the of Bessel function of the first kind of order zero \(J_0(\kappa \rho)\) (17):
\[
\frac{1}{r} = \int_0^\infty J_0(\kappa \rho) e^{-\kappa z} \, d\kappa,
\]
(6.14)
and express the Green’s function \(G^{(1)}_J(\mathbf{r}|\mathbf{r}')\) in the form of Bessel functions of the first and the second kind
\[
G^{(1)}_J(\mathbf{r}|\mathbf{r}') = \frac{1}{4\pi} \times \left\{ \int_0^\infty \tilde{K}_0^{(1)}(\rho, \rho', \kappa) e^{-\kappa|\mathbf{z} - \mathbf{z}'|} \, d\kappa \right. \\
+ \left. 2 \sum_{t=1}^{\infty} \cos[t(\phi - \phi')] \int_0^\infty \tilde{K}_t^{(1)}(\rho, \rho', \kappa) e^{-\kappa|\mathbf{z} - \mathbf{z}'|} \, d\kappa \right\},
\]
(6.15)
where
\[
\tilde{K}^{(1)}_t(\rho, \rho', \kappa) = J_t(\kappa \rho) J_t(\kappa \rho') + A_J(\kappa) Y_t(\kappa \rho) J_t(\kappa \rho') \quad t = 0, 1, \ldots, \quad (6.16)
\]
\[
A_J(\kappa) = - \frac{(\epsilon_2 - \epsilon_1) J_t(\kappa a) J'_t(\kappa a)}{\epsilon_2 J'_t(\kappa a) Y_t(\kappa a) - \epsilon_1 J_t(\kappa a) Y'_t(\kappa a)}, \quad (6.17)
\]
\[
J'_t(\kappa a) = \frac{d J_t(\kappa \rho)}{d(\kappa \rho)}|_{\rho \to a} \quad \text{and} \quad Y'_t(\kappa a) = \frac{d Y_t(\kappa \rho)}{d(\kappa \rho)}|_{\rho \to a}.
\]
However, the denominator in (6.17) contains an infinite number of zeros for \( \kappa \) from 0 to \( \infty \), and increases the complexity in MoM numerical implementations. Therefore, the Green’s function in the form of modified Bessel functions, (6.13), is a better choice here for calculating the sensor output capacitance.

6.4 Numerical Implementation

The capacitance \( C \) between the two arc-electrodes is calculated numerically as follows. The Green’s function derived above is used to set up the integral equation in MoM calculations, which leads to the solution for the surface charge density on each electrode. The two electrodes are oppositely charged in the numerical calculations. Because of the axisymmetry of the problem, it is only necessary to calculate the surface charge density on one of the electrodes. The output capacitance \( C \) is then calculated from
\[
C = \frac{Q}{V}, \quad (6.18)
\]
where the total charge \( Q \) on each electrode is obtained by integrating the surface charge density over the electrode surface and \( V \) is the potential difference between the electrodes.

6.4.1 Calculation Method

Figure 6.3 shows the discretization of the arc-electrode surfaces into \( M \times N \) elements of assumed constant surface charge density. Each electrode is discretized into \( M \) elements in the \( \phi \) direction and \( N \) elements in the \( z \) direction. Denote the surface charge density on the left electrode as \( \sigma_s(\phi', z') \) and that on the right electrode as \( \sigma_s(\phi' + \pi, z') \). The potential at the observation point \( r = (\rho_0, \phi, z) \) on the electrode surface due to the charged arc-electrodes can be expressed by integrating (6.13) over the electrode surfaces:
Figure 6.3 Discretization of the arc-electrode surfaces into $M \times N$ elements of assumed constant surface charge density.

$$
\Psi(\phi, z) = \frac{1}{\epsilon_0} \int \int_{\text{Left electrode}} G^{(1)}(r|r') \sigma_s(\phi', z') \rho_0 \, d\phi' \, dz' - \frac{1}{\epsilon_0} \int \int_{\text{Right electrode}} G^{(1)}(r|r') \sigma_s(\phi' + \pi, z') \rho_0 \, d\phi' \, dz'.
$$

(6.19)

In the MoM calculations, the following expansion is used to approximate the continuous function $\sigma_s(\phi', z')$:

$$
\sigma_s(\phi', z') = \sum_{j=1}^{MN} \sigma_j b_j(\phi', z'),
$$

(6.20)

where $\sigma_j$ is the unknown constant surface charge density on element $j$ and $b_j(\phi', z')$ is the pulse basis function

$$
b_j(\phi', z') = \begin{cases} 
1 & \text{on element } j \\
0 & \text{elsewhere.}
\end{cases}
$$

(6.21)

To solve for the $MN$ unknown coefficients $\sigma_j$, weighting (or testing) functions $w_i(\phi, z)$ are introduced to force that the boundary condition for the potential in (6.19) is satisfied for each element on the sensor surface. The point-matching method is used, in which the weighting functions are Dirac delta functions:

$$
w_i(\phi, z) = \delta(\phi - \phi_i) \delta(z - z_i) \quad \text{on element } i,
$$

(6.22)
where \( i = 1, 2, ..., MN \). Discretizing the integral equation using weighting functions in each of the \( MN \) elements, (6.19) is expressed as the following matrix equation:

\[
\begin{pmatrix}
G_{11} & G_{12} & \cdots & G_{1L} \\
G_{21} & G_{22} & \cdots & G_{2L} \\
\vdots & \vdots & \ddots & \vdots \\
G_{L1} & G_{L2} & \cdots & G_{LL}
\end{pmatrix}
\begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\vdots \\
\sigma_L
\end{pmatrix}
= \mathbf{V},
\]

(6.23)

where \( L = MN \) and

\[
G_{ij} = \frac{1}{\varepsilon_0} \int \int_{\text{element } j} G^{(1)}(r_i|r_j') b_j(\phi', z') \rho_0 d\phi' dz'.
\]

(6.24)

All the elements in \( \mathbf{V} \) share the same potential \( v \) that is the potential applied to one of the electrodes. The other electrode has potential \(-v\). From (6.24) the surface charge density \( \sigma_s(\phi', z') \) on one of the electrodes is solved, and that for the other electrode is simply \(-\sigma_s(\phi' + \pi, z')\). The total charge \( Q \) on each electrode can be found by integrating \( \sigma_s(\phi', z') \) over the electrode surface. The sensor output capacitance \( C \) is ultimately calculated through (6.18).

### 6.4.2 Example Calculations

When numerically calculating the matrix element given in (6.24), the zero to infinity summation and integral in \( G^{(1)}(r|r') \) (see (6.13)) need to be truncated. The convergence of the Green’s function depends on values of \( \varepsilon_2/\varepsilon_1 \), \( a/\rho \) and \( \phi_0 \). When these values are large, large truncation ranges for the summation and integral in (6.13) are needed. It is found that, for the case \( \varepsilon_2/\varepsilon_1 = 5 \), \( a/\rho = 1 \), \( \phi_0 = 177^\circ \) and \( l = 4 \) cm, if one truncates the summation in \( G^{(1)}(r|r') \) with 40 terms and the integral with the range from 0 to 2000 for the off-diagonal components in (6.24), and the summation with 300 terms and the integral with the range from 0 to 2000 for the diagonal components, accuracy to three significant figures can be achieved in the final calculated sensor output capacitance \( C \). The cases calculated in Figures 6.4 to 6.6 and 6.8 have smaller \( \varepsilon_2/\varepsilon_1 \), \( a/\rho \) and \( \phi_0 \) values than those in the case calculated above. The truncation standard used here is adopted in all numerical calculations of sensor capacitance value in this paper. It guarantees achieving convergence with accuracy to the third significant digit in all the cases discussed below.
The dependence of sensor output capacitance on the electrode configuration is investigated as follows. In Figure 6.4, sensor output capacitance $C$ is plotted as a function of the electrode length $l$ and the arc-angle $\phi_0$. In this example calculation, the infinitely long dielectric rod is assumed to be in free space, with relative permittivity $\epsilon_r = 2.5$ and radius $a = 9.525$ mm (chosen to be similar to the radii of the rods used for experiments described in Section 6.5.)

The arc-electrodes share the same radius as the cylindrical rod. It is seen from Figure 6.4 that for any fixed electrode arc-angle $\phi_0$, there exists a linear relationship between the sensor capacitance $C$ and the electrode length $l$. On the other hand, for any given electrode length $l$, the sensor output capacitance $C$ increases as the electrode arc-angle $\phi_0$ increases, and tends to infinity as $\phi_0$ tends to $180^\circ$. This is explained by the fact that the output capacitance $C$ results from interaction between the sensor electrodes. The charge density on the electrodes is highest at the electrode edges, and increases as the electrode edges come closer together. As $\phi_0$ tends to $180^\circ$, the gaps between the edges of the two electrodes become infinitesimally small and therefore the resulting capacitance tends to infinity, in accordance with the singular behavior of the charge density at the electrode edges. Figure 6.4 shows that in order to achieve maximum sensor output signal, the ideal sensor electrodes would be as long as practically possible and with large arc-angle $\phi_0$.

Figure 6.5 shows an example of the sensor output capacitance $C$ as a function of the ratio $a/\rho_0$ (see Figure 6.1). Rod parameters are as for Figure 6.4. The arc-electrodes each have fixed radius $\rho_0 = 9.525$ mm, arc-angle $\phi_0 = 174.44^\circ$, and length $l = 4$ cm. In other words, Figure 6.5 shows the dependence of sensor capacitance on the cylindrical test-piece diameter, for a fixed arc-electrode sensor configuration. It is seen from Figure 6.5 that as the ratio $a/\rho_0$ increases, sensor output capacitance increases dramatically, especially when this ratio tends to 1. This is because as $a/\rho_0$ increases, the average permittivity interior to the arc-electrodes increases and therefore $C$ increases. On the other hand, the sensor’s most sensitive area lies in the region close to the gaps between the two electrodes. As $a/\rho_0$ tends to 1, the arc-electrodes are more likely to detect increases in the average permittivity surrounding the sensor. This is why the sensor output capacitance $C$ changes more rapidly as the ratio $a/\rho_0$ approaches 1. The theoretical calculation in Figure 6.5 demonstrates that, during measurements, unidentified small air gaps
Figure 6.4 Calculated sensor output capacitance as a function of electrode length $l$ and arc-angle $\phi_0$. The dielectric rod is in free space, with a relative permittivity of 2.5 and a radius of 9.525 mm.

existing between the arc electrodes and the dielectric rod under test can introduce relatively large uncertainty in the measured $C$, especially as $a \to \rho_0$. Therefore, in order to achieve the strongest sensor output signal and the smallest uncertainty due to possible air gaps between the electrodes and test-piece, it is desirable to have the arc-electrodes in tight surface contact with the test-piece.

The sensor output capacitance $C$ as a function of dielectric rod relative permittivity $\epsilon_{r,2}$ is plotted in Figure 6.6, in which different sensor configurations are considered. A linear relationship between the sensor output capacitance and the test-piece permittivity is observed and has been verified numerically, by computation of a sufficient number of data points (seven in this case). It is seen that the slope of sensor output capacitance versus rod permittivity depends on both the sensor configuration and the ratio $a/\rho_0$. For a given $a/\rho_0$, the value of the slope increases as the electrode length $l$ and arc-angle $\phi_0$ increase. This is because the value of the slope represents changes in the absolute values of the capacitance for any rod permittivity increment. These absolute value changes in capacitance are most obvious for sensors with large electrode length $l$ and arc-angle $\phi_0$ values. This also explains why the value of the slope, for fixed $l$ and $\phi_0$ values, increases as $a/\rho_0$ increases. However, it is worth pointing out that for
fixed electrode radius $\rho$, arc-angle $\phi_0$, and $a/\rho_0$ values, although increasing electrode length $l$ increases the value of the slope, relative changes in capacitance as $\epsilon_{r2}$ changes stay the same, because of the linear relationship between the sensor output capacitance $C$ and electrode length $l$ (see Figure 6.4).

6.5 Experimental Verification

Capacitance experiments were performed to verify the validity of the developed theory. Two sets of rectangular planar electrodes (shown in Figure 6.7) were fabricated using photolithography by American Standard Circuits Inc.. The sensor shape was achieved by selectively etching a 18 $\mu$m thick copper cladding (14 mL standard) off a flat 25.4 $\mu$m thick Kapton® type 100 CR polyimide film. These flexible electrodes were fixed onto different cylindrical dielectric testpieces later to form the arc-electrode capacitance sensors. The sensor dimensions are $w = 29$ mm and $l = 20$ mm for one set and $w = 29$ mm and $l = 40$ mm for the other (see Figure 6.1). A Nikon EPIPHOT 200 microscope was used to independently measure the fabricated sensor dimensions, for the purpose of checking the difference between the fabricated dimensions and the specified ones, and therefore the accuracy of the fabrication process. The “traveling mi-
Figure 6.6 Calculated sensor output capacitance as a function of dielectric rod relative permittivity. $a/\rho_0 = 1$, $l = 4$ cm and $\phi_0 = 174.44^\circ$ except where indicated. All the sensor electrodes have fixed radius $\rho_0 = 9.525$ mm.

croscope” measurement method, with accuracy of 0.01 mm, was used to measure the relatively large sensor electrode dimensions. It was found that the measured dimensions of the fabricated electrodes are identical with the nominal values under such measurement accuracy.

Three 304.8-mm-long dielectric rods are used in the measurements to simulate the infinitely long cylindrical dielectric rod. The dielectric rods are long compared with the electrode lengths (factors of approximately 8 and 15 longer), and the edge effect due to finite rod length can be neglected if the sensor electrodes are placed at the center of the rods. The rod materials are Acetal Copolymer (Tecaform™), Cast Acrylic, and Virgin Electrical Grade Teflon® PTFE. A digital caliper, with accuracy of ±0.01 mm, was used to independently measure the diameter of each rod. The permittivity of each rod was independently determined by cutting a slice from the end of each rod, and then measuring the permittivity of each slice using a Novocontrol Alpha Dielectric Spectrometer at 1 MHz. In the Novocontrol measurements, both sides of each slice were brushed with silver paint to form the measuring electrodes.

The rectangular planar electrodes were attached to each dielectric rod by taping the thin Kapton® sensor substrate tightly against the rod material, as shown in Figure 6.7. The electrodes were aligned carefully so that the upper and lower edges of the two electrodes were at
the same height, the vertical edges of both electrodes were in parallel, and the two vertical gaps between the two electrodes were of the same size, as assumed in the theoretical model. Another layer of 25.4-µm-thick Kapton® film was wrapped tightly onto the outsides of the electrodes in order to minimize the air gap between the electrodes and the dielectric rod. Because the Kapton® films used were quite thin, influences from their permittivity on the measurement signal were negligible.

For each dielectric test-piece used in the benchmark experiments, the test-piece material, test-piece diameter, independently measured test-piece relative permittivity, electrode radius $\rho_0$, and electrode arc-angle $\phi_0$ are listed in Table 6.1. The electrode radius $\rho_0$ for each rod is obtained by summing the dielectric rod radius and the Kapton® substrate thickness. The electrode arc-angle $\phi_0$ in Table 6.1 is different for each test-piece because of the fact that the diameter of each rod is different while the electrode width $w$ is the same. The parameters shown in the table were used as the inputs in the numerical calculations.

An Agilent E4980A precision LCR meter was used to measure the sensor output capacitance. The LCR meter operating frequency was set as 1 MHz, so that the measurement error from the LCR meter was less than 0.3% for a 1 pF capacitance while at the same time being
Table 6.1 Parameters of the dielectric test-pieces and the arc-electrode sensors used in bench-mark experiments. The areas of the two sets of sensor electrodes are $29 \times 20 \text{ mm}^2$ and $29 \times 40 \text{ mm}^2$, respectively.

<table>
<thead>
<tr>
<th>Test-piece material</th>
<th>Test-piece diameter (mm)</th>
<th>Measured test-piece permittivity</th>
<th>Electrode radius $\rho_0$ (mm)</th>
<th>Electrode arc-angle $\phi_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tecaform\textsuperscript{TM}</td>
<td>19.08</td>
<td>3.77</td>
<td>9.565</td>
<td>173.71°</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.01$</td>
<td>$\pm 0.05$</td>
<td>$\pm 0.005$</td>
<td>$\pm 0.10^\circ$</td>
</tr>
<tr>
<td>Cast Acrylic</td>
<td>19.03</td>
<td>2.88</td>
<td>9.540</td>
<td>174.17°</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.01$</td>
<td>$\pm 0.05$</td>
<td>$\pm 0.005$</td>
<td>$\pm 0.09^\circ$</td>
</tr>
<tr>
<td>Teflon\textsuperscript{®}</td>
<td>19.10</td>
<td>2.23</td>
<td>9.575</td>
<td>173.53°</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.01$</td>
<td>$\pm 0.05$</td>
<td>$\pm 0.005$</td>
<td>$\pm 0.09^\circ$</td>
</tr>
</tbody>
</table>

A good approximation for the electrostatic assumption in the numerical model (results of the calculation do not depend on frequency). A static model can be applied for this configuration even at $f = 1 \text{ MHz}$ because, at this frequency, the corresponding wavelength $\lambda = 300 \text{ m}$. The diagonal dimensions of the electrodes in the measurement are smaller than 5 cm, which means the maximum phase change over the electrode surfaces is less than 0.06° and the effect of scattering is therefore negligible. A 0.06° phase change is not detectable here, being below the measurement sensitivity of the Novocontrol dielectric spectrometer and the LCR meter. If a lower operating frequency is desired for practical capacitance measurements, an impedance measurement instrument with higher accuracy when measuring large impedance values should be used. (According to the relationship $Z = 1/j2\pi f C$, the impedance $Z$ resulting from measuring a given capacitance $C$ under a lower frequency $f$ will be larger).

Sensor output capacitance $C$ was measured by placing the probe of an Agilent probe test fixture 16095A across the two sensor electrodes, as shown in Figure 6.7. The parts on the electrodes where the probe is in surface contact were not covered by Kapton\textsuperscript{®} films. This probe test fixture was connected to the LCR meter and the measured capacitance was read from the LCR meter screen. Figure 6.8 shows the comparison between the calculated and measured sensor output capacitance for each test-piece material and the two different electrode configurations. Experimental data show excellent agreement with numerical results (to within 3%), and the maximum absolute difference in capacitance is less than 0.1 pF. It is worth...
Figure 6.8 Measured and calculated $C$ for various sensor configurations (see Table 6.1) in contact with different dielectric test-pieces. Measurement results and error bars are denoted by the black symbol.

pointing out that even if Kapton® films are tightly wrapped around the electrodes to attempt to eliminate the air gaps between electrodes and the test-piece, small gaps still exist. In particular, the vertical edges of the electrodes tend to bend up, giving rise to small air gaps, where the sensor is most sensitive. This points to the fact that the ideal way to achieve best agreement between theory and measurements is to deposit the arc-electrodes directly onto the cylindrical test-pieces. Thus errors coming from the misalignment of sensor electrodes and the existence of air gaps will be eliminated. On the other hand, deposition of electrodes directly onto the test-piece is costly, time-consuming and undesirable for most practical purposes.

One purpose for these arc-electrode sensors is the inverse determination of the permittivity of materials under test from measured sensor capacitance. Comparisons are made in Table 6.2 to assess the capability and accuracy of the arc-electrode sensors in material dielectric property characterization. The sample permittivities inferred from measured capacitance values shown in Figure 6.8 are compared with those measured by a Novocontrol dielectric spectrometer, in the manner discussed in the second paragraph in Section 6.5. Again, excellent agreement between inferred and independently measured test-piece permittivities is obtained. Table 6.2 demonstrates the great potential of using the arc-electrode capacitive sensors for accurate and
Table 6.2 Comparison of test-piece permittivity values between independently measured ones and inversely determined ones from measured capacitance using the arc-electrode sensors.

<table>
<thead>
<tr>
<th>Test-piece material</th>
<th>Independently measured permittivity</th>
<th>Electrode length (mm)</th>
<th>Inversely determined permittivity</th>
<th>Relative difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tecaform™</td>
<td>3.77 ± 0.05</td>
<td>20</td>
<td>3.76 ± 0.07</td>
<td>0.3</td>
</tr>
<tr>
<td>Cast Acrylic</td>
<td>2.88 ± 0.05</td>
<td>20</td>
<td>2.88 ± 0.01</td>
<td>0.0</td>
</tr>
<tr>
<td>Teflon®</td>
<td>2.23 ± 0.05</td>
<td>20</td>
<td>2.16 ± 0.11</td>
<td>3.1</td>
</tr>
</tbody>
</table>

6.6 Conclusion

A numerical model, based on the electrostatic Green’s function due to a point source exterior to an infinitely long cylindrical dielectric, has been developed to quantitatively evaluate the dielectric property of cylindrical dielectric test-pieces. The quantitative dependence of the sensor output capacitance on the test-piece permittivity and radius has been demonstrated numerically and verified experimentally. The permittivity of various cylindrical test-pieces has been inferred from measured capacitance to within 1% accuracy, on average. The numerical model developed here will be extended in future to deal with other cylindrical problems such as the nondestructive evaluation of wire insulation and dielectric tubes. Practical clip-type sensors based on these theoretical models will also be developed.

6.7 Acknowledgments

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6.8 References


7.1 Abstract

This letter presents an analytical expression for the capacitance of a curved patch capacitor that conforms to the curvature of an infinitely-long, homogeneous, cylindrical dielectric rod. The capacitor is composed of two symmetric and infinitely-long curved electrodes. The resulting capacitance per unit length depends on both the dielectric properties of the material under test and the capacitor configuration. A practical capacitance measurement system based on the theory has also been described. The validity of the theory has been verified by very good agreement between measured and theoretically-predicted capacitances (to within 4%). The analytical solution in this letter has the potential be applied to many scientific and engineering fields.

7.2 Body of the Letter

Capacitive methods electronically measure the capacitance between two or more conductors, and have been applied to solve many different types of sensing and measurement problems (1). Capacitive touchscreen based on pressure sensing (2) is probably the application most related to people’s life. In addition, capacitive techniques find applications in areas such as micrometer
Figure 7.1 Configuration of the curved patch capacitor. The symmetric electrodes are defined in the φ direction as: \( \phi_0 \leq \phi \leq \phi_1 \) (top) and \( -\phi_1 \leq \phi \leq -\phi_0 \) (bottom).

development (3), proximity and position sensing (4; 5), displacement measurement (6; 7), and materials characterization (8; 9; 10; 11).

Previously, we have demonstrated the feasibility of utilizing finited-sized curved patch capacitors for materials characterization of cylindrical structures (12; 13). The sensor modeling was achieved numerically using the method of moments. In this letter, a two-dimensional analytical solution is provided for the rapid and accurate calculation of the capacitance of curved patch capacitors that conform to the curvature of cylindrical homogeneous dielectric rods. Theoretical derivations in this letter are extended from discussions on of the capacitance between axially slotted open circular cylinders in free space (14).

Figure 7.1 shows the configuration of the problem. The capacitor consists of two infinitely long curved patches that are symmetric with respect to the x axis. The radius of the electrodes is equal to 1. The upper electrode is defined by \( \phi_0 \leq \phi \leq \phi_1 \), and the lower one is defined by \( -\phi_1 \leq \phi \leq -\phi_0 \). These two electrodes are charged to 1 V and -1 V, respectively. The material under test is an infinitely long dielectric rod having the same radius as the curved electrodes. The dielectric constant of the dielectric test-piece is assumed to be \( \epsilon_2 \) and the background material \( \epsilon_1 \).

Considering the symmetry of the problem, one only has to solve for the potential in the upper half plane in Figure 7.1 to obtain the capacitance. The electric potential \( \Psi \) resulting
from the charged capacitor satisfies 2D Laplace equation:

$$\left( \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} \right) \Psi(\rho, \phi) = 0, \quad (7.1)$$

and can be expressed as

$$\Psi_i(\rho, \phi) = \sum_{n=1}^{\infty} a_n \rho^{(-1)^i n} \sin n\phi, \quad i = 1, 2, \quad (7.2)$$

where the sub- and superscripts 1 and 2 correspond to the regions defined by $\rho > 1$ and $\rho \leq 1$, respectively. The interface conditions for the potentials at $\rho = 1$ in the upper half plane are

$$\Psi_1(\rho, \phi) = \Psi_2(\rho, \phi) = 1, \quad \phi \in (\phi_0, \phi_1), \quad (7.3)$$

$$\epsilon_1 \frac{\partial \Psi_1(\rho, \phi)}{\partial \rho} = \epsilon_2 \frac{\partial \Psi_2(\rho, \phi)}{\partial \rho}, \quad \phi \in (0, \phi_0) \cup (\phi_1, \pi). \quad (7.4)$$

Inserting equation (7.2) into (7.3) and (7.4), we find the trigonometric series equations to determine the coefficients $a_n$

$$\sum_{n=1}^{\infty} n a_n \sin n\phi = 0, \quad \phi \in (0, \phi_0) \cup (\phi_1, \pi), \quad (7.5)$$

$$\sum_{n=1}^{\infty} a_n \sin n\phi = 1, \quad \phi \in (\phi_0, \phi_1). \quad (7.6)$$

These nonsymmetrical triple series equations can be transformed into symmetrical triple series equations and considered in terms of dual series equations (14)

$$\sum_{n=0}^{\infty} (n + \frac{1}{2}) b_{2n+1} \sin(n + \frac{1}{2}) \theta = 0 \quad \theta \in (0, \theta_0), \quad (7.7)$$

and

$$\sum_{n=0}^{\infty} b_{2n+1} \sin(n + \frac{1}{2}) \theta = E_0 \quad \theta \in (\theta_0, \pi), \quad (7.8)$$

where

$$E_0 = \left( \tan \frac{1}{2} \phi_0 \tan \frac{1}{2} \phi_1 \right)^{\frac{1}{2}}, \quad (7.9)$$

$$\theta = 4 \arctan \left[ \tan \frac{1}{2} \phi \left( \tan \frac{1}{2} \phi_0 \tan \frac{1}{2} \phi_1 \right)^{-\frac{1}{2}} \right], \quad (7.10)$$

$$\theta_0 = 4 \arctan \left[ \left( \tan \frac{1}{2} \phi_0 \cot \frac{1}{2} \phi_1 \right)^{\frac{1}{2}} \right]. \quad (7.11)$$
The transformation relationship between \( a_n \) and \( b_n \) is described in (14). \( b_{2n+1} \) in equations (7.7) and (7.8), however, cannot be easily solved through orthogonality relationship for the sine functions, because of the \((n + 1/2)\) term in front of \( b_{2n+1} \) in (7.7). The scheme adopted in this letter is to transform the dual series equations (7.7) and (7.8) into equations containing the product of \( b_{2n+1} \) and Legendre functions, with intermediate steps in terms of the product of \( b_{2n+1} \) and Jacobi polynomials, and solve for \( b_{2n+1} \) utilizing the orthogonality relationship for Legendre functions. First, equations (7.7) and (7.8) are rewritten in terms of Jacobi polynomials as

\[
\sum_{n=0}^{\infty} \frac{(n + \frac{1}{2})\Gamma(n + 1)}{\Gamma(n + \frac{1}{2})} b_{2n+1} P_{n}^{\left(\frac{1}{2},-\frac{1}{2}\right)}(z) = 0 \quad z \in (z_0, 1) \tag{7.12}
\]

and

\[
\sqrt{\frac{\pi}{2}} \sum_{n=0}^{\infty} \frac{\Gamma(n + 1)}{\Gamma(n + \frac{1}{2})} (1 - z)^{\frac{1}{2}} P_{n}^{\left(\frac{1}{2},-\frac{1}{2}\right)}(z) = E_0 \quad z \in (-1, z_0), \tag{7.13}
\]

where \( z = \cos \theta \) and \( z_0 = \cos \theta_0 \), by replacing the sine function with Jacobi polynomials using the identity

\[
\sin(n + \frac{1}{2})\theta = \frac{\Gamma(\frac{1}{2})\Gamma(n + 1)}{\Gamma(n + \frac{1}{2})} \sin \frac{\theta}{2} P_{n}^{\left(\frac{1}{2},-\frac{1}{2}\right)}(\cos \theta). \tag{7.14}
\]

To transform equation (7.13) into an equation containing the product of \( b_{2n+1} \) and Legendre functions, multiply its both sides by \((1 + z)^{-\frac{1}{2}}(1 - z)^{-\frac{1}{2}}\), integrate from \(-1\) to \(z\), and make use of the following identity

\[
(1 + t)^{\frac{1}{2}} P_{n}^{\left(-\frac{1}{2},\frac{1}{2}\right)}(t) = (n + \frac{1}{2}) \int_{-1}^{t} (1 + x)^{-\frac{1}{2}} P_{n}^{\left(\frac{1}{2},-\frac{1}{2}\right)}(x) \, dx \tag{7.15}
\]

to obtain

\[
\sum_{n=0}^{\infty} b_{2n+1} \frac{\Gamma(n + 1)}{\Gamma(n + \frac{1}{2})} \frac{(1 + z)^{\frac{1}{2}}}{n + \frac{1}{2}} P_{n}^{\left(-\frac{1}{2},\frac{1}{2}\right)}(z) = E_0 \sqrt{\frac{\pi}{2}} \left( \frac{\pi}{2} + \arcsin z \right) \quad z \in (-1, z_0). \tag{7.16}
\]

Equation (7.16) can be further written in the form of Abel’s integral equation (14) as

\[
\int_{-1}^{z} \frac{\sum_{n=0}^{\infty} b_{2n+1} P_{n}(x)}{(z - x)^{\frac{1}{2}}} \, dx = \sqrt{2} E_0 \left( \frac{\pi}{2} + \arcsin z \right) \quad z \in (-1, z_0), \tag{7.17}
\]

by applying the identity

\[
P_{n}^{\left(-\frac{1}{2},\frac{1}{2}\right)}(z) = \frac{(1 + z)^{-\frac{1}{2}} \Gamma(n + \frac{3}{2})}{\Gamma\left(\frac{1}{2}\right) \Gamma(n + 1)} \int_{-1}^{z} \frac{P_{n}(x)}{(z - x)^{\frac{1}{2}}} \, dx. \tag{7.18}
\]
where $P_n(z)$ is Legendre polynomials of order $n$. After making use of the inversion formula for Abel’s integral equation for equation (7.17), the following relationship is obtained
\[
\sum_{n=0}^{\infty} b_{2n+1} P_n(x) = \frac{2E_0}{\pi} K \left( \sqrt{\frac{1+x}{2}} \right) \quad x \in (-1, z_0),
\] (7.19)
where $K(z)$ is the complete elliptic integral of the first kind. A similar equation containing the product of $b_{2n+1}$ and $P_n(x)$ may be obtained for $x \in (z_0, 1)$ based on equation (7.12). Now, multiply both sides of equation (7.12) by $(1+z)^{-\frac{1}{2}}$, integrate from $-1$ to $z$, and make use of the identity
\[
(1+t)^{\frac{1}{2}} P_n\left(\frac{-1}{2}, \frac{1}{2}\right)(t) = \left(n + \frac{1}{2}\right)^{-\frac{1}{2}} \int_{-1}^{t} (1+x)^{-\frac{1}{2}} P_n\left(\frac{1}{2}, \frac{-1}{2}\right)(x) \, dx
\] (7.20)
to express equation (7.12) as
\[
\sum_{n=0}^{\infty} \frac{\Gamma(n+1)}{\Gamma(n+\frac{3}{2})} b_{2n+1} P_n\left(\frac{-1}{2}, \frac{-1}{2}\right)(z) = F_0 (1+z)^{-\frac{1}{2}} \quad z \in (z_0, 1),
\] (7.21)
where $F_0$ is a constant will be determined later. Next, multiply both sides of equation (7.21) by $(1-z)^{-\frac{1}{2}}$, integrate from $z$ to $1$, and make use of the identity
\[
(1-t)^{\frac{1}{2}} P_n\left(\frac{1}{2}, \frac{-1}{2}\right)(t) = \left(n + \frac{1}{2}\right)^{-\frac{1}{2}} \int_{t}^{1} (1-x)^{-\frac{1}{2}} P_n\left(\frac{1}{2}, \frac{-1}{2}\right)(x) \, dx
\] (7.22)
to obtain
\[
\sum_{n=0}^{\infty} \frac{\Gamma(n+1)}{\Gamma(n+\frac{3}{2})} (1-z)^{\frac{1}{2}} b_{2n+1} P_n\left(\frac{1}{2}, \frac{-1}{2}\right)(z) = F_0 \arccos z \quad z \in (z_0, 1).
\] (7.23)
Similarly, equation (7.23) can be expressed in the form of Abel’s integral equation as
\[
\int_{z}^{1} \frac{\sum_{n=0}^{\infty} b_{2n+1} P_n(x)}{\sqrt{x-z}} \, dx = \sqrt{\pi} F_0 \arccos z \quad z \in (z_0, 1)
\] (7.24)
by making use of the identity
\[
P_n\left(\frac{1}{2}, \frac{-1}{2}\right)(z) = \frac{(1-z)^{-\frac{1}{2}} \Gamma(n + \frac{3}{2})}{\Gamma(\frac{1}{2}) \Gamma(n+1)} \int_{z}^{1} \frac{P_n(x)}{(x-z)^{\frac{1}{2}}} \, dx.
\] (7.25)
The following relationship
\[
\sum_{n=0}^{\infty} b_{2n+1} P_n(x) = \sqrt{\frac{\pi}{7}} F_0 K \left( \sqrt{\frac{1-x}{2}} \right) \quad x \in (z_0, 1)
\] (7.26)
is obtain by applying the inversion formula of Abel’s integral equation to equation (7.24). Equations (7.19) and (7.26) have to be continuous at $x = z_0$ because of the continuity condition
for the electric potential. Therefore, it is found that

$$F_0 = \sqrt{\frac{2}{\pi}} \left( \tan \frac{\phi_0}{2} \tan \frac{\phi_1}{2} \right)^{1/2} K \left( \sqrt{\frac{1+z_0}{2}} \right) / K \left( \sqrt{\frac{1-z_0}{2}} \right).$$ (7.27)

The coefficients $b_{2n+1}$ are determined by applying the orthogonality relationship for Legendre polynomials to equations (7.19) and (7.26). The result is

$$b_{2n+1} = \left( \tan \frac{\phi_0}{2} \tan \frac{\phi_1}{2} \right)^{1/2} \left\{ (n+1/2) K \left( \sqrt{\frac{1-z_0}{2}} \right) \right\}^{-1} P_n(z_0).$$ (7.28)

After knowing the coefficients $b_{2n+1}$, the electric potential in space is obtained.

In order to solve for the capacitance, the surface charge density $\sigma_s(\phi)$ on the electrodes is derived. Applying the interface condition for the tangential components of electric flux density $D$ at $\rho = 1$, which can be derived from equation (7.2), gives rise to

$$\sigma_s(\phi) = (\epsilon_1 + \epsilon_2) \sum_{n=1}^{\infty} n a_n \sin n\phi.$$ (7.29)

Since an explicit expression is given for $b_n$ instead of $a_n$, equation (7.29) is expressed in terms of $b_n$ as

$$\sigma_s(\theta) = (\epsilon_1 + \epsilon_2) \left( \frac{\cos^2 \theta}{4} + \tan \frac{\phi_0}{2} \tan \frac{\phi_1}{2} \sin^2 \theta \right) \right) \left( \tan \frac{\phi_0}{2} \tan \frac{\phi_1}{2} \right)^{-1} \sum_{n=1}^{\infty} nb_n \sin \frac{n\theta}{2}$$ (7.30)

based on the transformation relationship (14) between $a_n$ and $b_n$. Next, insert equation (7.28) into (7.30) and express $\theta$ in terms of $\phi$ to express the electrode surface charge density in terms of the original capacitor configuration. After some straightforward manipulation, it is found that

$$\sigma_s(\phi) = \frac{\epsilon_1 + \epsilon_2}{K(\sqrt{1-t^2})} \frac{\sin \frac{1}{2}(\phi_0 + \phi_1)}{(\cos \phi_0 - \cos \phi)(\cos \phi - \cos \phi_1)},$$ (7.31)

where $t = \sin \frac{\phi_1-\phi_0}{2} / \sin \frac{\phi_1+\phi_0}{2}$. The surface charge distribution on the lower electrode is equal and opposite in sense as in equation (7.31).

The capacitance per unit length $C$ is calculated using the formula $C = Q/V$, where $Q$ is the total charge per unit length on each electrode and $V$ is the potential difference between the two electrodes. $Q$ can be obtained by integrating equation (7.29) with respect to $\phi$ from 0 to $\pi$ and following the same transformation method by which $\sigma_s(\phi)$ was obtained. Note that the identity

$$\sum_{0}^{\infty} \frac{P_n(z_0)}{n + 1/2} = K \left( \sqrt{\frac{1+z_0}{2}} \right)$$ (7.32)
Table 7.1 Parameters of the dielectric test-pieces and curved patch capacitors used in benchmark experiments. Capacitor dimensions are $l_D = 50.24 \pm 0.01$ mm, $l_P = 40.00 \pm 0.01$ mm, $g = 1.00 \pm 0.01$ mm, and $s = 0.12 \pm 0.01$ mm except where indicated (see Figure 7.2).

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$</th>
<th>$\sigma$</th>
<th>$\rho_g$</th>
<th>$\sigma_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TecaformTM</td>
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<td>3.77</td>
<td>10.00</td>
<td>2.44</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.01$</td>
<td>$\pm 0.05$</td>
<td>$\pm 0.01$</td>
<td>$\pm 0.01$</td>
</tr>
<tr>
<td>Acrylic</td>
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<td>2.88</td>
<td>20.00</td>
<td>2.47</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.01$</td>
<td>$\pm 0.05$</td>
<td>$\pm 0.01$</td>
<td>$\pm 0.01$</td>
</tr>
<tr>
<td>Teflon®</td>
<td>19.10</td>
<td>2.33</td>
<td>15.00</td>
<td>1.85</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.01$</td>
<td>$\pm 0.05$</td>
<td>$\pm 0.01$</td>
<td>$\pm 0.01$</td>
</tr>
</tbody>
</table>

is used in deriving $Q$. Finally, the capacitance per unit length $C$ for a curved patched capacitor that conforms to the curvature of a cylindrical dielectric rod is obtained as

$$C = (\epsilon_1 + \epsilon_2)K(t)/K\left(\sqrt{1-t^2}\right).$$

A practical capacitance measurement setup based on the described theory is shown in Figure 7.2. Electrodes 1 and 2 are driving and pick-up electrodes, whereas 3 and 4 are the guard electrodes. Guard electrodes are introduced to eliminate the fringing effects not considered in the 2D model. In the measurements, the guard electrodes are kept at the same potential as the pick-up electrode, so that the electric fields go straight from the driving electrode to pick-up electrode without bending. Benchmark experiments were carried out to verify the theory. Three groups of capacitors of different dimensions (see Table 7.1) were fabricated using photolithography. These capacitors were attached to cylindrical dielectric rods of different materials and configurations (Table 7.1). Capacitance measurements were performed in free space at 1 MHz and room temperature using an Agilent LCR meter E4980A and an Agilent probe 16095A. The high potential pin of the probe was placed on the driving electrode while the low potential pin (potential of virtual ground) on the pick-up electrode. The guard electrodes were connected electrically to the guard port of the LCR meter (potential of virtual ground). More details on sensor fabrication, test-piece information and measurement procedures can be found in (12). Table 7.1 shows the very good agreement between measured and theoretically-predicted capacitances (to within 4%). Note that the calculated capacitances were obtained easily by rescaling equation (7.33).
Figure 7.2  A curved patch capacitor in surface contact with a dielectric test-piece. All the capacitor electrodes have a width \( w \). The gap between the driving and pick-up electrodes and the separation between the guard and pick-up electrodes are denoted \( g \) and \( s \). The lengths of the driving and pick-up electrodes are denoted \( l_D \) and \( l_P \), respectively. The length of the guards is \((l_D - l_P - 2s)/2\).

To summarize, an analytical solution for the capacitance of a curved patched capacitor in surface contact with a homogeneous cylindrical dielectric rod has been derived. Very good agreement between theoretically-predicted and measured capacitances was observed. Results described in this letter has the potential to be applied to many different science and engineering fields.
7.3 References


A paper accepted for publication in the Measurement Science and Technology

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8.1 Abstract

A capacitive sensor has been developed for the purpose of measuring the permittivity of a cylindrical dielectric that coats a conductive core cylinder. The capacitive sensor consists of two identical curved patch electrodes that are exterior to and coaxial with the cylindrical test-piece. The permittivity of the cylinder is determined from measurements of capacitance by means of a physics-based model. In the model, an electroquasistatic Green’s function due to a point source exterior to a dielectric-coated conductor is derived, in which the permittivity of the dielectric material may take complex values. The Green’s function is then used to set up integral equations that relate the unknown sensor surface charge density to the imposed potentials on the electrode surfaces. The method of moments is utilized to discretize the integral equation into a matrix equation that is solved for the sensor surface charge density and eventually the sensor output capacitance. This model enables the complex permittivity of the dielectric coating material, or the geometry of the cylindrical test-piece, to be inferred from the measured sensor capacitance and dissipation factor. Experimental validation of the numerical model has been performed on three different cylindrical test-pieces for two different electrode configurations. Each of the test-pieces has the structure of a dielectric coated brass rod. Good agreement between measured and calculated sensor capacitance (to an average of
7.4\%) and dissipation factor (to within 0.002) was observed. Main sources of uncertainty in the measurement include variations in the test-piece geometry, misalignment of sensor electrodes, strain-induced variation in the test-piece permittivity, and the existence of unintended air gaps between electrodes and the test-piece. To demonstrate the effectiveness of the sensor, measurements of capacitance have been made on aircraft wires and the permittivity of the insulation inferred. A significant change in permittivity was observed for thermally degraded wires.

### 8.2 Introduction

This paper describes development and benchmark testing of a model-based capacitive method for complex permittivity measurement of a cylindrical dielectric that coats a conductive core cylinder. The work is motivated by the need for effective nondestructive evaluation of degradation status of air- and space-craft wiring insulation. Degradation in electrical wiring insulation has the potential to cause aviation catastrophe due to consequent short-circuiting or loss of control function (1). One effective approach of evaluating insulation degradation state is through permittivity measurements, which can be achieved using capacitive methods.

Capacitive methods offer a favorable solution to the accurate characterization of material dielectric properties at low costs. For example, model-based interdigital sensors allow the inverse determination of test-piece material properties from measured sensor output capacitance (2). Applications of interdigital sensors include humidity and moisture sensing, electrical insulation properties sensing, chemical sensing, biosensing, and others. Rectangular coplanar capacitive sensors have been developed to detect water intrusion in composite materials, on the physical basis that changes in material dielectric properties lead to variations in the sensor capacitance (3). Rectangular capacitance sensors also find application in damage detection in laminated composite plates (4), evaluation of moisture content in reinforced concrete covers (5), and rain sensing (6). Rectangular capacitive sensor arrays have been reported in (7), and shown to be capable of detecting surface and subsurface features of dielectrics. Circular shaped capacitive sensors have also been developed for the quantitative characterization of material properties. Multichannel fringing electric field sensors (8), cylindrical geometry electroquasistatic dielec-
trometry sensors (9) and concentric coplanar capacitive sensors (10) are some examples of these. In addition, parallel plate capacitors formed by two circular discs with coplanar guard electrodes have been used to detect strength-limiting defects (large voids) in cellular glasses (11). Capacitive techniques that have the potential to be integrated into aircraft wiring test systems are presented in (12; 13; 14). Arc-electrode capacitive sensors have been developed to characterize material and structural properties of cylindrical dielectric rods (12). A so-called ‘meander’ coil and a quarter-circular interdigital sensor have been used in detection of cable insulation damage (13). In (14), linear relationships between the capacitance of open-circuited aircraft wires and their length have been demonstrated and enable the determination of wiring conductor length from measured capacitance values. Further references on capacitive methods can be found in (10) and (12).

In addition to capacitive techniques, other electrical testing methods have been developed to evaluate the wiring conductor condition. Reflectometry is one of the most commonly used techniques for aircraft wiring testing, in which a high frequency electrical signal is transmitted along the wire and any impedance discontinuities result in reflected signals whose interpretation may give an estimate of the flaw position. An excellent review paper that compares different reflectometry methods is (15). Low-voltage resistance tests and dielectric-withstand-voltage tests are two qualitative methods that can detect faults, but are not suitable for inspection of aging aircraft wiring because of the difficulty of miniaturization and pinpointing the fault (16).

The purpose of this paper is to present fundamental relationships between the capacitance of a curved patch capacitive sensor and the complex permittivity of its test-piece. The test-piece discussed here has the structure of a dielectric-coated conductor. The sensor consists of two identical and symmetric curved patch electrodes, which are located exterior to and coaxially with the cylindrical test-piece. In the modeling of this system, an electroquasistatic Green’s function due to a point source exterior to a cylindrical dielectric-coated conductor is derived in cylindrical coordinates, in which the permittivity of the dielectric material may take complex values, Section 8.3. The Green’s function is then used to set up integral equations that relate the unknown sensor surface charge density to the imposed potential difference, $V$, on the sensor electrodes. The method of moments (MoM) is utilized to discretize the integral
equation into a matrix equation that is solved for the sensor surface charge density, Section 8.4. The total charge $Q$ on each electrode is then calculated and the capacitance between the two curved patch electrodes is obtained through $C = Q/V$. Experimental validation of the theory has been carried out on brass rods coated with different dielectric materials, for two different electrode configurations, Section 8.5. Measured and numerically calculated sensor capacitance values agree to an average of 7.4%, whereas the dissipation factors agree to within 0.002. Major factors contributing to the measurement uncertainty are variations in the test-piece geometry, misalignment of sensor electrodes, strain induced variation in the test-piece permittivity, and the existence of unintended air gaps between the sensor electrodes and cylindrical test-pieces. These are discussed in detail in Section 8.5.

This paper is focused on the development and verification of a physics-based measurement method. In related research that will be published in a later article, a prototype capacitive probe has been built based on the method presented in this paper (17). This probe has been applied for quantitative characterization of insulation degradation on actual aircraft wires. Changes in the insulation complex permittivity, induced by thermal and hydrolytic exposures, have been clearly detected. One of these results is shown here, Section 8.6, in order to demonstrate the feasibility of quantitative evaluation of wiring insulation permittivity using the model described in this paper. The capacitance technique developed here has the potential to be built into smart embedded wiring test systems of the future. It is complementary to large-scale inspection techniques. For instance, two ultrasonic transducers can be used in a pitch/catch configuration to generate and receive an ultrasonic guided wave in a wire, and obtain an overall indication of the wire insulation condition (18). When faults are indicated by such system-level inspection techniques, high accuracy local inspections using the curved patch capacitive sensors may follow.

### 8.3 Modeling

The method of moments (MoM) is utilized in the modeling process instead of the finite element method. The adopted numerical method has the advantage of needing only to discretize surfaces, rather than the volume, to obtain the sensor capacitance. The number of unknowns
to solve is proportional to $N^2$, where $N$ is the number of unknowns in one dimension. Most commercially available finite element tools, however, have to discretize a truncated space, in which the number of unknowns to solve is proportional to $N^3$. This means using the MoM in the modeling significantly reduces the number of unknowns to solve and therefore the required computation time and computer memory. This feature is important in quantitative evaluation of wiring insulation permittivity using the capacitive method, because faster numerical models can significantly reduce the amount of time required to inversely determine the insulation permittivity from measured capacitance.

For instance, it takes 40 to 60 minutes to compute a single capacitance to accuracy of three significant figures for the cases discussed in this paper, using a laptop with a 1.6 GHz single core CPU and 256 MB memory. Commercial finite element tools may take days to achieve the same numerical accuracy.

### 8.3.1 Sensor Configuration

The configuration of the curved patch capacitive sensor is shown in Figure 8.1. The two identical and symmetric curved sensor electrodes are assumed in the theory to be infinitesimally thin. The cylindrical test-piece is modeled as an infinitely long dielectric-coated perfect conductor. The conductor is kept at ground potential in the modeling and the patch electrodes are held at equal and opposite potentials, $\pm V/2$. The sensor output capacitance is calculated in the electroquasistatic regime, in which the permittivity of the dielectric coating may take complex values while the resulting potential still satisfies the Laplace equation.

### 8.3.2 Derivation of Green’s Function in Cylindrical Coordinates

Figure 8.2 shows a point source placed at $(\rho', \phi', z')$ exterior to a cylindrical dielectric-coated conductor. This configuration is used in the following Green’s function derivation. The outer radius of the cylindrical test-piece is $b$ and the radius of the inner conductor is $a$. The complex permittivities of the dielectric coating and the background medium are $\epsilon_2^* = \epsilon_2' - j\epsilon_2''$ and $\epsilon_1^* = \epsilon_1' - j\epsilon_1''$, respectively, where $j = \sqrt{-1}$. Considering the canonical shapes of the sensor electrodes and the test-piece, the electroquasistatic Green’s function is derived in cylindrical
Figure 8.1 Curved patch capacitive sensor. The radii of the sensor electrodes, the conductor, and the cylindrical test-piece are denoted $\rho_0$, $a$, and $b$, respectively. The arc-angle of each sensor electrode is $\phi_0$ (rad). The length of each electrode in the vertical direction is $l$ and the width in the horizontal direction is $w = \phi_0 \times \rho_0$.

coordinates. The electric potential $\Psi$ at an observation point $(\rho, \phi, z)$ due to the point source in Figure 8.2 satisfies the Laplace equation in each homogeneous region exterior to the conductor:

$$
\left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right] \Psi^{(i)}(\rho, \phi, z) = -\frac{1}{\rho} \delta(\rho - \rho') \delta(\phi - \phi') \delta(z - z') \quad i = 1, 2, \tag{8.1}
$$

is subject to a Dirichlet boundary condition at the surface defined by $\rho = a$:

$$
\Psi^{(2)}(\rho = a, \phi, z) = 0, \tag{8.2}
$$

and the interface conditions at the surface defined by $\rho = b$:

$$
\Psi^{(1)}(\rho = b, \phi, z) = \Psi^{(2)}(\rho = b, \phi, z)
$$

$$
\varepsilon_1^* \frac{\partial}{\partial \rho} \Psi^{(1)}(\rho = b, \phi, z) = \varepsilon_2^* \frac{\partial}{\partial \rho} \Psi^{(2)}(\rho = b, \phi, z), \tag{8.3}
$$

where superscripts (1) and (2) correspond to the regions defined by $\rho \geq b$ and $a \leq \rho < b$, respectively. The Green’s function for the electroquasistatic potential can be found by following a procedure similar to that presented in (12), in which the Green’s function due to a point source exterior to a homogeneous dielectric cylinder is derived. Steps in the derivation may be summarized as follows.
Figure 8.2 Point source exterior to an infinitely long dielectric-coated conductor.

Begin with the free space Green’s function in cylindrical coordinates due to a point charge at the origin. The potential at an observation point \((\rho, \phi, z)\) is

\[
G_0(\rho, z) = \frac{1}{4\pi r} = \frac{1}{2\pi^2} \int_0^{\infty} K_0(\kappa \rho) \cos(\kappa z) \, d\kappa,
\]

(8.4)

where \(r = \sqrt{\rho^2 + z^2}\) and \(K_0(\kappa \rho)\) is the modified Bessel function of the second kind of order zero. The potential due to a source point at arbitrary location \((\rho', \phi', z')\) can be obtained by applying the addition theorem given in (8.5)

\[
K_0(\kappa \zeta) = I_0(\kappa \rho_<) K_0(\kappa \rho_>),
\]

(8.5)

to \(K_0(\kappa \rho)\) in (8.4) and replacing \(z\) by \(z - z'\), where \(\zeta = \sqrt{\rho^2 + \rho'^2 - 2\rho \rho' \cos(\phi - \phi')}\), \(\rho_<\) is the lesser of \(\rho\) and \(\rho'\), and \(\rho_>\) is the greater. Now the Green’s function in free space is rewritten as

\[
G_0(\rho, \phi, z | \rho', \phi', z') = \frac{1}{2\pi^2} \times \left\{ \int_0^{\infty} \tilde{G}_0(\rho, \rho', \kappa) \cos[\kappa(z - z')] \, d\kappa + 2 \sum_{m=1}^{\infty} \cos[m(\phi - \phi')] \int_0^{\infty} \tilde{G}_m(\rho, \rho', \kappa) \cos[\kappa(z - z')] \, d\kappa \right\},
\]

(8.6)

where

\[
\tilde{G}_p(\rho, \rho', \kappa) = I_p(\kappa \rho_<) K_p(\kappa \rho_>),
\]

(8.7)

and \(I_p(\kappa \rho_<)\) is the modified Bessel function of the first kind. As pointed out in (12), the difference between the Green’s function due to a point charge in free space and that due to a point charge exterior to an infinitely long dielectric-coated conductor (Figure 8.2) is due to
interface conditions that are applied at surfaces of constant \( \rho \), corresponding to the physical interfaces of the test-piece. To find the Green’s function for this case, one needs only to modify the integral kernel \( \tilde{G}_p(\rho, \rho', \kappa) \) in (8.6) so that the interface conditions in (8.2) and (8.3) are satisfied:

\[
G_p^{(2)}(\rho = a, \rho', \kappa) = 0 \quad (8.8)
\]

\[
\tilde{G}_p^{(1)}(\rho, \rho', \kappa) = \tilde{G}_p^{(2)}(\rho, \rho', \kappa) \quad (8.9)
\]

\[
e_1 \frac{\partial}{\partial \rho} \tilde{G}_p^{(1)}(\rho, \rho', \kappa) = e_2 \frac{\partial}{\partial \rho} \tilde{G}_p^{(2)}(\rho, \rho', \kappa). \quad (8.10)
\]

Following the same steps as in (12), the Green’s function \( G(\rho, \phi, z|\rho', \phi', z') \) at an observation point \((\rho, \phi, z)\) due to a point charge at \((\rho', \phi', z')\) that is also exterior to the dielectric-coated conductor is derived as:

\[
G(\rho, \phi, z|\rho', \phi', z') = \frac{1}{2\pi^2} \times \left\{ \int_0^\infty \tilde{G}_0(\rho, \rho', \kappa) \cos[\kappa(z - z')] d\kappa \\
+ 2 \sum_{t=1}^\infty \cos[t(\phi - \phi') \int_0^\infty \tilde{G}_p(\rho, \rho', \kappa) \cos[\kappa(z - z')] d\kappa \right\}, \quad (8.11)
\]

where

\[
\tilde{G}_p(\rho, \rho', \kappa) = I_p(\kappa \rho<)K_p(\kappa \rho>) + A_p(\kappa)K_p(\kappa \rho<)K_p(\kappa \rho') \quad p = 0, 1, \cdots, \quad (8.12)
\]

\[
A_p(\kappa) = - \left[ \frac{e_2^* - e_1^*}{e_2^* I_p(\kappa b)K_p(\kappa b)} \left[ I_p(\kappa b)K_p'(\kappa b) - e_1^* I_p(\kappa b)K_p'(\kappa b) \right] - \alpha_p(\kappa) \left[ I_p(\kappa b)K_p'(\kappa b) - \alpha_p(\kappa) (e_2^* - e_1^*)K_p(\kappa b)K_p(\kappa b) \right] \right], \quad (8.13)
\]

\[
\alpha_p(\kappa) = I_p(\kappa a)/K_p(\kappa a), \quad (8.14)
\]

\[
I_p'(\kappa b) = dI_p(\kappa p)/d(\kappa p)|_{\rho \to b} \text{ and similarly for } K_p'(\kappa b). \]

The Green’s function in the form of modified Bessel functions, (8.11), is used in the following calculations of the sensor output capacitance. The sensor capacitance is computed later using the derived test-piece geometry and permittivity dependent Green’s function.

Note that (8.11) can be simplified to the case of a homogeneous dielectric rod, described in (12), by assigning \( a = 0 \). The Green’s function (8.11) can also be expressed in the form of Bessel functions of the first and second kind. However, the denominator of the integrand in the Green’s function contains an infinite number of zeros and increases the complexity in the numerical implementations.
8.4 Numerical implementation

8.4.1 Calculation Method

The sensor output capacitance $C$ is calculated numerically using the method of moments (MoM). Calculation procedures used here are similar to those in (12). In summary, the following steps are taken to compute the sensor capacitance.

First, the Green’s function (8.11) is used to set up the integral equation that relates the unknown sensor surface charge density $\sigma_s(\phi', z')$ to the imposed potential $\Psi(\phi, z)$ on the sensor electrodes

$$
\Psi(\phi, z) = \frac{1}{\epsilon_0} \int \int_{\text{Left electrode}} G(\rho, \phi, z | \rho', \phi', z') \sigma_s(\phi', z') \rho_0 d\phi' dz' - \frac{1}{\epsilon_0} \int \int_{\text{Right electrode}} G(\rho, \phi, z | \rho', \phi', z') \sigma_s(\phi' + \pi, z') \rho_0 d\phi' dz'.
$$

(8.15)

In order to solve for the sensor surface charge density numerically, i.e., to use discrete functions approximating the continuous function $\sigma_s(\phi', z')$, each electrode in Figure 8.3 is discretized into $M \times N$ rectangular elements. The charge density on each element is assumed to be constant and can be different from others. Mathematically, this approximation is expressed as

$$
\sigma_s(\phi', z') \approx \sum_{j=1}^{MN} \sigma_j b_j(\phi', z'),
$$

(8.16)

where $b_j(\phi', z')$ is the selected pulse basis function and $\sigma_j$ is the unknown coefficient to be determined. Note the axisymmetry of the problem, it is only necessary to calculate the surface charge density on one of the electrodes.

To solve for the $MN$ unknown coefficients $\sigma_j$, weighting functions $w_i(\phi, z)$ are introduced to force the integral equation (8.15) to be satisfied for each element on the sensor surface. The point-matching method is used in this process, in which the weighting functions are Dirac delta functions. Expressions for $b_j(\phi', z')$ and $w_i(\phi, z)$ can be found in (12). Discretizing the integral equation using weighting functions in each of the $MN$ elements, (8.15) is expressed as
the following matrix equation:

\[
\begin{pmatrix}
G_{11} & G_{12} & \ldots & G_{1L} \\
G_{21} & G_{22} & \ldots & G_{2L} \\
\vdots & \vdots & \ddots & \vdots \\
G_{L1} & G_{L2} & \ldots & G_{LL}
\end{pmatrix}
\begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\vdots \\
\sigma_L
\end{pmatrix}
= \mathbf{V},
\] (8.17)

where

\[
G_{ij} = \frac{1}{\epsilon_0} \int \int_{\text{element } j} G(\rho_i, \phi_i, z_i | \rho_j', \phi_j', z_j') b_j(\phi', z') \rho_0 d\phi' dz',
\] (8.18)

\(L = MN\) and all the elements in \(\mathbf{V}\) share the same potential that is applied to the electrode. The unknown coefficients \(\sigma_j\) are obtained by solving the matrix equation, and the total charge \(Q\) on each electrode is calculated by integrating the surface charge density over the electrode surface. The capacitance \(C\) between the two electrodes is obtained using the relationship \(C = Q/V\).

### 8.4.2 Example Calculations

The dielectric-coated conductor is assumed to be in free space in the following calculations, i.e., \(\epsilon_1^* = 1\). When numerically evaluating elements in the \(\mathbf{G}\) matrix, the zero to infinity summation and integral in \(G(\rho, \phi, z | \rho', \phi', z')\) (see (8.11)) need to be truncated. The convergence of the Green’s function depends on values of \(\epsilon_2^*\), \(\tan \delta = \epsilon_2''/\epsilon_2', a/b, b/\rho\) and \(\phi_0\). When these
values are large, large truncation ranges for the summation and integral in (8.11) are needed. It is found that, for the case $\epsilon'_2 = 5$, $\tan \delta = 0.02$, $a/b = 0.9$, $b/\rho = 1$, $\phi_0 = 178^\circ$ and $l = 4$ cm, if one truncates the summation in $G(\rho, \phi, z | \rho', \phi', z')$ to 80 terms and the integral with the range from 0 to 6000 for the off-diagonal components in $\overline{G}$ matrix, and the summation to 400 terms and the integral with the range from 0 to 6000 for the diagonal components, accuracy to three significant figures can be achieved in the final calculated complex sensor output capacitance $C$, for both real and imaginary parts. The cases calculated in Figures 8.4 to 8.6 and Section 8.5 have smaller $\epsilon'_2$, $a/b$, $b/\rho$ and $\phi_0$ values than those in the case calculated above. The truncation standard used here is adopted in all of the following numerical calculations, which guarantees convergence to three significant figure accuracy for all the numerically calculated capacitance values in this paper.

The dependence of sensor capacitance on the electrode configuration is shown in Figure 8.4. The sensor output capacitance is plotted as a function of the electrode length $l$ and the arc-angle $\phi_0$. The dielectric coating has a relative permittivity $\epsilon_{r2} = 2.5$. The radius of the core conductor $a = 8$ mm and the outer radius of the dielectric-coated conductor $b = 9$ mm. The sensor electrodes are assumed to be right on the cylindrical test-piece: $\rho_0 = b$. In Figure 8.4, a linear relationship between the sensor capacitance $C$ and the electrode length $l$ is observed for any fixed electrode arc-angle $\phi_0$. On the other hand, the sensor output capacitance $C$ increases as the electrode arc-angle $\phi_0$ increases for any given electrode length $l$, and tends to infinity as $\phi_0$ tends to $180^\circ$. This is explained by the fact that as $\phi_0$ tends to $180^\circ$, the gaps between the two electrodes become infinitesimally small and the resulting capacitance tends to infinity.

Figure 8.4 shows that the sensor output capacitance changes dramatically when $\phi_0$ and $l$ have large values. When performing dielectric measurements in practice, it is usually helpful to have large sensor output signal and therefore to have large $\phi_0$ and $l$ values. However, when $l$ and $\phi_0$ are large, $C$ changes rapidly, and it is important to have accurate sensor configuration information to infer accurately material dielectric properties from measured $C$.

The dependence of sensor capacitance on the test-piece geometry is shown in Figure 8.5. The sensor output capacitance $C$ is plotted as a function of the ratios $b/\rho_0$ and $a/b$ (Figure 8.1). The dielectric coating permittivity $\epsilon_{r2}$ is as for Figure 8.4. The sensor electrodes have fixed
radius $\rho_0 = 9$ mm, arc-angle $\phi_0 = 174^\circ$ and length $l = 4$ cm. It is observed that, for fixed $b/\rho_0$, the sensor capacitance increases as the ratio $a/b$ increases. Such a trend is more obvious when the ratio $b/\rho_0$ tends to 1. This is because the calculated capacitance $C$ is actually the series capacitance of the capacitance between one electrode and the core conductor and the capacitance between the core conductor and the other electrode. When the ratio $a/b$ increases, the distance between the sensor electrodes and the core conductor decreases and the resulting total capacitance increases. In particular, when $b/\rho_0 = 1$, the output capacitance tends to infinity as $a/b$ tends to 1, in which case the gap between the sensor electrodes and the core conductor approaches zero. This also explains why the sensor capacitance increases as the ratio $b/\rho_0$ increases for given $a/b$ values, and why such changes in capacitance are more rapid for large $a/b$ values. The fact that the overall permittivity of the region between the electrodes and the conductive core increases as $b/\rho_0$ increases also contributes to increases in the sensor output signal. In summary, the existence of the conductive core in the test-piece increases the output capacitance for given sensor configurations, and as the conductive core radius $a$ approaches zero, the test-piece reduces to a homogeneous dielectric rod.

Figure 8.6 shows the sensor capacitance $C$ and dissipation factor $D$ as functions of the dielectric coating real permittivity $\epsilon'_r$ and imaginary permittivity $\epsilon''_r$, respectively. Different
Figure 8.5  Calculated sensor output capacitance $C$ as a function of the ratio of cylindrical test-piece outer radius $b$ to electrode radius $\rho_0$ and the ratio of conductive core $a$ to cylindrical test-piece outer radius $b$. The electrode radius, arc-angle and length are 9 mm, 174° and 4 cm, respectively.

sensor configurations are considered. In Figure 8.6 a), a linear relationship between $C$ and $\epsilon'_{r,2}$ is observed for all sensor configurations. It is seen that the slope of sensor capacitance versus dielectric coating real permittivity, i.e., the sensor sensitivity, depends on both the sensor configuration and the geometry of the cylindrical test-piece. The largest slope in Figure 8.6 occurs when $a/b$, $b/\rho_0$, electrode length $l$ and electrode arc-angle $\phi_0$ are the largest of the values considered. However, it is worth pointing out that although increasing electrode length $l$ increases the value of the slope, relative changes in capacitance stay the same, because of the linear relationship between the sensor output capacitance $C$ and electrode length $l$ (see Figure 8.4). For practical inspection of wires, $a/b$ is fixed, and it is therefore important to keep $b/\rho_0$ close to 1 to achieve the highest sensitivity. In the selection of $\phi_0$, a trade-off exists. Larger $\phi_0$ gives rise to larger sensitivity as well as capacitance. On the other hand, larger $\phi_0$ means that the inter-electrode gap decreases and the penetration of the field into the insulation decreases as a consequence.

Similar relationships between $D$ and $\epsilon''_{r,2}$ are observed in Figure 8.6 b). The major difference between the response of $C$ and $D$ to the investigated factors is that $D$ is less sensitive than $C$ to changes in $l$, $\phi_0$ and $a/b$. 
8.4.3 Dependence of Capacitance on Test-piece Permittivity and Sensor Configuration

If $l \to \infty$, $a \to 0$ and $b/\rho_0 = 1$, the case of Figure 8.1 becomes a two-dimensional problem. An analytical expression for the capacitance per unit length between the two curved patches has been derived in (20) and takes the following general form

$$C = F_s(\epsilon_1 + \epsilon_2), \quad (8.19)$$

where $F_s$ is a shape factor that depends solely on the capacitor geometry. Considering the linear plots of Figure 8.6 a), the following relationship is found to hold in general for the problem discussed in this paper (Figure 8.1):

$$C = F_s(\epsilon_1 + \alpha \epsilon_2), \quad (8.20)$$

where $\alpha$ is a constant showing the dependence of $C$ on $\epsilon_1$ and $\epsilon_2$. $\alpha > 1$ means $C$ is more dependent on $\epsilon_2$ than $\epsilon_1$, and vice versa. $\alpha = 1$ means $C$ depends equally on $\epsilon_1$ and $\epsilon_2$. The factors $\alpha$ and $F_s$ in (8.20) may be determined by selecting two data points on any of the lines in Figure 8.6 a). It is found that $\alpha$ and $F_s$ are constant for any given sensor configuration, independent of the particular data points selected for the calculation.
As an example shown in Figure 8.6 a), $C$ obtained based on (8.20) fits nicely on the dashed line for $\alpha = 2.61$ and $F_s = 1.01$ m. Similar results are observed for all the other sensor configurations as well. It is found that $\alpha$ increases as $b/\rho_0$, $a/b$ and $l$ increase, and as $\phi_0$ decreases. The latter relationship can be explained by the fact that as $\phi_0$ decreases, more electric field penetrates the dielectric coating, and $\epsilon_2$ will therefore have larger impact on $C$. The product $\alpha F_s$ is the slope in the $C$ versus $\epsilon_2$ plot and shows the sensor sensitivity, whose dependency on the sensor configuration has been discussed earlier.

8.5 Validation Experiment

8.5.1 Experiment Arrangement and Measurement Procedures

Benchmark experiments comparing the measured sensor capacitance with numerically-predicted values were performed to verify the validity of the developed theory. Experiments were conducted at frequency 1 MHz. Note that although the numerical model is developed in the electroquasistatic regime, i.e., the wave length $\lambda$ (approximately 300 m at 1 MHz) is much greater than the dimension of the test-pieces in the experiment, the dielectric coatings still have complex permittivities. This is due to losses arising in the materials due to the polarization response of the polymers lagging behind the switching of the applied electric field at 1 MHz. For this reason, complex permittivities are considered in the following benchmark experiments.

Two sets of rectangular planar electrodes (shown in Figure 8.7) were fabricated using photolithography by selectively etching a 18-$\mu$m-thick brass cladding (14 mL standard) off a flat 25.4-$\mu$m-thick Kapton® type 100 CR polyimide film. These flexible electrodes were attached to different cylindrical test-pieces later to form the capacitance sensors. The sensor dimensions are $w = 29$ mm and $l = 20$ mm for one set and $w = 29$ mm and $l = 40$ mm for the other (see Figure 8.1). A Nikon EPIPHOT 200 microscope was used to measure the fabricated sensor dimensions, for the purpose of checking the difference between the fabricated dimensions and the nominal ones, and therefore the accuracy of the fabrication process. The “traveling microscope” measurement method, with accuracy of $\pm 0.01$ mm, was used to measure the dimensions of the relatively large sensor electrodes. It was found that the measured dimensions of the fabricated
Table 8.1 Measured complex permittivity values of the dielectric coating materials.

<table>
<thead>
<tr>
<th>Dielectric material</th>
<th>$\varepsilon'_r$</th>
<th>$\varepsilon''_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tecaform&lt;sup&gt;TM&lt;/sup&gt;</td>
<td>3.77 ± 0.05</td>
<td>0.014 ± 0.002</td>
</tr>
<tr>
<td>Acrylic</td>
<td>3.02 ± 0.05</td>
<td>0.055 ± 0.009</td>
</tr>
<tr>
<td>Teflon®</td>
<td>2.21 ± 0.04</td>
<td>below instrument capability</td>
</tr>
</tbody>
</table>

electrodes are the same as the nominal ones under such measurement accuracy.

Three cylindrical test-pieces, each being a brass rod coated with a dielectric tube, were used in the measurements to simulate the infinitely long dielectric-coated conductors modeled in theory (Figure 8.1). The dielectric tube materials were Acetal Copolymer (Tecaform<sup>TM</sup>), Acrylic, and Virgin Electrical Grade Teflon® PTFE, respectively. They were hollowed from homogeneous rods so that the inner diameter of the tube matched the diameter of the brass rod as closely as possible. After assembly, all three dielectric tubes were in tight surface contact with the central brass rods. The cylindrical test-pieces used were 152 mm long (factors of approximately 4 and 7 times longer than their electrode lengths). The edge effect due to finite rod length can be neglected if the sensor electrodes are placed sufficiently far from the ends of the test-piece. Prior to being hollowed out, the permittivity of each dielectric tube was measured, by cutting a disc from the end of each rod and measuring its permittivity using a Novocontrol Alpha Dielectric Spectrometer at $f = 1$ MHz and room temperature. The measured test-piece permittivity values, together with uncertainty, are provided in Table 8.1. In the Novocontrol measurements, both sides of each disc (around 19 mm in diameter) were brushed with silver paint to form the measuring electrodes. The measured capacitance values lay between 2 and 5 pF. The impedance measurement accuracy of the Novocontrol at 1 MHz and room temperature for such capacitance values is 0.1% for the magnitude and 0.06° for the absolute phase accuracy. Note that since Teflon® is low loss material (loss tangent below $10^{-4}$ in the frequency range 20 kHz to 1 MHz at room temperature (21)), accurate measurements of its imaginary permittivity was not achieved using either the Novocontrol or the Agilent E4980A LCR meter.

The diameter of the brass rods, the outer diameter of the dielectric-coated conductors and the thickness of the dielectric coatings were measured using a digital caliper with uncertainty
± 0.01 mm. Each of these values was measured at ten different locations on the test-piece. An average value and the corresponding deviation were obtained, in which the deviation is defined as the maximum absolute difference between the average value and the ten measured values.

The experimental arrangement for the capacitance measurements is shown in Figure 8.7. The rectangular patch electrodes were conformed to each cylindrical test-piece by taping the thin Kapton® sensor substrate tightly against the dielectric material. The thickness of the film (25 µm) is accounted for in the numerical modeling while effects of its permittivity are neglected. This is because the sensor capacitance is much more sensitive to small variations in $b/\rho_0$, when this ratio is very close to 1, than those in the test-piece permittivity. The permittivity of the substrate was hence assumed to be that of the test-piece, an assumption that introduces negligible uncertainty. The electrodes were aligned carefully in order to achieve the sensor configuration in the theoretical model as closely as possible. The goal in the alignment was to keep the upper and lower edges of the two electrodes at the same height, the vertical edges of both electrodes in parallel, and the two vertical gaps between the two electrodes of the same size. Another layer of 25.4-µm-thick Kapton® film was wrapped tightly onto the outsides of the electrodes to further minimize the air gap between the electrodes and the cylindrical test-piece, leaving part of each electrode exposed to make electrical contact with the probe test fixture later.

In the experimental verifications, two groups of capacitance measurements were performed
for each cylindrical test-piece: one group using the 20-mm-long electrodes and the other using the 40-mm-long electrodes. For each cylindrical test-piece, the tube material, brass rod diameter, outer diameter of the dielectric-coated brass, variation in the dielectric tube thickness, electrode radius $\rho_0$ and electrode arc-angle $\phi_0$ are provided in Table 8.2, with uncertainties. Because the two types of electrodes were attached at different locations on each test-piece, the outer diameters of the test-pieces in Table 8.2 were measured at those individual locations and vary slightly. The electrode radius $\rho_0$ for each cylindrical test-piece is obtained by summing the outer radius of the dielectric-coated brass rod and the Kapton® substrate thickness. The electrode arc-angle $\phi_0$ and its uncertainty in Table 8.2 are calculated from the electrode width $w$, electrode radius $\rho_0$ and its uncertainty. Each test-piece has its own electrode arc-angle $\phi_0$ because the fabricated electrodes share a fixed width $w$ while the cylindrical test-pieces have different radii. The parameters shown in Tables 8.1 and 8.1 were used as the inputs in the numerical calculations.

An Agilent E4980A precision LCR meter was used to measure the sensor output capacitance at room temperature. The LCR meter operating frequency was set at 1 MHz to approximate the electroquasistatic assumption in the numerical model. Under these conditions, the measurement accuracy of the LCR meter for a 4 pF capacitance is 0.15% and the absolute accuracy for the dissipation factor is 0.0015, whereas those for a 13 pF capacitance are 0.13% and 0.0013, respectively. The measured capacitance values in this paper are all within 4 and 13 pF. If a lower operating frequency is desired in capacitance measurements, an impedance measurement instrument with higher accuracy when measuring large impedance values should be used. (According to the relationship $Z = 1/j2\pi fC$, the impedance $Z$ resulting from measuring a given capacitance $C$ under a lower frequency $f$ will be larger). The sensor capacitance $C$ was measured by placing an Agilent probe test fixture 16095A across the two sensor electrodes, as shown in Figure 8.7. This probe test fixture was connected to the LCR meter and the measured capacitance was read from the LCR meter screen. Note that in the modeling the two electrodes are assumed oppositely charged and the conductive core of the test-piece is kept at ground potential. The calculated capacitance is the series capacitance of the capacitance between one electrode and the conductive core and the capacitance between the conductive core and the
Table 8.2  Parameters of the test-pieces and the capacitive sensors used in benchmark experiments. The three brass rods used as the conductive cores in the cylindrical test-pieces had a uniform diameter of 15.90 ± 0.01 mm.

<table>
<thead>
<tr>
<th>Dielectric material</th>
<th>l (mm)</th>
<th>2b (mm)</th>
<th>Variation in (b − a) (mm)</th>
<th>ρ₀ (mm)</th>
<th>φ₀(°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tecaform™</td>
<td>20</td>
<td>19.28 ± 0.03</td>
<td>0.06</td>
<td>9.64 ± 0.02</td>
<td>172.4 ± 0.3</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>19.22 ± 0.05</td>
<td>0.06</td>
<td>9.61 ± 0.03</td>
<td>172.9 ± 0.5</td>
</tr>
<tr>
<td>Acrylic</td>
<td>20</td>
<td>19.18 ± 0.04</td>
<td>0.03</td>
<td>9.59 ± 0.02</td>
<td>173.3 ± 0.4</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>19.17 ± 0.07</td>
<td>0.03</td>
<td>9.59 ± 0.04</td>
<td>173.4 ± 0.6</td>
</tr>
<tr>
<td>Teflon®</td>
<td>20</td>
<td>19.31 ± 0.02</td>
<td>0.07</td>
<td>9.65 ± 0.01</td>
<td>172.1 ± 0.2</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>19.3 ± 0.1</td>
<td>0.07</td>
<td>9.67 ± 0.07</td>
<td>171.9 ± 1.2</td>
</tr>
</tbody>
</table>

other electrode. When performing capacitance measurements, however, one needs only to place the probe test fixture across the two sensor electrodes, as shown in Figure 8.7. The potential on the conductive core is the average of the potentials on the two sensor electrodes, due to the symmetry of the problem, and the capacitance picked up by the probe is the series capacitance calculated in the numerical model.

8.5.2 Results and Discussion

Comparison between calculated and measured sensor capacitance $C$ and dissipation factor $D$ for each cylindrical test-piece and the two different electrode configurations is made in Table 8.3. Because of material complex permittivity, complex sensor output capacitance is expected. The sensor dissipation factor is defined as the ratio of the imaginary part of the capacitance to its real part. Since accurately measured imaginary permittivity was not achieved for Teflon®, the sensor dissipation factor cannot be calculated and comparison between its calculated and measured $D$ is not made in Table 8.3.

Measured and calculated capacitance $C$ agree to an average of 7.4% in Table 8.3. All the measurement results are smaller than the numerically-predicted ones, by between 0.3 to 0.9 pF. Two factors contributing to lower measured $C$ are identified. First, during sample preparation, the brass rods were inserted into hollowed dielectric rods to achieve tight surface contact. After insertion, the dielectric coatings were subjected to normal stress exerted by the brass rods and circumferential strain was introduced in the dielectrics. In an independent study, it is found
Table 8.3  Measured and calculated capacitance for various sensor configurations in contact with different cylindrical test-pieces.

<table>
<thead>
<tr>
<th>Dielectric material</th>
<th>$l$ (mm)</th>
<th>Calculated $C$ (pF)</th>
<th>Measured $C$ (pF)</th>
<th>Diff. (%)</th>
<th>Calculated $D$</th>
<th>Measured $D$</th>
<th>Abs. diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tecaform$^\text{TM}$</td>
<td>20</td>
<td>6.58</td>
<td>6.09 ± 0.12</td>
<td>-7.5</td>
<td>0.003</td>
<td>0.002 ± 0.002</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>12.34</td>
<td>11.47 ± 0.19</td>
<td>-7.1</td>
<td>0.003</td>
<td>0.002 ± 0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>Acrylic</td>
<td>20</td>
<td>5.64</td>
<td>5.30 ± 0.11</td>
<td>-6.0</td>
<td>0.015</td>
<td>0.014 ± 0.006</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>10.36</td>
<td>9.47 ± 0.17</td>
<td>-8.6</td>
<td>0.015</td>
<td>0.015 ± 0.007</td>
<td>0.000</td>
</tr>
<tr>
<td>Teflon$^\text{®}$</td>
<td>20</td>
<td>4.20</td>
<td>3.92 ± 0.13</td>
<td>-6.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>7.63</td>
<td>6.98 ± 0.16</td>
<td>-8.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

that the real permittivity of Teflon$^\text{®}$(PTFE) decreases as its strain increases (22). This means the actual permittivity of the Teflon$^\text{®}$ coating the brass rod is likely to be lower than the value used in the numerical calculations, which was measured on a sample slice cut from the rods prior to their being hollowed out, i.e., before strain was introduced. Consequently, measured $C$ will be smaller than that predicted numerically for dielectrics with strain-induced reduction in permittivity. Second, although Kapton$^\text{®}$ films are tightly wrapped around the electrodes to attempt to eliminate air gaps between the electrodes and the test-piece, small gaps still exist due to the nonuniform diameters (surface roughness) of the cylindrical test-pieces. For example, the two largest percentage differences between calculated and measured capacitance values in Table 8.3 are observed for the cases of Acrylic and Teflon$^\text{®}$ coated test-pieces with 40-mm-long electrodes, which also show the largest variations in test-piece outer diameter (Table 8.2). It is seen from Figure 8.5 that the sensor output capacitance changes rapidly when the ratios $b/\rho_0$ and $a/b$ are close to one. This indicates that for $a/b \approx 0.82$ as in these measurements, the existence of air gaps can affect measurement results significantly.

Apart from the above two factors, two sources contributing to measurement uncertainty (not necessarily lower $C$) exist. One is the misalignment of sensor electrodes. For example, the vertical edges of the electrodes tend to bend up, giving rise to small air gaps in the vicinity of inter-electrode gaps, where the sensor is most sensitive. This points to the fact that the ideal way to achieve best agreement between theory and measurements is to deposit the sensor electrodes directly onto the cylindrical test-pieces, and errors coming from the misalignment of sensor electrodes and the existence of air gaps will be eliminated. However, deposition of
electrodes directly onto the test-piece is costly, time-consuming and undesirable in some cases although may be useful for real-time monitoring of structures by in-situ sensors. The other source of uncertainty is variation in the dielectric tube thickness, Table 8.2. These variations can be traced to surface roughness of the dielectric tubes and/or non-concentricity between the conductive core and the dielectric tube. The effects of rough test-piece surfaces have been discussed above. The effects of eccentricity are discussed here. For the ideal case that the conductor and the dielectric are concentric, as shown in Figure 8.1, the total sensor capacitance may be approximated by a relationship of the following form:

\[ C = \frac{C_0^2}{C_0 + C_0} = \frac{C_0}{2}, \]  

(8.21)

where \( C_0 \) is the capacitance between the conductive core and either electrode. When the conductor moves towards either of the electrodes, the conductor and the dielectric become eccentric. The capacitance between one electrode and the conductor changes to \( C_0 + mC_0 \) while the capacitance between the conductor and the other electrode changes to \( C_0 - nC_0 \), where \( m, n > 0 \). The total capacitance \( C' \) changes to

\[ C' = \frac{(C_0 + mC_0)(C_0 - nC_0)}{C_0 + mC_0 + C_0 - nC_0} = C_0 \frac{1 + m - n + mn}{2 + m - n}. \]  

(8.22)

It is found that when \( m \leq n \), \( C' \) is always smaller than \( C \). However, when \( m > n \), \( C' \) is not necessarily smaller than \( C \). In other words, non-concentricity between the brass rods and the dielectric coatings does not necessarily result in smaller sensor output capacitance, but does introduce uncertainty to the capacitance measurement.

As shown in Table 8.3, the absolute difference between the numerically predicted and the measured sensor dissipation factor \( D \) is found to be 0.002 or less. This is close to the LCR meter measurement accuracy, e.g., the measurement accuracy of the dissipation factor of a 5 pF capacitance is \( \pm 0.001 \) at 1 MHz and room temperature. Large variation in measured \( D \) is observed, however. One important reason for this is that large variation (\( \pm 16\% \)) in the measured test-piece \( \epsilon''_{r2} \) is observed, Table 8.1, due to the measurement accuracy of the instrument. This fact also introduces uncertainty into the calculated \( D \) and contributes to the difference between numerically-predicted and measured results, given in Table 8.3.
8.6 Test on Wires

In order to demonstrate the feasibility of using the described capacitive method for the quantitative nondestructive evaluation of wiring insulation, proof-of-concept experiments have been performed on actual aircraft wires of type MIL-W-81381/12 (provided by the NASA Langley Research Center). The wire samples are 2.5 mm in diameter and coated by Kapton® insulation. A prototype capacitive probe that clamps to the wire under test has been fabricated according to the model in Figure 8.1. Wire samples were thermally exposed at 400°C, 425°C, 450°C and 475°C for 1 to 5 hours to induce degradation in the insulation and capacitance measurements on these degraded wires were made. The insulation complex permittivity was inferred from measured probe capacitance and dissipation factor, and is compared with that of the control wires in Figures 8.8 and 8.9. Measurements were performed on multiple samples and error bars are included that account for variations in the dimensions, surface roughness, roundness and bending of actual wires. It is observed from Figures 8.8 and 8.9 that changes in the insulation complex permittivity due to thermal degradation are clearly detected using the described capacitive approach for this wire type. In addition, these changes are in accordance with results of previously conducted research on thermally exposed Kapton® films (23), in which capacitance measurements were made utilizing a parallel plate capacitor. Detailed work on this topic will be presented in (17).
8.7 Conclusion

The electroquasistatic Green’s function due to a point source exterior to a dielectric-coated conductor has been derived in cylindrical coordinates. The capacitance of the curved patch capacitive sensor has been calculated numerically using the method of moments based on the Green’s function. The quantitative dependence of the sensor capacitance on test-piece geometry and the dielectric coating permittivity has been demonstrated numerically and verified experimentally. A discussion of measurement uncertainty is provided. The curved patch capacitive sensor developed in this paper has the potential to evaluate effectively the condition of wiring insulation and is complementary to other inspection methods that are focused on evaluating the conductor’s condition.

8.8 Acknowledgment

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8.9 References


CHAPTER 9. A CAPACITIVE PROBE FOR QUANTITATIVE NONDESTRUCTIVE EVALUATION OF WIRING INSULATION

A paper submitted to the *NDT&E International*

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9.1 Abstract

A capacitive probe has been developed for quantitative evaluation of wiring insulation permittivity. The probe consists of two patch electrodes that conform to the curvature of the wire under test. A numerical model is utilized for inverse determination of insulation complex permittivity from measured probe response. Experimental studies on thermally and hydrolytically degraded wire samples show that the resulting permittivity change of the insulation is successfully detected using the described capacitive probe, for the wire type MIL-W-81381/12, which is predominantly Kapton® coated. Changes in the permittivity of the wiring insulation, detected by the capacitive probe, are shown to be in accordance with results of research conducted previously on Kapton® film samples degraded by thermal and hydrolytic exposure. Thus, the feasibility of assessing wiring insulation degradation status by quantitative capacitive techniques is demonstrated, which is of particular interest to the aerospace industry.

9.2 Introduction

Efforts have been made for years to guarantee the functionality of key electrical systems on aircraft. The performance of the wiring that connects these key systems was not, however, a strong focus of attention until the crashes of TWA 800 and Swissair 111 (1), attributed to aging wiring. In (2), causes and modes of failure in legacy aircraft wiring have been categorized.
These causes include chemical degradation such as corrosion of current carriers and hydrolytic scission of polymer chains in the insulation; electrical degradation of the insulation that may be due to concentrated electric fields at sites of electrical stress; and mechanical degradation due to vibration, over-bending and other kinds of mechanical stress.

Visual inspection is the most widely used method for aircraft wiring inspection. It is highly laborious while giving little quantitative information about the condition of the wires. Different physics-based wire inspection techniques have been developed over the past decade to replace this traditional inspection method, of which a summary is given here.

One of the most commonly used physics-based techniques for the inspection of the conductor in aircraft wiring is reflectometry. A high frequency electrical signal is transmitted along the wire and any impedance discontinuities in the conductor result in reflected signals. The location of the fault can be determined from the time and/or phase delay between the incident and reflected signals. An excellent review paper that compares different reflectometry methods is (3). Reflectometry methods are distinguished by the types of incident voltage applied. Time domain reflectometry (TDR) uses a short rise time voltage step as the incident voltage. This method is susceptible to noise and is not optimal for live wire testing (4; 5; 6). Frequency domain reflectometry (FDR) uses a set of stepped-frequency sine waves as the incident voltage. A conceptual design of a smart wiring system based on FDR methods that can be used for on-board testing of aging aircraft wiring has been described in (7). Phase-detection frequency-domain reflectometry (PD-FDR) has also been applied for locating open and short circuits in a Navy F-18 flight control harness (8). Sequence time domain reflectometry (STDR) and spread spectrum time domain reflectometry (SSTDR) use pseudo noise sequence and sine wave modulated pseudo noise code as the incident voltage, respectively. Testing systems based on these two techniques are capable of testing live wires and therefore have the potential to be used on energized aircraft to locate intermittent faults. Parameters that control the accuracy, latency, and signal to noise ratio for SSTDR in locating defects on live cables have been examined in (9), and the feasibility of spread-spectrum sensors for locating arcs on realistic aircraft cables and live wire networks has been demonstrated in (10) and (11). Aside from reflectometry methods, capacitive and inductive methods have also been applied for assessment.
of wiring conductor condition. In (12), linear relationships between the capacitance/inductance of open-/short-circuited wires (parallel insulated round wires, twisted-pair wires, and coaxial cables) and their conductor length have been demonstrated. This relationship enables the inverse determination of wire length from measured capacitance/inductance values. These techniques all inspect for so-called ‘hard’ faults in the metal wire conductor itself and are not capable of inspecting the insulation conditions.

Techniques have also been developed for the assessment of wiring insulation condition. Infrared thermography and pulsed X-ray systems have been developed for nondestructive testing of wiring insulation (13). Infrared thermography offers the advantage of rapidly examining large areas of wiring and can serve as a global testing method, whereas a portable pulsed X-ray system can be used to obtain a radiographic image of a portion of the wire or cable. Ultrasonic methods have also been developed for quantitative assessment of degradation in wiring insulation condition caused by heat damage, by modeling insulated wires as cylindrical waveguides (14). Moreover, acoustic and impedance testing methods aimed at locating intermittent faults in aircraft wires have been reported in (15). Partial discharge (PD) analysis methods for diagnosing aircraft wiring faults are explored in (16), in which a simulation of PD signal based on a high-voltage insulation testing standard (17) has been detailed, followed by wavelet-based analysis to de-noise the PD signals. Deficiencies of the above methods include the need for complex instrumentation and their inability to provide quantitative information about the insulation condition at specific locations. A favorable solution to these deficiencies is the capacitive method, from which quantitative measurements of the permittivity of wiring insulation at specific locations can be made, from which its condition can be inferred, using relatively simple equipment.

This paper describes a prototype capacitive probe that has been fabricated and applied for the inspection of wiring insulation in wires of type MIL-W-81381/12, which can be adapted for application to any single-conductor wire. The probe and test-piece interaction is describe by a physical model developed previously, in which a curved patch capacitor is located exterior to and coaxial with a cylindrical dielectric that coats a conductive core cylinder (18). In the model, a quantitative relationship between the complex permittivity of the dielectric coating
material and the complex capacitance of the capacitor is described. This relationship is utilized here for quantitative assessment of wiring insulation permittivity, based on measured probe capacitance. To demonstrate the feasibility of this proposed technique, groups of wire samples have been thermally and hydrolytically exposed, under different conditions, to induce dielectric property changes in the insulation. Capacitance measurements were performed on the samples, and complex permittivity values of their insulation determined inversely by means of the numerical model. Comparisons made between the complex permittivity of the damaged wires and the control wires show that both the real and imaginary parts of the insulation permittivity of the damaged wires increase as the thermal exposure temperature/time and hydrolytic exposure time increase, and are higher than those of the control wires. Especially, changes in the imaginary permittivity are more significant than those in the real part. For example, following thermal exposure, the imaginary permittivity was observed to increase by up to 39% and the real part by up to 17%, for exposures at temperatures between 400 and 475°C for various times up to 5 hours. In the hydrolytic exposure experiment, the imaginary permittivity was observed to increase by up to 75% and the real part by up to 12%, for exposure in water for various times up to 4 days. Permittivity changes in the wire insulation detected by the capacitive probe are in accordance with results of independent measurements conducted previously on planar thermally and hydrolytically exposed Kapton® film (19). These proof-of-concept experiments have demonstrated the excellent capability of the capacitive probe for quantitative evaluation of insulation condition for wires of type MIL-W-81381/12 and which, in principle, can be adapted for application to any single-conductor wire.

Apart from the capacitive probe discussed in this research, many other capacitive techniques have been developed and applied for NDE of dielectric materials (20). For example, capacitive arrays have been developed for robotic sensing using ‘scanning’ and ‘staring’ modes, (21; 22). Detailed literature surveys of capacitive NDE methods can be found in (23) and (24).

### 9.3 Summary of the Physical Model

The capacitive probe in this paper is designed based on the sensor model described in (18). The curved patch capacitive sensor consists of two identical and symmetric electrodes,
2.2. Derivation of Green's function in cylindrical coordinates

Figure 2 shows a point source placed at \((\rho', \phi', z')\) exterior to a cylindrical dielectric-coated conductor. This configuration is used in the following Green's function derivation. The outer radius of the cylindrical test-piece is \(b\) and the radius of the inner conductor is \(a\). The complex permittivities of the dielectric coating and the background medium are \(\epsilon^*_2 = \epsilon'_2 - j\epsilon''_2\) and \(\epsilon^*_1 = \epsilon'_1 - j\epsilon''_1\), respectively, where \(j = \sqrt{-1}\). Note that, by adopting this model, an insulation layer that consists of multiple dielectric layers is modeled as a one-layer homogenized dielectric. This means the insulation status assessed by the capacitive probe will be the overall condition of all the insulation layers.

In the numerical model described in (18), the insulation real permittivity \(\epsilon'\) is related linearly to the sensor capacitance \(C\), and the insulation loss tangent \(\tan\delta = \epsilon''/\epsilon'\) is related linearly to the sensor dissipation factor \(D\). The constants of proportionality increase as the following factors increase: \(b/\rho_0\), \(l\), \(\phi_0\) and \(a/b\). In this paper, complex permittivity values of wire samples are inferred from measured probe capacitance and dissipation factor values, based on this numerical model.

Without loss of generality, the conductor is kept at ground potential in the model and the potential on the two electrodes is +1 V and -1 V. The calculated capacitance is in fact the series capacitance of two contributions: i) the capacitance between the first electrode and the...
conductive core, and ii) the capacitance between the conductive core and the second electrode. In the measurements, however, the conductive core does not have to be grounded. This is because the potential on the conductive core will be the average of that on the two electrodes, if no other potential is applied directly to the core. The validity of this physics-based model has been verified by good agreement between numerical calculations and results of benchmark experiments (18). A detailed description of numerical modeling and experimental verification can be found in (18).

9.4 Probe and Measurement System

9.4.1 Probe

9.4.1.1 Assembly

Figure 9.2 shows the capacitive probe. The probe consists of two 2 cm × 4 cm acrylic plates and an acrylic rod on which the two plates are mounted so that their surfaces remain parallel. The lower plate is attached to the acrylic rod using a plastic screw, whereas the upper plate can glide up and down by adjusting another plastic screw perpendicular to the two plates. The curved sensor electrodes were formed by brushing a layer of silver paint onto the symmetric grooves in the two plates. The two electrodes are connected to two pins, which are then connected to an LCR meter for capacitance measurements. Two acrylic dowels were attached to the upper plate using epoxy glue, and inserted into holes drilled through the lower plate. Together with the plastic screw, these dowels assure that the two acrylic plates remain parallel during measurements. The subfigure in Figure 9.2 shows the probe holding a wire under test. The probe and the wire are in tight surface contact with each other.

9.4.1.2 Parameters

Parameters of the probe are: electrode radius $\rho_0 = 2.50 \pm 0.13$ mm and electrode length $l = 20.0 \pm 0.6$ mm, Figure 9.1. The electrode radius is taken to be the same as the specified wire sample outer radius. The electrode length is measured directly from the probe using a digital caliper.
The region exterior to the curved electrodes in Figure 9.1 is assumed in the model to be homogeneous, e.g., air. This assumption is not satisfied, however, for the probe depicted in Figure 9.2 due to the existence of the acrylic plates exterior to the electrodes. To account for this effect, along with possible air gaps between the electrodes and the testing wire due to surface roughness, an effective electrode arc-angle $\phi_0$ is introduced for the capacitive probe. Steps taken to determine $\phi_0$ are shown in Figure 9.3. First, an assumed electrode arc-angle $\phi_0$, the wire sample geometry information, and the assumed wire insulation real permittivity ($\epsilon' = 2.7$) are input into the numerical model, from which a computed probe capacitance $C_{\text{model}}$ is obtained. Second, capacitance measurements are performed on the control wire samples for which the dimensions and insulation permittivity are known. Third, $C_{\text{model}}$ is compared to the measured probe capacitance $C_{\text{meas}}$. If $C_{\text{model}}$ and $C_{\text{meas}}$ agree to within three significant figures, then $\phi_0$ is adopted as the effective electrode arc-angle. Otherwise, a different value is assumed for the effective electrode arc-angle and the above steps are repeated until the stop criterion is satisfied. For testing wires of type MIL-W-81381/12, the effective arc-angle is determined as $\phi_0 = 80.1^\circ \pm 0.5^\circ$. This inferred $\phi_0$ is quite close to the physical electrode arc-angle, which, by visual inspection, lies in the range 80 to 90$^\circ$. Uncertainty in $\phi_0$ is due to variations in the measured capacitance ($2.11 \pm 0.01$ pF). Probe parameters are listed in Table 9.1.
Determination of Effective Electrode Arc-angle

Select effective electrode arc-angle $\phi_0$

Wire geometry information

Wire insulation real permittivity $\varepsilon'$

Numerical model

Calculated sensor capacitance

Compare

Do not agree?

Agree $\phi_0$ is the effective arc-angle

Measured probe capacitance on a virgin wire

Figure 9.3 Algorithm for determination of the effective electrode arc-angle $\phi_0$.

Table 9.1 Probe parameters, Figure 9.2

<table>
<thead>
<tr>
<th>$l$ (mm)</th>
<th>$\rho_0$ (mm)</th>
<th>$\phi_0$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.0 ± 0.6</td>
<td>2.50 ± 0.13</td>
<td>80.1 ± 0.5</td>
</tr>
</tbody>
</table>

9.4.2 Measurement System and Uncertainty Analysis

For capacitance measurements, the probe was connected to an Agilent LCR meter 4980A by an Agilent probe test fixture 16095A, whose probe pins were connected to the curved patch electrodes of the prototype probe. Capacitance measurements in this paper were performed at 1 MHz and room temperature. Selecting the operating frequency as 1 MHz allows for very low uncertainty resulting from the LCR meter, less than 0.2% for the cases discussed here, while at the same time being a good approximation to the quasistatic assumption of the numerical model. Further discussion on the selection of the operating frequency is provided in (18).

Uncertainties in the measurement procedure are attributed to achievable precision in the measurement equipment, uncertainty in $\phi_0$, and variation in the geometry of the wires under test. These uncertainties contribute to uncertainty in the insulation complex permittivity determined inversely utilizing the numerical model.

The achievable precision of the LCR meter depends on the magnitude of the capacitance measured. In the LCR meter operation manual, relative uncertainty is provided for the measured capacitance whereas absolute uncertainty is provided for the measured dissipation factor.
Measured capacitance values in this paper are within the range 2 to 3 pF and for these values the corresponding uncertainty is 0.18% for the capacitance and 0.0018 for the dissipation factor. This means that, in order to detect insulation degradation by capacitive measurements, the degradation-induced changes in the measured probe capacitance and dissipation factor must be that greater than these values.

As seen in Table 9.1, the uncertainty in \( \rho_0 \) and \( l \) are approximately 5% and 3%, respectively. When the arc-angle of the curved patch electrodes is far from 180°, i.e. the distance between the edges of the oppositely-charged electrodes is relatively large, the capacitance and dissipation factor change slowly as \( \phi_0 \) varies (18). This is because the interaction between the electrodes is not intense under this circumstance. For the degradation cases studied in this paper, the uncertainty in the inferred insulation real permittivity \( \epsilon' \) resulting from the uncertainty in \( \phi_0 \) is within 0.01, i.e., less than 0.5%, whereas that in \( \epsilon'' \) is 0.0002, i.e., less than 0.2%.

Real wires exhibit variations in their dimensions, surface roughness, roundness and curvature even when they may appear macroscopically similar. For this reason, in the experiments that follow, capacitance measurements were performed on three samples, for each degradation condition. Error bars are included in all the following measurement results. It is worth pointing out that the outer diameters of the thermally and hydrolytically exposed wire samples still lie in the range between 2.41 and 2.63 mm (same range as for the control wire). As will be seen in Section 9.7, the standard deviation in the measurements derived from the variation in wire diameter, surface roughness, roundness and curvature is the dominant source of uncertainty in the measurements, larger than that due to the other sources discussed above. Nonetheless, changes in the insulation complex permittivity, due to thermal and hydrolytic exposure, are clearly observed above the level of uncertainty.

### 9.5 Parameters of the Wire Under Test

Capacitance measurements throughout this paper are performed on aircraft wire samples of type MIL-W-81381/12. The wire is composed of a nickel-coated copper conductor, wrapped with two layers of polyimide 150FW-N019 film and one layer of aromatic polyimide coating. Each layer of the polyimide 150FWN019 film is constructed by bonding a 13-µm-thick flu-
orinated ethylene propylene (FEP) fluoropolymer film to a 25-µm-thick polyimide FN film, Figure 9.4. Nominal conductor and outer diameters of the wire are 2.09 and 2.50 mm, respectively. Actual measured wire outer diameters for wires examined in this study range between 2.41 and 2.63 mm.

9.5.1 Real permittivity

In order to infer the permittivity of the thermally/hydrolytically exposed wires to that of the control wires and thereby obtain quantitative assessment of the condition of the insulation, an initial value for the permittivity of the control wires must be assigned.

The real part of the undamaged wire insulation permittivity is assumed to be 2.7 at 1 MHz and room temperature, the condition under which capacitance measurements reported here were performed. Reasons for this assumption are as follows. Considering the schematic shown in Figure 9.4, we note that there are two main components: polyimide 150FWN019 and an aromatic polyimide outer layer. The manufacturer, DuPont™, does not provide a specific permittivity value for polyimide 150FWN019 films, mentioning only that it is less than 3 at 1 kHz and room temperature. According to DuPont™, the processing conditions of
polyimide 150FWN019 film is, however, very similar to those of the standard polyimide FN films, whose real permittivity is 2.7 at 1 kHz and room temperature. Therefore, we assume the real permittivity of polyimide 150FWN019 to be 2.7 at 1 MHz and room temperature, taking into account the fact that the real permittivity of polyimide does not change significantly over the frequency range from 1 kHz to 1 MHz. Regarding the aromatic polyimide portion of the insulation, the dielectric behavior of aromatic polyimide, types \(a\) through \(f\), is presented in (25). As will be seen in Section 9.7, degradation of the wire insulation initiates between 400°C and 450°C. Only type \(f\) of the aromatic polyimides studied in (25) presents a thermal degradation initiation temperature \((T_i = 430°C)\) within this range, whereas \(T_i\) of the other types are greater than 450°C. The aromatic polyimide coating used in the wire insulation is therefore inferred to be type \(f\), whose measured real permittivity is 2.7 at 1 MHz and room temperature (25), the same as that assumed for the polyimide 150FWN019 films. Finally, since the FEP adhesive layer is a relatively minor constituent of the insulation, accounting for approximately 12% of the insulation cross sectional area, and exhibits no unusual dielectric properties \((\epsilon' = 2.01\) at 1 MHz and room temperature according to DuPont\textsuperscript{TM}), it is reasonable to assume that overall real permittivity of the undamaged insulation is close to 2.7 at 1 MHz and room temperature.

### 9.5.2 Imaginary permittivity

Again, in order to infer the imaginary permittivity of thermally and hydrolytically exposed wires, the imaginary permittivity \(\epsilon''\) for the control wires has to be determined. Figure 9.5 shows the steps taken to determine the imaginary permittivity \(\epsilon''\) for these wires. The steps are similar to those described in Figure 9.3, except that the quantity compared here is the probe dissipation factor \(D\), instead of the capacitance \(C\). The inversely determined \(\epsilon''\) for the control wires is \(0.016 \pm 0.002\). Uncertainty in \(\epsilon''\) is due to variations in the measured dissipation factor of the control wires \((0.0055 \pm 0.0006)\) and in \(\phi_0\). The inferred \(\epsilon''\) is similar to the value given for type \(f\) aromatic polyimide in (25) \((\epsilon'' = 0.010 \pm 0.005\) at 1 kHz and room temperature). Values for polyimide 150FWN019 and FEP adhesive given by DuPont\textsuperscript{TM} are \(\epsilon'' = 0.006 \pm 0.001\) at 1 kHz and room temperature and \(\epsilon'' = 0.004 \pm 0.001\) at 1 MHz and room temperature, respectively.
Determination of Insulation Imaginary Permittivity $\varepsilon''$

Select insulation imaginary permittivity $\varepsilon''$

Wire geometry information

Effective electrode arc-angle $\phi_0$ and insulation real permittivity $\varepsilon'$

Numerical model

Calculated sensor dissipation factor

Measured probe dissipation factor on a virgin wire

$\varepsilon''$ is the inferred insulation imaginary permittivity

Figure 9.5  Algorithm for determination of the imaginary permittivity $\varepsilon''$ for the control wires.

9.6 Case Study: Evaluation of Polyimide-coated Wires After Thermal and Hydrolytic Exposure

The influence of thermal degradation and saline exposure on the complex permittivity of polyimide HN films has been studied in (23). The samples studied were 125 $\mu$m thick. Explanations of the mechanisms by which thermal degradation and saline exposure affect the complex permittivity of polyimide are provided. The work of (23) guided the choice of experimental parameters for this study.

For the thermal exposure experiment, the exposure temperatures were selected as 400, 425, 450 and 475 $^\circ$C. For each exposure temperature, five groups of wires, each with three 4-cm-long samples, were isothermally heated in a muffle furnace, Figure 9.6, for 1, 2, 3, 4 and 5 hours.

Note, the temperature distribution in the furnace is not uniform. To ensure that all the samples are heated at the selected temperature in each experiment, a small ceramic bowl was used to accommodate the wires, and the temperature in the ceramic bowl was measured independently using a thermometer. The upper right figure in Figure 9.6 shows some of the heat-damaged wire samples in comparison with a control wire. Thermally exposed wires were sealed in plastic bags immediately after being taken out of the furnace to avoid moisture absorption. These samples were then allowed to cool down to room temperature naturally.

The lower right figure in Figure 9.6 shows a wire sample used in the hydrolytic exposure
experiment. Both ends of the wire were sealed with wax to prevent water from migrating into the samples at its ends via the conductor insulator interface. Five groups of wires, each having three 4-cm-long samples, were immersed in water at room temperature for 0.5, 1, 2, 3, and 4 days, respectively. Capacitance measurements were performed immediately after the samples were removed from the water and their surfaces were wiped dry with a soft cloth.

9.7 Results and discussion

9.7.1 Thermal exposure

Figure 9.7 shows probe capacitance and dissipation factor measured on the thermally exposed wires, as a function of exposure temperature and time. Measurements were performed on three nominally identical wire samples in each group and the mean value and standard deviation are presented.

As can be seen in Figure 9.7, measured capacitance increases as exposure time and temperature increase. A similar behavior was observed in the measured dissipation factor. In (23), it was observed that the dissipation factor of polyimide HN film did not change significantly unless exposed at 475°C for more than 3 hours. This suggests that increases in the dissipation factor of the thermally degraded wires observed here results primarily from degradation of the
Figure 9.7  a) Measured capacitance and b) dissipation factor for thermally exposed wires. Uncertainties derive from the standard deviation of measurements on three nominally identical samples. Physical degradation of the sample heated beyond 2 hours at 475 °C prevented accurate capacitance measurement for those conditions.

aroconomic polyimide outer coating, rather than the polyimide 150FWN019 layers.

Complex permittivity of the wire samples was inferred from the measured probe capacitance and dissipation factor in the following way. First, an initial guess of the sample complex permittivity $\epsilon^*$ is input into the numerical model, from which particular values of probe capacitance $C_{thry}$ and dissipation factor $D_{thry}$ are obtained. These values are compared with the measured values $C_{exp}$ and $D_{exp}$ and $\epsilon^*$ adjusted until $C_{thry}$ ($D_{thry}$) and $C_{exp}$ ($D_{exp}$) agree to within three significant figures. Then, $\epsilon^*$ is adopted as the inferred sample complex permittivity.

Figure 9.8 shows the inferred insulation real permittivity values compared with results presented in (23) for polyimide HN film. In (23) the permittivity of the control film was measured as $\epsilon^* = 3.15 - j0.013$ at 1 MHz and room temperature, larger than the values assumed
here, but the increase in $\epsilon'$ with time and temperature of thermal exposure are similar for the wire samples studied here and for the polyimide HN film studied in (23). The inferred imaginary permittivity values of the wires are presented in Figure 9.9. The imaginary permittivity of the polyimide HN film studied in (23), however, showed insignificant change for the exposure conditions in Figure 9.9. The major difference between the wire insulation material and the polyimide HN film lies in the aromatic polyimide outer coating. Therefore, it is concluded that the existence of the aromatic polyimide coating contributed predominately to the increases in the insulation imaginary permittivity observed here. As can be seen from these two figures, both the real and imaginary parts of the insulation increase as heat exposure temperature and time increase. Especially, the relative change in the insulation imaginary permittivity is greater
than that in the real permittivity; the imaginary part increases by up to 39% and the real part by up to 17% compared with the permittivity of the control wires, suggesting that imaginary permittivity is the stronger indicator of insulation condition.

9.7.2 Hydrolytic exposure

The capacitance and dissipation factor for hydrolytically exposed wires are shown in Figure 9.10. In accordance with results of previous studies (26; 27), it is observed that both the measured capacitance and dissipation factor increase as water immersion time increases in the first two days, and do not change significantly afterwards. Inferred wire insulation complex permittivity is shown in Figure 9.11 along with measurement results presented in (23) for 125-\(\mu\)m-thick polyimide HN film. The real part of the wire insulation permittivity increased by up to 12% and the imaginary part by up to 75% compared with the permittivity of the control wires. Note that although the complex permittivity values for the polyimide HN film differ from those of the wire insulation, Figure 9.11, the increasing trend of \(\varepsilon'\) and \(\varepsilon''\) with hydrolytic exposure time is similar. In addition, the increase in \(\varepsilon''\) as a result of hydrolytic exposure is larger for the wire insulation than for the polyimide HN film, approximately 0.01 (75%) for the wire insulation compared with approximately 2.5\(\times\)10\(^{-3}\) (54%) for the polyimide HN film. These relatively large changes suggest, as for the thermally degraded wires, that the value of imaginary permittivity (or the measured dissipation factor \(D\)) is a strong indicator of insulation condition.
Figure 9.10  As for Figure 9.7 but for hydrolytically exposed wires.

9.8 Conclusion

A prototype capacitive probe for quantitative NDE of wiring insulation has been designed and fabricated. A numerical model allows for inverse determination of insulation complex permittivity from measured probe response. Complex permittivity, especially the imaginary part, is an effective indicator of wiring insulation condition. Changes in wiring insulation permittivity, induced by thermal degradation and hydrolytic exposure, have been successfully detected using the capacitive probe presented here, and observed trends in $\epsilon^*$ are in accordance with results of previous research on insulating films (23). Experimental studies on both the damaged and undamaged wire samples demonstrated that insulation condition changes for wires of simple construction (cylindrically-concentric conductor coated with dielectric) can
Figure 9.11  Inferred complex permittivity, a) real part and b) imaginary part, of the hydrolytically exposed wires in comparison with that of polyimide HN film (23). Note the very different scales on the vertical axes of Figure 9.11 b), presented this way for clarity.

be successfully detected and quantified using the capacitive probe described in this research. Furthermore, the one-to-one correspondence that exists between the insulation permittivity and measured capacitance suggests monitoring changes in insulation condition by detecting changes in insulation permittivity, even if the absolute value of the control wire permittivity is unknown.

This paper focuses on quantitative evaluation of the insulation permittivity after degradation. In some practical circumstances, it may be more convenient to make comparative measurements between the capacitance, and in particular the dissipation factor $D$, measured on a reference wire and measured on the wire under test. With simple adaptation, the probe presented here is capable of both tasks. Additionally, for aging aircraft, prior knowledge of
the locations at which wiring insulation degradation is most common is usually available. The capacitive NDE approach described in this paper can be applied at those locations for accurate characterization of localized insulation degradation. For improved ease for practical application, the capacitor electrodes can be mounted on a spring loaded mechanical clip. This is the subject of future work.

9.9 Acknowledgments

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9.10 References


CHAPTER 10. GENERAL CONCLUSIONS

10.1 General Discussion

The work in this thesis is motivated by the need for quantitative NDE of planar and cylindrical low-conductivity structures. All the capacitive sensors are developed following the progression of analytical or numerical modeling, followed by experimental verification and prototype probe fabrication. One practical motivation behind the development of coplanar capacitive sensors is detecting water or excessive inhomogeneities caused by repairs in aircraft radome structures. As demonstrated in Chapter 4, the fabricated prototype probe based on the coplanar sensor structure has successfully detected 1 cc of injected water and 1 cc of injected olive oil, whose dielectric property is close to that of excessive resin caused by repair, in simulated radomes. The curved patch capacitive sensors are motivated by the need for quantitative evaluation of aircraft wiring insulation condition. The prototype probe developed based on the curved patched sensor models, Chapter 9, has successfully distinguished thermally and hydrolytically degraded wires from the control ones. Note that although the work presented in this thesis is motivated by needs in the aviation industry, results this work can be applied to other engineering fields.

10.2 Recommendations for Future Research

The coplanar concentric capacitive sensors developed in this work have the potential to be extended for accurate moisture content monitoring of soil and agricultural products. Soil water content is important information to crop growers, and the quality of several fruits and vegetables can be determined by their moisture content (1). Moisture content monitoring is also important to the safe storage of grains. Water has a real permittivity approximately 80 at
low frequencies, which is much higher than that of soil and most agricultural products. In other
words, higher product moisture content means higher bulk permittivity and can be detected
through capacitive measurements (2). It is worth pointing out that if the material under
test is a mixture of different materials, e.g., soil, it is helpful to incorporate dielectric mixing
models (3) in the bulk moisture content monitoring process. The advantage of the coplanar
concentric capacitive sensors in such applications lies in the fact that the sensor capacitance is
independent of the relative orientation between the sensor and the mixture, thus more suitable
for the characterization of bulk permittivity.

Ongoing research on curved patch capacitive sensor design includes the development of
interdigital electrode configurations. The interdigital configuration allows for higher sensor
sensitivity and signal-to-noise ratio. In the area of evaluation of wiring insulation condition,
instead of operating the capacitive sensors at a given frequency, frequency sweep approaches
can be adopted in the near future. As the complex permittivity of most materials changes
as a function of frequency (4), frequency sweep allows for the identification of the optimal
measurement frequency, at which the complex permittivity of the insulation is affected the
most by degradation.

The ultimate goal for the evaluation of aircraft wiring condition is to develop a smart
embedded test system that can accurately detect and locate faulty wires during flight; and
even predict potential wiring failures. This involves developing inspection techniques that
are capable of diagnosing the condition of live wires at a distance (without disconnecting or
contacting the wires), and accurate life prediction methods for different aircraft wires. The
work of this thesis provides part of the body of knowledge upon which such advances can be
built.
10.3 References


APPENDIX A. RELATIONSHIP BETWEEN THE PENETRATION DEPTH OF CONCENTRIC COPLANAR CAPACITIVE SENSORS AND THE TEST-PIECE PERMITTIVITY

The following derivation is to proof the penetration depth of concentric coplanar capacitive sensors, defined in (4.1) in Chapter 4, increases as test-piece permittivity increases.

For a half-space dielectric test-piece with permittivity $\epsilon_r$, the integral equation that relates the potential at a certain observation point on the sensor electrodes and the sensor surface charge density is given as 

$$
\Psi_H = \frac{1}{2\pi(\epsilon_0 + \epsilon_r)} \int_S \int_0^{2\pi} \frac{\sigma_H(\rho')}{|\mathbf{r} - \mathbf{r}'|} d\phi' d\rho',
$$

(A.1)

where $S$ denotes the sensor surface, $\sigma_H(\rho')$ is the corresponding sensor surface charge density and is independent of the azimuthal angle $\phi'$ because of the cylindrical symmetry of the sensor. $|\mathbf{r} - \mathbf{r}'|$ is the distance between a source point and the observation point.

Similarly, for a one-layer test-piece with the same permittivity value, the potential at a certain observation point on the sensor electrodes is expressed as

$$
\Psi_o = \frac{1}{2\pi(\epsilon_0 + \epsilon_r)} \int_S \int_0^{2\pi} \sigma_o(\rho') \left[ \frac{1}{|\mathbf{r} - \mathbf{r}'|} + \sum_{n=0}^{\infty} \frac{(1 + \alpha)\alpha^{2n+1}}{\sqrt{|\mathbf{r} - \mathbf{r}'|^2 + [2(n+1)T]^2}} \right] d\phi' d\rho',
$$

(A.2)

where $\sigma_o(\rho')$ is the corresponding sensor surface charge density, $\alpha = (\epsilon_r + \epsilon_0)/(\epsilon_r - \epsilon_0)$, and $T$ is the test-piece thickness. Because the potentials applied to the sensor electrodes for these two cases are identical, the integral kernels in A.1 and A.2 must be the same:

$$
\frac{\sigma_H(\rho')}{|\mathbf{r} - \mathbf{r}'|} = \sigma_o(\rho') \left[ \frac{1}{|\mathbf{r} - \mathbf{r}'|} + \sum_{n=0}^{\infty} \frac{(1 + \alpha)\alpha^{2n+1}}{\sqrt{|\mathbf{r} - \mathbf{r}'|^2 + [2(n+1)T]^2}} \right].
$$

(A.3)

From (A.3), $\sigma_H(\rho')$ and $\sigma_o(\rho')$ can be written as

$$
\sigma_H(\rho') = A \times |\mathbf{r} - \mathbf{r}'|,
$$

(A.4)

---

$$\sigma_o(\rho') = A \times \left| r - r' \right| + \sum_{n=0}^{\infty} \frac{(1 + \alpha)^{2n+1} \alpha^{2n+1}}{\sqrt{|r - r'|^2 + |2(n+1)T|^2}}, \quad (A.5)$$

where $A$ in (A.4) and (A.5) stands for a constant.

The sensor output capacitance is proportional to the sensor surface charge distribution if the potential on the sensor is fixed, and the sensor penetration depth defined in (4.1) is expressed as:

$$\frac{|C - C_\infty|}{C_\infty} \sim \frac{\left| \sigma_o(\rho') - \sigma_H(\rho') \right|}{\sigma_H(\rho')} = \frac{1 + \alpha}{\left| r - r' \right|} \sum_{n=0}^{\infty} \frac{\alpha^{2n+1}}{\sqrt{|r - r'|^2 + |2(n+1)T|^2}} \quad (A.6)$$

In order to achieve the same 10% difference between $C$ and $C_\infty$ in Figure 4.4 c), test-pieces with high $\epsilon_r$ values ($\alpha \to 1$) need to have large $T$ in the denominator in (A.6), whereas test-pieces with low $\epsilon_r$ values ($\alpha \to 0$) can have small $T$ to achieve the same percentage of difference. This implies that penetration depth is bigger for test-pieces with larger $\alpha \to 1$. However, it is worth pointing out that as $\alpha \to 1$ becomes large, $\alpha$ is very close to one, and increments in $\alpha \to 1$ do not result in significant increases in the sensor penetration depth.
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