EDDY CURRENT SCAN SIMULATION WITH COUPLED FEM/BEM

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INTRODUCTION

The finite element method (FEM), boundary element method (BEM), and volume integral method (VIM) are well-known numerical techniques that have been successfully applied to electromagnetic calculations. At Southwest Research Institute (SwRI), we routinely use these calculations to design eddy current probes. In our applications, the desired information is the impedance of a probe as it is scanned across a structure containing a flaw.

In order to evaluate more complicated probe designs, procedures for coupling FEM with VIM, and FEM with BEM, are currently under development. The FEM/VIM procedure has been completed and is now in use to generate simulated scans over notches in planar, layered materials [1]. When complete, the FEM/BEM technique will extend our capability to simulate scans over surfaces with irregular geometry.

BACKGROUND

With FEM, the impedance is calculated for the probe and structure at one fixed position. To get the impedance for all the points in a scan would require a separate model and separate calculation at each scan point. Developing the FEM model is tedious; it is usually too time-consuming to generate eddy current scan data from multiple FEM calculations in this manner.

The boundary element method (BEM) is another technique that we have used to calculate probe impedance for an eddy current scan. In our formulation, the magnetic scalar potential is first determined for a probe in air with no other material or structure present. This solution is then used with the boundary integral equation and reciprocity relationship to calculate the probe impedance at every point in a scan over the structure containing a flaw.

One of the limitations of our BEM formulation has been the inability to model probes with complicated geometry or shielding materials. For example, we often use a ferrite core in our probes; at times, we may also use copper shielding. In the past, we were not able to evaluate probes of this type with our BEM code. All of our BEM calculations were for probes with windings that could be evaluated analytically.

In this paper, we present the status of ongoing work to combine FEM and BEM modeling so that we can generate scan data for probes with more complicated geometry. With the combined procedure, the FEM solution for the probe in air is only calculated once, after which it can be used with our BEM code to generate scan data for any number of flawed structures. The advantage of our BEM code is that once the probe field in air is calculated, a scan over a flaw can be generated faster and more efficiently than with FEM. In the sections that follow, we discuss difficulties that were encountered with the existing BEM code, and present solution methods to eliminate those difficulties.
THEORY

Since \( \nabla \times \mathbf{H} = 0 \) in air (outside of the probe and structure being scanned), it follows that \( \mathbf{H} = -\nabla \phi \). Using \( \nabla \cdot \mathbf{H} = 0 \), we obtain Laplace’s equation, \( \nabla^2 \phi = 0 \), for which the boundary integral equation is well known:

\[
\phi(\mathbf{x}) = \phi_0(\mathbf{x}) + \int \left[ G_0(\mathbf{x}, \mathbf{x}') \frac{\partial \phi(\mathbf{x}')}{\partial n} - (\phi(\mathbf{x}') - \phi(\mathbf{x})) \frac{\partial G_0(\mathbf{x}, \mathbf{x}')}{\partial n} \right] dS
\]

where the integral is over the surface of the structure or material being scanned, \( G_0 \) is the free space Green’s function, and \( \phi_0 \) is the solution for the probe in air with no structure or material present, given by [2]:

\[
\phi_0(\mathbf{r}, z) = \frac{-i}{2\pi\mu_0} \int e^{i\mathbf{k}\cdot\mathbf{r}} \mathbf{k} \times \mathbf{A} \frac{d^2k}{k}
\]

where \( \mu_0 \) is the permeability of free space, and \( \mathbf{A} \) is the 2D Fourier transform of the magnetic vector potential. The boundary integral equation is exact for the field everywhere outside of the structure or material being scanned, and outside of the probe itself.

The discretized integral equation is

\[
\sum_j M_{ij} \phi_j = \phi_i^0
\]

where \( M_{ij} \) is independent of probe geometry.

In order to calculate the change in probe impedance caused by the presence of the structure being scanned, we use the reciprocity relationship [2]

\[
\Delta Z = \frac{-i\omega\mu_0}{I^2} \int \left[ \phi \frac{\partial \phi_0}{\partial n} - \phi_0 \frac{\partial \phi}{\partial n} \right] dS
\]

where \( \omega \) is excitation frequency, \( \mu_0 \) is free-space permeability, \( I \) is the excitation current, \( \phi \) is the solution to the boundary integral equation (1), and the integral is over the surface being scanned.

Our new approach involves the calculation of \( \phi_0 \), which is the matrix that contains all of the probe information. Instead of determining \( \phi_0 \) analytically, we first model the probe in air using FEM. The magnetic vector potential \( \mathbf{A} \) on a plane is extracted from the FEM solution, and a Fourier Transform is performed to give \( \mathbf{A} \). \( \phi_0 \) can then be calculated in a straightforward manner. No other changes in the procedure for determining \( \Delta Z \) were necessary.

The advantage to the new approach for determining \( \phi_0 \) is that any geometry and material can be included in the probe design, and we are no longer restricted to air core coils.

APPLICATIONS

In order to check out the BEM code before beginning modifications, we performed a calculation for a 5-mm-diameter air-core coil and compared the results with those from our VIM (volume integral method) code. We have used VIM extensively, and the results of VIM calculations have compared favorably with benchmark data [1]. Thus, we have reason to believe that the VIM calculation is reliable; and, in the absence of experimental data, we have used the VIM solution as a basis for comparison.
In our first set of calculations, the BEM solution for a scan over a 1.2(1) by 0.04(w) by 0.4(d) cm flaw at 20 kHz showed poor agreement with the VIM solution. At higher frequencies, the difference between the two calculations was even greater, leading us to conclude that the BEM code was not as accurate as we had believed.

Initially, we believed the inaccuracy was due to approximations to $\frac{\partial \phi}{\partial n}$ in equation (1). In our scalar potential formulation of the boundary integral equation, $\frac{\partial \phi}{\partial n}$ is an unknown. In order to solve the equation, we invoke the impedance boundary condition (IBC), which is exact only if the incident field is uniform and the surface is an infinite plane. Use of the IBC leads to the simple expression:

$$ \frac{\partial \phi}{\partial n} = \left( \frac{1+i}{2} \right) \mu_s \delta \frac{\partial^2 \phi}{\partial n^2}. $$

(5)

Laplace’s equation and the introduction of boundary element shape functions leads to [2]:

$$ \frac{\partial \phi}{\partial n} = \sum_q N_{q} \phi_{q}. $$

(6)

To obtain a correction to the IBC, we made use of a more general relationship, which holds for a nonuniform field on a plane surface [2],

$$ \frac{\partial \phi(\vec{x})}{\partial n} = \frac{\mu_s}{2\pi} \frac{r \frac{\partial^2 \phi(\vec{x}')}{\partial n^2}}{d^2} \ d^2 x' $$

(7)

where $q = (i - 1)/\delta$, $\delta$ is the skin depth, $r = |\vec{x} - \vec{x}'|$, and the integral is over the plane surface of the conductor. If the field is approximately constant over distances of the order of $\delta$, then the second derivative inside the integral can be approximated by its value at $\vec{x}$, which leads to equation (5), the IBC. A higher order approximation is obtained from the correction term

$$ \frac{\mu_s}{2\pi} \left[ \frac{\partial^2 \phi(\vec{x}')}{\partial n^2} - \frac{\partial^2 \phi(\vec{x})}{\partial n^2} \right] \left( \frac{\mu_s}{r} \right) \ d^2 x' $$

(8)

by assuming that $\frac{\partial^2 \phi(\vec{x}')}{\partial n^2}$ varies with position according to the same boundary element shape functions used in equation (6), expanding the result through second order about $\vec{x}' = \vec{x}$, and carrying out the indicated integration. The end result has the same form as equation (6), but with a correction term added to the IBC coefficients.

A comparison between the BEM with corrected IBC term and the VIM solutions is shown in Figure 1. The complex impedance is shown as a function of scan position, starting in the middle of the flaw and scanning along the length. Even with the correction term, the solutions do not agree.

This led us to suspect that the inaccuracy of the IBC was much worse than we thought. However, after this work was completed, we learned of another difficulty with our implementation of the boundary integral equation [3]. In our original formulation, we ignored the fact that we must be able to add a constant in Laplace’s equation. In addition to

$$ \nabla^2 \phi = 0 $$

(9)

we should have

$$ \nabla^2 (\phi + C) = 0. $$

(10)
Figure 1. Results of simulated scan over a 1.2(l) by 0.04(w) by 0.4(d) cm surface-breaking flaw with a 5-mm-diameter air core coil. Impedance change in the coil is calculated using both VIM and the BEM with a correction term for the impedance boundary condition (IBC). Since the VIM code has been validated experimentally for similar problems, we conclude that the BEM calculation is incorrect.

And the equivalent BEM matrix equation

\[ M\phi = \phi_0 \]  

becomes

\[ M(\phi + C) = \phi_0 \text{ and } MC = 0. \]  

The fact that \( MC = 0 \) means that the matrix is singular, a condition we failed to consider in our earlier formulation. We are now modifying our code to include the necessary special handling of a singular matrix.

Because of the ambiguity implied by equation (12), we are also required to add an auxiliary condition, such as \( \nabla \cdot B = 0 \), in order to obtain a solution.

**SUMMARY**

The coupled FEM/BEM method offers the flexibility of the FEM in modeling complex probe configurations with the scan simulation efficiency of the BEM. With the scalar potential formulation of the BEM, it is, however, necessary to invoke the impedance boundary condition (IBC) or some similar approximation to the normal derivative of the potential. Results reported here indicate that the validity of the IBC is subject to question, but another recently discovered deficiency in the BEM formulation suggests that further study is required.
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REFERENCES

3. Private communication with John Bowler, University of Surrey; Steve Burke, ARL Melbourne; and Norio Nakagawa, CNDE at Iowa State University.