CONTRAST MECHANISMS IN THE THERMOACOUSTIC MICROSCOPE

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INTRODUCTION

The thermoacoustic microscope [1-9] uses a modulated particle beam (electrons, photons or ions) as a heat source on (or near) the surface of the sample, and a piezoelectric transducer attached elsewhere on the sample as a detector of acoustic waves generated by the beam. The modulation frequency is typically $10^3$ to $10^7$ Hz, so the acoustic signal is in the sonic to ultrasonic range. These acoustic signals are used to produce images of surface and near surface features of the solid by scanning the source over the face of the sample. In order to make full use of this type of microscope as a quantitative NDE tool, one must be able to interpret the images in terms of the physical properties of the features being imaged. The interpretation of an image resulting from variations in, say, the elastic constants as if it were caused by variations in, say, thermal expansion coefficient, could lead to totally incorrect conclusions about the nature of a defect. This paper summarizes a theoretical analysis which can form a basis for assessing the relative importance of different contrast mechanisms.

THEORY

The coupled differential equations which describe the acoustic displacement, $\xi$, and the temperature, $T$, in an elastic solid are just representations of the conservation of energy and momentum. These equations and their solutions have been described elsewhere [10,11]. These solutions were obtained by separating the coupled equations into equations for their uncoupled normal modes. These modes, in the absence of nearby surfaces, consist of an acoustic shear wave (whose source is the momentum deposited by the beam), a compressional acoustic wave (for which both the momentum and the energy deposition act as sources), and a thermal wave (whose sources also are both momentum and energy). The wave numbers for these modes are $k_S = \left[ \frac{\rho \omega^2}{\mu} \right]^{1/2}$, $k_C = \left[ \frac{\rho \omega^2}{(\lambda + 2\mu)} \right]^{1/2}$, and $q = \left[ \frac{\rho \omega^2}{K} \right]^{1/2}$, respectively. Here, $\omega$ is the angular frequency, $\lambda$ is the Lame constant, $\mu$ the shear modulus, $\rho$ the density, $c$ the specific heat at constant strain, and $K$ is the thermal conductivity. In most situations, the momentum contributions are small, and one obtains only a thermal wave and a compressional acoustic wave. The one-dimensional expressions for the two waves are given by
\[ T = \frac{Q}{2\Omega q} e^{iqx} \quad \text{Thermal wave (1)} \]

\[ \xi = r_1 \frac{Q}{2\Omega q} e^{iqx} \]

and

\[ \xi = -r_1 \frac{Q}{2\Omega q} e^{ikc} \quad \text{Acoustic wave (2)} \]

\[ T = -r_1 r_2 \frac{Q}{2\Omega q} e^{ikc} \]

with

\[ r_1 = \frac{-iq(3\lambda+2\mu)}{(\lambda+2\mu)(q^2-k_c^2)} \quad \text{(3)} \]

and

\[ r_2 = \frac{\omega T_0 \alpha(3\lambda+2\mu)k_c}{\kappa(q^2-k_c^2)} \quad \text{(4)} \]

Here, Q is a measure of the source strength, \( \xi \) is the particle displacement, \( \alpha \) is the thermal expansion coefficient, \( T_0 \) is the absolute dc temperature, and \( r_1 \) and \( r_2 \) characterize the displacement associated with the thermal wave and the temperature associated with the acoustic wave, respectively. These additional terms in each wave are associated with the presence of thermal expansion. While the thermal wave mode involves elastic displacements as well as variations in temperature, these displacements are an intrinsic part of the thermal wave and damp out along with the temperature variation in a distance determined by the thermal diffusion length. These displacements therefore are not detected by the piezoelectric transducer, which is usually relatively distant on this scale. The acoustic wave, in turn, has an associated intrinsic temperature variation, but one which propagates with the acoustic wave vector and which is completely unrelated to the thermal wave. In the three-dimensional version of these solutions, both the thermal and acoustic wave are seen to originate in the small region in which the beam deposits its energy. This localization of the sources of the waves has important ramifications when questions of resolution are considered. Of course, the situation will be complicated in the presence of a boundary, where mode conversion will occur among thermal waves and both bulk and surface acoustic waves. Of these processes, we will only consider the thermal wave-compressional acoustic wave mode conversion.

Using the one dimensional version of this normal mode description, we have evaluated the effects of the presence of a free surface and have calculated [10] the relative efficiencies of four distinct processes for imaging of a discontinuity such as a grain boundary. It is assumed that the source is an electron or photon beam.
for which the momentum contribution to the waves is small compared to the energy contribution. These processes are represented schematically in Figs. 1-4. For each figure, dashed wavefronts indicate acoustic waves, and solid wavefronts indicate thermal waves. Figure 1 represents the scattering of the directly generated acoustic radiation. The process illustrated in Fig. 2 involves the mode-conversion of the thermal wave to an acoustic wave at the sample surface, followed by acoustic scattering. In Fig. 3, the thermal wave scatters first, then mode-converts to an acoustic wave at the surface. Finally, in Fig. 4, the thermal wave mode-converts to an acoustic wave at the scatterer rather than at the sample surface. While other

Figure 1. Schematic diagram of an imaging process which involves only scattering of acoustic radiation (dashed wavefronts).

Figure 2. Schematic diagram of an imaging process which involves mode conversion of thermal waves (solid wavefronts) to acoustic waves (dashed wavefronts) at the surface, followed by scattering of the acoustic waves.
processes can occur, these processes have the distinction of ending up with an acoustic wave which can be detected by the piezoelectric transducer. The relative efficiencies of these processes (in the one dimensional model) can be expressed as

\[ \xi_1 \sim A \left( k c d \right) \delta \left[ \rho (\lambda + 2\mu) \right] \]  
\[ \xi_2 \sim A \left( k_c/q \right) \delta \left[ \rho (\lambda + 2\mu) \right] \]
Here, \( A \) is an overall amplitude factor, \( d \) is the depth of the source, and \( [x] = \frac{x_1 - x_2}{x_1 + x_2} \) represents the fractional discontinuity in the quantity \( x \) across the boundary of the scatterer. Note that in Eq. (8) the elastic constants enter only through the discontinuity in Poisson's ratio. The factor \( k_{cd} \) in Eq. (5) results from the interference of the direct acoustic radiation and the reflected acoustic wave from the surface and typically is much less than one.

In the three dimensional case, however, this interference is less and less complete for angles away from the forward direction, and the factor is no longer small. The factor \( k_{c} / q \), which appears in Eqs. (6) and (7), represents the effects of the mode conversion process at the surface of the sample. This factor, which is essentially the ratio of the thermal wavelength to the acoustic wavelength, is also much less than one for most experimental situations.

\[ \xi_3 \sim A \left( k_c / q \right) \delta(\beta_c \rho) \]  
(7)

\[ \xi_4 \sim A \delta \left[ \frac{\alpha(3\lambda + 2 \mu)}{(\lambda + 2 \mu) \beta_c \rho} \right] \]  
(8)

The contrast of the image produced by the thermoacoustic microscope will depend upon the nature of the material properties associated with the feature being imaged, as well as the instrumental parameters. Different combinations of these properties and parameters can cause one or the other of the processes described above to be dominant. Thus a thermoacoustic microscope could operate in a variety of imaging modes. If Process 1 (see Fig. 1) were dominant, the microscope would be an acoustic microscope instead of a thermal wave microscope. In three dimensional scattering, where the factor \( k_{cd} \) in Eq. (5) is not present, Process 1 is expected to be dominant over Process 2. Both involve scattering of acoustic waves from the defect, and hence depend on the fractional change in the density and the elastic constants for contrast. However, Eq. (6), which corresponds to Process 2, contains the additional small factor \( k_c / q \), which is associated with the mode conversion at the surface. The scattering in Process 3 involves only thermal waves, so that for this process one obtains a pure thermal wave image from the thermoacoustic microscope.

This is the scattering process which is commonly assumed to be responsible for the contrast in a thermoacoustic microscope. However, Eq. (7), which corresponds to this process, also contains the small factor \( k_c / q \). Therefore Process 3 is likely to be dominated by Process 4 (Eq. (8) contains no such small prefactor) unless the changes in thermal conductivity at the scatterer are at least of the order of \( q / k_c \) times larger than the corresponding changes in the other material parameters. Process 4, because it represents mode conversion at the scatterer, will have a relative importance which will be determined by the fractional changes in both elastic and thermal properties, and hence corresponds neither to a pure acoustic nor a pure thermal wave microscope. With Process 4, one would have a "mode conversion microscope", with the incoming thermal wave's being mode-converted to an acoustic wave at the scatterer. Thus, the thermoacoustic microscope is likely not to be a thermal wave microscope, at least as normally conceived, because of the absence of the scattered thermal wave in the processes (1 and 4) which are likely to be dominant. The experimental determination of which process is...
dominant in a particular situation may require consideration of contrast variations with frequency, energy and momentum of the beam. In addition, experimental observations of the corresponding variations in spatial resolution for different subsurface scatterers can be very useful in making such an assessment.

Because the thermoacoustic microscope utilizes a highly focused particle beam, the source of the waves is normally much smaller than either the thermal or acoustic wavelength and is beneath the surface. This introduces the possibility of achieving a situation in which the source is both very small compared to a wavelength and very close to the scatterer. This is a situation which is not normally encountered in scattering theory. Scattering theories normally start with Green's function for the Helmholtz equation, and expand the phase of the wave in powers of the ratio of some characteristic length (say, the size of the scatterer) to some distance (say, the source-scatterer distance or the scatterer-detector distance). Depending upon the degree of approximation necessary in this expansion, one talks about the Fraunhofer (far field) limit or the Fresnel (near field) limit. The inverse power of the distance which appears in the Green's function is assumed to be slowly varying in both of these limits. However, in the situation which we described above, with a small source which can be brought close to the scatterer, the variation of the phase factor is dominated by that of the inverse power of the distance, and the scattering becomes quasi-static (extreme-near-field limit) [12]. In this limit the incident energy is effectively localized to a region of the scatterer whose dimensions are comparable to the size of the source or the source-scatterer distance, whichever is larger. Therefore adjacent regions of the scatterer can be imaged with a resolution which depends only on these dimensions, and not on the wavelength. Since in the thermoacoustic microscope one has control of both the lateral position, and to some extent the depth (by varying the particle energy) of the source, it is often possible to achieve an image resolution which is much better than either the thermal or acoustic wavelength, regardless of the scattering mechanism involved. This is also true for microscopes (thermal wave or acoustic) which use lasers as a source, except that because the source is on the surface, the resolution is limited by the depth of the scatterer.

It is interesting to note that two recent experiments have been carried out using thermoacoustic microscopes with scatterer depths which were tens of thermal diffusion lengths from the source [13,14]. At these depths, the scattering certainly is acoustic. In one of these experiments [13] the scatterer has a continuously varying depth. In this experiment the observed resolution is two or three orders of magnitude better than the acoustic wavelength and deteriorates linearly with depth, as predicted by the extreme-near-field theory [12]. Another recent experiment using such a microscope has been carried out on an aluminum bicrystal having a nearly vertical grain boundary [15]. The apparent width of the image of that boundary was found to vary as the inverse square root of the frequency. Such an observation would rule out Process 1 as the principal contrast mechanism in that case. Although detailed calculations have not been carried out, this appears to be consistent with Process 4, or, except for the small factor $k_c/q$, with Process 3. The wavelength dependence is expected in this case because of the special geometry of the scatterer (it is extended in depth). Thus the microscope may be operating either as a mode conversion microscope or as a thermal wave microscope in this situation.
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