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A statistical analysis of some of the admissible hypotheses underlying the demand for food products

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A STATISTICAL ANALYSIS OF SOME OF THE ADMISSIBLE
HYPOTHESES UNDERLYING THE DEMAND FOR FOOD PRODUCTS

by

John Richard Tedford

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
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INTRODUCTION

One striking feature in the United States today is the large degree of interdependence existing within agriculture and among agriculture and other sectors of the economy. Government and non-agricultural groups currently perform much more important roles and have greater impact on the organization and operation of agricultural firms, markets, and programs than they did in past generations. With the increasing dependence of agriculture on government and other sectors, the formulation of economic policies, either by government or by firms, has become more complex and requires more knowledge and intricate analysis today than previously. If the over-all repercussions of policies are to be accounted for, it will be necessary that policy-makers obtain and use knowledge of the interdependence existing within and among relevant sectors and that we obtain the most we possibly can from resources devoted to the policy-making activities.

One possible way to obtain more from resources employed in policy-making is by treating the broader problems of policy in a systematic manner and by use of quantitative economic analysis wherever possible. It appears that wider application of existing economic tools of analysis would be preferable to the usual practices of today. Certain analytical tools, such as the main elements of the theory
of choice, have been available for quite some time and could have been used in policy making to better advantage than they were. Recent improvements in the fields of economics, statistics, and econometrics have added to our knowledge of concepts and techniques and have extended the range of application for certain types of analysis. Developments in these and other fields of study have added to our understanding of basic relationships and have brought about more and better data. In recent years, Tinbergen [120], [119] has contributed to our knowledge on the logical structure of general problems of economic policy and Theil [116] has added to our knowledge on some of the basic problems involved in policy making. These developments enable the problem to be set out more clearly today than formerly and point out the contributions that can be made by economic analysis.

Specific Purpose of the Study

One of the practical purposes underlying the construction of econometric models is to provide information on the quantitative characteristics of the economic system or relevant sectors within the system. When such information can be obtained, it will be possible to predict with a

---

1Numbers in brackets refer to references cited in the attached bibliography.
specified level of probability the future course of certain economic variables and the effects upon the system of various courses of action under given conditions.

Currently, there are no well defined procedures to follow in model building. The construction and choice of a suitable model is mainly a problem of knowing something about the economic and environmental phenomena and of making realistic assumptions about them. Although economic theory provides reasonably detailed specifications for certain sectors of the economic system, many competing sets of assumptions can be employed in the specification of certain structural equations without contradicting present knowledge of human behavior and the environmental conditions [17, pp. 570-2], [93, pp. 276-7]. When constructing models suitable for purposes of prediction and making policy recommendations, it is necessary therefore to consider several alternative economic theories as admissible hypotheses.

In dealing with many types of economic policy relating to agriculture, information is generally needed on the nature of consumers' responses to selected economic changes in the economy [97]. It has been a common practice in past studies to employ the classical static theory of consumer choice in the specification of demand equations, i.e., it has been a common practice to assume that the variables of primary importance in determining consumer behavior are prices and income. The use of such specifications, however,
do not reflect the time-incidence of the consumers' reactions to selected economic changes.

The main objective of this study is to test different hypotheses about consumers' adjustment over time to changes in certain economic variables. More specifically the study attempts to test hypotheses about (1) liquid assets, (2) long-run elasticities, and (3) variable preferences as additional factors that are important in determining consumers' consumption of the major dairy, meat, poultry, fish, and fat and oil food products.

In order to obtain estimates of the parameters in the postulated demand equations and hence to test the alternative hypotheses, it is necessary to construct an econometric model. A secondary objective of the study is to construct a complete shock\(^a\) model to present in simple form the underlying relations which reflect observable economic phenomena in the dairy, meat, poultry, fish, fat and oil, and other sectors of the United States economy.

Need for Knowledge of Economic Relations in Economic Policy-making

The formulation of policy can be considered as arising from a divergence between the actual and some desirable or

\(^a\)In shock models the errors are associated with the equations.
optimum situation. The process of policy-making consists of the deliberate selection and adaptation of available means in order to attain certain ends under different conditions. The ends of policy, following the arguments of Tinbergen, are determined by the preferences of the policy-makers. In cases of governmental policy in a democracy the preferences of policy-makers are considered to be similar, at least to a certain degree, with the preferences of the citizens. The alternative means of policy at the policy-makers' disposal are defined as those variables that can be controlled to a certain extent by the policy-makers. Changes in the available means used to offset the effects of changes in non-controlled variables are called courses of action.

The policy-making process can be described alternatively as decision-making under uncertainty. The general procedures involved in such decision-making can be outlined briefly as follows: (1) The changing actual situation has to be estimated. That is, knowledge is needed of the initial level and future course of the economic and other conditions comprising the situation in absence of changes in the alternative means of policy. (2) After the initial and future conditions have been estimated, it is necessary to find out if the estimated situation differs from what is considered to be the most desirable situation as defined by the preferences of policy-makers. When a divergence is
found to exist, the effects of alternative courses of action have to be estimated. That is, since several means of policy making may be available and adaptable to attain the selected ends, it is important that policy-makers have information on the changes in future conditions resulting from their behavior under alternative courses of action. (3) The estimated outcomes associated with alternative courses of action have to be appraised relative to the selected ends. The appraisal activities should also consider the costs of using different means because it is not a matter of indifference as to which means of policy are employed. The adaptation of different means involves costs which in general depend upon the level at which the means are used. Both the estimated outcomes and costs can influence the weights assigned to the alternative proposals by policy-makers. On the basis of the appraisal a choice has to be made among all of the estimated outcomes as to the course of action that is expected to yield the most desirable outcome. (4) The final activity consists of executing and supervising the selected course of action. This activity can perform the service of determining to what extent the adopted course of action does in fact yield desirable results and can provide information for future decision-making.

The effectiveness of the decision-making process depends in part upon the policy-makers' judgments about the relevant
ends and alternative courses of action. These judgments relate to the appraisal of the varying conditions and therefore are influenced by the data and methods of analysis available for use in providing estimates of the changing situations and of the effects of different means of policy. For example, given the ends of economic policy, the appraisal of alternative courses of action depends upon judgments made about (1) the estimated future conditions in the economic system and (2) how the course of economic conditions are altered by adaptation of the available means relative to what it would have been without them. Due to the element of uncertainty, the description of the changing future conditions in the system, resulting from variations in the controlled and non-controlled variables, is necessarily based upon expectations and predictions. When errors arise in the prediction process, the policy proposal will in general be suboptimal.

As indicated above, the effectiveness of policy-making depends partly on the policy-makers' judgments and partly upon statements made available to policy-makers about the future conditions. All statements about the future must necessarily employ data on past events. If these statements are to be useful for predictive purposes, however, it is necessary that systematic or orderly patterns existing in the past be determined and that relationships likely to hold true between past and future patterns can be identified and
established [79]. The determination of systematic patterns for the past requires the accumulation and analysis of data and the existence of effective theories or working hypotheses. A well-founded theory not only helps to determine the relevant variables and relationships existing between past values of these variables, but can also serve as a general model to describe the patterns of change over time.

Contribution of Economic Analysis in Economic Policy-making

By use of economic theory and quantitative economic analysis the economist can provide policy-makers with knowledge about the past patterns and with predictions about the future course of certain qualitative and quantitative economic variables under assumed conditions [62]. Tinbergen [119, pp. 8-9] states,

The logic of finding the best economic policy, that is, of finding the extent to which certain means should be used in order to achieve certain aims, is, in a sense, an inversion of the logic to which the economist is accustomed. The task of economic analysis is to consider the data (including the means of economic policy) as given or known, and the economic phenomena and variables (including the aims of economic policy) as unknown. The problem of economic policy considers the aims as given and the means as unknown, or at least partly unknown. . . economic analysis cannot provide a complete treatment of problems of economic policy. 'Extra economic' elements are involved: especially the choice of aims, and, to some extent, the choice of means. But, nevertheless, analysis can make some important contributions. It can (a) help to
judge the consistency of the aims assumed, and of the aims and means as a combination. . . . By detecting inconsistencies it may be (b) narrow down the possibilities and so contribute to a solution. Finally, it can also (c) determine the values of instrument variables in problems where targets or more general aims have been sufficiently specified and cannot be shown to be inconsistent.

The way in which the economy adjusts to changes in non-controlled variables and the way in which different means of economic policy influence future conditions depends upon how individuals, institutions and other phenomena respond under different conditions. As a result of the theoretical discussions in economics and developments in statistical inference during the past two decades [50], it has become possible to formulate more sharply the structure of the economy or sectors thereof by use of econometric models. Recent developments in quantitative economic research enable the economist to provide policy-makers with reasonably accurate predictions of the future course of economic variables under certain conditions [116], [170, pp. 5-6]. Judge [60, p. 4] states,

... recent research advances in econometrics have contributed to a sharper formulation and treatment of the choice of a model and the development of methods of estimation which are logically consistent with the abstract schemes proposed by economic theory.

The basic purpose in constructing econometric models [60, pp. 4-20], [62, pp. 1-12] is to describe the way in which the economy actually operates and to represent in a
simplified way the mechanisms which underlie the phenomena observed in the real world. In these models [67, pp. 27-48] certain economic variables, classified as endogenous variables, are considered to be determined by a complete system of structural equations which is consistent with the a priori knowledge and assumptions of the economist. These equations represent the basic economic relations [104, pp. 7-20] and reflect the direct logical ties between variables introduced by economic behavior or by the logic of definition or technique.

Usually distinctions are made between four types of economic relations, namely behavioral, technical, institutional, and definitional. The behavioral relations, such as supply and demand equations, serve to describe the behavior of individuals or groups of individuals. The relationships imposed by technical and physical conditions are reflected in the technical relations. The institutional relations indicate the relationships holding among variables that are due to the social and institutional framework of the economy. The definitional relations describe the inter-relationships among variables that follow simply from their definitions.

All of the structural equations taken together form the economic model and provide a simplified representation of the economy or relevant sectors under consideration. It would be too unrealistic, however, to look upon the
structural equations as being exact. To obtain predictions of the future course of the endogenous variables or estimates of the structural parameters, it is necessary to consider the character of the process generating the observations [69]. That is, in addition to the specification of the economic model, random shocks and/or random observation errors have to be specified to represent the random or stochastic elements in human behavior or in model specification. Marschak [79, p. 12] states,

Even if, in describing the behavior of buyers, we had included, in addition to the price and to the quantity demanded, a few more variables deemed relevant . . . , an unexplained residual would remain. It is called 'disturbance' or 'shock', and can be regarded as the joint effect of numerous separately insignificant variables that we are unable or unwilling to specify but presume to be independent of observable exogenous variables. Similarly, numerous separately insignificant variables add up to produce errors in the measurement of each observable variable (observation errors). Shocks and errors can be regarded as random variables . . . the probability that the observation on a certain endogenous variable will take a certain value, or will fall within a certain range of values, can be stated, provided that the probability distribution of observation errors of the variables is known. Similarly, no exact prediction but, in general, only probability statements, can be made if at least one of the structural relations is subject to random disturbances (shocks), even if all observations are exact.

By associating the disturbance with each equation or the errors with observations and treating them as random variables with a given probability distribution, we can apply the methods of statistical inference on the problem of parameter
estimation. The specification of such shocks or errors play a major role in determining the method of estimation and the probability distribution of the endogenous variables. When the set of structural equations is completely specified [69], [109] and consistent with all of the assumptions of the economist, the set will be referred to as the econometric model.

Accurate specification of the structural equations, relevant variables and statistical properties of the model are needed to avoid misunderstanding. It is only on the basis of such specifications that the economist can formulate precise questions and provide reasonable answers. The implications and statistical inferences drawn from the estimated parameters and economic relations are necessarily conditioned by the validity of the specification of the model. Economic theory and the economists a priori knowledge of the underlying economic structure can aid in the determination of relevant equations and variables and can provide a guide as to the appropriateness of the assumptions he might make about these aspects of his model. The choice of variables to include in the equations will also be affected by the availability of data. The specifications of the form of the equations and of the stochastic properties are often chosen partly to simplify the statistical analysis and are to some extent arbitrary. For a discussion of the details
and problems of specification see [17], [48], [50], [55],
[60], [62], [64], [67], [69], and [79].

By constructing econometric models, estimating the
structural parameters and obtaining predictions for economic
variables relevant to the problem under consideration, the
economist can provide policy-makers with such knowledge to
serve as a basis for evaluating the effects of alternative
courses of action. Tinberger [119, p. 27] states,

Problems of practical policy have to be in­
terpreted in terms of such simplified models and, af­
after the analysis has been made, an interpreta­
tion back (i.e., an application of the findings
of the model back to the real situation) has to be
attempted. Here, of course, divergencies of
opinion may, and necessarily will, arise. It is
an initial advantage for mutual understanding,
however, if consensus of opinion can be obtained
on the precise problems and answers constructed
with the aid of the models; this helps to narrow
down differences of opinion. And, if somebody
believes that model A does not fairly represent
the actual situation to be discussed, he will be
forced to indicate in what respect that model has
to be changed.
THEORETICAL CONSIDERATIONS

Examination of the literature reveals that numerous empirical studies have been made to derive demand curves and to obtain statistical estimates of price and income elasticities. It can be argued that most of these studies have been concerned with estimates of short-run elasticities almost to the complete neglect of the long-run aspects of demand. Consideration and estimation of the long-run aspects of demand appear to be of interest to economists. Working [179, pp. 972-3] states,

Perhaps the most unsatisfactory part of our knowledge of demand is in the area of demand dynamics. We have long recognized that there may be differences between short-run and long-run elasticities of demand, but little progress has been made in measuring them. Attempts to deal statistically with the dynamics of demand are, of course, nothing new. Lagged variables have been used in many studies, but for the most part, analyses have not been designed or interpreted in the light of their possible meaning as to differences between short-run and long-run elasticities. Probably the principal reason for our lack of quantitative knowledge of the dynamics of demand lies in the inherent difficulty, if not impossibility, of obtaining reliable estimates of long-run elasticities of demand.

Although the theoretical distinction between short-run and long-run elasticities is still rather arbitrary, a few recent studies have been designed and directed to obtain quantitative estimates of long-run elasticities of demand. This section outlines some of the underlying theoretical concepts that may be employed in the specification of demand
functions and discusses some of the conceptual and estimation problems involved in the empirical analysis of short-run and long-run elasticities of demand.

Static Theories of Demand

When constructing economic models for use in empirical studies, it is generally assumed that groups of individuals such as producers and consumers behave according to some fundamental pattern and that this behavior is reflected in the structural equations. The specification of demand functions or structural equations representing consumers' behavior has generally been based upon the static theory of consumers' choice. Since the static theory is discussed rather extensively in the literature on the theory of consumer choice, only a brief review will be given. Under traditional theory, the consumers' market behavior is explained in terms of preferences or a utility map. Each consumer is viewed as possessing a well-defined system of preferences represented by a utility function, such as $u = u(q_1, ..., q_n)$, which he is assumed to maximize subject to his income. The concept of utility and utility maximization is introduced to provide a basis for the development of laws of demand for consumer goods.

Early economists assumed utility was a measurable quality of a good and that it was additive. Modern theorists argue
that it was an unnecessary and unwarranted assumption for utility to exist as a cardinal magnitude. It is necessary only that an ordinal preference field exist because only more or less comparisons are needed when dealing with consumers' behavior. They argue that only the shape of the indifference map is important in deriving the main results of demand theory. As proof, modern theorists have shown that all of the results of the utility maximization procedure are invariant under a monotonic transformation of $u$. Equivalent results will be obtained if a transformed utility function $F(u)$ is maximized instead of $u$ when $F'(u) > 0$. This transformation changes the theory to one of ordinal utility where the indifference map can be defined on a psychological behavioristic basis without using the concept of measurable utility.

Underlying the indifference map, the consumer is assumed to possess a scale or field of preferences. The preference field is represented by a utility or preference-index function $u = u(q_1,\ldots,q_n; \theta_1,\ldots,\theta_n)$ which depends upon the consumers' budget $q = (q_1,\ldots,q_n)$ consisting of $n$ goods and services. The $\theta_i$ are parameters describing the form of the function or the consumer's preferences and are often assumed to be given. When in the preference field the axioms of comparison and transitivity are fulfilled, the preference field is said to have an ordering. This ordering enables
the consumer to compare budget $q = (q_1, \ldots, q_n)$ with alternative budgets $q' = (q'_1, \ldots, q'_n)$, i.e., different combinations of the quantities of $n$ goods and services, and to decide if $q$ is preferred to $q'$, $q'$ is preferred to $q$, or $q$ and $q'$ are indifferent. The locus of all points $q'$ which are indifferent to $q$ constitute the indifference surface running through $q$. The system of all indifference surfaces constitute the indifference map of the consumer.

The consumers' selection among the alternative budgets, i.e., his purchase region, is limited by the budget constraint

$$ (2.1) \quad p_1 q_1 + \ldots + p_n q_n = \sum_{i=1}^{n} p_i q_i \leq y $$

where $p_i$ is the $i^{th}$ price, $q_i$ is the quantity of the $i^{th}$ good and $y$ is income. Each budget satisfying this condition is called an obtainable budget. If among all possible obtainable budgets, there is one budget preferred to all other obtainable budgets it is called an optimal budget. Budget $q$ can be an optimum only if the indifference surface is tangent to the budget equation at $q$. All optimal budgets must satisfy

$$ (2.2) \quad p_1 q_1 + \ldots + p_n q_n = \sum_{i=1}^{n} p_i q_i = y $$

---

\[ ^a \text{The Latin capital } S \text{ is used to denote the summation sign in this study.} \]
which is called the budget equation. The choice criterion underlying the static theory of consumer choice is that the consumer always selects an optimal budget if one exists. This is simply a restatement of the assumption that utility is maximized.

The consumers' demand functions are derived from the indifference map under the traditional theory of consumer choice in the following manner. Suppose the assumptions of non-satiety, continuity, and differentiability, stated by Wold [176, p. 82], are satisfied in order to enable the use of mathematical analysis. During the defined budget period the consumer is considered in a market buying goods and services at prices which he cannot appreciably affect. It is assumed there is a fixed number of goods and services designated by \( q_1, \ldots, q_n \) which are clearly defined, homogeneous and divisible. The respective prices \( p_1, \ldots, p_n \) are assumed to be positive and measured in terms of the monetary unit. It is also assumed that the consumer has a positive sum of income \( y \). Following the choice criterion, the consumer is assumed to behave so as to maximize his utility function

\[
U = F[u(q_1, \ldots, q_n)]
\]

subject to the budget equation

\[
p_1q_1 + p_2q_2 + \ldots + p_nq_n = \sum_{i=1}^{n} p_iq_i = y
\]
where total expenditures equal income. It is necessary, as mentioned earlier, that the indifference surface passing through q has (2.2) for its tangent plane if q is to be the optimal budget in region (2.1). Without this restriction the consumer would be able to purchase an unlimited quantity of goods and services up to the point of satiation and hence the axiom of selection would not be fulfilled.

This is a constrained maximum problem for which the maximal or optimum budget solution is obtained by introducing the La Grange multiplier L and forming the function

\[(2.4) \quad C = U + L \sum_{i=1}^{n} p_i q_i - y\] a

Upon maximizing (2.4) by differentiating and setting the first order partial derivatives equal to zero, the conditions for consumer equilibrium (i.e., optimal budget) are obtained

\[(2.5) \quad U_1 = L p_1 \; ; \; i = 1, \ldots, n\]

where

\[U_1 = \frac{\partial U}{\partial q_1} \quad \text{and} \quad L = \frac{S U_1 q_1}{S p_1 q_1} .\]

When L is eliminated, Equation (2.5) is equivalent to

\[(2.6) \quad \frac{U_1}{p_1} = \frac{U_2}{p_2} = \ldots = \frac{U_n}{p_n}\]

aIt is a common practice to denote known or unknown constants by Greek characters, however, in this manuscript they will be denoted by Latin characters.
The ratio \( \frac{U_i}{U_1} \) is called the marginal rate of substitution. In equilibrium (i.e., for budget \( q \) to be optimal) the marginal rates of substitution between all goods must equal their respective price ratios. The budget equation and the conditions for consumer equilibrium as expressed in Equations (2.5), (2.6), or (2.7) constitute the necessary conditions for a maximum. In order that \( U \) should be a true maximum, the sufficient conditions must also be satisfied. That is, the indifference surface must be convex to the origin at the budget point \( q \). The sufficient conditions are met when

\[
(2.8) \quad d^2U = \sum_{i=1}^{n} \sum_{j=1}^{n} U_{ij} t_it_j < 0
\]

for all nonzero values of \( t_i \) and \( t_j \) satisfying \( p_1 t_1 + \ldots + p_n t_n = 0 \). From the above discussion we see that the necessary conditions of consumers' equilibrium can be represented by any one of the various forms of Equations (2.5), (2.6), or (2.7) in combination with Equation (2.2). The consumers' demand functions are derived below by use of Equations (2.5) and (2.2) as this formulation maintains the symmetry of all variables.

Suppose the indifference map is such that for a given combination of prices the consumer spends his total income
upon an optimal budget \( \mathbf{q} = (q_1, \ldots, q_n) \). Let the equilibrium conditions be represented as

\[
\sum_{i=1}^{n} p_i q_i - y = 0
\]

\[
U_i - Lp_i = 0; \quad i = 1, \ldots, n.
\]

By solving these equations for \( L, q_1, q_2, \ldots, q_n \) in terms of \( p_1, p_2, \ldots, p_n, y \), we obtain

\[
q_i = q_i (p_1, \ldots, p_n, y); \quad i = 1, \ldots, n
\]

and

\[
L = f (p_1, \ldots, p_n, y)
\]

where each of the equilibrium demand quantities \( q_i \) are expressed as functions of prices and income. Allowing the prices and income to vary the equations in (2.10) represents the demand equations indicating how the equilibrium demand quantities \( q_i \) change with such variations in prices and income. The demand function is the locus of the consumer equilibrium positions and expresses demand quantity for each consumer's good and service as a function of all prices and income. The variable \( L \) is called the marginal utility of money and can be eliminated.

Modern theorists show that certain restrictions apply for the demand functions. First, the demand functions are
said to be homogeneous of degree zero. That is, the equilibrium solution remains unchanged when both prices and income experience an equal proportionate change. This implies that the equilibrium quantities \( q_i \) are functions of relative prices and relative income; therefore, the demand functions (2.10) may also be expressed as

\[
(2.12) \quad \bar{q}_i = d_i\left(\frac{p_1}{y}, \frac{p_2}{y}, \ldots, \frac{p_n}{y}\right); \quad i = 1, \ldots, n
\]

or

\[
\bar{q}_i = d_i\left(\frac{p_2}{p_1}, \frac{p_3}{p_1}, \ldots, \frac{p_n}{p_1}, \frac{y}{p_1}\right).
\]

The second restriction is that the integrability conditions hold. This restriction indicates that the substitution effects of a compensated change in the \( j \)th price upon the \( i \)th demand quantity is equal to the effect of a compensated change in the \( i \)th price upon the \( j \)th demand quantity.

Nothing has been said thus far about the direction of changes in equilibrium demand quantities resulting from changes in income or prices. The nature of such changes in the equilibrium demand quantities may be determined from the marginal responses \( \frac{\partial \bar{q}_i}{\partial y} \) and \( \frac{\partial \bar{q}_i}{\partial p_j} \). First, consider how a change in income with prices remaining constant will affect the equilibrium demand quantities. By differentiating the equations in (2.9) partially with respect to \( y \) we obtain
where the partial derivatives $\frac{\partial \bar{q}_i}{\partial y}$; $i = 1, \ldots, n$, indicate the marginal responses in the equilibrium demand quantities to changes in income. Substituting $p_i = \frac{U_i}{L}$ from the equations in (2.5) and applying Cramer's rule to this system of equations, the marginal response of the $i$th equilibrium demand quantity to the change in income is given by

$$\frac{\partial \bar{q}_i}{\partial y} = \frac{L\Delta_i}{\Delta}; \ i = 1, \ldots, n$$

where $\Delta$ is the determinant of the coefficients for the system of equations and $\Delta_i$ is the cofactor of $U_i$. Without knowing something about the sign and relative size of the $U_{ij}$ terms in system (2.13), nothing can be said about the marginal response in the $i$th demand quantity resulting from the change in income. On the basis of empirical research, however, the equilibrium demand quantities are expected to increase
with increases in income except for the case of inferior goods.

Let us now consider the effects, upon the equilibrium demand quantities of a change in the \( j \)th price, assuming all other prices and income remain fixed. Differentiating the equations in (2.9) with respect to \( p_j \) we obtain

\[
\begin{align*}
& p_1 \frac{\partial \bar{q}_1}{\partial p_j} + p_2 \frac{\partial \bar{q}_2}{\partial p_j} + \ldots + p_n \frac{\partial \bar{q}_n}{\partial p_j} = - \bar{q}_j \\
& - p_1 \frac{\partial L}{\partial p_j} + U_{11} \frac{\partial \bar{q}_1}{\partial p_j} + U_{12} \frac{\partial \bar{q}_2}{\partial p_j} + \ldots + U_{1n} \frac{\partial \bar{q}_n}{\partial p_j} = 0 \\
& \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
& - p_n \frac{\partial L}{\partial p_j} + U_{n1} \frac{\partial \bar{q}_1}{\partial p_j} + U_{n2} \frac{\partial \bar{q}_2}{\partial p_j} + \ldots + U_{nn} \frac{\partial \bar{q}_n}{\partial p_j} = 0 \\
\end{align*}
\]

(2.15) 

The partial derivatives \( \frac{\partial \bar{q}_i}{\partial p_j} ; i = 1, \ldots, n \), indicate the marginal response in the \( i \) equilibrium demand quantities, to changes in the \( j \)th price. By simplifying and solving the system of equations as before

\[
(2.16) \quad \frac{\partial \bar{q}_i}{\partial p_j} = \left( - \frac{\bar{q}_j L \Delta_i + L \Delta_{ij}}{\Delta} \right)
\]

Substituting Equation (2.14) into Equation (2.16), we obtain
Equation (2.17) indicates the effect of changes in the price of the \( j^{th} \) good upon the equilibrium demand quantity of the \( i^{th} \) good. The effects of the price change are split into two components, namely the income and substitution effects. The first term \(- \bar{q}_j \frac{\partial \bar{q}_i}{\partial y}\) represents the income effect where a rise in the price of one good, other prices and income remaining fixed, is partially equivalent to a decrease in money income. The size of the income effect depends upon how important \( \bar{q}_j \) is in the consumers' budget.

The substitution term \( \frac{L \Delta_{ij}}{\Delta} \) represents the effect on equilibrium demand quantity \( \bar{q}_i \) of a change in the price of \( \bar{q}_j \) when income is adjusted so that the individual would be able to purchase the original budgets.

It was pointed out earlier that nothing can be said about the sign or relative size of \( \frac{\partial \bar{q}_i}{\partial y} \) and hence nothing can be said about the size of \( \frac{\partial \bar{q}_i}{\partial p_j} \) from Equation (2.17). It is possible, by ignoring the income effects, to obtain some information from the substitution term as to the complementarity between goods. If \( \frac{L \Delta_{ij}}{\Delta} \) is positive, then a reduction in the price of \( \bar{q}_j \) leads to a decrease in the demand for \( \bar{q}_i \) and the goods are called substitutes. If the substitution term is negative, then a reduction in the price of \( \bar{q}_j \) leads to an increase in the demand for \( \bar{q}_i \) and the
goods are called complements. When the term is equal to zero the goods are said to be independent. These results depend upon the properties of the indifference map and not upon the utility index representing the indifference map.

The above discussion covers the main aspects of static demand theory generally presented in the literature. This theory may be extended or generalized in various ways to provide a basis for the study of particular aspects of consumer behavior. One of the plausible generalizations is that the utility function need not depend only upon consumption goods as outlined above but might also depend upon the holding of assets. Klein [62, pp. 45-50], Bushaw and Clower [19, pp. 128-134], and Chow [20] have generalized demand theory in this direction. Under these alternative theories the consumer is assumed to maintain some desirable structure of assets while at the same time to maximize his utility from consumption goods. That is, the consumer is assumed to decide upon the proportion of his income to spend on consumption goods for current use relative to savings and to decide upon the form in which his savings will be held. Such generalizations are made by introducing new variables and parameters into the utility index as well as assumptions defining the roles played by the new variables.

The generalized theory may be developed by considering the two types of decisions independently, as mentioned by
Klein [62, p. 46], or by combining and treating the two decisions as a single decision. The theory to be outlined follows the latter approach and has drawn heavily upon Bushaw and Clower's generalization of demand theory to stock-flow commodities [19, pp. 128-134]. In fact the following discussion in large part repeats their arguments with a few modifications.

Under the static theory of demand discussed above, the consumer was assumed to possess a utility function which depends upon the quantities of n goods and services purchased for current consumption. When the consumer is assumed to maintain some desirable structure of assets and to maximize his satisfaction from goods used in current consumption, the utility function can be represented by

\[ U = U (q_1, \ldots, q_n, s_1, \ldots, s_r; \theta_1, \ldots, \theta_n, \theta'_1, \ldots, \theta'_r) \]

where utility depends upon \( q_1, \ldots, q_n \) the quantities of \( n \) goods and services purchased for current consumption, and \( s_1, \ldots, s_r \) the quantities of the \( r \) goods and assets (i.e., durable goods, securities, and/or money) which the consumer desires to hold for future use or sale. The parameters \( \theta_1, \ldots, \theta_n, \theta'_1, \ldots, \theta'_r \) describe the form of the utility function or the consumer's preference system which is assumed to be given.

Bushaw and Clower [19, p. 128] state,
While the decision to hold units of a given commodity is essentially independent of the decision to use units of the same or another commodity, there will usually be links between the two decisions since commodities are not normally held for their own sake; and the properties of the utility index will reflect this fact (e.g., the demand to hold houses may be closely related to purchases of saving bonds, etc.). Furthermore, the decision to alter current asset holdings will usually have a direct influence upon current purchases generally since the consumer cannot acquire additional stocks of one good without either . . . diminishing current purchases for use, or disposing of units of other assets. As a matter of logic, however, changes in the demand for assets to hold cannot be related directly to commodity purchases because the former quantity is a stock while the latter is a flow.

In order to develop the budget constraint appropriate for this situation so that all demands will be backed by purchasing power, Bushaw and Clower proceed in the following manner. Defining \( h_i; i = 1, \ldots, r \), as the quantity of the \( i \)th good or asset actually held by the consumer, it is assumed that the consumer will wish to change his actual holdings of the \( i \)th good or asset according to his desired excess demand, \( x_i = g_i - h_i \) for the \( i \)th good or asset. That is, \( s_i \), the desired time rate of change of actual asset holdings for the \( i \)th good is assumed to vary in the same way as \( x_i \), as

\[
(2.19) \quad s_i \leq 0 \text{ when } x_i \leq 0.
\]

The extent to which the desired changes in asset holdings does in fact occur is assumed to depend upon many considerations which are reflected in the equations.
(2.20) \[ q_i = q_i(x_i) ; i = 1, \ldots, r \]

where \( q_i \) represents the actual quantity of the \( i^{th} \) good purchased to add to asset holdings. In Equations (2.20) it is assumed that the actual and desired changes in quantities of asset holdings, \( q_i \) and \( s_i \), are nearly equal, so that

(2.21) \[ \frac{dq_i}{dx_i} > 0, q_i = 0 \text{ if and only if } x_i = 0 \]

Building upon these properties and assumptions and supposing that the equations in (2.20) are represented by the simpler form

(2.22) \[ q_i = k_i x_i ; i = 1, \ldots, r \]

where \( k_i \) is a fixed positive number, the equations in (2.22) state that the consumer's actual and desired purchases of the \( i^{th} \) good to add to asset holdings is constantly proportional to his desired excess demand for that good or asset. This formulation enables one to consider both purchases and sales for the \( i^{th} \) good or asset, i.e., when \( q_i \) is positive the consumer is assumed to purchase the good and when negative to sell the good. Bushaw and Clower argue that the above formulation is appropriate under the purely competitive hypothesis, because under these conditions desired and actual purchases (sales) in any asset are always equal.
Following the above arguments, the consumer's budget equation is defined as

\[(2.23) \quad \sum_{i=1}^{n} p_i q_i + \sum_{i=n+1}^{n+r} p_i k_i (g_i - h_i) = y\]

where total purchases are \((q_i + q_i')\), the quantities of \(n\) goods purchased for current use and the quantities of \(r\) goods purchased to add to asset holdings.

The consumer's optimal conditions for the case of current consumption goods and asset holdings are obtained by introducing the LaGrange multiplier, by forming the function

\[(2.24) \quad C = U + L \left[ \sum_{i=1}^{n} p_i q_i + \sum_{i=n+1}^{n+r} p_i k_i (g_i - h_i) - y \right]\]

and then maximizing \((2.24)\) where the \(h_i's\), \(p_i's\), and \(y\) are regarded as fixed. By differentiating \((2.24)\) and setting the first order partial derivatives equal to zero the necessary conditions, in the sense of a current consumption goods and asset holdings decision process, are given by

\[(2.25) \quad \begin{align*}
U_i + L p_i &= 0 \; ; \; i = 1, \ldots, n \\
U_i + L k_i p_i &= 0 \; ; \; i = n+1, \ldots, n+r \\
\sum_{i=1}^{n} p_i q_i + \sum_{i=n+1}^{n+r} p_i k_i (g_i - h_i) - y &= 0
\end{align*}\]

where
The sufficient conditions are met when

\begin{equation}
(2.26) \quad d^2U = S_i S_j U_{ij} t_i t_j \quad i=1,\ldots,n ; \quad n+1,\ldots, n+r
\quad j=1,\ldots,n ; \quad n+1,\ldots, n+r
\end{equation}

is negative definite for values of the t's satisfying

\begin{equation}
(2.27) \quad \sum_{i=1}^{n} p_i t_i + \sum_{i=n+1}^{n+r} k_i p_i t_i = 0
\end{equation}

where

\begin{align*}
U_{ij} &= \frac{\partial^2 U}{\partial U_i} \\
U_i &= q_i \quad \text{for } 1 \leq i \leq n \\
U_i &= g_i - n \quad \text{for } n < i \leq n+r
\end{align*}

Following Bushaw and Clower's arguments, the condition defined by (2.22), (2.23), (2.25), and (2.26) is called an equilibrium or optimal plan. Consumers' equilibrium is then defined by the further condition that

\begin{equation}
(2.28) \quad g_i - h_i = 0 ; \quad i = 1,\ldots, r
\end{equation}

which implies and is implied by \( q_i = 0 \) (i=1,\ldots,r). That is, consumer equilibrium occurs if the equilibrium plan
(q_1,\ldots, q_n; g_1,\ldots, g_r) or equilibrium budget (q_1',\ldots, q_n'; q_1',\ldots, q_r') is such that desired and actual holdings of goods and assets are equal. Bushaw and Clower argue that the restrictions in (2.28) appear to be justified since some quantities of the asset holdings will be changing over time unless these restrictions are satisfied.

Suppose the indifference map is such that for a given combination of prices p_i and actual asset holdings h_i the consumer spends his total income y upon an optimal plan \( \bar{q} = (\bar{q}_1,\ldots, \bar{q}_n, \bar{g}_1,\ldots, \bar{g}_r) \) as defined by

\[
(2.29) \quad \sum_{i=1}^{n} p_i \bar{q}_i + \sum_{i=n+1}^{n+r} p_i k_i (\bar{g}_i - h_i) - y = 0
\]

\[
U_i + L p_i = 0 \quad ; \quad i=1,\ldots,n
\]

\[
U_i + L k_i p_i = 0 \quad ; \quad i=n+1,\ldots,n+r
\]

By solving these equations in terms of the \( p_i, h_i \), and \( y \), where for the \( p_i, i=1,\ldots,n \) and for the \( h_i, i=n+1,\ldots,n+r \), the respective demand functions for goods for current consumption and for asset holdings are obtained as:

\[
(2.30) \quad \bar{q}_i = \bar{q}_i'(p_1,\ldots,p_n, h_1,\ldots,h_r,y), \quad i=1,\ldots,n
\]

\[
\bar{g}_i = \bar{g}_i'(p_1,\ldots,p_n, h_1,\ldots,h_r,y), \quad i=n+1,\ldots,n+r
\]

The use of equations contained in (2.30) and (2.22) yield
Bushaw and Glower show that the demand equations in (2.30) are homogeneous of order zero in the prices $p_i$ and income $y$.

What can be said about the direction of the changes in optimal quantities $\bar{q}_i$ and $\bar{g}_i$ resulting from changes in prices and income? Information on these properties of the demand functions can be obtained by direct analogy with the development of Equations (2.13) through (2.17) relating to the traditional static theory of demand. Assuming that the values of the $h_i$'s and other relevant variables are fixed, differentiating the equations in (2.29) and (2.31) with respect to the $j^{th}$ price, and by substituting and applying Cramer's rule, we obtain the marginal responses to a change in the $j^{th}$ price as

\[
(2.32) \quad \frac{\partial \bar{q}_i}{\partial p_j} = - (\bar{q}_j + \bar{q}_j') \frac{\partial \bar{q}_i}{\partial y} + L \frac{\Delta j_i}{\Delta} + k_j \frac{\Delta_{n+j,i}}{\Delta}
\]

\[
\frac{\partial \bar{q}_i}{\partial p_j} = - (\bar{q}_j + \bar{q}_j') \frac{\partial \bar{q}_i}{\partial y} + L \frac{\Delta j_i, n+1}{\Delta} + k_j \frac{\Delta_{n+j, n+1}}{\Delta}
\]

\[
\frac{\partial \bar{q}_i}{\partial p_j} = k_i \frac{\partial \bar{g}_i}{\partial p_j}
\]

where
A = -\Pi_n U_{n,n} U_{n+1,n} U_{n+1,n+1} U_{n+2,n+2}

\Delta = -p_n U_{n+1,n} U_{n+1,n+1} U_{n+2,n+2}

and \Delta_{ij} \text{ is the cofactor. Bushaw and Clower argue that since the quantities of desired asset holdings, } g_i, \text{ are generally nonobservable, empirical demand behavior is described by the marginal responses } \frac{\partial q_i}{\partial p_j} \text{ and } \frac{\partial q_i}{\partial p_j}. \text{ The interpretation of these expressions is quite similar to those given for (2.14) and (2.17). Bushaw and Clower [19, p. 133] state, "... that is to say, purchases of a commodity for current consumption may be either a substitute for or a complement to purchases of the ... commodity to add to stocks".}

Dynamic Adjustment Theory

The theory discussed thus far like most of the theoretical literature on consumer demand is of a statical nature. In such theory the underlying factors are treated
as given and the demand functions are assumed to apply as of a given situation, i.e., for a given level of income, prices, and other explanatory variables. When price or one of the other factors change, the demand functions indicate the respective changes in the consumers' equilibrium demand quantities. In empirical applications where the assumed conditions are relatively stable, demand functions postulated from static theory provide satisfactory results. Static demand theory does not provide appropriate results, however, in analysis directed at the explanation of demand adjustment processes for changing conditions or where the influence of consumers' past behavior is important. This is to be expected because static demand theory is concerned only with the determination of equilibrium budgets under different situations and not with the processes by which the equilibrium demand quantities are approached or attained. The passage of time is ignored in such theory because by definition the individual's adjustment to a change in price or other explanatory factors is completed within the defined budget period of time.

The consideration of time in demand analysis has at least two implications for the demand functions. First, when prices or income fluctuate over time the consumers' demand pattern may be influenced by anticipated as well as current values of these variables. In such cases
consideration must be given to the problem of uncertainty and questions relating to the nature of the individual's expectations as well as to how they are formed. The second implication is concerned with the effects that differences in the length of the time period have upon the elasticities of the demand functions. It is generally accepted in the theoretical literature that the elasticity of demand for a good increases as the time period is extended, however, only vague distinctions have been made about the differences between short-run and long-run demand functions and their respective elasticities. It is only in the past few years that attention has been directed at the theoretical considerations underlying such differences and that studies have been designed and directed to obtain quantitative estimates of short-run and long-run elasticities of demand. The purpose of this section is to briefly outline the basic hypothesis and theoretical considerations underlying the study of long-run elasticities and to discuss some of the estimation procedures suggested for use in obtaining quantitative estimates of short-run and long-run demand elasticities.

Definitions

When the influence of time is explicitly introduced in

The thesis presented in this section is based on the arguments of Ladd and Tedford [77].
demand analysis, it is necessary to turn to dynamics. In dynamic systems values of the variables are partly or wholly determined by the past behavior of the system. In a static system, on the other hand, the variables are determined by values of other variables in the system during the same time period. The relation between statics and dynamics is often expressed in terms of the adjustment period of a variable relative to its equilibrium position. Static demand theory represents a limiting case in dynamics where the equilibrium demand quantities are so sensitive to changes in the explanatory variables that the adjustment process is instantaneous, i.e., the time rate of change is infinite. Due to the infinite time rate of change of the consumers' reactions in static theory, the passage of time can be ignored.

The estimation of long-run elasticities presupposes a model specified in line with dynamic economic theory as determining the variables behavior through time. The basic hypothesis made in models employed in the study of long-run demand functions and their elasticities is that there is a perceptible lag in the consumers' adjustments to changes in price, income or other factors. This means that the consumers do not adjust immediately from one equilibrium position to another when an explanatory variable changes, i.e., the time rate of change is finite. The hypothesis that the
complete adjustment could be spread out over time appears reasonable in that institutional and technological limitations, consumers' ignorance, uncertainty and similar factors may prevent the consumer from increasing his satisfaction at the greatest rate over time. For example, lags may occur in the consumers reaction to a change in price when: (1) habit plays an important part in the decision process and time is required before consumers can appraise the change and completely adjust their budgets; (2) some consumers are unaware of the change and this prohibits the adjustment from being completed within the defined budget period; (3) the consumption of a good requires complementary goods; (4) the good is a stock and must be worn out before the consumers shift their consumption patterns; or (5) debts incurred in the past prevent changes in current budgets.

The difference between hypotheses concerning the consumers' adjustment processes underlying static demand functions and long-run demand functions is shown for one good in Figure 1. To simplify the exposition, suppose there are no changes in income and prices other than the one in question. Further assume that this price is exogenous. The time path of price is represented in Figure 1 by the solid line $p_t$. In this case it is assumed that price has been constant long enough previous to period $t = -1$ so that the consumers' adjustment to this price level has been completely worked out. Between periods $-1$ and $0$, $p_t$ increases.
and then remains constant at the new level for all periods \( t \geq 0 \). The course of equilibrium demand quantity over time is represented by the solid line labelled \( \bar{q} \) which indicates that the equilibrium level changes simultaneously with price. If the consumers' adjustment to the change in price is completed immediately, the path of actual consumption will coincide with the time path of \( \bar{q} \). If the consumers' adjustment is not completed immediately but spread out over \( n + 1 \) time periods as assumed in Figure 1, the course of actual demand quantity would be represented by a curve similar to the dotted line \( q_t \). When the new equilibrium level is stable, the actual quantity demanded will approach equilibrium gradually over time. As illustrated in Figure 1, it is considered that by period \( n \) the difference between actual demand quantity and equilibrium demand quantity is so small that little error is introduced by assuming the total change in actual demand quantity from period \(-1\) to period \( n \) is equal to the change in equilibrium demand quantity over the same period.

The hypothesis that there is a perceptible lag in the consumers' adjustment to a change in price can be represented analytically as

\[
(2.33) \quad q_t = f(p_t, p_{t-1}, \ldots, p_{t-n})
\]

or by the linear approximation
FIGURE 1. HYPOTHETICAL CONSUMERS' ADJUSTMENT RESULTING FROM A ONCE-FOR-ALL CHANGE IN PRICE.
where current demand quantity $q_t$ is expressed as a function of current and lagged prices. This formulation indicates that the current level of demand quantity is composed of the consumers' adjustment to current price and of adjustments to prices existing in past periods. Equation (2.34) provides an approximation of the consumers' adjustment process over time to the price change as the consumers' reaction is reflected by the coefficients associated with the respective prices. This can be shown by following the arguments of Ladd and Tedford [77, pp. 223-226] and by use of the case postulated in Figure 1. To simplify the presentation the units of measurements are so chosen that $p_t = 0$ for $t \leq -1$ and $p_t = 1$ for $t \geq 0$. Assuming that demand can be represented by a function linear in arithmetic values of current and past prices

$$(2.34) \quad q_t = a + a_0 p_t + a_1 p_{t-1} + \ldots + a_n p_{t-n} .$$

For the postulated situation the initial level of demand quantity is given by

$$(2.35) \quad q_{-1} = a + a_0 p_{-1} = a .$$

Writing only the nonnegative terms in the equations for $t \geq 0$ we obtain
(2.36.0) \( q_0 = a + a_0 p_0 = a + a_0 \)

(2.36.1) \( q_1 = a + a_0 p_1 + a_1 p_0 = a + a_0 + a_1 \)

(2.36.2) \( q_2 = a + a_0 p_2 + a_1 p_1 + a_2 p_0 = a + a_1 + a_1 + a_2 \)

\[ \vdots \]

(2.36.n) \( q_n = a + a_0 p_n + a_1 p_{n-1} + \ldots + a_n p_0 \)

\[ = a + a_0 + a_1 + \ldots + a_n \]

The change in demand quantity \( q_t \) from the initial level to some time period is obtained by taking the relevant difference. Defining \( \Delta^j x_i = x_i - x_{i-j} ; j > i \), the quantity change from period -1 to period 0 is \( \Delta^1 q_0 = a_0 \Delta^1 p_0 \). Likewise, the quantity change from period -1 to period \( j \) is \( \Delta^{j+1} q_j = \sum a_i \Delta^1 p_0 ; i = 0, 1, \ldots, j \).

By taking the relevant differences with respect to the relative change in price between period -1 and period 0 we obtain

(2.37) \[ \frac{\Delta^1 q_0}{\Delta^1 p_0} = a_0 \]

\[ \frac{\Delta^2 q_1}{\Delta^1 p_0} = a_0 + a_1 \]

\[ \vdots \]

\[ \vdots \]
\[
\frac{\Delta^{j+1}q_j}{\Delta^{i}p_0} = a_0 + a_1 + \ldots + a_j = Sa_1;
\]

\[i = 0,1,\ldots,j\]

The coefficient \(a_0\) indicates the relative change in demand quantity with respect to the relative change in price between periods \(-1\) and \(0\). Likewise, the coefficients \(a_0 + a_1\) relate the relative quantity change between period \(-1\) and period \(1\) to the relative change in price occurring between period \(-1\) and period \(0\). When it takes \(n+1\) time periods for the consumers' adjustment to be completed, the sum of the coefficients, \(Sa_1\); \(i = 0,\ldots,n\), indicate the relative change in demand quantity over this period relative to the price change between periods \(-1\) and \(0\). Although the above discussion applies for a situation where it is assumed there is only one change in price, the same conclusions can be derived whether we assume a once-for-all change or continuous changes in price.

The concept of elasticity is often used in demand analysis to indicate the consumers' response to changes in price, income or other factors as elasticities are independent of the units of measurement. The price or other factor elasticities of demand are defined as the partial derivative of the logarithm of demand quantity with respect to the logarithm of the respective factor. Following this
definition the price elasticity of demand is

\[
(2.38) \quad \frac{\Delta \log q_t}{\Delta \log p_t} = \frac{\Delta q_t}{\Delta p_t} \cdot \frac{p}{q}.
\]

Applications of this definition have generally followed classical static theory where the influence of time is ignored.

When time is explicitly specified in the demand function, the definition of elasticity as given in (2.38) is too general. Considering the definition of the demand curve as the locus of equilibrium demand quantities, it is possible to restate the definition of elasticity as the partial logarithmic derivative of equilibrium demand quantity with respect to the relevant explanatory variable. This definition is in keeping with the equilibrium nature of static demand theory. Using this definition and the notation of Figure 1, the price elasticity of demand is

\[
(2.39) \quad \frac{\Delta \log \bar{q}}{\Delta \log p_t} = \frac{\Delta \bar{q}}{\Delta p_t} \cdot \frac{p}{q}
\]

which indicates the response in equilibrium demand quantity to a change in its own price. The equilibrium quantity refers to a point on one of the family of static demand curves as defined in Equations (2.9) and (2.10) under the static theory discussed above.

By use of the definition underlying (2.39) and assuming
the new equilibrium is stable and it takes \( n + 1 \) time periods for the consumers to completely adjust to the change in price, the long-run elasticity is

\[
(2.40) \quad e_{ir} = \frac{\partial \bar{q}}{\partial p_t} \cdot \frac{p}{q} = \frac{p}{q} S_{a1}.
\]

The specification given in (2.40) follows from the discussion presented earlier where it was argued that the sum of the coefficients, \( S_{a1}, i=0,1,\ldots,n \), associated with the current and past prices provide an approximation of the consumers' adjustment process over time to the change in price. That is, [77, pp. 224-226] if the new equilibrium level \( \bar{q}_0 \) is stable, actual demand quantity \( q_t \) will approach \( \bar{q}_0 \) gradually over time. After \( n + 1 \) periods \( q_t \) will equal \( \bar{q}_0 \), or will be so close that little error is introduced by assuming them to be equal so that change in actual demand quantity from period \(-1\) to period \( n \) will equal the change in the equilibrium demand quantity from period \(-1\) to period \( 0 \). The change in actual demand quantity will be \( S_{a1}, i=0,1,\ldots,n \) and in accordance with the previous definition the long-run elasticity is (2.40) as

\[
(2.40) \quad e_{ir} = \frac{\partial \bar{q}}{\partial p_t} \cdot \frac{p}{q} = \frac{\bar{q}_0 - \bar{q}_{-1}}{p_0 - p_{-1}} \cdot \frac{p}{q} = \frac{q_n - q_{-1}}{p_0 - p_{-1}} \cdot \frac{p}{q} \cdot \frac{p}{q} S_{a1}.
\]
where \( \bar{q}_0 = \bar{q} = q_n \) and \( \bar{q}_{-1} = q_{-1} \).

The use of the above arguments enable one to obtain elasticities of demand for the time period to which the consumers' reaction relates. For example, in the situation postulated in Figure 1 and from Equation (2.34), it follows that the first period elasticity of demand for the price change between periods -1 and 0 is

\[
(2.41) \quad e_1 = \frac{\lambda \log q_0}{\delta \log p_0} = a_0 \cdot \frac{p}{q}.
\]

The elasticity of demand for the first two periods is

\[
(2.42) \quad e_2 = \frac{\lambda \log q_1}{\delta \log p_0} = (a_0 + a_1) \frac{p}{q}
\]

which relates the consumers' reaction over the first two periods to the price change between periods -1 and 0. Price elasticities may be obtained for longer periods of time so long as the coefficients associated with the relevant lagged prices are non-zero in value.

Defining the short-run response as the contemporaneous change in demand quantity associated with the change in price, the short-run price elasticity of demand is

\[
(2.43) \quad e_{sr} = \frac{\lambda \log q_t}{\delta \log p_t} = a_0 \frac{p}{q}.
\]

Estimates of short-run elasticities obtained from Equation
(2.43) may not be unique as Nerlove [95, p. 304] states, "... the short-run elasticity differs depending on the position from which we start and the length of time we allow for adjustment".

**Procedures for estimating long-run elasticities**

It was mentioned above that estimates of long-run demand elasticities can be obtained from equations such as

\[
q_t = a + a_0 p_t + a_1 p_{t-1} + ... + a_n p_{t-n}
\]

when the relationships are properly formulated. That is, long-run demand elasticities may be estimated from equations where demand quantity is specified as a function of a variable taken with a distributed lag. Nerlove [94, p. 307] states,

... the formulation of economic relationships containing distributed lags is related to the problem of formulating meaningful relationships among variables we can observe, and the problem of estimating distributions of lag is really the problem of estimating long-run elasticities.

Following the arguments and definitions presented earlier we find that the problem is one of estimating the series of non-zero coefficients associated with current and lagged values of the relevant explanatory variable.

Nerlove [94, pp. 7-8] mentions three general approaches that can be employed in estimation: first, make no assumptions as to the form of the distribution of lag and estimate the coefficients for an equation such as (2.34) directly;
second, make an assumption as to the form of the distributed lag and estimate the relevant parameters; or third, develop a dynamic model where the various causes assumed to bring about rigidities in consumers' behavior are explicitly introduced.

The first approach provides unsatisfactory results when the consumers' adjustment is spread over many time periods. Estimates for many coefficients in an equation like (2.34) are subject to wide errors due to such problems as autocorrelation, multicollinearity, and the small number of degrees of freedom. The second approach reduces the number of relevant parameters and probably eliminates some of the statistical problems. Specification errors may arise in the second approach due to the arbitrary nature of assumptions introduced to approximate the time path of consumers' behavior. The use of different assumptions leads to different estimation procedures. In the third approach estimates of long-run elasticities are obtained from the behavioral or reduced equations where different factors are assumed to cause the rigidities in consumers' reactions. Specification errors may also arise in this approach. This section outlines some of the specific procedures that have been suggested for use in estimating long-run elasticities of demand.
Koyck procedure

Koyck argues it might be expected that hindrances causing the lag in consumers' adjustments will be gradually overcome and that the effect of the change in price or other variables will decrease as the time of the change recedes further into the past [70, p. 12]. To reduce the number of coefficients to be estimated and some of the statistical difficulties he assumes that from some period $i = k$ the series of coefficients $a_i$ associated with the relevant variable follow a converging geometric series

$$a_{k+1} = r a_k$$

where $0 \leq r < 1$. That is, the coefficients are assumed to decrease by a constant proportion.

Substituting (2.44) into demand Equation (2.34) for the hypothetical case where $k = 0$ and price increases from a constant level $p_t = 0$ for $t < 0$ to a new level $p_t = 1$ for $t \geq 0$, it follows that

$$q_t = a + a_0 p_t + a_0 rp_{t-1} + a_0 r^2 p_{t-2} + \cdots$$

$$= a + a_0 S r^i p_{t-i}; \quad i = 0, 1, \cdots$$

In Equation (2.45) demand quantity is expressed as a function of a series of geometrically weighted prices. The consumers' adjustment to the change in price is approximated by
\[ a_0 S^{t^i} = a_0 \frac{1-r^t}{1-r} ; \quad i = 0,1, \ldots, t \]

The new level of equilibrium demand quantity is

\[ (2.46) \quad \bar{q} = \lim_{t \to \infty} q_t = a + \lim_{t \to \infty} a_0 \frac{1-r^t}{1-r} \]

or \[ \bar{q} = a + \frac{a_0}{1-r} \]

To further simplify the problem of estimating the series of coefficients, a reduced equation is derived in the following way. Consider Equation

\[ (2.45) \quad q_t = a + a_0 p_t + a_0 r p_{t-1} + a_0 r^2 p_{t-2} + \ldots \]

Lagging (2.45) one period and multiplying by \( r \) yields

\[ (2.47) \quad r q_{t-1} = ar + a_0 r p_{t-1} + a_0 r^2 p_{t-2} + \ldots \]

Subtracting (2.47) from (2.45) we obtain

\[ (2.48) \quad q_t - r q_{t-1} = a(1-r) + a_0 p_t \]

or

\[ (2.48.a) \quad q_t = a (1-r) + a_0 p_t + r q_{t-1} \]

which provides the coefficients \( a_0 \) and \( r \) needed to obtain estimates of the price elasticities of demand and the consumers' speed of adjustment.

From the definitions presented earlier and Equation
(2.45) or (2.48.a), the short-run price elasticity is \( a_0 \frac{p}{q} \). Under assumption (2.44) the long-run consumers' reaction to the price change is \( a_0 / 1 - r \) if \( 0 \leq r \leq 1 \). From Equation (2.40) and the above arguments, the long-run price elasticity is

\[
(2.49) \quad \frac{a_0}{1 - r} \cdot \frac{p}{q}.
\]

That (2.49) is the long-run price elasticity can be illustrated in another way. At the new level of equilibrium \( q_t = q_{t-1} = \bar{q} \). By adding \( rq_{t-1} \) to and subtracting \( q_{t-1} \) from both sides of Equation (2.48) or by merely subtracting \( q_{t-1} \) from both sides of (2.48.a), we obtain

\[
(2.50) \quad \Delta q_t = q_t - q_{t-1} = a (1 - r) + a_0 p_t - (1/r) q_{t-1}.
\]

When equilibrium is attained \( \Delta q_t = 0 \), \( q_{t-1} = \bar{q} \), and it follows from (2.50) that

\[
(2.51) \quad (1 - r)\bar{q} = a (1 - r) + a_0 p_t
\]

\[
\bar{q} = a + \frac{a_0}{1 - r} p_t
\]

which is equivalent to Equation (2.46) at the new equilibrium level where it is assumed that \( p_t = 1 \). By taking the logarithmic derivative of \( \bar{q} \) with respect to \( p_t \) in (2.51), we obtain (2.49) as the long-run price elasticity of demand.

In addition to the estimates of long-run and short-run
elasticities, it is also possible to obtain an approximation of the consumers' speed of adjustment. Let the distance between \( q_t \), the level of demand quantity at period \( t \), and \( \bar{q} \), the new level of equilibrium demand quantity, be represented by \( d_t \). By use of Equations (2.45) and (2.46) and the approximation

\[
a_0 r^t = a_0 \frac{1 - r^t}{1 - r},
\]

the distance is represented by

\[
(2.52) \quad d_t = \bar{q} - q_t = \frac{a_0 r^t}{1 - r}.
\]

By taking the first difference \( \Delta q_{t+1} \) from Equation (2.45) and by use of the approximation for \( a_0 r^t \), we obtain

\[
(2.53) \quad \Delta q_{t+1} = q_{t+1} - q_t = a + \frac{a_0}{1 - r} - \frac{a_0 r^{t+1}}{1 - r} - \frac{a_0 r^t}{1 - r} \quad (1 - r) = a_0 r^t
\]

where \( \Delta q_{t+1} \) represents the change in demand quantity between time periods \( t \) and \( t+1 \). Substituting (2.53) into (2.52) yields

\[
(2.54) \quad \Delta q_{t+1} = (1 - r) d_t ; 0 \leq r < 1
\]

indicating that the consumers' adjustment between period \( t \) and \( t + 1 \) is proportional to the distance of demand quantity at period \( t \) from the new level of equilibrium demand quantity.
The speed of the consumers' adjustment to the price change is represented by \( (1-r) \). When \( r = 0 \) the new equilibrium level is attained in one time period whereas when \( r \to 1 \) numerous periods must pass before the new equilibrium will be attained.

**Nerlove procedure**

Nerlove indicates that results similar to Koyck's can be derived from dynamic models based upon different assumptions as to the cause of the lag in adjustment [95, pp. 14-46]. Although numerous dynamic models can be formulated, Nerlove considers two general classes in which he distinguishes between causal factors grouped according to (1) those of an institutional or technological nature and (2) those dealing with uncertainty about the future.

In the dynamic models where institutional or technological factors are considered to generate the distributed lag it is assumed that the consumers' expectations about the future values of explanatory factors are static in nature. That is, a change in price or other factors is expected to be permanent so that only one equilibrium demand quantity is uniquely determined for the given situation. Following the arguments of Nerlove if all prices other than the price of the good in question are held constant, the long-run demand function can be approximated as

\[
(2.55) \quad \bar{q}_t = a + a_0 p_t + b_0 y_t
\]
where $\bar{q}_t$ is the new level of equilibrium demand quantity resulting when the situation in period $t$ prevails indefinitely.

As mentioned earlier the relation between short-run and long-run demand functions depends upon the assumptions introduced in approximating the course of demand quantity over time. Nerlove argues that the shape and form of the time path is determined by the type of existing institutional or technological rigidity. The nature of such time paths can be represented by the differential equation

$$\frac{dq}{dt} = r(t) [\bar{q}_t - q_t]$$

or by the difference equation

$$q_t = q_{t-1} = r [\bar{q}_t - q_{t-1}]$$

where $r(t)$ is a constant $r$.

To derive the demand function with a distributed lag, let $\bar{q}_t$ be a function of time. By solving (2.57) for $q_t$ in terms of $\bar{q}_t$ we obtain

$$q_t = \sum_{i=0}^{t} r (1-r)^{i-1} \bar{q}_{t-i}$$

when period 0 relates to the distant past. The sum of the weights equals one if $0 < r < 1$. Substituting Equation (2.55) in (2.58) we obtain
\begin{equation}
(2.59) \quad q_t = \sum_{i=0}^{t} r^{(1-r)^i} \left[ a + a_0 p_{t-1} + b_0 y_{t-1} \right] \\
= a^* + a_0 \sum_{i=0}^{t} r^{(1-r)^i} p_{t-1} \\
+ b_0 \sum_{i=0}^{t} r^{(1-r)^i} y_{t-1}
\end{equation}

as the long-run demand function which has a distribution of lag similar to that assumed by Koyck. In this case the same distribution of lag applies to both price and income.

To simplify the estimation of long-run elasticities, Nerlove derives a reduced equation directly from Equations (2.55) and (2.57). Substituting (2.55) into (2.57) and adding \( q_{t-1} \) to both sides of the equation we obtain the reduced equation:

\begin{equation}
(2.60) \quad q_t = a r + a_0 r p_t + b_0 r y_t + (1-r) q_{t-1}
\end{equation}

which is similar in form to that derived by Koyck's method of reduction. From (2.60) the short-run price and income elasticities are \( a_0 r p/q \) and \( b_0 r y/q \), respectively, and the long-run price and income elasticities are

\[
\frac{a_0 r}{1-(1-r)} \cdot \frac{p}{q} \quad \text{and} \quad \frac{b_0 r}{1-(1-r)} \cdot \frac{y}{q}.
\]

Dynamic models leading to a distributed lag of an expectational nature are not as easy to formulate as those
resulting from institutional or technological rigidities. Numerous factors may influence consumers' anticipations and the multi-valued nature of expectations of future prices and incomes add complexity to the problem. By considering only that part of a change in expected future prices or incomes induced by a change in current values of price or income, it is possible to treat the expectations as being single-valued. Nerlove [95, pp. 302-3] states,

A meaningful concept of expectations (yet one which treats them as single-valued) is that of expected 'normal' price or income, i.e., the level about which future prices or incomes are expected to fluctuate. If changes in expected 'normal' price or income are induced by changes in current prices or incomes, simple but meaningful models of expectation formation may be constructed.

The construction of such models rests upon the following arguments. Changes in the current values of price or income are considered to consist of a permanent component and a transitory component. The permanent component affects all expected future prices or incomes whereas the transitory component affects only some or none. Denoting expected normal price by $p^*_t$ and expected normal income by $y^*_t$, changes in $p^*_t$ or $y^*_t$ are induced only by the permanent components of the changes in current price or current income. The transitory components affect only the deviations about the expected normal values. The relation between the current value of price or income and their respective expected normal values is introduced through a modification of Hick's definition.
[54, p. 205] of the coefficient of expectations. Following
these considerations Nerlove defines the coefficient of
expectations as the ratio of the change in expected normal
values for a variable between periods t-1 and t relative to
the change in current value expressed as a deviation from
the expected normal value in period t-1. That is

\[(2.61) \quad p^*_t - p^*_{t-1} = B(p_t - p^*_{t-1})\]

\[(2.62) \quad y^*_t - y^*_{t-1} = A(y_t - y^*_{t-1})\]

where B and A are the coefficients of expectation for price
and income. By treating the change in current value of price
as a deviation from the previous value of expected normal
price, Nerlove argues that B represents the proportion of
the change in current value which is regarded as the
permanent component and that 1-B represents the proportion
regarded as the transitory component.

Suppose all prices other than the price of the good in
question are held constant and assume the demand equation can
be approximated as

\[(2.63) \quad q_t = a + a_0 p_t + a_1 p^* + b_0 y_t + b_1 y^*\]

where demand quantity is expressed as a function of current
and expected normal values of price and income. By solving
(2.61) and (2.62) for \(p^*_t\) and \(y^*_t\) as functions of \(p_t\) and \(y_t\),
respectively, we obtain

$$(2.64) \quad p^*_t = \sum_{i=0}^{t} B (1-B)^i p_{t-i}$$

and

$$(2.65) \quad y^*_t = \sum_{i=0}^{t} A (1-A)^i y_{t-i}$$

where period 0 is in the distant past. Assume B and A are constants where $0 \leq B < 1$ and $0 \leq A < 1$. By substituting (2.64) and (2.65) into (2.63), the demand equation is

$$(2.66) \quad q_t = a + a_0 p_t + a_1 \sum_{i=0}^{t} B (1-B)^i p_{t-i} + b_0 y_t + b_1 \sum_{i=0}^{t} A (1-A)^i y_{t-i}$$

$$= a + (a_0 + a_1 B) p_t + a_1 \sum_{i=0}^{t-1} B (1-B)^i p_{t-i} + (b_0 + b_1 A) y_t + b_1 \sum_{i=0}^{t-1} A (1-A)^i y_{t-i}$$

where demand quantity is expressed as a function of current values of price and income and the series of lagged values of price and income with a distributed lag. Although the distribution of lag for each series of lagged values of price and income is similar in form to that considered by Koyck, the distributed lags may differ for each variable.
depending upon the values of B and A. The distributions will be identical only when the coefficients of expectation are equal.

To further simplify the statistical considerations it is important to consider the reduced equations. Nerlove argues that when the lag in consumers' adjustments are of an expectational nature, his method of reduction is more direct than Koyck's and can be applied in certain cases where the use of Koyck's method of reduction would be extremely difficult if not impossible. One of the advantages of Nerlove's method of reduction is that it can be applied to general demand equations as well as simple equations where only one variable is considered. To simplify the solution but at the same time illustrate Nerlove's method of reduction, assume that only the price of the good in question is variable and the demand equation is

\[(2.67) \quad q_t = a_1p^*_t\]

where demand quantity is expressed as a function of expected normal price. Following Nerlove's procedure [95, pp. 26-27], from Equation (2.61) we obtain the price expectational equation,

\[(2.68) \quad -Bp_t = -p^*_t + (1-B) p^*_{t-1}\]

Lagging (2.67) and (2.68) one time period yields
(2.69) \[ q_{t-1} = a_1 p^*_t \]
\[ -B p_{t-1} = -p^*_{t-1} + (1-B) p^*_{t-2} \]

Solving the two equations in (2.69) for \( p^*_{t-1} \) we obtain

(2.70) \[ p^*_{t-1} = \frac{\Delta_1}{\Delta} = \frac{q_{t-1} (1-B)}{a_1 (1-B)} = \frac{q_{t-1}}{a_1} \]

where

\[ \Delta_1 = \begin{vmatrix} q_{t-1} & 0 \\ -B p_{t-1} & (1-B) \end{vmatrix} = q_{t-1} (1-B) \]

and

\[ \Delta = \begin{vmatrix} a_1 & 0 \\ -1 & (1-B) \end{vmatrix} = a_1 (1-B) \]

A solution similar to that obtained in (2.70) for the simple case under consideration can be obtained directly from the first equation in (2.69) since the coefficient of \( p^*_{t-2} \) is zero in this equation. Substitution of the solution for \( p^*_{t-1} \) obtained from (2.70) into (2.68) yields

(2.71) \[ p^*_t = B p_t + \frac{(1-B)}{a_1} q_{t-1} \]

By rearranging and substituting (2.71) into (2.67) we obtain

(2.72) \[ q_t = a_1 B p_t + (1-B) q_{t-1} \]

as the reduced equation. All of the information needed to obtain estimates of the short-run and long-run elasticities
can be obtained from (2.72).

**Generalized Working procedure**

Estimates of long-run elasticities from the procedures described above are based upon the explicit or implicit assumption that consumers' adjustments are approximated by a converging geometric series. Working [179, pp. 46-52] has proposed another procedure based upon a different assumption. Ladd and Tedford [77, pp. 226-229] have presented a generalization of the Working procedure which indicates the specific assumptions implied in the Working procedure.

Suppose the long-run equation is approximated by

\[(2.73) \quad q_t = a + a_0 p_t + a_1 p_{t-1} + \ldots + a_n p_{t-n} + b_0 y_t + b_1 y_{t-1} + \ldots + b_m y_{t-m} \]

To simplify the estimation procedures assume that the series of coefficients associated with the lagged values of price and income follow the arithmetic progressions

\[(2.74) \quad a_i = a_{i-1} + d = a_1 + (i-1)d \quad ; \quad i = 2, \ldots, n \]

and

\[(2.75) \quad b_j = b_{j-1} + e = b_1 + (j-1)e \quad ; \quad j = 2, \ldots, m \]

where \(n\) and \(m\) are finite and may or may not be equal. In order to obtain estimates of long-run elasticities, equilibrium must be approached and hence the series of
coefficients must converge to zero. If the new equilibrium levels resulting from a change in price or income are approached by period \( n \) and \( m \), respectively, we would expect

\[
\begin{align*}
(2.76) & \quad a_{n+1} = a_1 + nd = 0 \\
& \quad \text{or} \quad a_1 = -nd
\end{align*}
\]

and

\[
(2.77) \quad b_{m+1} = b_1 + me = 0
\]

\[
\text{or} \quad b_1 = -me
\]

Since \( n \) and \( m \) are finite and positive in value, \( a_1 \) must be opposite in sign to \( d \) and \( b_1 \) must be opposite in sign to \( e \). These relationships can be tested statistically.

Upon substituting assumptions (2.74) and (2.75) into (2.73), we obtain

\[
(2.78.a) \quad q_t = a + a_0 p_t + a_1 p_{t-1} + (a_1 + d) p_{t-2} + \ldots \\
+ [a_1 + (n-1)d]p_{t-n} + b_0 y_t + b_1 y_{t-1} \\
+ (b_1 + e)y_{t-2} + \ldots + [b_1 + (m-1)e]y_{t-m}
\]

\[
(2.78.b) \quad q_t = a + a_0 p_t + na_1 \sum_{i=1}^{n} \frac{p_{t-i}}{n} \\
+ d \sum_{i=2}^{n} \frac{(i-1) p_{t-i}}{n} \\
+ \sum_{i=2}^{n} \frac{S (i-1) p_{t-i}}{n}
\]
\[
q_t = a + a_0 p_t + a^*_i p_{at} + d^* p_{wt} + b_0 y_t \\
+ b^*_i y_{at} + e^* y_{wt}
\]

In (2.78.c) demand quantity is expressed as a function of current values of price and income as well as simple and weight averages of lagged values for price and income. The introduction of assumptions (2.74) and (2.75) reduces the number of parameters to be estimated and provides all of the data needed to obtain estimates of short-run and long-run elasticities.

The respective short-run price and income elasticity estimates obtained from (2.78.c) are \(a_0 \, p/q\) and \(b_0 \, y/q\). Estimates of the long-run price and income elasticity are

\[
\sum_{i=0}^{n} a_i = a_0 + n a_i + d^* \sum_{i=2}^{n} (i-1)
\]

\[
= a_0 + a_i^* + d^*
\]
\begin{equation}
\sum_{j=0}^{n} b_j = b_0 + mb_1 + e \sum_{j=2}^{m} (j-1) = b_0 + b^*_1 + e^* \; .
\end{equation}

**Working procedure**

The method described above is essentially a generalization of the Working procedure. Equations similar in form to those used by Working [179, pp. 46-52] can be derived by making additional assumptions about the series of coefficients and by performing an algebraic transformation upon the resulting equation.

Let the demand equation be approximated by (2.73). Assume the constants \(d=e=0\) and the periods \(n\) and \(m\) are finite so that from (2.74) and (2.75) we obtain \(a_1 = a_2 = \ldots = a_n\) and \(b_1 = b_2 = \ldots = b_m\). Substitution of this assumption into (2.73) yields

\begin{equation}
q_t = a + a_0 p_t + na_1 \frac{\sum_{i=1}^{n} p_{t-i}}{n} + b_0 y_t \\
+ mb_1 \frac{\sum_{j=1}^{m} y_{t-j}}{m}
\end{equation}

or

\begin{equation}
q_t = a + a_0 p_t + a^*_1 p_{at} + b_0 y_t + b^*_1 y_{at}
\end{equation}

where demand quantity is expressed as a function of current values of price and income and simple averages for the lagged
values of price and income. From (2.80.b) the short-run price and income elasticity estimates are $a_0 \frac{p}{q}$ and $b_0 \frac{y}{q}$. The long-run price and income elasticities are $(a_0 + a^*_1) \frac{p}{q}$ and $(b_0 + b^*_1) \frac{y}{q}$.

Working does not estimate an equation such as (2.80.b). By performing a transformation, Equation (2.80.b) becomes

$\begin{align*}
(2.81) \quad q_t &= a + a_0 \frac{p_t}{p_{at}} + (a_0 + a^*_1) p_{at} \\
&+ b_0 \frac{y_t}{y_{at}} + (b_0 + b^*_1) y_{at}
\end{align*}$

which is similar in form to the equations estimated by Working. He estimates the long-run elasticities directly from the coefficients associated with the simple averages of price and income.

Variable Preferences and Lagged Consumption

The concept of long-run elasticity of demand outlined above is based upon the hypothesis that there is a perceptible lag in the consumers' adjustment process for a change in price or other explanatory factors. The existence of long-run elasticities also depends upon the assumption that consumers' preferences remain fixed over time. As mentioned earlier, equations such as (2.49), (2.60), (2.72), (2.78.c), or (2.80.b) can be used to test the long-run elasticity
hypothesis and to provide approximations of long-run elasticities of demand when the necessary assumptions are satisfied. Ladd [73, pp. 13-23] shows that these equations can also be derived and used to test a different hypothesis, namely, that consumers' preferences for a good change over time. The fact that equations such as (2.49), (2.60), etc., can be used to test two extremely different hypotheses raises a dilemma and points out some of the limitations of existing techniques for dealing with the dynamic aspects of demand.

Ladd [73, p. 13] states,

This alternative derivation is also pertinent to the problem of the 'contrast between the carefully elaborated theory of the influence on demand of income and prices in a static situation and the extreme vagueness about the way in which changes in tastes and habits affect consumers' behavior'.

This contrast may in part be due to the fact that existing economic theory treats the parameters \( \theta_i \) in the consumers' utility function \( u = u(q_1, \ldots, q_n); \theta_1, \ldots, \theta_n \) as being given. Economists generally consider that the study of the determination of these parameters lies outside of their domain.

In an attempt to account for changes in tastes and habits in consumer behavior, some analysts have used first

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1This portion of the quotation is a direct quotation from Stone [113, p. 272].
differences or introduced time [60], [97, p. 1009], [176, pp. 240-242]. It can be argued that the use of lagged demand quantity is an alternative to the use of time. The purpose of this section is to present Ladd's arguments and to discuss the thesis that lagged demand quantity is an alternative for the use of time on the trend problem in demand analysis.

Ladd shows that existing demand theory can be generalized to cover the study of the influence of changes in tastes, habits, and other long-term variables upon consumers' behavior. This extension is made by introducing additional assumptions and variables into the Paretoan preference function approach. Following the arguments of Duesenberry [30, Chaps. 2 and 3] and Clark [21, pp. 347-353], Ladd claims that the preference system of a consumer is influenced by a trial and error or learning process which depends upon his own and other consumers' experiences. He [73, p. 16] states,

Whenever one's experiences show him that a particular commodity is more or less satisfactory than he thought it would be when he bought it, his preferences have been altered by experience. A cumulative and systematic shift in preferences could result as greater satisfaction leads to greater use, leading in turn to greater satisfaction, and so on.

These arguments are introduced into the theory by assuming that the parameters \( \theta_i \) in the consumers' utility function depend upon previously attained consumption levels of the
consumer and of other consumers he has observed. That is,

\[(2.82) \theta_i = \theta_i (q_{t-1}^0, q_{t-2}^0, \ldots, q_{t-1}^1, q_{t-2}^1, \ldots) ; \]

\[i = 1, \ldots, n \]

where \(q_{t-j}^0\) represents the quantity of the \(i^{th}\) good consumed in the previous \(j^{th}\) period by the consumer and \(q_{t-j}^1\) represents the quantity of the \(i^{th}\) good consumed during the previous \(j^{th}\) period by others whom the consumer has observed. Assuming the utility function is continuous, possesses first and second order partial derivatives and the necessary and sufficient conditions of consumer equilibrium are satisfied, the demand function and other relevant relationships can be derived by following Basmann's [7, pp. 47-58] analysis.

In order to illustrate the influences of changes in preferences upon the adjustment process of consumers, Ladd proceeds in the following manner. The concept of long-run elasticity depends upon the existence of a stable equilibrium. The two conditions required for stable equilibrium are:

1. the point which actual demand quantity moves toward and which would ultimately be attained, and
2. absence of further changes in the given situation, i.e., the assumption of ceterius paribus which implies that there are no changes in tastes. If the consumers' preferences for a good or for closely related goods vary, the consumers' equilibrium positions will also vary. The direction and magnitude of
the changes in equilibrium positions will differ when different factors are assumed to bring about changes in preferences. These differences have particular significance for the stability conditions of the new equilibrium positions, and therefore affect the way in which they can be treated in the analysis. Ladd argues that when changes in preferences result from the introduction of a new product, an improvement in old products, or some other factor exogenous to the consumer, the new equilibrium position may be treated as being stable. Analysis of changes in equilibrium demand quantities resulting from these changes can be handled in the same way as changes in equilibrium demand quantities brought about by a once-for-all change in price or income if the two required conditions are satisfied. When, however, the consumer's preference for a good is altered through his own and others learning experiences as specified in (2.82), the two required conditions for a stable equilibrium are not satisfied.

The influence on equilibrium demand quantities of changes in preferences due to the consumers' learning process is illustrated by use of Figure 2 where $p_t$ represents the price of the good in question, $\bar{q}_t$ represents equilibrium demand quantity and $q_t$ represents actual demand quantity. Prices for all other goods and income are assumed to remain fixed. Suppose $p_t$ decreases in period $t = 0$ and remains
FIGURE 2. HYPOTHETICAL CONSUMERS' ADJUSTMENT RESULTING FROM A CHANGE IN CONSUMERS' PREFERENCES.
fixed at the new level for all periods where \( t \geq 1 \). Further suppose \( q_t \) changes simultaneously from \( q_0 \) to the new equilibrium level \( q_1 \) as a result of the price change. Applying assumption (2.82), as \( q_t \) moves toward \( q_1 \) the consumers' preference system shifts and hence \( q_1 \) is no longer the relevant equilibrium level. Position \( q_2 \) represents the equilibrium level associated with the situation prevailing at period \( t = 2 \). If the consumer's preference system continues to change as a result of his learning experiences, then as \( q_t \) moves for \( q_1 \) to \( q_2 \) we find that \( q_2 \) is not a stable equilibrium position. Ladd [74, p. 19] argues that in such cases when the satisfaction of condition (1) causes the violation of (2) it cannot be meaningful to call \( q_1 \) and \( q_2 \) equilibrium levels of consumption. Likewise the argument against calling \( q_0 \) an equilibrium is that the previous violation of condition (2) is required for the satisfaction of condition (1). Since the positions commonly referred to as equilibrium levels in cases of a once-for-all change in price, income, or other explanatory variables cannot be considered as equilibrium levels of consumption in cases where the self-generating process continues to change the consumers' preferences, there is no long-run elasticity.

The following models are employed by Ladd to clarify the significance of his arguments and to derive the relevant relationships. Suppose equilibrium demand quantity is
determined by

\[(2.83) \quad \bar{q}_t = a' + b_0p_t + c_0y_t \]

and actual demand quantity is determined by

\[(2.84) \quad q_t = a + \sum_{i=1}^{t} S b_i p_{t-i} + \sum_{i=1}^{t} c_i y_{t-i} + (1-r) \bar{q}_t \]

\[q_t = a^* + b_0p_t + \sum_{i=1}^{t} S b_i p_{t-i} + c_0y_t + \sum_{i=1}^{t} c_i y_{t-i} \]

where \(a^* = a + a'(1-r), b_0 = b_0'(1-r), \) and \(c_0 = c_0'(1-r).\)

If equilibrium demand quantity is stable, long-run price and income elasticities exist and estimates can be obtained. Following the discussion of Koyck's procedure assume that the series of coefficients follow a converging geometric progression; i.e., \(b_i = rb_{i-1} \) and \(c_i = rc_{i-1} \) for all \(i = 1, 2, \ldots, t\) where \(-1 < r < 1\). By introducing this assumption into (2.84) and applying Koyck's method of reduction we obtain

\[(2.85) \quad q_t = a^* (1-r) + b_0p_t + c_0y_t + rq_{t-1} \]

which provides all of the coefficients needed to obtain estimates of the long-run elasticities. The long-run price and income elasticities are given by

\[(2.86) \quad \frac{\delta \bar{q}_t}{\delta p_t} = \frac{b_0}{1-r} \frac{p}{\bar{q}} = b'_0 \frac{p}{q} \]
and

\[ \frac{\partial \bar{q}_t}{\partial y_t} = \frac{c_0}{1-r} \frac{y}{q} = c_0' \frac{y}{q} \]

respectively, when the equation is linear in arithmetic values. The existence of such elasticities depends upon the realization of conditions discussed in section (2.b.).

Ladd shows that the stable level of equilibrium demand quantity can also be derived from (2.85) by use of different arguments. By fixing the price and income variables in (2.85) at the stationary levels \( p_t = \overline{p} \) and \( y_t = \overline{y} \) we obtain

\[ (2.87) \quad q_t = a^* (1-r) + b_0 \overline{p} + c_0 \overline{y} + rq_{t-1} \]

\[ q_t = \overline{w} + rq_{t-1} \]

where actual demand quantity is generated by the previous levels of actual demand quantity. That is, Equation (2.87) is a first order autoregressive process. Representing the initial level of actual demand quantity as \( q_0 \), the general solution for \( q_t \) is

\[ (2.88) \quad q_t = \overline{w} (1 + r + \ldots + r^{t-1}) + r^t q_0 \]

\[ = \overline{w} \left( \frac{1-r^t}{1-r} \right) + r^t q_0 \]

The equilibrium demand quantity is
\[ (2.89) \quad \bar{q}_t = \lim_{t \to \infty} q_t \]

\[ = \lim_{t \to \infty} \left[ \bar{w} \left( \frac{1-r^{t-1}}{1-r} \right) + r^t q_0 \right] \]

\[ = \frac{\bar{w}}{1-r} \]

\[ = a' + \frac{b_0}{1-r} p + \frac{c_0}{1-r} y \]

and is stable if \(-1 < r < 1\).

Let us now consider changes in the consumer's preferences for a good which are assumed to be induced by the consumer's consumption experiences in past periods. That is, suppose the parameters \( \theta_i \) depend upon consumption levels attained in the previous period by the consumer so that equilibrium demand quantity is approximated by

\[ (2.90) \quad \bar{q}_t = a' + b_0 p_t + c_0 y_t + d_0 q_{t-1} . \]

Obtaining \( q_{t-1} \) from (2.84) and substituting into (2.90) yields

\[ (2.91) \quad \bar{q}_t = (a' + d_0 a) + b_0 p_t + \sum_{i=2}^{t} b_i p_{t-i} \]

\[ + c_0 y_t + d_0 \sum_{i=2}^{t} c_i y_{t-i} + d_0 (1-r) \bar{q}_{t-1} . \]

Assigning stationary values for \( p_t = \bar{p} \) and \( y_t = \bar{y} \), collecting the constants and letting them equal \( \bar{V} \), (2.91) becomes
Let \( d_0^i \) (1-r) = K and introduce the initial conditions so that we obtain the general term

\[
q_t = v + d_0^i (1-r) q_{t-1}.
\]

On the basis of these equations Ladd argues that the positions \( q_t \) cannot be interpreted as stable equilibrium positions so long as preferences undergo self-generating changes. Due to the presence of \( q_{t-1} \) in (2.90) the movement of actual demand quantity \( q_t \) toward \( q_t \) brings about shifts in the equilibrium demand quantities, i.e., the satisfaction of condition (1) causes condition (2) to be violated. Ladd [73, p. 21] states,

... during each time period actual consumption \( q_t \) is approaching a different 'equilibrium' level \( q_t \) and this level changes over time in response to previous movements of \( q_t \) toward previous levels of \( q_t \). Equation (2.93) shows that the level of \( q_t \) goes on changing indefinitely even though prices and income remain constant. Equation (2.90) shows why it was previously stated that \( q_t \) could not be considered as the equilibrium level defined in our static theory. By (2.93) \( q_t \) continues to vary as \( t \to \infty \). In reality \( q_t \) cannot go increasing or decreasing indefinitely.

It appears reasonable to assume that the effects of the consumer's learning process will gradually decrease with the passage of time especially when dealing with one good.

In empirical analysis only data on the actual demand quantities are available. To test the hypothesis of a
self-generating change in preferences, the hypothetical demand equation should contain actual demand quantity lagged one or more periods. The relevant equation to use in performing this test can be derived from equations (2.84) and (2.90). Substituting (2.90) into (2.84) yields

\[(2.94) \quad q_t = [a + (1-r) a'] + (1-r) b_i p_t + \sum_{i=1}^{t} S b_i p_{t-i} + (1-r) c_i y_t - \sum_{i=1}^{t} S c_i y_{t-i} + (1-r) d_i q_{t-i} \]

By fixing stationary values for \( p_t = \bar{p} \) and \( y_t = \bar{y} \) we obtain

\[(2.95) \quad q_t = a* + [(1-r) b_0 \bar{p} + \sum_{i=1}^{t} S b_i \bar{p}] + [(1-r) c_0 \bar{y} + \sum_{i=1}^{t} S c_i \bar{y}] + (1-r) d_0 q_{t-1} \]

or

\[ q_t = a* + b_0 \bar{p} + c_0 \bar{y} + K q_{t-1} \]

which is similar in form to (2.49), (2.60), or (2.72).

The hypothesis of self-generating changes in preferences can be tested by testing the significance of \( K \), the coefficient associated with the lagged value of actual demand quantity in (2.95). If \( K \) is significantly different from zero we would accept the hypothesis of a self-generating change in preferences. Following Ladd's arguments, such changes in preferences would invalidate the stability conditions and hence we would reject the existence of a
long-run elasticity. If, however, the coefficients associated with the lagged value of demand quantity in (2.49), (2.60), or (2.72) are significantly different from zero, we would accept the long-run elasticity hypothesis. That is, finding significant values for the coefficients associated with the demand determining variables and demand quantity lagged one time period enables the acceptance of two contradictory hypotheses.

This dilemma appears to have particular significance for forecasts or predictions in policy formulation especially where judgments have to be made about the cause of changes in the situation. Ladd [73, p. 22] states,

A price or income elasticity estimated from a situation where self-generated preference changes follow a price change, will certainly over-state the response to any price change made in a later period when preferences have become constant.

The problem is that we do not know when the equilibrium position is stable or not.
ECONOMETRIC MODEL AND EMPIRICAL RESULTS

General Considerations Underlying the Model and Classification of the United States' Economy

As pointed out earlier, this study attempts to describe certain aspects of consumer behavior by postulating and testing different hypotheses about consumers' response over time to changes in certain economic variables. In the process of verifying the hypotheses, it is necessary to obtain estimates of parameters in the postulated demand equations and approximations of short-run and long-run elasticities. The choice and usefulness of the method of estimation and of the test for each hypothesis necessarily depend both on the existing observational data and on the assumptions made about the process generating the observations. These assumptions represent the statistical specifications and their choice is particularly difficult when applying statistical analysis in economics [69, pp. 113-126].

It is seldom possible to use designed and controlled experiments in economic investigations, and therefore, the investigator must choose a specification that is reasonably consistent with the process generating his data in the real world. Koopmans and Hood [69, p. 115] argue that the statistical specifications must be derived from information or assumptions concerning the underlying economic structure,
especially when the immediate or ultimate purpose of statistical estimation is to serve as a basis for predicting the effects of given changes in the structure. An econometric model has to be constructed, therefore, to represent the mechanism assumed to generate the observations. Hildreth and Jarrett [55, p. 6] state,

In the language that has been developed to consider statistical analysis of economic relations, the process by which a set of economic variables is generated is called a structure. The variables whose values are explained by the structure are called endogenous variables whereas those whose values are determined outside of the structure are called exogenous. The set of structures compatible with the investigator's statistical specification is called a model.

The model can be visualized as a complete set of structural equations with specifications about the form of the equations (for instance, their linearity and a designation of the variables occurring in each equation) and the class of functions to which the distribution functions of the unobserved shock or error variables belong.

Although there is no well-defined procedure to follow when constructing models, the economist can use existing theories of economic behavior and his knowledge of the characteristics of relevant economic units or sectors of the economy as a basis for specifying some of the economic properties of the model. For example, economic theory and a priori knowledge provide a basis for specifying (1) the kinds of equations to consider, (2) the variables to include
and their classification in each equation, and (3) certain restrictions to be placed on different types of equations. Economic considerations, however, do not provide a strong basis for choosing a particular form for the equations, and therefore, choice is generally made on the basis of simplicity or convention. Specifications about the statistical properties of the model (i.e., assumptions about shocks or errors, their relationship to the economic variables, their distributions, their serial correlations, etc.) generally rest heavily upon the assumptions that the statistician feels justified in making because information relating to the probability distributions and the effects of the unobserved disturbances is seldom available. For detailed discussions on specifications relating to the economic and statistical properties of models see [48], [50], [62], [64], [68], [67], and [69].

The econometric model necessarily has to be simplified representation of the real world as it tries to explain observed facts by postulating plausible behavior for firms and households under given conditions. The model to be presented was constructed to describe the aggregate behavior of economic units, in certain sectors, operating at different levels of the food marketing system in the United States. The model attempts to represent in simple form the underlying relationships relating to the production, process-
distributing, retailing and consumption of the major dairy, meat, fish, poultry, and fat and oil food products. Aside from the demand equations postulated to explain certain aspects of consumers' behavior, additional structural equations were specified to complete the system and thus enable the use of a simultaneous equations method of estimation.

Although the equations and variables are of an aggregative nature, existing microeconomic theory and a priori knowledge served as a basis for the selection of equations, the specification and classification of relevant variables in the equation and for analysis of the macroeconomic phenomena. The postulated equations are of necessity aggregative in nature. Klein [62, p. 13] states,

... there are so many individual units in our economic system that it is hopeless to consider estimating a complete set of equations involving the variables of microeconomics. As an alternative, we must sacrifice detailed information and develop systems of macroeconomic equations which involve a much smaller number of (aggregative) variables. It is a very difficult problem to pass from the theories of microeconomics to the theories of macroeconomics. The principal vehicles of this transformation are index numbers and other similar aggregates.

Possibly one of the major limitations of the model is that it is highly aggregative and requires complex aggregation. Many aspects of the aggregation problem and questions of the relation between micro and macro relationships and theories are yet unsolved [64] and [118]. Due to problems of aggregation, the use of a linear model and other simplifying
assumptions, the specifications are at best only approximations. It is not claimed that real conditions in the economy are actually or completely described by the model. It is hoped, however, that use of the model in testing different hypotheses about consumers' behavior will provide specifications which yield a fairly close approximation to the consumers' observed consumption behavior for specific food products.

In the model the economy is considered to be composed of households and firms, and their behavior and interaction in the farm and food product markets are assumed to explain the way in which certain observed variables are generated. Activities of the United States government relating to farm and food products are not explicitly considered, although government actions on farm production and food marketing have become quite important in the post World War II period. To simplify the construction and description of the model, the economy was arbitrarily assumed to be composed of five basic sectors, namely (1) the consumer sector, (2) the retailer sector, (3) the food marketing sector, (4) the non-food commercial sector, and (5) the farm sector.

The consumer's sector consists of households which offer their labor services to all other sectors for income and purchase goods and services with this income. The households or consumers are assumed to spend their income upon
specific food and non-food consumer goods or to save some of it for expenditure upon food and non-food goods in later time periods.

The retailer sector consists of all firms that sell food and non-food products to consumers. The retail firms are assumed basically to provide retailing services as they are considered (1) to purchase food and non-food products from the food marketing sector and non-food commercial sector, respectively, (2) to employ labor services from the consumer sector, and (3) to obtain other factor inputs from the non-food commercial sector.

The food marketing sector is assumed to include all firms that procure, assemble, process, distribute, import, and export food products. That is, the food marketing sector's activities encompass all of the operations involved: (1) in buying and moving domestic farm-food products from the farm sector, (2) in processing farm products into food products and maintaining inventories, (3) in distributing the food products to retailers or other sectors, and (4) in importing and exporting farm food products. All of the firms performing these operations are assumed to employ labor services from the consumer sector and to obtain other factor inputs from the non-food commercial sector.

The non-food commercial sector is assumed to consist of all firms concerned with the production, distribution, and
all other operations relating to non-food products. This sector was introduced mainly to simplify the model and all variables relating to the non-food commercial sector were assumed to be exogenous.

The farm sector is assumed to include all farm firms producing farm products for food and non-food uses. These firms are assumed: (1) to purchase farm products required in the production of a specific farm commodity from other firms in the farm sector, (2) to employ labor services from the consumer sector, (3) to obtain other factor inputs from the non-food commercial sector, (4) to sell non-food farm products, such as timber, to the non-food commercial sector, and (5) to sell farm products disposed of in food product uses to the food marketing sector. Although in reality some farmers process farm products and distribute their products directly to consumers, these activities are considered to be carried out by the relevant sectors described above.

For purposes of this investigation distinction has been made among the following 20 commodity groupings: butter, cheese, evaporated and condensed milk, fluid milk and cream, other processed dairy products, beef, veal, lamb and mutton, pork, chicken, eggs, other poultry products, canned fish, other fish products, lard, margarine, shortening, other fat and oil food products, other food products and non-food products. The consideration of these products necessarily
represents a compromise where some of the commodity groups are quite realistic and where other groups represent a lumping together of certain variables and structural relationships that would appear in a more general model. Although some of the above commodity groupings are necessarily quite aggregative and their selection arbitrary, they appear meaningful for the purpose at hand. Wold [176, p. 108] states,

... the commodities dealt with in demand theory are regarded as well-defined and distinct. This, however, is a simplified abstraction such as is met in any theory about real phenomena. Almost every commodity is the group label of a more or less vague aggregate of different items and qualities. In practice, what makes it realistic to disregard the group character of the commodities is the general experience that price changes within an aggregate are approximately uniform.

Many of the variables in the model, such as demand quantity, prices, etc., are assumed to have been derived following Wold's arguments [176, pp. 108-110], [176, pp. 243-244]. For example, the quantity of cheese demanded by consumers at any given period of time is assumed to include all types and qualities of cheese products and the aggregate quantity formed by a simple or weighted summation process. The use of such aggregate groupings as the other food products group or the non-food product group might be questioned as the prices for each of the products contained within these groupings generally do not experience proportional changes.
They were introduced, however, to keep the model within manageable limits. If it should appear desirable to consider products contained in these groupings, it would be quite easy to expand the model by disaggregating the commodity groupings into less heterogeneous product groups and to specify relevant equations for the redefined products.

Generally, a specific farm or food product can be disposed of in many ways and this adds complexity to their study. Difficulties arise in part from the diverse nature of the product and in part from the fact that their price structure is interrelated with the pricing structure for all other products. Due to differences in the characteristics of specific products, the specification of equations and variables relating to the production, processing, and distribution for specific products necessarily differs. It was necessary to make additional assumptions about the nature of the food marketing sector and to account for some of the characteristics associated with the production and marketing activities when specifying equations for the food products mentioned above. It was assumed that each of the specific food products, except pork and lard, is handled by a distinct industry in the food marketing sector. This assumption was made in order to keep the model relatively simple. Although in reality one finds that marketing firms often process or handle more than one line of products,
direct consideration of multiple product forms would be too difficult to handle in the present model. The variables relating to a specific food marketing industry were assumed to have been derived from aggregation over commodities, over firms, and over different types of marketing functions. For example, the relevant variables in the cheese marketing industry are considered to be composites obtained by aggregating over various types and qualities of cheese products, by aggregating over firms performing a specific marketing function and by aggregating over firms performing different marketing functions. Although the above assumptions are unrealistic, direct consideration of multiple product firms raises certain questions about interdependence in the buying and selling activities of firms as well as associated problems of joint costs. The above assumptions also overlook problems introduced by differences in the nature of integration existing at different levels of the marketing system for different food products. It is questionable if such problems can be treated adequately with the time series data available at the present time.

Discussion of the General Econometric Model

In specifying the equations relating to economic units in the sectors described earlier, it was assumed that the economic units behave according to some fundamental pattern
which can be written in equation form. For example, firms are assumed to maximize profits subject to technological possibilities and consumers are assumed to maximize their satisfaction or preferences subject to budgetary restraints. It was also assumed that some of the equations depend upon subjective anticipations. Following the arguments of Klein [62, pp. 13-58], the method of expressing anticipations by functions of lagged variables plus random disturbances was adopted to account for the subjective variations.

In the model, all variables are annual time series data and each is specified for time period \( t \) unless otherwise indicated. The \( j^{th} \) endogenous variable is denoted by \( y_j \), the \( j^{th} \) predetermined variable (i.e., the \( j^{th} \) endogenous variable lagged \( r \) time periods) by \( y_{j,t-r} \) and the \( j^{th} \) exogenous variable by \( z_j \). The constants \( B_{ij}, A_{ij} \) and \( A_{ij} \) represent the unknown parameters associated with the respective \( j^{th} \) endogenous, \( j^{th} \) predetermined and \( j^{th} \) exogenous variables contained in equation \( i \). As customary, the \( u_i \)'s represent the random disturbances in equation \( i \) and are assumed to possess the characteristics generally specified in shock models [69, pp. 117-121], [170, p. 24]. That is, the unobserved random disturbances \( u_i \) are assumed to come from a multivariate normal distribution with zero mean and a finite covariance matrix and to be independent over time. The variables \( y_{j,t-r} \) and \( z_j \) are considered to be exogenous.
in the sense that they are stochastically independent of the random disturbances $u_1$.

**Definitions for the endogenous and exogenous variables**

The endogenous variables contained in the model are as follows:

- $y_1$ per capita quantity of butter purchased at retail by consumers
- $y_2$ per capita quantity of cheese purchased at retail by consumers
- $y_3$ per capita quantity of evaporated and condensed milk purchased at retail by consumers
- $y_4$ per capita quantity of fluid milk and cream purchased at retail by consumers
- $y_5$ per capita quantity of other dairy products purchased at retail by consumers
- $y_6$ per capita quantity of beef purchased at retail by consumers
- $y_7$ per capita quantity of veal purchased at retail by consumers
- $y_8$ per capita quantity of lamb and mutton purchased at retail by consumers
- $y_9$ per capita quantity of pork purchased at retail by consumers
- $y_{10}$ per capita quantity of lard purchased at retail by consumers
- $y_{11}$ per capita quantity of chicken purchased at retail by consumers
- $y_{12}$ per capita quantity of eggs purchased at retail by consumers
- $y_{13}$ per capita quantity of other poultry products purchased at retail by consumers
\[ Y_{14} \] per capita quantity of canned fish products purchased at retail by consumers

\[ Y_{15} \] per capita quantity of other fish products purchased at retail by consumers

\[ Y_{16} \] per capita quantity of margarine purchased at retail by consumers

\[ Y_{17} \] per capita quantity of shortening purchased at retail by consumers

\[ Y_{18} \] per capita quantity of other fat and oil food products purchased at retail by consumers

\[ Y_{19} \] per capita quantity of food products other than dairy, meat, fish, poultry, eggs, or fat and oil products purchased at retail by consumers

\[ Y_{20} \] per capita quantity of non-food products purchased at retail by consumers

\[ Y_{21} \] retail price of butter deflated by the consumer price index

\[ Y_{22} \] retail price of cheese deflated by the consumer price index

\[ Y_{23} \] retail price of evaporated and condensed milk deflated by the consumer price index

\[ Y_{24} \] retail price of fluid milk and cream deflated by the consumer price index

\[ Y_{25} \] retail price of other dairy products deflated by the consumer price index

\[ Y_{26} \] retail price of beef deflated by the consumer price index

\[ Y_{27} \] retail price of veal deflated by the consumer price index

\[ Y_{28} \] retail price of lamb and mutton deflated by the consumer price index

\[ Y_{29} \] retail price of pork deflated by the consumer price index

\[ Y_{30} \] retail price of lard deflated by the consumer price index
retail price of chicken deflated by the consumer price index

retail price of eggs deflated by the consumer price index

retail price of other poultry products deflated by the consumer price index

retail price of canned fish products deflated by the consumer price index

retail price of other fish products deflated by the consumer price index

retail price of margarine deflated by the consumer price index

retail price of shortening deflated by the consumer price index

retail price of other fat and oil food products deflated by the consumer price index

retail price of food products other than dairy, meat, fish, poultry, eggs, and fat and oil food products deflated by the consumer price index

retail price of non-food products deflated by the consumer price index

per capita disposable income deflated by the consumer price index

aggregate consumer expenditures deflated by the consumer price index

quantity of fluid milk and cream supplied at retail by retailers

quantity of fluid milk and cream supplied at retail by the fluid milk and cream marketing industry

wholesale price of butter deflated by the consumer price index

wholesale price of cheese deflated by the consumer price index
\[ y_{47} \] wholesale price of evaporated and condensed milk deflated by the consumer price index

\[ y_{48} \] wholesale price of fluid milk and cream deflated by the consumer price index

\[ y_{49} \] wholesale price of other dairy products deflated by the consumer price index

\[ y_{50} \] wholesale price of beef deflated by the consumer price index

\[ y_{51} \] wholesale price of veal deflated by the consumer price index

\[ y_{52} \] wholesale price of lamb and mutton deflated by the consumer price index

\[ y_{53} \] wholesale price of pork deflated by the consumer price index

\[ y_{54} \] wholesale price of lard deflated by the consumer price index

\[ y_{55} \] wholesale price of chicken deflated by the consumer price index

\[ y_{56} \] wholesale price of eggs deflated by the consumer price index

\[ y_{57} \] wholesale price of other poultry products deflated by the consumer price index

\[ y_{58} \] wholesale price of canned fish products deflated by the consumer price index

\[ y_{59} \] wholesale price of other fish products deflated by the consumer price index

\[ y_{60} \] wholesale price of margarine deflated by the consumer price index

\[ y_{61} \] wholesale price of shortening deflated by the consumer price index

\[ y_{62} \] wholesale price of other fat and oil food products deflated by the consumer price index

\[ y_{63} \] wholesale price of animal and vegetable fats and oils deflated by the consumer price index
$y_{64}$ wholesale price of food products other than dairy, meat, fish, poultry, eggs, and fat and oil food products deflated by the consumer price index

$y_{65}$ quantity of butter supplied from current production by the butter marketing industry

$y_{66}$ quantity of cheese supplied from current production by the cheese marketing industry

$y_{67}$ quantity of evaporated and condensed milk supplied from current production by the evaporated and condensed milk marketing industry

$y_{68}$ quantity of fluid milk and cream supplied from current production by the fluid milk and cream marketing industry

$y_{69}$ quantity of other dairy products supplied from current production by the other dairy products marketing industry

$y_{70}$ quantity of beef supplied from current production by the beef marketing industry

$y_{71}$ quantity of veal supplied from current production by the veal marketing industry

$y_{72}$ quantity of lamb and mutton supplied from current production by the lamb and mutton marketing industry

$y_{73}$ quantity of pork supplied from current production by the pork and lard marketing industry

$y_{74}$ quantity of lard supplied from current production by the pork and lard marketing industry

$y_{75}$ quantity of chicken supplied from current production by the chicken marketing industry

$y_{76}$ quantity of eggs supplied from current production by the egg marketing industry

$y_{77}$ quantity of other poultry products supplied from current production by the other poultry product marketing industry

$y_{78}$ quantity of canned fish products supplied from current production by the canned fish product marketing industry
\( y_{79} \) quantity of other fish products supplied from current production by the other fish product marketing industry

\( y_{80} \) quantity of margarine supplied from current production by the margarine marketing industry

\( y_{81} \) quantity of shortening supplied from current production by the shortening marketing industry

\( y_{82} \) quantity of other fat and oil food products supplied from current production by the other fat and oil food product marketing industry

\( y_{83} \) quantity of animal and vegetable fats and oils supplied from current production by the fat and oil mill processing industry

\( y_{84} \) quantity of food products other than dairy, meat, fish, poultry, eggs, and fat and oil food products supplied from current production by the other food products marketing industries

\( y_{85} \) price received by farmers for milk and cream deflated by the consumer price index

\( y_{86} \) price received by farmers for beef animals deflated by the consumer price index

\( y_{87} \) price received by farmers for veal animals deflated by the consumer price index

\( y_{88} \) price received by farmers for lambs deflated by the consumer price index

\( y_{89} \) price received by farmers for hogs deflated by the consumer price index

\( y_{90} \) price received by farmers for chicken deflated by the consumer price index

\( y_{91} \) price received by farmers for eggs deflated by the consumer price index

\( y_{92} \) price received by farmers for other poultry products deflated by the consumer price index

\( y_{93} \) price received by farmers for farm products purchased by the other food products marketing industries deflated by the consumer price index
$Y_{94}$ price received by farmers and the non-food commercial sector for raw materials used in the fat and oil mill processing industry deflated by the consumer price index

$Y_{95}$ price received for fish at docks and piers deflated by the consumer price index

$Y_{96}$ quantity of butter held in inventory by the butter marketing industry at the end of period $t$

$Y_{97}$ quantity of cheese held in inventory by the cheese marketing industry at the end of period $t$

$Y_{98}$ quantity of evaporated and condensed milk held in inventory by the evaporated and condensed milk marketing industry at the end of period $t$

$Y_{99}$ quantity of other dairy products held in inventory by the other dairy products marketing industry at the end of period $t$

$Y_{100}$ quantity of beef held in inventory by the beef marketing industry at the end of period $t$

$Y_{101}$ quantity of veal held in inventory by the veal marketing industry at the end of period $t$

$Y_{102}$ quantity of lamb and mutton held in inventory by the lamb and mutton marketing industry at the end of period $t$

$Y_{103}$ quantity of pork held in inventory by the pork and lard marketing industry at the end of period $t$

$Y_{104}$ quantity of lard held in inventory by the pork and lard marketing industry at the end of period $t$

$Y_{105}$ quantity of chicken held in inventory by the chicken marketing industry at the end of period $t$

$Y_{106}$ quantity of eggs held in inventory by the egg marketing industry at the end of period $t$

$Y_{107}$ quantity of other poultry products held in inventory by the other poultry products marketing industry at the end of period $t$

$Y_{108}$ quantity of canned fish held in inventory by the canned fish marketing industry at the end of period $t$
$y_{109}$ quantity of other fish products held in inventory by the other fish products marketing industry at the end of period $t$

$y_{110}$ quantity of margarine held in inventory by the margarine marketing industry at the end of period $t$

$y_{111}$ quantity of shortening held in inventory by the shortening marketing industry at the end of period $t$

$y_{112}$ quantity of other fat and oil food products held in inventory by the other fat and oil food products marketing industries at the end of period $t$

$y_{113}$ quantity of animal and vegetable fats and oils held in inventory by the fat and oil mill processing industry at the end of period $t$

$y_{114}$ quantity of food products other than dairy, meat, fish, poultry, eggs, and fat and oil food products held in inventory by the other food products marketing industry at the end of period $t$

$y_{115}$ quantity of milk and cream purchased from farms by the butter marketing industry

$y_{116}$ quantity of milk and cream purchased from farms by the cheese marketing industry

$y_{117}$ quantity of milk and cream purchased from farms by the evaporated and condensed milk marketing industry

$y_{118}$ quantity of milk and cream purchased from farms by the fluid milk and cream marketing industry

$y_{119}$ quantity of milk and cream purchased from farms by the other dairy products marketing industry

$y_{120}$ quantity of beef animals purchased from farms by the beef marketing industry

$y_{121}$ quantity of veal animals purchased from farms by the veal marketing industry

$y_{122}$ quantity of lambs and sheep purchased from farms by the lamb and mutton marketing industry

$y_{123}$ quantity of hogs purchased from farms by the pork and lard marketing industry
\[ Y_{124} \] quantity of chickens purchased from farms by the chicken marketing industry

\[ Y_{125} \] quantity of eggs purchased from farms by the egg marketing industry

\[ Y_{126} \] quantity of other poultry products purchased from farms by the other poultry product marketing industry

\[ Y_{127} \] quantity of fish purchased at docks and piers by the canned fish marketing industry

\[ Y_{128} \] quantity of fish purchased at docks and piers by the other fish product marketing industry

\[ Y_{129} \] quantity of animal and vegetable fats and oils purchased from the fat and oil mill processing industry by the margarine marketing industry

\[ Y_{130} \] quantity of butter purchased from the butter marketing industry by the other margarine marketing industry

\[ Y_{131} \] quantity of lard purchased from the pork and lard marketing industry by the margarine marketing industry

\[ Y_{132} \] quantity of animal and vegetable fats and oils purchased from the fat and oil mill processing industry by the shortening marketing industry

\[ Y_{133} \] quantity of lard purchased from the pork and lard marketing industry by the shortening marketing industry

\[ Y_{134} \] quantity of animal and vegetable fats and oils purchased from the fat and oil mill processing industry by the other fat and oil food products marketing industry

\[ Y_{135} \] quantity of raw materials purchased from farms and non-food commercial sectors by the fat and oil mill processing industry

\[ Y_{136} \] quantity of farm products purchased from farms by the other food products marketing industries

\[ Y_{137} \] quantity of milk and cream supplied from current production by the farm sector

\[ Y_{138} \] quantity of fish supplied at docks and piers
The exogenous variables contained in the model are as follows:

$z_1$ per capita quantity of liquid assets held by consumers at the end of period $t-1$ deflated by the consumer price index

$z_2$ prices paid for factors of production used by retailers in retailing butter deflated by the consumer price index

$z_3$ prices paid for factors of production used by retailers in retailing cheese deflated by the consumer price index

$z_4$ prices paid for factors of production used by retailers in retailing evaporated and condensed milk deflated by the consumer price index

$z_5$ prices paid for factors of production used by retailers in retailing fluid milk and cream deflated by the consumer price index

$z_6$ prices paid for factors of production used by retailers in retailing other dairy products deflated by the consumer price index

$z_7$ prices paid for factors of production used by retailers in retailing beef deflated by the consumer price index

$z_8$ prices paid for factors of production used by retailers in retailing veal deflated by the consumer price index

$z_9$ prices paid for factors of production used by retailers in retailing lamb and mutton deflated by the consumer price index

$z_{10}$ prices paid for factors of production used by retailers in retailing pork deflated by the consumer price index

$z_{11}$ prices paid for factors of production used by retailers in retailing lard deflated by the consumer price index

$z_{12}$ prices paid for factors of production used by retailers in retailing chicken deflated by the consumer price index

$z_{13}$ prices paid for factors of production used by retailers in retailing eggs deflated by the consumer price index
prices paid for factors of production used by retailers in retailing other poultry products deflated by the consumer price index

prices paid for factors of production used by retailers in retailing canned fish deflated by the consumer price index

prices paid for factors of production used by retailers in retailing other fish products deflated by the consumer price index

prices paid for factors of production used by retailers in retailing margarine deflated by the consumer price index

prices paid for factors of production used by retailers in retailing shortening deflated by the consumer price index

prices paid for factors of production used by retailers in retailing other fat and oil food products deflated by the consumer price index

prices paid for factors of production used by retailers in retailing food products other than dairy, meat, fish, poultry, eggs, and fat and oil food products deflated by the consumer price index

prices paid for factors of production used by retailers in retailing non-food products deflated by the consumer price index

wages paid to labor employed in the butter marketing industry deflated by the consumer price index

prices paid for factors of production other than labor and butterfat used in the butter marketing industry deflated by the consumer price index

index of technical productivity for the butter marketing industry

cost of holding butter in inventory by the butter marketing industry deflated by the consumer price index

wages paid to labor employed in the cheese marketing industry deflated by the consumer price index
prices paid for factors of production other than labor and butterfat by the cheese marketing industry deflated by the consumer price index

index of technical productivity for the cheese marketing industry

cost of holding cheese in inventory by the cheese marketing industry deflated by the consumer price index

wages paid to labor employed in the evaporated and condensed milk marketing industry deflated by the consumer price index

prices paid for factors of production other than labor and butterfat by the evaporated and condensed milk marketing industry deflated by the consumer price index

index of technical productivity for the evaporated and condensed milk marketing industry

cost of holding evaporated and condensed milk in inventory by the evaporated and condensed milk marketing industry deflated by the consumer price index

wages paid to labor employed in the fluid milk and cream marketing industry deflated by the consumer price index

prices paid for factors of production other than labor and butterfat by the fluid milk and cream marketing industry deflated by the consumer price index

index of technical productivity for the fluid milk and cream marketing industry

wages paid to labor employed in the other dairy product marketing industry deflated by the consumer price index

prices paid for factors of production other than labor and butterfat by the other dairy product marketing industry deflated by the consumer price index

index of technical productivity for the other dairy product marketing industry

cost of holding other processed dairy products in inventory by the other dairy product marketing industry deflated by the consumer price index
| Z_{41} | wages paid to labor employed in the beef marketing industry deflated by the consumer price index |
| Z_{42} | prices paid for factors of production other than labor and beef animals by the beef marketing industry deflated by the consumer price index |
| Z_{43} | index of technical productivity for the beef marketing industry |
| Z_{44} | cost of holding beef products in inventory by the beef marketing industry deflated by the consumer price index |
| Z_{45} | wages paid to labor employed in the veal marketing industry deflated by the consumer price index |
| Z_{46} | prices paid for factors of production other than labor and veal animals by the veal marketing industry deflated by the consumer price index |
| Z_{47} | index of technical productivity for the veal marketing industry |
| Z_{48} | cost of holding veal products in inventory by the veal marketing industry deflated by the consumer price index |
| Z_{49} | wages paid to labor employed in the lamb and mutton marketing industry deflated by the consumer price index |
| Z_{50} | prices paid for factors of production other than labor, lambs, and sheep by the lamb and mutton marketing industry deflated by the consumer price index |
| Z_{51} | index of technical productivity for the lamb and mutton marketing industry |
| Z_{52} | cost of holding lamb and mutton in inventory by the lamb and mutton marketing industry deflated by the consumer price index |
| Z_{53} | wages paid to labor employed in the pork and lard marketing industry deflated by the consumer price index |
| Z_{54} | prices paid for factors of production other than labor and hogs by the pork and lard marketing industry deflated by the consumer price index |
| Z_{55} | index of technical productivity for the pork and lard marketing industry |
cost of holding pork in inventory by the pork and lard marketing industry deflated by the consumer price index

wages paid to labor employed in the chicken marketing industry deflated by the consumer price index

prices paid for factors of production other than labor and chickens by the chicken marketing industry deflated by the consumer price index

index of technical productivity for the chicken marketing industry

cost of holding chicken in inventory by the chicken marketing industry deflated by the consumer price index

wages paid to labor employed in the egg marketing industry deflated by the consumer price index

prices paid for factors of production other than labor and eggs by the egg marketing industry deflated by the consumer price index

index of technical productivity for the egg marketing industry

cost of holding eggs in inventory by the egg marketing industry deflated by the consumer price index

wages paid to labor employed in the other poultry product marketing industry deflated by the consumer price index

prices paid for factors of production other than labor and poultry by the other poultry product marketing industry deflated by the consumer price index

index of technical productivity for the other poultry product marketing industry

cost of holding other poultry products in inventory by the other poultry product marketing industry deflated by the consumer price index

wages paid to labor employed in the canned fish marketing industry deflated by the consumer price index
prices paid for factors of production other than labor and fish by the canned fish marketing industry deflated by the consumer price index

index of technical productivity for the canned fish marketing industry

cost of holding canned fish in inventory by the canned fish marketing industry deflated by the consumer price index

wages paid to labor employed in the other fish product marketing industry deflated by the consumer price index

prices paid for factors of production other than labor and fish by the other fish product marketing industry deflated by the consumer price index

index of technical productivity for the other fish product marketing industry

cost of holding other fish products in inventory by the other fish product marketing industry deflated by the consumer price index

wages paid to labor employed in the margarine marketing industry deflated by the consumer price index

prices paid for factors of production other than labor and margarine ingredients by the margarine marketing industry deflated by the consumer price index

index of technical productivity for the margarine marketing industry

cost of holding margarine in inventory by the margarine marketing industry deflated by the consumer price index

wages paid to labor employed in the shortening marketing industry deflated by the consumer price index

prices paid for factors of production other than labor and shortening ingredients by the shortening marketing industry deflated by the consumer price index

index of technical productivity for the shortening marketing industry

cost of holding shortening in inventory by the shortening marketing industry deflated by the consumer price index
$z_{86}$ wages paid to labor employed in the other fat and oil food products marketing industry deflated by the consumer price index

$z_{87}$ prices paid for factors of production other than labor and ingredients by the other fat and oil food products marketing industry deflated by the consumer price index

$z_{88}$ index of technical productivity for the other fat and oil food products marketing industry

$z_{89}$ cost of holding other fat and oil food products in inventory by the other fat and oil food products marketing industry deflated by the consumer price index

$z_{90}$ wages paid to labor employed in the fat and oil mill processing industry deflated by the consumer price index

$z_{91}$ prices paid for factors of production other than labor and raw materials by the fat and oil mill processing industry deflated by the consumer price index

$z_{92}$ index of technical productivity for the fat and oil mill processing industry

$z_{93}$ cost of holding animal and vegetable fats and oils in inventory by the fat and oil mill processing industry deflated by the consumer price index

$z_{94}$ wages paid to labor employed in the other food products marketing industries deflated by the consumer price index

$z_{95}$ prices paid for factors of production other than labor and farm products by the other food products marketing industries deflated by the consumer price index

$z_{96}$ index of technical productivity for the other food products marketing industries

$z_{97}$ cost of holding other food products in inventory by the other food products marketing industries deflated by the consumer price index

$z_{98}$ quantity of butter imported during period $t$ by the butter marketing industry
$$z_{99} \quad \text{quantity of cheese imported during period } t \text{ by the cheese marketing industry}$$

$$z_{100} \quad \text{quantity of evaporated and condensed milk imported during period } t \text{ by the evaporated and condensed milk marketing industry}$$

$$z_{101} \quad \text{quantity of other dairy products imported during period } t \text{ by the other dairy product marketing industry}$$

$$z_{102} \quad \text{quantity of beef imported during period } t \text{ by the beef marketing industry}$$

$$z_{103} \quad \text{quantity of veal imported during period } t \text{ by the veal marketing industry}$$

$$z_{104} \quad \text{quantity of lamb and mutton imported during period } t \text{ by the lamb and mutton marketing industry}$$

$$z_{105} \quad \text{quantity of pork imported during period } t \text{ by the pork and lard marketing industry}$$

$$z_{106} \quad \text{quantity of chicken imported during period } t \text{ by the chicken marketing industry}$$

$$z_{107} \quad \text{quantity of eggs imported during period } t \text{ by the egg marketing industry}$$

$$z_{108} \quad \text{quantity of other poultry products imported during period } t \text{ by the other poultry product marketing industry}$$

$$z_{109} \quad \text{quantity of canned fish imported during period } t \text{ by the canned fish marketing industry}$$

$$z_{110} \quad \text{quantity of other fish products imported during period } t \text{ by the other fish product marketing industry}$$

$$z_{111} \quad \text{quantity of shortening imported during period } t \text{ by the shortening marketing industry}$$

$$z_{112} \quad \text{quantity of other fat and oil food products imported during period } t \text{ by the other fat and oil food product marketing industry}$$

$$z_{113} \quad \text{quantity of animal and vegetable fats and oils imported during period } t \text{ by the fat and oil mill processing industry}$$
\[ z_{114} \] \text{quantity of other food products imported during period } t \text{ by the other food products marketing industries}

\[ z_{115} \] \text{quantity of butter exported during period } t \text{ by the butter marketing industry}

\[ z_{116} \] \text{quantity of cheese exported during period } t \text{ by the cheese marketing industry}

\[ z_{117} \] \text{quantity of evaporated and condensed milk exported during period } t \text{ by the evaporated and condensed milk marketing industry}

\[ z_{118} \] \text{quantity of other dairy products exported during period } t \text{ by the other dairy product marketing industry}

\[ z_{119} \] \text{quantity of beef exported during period } t \text{ by the beef marketing industry}

\[ z_{120} \] \text{quantity of veal exported during period } t \text{ by the veal marketing industry}

\[ z_{121} \] \text{quantity of lamb and mutton exported during period } t \text{ by the lamb and mutton marketing industry}

\[ z_{122} \] \text{quantity of pork exported during period } t \text{ by the pork and lard marketing industry}

\[ z_{123} \] \text{quantity of lard exported during period } t \text{ by the pork and lard marketing industry}

\[ z_{124} \] \text{quantity of chicken exported during period } t \text{ by the chicken marketing industry}

\[ z_{125} \] \text{quantity of eggs exported during period } t \text{ by the egg marketing industry}

\[ z_{126} \] \text{quantity of other poultry products exported during period } t \text{ by the other poultry products marketing industry}

\[ z_{127} \] \text{quantity of canned fish exported during period } t \text{ by the canned fish marketing industry}

\[ z_{128} \] \text{quantity of other fish products exported during period } t \text{ by the other fish product marketing industry}

\[ z_{129} \] \text{quantity of margarine exported during period } t \text{ by the margarine marketing industry}
$Z_{130}$ quantity of shortening exported during period $t$ by the shortening marketing industry

$Z_{131}$ quantity of other fat and oil food products exported during period $t$ by the other fat and oil food product marketing industry

$Z_{132}$ quantity of animal and vegetable fats and oils exported during period $t$ by the fat and oil mill processing industry

$Z_{133}$ quantity of other food products exported during period $t$ by the other food product marketing industries

$Z_{134}$ number of milk cows and heifers on farms at the beginning of period $t$

$Z_{135}$ number of beef animals on farms at the beginning of period $t$

$Z_{136}$ number of veal animals on farms at the beginning of period $t$

$Z_{137}$ number of lambs and sheep on farms at the beginning of period $t$

$Z_{138}$ number of hogs on farms at the beginning of period $t$

$Z_{139}$ number of chickens on farms at the beginning of period $t$

$Z_{140}$ number of hens and pullets on farms at the beginning of period $t$

$Z_{141}$ number of other poultry on farms at the beginning of period $t$

$Z_{142}$ quantity of feeds fed to milk cows and heifers during period $t$

$Z_{143}$ quantity of feeds fed to beef animals during period $t$

$Z_{144}$ quantity of feeds fed to veal animals during period $t$

$Z_{145}$ quantity of feeds fed to lambs and sheep during period $t$

$Z_{146}$ quantity of feeds fed to hogs during period $t$

$Z_{147}$ quantity of feeds fed to chickens during period $t$
quantity of feeds fed to hens and pullets during period $t$

quantity of feeds fed to other poultry during period $t$

wages paid to labor employed in milk production operations on farms deflated by the consumer price index

prices paid for factors of production other than labor and feeds used in milk production operations on farms deflated by the consumer price index

index of technical productivity for milk production operations on farms

wages paid to labor employed in beef cattle production operations on farms deflated by the consumer price index

prices paid for factors of production other than labor and feed used in beef cattle production operations on farms deflated by the consumer price index

index of technical productivity for beef cattle production operations on farms

wages paid to labor employed in veal animal production operations on farms deflated by the consumer price index

prices paid for factors of production other than labor and feeds used in veal animal production operations on farms deflated by the consumer price index

index of technical productivity for veal animal production operations on farms

wages paid to labor employed in lamb and sheep production operations on farms deflated by the consumer price index

prices paid for factors of production other than labor and feeds used in lamb and sheep production operations on farms deflated by the consumer price index

index of technical productivity for lamb and sheep production operations on farms

wages paid to labor employed in hog production operations on farms deflated by the consumer price index
prices paid for factors of production other than labor and feeds used in hog production operations on farms deflated by the consumer price index

index of technical productivity for hog production operations on farms

wages paid to labor employed in chicken production operations on farms deflated by the consumer price index

prices paid for factors of production other than labor and feeds used in chicken production operations on farms deflated by the consumer price index

index of technical productivity for chicken production operations on farms

wages paid to labor employed in egg production operations on farms deflated by the consumer price index

prices paid for factors of production other than labor and feeds used in egg production operations on farms deflated by the consumer price index

index of technical productivity for egg production operations on farms

wages paid to labor employed in other poultry production operations on farms deflated by the consumer price index

prices paid for factors of production other than labor and feeds used in other poultry production operations on farms deflated by the consumer price index

index of technical productivity for other poultry production operations on farms

wages paid to labor employed in fishing and handling fish at docks and piers deflated by the consumer price index

prices paid for factors of production used in fishing and handling fish at docks and piers deflated by the consumer price index

quantity of seeds and farm products sold by farmers to the fat and oil mill processing industry
quantity of animal and vegetable fats and oils sold to the non-food commercial sector by the fat and oil mill processing industry

quantity of farm products sold by farmers to the other food products marketing industries

wholesale price of all non-food products deflated by the consumer price index

civilian population in the United States as of July 1 for time period t adjusted for underenumeration

government expenditures on goods and services component of the Gross National Product

gross private domestic investment and the net foreign investment components of the Gross National Product

Structural equations relating to the consumers' sector

As mentioned earlier one of the practical purposes underlying the construction and estimation of structural equations is to describe how particular economic units, institutions, and other phenomena respond to changes in certain economic variables under given conditions. The behavior equations relevant to the consumer sector are the demand equations for specific products and the consumption function.

Consumers' demand equations for butter. The simplest type of demand equation derived from the microeconomic theory of consumer choice is a linear function where demand quantity is expressed as a function of prices and income, such as in Equation (2.10). By use of the theoretical considerations underlying the traditional static theory of demand and the linear approximation where demand quantity is
expressed as a function in arithmetic values of real prices and real per capita disposable income, the demand equation for butter is postulated as

$$\text{(3.1.a)} \quad B_{1,i}y_i + \sum_{i=21}^{40} B_{1,i}y_i + B_{1,41}y_{41} + B_{1,0} = u_1$$

where $y_i$ represents per capita consumption of butter, $y_i$, $i=21, \ldots, 40$, represent the real prices for butter and for each of the other commodity groups as previously defined, and $y_{41}$ represents real per capita disposable income of consumers. $u_1$ is the random disturbance variable introduced to represent factors not specified in the equation.

As mentioned earlier the use of specifications such as (3.1.a) do not provide appropriate results when the analysis is directed at the explanation of consumers' adjustment processes for changing conditions or where the influence of consumers' past behavior is important. By use of the theoretical considerations discussed in sections b and c of Chapter 2, it is possible to test hypotheses about consumers' short-run and long-run responses to changes in certain economic factors and/or about self-generating changes in consumers' preferences for specific goods. Using the theoretical considerations underlying the derivation of equations (2.48) and (2.95) and assuming the demand function for butter is approximated as a linear function in
arithmetic values, the derived equation is postulated as

\[(3.1.b) \quad B_{1,1}Y_1 + \sum_{i=2}^{40} B_{1,i}Y_i + B_{1,41}Y_{41} + A_{1,1}Y_{1,t-1} + B_{1,0}^{*} = u_{1}^{*}\]

where \(y_{1,t-1}\) represents per capita consumption of butter lagged one time period. The constant \(B_{1,0}^{*}\) and the random disturbance variable \(u_{1}^{*}\) in (3.1.b) are not the same as \(B_{1,0}\) and \(u_{1}\) in Equation (3.1.a), as Equation (3.1.b) is a reduced equation derived by use of Koyck's assumption about the adjustment path or by use of assumptions about self-generating changes in preferences. One of the difficulties encountered with Koyck's or Nerlove's procedures for estimating long-run elasticities is the problem of serial correlation in the residuals [70, Chap. 2], [94, pp. 47-82].

Aside from considering prices and income as the important factors influencing consumers' consumption patterns for specific products, it is fairly realistic to assume that liquid asset holdings also play a dominate role. By using the generalization of static demand theory where only liquid asset holdings are included instead of all asset holdings as discussed in Section 2.a, the demand equation for butter can be approximated by the linear function in arithmetic values as
(3.1.c) \[ B_{1,1}y_1 + \sum_{i=21}^{40} B_{1,i}y_i + B_{1,41}y_{41} \]

\[ + A_{1,1}z_1 + B_{1,0} = u_1 \]

where \( z_1 \) represents real per capita liquid asset holdings of consumers.

The specification given by Equation (3.1.c), like that of (3.1.a), does not reflect the time-incidence upon consumers' behavior of changes in certain economic variables. By using the theoretical considerations underlying the specification of Equation (3.1.c) as the basic theory and assuming the long-run or variable preference hypothesis, the demand equation for butter (reduced equation) can also be postulated as a linear function in arithmetic values as follows

(3.1.d) \[ B_{1,1}y_1 + \sum_{i=21}^{40} B_{1,i}y_i + B_{1,41}y_{41} \]

\[ + A_{1,1}y_{1,t-1} + A_{1,1}z_1 + B_{1,0}^* = u_1^* \]

where the variables are as defined earlier. The constant \( B_{1,0}^* \) and the random disturbance \( u_1^* \) are similar in form to those in Equation (3.1.b) but are derived from different equations. That is, Equation (3.1.d) can be derived from a demand equation which is specified as a function of prices, income and asset holdings where each explanatory variable is taken with a distributed lag and the adjustment
path is approximated by Koyck's assumption or where changes in demand quantity are in part induced by self-generating changes in consumers' preferences.

The four forms of equations, specified above as representing the demand equations for butter, are also considered for each of the other product groupings. To avoid considerable repetition, however, only equations of form (a) are presented for each of the other commodity groups. As illustrated earlier form (b) differs from form (a) only by the addition of $y_{t-1}$; form (c) differs from (a) only by the addition of $z_i$; and form (d) differs from (a) by the addition of $y_{t-1}$ and $z_i$.

**Consumers' demand equations for cheese**

\[(3.2.a) \quad B_2,2y_2 + \sum_{i=21}^{40} B_2,1y_1 + B_2,41y_{41} + B_2,0 = u_2\]

**Consumers' demand equations for evaporated and condensed milk**

\[(3.3.a) \quad B_3,3y_3 + \sum_{i=21}^{40} B_3,1y_1 + B_3,41y_{41} + B_3,0 = u_3\]

**Consumers' demand equations for fluid milk and cream**

\[(3.4.a) \quad B_4,4y_4 + \sum_{i=21}^{40} B_4,1y_1 + B_4,41y_{41} + B_4,0 = u_4\]

**Consumers' demand equations for other dairy products**

(i.e., dairy products other than butter, cheese, evaporated
and condensed milk and fluid milk and cream)

\[(3.5.a) \quad B_5, v_5 + \sum_{i=21}^{40} B_5, i v_i + B_5, 41 v_{41} + B_5, 0 = u_5\]

Consumers' demand equations for beef

\[(3.6.a) \quad B_6, v_6 + \sum_{i=21}^{40} B_6, i v_i + B_6, 41 v_{41} + B_6, 0 = u_6\]

Consumers' demand equations for veal

\[(3.7.a) \quad B_7, v_7 + \sum_{i=21}^{40} B_7, i v_i + B_7, 41 v_{41} + B_7, 0 = u_7\]

Consumers' demand equations for lamb and mutton

\[(3.8.a) \quad B_8, v_8 + \sum_{i=21}^{40} B_8, i v_i + B_8, 41 v_{41} + B_8, 0 = u_8\]

Consumers' demand equations for pork

\[(3.9.a) \quad B_9, v_9 + \sum_{i=21}^{40} B_9, i v_i + B_9, 41 v_{41} + B_9, 0 = u_9\]

Consumers' demand equations for lard

\[(3.10.a) \quad B_{10}, v_{10} + \sum_{i=21}^{40} B_{10}, i v_i + B_{10}, 41 v_{41}
+ B_{10}, 0 = u_{10}\]

Consumers' demand equations for chicken

\[(3.11.a) \quad B_{11}, v_{11} + \sum_{i=21}^{40} B_{11}, i v_i + B_{11}, 41 v_{41}
+ B_{11}, 0 = u_{11}\]
Consumers' demand equations for eggs

\[(3.12.a) \quad B_{12,12}y_{12} + \sum_{i=21}^{40} B_{12,i}y_i + B_{12,41}y_{41} + B_{12,0} = u_{12}\]

Consumers' demand equations for other poultry products
(i.e., all poultry products other than chickens and eggs)

\[(3.13.a) \quad B_{13,13}y_{13} + \sum_{i=21}^{40} B_{13,i}y_i + B_{13,41}y_{41} + B_{13,0} = u_{13}\]

Consumers' demand equations for canned fish

\[(3.14.a) \quad B_{14,14}y_{14} + \sum_{i=21}^{40} B_{14,i}y_i + B_{14,41}y_{41} + B_{14,0} = u_{14}\]

Consumers' demand equations for other fish products
(i.e., for all types and qualities of fish products other than canned fish)

\[(3.15.a) \quad B_{15,15}y_{15} + \sum_{i=21}^{40} B_{15,i}y_i + B_{15,41}y_{41} + B_{15,0} = u_{15}\]

Consumers' demand equations for margarine

\[(3.16.a) \quad B_{16,16}y_{16} + \sum_{i=21}^{40} B_{16,i}y_i + B_{16,41}y_{41} + B_{16,0} = u_{16}\]
Consumers' demand equations for shortening

\[ (3.17.a) \quad B_{17,17} Y_{17} + \sum_{i=21}^{40} \left( B_{17,i} Y_{i} + B_{17,41} Y_{41} \right) + B_{17,0} = u_{17} \]

Consumers' demand equations for other fat and oil food products (i.e., all fat and oil food products other than margarine, shortening, butter, and lard)

\[ (3.18.a) \quad B_{18,18} Y_{18} + \sum_{i=21}^{40} \left( B_{18,i} Y_{i} + B_{18,41} Y_{41} \right) + B_{18,0} = u_{18} \]

Consumers' demand equations for other food products (i.e., all food products other than dairy, meat, fish, poultry, and fat and oil food products)

\[ (3.19.a) \quad B_{19,19} Y_{19} + \sum_{i=21}^{40} \left( B_{19,i} Y_{i} + B_{19,41} Y_{41} \right) + B_{19,0} = u_{19} \]

Consumers' demand equations for non-food products

\[ (3.20.a) \quad B_{20,20} Y_{20} + \sum_{i=21}^{40} \left( B_{20,i} Y_{i} + B_{20,41} Y_{41} \right) + B_{20,0} = u_{20} \]

The consumption function. The demand equations specified above, in general, indicate how the consumers allocate some of their resources among various products. Following
the practice commonly employed in models of Keynesian economics, the aggregate behavior of households for all consumer goods is described by use of the consumption function. Klein [63, p. 58] states,

From the accepted theories of consumer behavior, it is learned that if a household maximizes its satisfaction (or preferences) subject to the constraint that its budget does not exceed its income, then the demand for each type of good consumed by a particular household will depend upon the household income and the prices of all goods in the household budget. By appropriate aggregation methods, one can develop the analogue of these demand schedules which says that the demand of each household for real consumer goods depends on the general price level of consumer goods, the interest rate (which relates the price of future consumer goods to the prices of current consumer goods), and the households' money income. Matters can further be simplified by assuming that households would not alter their expenditures on consumer goods if all prices and incomes were to change by the same proportion. Then the relevant variable affecting consumption is real income, i.e., income correlated for price changes, rather than money income and the price level separately.

Following the above arguments, the consumption function can be approximated as a linear function between consumers' expenditures and real income. The results of recent investigations, however, suggest that the simple linear relationship is not appropriate for the post World War II period. For example, Klein and Goldberger [66] argue that the characteristics of the size distribution of incomes would be desirable variables to include in the consumption function. Klein [63] claims that the influence of liquid assets might also be introduced as the consumers may very
well spend more than current incomes by drawing upon their accumulated liquid assets.

It would be desirable to benefit from the results of recent studies and to employ specifications similar to Klein and Goldberger [66, pp. 4-10] for the consumption function. The econometric model employed in the present investigation, however, was designed primarily to enable the estimation of demand equation for certain food products. For purposes of the present investigation, therefore, the consumption function was approximated by the following linear relationship.

\[
(3.21) \quad B_{21,42}y_{42} + B_{21,41}(y_{41} \cdot z_{180}) \\
+ A_{21,41}(y_{41,t-1} \cdot z_{180,t-1}) \\
+ A_{21,1}(z_{1} \cdot z_{180}) + B_{21,0} = u_{21}
\]

where \(y_{42}\) represents aggregate consumer expenditures on all goods and services and is assumed to be derived by aggregating over-all households, \(y_{41} \cdot z_{180}\) represents deflated aggregate disposable personal income, and \(z_{1} \cdot z_{180}\) represent aggregate quantity of liquid assets held by consumers at the end of the previous period.

**Structural equations relating to the other sectors**

In order to provide an approximate description of the
farm production, marketing and retailing activities for each of the classified product groupings in the economy, structural equations were specified to represent the aggregate behavior of firms operating in the retailer sector, food product marketing industries, non-food commercial sector and the farm sector. In general the economic theory of the firm and some of the considerations outlined by Hildreth and Jarrett [55], Klein [62, pp. 14-40], Nicholls [96], Samuelson [104, pp. 57-89] and Weintraub [171] served as a basis for the specification of these structural equations. Since general equilibrium was assumed in the model, the equilibrium method discussed by Baumol [8, pp. 126-130] was adopted. In general it was assumed that prices are determined within a given time period by the interaction of supply and demand. That is, at the beginning of each period, the decision makers of the firms are assumed to decide on their production output on the basis of expected demand. In the period, planned production takes place and prices are assumed to be reached which equate supply and demand in the market. Although the supply curve for products supplied from current production in any given period is totally inelastic, the total supply curve is not perfectly inelastic as it is possible to vary the quantity held in inventories. The main consideration underlying the choice of equations and classification of variables relating to firms and market
behavior was to keep the model relatively simple so as to enable the estimation of parameters in some of the demand equations by use of a method of simultaneous equations estimation.

As mentioned earlier, the total quantity of each product, other than fluid milk and cream, was assumed to be sold directly to consumers by the retailer sector. A retailer's supply equation has been specified for each of the food product groupings and the non-food product grouping to represent behavior in the retailer sector of the economy. It was assumed that the quantity of products demanded by retailers from the respective food product marketing industries and the non-food commercial sector in a given time period is equal to the quantity of products sold to consumers. That is, it was assumed that retailers' inventories are negligible and that the retailers adjust to changes in demand by varying their purchases. In general, the retailers' supply equation for a product is assumed to depend on the retail and wholesale price of the production, retail prices of other products and prices paid for factors of production used in retailing the product. Due to the retailers' subjective anticipations about demand, lagged retail prices were introduced.

In selecting and specifying equations for each of the food product marketing industries, an attempt was made to
account for some of the major characteristics of the products and the general nature of processing and distributing activities. In general four types of equations were specified which are as follows: the specific food product marketing industry's supply equation for products supplied from current production, the identity equating total supply with total demand at the wholesale level of the marketing system, the marketing industry's inventory demand equation and the marketing industry's demand equation for farm products or raw materials. Due to the high proportion of fluid milk and cream sold directly to consumers by fluid milk and cream processing-distributing firms and the relative unimportance of inventories, different types of equations were postulated for the fluid milk and cream marketing industry.

The following considerations served as a basis for the selection of variables included in the four types of equations specified for the food product marketing industries. The supply from current production equation was assumed to be derived by aggregating over food product processing firms operating under equilibrium conditions and pure competition. Weintraub [171, pp. 113-114] states,

The MC (marginal cost) curve is thus literally the firm's supply curve under competitive conditions. This condition will be true of each firm. By aggregating at each price the quantity offerings of the individual firms we can construct the market-supply curve: this is compounded out of the lateral output distance on each firm's MC curve at each particular price. It should be
recognized, however, that this unique association of the market supply curve and the aggregate of the individual-firm MC curves is valid only on the maximum-profit hypothesis and in stationary conditions, . . . .

Following those considerations and assumptions made earlier about the structure of the economy, the marketing industry's supply equation for products from current production was assumed to depend on the wholesale price of the food product, wholesale prices for other products, prices for the farm product and other factors of production, and the level of technology. Lagged values of wholesale prices were introduced to represent the entrepreneurs' demand expectations at the wholesale level.

The above assumptions and considerations concerning the supply curve are quite unrealistic and at best might be looked upon as providing a first approximation. Few, if any, of the marketing firms actually operate under conditions approaching pure competition and no supply curve exists for an imperfectly competitive firm. Nevertheless, some function relating output to price and to other variables does exist and for purposes of this study it will be referred to as the supply curve.

Identities were introduced to show that market equilibrium exists at the wholesale level of the marketing system for each commodity group. That is, we are assuming that the wholesale price for a good adjusts itself so that
the market is cleared during the period of time. The identities $Q^d_t = Q^s_t$ imply that the market is cleared during each time period following the equilibrium method discussed by Baumol [8, pp. 126-130] and were introduced to simplify the model. The total demand $Q^d_t$ is considered to consist of the retailers' demand, the specific marketing industry's inventory demand, other marketing industry's demand and the export demand for the commodity in question. On the other hand the total supply is assumed to consist of the quantities supplied from current production, input supplies and from stocks held in inventory at the beginning of the period by the commodity marketing industry.

More than likely the quantity of a product supplied from current production and from imports will not equal the total demand during a given time period. Therefore, inventories are assumed to be held partly to perform the function of maintaining equilibrium and partly to provide for the smooth operation of firms comprising the specific marketing industry. Some inventories are held by firms so that they have enough goods on hand to cover their current sales; other inventories are purposely held by firms for price speculation. Those inventories which entrepreneurs desire to hold for rational reasons are merely factors of production, i.e., working capital demanded by the firm for the smooth operation of the business. The quantities of a
product held in inventories for this purpose can be formulated on the basis of expected total demand for the product and the theory of profit maximization. However, the decision-makers' expectations about total demand may deviate from actual demand and then more or less will be held in inventory than anticipated. For example, when the actual price turns out to be greater than expected (i.e., excess demand exists), it is assumed that firms in a specific marketing industry will sell out of inventories and when prices are less than expected (excess supply exists), some of the period's output is assumed to be added to inventories. Following Klein's discussion [62, pp. 53-55], inventories may be divided into two groups, namely (1) desired inventories needed for the regular operations of firms and (2) undesired inventories held as a result of erroneous expectations. If, in line with the assumption of general equilibrium, it is assumed that the wholesale market for each product is always cleared except for random disturbances, the random disturbance term in the inventory demand equation will reflect the undesired inventories (i.e., indicating excess demand when negative and excess supply when positive). The desired inventories are reflected by the other variables postulated in the inventory demand equation which are as follow: the quantity of the product supplied from current production, current and lagged values of wholesale price, beginning of period inventories and the
cost of holding the product in inventory.

Each of the specific food product marketing industry's demand equations for farm products or raw materials is based on the theory of profit maximization. These demand equations are assumed to depend on: the wholesale price of the product, price for the farm products or raw material, prices of other factors of production and the level of technology.

The farm sector is assumed to supply farm products to the specific food product marketing industries and to the commercial sector. The quantities of farm products supplied during any given time period are considered to result from the decisions of farm firms following the concept of profit maximization. The supply equations for each of the farm products is assumed to depend on: the price of the farm product, prices of other farm products, number of animals on farms at the beginning of the period, quantity of feed fed to animals during the period, prices of other factors of production and the level of technology.

The income equation is specified to complete the model. The equations relating to the other sectors described above are given in functional form as follow:

Retailer's supply equation for butter

\[ y_1 = f(y_1, y_4, y_5; y_i, t-1, z_2, u_22); \ i=21, \ldots, 40 \]

Butter marketing industry's supply equation for butter from current production
Identity equating total supply with total demand for butter at wholesale

\[(3.24) \quad y_{65} + y_{95,t-1} + z_{95} = y_1 \cdot z_{180} + y_{95} + y_{180} + z_{115}\]

Butter marketing industry's inventory demand equation

\[(3.25) \quad y_{95} = f(y_{45}, y_{65} ; y_{45,t-1}, y_{95,t-1}, z_{25}, u_{25})\]

Butter marketing industry's demand equation for milk and cream

\[(3.26) \quad y_{115} = f(y_{45}, y_{65} ; y_{45,t-1}, z_k, u_{2e}) ; \quad k=22,23,24\]

Retailer's supply equation for cheese

\[(3.27) \quad y_2 = f(y_1, y_{46} ; y_{1,t-1}, z_3, u_{27}) ; \quad i=21,\ldots,40\]

Cheese marketing industry's supply equation for cheese from current production

\[(3.28) \quad y_{66} = f(y_1, y_{85} ; y_{1,t-1}, z_k, u_{28}) ; \quad i=45,\ldots,49, \quad k=26,27,28\]

Identity equating total supply with total demand for cheese at wholesale

\[(3.29) \quad y_{66} + y_{97,t-1} + z_{99} = y_2 \cdot z_{180} + y_{97} + z_{116}\]

Cheese marketing industry's inventory demand equation

\[(3.30) \quad y_{97} = f(y_{46}, y_{66} ; y_{46,t-1}, y_{97,t-1}, z_{29}, u_{30})\]
Cheese marketing industry's demand equation for milk and cream

\[(3.31) \quad y_{116} = f(y_{46}, y_{85}; y_{46,t-1}, z_k, u_{31}); \quad k=26,27,28\]

Retailer's supply equation for evaporated and condensed milk

\[(3.32) \quad y_3 = f(y_i, y_{47}; y_{i,t-1}, z_4, u_{32}); \quad i=21,...,40\]

Evaporated and condensed milk marketing industry's supply equation for evaporated and condensed milk from current production

\[(3.33) \quad y_{67} = f(y_1, y_{65}; y_{1,t-1}, z_k, u_{33}); \quad i=45,...,49 \quad k=30,31,32\]

Identity equation total supply with total demand for evaporated and condensed milk at wholesale

\[(3.34) \quad y_{67} + y_{68,t-1} + z_{100} = y_3 \cdot z_{180} + y_{98} + z_{117}\]

Evaporated and condensed milk marketing industry's inventory demand equation

\[(3.35) \quad y_{98} = f(y_{47}, y_{87}; y_{47,t-1}, y_{88,t-1}, z_{33}, u_{33})\]

Evaporated and condensed milk marketing industry's demand equation for milk and cream

\[(3.36) \quad y_{117} = f(y_{47}, y_{85}; y_{47,t-1}, z_k, u_{36}); \quad k=30,31,32\]

Retailer's supply equation for fluid milk and cream

\[(3.37) \quad y_{43} = f(y_1, y_{48}; y_{i,t-1}, z_5, u_{37}); \quad i=21,...,40\]
Fluid milk and cream marketing industry's supply equation for fluid milk and cream from current production

\[(3.38) \ y_{68} = f(y_{24}, y_1, y_{85}; y_{1,t-1}, y_{48,t-1}, z_k, u_{48}); \]
\[i=45,...,49 \]
\[k=34,35,36 \]

Identity indicating the fluid milk and cream marketing industry's retail supply of fluid milk and cream

\[(3.39) \ y_{44} = y_{68} - y_{43} \]

Retail price equation for fluid milk and cream

\[(3.40) \ y_{24} = f(y_4 \cdot z_{180} - y_{68}; u_{40}) \]

Wholesale price equation for fluid milk and cream

\[(3.41) \ y_{48} = f(y_{24}; u_{41}) \]

Fluid milk and cream marketing industry's demand equation for milk and cream

\[(3.42) \ y_{118} = f(y_{24}, y_{48}, y_{85}; y_{24,t-1}, y_{48,t-1}, z_k, u_{42}); \]
\[k=34,35,36 \]

Retailer's supply equation for other processed dairy products

\[(3.43) \ y_5 = f(y_1, y_{49}; y_{1,t-1}, z_6, u_{43}); \]
\[i=21,...,40 \]

Other dairy product marketing industry's supply equation for dairy products from current production

\[(3.44) \ y_{69}, f(y_1, y_{85}; y_{1,t-1}, z_k, u_{44}); \]
\[i=45,...,49 \]
\[k=37,38,39 \]
Identity equating total supply with total demand for other dairy products at wholesale

\[ (3.45) \quad Y_{69} + Y_{69,t-1} + z_{101} = Y_5 \cdot z_{180} + Y_{69} + z_{118} \]

Other dairy product marketing industry's inventory demand equation

\[ (3.46) \quad Y_{69} = f(Y_{49}, y_{69} ; Y_{49,t-1}, Y_{69,t-1}, z_{40}, u_{46}) \]

Other dairy product marketing industry's demand equation for milk and cream

\[ (3.47) \quad Y_{119} = f(Y_{49}, y_{65} ; Y_{49,t-1}, z_k, u_{47}) ; \quad k=37,38,39 \]

Identity equating total supply with total demand for milk and cream at the farm level

\[ (3.48) \quad Y_{137} = \sum_{i=115}^{119} Y_i \]

Farm sector's supply equation for milk and cream

\[ (3.49) \quad Y_{137} = f(Y_j ; Y_{j,t-1}, z_{134}, z_{142}, z_k, u_{49}) ; \quad j=85,...,92 \quad k=150,151,152 \]

Retailer's supply equation for beef

\[ (3.50) \quad Y_e = f(Y_1, y_{50} ; Y_{1,t-1}, z_7, u_{50}) ; \quad i=21,...,40 \]

Beef marketing industry's supply equation for beef from current production

\[ (3.51) \quad Y_{70} = f(Y_1, y_{86} ; Y_{1,t-1}, z_k, u_{51}) ; \quad i=50,...,57 \quad k=41,42,43 \]
Identity equating total supply with total demand for beef products at wholesale

\[(3.52) \quad y_{70} + y_{100,t-1} + z_{102} = y_e \cdot z_{180} + y_{100} + z_{119} \]

Beef marketing industry's inventory demand equation

\[(3.53) \quad y_{100} = f(y_{50}, y_{70} ; y_{50,t-1}, y_{100,t-1}, z_{44}, u_{53}) \]

Beef marketing industry's demand equation for beef animals

\[(3.54) \quad y_{120} = f(y_{50}, y_{86} ; y_{50,t-1}, z_k, u_{54}) ; \quad k=41,42,43 \]

Farm sector's supply equation for beef animals

\[(3.55) \quad y_{120} = f(y_j ; y_{j,t-1}, z_{135}, z_{143}, z_k, u_{55}) ; \quad j=85, \ldots, 92 \quad k=153,154,155 \]

Retailer's supply equation for veal

\[(3.56) \quad y_7 = f(y_1, y_{51} ; y_{1,t-1}, z_8, u_{56}) ; \quad i=21, \ldots, 40 \]

Veal marketing industry's supply equation for veal from current production

\[(3.57) \quad y_{71} = f(y_1, y_{87} ; y_{1,t-1}, z_k, u_{57}) ; \quad i=50, \ldots, 57 \quad k=45,46,47 \]

Identity equating total supply with total demand for veal at wholesale

\[(3.58) \quad y_{71} + y_{101,t-1} + z_{103} = y_7 \cdot z_{180} + y_{101} + z_{120} \]

Veal marketing industry's inventory demand equation

\[(3.59) \quad y_{101} = f(y_{51}, y_{71} ; y_{51,t-1}, y_{101,t-1}, z_{88}, u_{58}) \]
Veal marketing industry's demand equation for veal animals

\( (3.60) \quad y_{121} = f(y_{51}, y_{87}; y_{51,t-1}, z_k, u_{60}); \quad k=45,46,47 \)

Farm sector's supply equation for veal animals

\( (3.61) \quad y_{121} = f(y_j; y_j,t-1, z_{13e}, z_{14e}, z_k, u_{81}); \quad j=85,\ldots,92 \quad k=156,157,158 \)

Retailer's supply equation for lamb and mutton

\( (3.62) \quad y_8 = f(y_i, y_{52}; y_{i,t-1}, z_e, u_{63}); \quad i=21,\ldots,40 \)

Lamb and mutton marketing industry's supply equation for lamb and mutton from current production

\( (3.63) \quad y_{72} = f(y_i, y_{8e}; y_{i,t-1}, z_k, u_{83}); \quad i=50,\ldots,57 \quad k=49,50,51 \)

Identity equating total supply with total demand for lamb and mutton at wholesale

\( (3.64) \quad y_{72} + y_{102,t-1} + z_{104} = y_8 \cdot z_{13e} + y_{102} + z_{121} \)

Lamb and mutton marketing industry's inventory demand equation

\( (3.65) \quad y_{102} = f(y_{52}, y_{72}; y_{52,t-1}, y_{102,t-1}, z_{52}, u_{85}) \)

Lamb and mutton marketing industry's demand equation for lambs and sheep

\( (3.66) \quad y_{122} = f(y_{52}, y_{8e}; y_{52,t-1}, z_k, u_{86}); \quad k=49,50,51 \)

Farm sector's supply equation for lambs and sheep
(3.67) \( y_{122} = f(y_j; y_{j,t-1}, z_{127}, z_{145}, z_k, u_{67}) \);
\[ j=85,\ldots,92 \]
\[ k=159,160,161 \]

Retailer's supply equation for pork

(3.68) \( y_9 = f(y_1, y_{53}; y_{1,t-1}, z_{10}, u_{68}) \); \( i=21,\ldots,40 \)

Pork and lard marketing industry's supply equation for pork from current production

(3.69) \( y_{73} = f(y_1, y_{74}, y_{88}; y_{1,t-1}, z_k, u_{68}) \);
\[ i=50,\ldots,57 \]
\[ k=53,54,55 \]

Identity equating total supply with total demand for pork at wholesale

(3.70) \( y_{73} + y_{103,t-1} + z_{105} = y_9 \cdot z_{180} + y_{103} + z_{122} \)

Pork and lard marketing industry's inventory demand equation for pork

(3.71) \( y_{103} = f(y_{53}, y_{73}; y_{53,t-1}, y_{103,t-1}, z_{58}, u_{71}) \)

Retailer's supply equation for lard

(3.72) \( y_{10} = f(y_1, y_{54}; y_{1,t-1}, z_{11}, u_{72}) \); \( i=21,\ldots,40 \)

Pork and lard marketing industry's supply equation for lard from current production

(3.73) \( y_{74} = f(y_1, y_{73}, y_{88}; y_{1,t-1}, z_k, u_{73}) \);
\[ i=50,\ldots,57 \]
\[ k=53,54,55 \]

Identity equating total supply with total demand for lard at wholesale
(3.74) \[ y_{74} + y_{104, t-1} = y_{10} \cdot Z_{180} + y_{104} + Z_{124} + y_{131} + y_{133} \]

Pork and lard marketing industry's inventory demand equation for lard

(3.75) \[ y_{104} = f(y_{54}, y_{74}; y_{54, t-1}, y_{104, t-1}, Z_{57}, u_{75}) \]

Pork and lard marketing industry's demand equation for hogs

(3.76) \[ y_{123} = f(y_{53}, y_{54}, y_{89}; y_{53, t-1}, y_{54, t-1}, z_k, u_{76}); k=53,54,55 \]

Farm sector's supply equation for hogs

(3.77) \[ y_{123} = f(y_j; y_{j, t-1}, z_{138}, z_{146}, z_k, u_{77}); \]
\[ j=85,...,92 \]
\[ k=162,163,164 \]

Retailer's supply equation for chickens

(3.78) \[ y_{11} = f(y_1, y_{55}; y_{1, t-1}, z_{12}, u_{78}); i=21,...,40 \]

Chicken marketing industry's supply equation for chicken supplied from current production

(3.79) \[ y_{75} = f(y_1, y_{80}; y_{1, t-1}, z_k, u_{79}); i=50,...,57 \]
\[ k=58,59,60 \]

Identity equating total supply with total demand for chicken at wholesale

(3.80) \[ y_{75} + y_{105, t-1} + Z_{108} = y_{11} \cdot Z_{180} + y_{105} + Z_{124} \]

Chicken marketing industry's inventory demand equation

(3.81) \[ y_{105} = f(y_{55}, y_{75}; y_{55, t-1}, y_{105, t-1}, z_{61}, u_{81}) \]
Chicken marketing industry's demand equation for chickens
(3.82) \( y_{124} = f(y_{55}, y_{90}; y_{55}, t-1, z_k, u_{82}) ; k=58,59,60 \)

Farm sector's supply equation for chickens
(3.83) \( y_{124} = f(y_j, y_j, t-1, z_{139}, z_{147}, z_k, u_{83}) ; \)
\[ j=85, \ldots, 92 \]
\[ k=165,166,167 \]

Retailer's supply equation for eggs
(3.84) \( y_{12} = f(y_1, y_{56}; y_1, t-1, z_{138}, u_{84}) ; i=21, \ldots, 40 \)

Egg marketing industry's supply equation for eggs supplied from current production
(3.85) \( y_{76} = f(y_1, y_{91}; y_1, t-1, z_k, u_{85}) ; i=50, \ldots, 57 \)
\[ k=62,63,64 \]

Identity equating total supply with total demand for eggs at wholesale
(3.86) \( y_{76} + y_{106}, t-1 + z_{107} = y_{12} \cdot z_{136} + z_{106} + z_{125} \)

Egg marketing industry's inventory demand equation
(3.87) \( y_{106} = f(y_{56}, y_{76}; y_{56}, t-1, y_{106}, t-1, z_{95}, u_{87}) \)

Egg marketing industry's demand equation for eggs
(3.88) \( y_{125} = f(y_{56}, y_{91}; y_{56}, t-1, z_k, u_{88}) ; k=62,63,64 \)

Farm sector's supply equation for eggs
(3.89) \( y_{125} = f(y_j, y_j, t-1, z_{140}, z_{148}, z_k, u_{89}) ; \)
\[ k=168,169,170 \]
Retailer's supply equation for other poultry products

\[(3.90) \ y_{13} = f(y_1, y_{57} ; y_{1,t-1}, z_{14}, u_{90}) ; i=21,...,40 \]

Other poultry product marketing industry's supply equation for other poultry products supplied from current production

\[(3.91) \ y_{77} = f(y_1, y_{92} ; y_{1,t-1}, z_{k}, u_{91}) ; i=50,...,57 \ k=66,67,68 \]

Identity equating total supply with total demand for other poultry products at wholesale

\[(3.92) \ y_{77} + y_{107,t-1} + z_{108} = Y_{13} \cdot z_{180} + Y_{107} + z_{12e} \]

Other poultry product marketing industry's inventory demand equation

\[(3.93) \ y_{107} = f(y_{57}, y_{77} ; y_{57,t-1}, y_{107,t-1}, z_{69}, u_{93}) \]

Other poultry product marketing industry's demand equation for farm products

\[(3.94) \ y_{126} = f(y_{57}, y_{92}, y_{57,t-1}, z_{k}, u_{94}) ; k=66,67,68 \]

Farm sector's supply equation for other poultry products

\[(3.95) \ y_{126} = f(y_j ; y_{j,t-1}, z_{141}, z_{149}, z_{k}, u_{95}) ; \ j=85,...,92 \ k=171,172,173 \]

Retailer's supply equation for canned fish

\[(3.96) \ y_{14} = f(y_1, y_{58} ; y_{1,t-1}, z_{15}, u_{98}) ; i=21,...,40 \]

Canned fish marketing industry's supply equation for canned fish supplied from current production
(3.97) \( y_{78} = f(y_{58}, y_{59}, y_{95}, y_{58,t-1}, y_{59,t-1}, z_k, u_{97}) ; k=70,71,72 \)

Identity equating total supply with total demand for canned fish at wholesale

(3.98) \( y_{78} + y_{109,t-1} + z_{109} = y_{14} \cdot z_{180} + y_{109} + z_{127} \)

Canned fish marketing industry's inventory demand equation

(3.99) \( y_{108} = f(y_{58}, y_{78}, y_{58,t-1}, y_{108,t-1}, z_{78}, u_{99}) \)

Canned fish marketing industry's demand equation for fish at docks and piers

(3.100) \( y_{127} = f(y_{58}, y_{95}, y_{58,t-1}, z_k, u_{100}) ; k=70,71,72 \)

Retailer's supply equation for other fish products

(3.101) \( y_{15} = f(y_1, y_{58}, y_{1,t-1}, z_{18}, u_{101}) ; i=21,\ldots,40 \)

Other fish product marketing industry's supply equation for other fish products supplied from current production

(3.102) \( y_{78} = f(y_{58}, y_{58}, y_{95}, y_{58,t-1}, y_{59,t-1}, z_k, u_{102}) ; k=74,75,76 \)

Identity equating total supply with total demand for other fish products at wholesale

(3.103) \( y_{79} + y_{109,t-1} + z_{110} = y_{15} \cdot z_{180} + y_{109} + z_{128} \)

Other fish product marketing industry's inventory demand equation
Other fish product marketing industry's demand equation for fish at docks and piers

\[ y_{128} = f(y_{59}, y_{95} ; y_{59,t-1}, z_{k}, u_{109}) ; \quad k=74,75,76 \]

Identity indicating the total demand for fish at docks and piers

\[ Y_{1_{26}} = Y_{1_{27}} + Y_{1_{28}} \]

Supply equation for fish at docks and piers

\[ Y_{1_{38}} = f(y_{95} ; z_{174}, z_{175}, u_{109}) \]

Retailer's supply equation for margarine

\[ Y_{1_{6}} = f(y_{1}, y_{60} ; y_{1,t-1}, z_{17}, u_{108}) ; \quad i=20,\ldots,40 \]

Margarine marketing industry's supply equation for margarine supplied from current production

\[ Y_{80} = f(y_{1}, y_{45}, y_{54}, y_{63} ; y_{1,t-1}, z_{k}, u_{109}) ; \quad i=60,61,62 \quad k=78,79,80 \]

Identity equating total supply with total demand for margarine at wholesale

\[ Y_{80} + Y_{110,t-1} = Y_{1_{6}} \cdot z_{180} + Y_{110} + z_{128} \]

Margarine marketing industry's inventory demand equation
Margarine marketing industry's demand equation for fats and oils from the fat and oil mill processing industry

\[
(3.112) \quad y_{129} = f(y_{45}, y_{54}, y_{60}, y_{63}; y_{60,t-1}, z_k, u_{112}); \quad k=78,79,80
\]

Margarine marketing industry's demand equation for butter

\[
(3.113) \quad y_{130} = f(y_{45}, y_{54}, y_{60}, y_{63}; y_{60,t-1}, z_k, u_{113}); \quad k=78,79,80
\]

Margarine marketing industry's demand equation for lard

\[
(3.114) \quad y_{131} = f(y_{45}, y_{54}, y_{60}, y_{63}; y_{60,t-1}, z_k, u_{114}); \quad k=78,79,80
\]

Retailer's supply equation for shortening

\[
(3.115) \quad y_{17} = f(y_i, y_{61}; y_{1,t-1}, z_{18}, u_{115}); \quad i=21,\ldots,40
\]

Shortening marketing industry's supply equation for shortening supplied from current production

\[
(3.116) \quad y_{61} = f(y_1, y_{54}, y_{63}; y_{1,t-1}, z_k, u_{116}); \quad i=60,61,62
\]

Identity equating total supply with total demand for shortening at wholesale

\[
(3.117) \quad y_{61} + y_{111,t-1} + z_{111} = y_{17} \cdot z_{180} + y_{111} + z_{130} + y_{133}
\]
Shortening marketing industry's inventory demand equation

\[ y_{111} = f(y_{s1}, y_{s1}, y_{i1}, y_{i1}, t-1, z_{85}, u_{118}) \]

Shortening marketing industry's demand equation for fats and oils from the fat and oil mill processing industry

\[ y_{132} = f(y_{54}, y_{61}, y_{63}, y_{61}, t-1, z_k, u_{119}) ; \quad k=82,83,84 \]

Shortening marketing industry's demand equation for lard

\[ y_{133} = f(y_{54}, y_{61}, y_{63}, y_{61}, t-1, z_k, u_{120}) ; \quad k=82,83,84 \]

Retailer's supply equation for other fat and oil food products

\[ y_{18} = f(u_1, y_{62}, y_{i1}, t-1, z_{19}, u_{121}) ; \quad i=21,\ldots,40 \]

Other fat and oil food product marketing industry's supply equation for other fat and oil food products supplied from current production

\[ y_{62} = f(y_1, y_{63}, y_{1}, t-1, z_k, u_{122}) ; \quad i=60,61,62 \quad k=86,87,88 \]

Identity equating total supply with total demand for other fat and oil food products at wholesale

\[ y_{62} + y_{112,t-1} + z_{112} = y_{18} \cdot z_{180} + y_{112} + z_{131} \]
Other fat and oil food product marketing industry's inventory demand equation

\[(3.124) \quad y_{112} = f(y_{62}, y_{82}, y_{62,t-1}, y_{112,t-1}, z_{83}, u_{124}) \]

Other fat and oil food product marketing industry's demand equation for fats and oils from the fat and oil mill processing industry

\[(3.125) \quad y_{134} = f(y_{62}, y_{83}, y_{62,t-1}, z_{k}, u_{125}); \quad k=86,87,88 \]

Fat and oil mill processing industry's supply equation for animal and vegetable fats and oils supplied from current production

\[(3.126) \quad y_{83} = f(y_{83}, y_{84}; y_{63,t-1}, z_{k}, u_{126}); \quad k=90,91,92 \]

Identity equating total supply with total demand for animal and vegetable fats and oils

\[(3.127) \quad y_{83} + y_{113,t-1} + z_{113} = y_{123} + y_{132} + y_{134} + y_{113} + z_{132} + z_{177} \]

Fat and oil mill processing industry's inventory demand equation

\[(3.128) \quad y_{113} = f(y_{83}, y_{88}; y_{63,t-1}, y_{113,t-1}, z_{83}, u_{128}) \]

Fat and oil mill processing industry's demand equation for farm products
(3.129) \( y_{135} = f(y_{83}, y_{94}, y_{83,t-1}, z_k, u_{129}) \); 
\[ k=90,91,92 \]

Farm price equation for beans, seeds, and other farm products purchased by the fat and oil mill processing industry

(3.130) \( y_{94} = f(y_{135} - z_{176}; u_{130}) \)

Retailer's supply equation for other food products

(3.131) \( y_{19} = f(y_1, y_{64}; y_{1,t-1}, z_{20}, u_{131}) \); 
\[ i=21, \ldots, 40 \]

Other food products marketing industry's supply equation for other food products supplied from current production

(3.132) \( y_{84} = f(y_1, y_{83}; y_{1,t-1}, z_k, u_{132}) \); 
\[ i=45, \ldots, 64 \]  
\[ k=94,95,96 \]

Identity equating total supply with total demand for other food products at wholesale

(3.133) \( y_{84} + y_{114,t-1} + z_{114} = y_{19} \cdot z_{180} + y_{114} + z_{133} \)

Other food products marketing industry's inventory demand equation

(3.134) \( y_{114} = f(y_{94}, y_{84}; y_{84,t-1}, y_{114,t-1}, z_{97}, u_{134}) \)

Other food products marketing industry's demand equation for farm products
143

\[(3.135) \quad y_{136} = f(y_{64}, y_{36} ; y_{64,t-1}, z_k, u_{135}) ; \quad k=94,95,96 \]

Farm price equation for farm products sold to the other food products marketing industry

\[(3.136) \quad y_{93} = f(y_{136} - z_{178} ; u_{136}) \]

Retailer’s supply equation for non-food products

\[(3.137) \quad y_{20} = f(y_1 ; y_{1,t-1}, z_{21}, z_{178}, u_{137}) ; \quad i=21,\ldots,40 \]

The income equation. In the model disposable personal income has been classified as an endogenous variable, as it was assumed that income is in part directly effected by total consumers' expenditures within the period of a year. Earlier, income was defined as the sum of current consumers' expenditures and saving. Although the income generating mechanism can be developed in several ways, often in past studies income has been assumed to be generated by outlays on consumption and investment goods and the relation of savings to investment has been treated in different ways [24], [49], [52, pp. 219-225], and [66].

For purposes of this model investment expenditures are assumed to equal savings. That is, from the above definition, income is given by \( y=c+x \) and investment by \( x=y-c \) where \( y \) represents income, \( c \) represents consumer expenditures and \( x \) represents investment expenditures. In the model
total investment is assumed to be represented by the two exogenous variables $z_{181}$ and $z_{182}$, where $z_{181}$ represents the government expenditures on goods and services component of the Gross National Product and $z_{182}$ represents the gross private domestic investment and net foreign investment components of the Gross National Product. The total consumers' expenditures generating mechanism is represented by the consumption function

$$
(3.21) \quad B_{21,42}y_{42} + B_{21,41}y_{41} + A_{21,41}y_{41, t-1} + A_{21,1} (z_1 \cdot z_{180}) + B_{21,0} = u_{21}
$$

where all variables are as defined earlier. The disturbance term $u_{21}$ can be looked upon as the sum of disturbance terms contained in demand equations (3.1.d) through (3.20.d) and represents the random elements in consumers' behavior.

Substituting Equation (3.21) into the definition, the income equation is assumed to be represented by

$$
(3.138) \quad y_{41} = f(y_{41, t-1}, z_1 \cdot z_{180}, z_{181}, z_{182}, u_{138}).
$$

The specifications for Equation (3.138), like those for Equation (3.21), are necessarily quite simple and have been introduced primarily to complete the system. Although these specifications appear to be useful approximations for purposes of the present investigation, for other investigations it would be desirable to benefit from the results of
recent work in this area and to consider some of Haavelmo's [49], Klein and Goldberger's [66] and other arguments.

Empirical Results

In this section the results of the statistical analysis performed on the four forms of demand equations postulated for each of twelve food products will be presented and discussed in regard to their economic interpretation. That is, in addition to presenting the empirical results, the estimates found to be significantly different from zero will be examined for agreement or disagreement with theory in regard to sign and economic interpretation. Comparisons will also be made among the four forms of equations estimated for each product and the results interpreted in regard to the underlying hypotheses.

Since only the demand equations were estimated, all other equations contained in the model were assumed to be as specified in order to satisfy the identification requirements. Due to such statistical problems as multicollinearity, small number of degrees of freedom, the nature of the time series data, etc., more than likely we would not be able to obtain a unique solution or estimates for each of the coefficients if prices for all of the possible substitutes and complements were included in the equation. To reduce the number of coefficients to be estimated and probably
eliminate some of the statistical problems in the empirical analysis, only a few of the price variables \( y_{21}, \ldots, y_{40} \), defined earlier and specified in the model, were included in the estimated demand equations. Some of the price variables were omitted and others replaced by a more aggregative variable, following the arguments of Wold and Jureen [176, pp. 108-110] and Bergstrom [11], due to their high inter-correlation.

The Theil-Basmann method, outlined in Chapter 4 and discussed in [7] and [170], was used as the simultaneous equations technique of estimation under the assumption that the equations are linear in observed variables. The resulting estimates are based upon annual time series data for the sample period 1920-1941, and 1947-1949. The terminal year, 1949, was chosen because of the increasing importance of national agricultural programs after this period and changes in the nature of available data. The values of all variables employed in the empirical analysis and their source and method of construction are given in the Appendix.

Before proceeding to the discussion of the statistical results, it appears desirable to outline the general criteria underlying the comparisons made between the different forms of equations because the statistical comparisons provide a partial basis for making judgments about the alternative hypotheses concerning consumers' demand for a given product.
As mentioned earlier, equations of form a are demand equations postulated in line with the traditional theory of consumer demand, whereas the equations of form c are demand equations postulated in line with a generalized static theory of demand, i.e., one derived from the traditional static theory by following the arguments presented in the latter part of section entitled Static Theories of Demand, but where only the influence of liquid asset holdings is taken into account.

Equations of form b and form d, on the other hand, are reduced equations postulated in line with the specifications underlying the long-run elasticity hypothesis and Koyck's method of reduction, or demand equations postulated in line with the specifications underlying the hypothesis of self-generating changes in preferences.

Since the same economic theory of demand serves as a basis for deriving equations of forms b and c, the direct comparison of each of these equations with an equation of form a is appropriate. The comparison of a and c provides a partial basis for accepting or rejecting the hypothesis concerning liquid asset holdings. That is, this comparison yields part of the empirical basis for determining if the traditional static theory or the generalized static theory provides the better explanation of the factors important in determining consumers' consumption for a particular good. If the addition of the liquid asset holdings variable $z_1$
causes a significant increase in the value of R² and if the coefficient of z₁ is significantly different from zero, we would accept the hypothesis that liquid asset holdings are important in determining consumers' consumption for a particular good and would select form c over form a as providing the better explanation. On the other hand, if the addition of z₁ does not cause a significant increase in the value of R² and its coefficient is not significantly different from zero, we would reject the liquid asset holdings hypothesis and would not choose form c over form a. The interpretation of the statistical results for the other possible situations will not be as clear as those mentioned above due to a lack of compatible evidence. The conclusions drawn in these situations will necessarily be based upon judgments made from the available statistical evidence and possibly will be subject to greater error. In general, we would tend to accept the hypothesis of liquid asset holdings if the coefficient of z₁ is significant even though the addition of this variable to equations of form a does not cause a significant improvement in the goodness of fit. We would also tend to reject this hypothesis if the addition of z₁ causes a significant increase in the value of R² but the coefficient of z₁ is not significant. It appears that the incompatible evidence arises in large part from violations made in the assumptions underlying the statistical analysis, i.e., the
assumptions of non-serial correlation in the residuals, independence of the exogenous variables with the residuals, etc.

The comparisons of equations of forms a and b provide a partial basis for accepting or rejecting either the long-run elasticity hypothesis or the hypothesis of variable preferences. They also provide the empirical basis for determining if the dynamic theories yield a better explanation of consumers' consumption for a particular good than does the traditional static theory. If the addition of the lagged consumption variable \(y_{1,t-1}\) to an equation of form a causes a significant increase in the value of \(R^2\) and has a significant coefficient, we would accept either of the dynamic hypotheses and would choose form b over form a as providing the better explanation. We would reject the dynamic hypotheses and would not select equation b over a, however, if the addition of \(y_{1,t-1}\) does not cause a significant improvement in the goodness of fit and the coefficient of \(y_{1,t-1}\) is not significant. We would also reject the dynamic hypotheses if the coefficient is not significantly different from zero even though the addition of \(y_{1,t-1}\) to an equation of form a causes a significant increase in the value of \(R^2\). If, on the other hand, the addition of \(y_{1,t-1}\) does not cause a significant improvement in the goodness of fit but the coefficient of \(y_{1,t-1}\) in form b is significant, we would
tend to accept either of the dynamic hypotheses.

Even though the statistical conditions necessary for acceptance of the dynamic hypotheses are satisfied, there are few cases where it will be possible to determine whether the long-run elasticity hypothesis or the variable preference hypothesis is the appropriate dynamic factor underlying equations of form b. Following the arguments presented in section entitled Variable Preferences and Lagged Consumption, the only time that we can distinguish between the two dynamic hypotheses is when the coefficient of the explanatory variables other than $y_{1,t-1}$ are not significantly different from zero and when the coefficient of $y_{1,t-1}$ is significant. In this particular case we would accept the variable preference hypothesis. Definite conclusions about the appropriate dynamic hypothesis are further complicated by the fact that other hypotheses can also be tested by use of an equation of forms b or d. For example, Nerlove [94, pp. 109-116] shows that Friedman's permanent income hypothesis can also be tested by the use of such equations.

It is also possible to make a direct comparison between equations of forms c and d as this comparison can be rationalized on the basis of economic considerations. That is, the basic theory underlying the specifications for equations of forms c and d is the generalized static theory in contrast to the traditional static theory which underlies
equations of forms a and b. By making direct comparisons between equations of forms c and d, we are testing the same dynamic hypotheses that we do when we compare a and b. The main difference rests in the initial theory we consider as determining consumers' consumption behavior. The interpretation of the resulting estimates and statistical results for the comparison of c and d in regard to the dynamic hypotheses is similar to that mentioned above for equations of forms a and b.

No well defined economic rational is available for use as a basis in making comparisons between equations of forms b and d. Nevertheless, in cases where the goodness of fit does not differ very much between equations of forms b and c, we might make a choice between b and d by use of judgments about the other statistical results. That is, by use of the results for the direct comparisons made between equations of forms a and c and forms c and d, it is possible to make an indirect comparison of equations of forms b and d. For example, if we accept the generalized static theory as providing a better explanation of consumers' behavior than the traditional static theory and if the dynamic hypotheses underlying form d are accepted, we would generally choose form d over form b even if the hypotheses underlying form b are acceptable. On the other hand, if form d were preferable to form c but the generalized static
theory underlying form c were rejected, we would choose form b over form d whenever either of the dynamic hypotheses underlying form b are accepted. This line of reasoning can also be applied to the other possible results.

In presenting the estimates and other statistical results, the standard errors of estimates are given directly below the coefficients in parentheses. When the coefficients were significantly different from zero, the level of significance is indicated by superscripts as follows: * significant at 10 per cent level of probability, ** significant at 5 per cent level, and *** significant at 1 per cent level. A double asterisk following the value of d indicates that based on the Durbin-Watson test, for a discussion of this test see [43, p. 77] and [55, pp. 77-78], the hypothesis of serially independent residuals was accepted at the 5 per cent level, a + superscript indicates that the test of d was inconclusive, and no superscript indicates that the hypothesis was rejected. In testing the significance of the difference in $R^2$, a double asterisk following F indicates significance at the 1 per cent level and a single asterisk significance at the 5 per cent level. In using Hotelling's t test, the level of significance is indicated by the same superscripts employed in testing the significance of the coefficients.

In predicting or estimating a particular variate there is frequently a large number of other variates which can
serve as independent variables or predictors. Hotelling's t test, see [57] and Appendix B, provides a basis for making a choice among the available predictors, after some have already been selected, which has the highest partial correlation with the predictand (dependent variable). Hotelling's t test was used to determine whether the addition of $y_{t-1}$ or $z_1$ to equations of form a and associated hypotheses provided the better explanation for the consumption of the commodity in question. The values predicted for the normalized dependent variables for the post-estimation period and the estimated residual are presented to indicate the biased or unbiased nature of the equations estimated for particular products.

**Demand for butter**

Estimates for the parameters in the four forms of butter demand equations are presented in Table 1. The endogenous variable $y_1$, per capita civilian consumption of butter adjusted for government purchase and distribution programs for 1934-1941, was chosen as the normalized dependent variable. The explanatory variables used are: $y_{21}$, deflated retail price of butter; $y_{36}$, deflated retail price of margarine; $y_{41}'$, deflated per capita disposable income in states prohibiting the sale of colored margarine; $y_{41}''$, deflated per capita disposable income in states permitting
Table 1. Estimation results for the butter demand equations (Equation 3.1)

<table>
<thead>
<tr>
<th></th>
<th>$B_{1,0}$</th>
<th>$Y_{21}$</th>
<th>$Y_{31}$</th>
<th>$Y_{41}$</th>
<th>$Y''_{11}$</th>
<th>$Y_{1,t-1}$</th>
<th>$z_{1}$</th>
<th>$R^2$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>24.1449</td>
<td>-0.0356</td>
<td>0.0165</td>
<td>-0.0037</td>
<td>-0.0325***</td>
<td>0.0165</td>
<td>-0.0037</td>
<td>-0.0037</td>
<td>0.0165</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0536)</td>
<td>(0.0352)</td>
<td>(0.0074)</td>
<td>(0.0070)</td>
<td>(0.0074)</td>
<td>(0.0070)</td>
<td>(0.0070)</td>
<td>(0.0070)</td>
</tr>
<tr>
<td>b</td>
<td>5.0507</td>
<td>0.0317</td>
<td>-0.0179</td>
<td>-0.0067*</td>
<td>-0.0066</td>
<td>0.0181</td>
<td>-0.0067*</td>
<td>-0.0066</td>
<td>-0.0067*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0282)</td>
<td>(0.0181)</td>
<td>(0.0036)</td>
<td>(0.0052)</td>
<td>(0.1242)</td>
<td>(0.0181)</td>
<td>(0.0052)</td>
<td>(0.1242)</td>
</tr>
<tr>
<td>c</td>
<td>28.6440</td>
<td>0.0087</td>
<td>-0.0229</td>
<td>-0.0068</td>
<td>-0.0202***</td>
<td>-0.0797***</td>
<td>-0.0229</td>
<td>-0.0068</td>
<td>-0.0202***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0338)</td>
<td>(0.0231)</td>
<td>(0.0045)</td>
<td>(0.0051)</td>
<td>(0.0189)</td>
<td>(0.0231)</td>
<td>(0.0051)</td>
<td>(0.0189)</td>
</tr>
<tr>
<td>d</td>
<td>9.8312</td>
<td>0.0361</td>
<td>-0.0260</td>
<td>-0.0072*</td>
<td>-0.0065</td>
<td>0.6753***</td>
<td>-0.0260</td>
<td>-0.0072*</td>
<td>-0.0065</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0280)</td>
<td>(0.0187)</td>
<td>(0.0036)</td>
<td>(0.0052)</td>
<td>(0.1553)</td>
<td>(0.0187)</td>
<td>(0.0052)</td>
<td>(0.1553)</td>
</tr>
</tbody>
</table>

*Significantly different from zero at the 10 per cent level.

***Significantly different from zero at the 1 per cent level.

+Inconclusive test for autocorrelation in the residuals.
the sale of colored margarine; \( y_{1,t-1} \), per capita civilian consumption of butter lagged one time period; and \( z_1^1 = 100 \frac{z_1}{z_{41}} \), the ratio of deflated per capita liquid assets held by consumers at the end of period \( t-1 \) to deflated per capita disposable income for period \( t \). The two variables \( y_{41} \) and \( y_{41}' \) were used in the butter and margarine demand equations rather than \( y_{41} \) in an attempt to account for some of the institutional factors affecting the sales and purchases of these products. The arguments underlying the use of \( y_{41} \) and \( y_{41}' \) are given by Ladd [73]. The variable \( z_1 \) was used in all demand equations rather than \( z_1 \) as specified in the model because of the high intercorrelation between \( z_1 \) and \( y_{41} \). Many of the price variables specified in the demand equations in the model were omitted due to the high intercorrelation among the price variables and due to their high simple product-moment correlation with other explanatory variables.

Table 1 indicates that only the point estimates for the coefficient of \( y_{41}' \) in Equation (3.1.a) and for the coefficients of \( y_{41}' \) and \( z_1 \) in Equation (3.1.c) are significantly different from zero at the 10 per cent or a lower probability level. In Equations (3.1.b) and (3.1.d) on the other hand, the coefficients associated with \( y_{41}' \) and \( y_{1,t-1} \) are significant. The negative sign of the estimated coefficients for \( y_{41}' \) and \( y_{41}'' \) are in conflict with a priori
considerations as they suggest that butter is an inferior good.

The addition of $z_1$ to (3.1.a) causes a significant increase in the value of $R^2$, as does the addition of $y_{1,t-1}$ to (3.1.a) and (3.1.c). These results coupled with the significance of the coefficient of $z_1$ in (3.1.c) and of $y_{1,t-1}$ in (3.1.b) and (3.1.d) provide ample grounds for accepting the hypotheses underlying each of the Equations (3.1.b), (3.1.c), and (3.1.d). Although we can accept the generalized static theory, the results for Hotelling's t test suggest that (3.1.b) provides a better explanation for consumers' consumption of butter than does (3.1.c). Results for the F tests indicate that (3.1.d) is also preferable to Equation (3.1.c). However, the coefficient of $z_1$ in (3.1.d) is not significantly different from zero, and Equation (3.1.d) does not appear to provide a better explanation than does (3.1.b).

The statistical results presented in Tables 1 and 2 do not provide sufficient evidence for making a choice between (3.1.b) and (3.1.d) nor for determining which of the dynamic hypotheses is appropriate in either equation. Part of the problem arises from the nature of the statistical evidence because the coefficient of $y_{41}'$ is significantly different from zero in (3.1.a) and (3.1.c) whereas the coefficient of $y_{41}'$ is significantly different from zero in
Table 2. Results of the F test and Hotelling's t test for significance of difference in $R^2$ for the butter demand equations

<table>
<thead>
<tr>
<th>Equations compared</th>
<th>d.f.</th>
<th>F or t</th>
<th>Added variable</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F tests</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a and b</td>
<td>1,19</td>
<td>63.85**</td>
<td>$y_{1,t-1}$</td>
</tr>
<tr>
<td>a and c</td>
<td>1,19</td>
<td>20.51**</td>
<td>$z_i$</td>
</tr>
<tr>
<td>c and d</td>
<td>1,18</td>
<td>23.81**</td>
<td>$y_{1,t-1}$</td>
</tr>
<tr>
<td><strong>Hotelling's t test</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b with c</td>
<td>18</td>
<td>1.82</td>
<td>$y_{1,t-1}$ vs. $z_i$</td>
</tr>
</tbody>
</table>

*The procedure used in computing Hotelling's t is described in Appendix B.

**Significant at the 1 per cent level.

(3.1.b) and (3.1.d). Upon testing the hypothesis $H_0 : b_{1,41}' = b_{1,41}''$ we find that the coefficients of $y_{41}'$ and $y_{41}''$ are significantly different at the 10 per cent or a lower probability level in (3.1.a) and (3.1.c) but are not significantly different in (3.1.b) and (3.1.d). These results suggest that in the process of testing the null hypothesis $H_0 : B_{1,41} = 0$, a type I error was committed, i.e., we reject $H_0$ when it is true. If, in fact, the coefficient of $y_{41}'$ is not significantly different from zero in (3.1.b) and (3.1.d), then following the arguments presented in section entitled Variable Preferences and Lagged
Table 3. Estimates of short-run and long-run elasticities of demand for explanatory variables that are significantly different from zero in the butter demand equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>( y_{21} )</th>
<th>( y_{36} )</th>
<th>( y_{41}^I )</th>
<th>( y_{41}^{II} )</th>
<th>( z_1^I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-run elasticities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.1.a)</td>
<td></td>
<td></td>
<td>-0.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.1.b)</td>
<td></td>
<td>-0.16</td>
<td>-0.17</td>
<td>-0.33</td>
<td></td>
</tr>
<tr>
<td>(3.1.c)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.1.d)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long-run elasticities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.1.b)</td>
<td></td>
<td></td>
<td>-0.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.1.d)</td>
<td></td>
<td></td>
<td>-0.52</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( a \) Short-run and long-run elasticity approximations are given only for the explanatory variables whose coefficients are significantly different from zero. When the coefficient of lagged consumption is not significantly different from zero, the long-run elasticity approximation is enclosed in parentheses and has been presented for illustrative purposes.

Consumption we would accept the variable preference hypothesis, as one of the conditions for acceptance is that the coefficients of the explanatory variables other than \( y_{1,t-1} \) should not be significantly different from zero. On the basis of the predicted values for \( y_1 \) in the post-estimation period, Equation (3.1.b) appears to provide a better explanation for the consumption of butter than does (3.1.d).

Demand for margarine

In the margarine demand equations, the normalized
Table 4. Predicted values for the normalized dependent endogenous variables and residuals in the butter demand equations for the post-estimation period, 1950-55<sup>a</sup>

<table>
<thead>
<tr>
<th>Year</th>
<th>$y_1$</th>
<th>$\hat{y}_1$</th>
<th>$y_1 - \hat{y}_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3.1.a)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1950</td>
<td>10.6</td>
<td>6.07</td>
<td>4.53</td>
</tr>
<tr>
<td>1951</td>
<td>9.4</td>
<td>2.68</td>
<td>6.72</td>
</tr>
<tr>
<td>1952</td>
<td>8.5</td>
<td>-0.79</td>
<td>9.29</td>
</tr>
<tr>
<td>1953</td>
<td>8.4</td>
<td>-3.30</td>
<td>11.70</td>
</tr>
<tr>
<td>1954</td>
<td>8.8</td>
<td>-3.08</td>
<td>11.88</td>
</tr>
<tr>
<td>1955</td>
<td>8.9</td>
<td>-4.29</td>
<td>13.19</td>
</tr>
<tr>
<td>(3.1.b)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1950</td>
<td>10.25</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>1951</td>
<td>10.28</td>
<td>-0.88</td>
<td></td>
</tr>
<tr>
<td>1952</td>
<td>9.74</td>
<td>-1.24</td>
<td></td>
</tr>
<tr>
<td>1953</td>
<td>8.75</td>
<td>-0.35</td>
<td></td>
</tr>
<tr>
<td>1954</td>
<td>8.35</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>1955</td>
<td>8.41</td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td>(3.1.c)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1950</td>
<td>9.32</td>
<td>1.28</td>
<td></td>
</tr>
<tr>
<td>1951</td>
<td>7.53</td>
<td>1.87</td>
<td></td>
</tr>
<tr>
<td>1952</td>
<td>7.00</td>
<td>1.50</td>
<td></td>
</tr>
<tr>
<td>1953</td>
<td>5.94</td>
<td>2.46</td>
<td></td>
</tr>
<tr>
<td>1954</td>
<td>5.65</td>
<td>3.15</td>
<td></td>
</tr>
<tr>
<td>1955</td>
<td>5.05</td>
<td>3.85</td>
<td></td>
</tr>
<tr>
<td>(3.1.d)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1950</td>
<td>10.75</td>
<td>-0.15</td>
<td></td>
</tr>
<tr>
<td>1951</td>
<td>10.77</td>
<td>-1.37</td>
<td></td>
</tr>
<tr>
<td>1952</td>
<td>10.78</td>
<td>-2.28</td>
<td></td>
</tr>
<tr>
<td>1953</td>
<td>10.05</td>
<td>-1.65</td>
<td></td>
</tr>
<tr>
<td>1954</td>
<td>9.57</td>
<td>-0.77</td>
<td></td>
</tr>
<tr>
<td>1955</td>
<td>9.63</td>
<td>-0.73</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup>The post-estimation period predicted values for the normalized dependent variables, $\hat{y}_1$ above, and estimated residuals, $y_1 - \hat{y}_1$ above, are presented to indicate the biased or unbiased nature of the estimated equations.
dependent variable is $y_{16}$, per capita civilian consumption of margarine, and the explanatory variables are $y_{21}$, $y_{36}$, $y'_{41}$, $y''_{41}$ and $z'_1$ as defined above and $y_{16,t-1}$, per capita civilian consumption of margarine lagged one time period. Results of the statistical analysis for the margarine demand equations are presented in Tables 5 through 8.

For the margarine demand equations, as with the butter demand equations, only the point estimates for coefficients associated with the two income variables, liquid asset holdings and lagged consumption are significantly different from zero at the 10 per cent or a lower level of probability. The positive signs for the estimated coefficients of $y'_{41}$, $y''_{41}$, and $z'_1$ suggest that margarine is not an inferior good.

Since the coefficients of $z'_1$ and $y_{16,t-1}$ are significantly different from zero and the addition of $y_{16,t-1}$ to (3.16.a) and (3.16.c) causes a significant increase in the values of $R^2$, there are ample grounds for accepting the hypotheses underlying Equations (3.16.b), (3.16.c), and (3.16.d). The acceptance of the liquid asset holding hypothesis underlying (3.16.c) suggests that the generalized static theory is preferable to the traditional static theory as an explanation for the consumption of margarine. The acceptance of the hypotheses underlying (3.16.b) and (3.16.d) suggests that either it requires more than one year for the consumers to complete their adjustments to changes in the
Table 5. Estimation results for the margarine demand equations (Equation 3.16)

<table>
<thead>
<tr>
<th></th>
<th>Bi&lt;sub&gt;e,0&lt;/sub&gt;</th>
<th>y&lt;sub&gt;21&lt;/sub&gt;</th>
<th>y&lt;sub&gt;36&lt;/sub&gt;</th>
<th>y&lt;sub&gt;41&lt;/sub&gt;</th>
<th>y&lt;sub&gt;41&lt;/sub&gt;</th>
<th>y&lt;sub&gt;1e,t-1&lt;/sub&gt;</th>
<th>z&lt;sub&gt;1&lt;/sub&gt;</th>
<th>R&lt;sup&gt;2&lt;/sup&gt;</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>-1.0191</td>
<td>-0.0047</td>
<td>0.0046</td>
<td>0.0039</td>
<td>0.0166***</td>
<td></td>
<td></td>
<td>0.8130</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>(0.0235)</td>
<td>(0.0155)</td>
<td>(0.0032)</td>
<td>(0.0030)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>-1.5890</td>
<td>-0.0002</td>
<td>0.0003</td>
<td>0.0047*</td>
<td>0.0074*</td>
<td>0.5613***</td>
<td></td>
<td>0.8889</td>
<td>1.22+</td>
</tr>
<tr>
<td></td>
<td>(0.0187)</td>
<td>(0.0123)</td>
<td>(0.0027)</td>
<td>(0.0037)</td>
<td>(0.1692)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>-2.8292</td>
<td>-0.0223</td>
<td>0.0203</td>
<td>0.0051</td>
<td>0.0118***</td>
<td>0.0318**</td>
<td>0.8772</td>
<td>0.98+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0232)</td>
<td>(0.0158)</td>
<td>(0.0030)</td>
<td>(0.0035)</td>
<td>(0.0129)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>-2.9193</td>
<td>-0.0150</td>
<td>0.0135</td>
<td>0.0055**</td>
<td>0.0048</td>
<td>0.4817***</td>
<td>0.0255**</td>
<td>0.9276</td>
<td>1.38+</td>
</tr>
<tr>
<td></td>
<td>(0.0191)</td>
<td>(0.0132)</td>
<td>(0.0025)</td>
<td>(0.0038)</td>
<td>(0.1668)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Significantly different from zero at the 10 per cent level.

**Significantly different from zero at the 5 per cent level.

***Significantly different from zero at the 1 per cent level.

+Inconclusive test for autocorrelation in the residuals.
Table 6. Results of the F test and Hotelling's t test for significance of difference in $R^2$ for the margarine demand equations

<table>
<thead>
<tr>
<th>Equations compared</th>
<th>d.f.</th>
<th>F or t</th>
<th>Added variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>F tests</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a and b</td>
<td>1,19</td>
<td>12.96**</td>
<td>$y_{1e,t-1}$</td>
</tr>
<tr>
<td>a and c</td>
<td>1,19</td>
<td>9.93**</td>
<td>$z_1$</td>
</tr>
<tr>
<td>c and d</td>
<td>1,18</td>
<td>12.54**</td>
<td>$y_{1e,t-1}$</td>
</tr>
<tr>
<td>Hotelling's t test</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b with c</td>
<td>18</td>
<td>0.95</td>
<td>$y_{1e,t-1}$ vs. $z_1$</td>
</tr>
</tbody>
</table>

**Significant at the 1 per cent level.

Explanatory variables or there have been changes in consumers' preferences for margarine. The results for the F tests and statistical results for (3.16.d) suggest that the static theory extended to account for liquid asset holdings and generalized to account for one of the dynamic factors, i.e., the theory underlying (3.16.d), is preferable to the theories underlying the other equations as an explanation for consumers' consumption of margarine.

Again the test for the significance of difference between the regression coefficients $y_{41}'$ and $y_{41}''$ reveals that
Table 7. Estimates of short-run and long-run elasticities of demand for explanatory variables that are significantly different from zero in the margarine demand equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>$y_{21}$</th>
<th>$y_{31}$</th>
<th>$y_{41}'$</th>
<th>$y_{41}''$</th>
<th>$z_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-run elasticities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.16.a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.16.b)</td>
<td>0.65</td>
<td>0.37</td>
<td>0.59</td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td>(3.16.c)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.16.d)</td>
<td>0.76</td>
<td>0.83</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long-run elasticities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.16.b)</td>
<td>1.49</td>
<td>0.84</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.16.d)</td>
<td>1.48</td>
<td></td>
<td></td>
<td>1.21</td>
<td></td>
</tr>
</tbody>
</table>

they are significantly different in (3.16.a) but not significantly different in (3.16.b), (3.16.c), or (3.16.d). These results appear to be consistent with the results obtained from the test of the null hypothesis in Equations (3.16.a) and (3.16.b) but not in Equations (3.16.c) and (3.16.d). Even with this additional information it is not possible to determine if it is the long-run elasticity hypothesis or the variable preference hypothesis that is the appropriate dynamic factor. Assuming the long-run elasticity hypothesis, however, the estimates of the long-run elasticities of demand for margarine in terms of disposable income in states prohibiting the sale of colored margarine and in terms of liquid asset holdings are about twice as large
Table 8. Predicted values for the normalized dependent endogenous variables and residuals in the margarine demand equations for the post-estimation period, 1950-55

<table>
<thead>
<tr>
<th>Year</th>
<th>$y_{1e}$</th>
<th>$\hat{y}_{1e}$</th>
<th>$y_{1e} - \hat{y}_{1e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3.16.a)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1950</td>
<td>6.0</td>
<td>7.54</td>
<td>-1.54</td>
</tr>
<tr>
<td>1951</td>
<td>6.5</td>
<td>9.10</td>
<td>-2.60</td>
</tr>
<tr>
<td>1952</td>
<td>7.8</td>
<td>10.38</td>
<td>-2.58</td>
</tr>
<tr>
<td>1953</td>
<td>7.9</td>
<td>11.61</td>
<td>-3.71</td>
</tr>
<tr>
<td>1954</td>
<td>8.3</td>
<td>11.72</td>
<td>-3.42</td>
</tr>
<tr>
<td>1955</td>
<td>8.0</td>
<td>12.35</td>
<td>-4.35</td>
</tr>
<tr>
<td>(3.16.b)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1950</td>
<td>6.45</td>
<td></td>
<td>-0.45</td>
</tr>
<tr>
<td>1951</td>
<td>7.01</td>
<td></td>
<td>-0.51</td>
</tr>
<tr>
<td>1952</td>
<td>7.53</td>
<td></td>
<td>0.27</td>
</tr>
<tr>
<td>1953</td>
<td>8.60</td>
<td></td>
<td>-0.70</td>
</tr>
<tr>
<td>1954</td>
<td>8.66</td>
<td></td>
<td>-0.36</td>
</tr>
<tr>
<td>1955</td>
<td>9.17</td>
<td></td>
<td>-1.17</td>
</tr>
<tr>
<td>(3.16.c)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1950</td>
<td>6.27</td>
<td></td>
<td>-0.27</td>
</tr>
<tr>
<td>1951</td>
<td>7.21</td>
<td></td>
<td>-0.71</td>
</tr>
<tr>
<td>1952</td>
<td>7.33</td>
<td></td>
<td>0.47</td>
</tr>
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<td>1953</td>
<td>7.99</td>
<td></td>
<td>-0.09</td>
</tr>
<tr>
<td>1954</td>
<td>8.31</td>
<td></td>
<td>-0.01</td>
</tr>
<tr>
<td>1955</td>
<td>8.70</td>
<td></td>
<td>-0.70</td>
</tr>
<tr>
<td>(3.16.d)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1950</td>
<td>5.58</td>
<td></td>
<td>0.42</td>
</tr>
<tr>
<td>1951</td>
<td>5.78</td>
<td></td>
<td>0.72</td>
</tr>
<tr>
<td>1952</td>
<td>5.48</td>
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<td>1.95</td>
</tr>
<tr>
<td>1955</td>
<td>6.69</td>
<td></td>
<td>1.31</td>
</tr>
</tbody>
</table>
as the estimates of the respective short-run elasticities. If, in fact, this is the appropriate hypothesis, these results would be of particular significance for decision-making because the long-run estimates are elastic whereas the short-run estimates are inelastic.

**Demand for cheese**

The statistical estimates and test results for the four forms of cheese demand equations are given in Tables 9 through 12. The normalized dependent variable is \( y_2 \), per capita civilian consumption of cheese, and the explanatory variables are: \( y_{22} \), deflated retail price of cheese; \( y_{32} \), deflated retail price of eggs; \( y_{41} \), deflated per capita disposable income in the United States; \( y_{2,t-1} \), per capita civilian consumption of cheese lagged one time period; and \( z_i \) which was defined earlier.

Only the estimates for the coefficients of \( y_{32} \) and of \( y_{41} \) are significant in (3.2.a) whereas the resulting estimates for each of the variables specified in (3.2.c) are significantly different from zero. The sign for the coefficients of \( y_{32} \) in each of these equations is in conflict with a priori reasoning as the negative sign suggests that eggs are complements in consumption for cheese. Although the significance of the coefficient of \( z_i \) in (3.2.c) leads to the acceptance of the underlying hypothesis, the
Table 9. Estimation results for the cheese demand equations (Equation 3.2)

<table>
<thead>
<tr>
<th></th>
<th>( B_{2,0} )</th>
<th>( y_{22} )</th>
<th>( y_{32} )</th>
<th>( y_{41} )</th>
<th>( y_{2,t-1} )</th>
<th>( z_{1}^{1} )</th>
<th>( R^2 )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>3.6555</td>
<td>-.0105</td>
<td>-.0238**</td>
<td>.0098***</td>
<td></td>
<td></td>
<td>.8781</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>(.0139)</td>
<td>(.0096)</td>
<td>(.0014)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>-0.4331</td>
<td>.0018</td>
<td>.0019</td>
<td>-.0014</td>
<td>1.1706***</td>
<td></td>
<td>.9651</td>
<td>1.47+</td>
</tr>
<tr>
<td></td>
<td>(.0066)</td>
<td>(.0055)</td>
<td>(.0014)</td>
<td>(.1472)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>3.2386</td>
<td>-.0198*</td>
<td>-.0159*</td>
<td>.0089***</td>
<td>.0152**</td>
<td></td>
<td>.8940</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>(.0112)</td>
<td>(.0080)</td>
<td>(.0010)</td>
<td>(.0062)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>-0.3959</td>
<td>-.0016</td>
<td>.0026</td>
<td>-.0009</td>
<td>1.1055***</td>
<td>.0043</td>
<td>.9811</td>
<td>1.35+</td>
</tr>
<tr>
<td></td>
<td>(.0076)</td>
<td>(.0057)</td>
<td>(.0017)</td>
<td>(.1668)</td>
<td>(.0043)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Significantly different from zero at the 10 per cent level.
**Significantly different from zero at the 5 per cent level.
***Significantly different from zero at the 1 per cent level.

+Inconclusive test for autocorrelation in the residuals.
Table 10. Results of the F test and Hotelling's t test for significance of difference in $R^2$ for the cheese demand equations

<table>
<thead>
<tr>
<th>Equations compared</th>
<th>d.f.</th>
<th>F or t</th>
<th>Added variables</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F tests</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a and b</td>
<td>1,20</td>
<td>49.90**</td>
<td>$y_{2,t-1}$</td>
</tr>
<tr>
<td>a and c</td>
<td>1,20</td>
<td>2.99**</td>
<td>$z_1$</td>
</tr>
<tr>
<td>c and d</td>
<td>1,19</td>
<td>87.87**</td>
<td>$y_{2,t-1}$</td>
</tr>
<tr>
<td><strong>Hotelling's t test</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b with c</td>
<td>19</td>
<td>20.01**</td>
<td>$y_{2,t-1}$ vs. $z_1$</td>
</tr>
</tbody>
</table>

*The procedure used in computing the value of Hotelling's t is described in Appendix B. The large value $t = 998.44$ for the cheese demand equations is due to the relatively large value for the numerator, i.e., 0.5833, for the difference between the partial correlation coefficients in forms b and c times 19. The number of degrees of freedom, and to the relatively small value, 0.0111, of the denominator. The small value of the denominator results from the fact that the coefficient of non-determination, $1 - R^2$, in form d is small, i.e., form d explains a large proportion of the variation in cheese consumption. As the denominator approaches zero, the value of t approaches infinity.

**Significant at the 1 per cent level.

generalized static theory does not provide a better explanation for the consumption of cheese than does the traditional static theory.

In Equations (3.2.b) and (3.2.d) on the other hand, the only significant coefficient is that for lagged