INTRODUCTION

In general, an interferometer may consist of coherent source split into a ‘signal’ beam and a ‘reference’ beam, with irradiances $I_1$ and $I_2$, respectively. The signal beam carries information about surface/thickness of the test object. By bouncing the signal beam off of the test object (highly reflective objects) or passing it through (transparent objects) the object, the phase of the signal beam is altered according to the surface (or thickness) variations of the object. The reference and signal beams may then be interfered to yield an irradiance distribution:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(k \delta(x,y) + \Phi_0)$$  \hspace{1cm} (1)

where $\delta(x,y)$ is the surface deformation, $\Phi_0$ is a constant phase shift, and $k$ is the wave number.

For the interferometers shown in Figure 1, the signal beam will see twice the surface deformation (once going toward the object and once returning from the object.)
The irradiance pattern would be:

\[ I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos(k_2\delta(x,y) + \Phi_0) \]  

(2)

Interferometers may be adapted for holographic interferometric studies by replacing the viewing ‘screen’ with a film plate, tilting the reference mirror (an off-axis reference beam approach,) and adjusting the arm lengths such that their difference falls within the coherence length of the source. This is shown in Figure 2.

The term ‘resolution’ is used to indicate how well a surface deformation can be resolved. For example, the Michelson interferometers shown in Figure 1 have twice the resolution as in interferometer where the signal beam passes through the test object once (ie. a Mach-Zender interferometer.)

Interferometric resolution may be increased by either amplifying the surface deformations, \( \delta(x,y) \), or the phase shift associated with those deformations, \( k\delta(x,y) \).

Various techniques have been proposed and implemented based on these concepts. Only a brief review of the techniques are presented here; it is suggested that the reader consult the referenced literature for more in depth detail.

Figure 1. Simple interferometers.

Figure 2. Holographic system based on interferometer design.
**Figure 3.** Polarization isolating interferometer.

**REVIEW OF PROGRESS IN RESOLUTION ENHANCEMENT**

**Polarization Isolation**

Polarization selective optics are used to steer the signal beam so that it traverses the object 4 times, yielding an increase in the resolution by a factor of 2 over a simple Michelson. An example of this interferometer is shown in Figure 3.

**Multipass Interferometry[1][2]**

A Multipass Twyman-Green (MTG) Interferometer is shown in Figure 4. It can be thought of as a Fizeau interferometer coupled with a Michelson. The Fizeau interferometer is composed to a partially reflecting optical flat and the test object. A portion of the light entering the Fizeau is reflected off the test surface and exits, while the remainder is reflected off the partial reflector, back towards the surface. This reflection will pick up an additional 2 times the surface deformation. This process repeats to generate a number of 'orders' exiting the Fizeau, such that each order has an additional 2 times the surface deformation. In order to extract the \( m \)th order (ie. \( 2m \) times the deformation,) the partial reflector is tilted, causing an angular separation of the orders. The orders can then be brought to a focus and the desired one selected using a spatial filter. The reference mirror of the Michelson is then tilted to coincide with the selected order.

**Figure 4.** Multipass Twyman-Green interferometer.
Temporal Gating[3]

A folded Tywman-Green configuration is shown in Figure 5. Both the reference and the signal arms of the interferometer contain cavities formed between a partial reflector and the reference mirror or test object, respectively. For a pulse launched into the interferometer, a pulse train will be returned from each of the arms. Much like the MTG, some light passes through the partial reflector, while some is reflected back to pick up additional surface information. By making the cavity lengths long enough, one can control the temporal spacing of the reflections, thus forming a pulse train where each pulse has an additional 2 times the surface deformation information as the previous pulse. The desired interferogram is then captured using a camera with a high speed shutter.

Short Coherence Length[4]

A Non-Walkoff MTG is shown in Figure 6. As with the MTG, a cavity is established between the test object and a partial reflector. The cavity length is chosen such that it is much greater than the coherence length of the source. In general, an interferogram can only be formed when the difference in the reference and signal arms of the interferometer is less than the coherence length of the source. With a short coherence source, the reference and signal arms must be matched very closely.
As with the temporal gating case, each reflection coming out the cavity has an additional factor of 2 times the surface information. However, in this case, the desired ‘order’ is selected by matching the reference arm length to the distance traveled by a desired ‘order,’ thus only that ‘order’ will form an interferogram. For example, in order to achieve twice the resolution of a simple Michelson, the reference arm is slid back a distance equal to two times the cavity length. This matches the reference arm length with the distance traveled by the first reflection, yields an interferogram showing 4x the surface deformation.

**Digital Fringe Multiplication**[5-8]

This is a image processing technique by which the existing number of fringes in an interferogram can be increased. A series of interferograms, resulting from a series of optical phase shifts, are combined so as to produce interferograms of higher resolution.

**Phase Difference Amplification**[10-13]

This technique uses a non-linear hologram to achieve a phase amplification. When a non-linear hologram is reconstructed, multiple orders are generated[9]. These higher orders possess amplified object phase fronts. By interfering the appropriate higher orders, interferometric resolution can be increased.

**PHASE AMPLIFICATION USING NON-LINEAR OPTICS**

Consider passing an intense beam through a non-linear optical material of thickness d. The intense beam will cause localized changes in the index of refraction of the material proportional to the intensity of the beam according to $n(I) = n_o + n_2 I$, where I is the input irradiance. The induced index changes can then be read by passing a weak probe beam through the material and measuring the induced phase shift.

If the intense beam is the output of an interferometer, it can be shown that the weak probe beam passing through the material will acquire an amplified version of the phase difference in the original (or ‘biasing’) interferogram.

**Theoretical Basis**

Consider the form of the biasing interferogram:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(k \delta(x,y) + \Phi_0)$$  \hspace{1cm} (3)

where k is the wave number, $\delta(x,y)$ is the surface deformation, and $\Phi_0$ is a uniform phase offset.

The induced phase shift on a probe beam is given by:

$$\Phi_{LC} = 2 \pi \beta I + 2 \pi I_0$$  \hspace{1cm} (4)

where $\beta = \frac{k n_o d}{2\pi}$ and $I_0 = \frac{k n_o d}{2\pi}$.  
How displacements are mapped to phase shifts can be found by determining the sensitivity of the probe phase shift with respect to surface deformations:

$$\frac{d\Phi_{LC}}{d\delta} = \frac{dI_{1C}}{df} \frac{df}{d\delta}$$  \hspace{1cm} (5)

Integrating (and assuming sub-fringe displacements) leads to an expression for the induced probe phase shift:

$$\Phi_{LC} = -4\pi\beta k \sqrt{I_1I_2} \left[ 0.5k \delta^2 \cos(\Phi_o) + \delta \sin(\Phi_o) \right] + \Phi_{0LC}$$  \hspace{1cm} (6)

Now, consider using the probe beam as the signal beam in another interferometer. The resulting 'probe' irradiance pattern is given by:

$$I_{LC} = I_{1LC} + I_{2LC} + 2\sqrt{I_{1LC}I_{2LC}} \cos(\Phi_{1LC})$$  \hspace{1cm} (7)

which may be equated with a 'general' interference pattern:

$$I_{LC} = I_{1LC} + I_{2LC} + 2\sqrt{I_{1LC}I_{2LC}} \cos(\delta_{LC}(x,y) + \Phi_{0LC})$$  \hspace{1cm} (8)

where \(\delta_{LC}(x,y)\) is the 'apparent' displacement, that is, the equivalent displacement that would be needed to induce the same irradiance pattern.

Equating the expressions, one can find an expression for the apparent displacement:

$$\delta_{LC} = -4\pi\beta \sqrt{I_1I_2} \left[ 0.5k \delta^2 \cos(\Phi_o) + \delta \sin(\Phi_o) \right]$$  \hspace{1cm} (9)

In the case that the phase offset of the biasing irradiance pattern is \(\Phi_o = \pi/2\), such that the mean irradiance is \(I_1 + I_2\), the apparent displacement reduces to:

$$\delta_{LC} = -4\pi\beta \sqrt{I_1I_2} \delta$$  \hspace{1cm} (10)

Thus, using this technique, the sub-fringe displacements captured by the biasing interferometer will see an amplification factor of \(4\pi\beta \sqrt{I_1I_2}\).

**Material Choice**

In order to achieve a reasonable amplification factor and keep the optical system parameters practical, the candidate non-linear optical material must possess a large optical non-linearity. One possible material is the nematic liquid crystal, 5CB (pentyl-cyano-biphenyl.) Durbin, et. al.[14], have shown that a thin (250 \(\mu\)m thick) cell filled with homeotropically aligned 5CB can induce large phase shifts in a probe beam at a relatively low optical powers (100's of \(\text{W/cm}^2\).) According to the data presented, the value of \(\beta\) is approximately 0.9 cm²\(\text{W}^{-1}\text{rad}^{-1}\) for normal incidence of the high power beam.
The literature suggests that it is possible to electrically bias the cell with a DC field in order to achieve the ‘giant’ optical non-linearity at relatively low optical powers[15]. In this case, assuming 1 W/cm² in both the signal and reference arm of the biasing interferometer, the sub-fringe displacement amplification factor is $= 11.3$. If the biasing interferometer is in a configuration such as in Figure 1, the amplification factor is doubled to $= 22.6$.

The choice of material does have certain drawbacks. Chief among these is the temporal response of the material. Liquid crystals are rather sluggish, taking milliseconds to seconds to respond to optical biasing, thus limiting the application to static or slow varying deformation studies. Other potential problems are the high temperature dependence of the index of refraction (‘hot spots’ in the biasing interferogram can induce temperature changes, resulting in index changes) and insuring a uniform thickness and distribution of the liquid crystal in the cell.

Experimental Configurations

A possible system for generating phase amplified interferograms is shown in Figure 7. The system is designed such that biasing interferometer and a probe beam pass through the LC cell. The needed optical power level is achieved by concentrating the biasing interferogram using a telescope configuration. Figure 7 shows a double exposure holography configuration. This configuration has the benefit of removing system aberrations from the final interferogram.

![Figure 7. Double exposure holography configuration.](image)

CONCLUSION

An interferometer configuration has been presented, based on liquid crystal as the non-linear medium, which could provide resolution enhancements on the order of 10 using reasonable optical power levels and space constraints. The systems presented, while targeting interferometry, can be adapted for full field holographic studies, using configuration alterations similar to those shown in Figure 2, allowing for the subsequent increase in holographic interferometric resolution.
REFERENCES