ULTRASONIC CRACK CHARACTERIZATION: A CONSTRAINED INVERSION ALGORITHM

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INTRODUCTION

Obtaining flaw geometry and orientation information from ultrasonic measurements often involves time consuming scanning and data processing functions. One possible way of reducing the amount of information needed for flaw characterization purposes is to require that the fine details of the flaw not be resolved but instead to obtain the best "equivalent" flaw geometry and orientation that fits a predefined simple flaw shape of unknown size and orientation. Hsu et al. [1] have implemented such a procedure for isolated voids and inclusions by using the Born approximation and obtaining a non-linear least squares estimation of the best ellipsoid that fits the data. Here, we will demonstrate that a similar constrained inversion procedure can be developed using the Kirchhoff approximation [2-4] for a flat elliptical crack. Using the results of our previous paper [5], an explicit coordinate-invariant expression that relates the time difference, \( \Delta t \), between the arrival of the waves diffracted from the flashpoints of the crack and the crack shape and orientation is obtained. This expression, together with \( \Delta t \) measurements in different scattering directions is placed into a regression analysis to obtain a set of equivalent flat elliptical crack parameters. A series of tests of this method using noisy synthetic data are considered and the sensitivity of the results to number of measurements and the viewing aperture of the transducer set-up is discussed.

THE CONSTRAINED INVERSION ALGORITHM

Through the use of the Kirchhoff approximation, the far-field impulse response of an elliptical crack can be obtained as explained in Ref. [5]. This solution can form the basis for obtaining the size and orientation of an unknown flat crack by performing a constrained inversion, i.e.,
Far-Field Scattered Impulse Response

Fig. 1. Antisymmetrical impulse response.

Fig. 2. Elliptical crack geometry.
with

\[ f = \frac{2q}{\sqrt{\tan^2 \theta_n + \cos^2(\phi_n - \phi_a)} \left[ \sin \alpha_b \cos(\phi_n - \phi_a) \right.} \]

\[ \left. - \cos \alpha_b \tan \theta_n \cos(\phi_n - \phi_a) \right] \right)^2
\]

\[ + \left( \frac{b}{d} \right)^2 [\cos \theta_n \sec \theta_n \sin^2 \theta_n \sin(\phi_n - \phi_a) + \cos \theta_n \cos \phi_n \sin(\phi_n - \phi_a) \cos(\phi_n - \phi_a) \]

\[ + \sin \alpha_b \sin \theta_n \sin(\phi_n - \phi_a) \left( \frac{b}{d} \right)^2 ]^2 \]

where the dimensions \( a \) and \( b \) are normalized by an arbitrary reference length \( d \). Equation (6) is the generalization of the results obtained in Ref. [5] to an arbitrary pitch-catch type of experimental set-up. In Ref. [5] we only considered the special case of a pulse-echo experiment, where the incident wave was a P-wave. By using as few as eight "exact" synthetic \( \Delta t \) measurements and a nonlinear least squares algorithm, it was demonstrated in Ref. [5] that an expression similar to Eq. (6) could be used to accurately extract all the pertinent unknown crack parameters. Here, we will examine the effects of adding "noise" to such synthetic data and describe the sensitivity of these corrupted results to the number of measurements and transducer orientations. The choice of crack orientation for all our test cases is shown in Table 1.

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( \theta_\gamma )</th>
<th>( \phi_\gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>-17.92°</td>
<td>14.04°</td>
</tr>
<tr>
<td>( b )</td>
<td>-13.34°</td>
<td>108.43°</td>
</tr>
<tr>
<td>( n )</td>
<td>67.38°</td>
<td>53.13°</td>
</tr>
</tbody>
</table>

To show the effects of the viewing aperture on the results we have taken sampled data at \( \phi = 20°, 40°, 60°, 80° \) and the number of \( \eta \) values varying from two to six over a total aperture of 70° where \( \phi \) and \( \eta \) are angles measured from a coordinate system oriented with the plane of the crack and its normal (Fig. 2). In particular, for two sampled \( \eta \)-values, \( \eta \) was taken as 60°, 70° and additional \( \eta \) values were added at 10° decreasing intervals. To simulate the effects of errors on the constrained inversion process, random errors of 5% were introduced into synthetic \( \Delta t \) "measurements" and Eq. (6) was used, together with a nonlinear least squares algorithm [5], to obtain predicted values for the unknowns \( \theta_n, \phi_n, \phi_a, a/d \) and \( b/d \). The results are shown in Fig. 4a-e for the case of an elliptical crack where \( a/d = 1.5, b/d = 1.0 \). Similar results are shown in Fig. 5a-e for \( a/d = 3.0, b/d = 1.0 \). As these figures show the \( \theta_n, a/d, b/d \) errors generally are less than the induced errors, regardless of the aperture size. The quantities \( \phi_a \) and \( \phi_n \) are significantly more sensitive to the errors, although in general their errors do not exceed 2-2.5 times the induced percentage errors at the widest apertures. It is interesting to note, therefore, that the method appears to provide sizing measurements accurate to within the induced errors for even the smallest apertures. A separate case (not shown) where \( a/d = 2.0, b/d = 1.0 \) also demonstrated this same result.
where we attempt to obtain the "best" equivalent flat elliptical crack that fits the scattered data. First, we note that the far-field impulse response exhibits the characteristic antisymmetrical waveform pattern shown in Fig. 1, where the two singularities are the crack flashpoints [4]. The inverse algorithm is based on measurements of the time interval, $\Delta t$, between these flashpoints, which is given by (Fig. 2).

$$\Delta t = \frac{2q_{a\beta} \cos \gamma / c_{\beta}}{[a^2 \cos^2 \psi + b^2 \sin^2 \psi]^{\frac{1}{2}}}$$

where $q_{a\beta} = \left|q_{a\beta}\right|$

This expression can be put into a coordinate invariant form that reduces to

$$\Delta t = \frac{2q_{a\beta} \cdot e_k / c_{\beta}}{[a^2 (e_k \cdot e_a)^2 + b^2 (e_k \cdot e_b)^2]^{\frac{1}{2}}}$$

where $e_k = e_n \times (q_{a\beta} \times e_n) / \left|q_{a\beta} \times e_n\right|$ and $e_a$ and $e_b$ are unit vectors along the major and minor axes of the ellipse (Fig. 2), with $e_b = e_n \times e_a$. The vector $q_{a\beta}$ is known from Ref. [5] to define the nature of the sender-receiver configuration for an incident wave of type $\alpha$ with propagation direction $\hat{p}_i$ to that of a scattered wave of type $\beta$ received at location $x_0 = R_0 \hat{x}_0$ as shown in Fig. 3. This vector $q_{a\beta}(\alpha, \beta = P, SV)$ can be written compactly in matrix form as

$$\begin{align*}
q_{pp} & q_{ps} \\
q_{sp} & q_{ss}
\end{align*} = \begin{bmatrix}
\hat{p}_p - \frac{\hat{e}_o}{p} & -\frac{1}{p} \hat{p}_p - \hat{e}_o \\
-\frac{\hat{e}_o}{p} & \hat{p}_p - \frac{\hat{e}_o}{p}
\end{bmatrix}$$

where the wavespeed ratio is $\kappa = c_p / c_{SV}$.

For example, for a pulse-echo set-up with an incident $P$-wave we see that $q_{pp} = -2e_0$. As in Ref. [5] for the case of incident SV-waves ($\alpha = SV$), we will assume that the critical angle where the reflected $P$-wave is at grazing incidence is never exceeded. Hence, no pulse distortion will be present. In Eq. 2 there are a total of 5 independent unknowns because of the constraints $e_n \cdot e_n = e_a \cdot e_a = 1$ and $e_n \cdot e_a = 0$. To make this relationship explicit, we write all of these unit vectors in terms of spherical coordinates within an arbitrary fixed coordinate system, i.e.:

$$e_\gamma = \cos \theta_\gamma e_1 + \cos \phi_\gamma \sin \theta_\gamma e_2 + \sin \theta_\gamma e_3$$

where $(\gamma = \alpha\beta, \alpha, \beta, n)$ with $e_{\alpha\beta} = q_{a\beta} / q_{a\beta}$ and $e_j (j = 1, 2, 3)$ are unit vectors along the coordinate axes. Thus, all the constraints are satisfied automatically except $e_n \cdot e_a = 0$. This constraint can be used to solve for $\theta_a$ through

$$\tan \theta_a = \frac{-\cos (\phi_n - \phi_a)}{\tan \theta_n}$$

Placing these results in Eq. 2 then yields, finally

$$\frac{c_{\beta} \Delta t}{d} = f(\theta_n, \phi_n, \phi_a, d, d'; \alpha\beta, \phi_a, \phi_b)$$
All of the above data suggests that the proposed constrained inversion algorithm performs well in the presence of synthetic errors, particularly for sizing estimates. This is encouraging because in the cases considered the number of "measurements" varied only from 8-24 values. Thus, there appear to be no severe restrictions on the amount of data sets needed by the method. Also, computational times were not excessive, varying from as low as 7 CPU seconds to as high as 316 CPU seconds on a time-sharing VAX-11/780.

SUMMARY

The proposed constrained inversion algorithm was shown to function well when tested with noisy synthetic data. In general, it appeared that sizing estimates were more accurate and stable than orientation estimates. Refinements in the nonlinear least squares program employed here are underway to see if all of these estimates can be significantly improved.

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Fig. 4a-e. Percent error versus aperture size for an ellipse with $a/b = 1.5$, 5% noise.
Fig. 5a-e. Percent error versus aperture size for an ellipse with $a/b = 3.0$, 5% noise.
REFERENCES


