Base recirculation effects due to interaction of jets

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BASE RECIRCULATION EFFECTS DUE TO INTERACTION OF JETS

by

Lawrence Hugh Stein

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I. INTRODUCTION

The first line deterrent force of the United States, and indeed of the free world, is concentrated in two ballistic missiles, the Minuteman and the Polaris. Both of these missiles use single, solid-propellant rocket motors exhausting through a cluster of four supersonic nozzles. Also, the future of manned exploration of space depends on the success of the Saturn and the Nova booster rockets, both of which use a cluster of eight liquid propellant rocket motors.

Clustered nozzles are shorter and lighter than one large nozzle, and can be deflected to provide roll control of a missile. They have, however, one serious disadvantage. At high altitudes where the ambient air pressure is many times less than the exit pressure of the nozzles, the jet plumes expand upon leaving the nozzle exit and intersect with adjacent plumes. The interaction of the intersecting jets causes part of the hot exhaust gases to reverse directions and flow into the base region between the nozzles. This base flow, called base recirculation, produces very high temperatures which have in the past resulted in destruction of several missiles.

Experimental studies conducted by The Boeing Company (13) and Allen (1) using model rocket motors have yielded a good definition of the temperature distribution in the base region, but have not resulted in a basic understanding of the flow mechanism that produces the temperatures. Theoretical studies of clusters of axisymmetric nozzles exhausting hot gases require a three-dimensional analysis including real gas effects which tends to mask the fundamental aspects of the base recirculation.
In order to develop a basic understanding of the base flow mechanism, this paper will be restricted to the study of a pair of two-dimensional supersonic air jets. The flow geometry and resulting base flow are determined theoretically and compared with experimental results.

Some assumptions have been made in order to simplify the theoretical model. The flow downstream of the nozzle exit is considered as a non-viscous core with a superimposed viscous jet boundary. In the non-viscous core, the flow is treated as isentropic, and in the viscous jet boundary the flow is assumed turbulent with a fully developed error function velocity profile. The effects of these approximations are discussed and the resulting conclusions are presented in Part IV of this paper.

The numerical calculations required to determine the flow field and base flow were carried out on the ISU IBM 7074 computer. The computer program and an example printout are presented in the appendix.

A. Historical Background

During a development test firing of a Polaris missile, the first stage exploded about one minute after an apparently successful launch. An investigation disclosed that unusually high temperatures in the base region of the first stage had caused failure of the auxiliary power unit which provides actuation of the nozzles for attitude control. It seemed likely that the Minuteman missile would experience the same problem because of its similar four-nozzle configuration. Therefore, an extensive wind tunnel program was conducted in an effort to either reduce the base heating to a tolerable level, or to determine the heating rates so that adequate base insulation could be provided to protect the rocket.
The former effort proved futile and, therefore, the insulation method had to be used. As a result, both missiles carry many pounds of insulation which decreases both their payload capability and range.

The general phenomenon associated with supersonic jets exhausting into ambient air is developed by Schlicting (16) using a velocity profile in the viscous mixing zone described by the error function. Chow and Korst (8) and Page and Dixon (15) have made detailed studies of the flow structure in constant pressure mixing zones using the error function velocity profile, although Maydew and Reed (12) indicate that this profile is not very accurate for an axisymmetric nozzle. However, Barton (4), Beheim et al. (5, 6, 7), Chow and Korst (9) and Gothert (10) have used the error function profile successfully to predict base pressures and temperatures for two-dimensional models with closed base regions using the theoretical model proposed by Korst (11).

Some theoretical studies by Barton (4) and Stein (17) have been concerned with the prediction of actual mass flow rates involved in base recirculation, but these studies consider the base region closed to the ambient air. However, the base regions of current rockets using clustered nozzles are not actually closed, but are connected aerodynamically to the ambient air via paths between adjacent nozzles. This paper concentrates on the base flow rates using a model with an open base region.

B. Purpose

The design of rockets which incorporate clusters of supersonic nozzles requires at the present time an extensive and expensive testing program in order to determine the base heating resulting from recirculation
of hot exhaust gases. These experimental studies have in the past only resulted in the temperature distribution to be expected in the base region of the rocket which then determines the amount of insulation required to protect the base. These studies have not led to a detailed understanding of the flow in the base region and they have not resulted in methods of alleviating the high temperatures. Also the experimental data are valid only for the particular geometry tested, and cannot, in general, be applied to different configurations.

It is the purpose of this paper to propose a theoretical model to explain the fundamental causes of base recirculation. Once these fundamentals are understood, then it will be possible to analyze theoretically new configurations in order to predict the base heating to be expected, as well as to explore methods of reducing the heating.

In order to verify the results predicted by the theoretical model, an experimental investigation has been conducted using a pair of two-dimensional supersonic air jets. While this model is somewhat removed from actual rocket configurations, the theoretical model may be analyzed without the added complications of three-dimensional geometry and real gas effects, and the data analysis from the experimental model is more straightforward and the data does not exhibit the extreme scatter observed in hot flow studies. Also, this simple model leads to a good understanding of the basic aspects of base flow. Once it has been verified for two-dimensional cold flow, it should be possible to extend the theoretical model to cover the case of clusters of axisymmetric hot flow nozzles.
II. EXPERIMENTAL INVESTIGATION

A. Test Facility

1. Nozzle assembly

The nozzle assembly consists of two aluminum chambers with removable stainless steel nozzle blocks, as shown in Figure 1. Blocks were available for exit Mach numbers of 1.56, 1.86, 2.48, 2.92, and 3.59. Because of the higher pressure required for the higher Mach numbers and because of the pressure limitations of the facility, only the 1.56 and 1.86 blocks were used.

The interior dimensions produced a constant exit area of one square inch for each chamber for all Mach numbers. Each chamber is held together by sixteen three-eighths inch diameter stainless steel bolts, five inches long, which are also used to mount the chambers to six angle iron supports. The system is adjustable to provide a distance between inside nozzle blocks of either 3.568 or 6.000 inches. The flow exhausts between two magnesium plates in order to achieve two dimensional flow as shown in Figure 2. Each plate has a flush mounted one half inch thick Plexiglass viewing window.

The nozzle assemblies are connected to the magnesium plates so as to allow the base region to be open to the ambient air as shown in Figure 2. Static pressure taps are located at each nozzle exit and at the entrance to the base region. Five static pressure taps are located in one of the Plexiglass plates as shown in Figure 3.

2. Air Regulation

Each nozzle assembly is connected to a quarter-turn ball valve by a
Figure 1. Nozzle assembly

(a) One unit shown disassembled

(b) Complete set shown assembled
Figure 2. Photographs of test assembly

(a) Top view

(b) Side view
Figure 2. Continued

(c) End view

(d) Top plate removed
Figure 3. Plexiglass viewing window mounted in magnesium plate
flexible hose. Each valve is controlled by a pneumatic actuator connected to the University air supply. The valves are connected to eight storage tanks through regulators which provide a constant total pressure during each run. Two compressors supply air to the storage tanks through an oil remover and dryer. Figure 4 shows the air regulation system.

Pressure taps are installed upstream and downstream of the regulators to measure the total pressure in the storage tanks and the total pressure to the nozzles, respectively. The maximum total pressure allowable in the tanks was 1500 pounds per square inch. The total pressure at the nozzles is determined by the regulator setting, and in order to insure sufficient run time to establish equilibrium and take adequate data (approximately 4 seconds), the maximum downstream total pressure was 500 pounds per square inch for the Mach 1.56 blocks and 800 pounds per square inch for the Mach 1.86 blocks.

The control valves are actuated from a remote control panel, and all of the system pressures are recorded by pressure gauges located at the remote location.

3. Base flow measurements

An orifice meter, as shown in Figure 5, was connected to the entrance to the base region in order to determine the base flow rates. Static pressure taps were located on each side of the orifice and connected to a differential pressure transducer which was, in turn, connected to a Brush recorder. The flow coefficient (k) was determined from ASME power test codes (2). The weight flow in the base region may then be determined from
(a) Compressors, oil remover, and dryer

(b) Storage tanks, regulators, and valves

Figure 4. Air regulation system
Figure 5. Orifice meter
\[ \dot{w}_b = 2.98 \, \text{ki}^2 \, \left[ \rho_a \Delta P \right]^{\frac{1}{2}} \]  

(1)

where \( \rho_a \) is the ambient air density in slugs per cubic foot, \( d \) is the orifice diameter in inches, \( \Delta P \) is the pressure change across the orifice in pounds per square inch, and \( \dot{w}_b \) is the weight flow in pounds per second.

For the orifice meter shown in Figure 5, Equation 1 may be reduced to

\[ \dot{w}_b = 5.02 \left[ \rho_a \Delta P \right]^{\frac{1}{2}} \]  

(2)

4. Flow visualization

Two flow visualization methods were employed during the experimental part of this investigation. Following is a discussion of the two methods and the information that was obtained from each.

The first method was a technique to produce information about the direction of the local flow. Before a run, the inside surface of each viewing window was given a thin coating of either oil, water, a solution of graphite and kerosene, Bon Ami glass cleaner, Plexiglass cleaner, talcum powder, or powder or foam from a fire extinguisher. Part of the coating would then be blown away during the run leaving a pattern on the windows which indicated the direction of the local flow.

Of the many coatings used, none proved to be entirely satisfactory. The heavy, viscous coatings seemed to represent a combination of the starting transients and the steady-state flow, while the lighter fluids and powders tended to reflect a combination of the steady-state flow and the ending transients. Another difficulty associated with this method was that the patterns left on the windows really represented only the flows near the plates and not necessarily flow in the center of the channel. All of the coatings, however, left
Figure 6. Sketch of results of coating
patterns that were consistent in the region between the intersecting jet plumes. Figure 6 shows a typical pattern in this region.

Two important facts were brought out by this method. In the first place, air did not flow backward from the base region to the ambient air behind the nozzles, but instead ambient air was drawn into the base region. This result is directly opposite to the results by the Boeing Company (13) for a cluster of four axisymmetric nozzles, and also much different than the results obtained by Barton (4) and Beheim (5) for two-dimensional models which had closed base regions. The theoretical study also showed a net flow into the base region. A detailed discussion of this result is included in Part IV of this paper.

Another interesting result produced by the coating technique was the fact that the air drawn into the base region appeared to flow into the jet mixing zones as shown in Figure 6. High speed motion pictures and direct visual observation during some of the runs verified that the direction of flow was into the base region and into the jet mixing zones and not outward.

The second flow visualization technique used in the experimental investigation was a six-inch schlieren system. It consisted of a Sylvania Sun Gun Lamp, focused to a slit source, two six-inch parabolic mirrors, two eight-by-ten inch flat mirrors, a knife edge, and an eight-by-ten inch rear projection screen. Pictures were obtained by a remote controlled four-by-five inch Speed-Graphic camera employing a Polaroid back. A sketch of the system is shown in Figure 7 and Figure 8 shows some typical schlieren pictures that were obtained.
Figure 7. Schlieren system
(a) \( M_e = 1.56 \frac{P_e}{P_a} = 4.42 \)
Base open

(b) \( M_e = 1.56 \frac{P_e}{P_a} = 5.98 \)
Base closed

Figure 8. Schlieren photographs
(c) $M_e = 1.56 \quad P_e/P_a = 5.52$
Base open

(d) $M_e = 1.86 \quad P_e/P_a = 4.26$
Base open
Figure 8. Continued
(e) $M_e = 1.86 \quad P_e/P_a = 5.28$
Base open

(f) $M_e = 1.56 \quad P_e/P_a = 1.56$
Base open

Figure 8. Continued
The purpose of the schlieren study was to verify the flow boundaries and the shock wave angles predicted by the theoretical model. A discussion of the results is included in Part IV of this paper. There is, however, one interesting part of the discussion that will be included here because it is not really classified as a result.

It may be observed from Figures 8b, c, d, e, and f, that there are two pairs of lines downstream of the intersection region. The inside pair of lines were verified as being shock waves by conducting a pressure survey across the lines. The location of the pressure taps can be seen in Figure 8f. A plot of the pressure rise across the wave is shown in Figure 9 as a function of the local compression angle and compares fairly well with theory. Also, the waves show the typical dark and light bands normally observed in schlieren pictures of shock waves.

A large amount of effort was expended in an attempt to determine an explanation of the outer pair of lines. The only definite facts that were determined about them were the following:

1. As indicated by the pressure surveys, there was no noticeable pressure change across them.
2. They did not display the dark and light bands typical of shock waves.
3. They generated a considerable number of spirited discussions. As a matter of fact, there seemed to be about the same number of different explanations of these lines as there were "experts" offering explanations.

A few of the possibilities suggested were:
Figure 9. Pressure ratio across trailing shock waves
1. They represent the second (stronger) oblique shock solution.
2. They are caused by condensation in the flow.
3. They are caused by vibration of the plates and represent node lines.
4. They are caused by interference produced by the boundary layer build-up on the viewing windows.

At the present time, none of the explanations have been verified. However, these outer lines were not included in the theoretical model.

B. Data Reduction

Three data records were obtained for each run: a photograph of the pressure gauges, a photograph of the schlieren image, and a Brush recorder trace indicating the pressure change across the orifice meter. Ambient air pressure and temperature were also determined for each run. Following is a discussion of the data reduction required for each data record.

Each pressure gauge was calibrated several times throughout the test series and were found to be consistently accurate to within 1% of their full scale value. Also, each pressure tap and the tubing that connected it to the gauge were checked periodically for leaks. Therefore, the data reduction required here was the addition of ambient pressure to convert the gauge reading to absolute pressure.

The schlieren systems was aligned and focused before each run. Angular distortion effects were found to be negligible. The linear magnification factor and the actual model dimensions can be found from the photographs since a horizontal centerline and two vertical lines
were marked on one viewing window. The vertical lines are one inch apart and the left hand line is three inches from the nozzle exit plane.

The pressure transducer used to indicate the static pressure change across the orifice meter was calibrated against a manometer and data were obtained to convert deflection of the Brush recorder pen to differential pressure. Also, the pressure transducer was found to be linear in the range of interest for this study. The base flow rate was then obtained from Equation 2.
III. THEORETICAL MODEL

The basic flow configuration presented in this section is shown in Figure 10 in which the following regions can be identified:

1. Flow from the stagnation chamber to the nozzle exit plane.
   Flow in this region is treated as a perfect Laval nozzle. The pressure at the nozzle exit is assumed to be larger than the pressure in the base region between the two nozzles.

2. Flow between the nozzle exit plane and cross section 1. A Prandtl-Meyer expansion occurs in this region in order to lower the static pressure along the jet boundary to a value equal to the base pressure.

3. Flow along the free jet boundary between cross section 1 and the shock wave. It is assumed that flow in the central core is isentropic and that free turbulent mixing takes place in the viscous jet boundary. The subscript 1 applies to values on the boundary of the potential core and are constant along this boundary.

4. Flow downstream of the trailing shocks. It is assumed that the intersection of the two jet plumes causes a pair of trailing shock waves which turn the flow downstream. Values for parameters downstream of the shocks are determined by the usual oblique shock wave relationships and are indicated by the subscript 2.

5. Flow in region 3. This is a constant pressure mixing zone between the two parallel flows downstream of the trailing shock.
Figure 10. Flow model
waves.

6. The base region between the jets. In this region the flow is essentially stagnant. The pressure and temperature in this region are nearly equal to the ambient values and, therefore, are referred to by the subscript a.

The analysis is divided into three parts. First the non-viscous parts of the flow will be considered, then the viscous jet mixing along the free jet boundary, and finally the resulting base flow.

A. Non-viscous Regime

Referring to Figure 10, it is assumed that the following values are known for the model: \( P_o \) and \( T_o \) (the total pressure and temperature in the stagnation chamber), \( M_e \) (the Mach number at the nozzle exit plane), \( P_a \) and \( T_a \) (the ambient pressure and temperature), \( \theta \) (the slope of the nozzle at the exit plane), and the gas constants \( R \) and \( \gamma \). Then the pressure and temperature at the exit plane are

\[
P_e = P_o \left[ 1 + \frac{\gamma - 1}{2} M_e^2 \right]^{\frac{1}{\gamma - 1}}
\]

\[
T_e = T_o \left[ 1 + \frac{\gamma - 1}{2} M_e^2 \right]^{-\frac{1}{\gamma - 1}}
\]

Since \( P_e \) is assumed greater than \( P_a \), the flow will experience a Prandtl-Meyer expansion in order to produce a static pressure along the jet boundary \( (P_1) \) equal to \( P_a \). Since the turn is isentropic, the total pressure is conserved and the Mach number along the jet boundary is

\[
M_1 = \left[ \left( \frac{P_o}{P_1} \right)^{\frac{1}{\gamma - 1}} - 1 \right]^{\frac{1}{2}} \left( \frac{\gamma}{\gamma - 1} \right)^{\frac{1}{2}}
\]

and the turning angle is
$$\Delta v = v_1 - v_e$$

(6)

where

$$v = \left[ \frac{\gamma+1}{\gamma-1} \right]^{\frac{1}{2}} \tan^{-1} \left[ \frac{\gamma-1}{\gamma+1} \left( M_1^2 - 1 \right) \right]^{\frac{1}{2}} - \cos^{-1} \left( \frac{1}{M} \right).$$

(7)

The static temperature along the jet boundary \((T_1)\) is

$$T_1 = T_0 \left[ 1 + \frac{\gamma-1}{2} M_1^2 \right]^{-1}.$$  

(8)

The flow then meets the jet plume from the adjacent nozzle and is turned by the shock wave through the angle

$$\theta = \theta_0 + \Delta v.$$  

(9)

The shock wave angle \((\beta)\) may be found from the implicit equation

$$M_1^2 = \frac{2 \left( \cot \beta + \tan \theta \right)}{\sin 2\beta - \tan \theta \left( \gamma + \cos 2\beta \right)}.$$  

(10)

There are, in general, two values of \(\beta\) that will satisfy Equation 10 for a given \(M_1\) and \(\theta\). However, the larger of the two values is unstable and is not usually experienced so the smaller value is used. The static pressure behind the shock wave \((P_2)\) may then be found from

$$P_2 = P_1 \left[ 2\gamma \frac{M_1^2 \sin^2 \beta}{M_1^2} + \frac{(\gamma-1)}{(\gamma+1)} \right].$$  

(11)

The Mach number behind the shock wave \((M_2)\) is

$$M_2 = \left[ \frac{(\gamma-1) \sin^2 \beta + 2}{2\gamma \sin^2 \beta - (\gamma-1)} \right]^{\frac{1}{2}} \sin \left( \beta - \theta \right)$$  

(12)

and because the compression turn is adiabatic, the total temperature is conserved and the static temperature behind the shock wave \((T_2)\) is

$$T_2 = T_0 \left[ 1 + \frac{\gamma-1}{2} M_2^2 \right]^{-1}.$$  

(13)

Also, the total pressure behind the shock wave \((P_{20})\) is
\[ P_{o_2} = P_2 \left[ 1 + \frac{Y}{2} M_2^2 \right] \frac{Y}{Y-1}. \]  

(14)

The density, speed of sound, and velocity may be calculated by the usual relationships

\[ \rho = \frac{P}{gRT} \]  \hspace{1cm} (15)

\[ V_a = \left[ \gamma gRT \right]^{\frac{1}{2}} \]  \hspace{1cm} (16)

\[ U = MV_a. \]  \hspace{1cm} (17)

**B. Viscous Regime**

The following assumptions are made for the flow in the viscous mixing zones adjacent to the jet boundaries as shown in Figure 10:

1. The static pressure is constant throughout the mixing zone.
2. The flow is turbulent.
3. The mixing zone is thin compared to the length of the jet plume.
4. The velocity profile in the mixing zone is given by the fully developed error function.

\[ U(\eta) = U_1 \left( 1 + \text{erf} \ \eta \right) / 2 \] where \( \text{erf} \ \eta = \frac{2}{\sqrt{\pi}} \int_{-\infty}^{\eta} e^{-\beta^2} \ d\beta \)  \hspace{1cm} (18)

which is similar in the parameter (16)

\[ \eta = \sigma \frac{y}{x} \]  \hspace{1cm} (19)

where \( x \) and \( y \) are coordinates in the reference coordinate system as indicated in Figure 11, and \( \sigma \) is an empirical constant.

Dixon and Page (9) indicate that a good approximation for \( \sigma \) is

\[ \sigma = 12 + 2.758 M_1. \]  \hspace{1cm} (20)

Barton (4) has shown that the above assumptions lead to a total temperature distribution given by
Figure 11. Control volume for momentum equation
\[ T_0 (\eta) = T_a + \left( T_0 - T_a \right) U (\eta)/U_1. \]  

(21)

The static temperature profile is

\[ T (\eta) = T_0 (\eta) - \frac{\gamma-1}{2} \frac{U^2(\eta)}{\gamma g R}. \]  

(22)

and because the pressure is assumed constant in the mixing zone, the density profile is

\[ \rho(\eta) = \frac{P_a}{g R \left[ T_0 (\eta) - \frac{\gamma-1}{2} \frac{U^2(\eta)}{\gamma g R} \right]}. \]  

(23)

In order that the above profiles satisfy the momentum equation, it is necessary to introduce an intrinsic set of coordinates \((x^*, y^*)\) as indicated in Figure 11, which is related to the reference coordinate system \((x, y)\) by

\[ x^* = x \]
\[ y^* = y + y_m. \]  

(24)

Figure 11 shows the control volume used for application of the momentum equation. Assuming no shearing stresses on the sides of the control volume, the momentum equation is

\[ \int_0^{Y_R} \rho (y) U^2(y) \left| \frac{dy}{x=0} \right. = \int_{-\infty}^{Y_R} \rho(y)U^2(y) \left| \frac{dy}{x=x} \right. \]  

(25)

where \(y_R\) is sufficiently large so that at \(\eta = \eta_R\)

\[ |1 - U/U_1| < \epsilon \]
\[ |1 - T_0/T_{01}| < \epsilon' \]

where \(\epsilon\) and \(\epsilon'\) are small quantities.
If $x$ is considered constant, then
\[
\frac{d\eta}{x} = \frac{\sigma y}{x} = \frac{\sigma y^*}{x^*} = d\eta^*. \tag{26}
\]
Equation 25 may then be written in the form
\[
\int_{-\infty}^{\eta_R} \rho(\eta) U^2(\eta) \left| \frac{d\eta}{x^*} \right. = \int_{-\infty}^{\eta_R} \rho(\eta) U^2(\eta) \left| d\eta \right. \tag{27}
\]
Then substituting Equations 24 and 26 into 27,
\[
\int_{-\infty}^{\eta^*_R} \rho(\eta) U^2(\eta) \left| \frac{d\eta^*}{x^*} \right. = \int_{-\infty}^{\eta^*_R} \rho(\eta) U^2(\eta) \left| d\eta^* \right. \tag{28}
\]
Notice that for $x^* = 0$
\[
\eta^*_R = \frac{\sigma y^*}{x^*} = \sigma \left( \frac{y_R + y_m}{x} \right) = (\eta_R + \sigma y) \tag{29}
\]
and since $\eta_R$ and $\eta^*_R$ are finite at $x^* = 0$, then $y_m$ is also zero at $x^* = 0$.
Equation 29 then becomes
\[
\eta^*_R \mid_{x=0} = \eta_R \mid_{x=0} \tag{30}
\]
From $x$ different than zero
\[
\eta^*_R \mid_{x^* = x^*} = \eta_R + \eta_m \mid_{x^* = x^*} \tag{31}
\]
Now substituting Equations 30 and 31 into Equation 28 and noting that for
\[
\eta > \eta_R^*, \text{ as well as for } x = 0, \rho(\eta) U^2(\eta) = \rho_1 U^2_1
\]
\[
\rho_1 U^2_1 \eta_R = \int_{-\infty}^{\eta_R^* + \eta_m} \rho(\eta) U^2(\eta) \left| d\eta \right. \tag{32}
\]
Then separating the integral on the right hand side of Equation 32 and dividing by \( \rho U \), one obtains

\[
\eta_m = \eta_R - \int_{-\infty}^{\eta_R} \frac{\rho(\eta) U(\eta)}{\rho U} \, d\eta. \tag{33}
\]

Other profiles of interest may be obtained from

\[
V_a(\eta) = [\gamma g \text{RT}(\eta)]^{\frac{1}{2}} \tag{34}
\]

\[
M(\eta) = U(\eta) / V_a(\eta) \tag{35}
\]

\[
P_o(\eta) = P_a [1 + \frac{\gamma - 1}{2} M^2(\eta)]^{\frac{\gamma}{\gamma - 1}}. \tag{36}
\]

C. Base Recirculation

Figure 13 shows that the net airflow into or out of the base region is composed of two parts. The first part is due to the fluid which must flow into the mixing zones in order to satisfy continuity of mass, and the second part is due to the fluid which is turned back by the adverse pressure gradient across the trailing shocks. Since the base region is open to the ambient air, any shortage or surplus of air will be relieved via the opening between the nozzles.

Figure 12 shows the control volume used for the application of the continuity equation to determine the amount of fluid which must be drawn into the mixing zone from the base region. \( y_j \) is defined as the jet boundary which separates the fluid of the external stream from the fluid entrained within the wake. Therefore,

\[
\int_0^{Y_R} \rho(y) U(y) \, dy_{x=0} = \int_{y_j}^{Y_R} \rho(y) U(y) \, dy_{x=x}. \tag{37}
\]
Figure 12. Control volume for continuity equation
Figure 15. Base flow model
Changing to the intrinsic coordinate set as before, Equation 37 becomes

$$\rho U_1 \eta_R = \int_{\eta^*_j}^{\eta_R} \rho(\eta)U(\eta) \, d\eta^* . \quad (38)$$

Separating the integral in Equation 38 and adding to each side

$$\int_{-\infty}^{\eta^*_j} \rho(\eta)U(\eta) \, d\eta^*$$

$$\rho U_1 \eta_R = \int_{-\infty}^{\eta_R} \rho(\eta)U(\eta) \, d\eta^* - \int_{-\infty}^{\eta^*_j} \rho(\eta)U(\eta) \, d\eta^* - \rho U_1 \eta_m \quad (39)$$

After multiplying each term in Equation 39 by $gx/\sigma$, the first integral may be recognized as $\dot{\omega}_x$, the weight flow per unit width in the mixing zone at distance $x$ from the nozzle exit and the second integral may be recognized as $\dot{\omega}_j$, the weight flow into the mixing zone per unit width. Using the above definitions and substituting Equation 33 into Equation 39 one obtains

$$\dot{\omega}_j = \dot{\omega}_x - \frac{\rho U_1 x g}{144 \sigma} \int_{-\infty}^{\eta_R} \frac{\rho U^2}{\rho U_1^2} \, d\eta . \quad (40)$$

The units for Equation 40 are pounds per second per inch of nozzle width where $x$ has the units of inches.

Next, in order to determine the amount of fluid recirculated, the assumption is made that only the fluid in the mixing zone which has a total pressure greater than the static pressure behind the trailing shocks will be able to successfully negotiate the adverse pressure gradient across the shocks. The rest of the fluid will be turned back into the base region. The jet boundary which separates the fluid which proceeds downstream from the fluid recirculated is called the discriminating boundary,
\( \eta_d \), and is the boundary along which the total pressure is just equal to the static pressure behind the shocks. \( \eta_d \) may be found from the implicit relationship

\[ P_c(\eta_d) = P_2. \] (36)

The weight flow recirculated is

\[ \dot{w}_d = \frac{\rho x}{1 + 44} \sigma \int_{-\infty}^{\eta_d} \rho(\eta)U(\eta) \, d\eta. \] (37)

Note that the sign convention for \( \dot{w}_j \) is positive for flow into the mixing zone (into the base region), and the sign convention for \( \dot{w}_d \) is positive for flow recirculated (out of the base region). For the particular model geometry being studied, there are two intersecting jet plumes each one inch wide, and the total flow required is

\[ \dot{w}_b = 2 (\dot{w}_j - \dot{w}_d). \] (41)

where the sign convention for \( \dot{w}_d \) is positive for flow into the base region.

The detailed numerical calculations required in the above analysis were carried out on the ISU IBM 7074 digital computer. The results of this analysis are presented in Figure 14 and will be discussed in the next part of this paper along with a comparison of theoretical and experimental results. Figure 15 shows some of the profiles in the mixing zone for a typical case.
Figure 14a. $M_1$ vs $P_e/P_a$ for $\gamma = 1.4$
Figure 14b. $M_2$ vs $P_e/P_a$ for $\gamma = 1.4$, $\theta_o = 0$
Figure 14c. $\theta$ vs $P_e/P_a$ for $\gamma = 1.4, \theta_0 = 0$

\begin{align*}
M_e &= 1.56 \\
M_e &= 1.86 \\
M_e &= 2.48 \\
M_e &= 2.92 \\
M_e &= 3.59 \\
\end{align*}
WEIGHT FLOW INTO BASE REGION, \( w^p \), Pounds Per Second

**Figure**: \( \frac{v}{P^p} \) vs \( \frac{v}{P^p} \), For \( \gamma = 1.4 \), \( \theta = 0 \), \( h = 1 \), \( \Delta P = \frac{P^p}{P} \), in inches

**Exit to Ambient Pressure Ratio**, \( \frac{P^p}{P} \)

![Graph with curves and labels](image-url)
WEIGHT FLOW INTO BASE REGION, $\dot{V}$, POUNDS PER SECOND
Figure 15. Typical profiles in the jet mixing zone
IV. DISCUSSION

A. Results

The data from the experimental investigation are shown in Figures 16, 17, and 18 along with the results predicted by the theoretical model. Following is a discussion of the conclusions that may be made from each of these figures.

It may be observed from Figure 16 that the deflection angle, $\theta$, which was measured from the schlieren photographs is within two degrees (approximately eight percent) of the predicted value for the two Mach numbers tested. The deflection angle was measured from the nozzle centerline to the centerline of the jet mixing zone and then averaged from the two jets. Also, a parabolic curve fit of the data by the method of least squares results in less than one degree maximum error. Since the errors associated with the determination of $\theta$ from the schlieren pictures (see Figure 8) are on the order of one degree, the assumptions made in predicting $\theta$ appear to be justified, at least over the Mach number and pressure ratio ranges tested.

The shock wave angle data shown in Figure 17, also display about two degrees maximum deviation from predicted values. The angle $\beta - \theta$ was measured from the centerline between nozzles to the centerline of the shock wave. Again, a curve fit of the data results in a maximum error of less than one degree which seems to verify the predicted shock wave angle.

Figure 18 shows that the experimental base flow data differ from the predicted values by approximately twenty percent in the low pressure ratio range and by about ten percent at the end of the pressure ratio
Figure 16. $\theta$ vs $P_e/P_a$ for $\gamma = 1.4$, $\theta_o = 0$
Figure 17. $\beta - \theta$ vs $P_e/P_a$ for $\gamma = 1.4$, $\theta_o = 0$
WEIGHT FLOW INTO BASE REGION, \( w^* \), POUNDS PER SECOND

Figure 18. \( p^* \) vs. \( p^/p^* \) for \( L = 1, m = 1.75 \), Joukowsky

\( p^\prime = \frac{p^*}{p^/p^*} \)

EXHIT TO AVERAGE PRESSURE RATIO, \( p^* \)

DATA

NOTE: VIACED SYMBOLS FOR RELATABLE \( M = 1.96 \), \( M = 1.96 \)

DATA

THEORY

POUNDS PER SECOND
range tested. A curve fit of this data does not appreciably reduce the error since the data are all lower than the predicted values. The data do, however, show the predicted trend. Note that the flagged symbols are not reliable data because the pressure transducer which was used to measure the pressure change across the orifice meter was operating out of its linear range. While the theoretical model has predicted results from ten to twenty percent higher than observed values, the fundamental assumptions are well established by the good agreement between the predicted and observed trend of the results. Differences between experimental and predicted values of the base flow rates may be due to the fact that, for this geometry:

1. The mixing zones are not very thin as assumed in the theoretical model.
2. The jet plumes are not exactly straight, but curve slightly near the intersection point.
3. The base pressure may be somewhat lower than ambient because of the pressure losses through the orifice meter.

The two-dimensional model does not, however, display a net backflow out of the base region as was observed by Boeing (13) for a cluster of four axisymmetric hot flow rocket nozzles because, for the model geometry being studied, the weight flow into the mixing zones is larger than the recirculated flow. Based on the fundamentals acquired from the two-dimensional study, the three-dimensional problem may now be analyzed.

The author in 1959 was a member of a team of Boeing engineers that conducted an experimental investigation which resulted in a considerable amount of information concerning flow in the base region of the Minuteman
Figure 19. Three dimensional model
rocket motor as well as a film (13) showing some of the detailed base flow. The author later performed a theoretical analysis of the Minuteman base region using the theory presented in this paper. While the movie (13) is not classified, the quantitative experimental data and the detailed theoretical analysis are classified and, therefore, will not be presented. The general qualitative results are not classified and will now be discussed.

Figure 19 shows a typical cluster of four axisymmetric nozzles. The figure represents a small solid propellant rocket motor mounted in a supersonic wind tunnel. In region A, the region between the four intersecting jet plumes, fluid will be drawn into the mixing zones \( \dot{V}_j(A) \) in order to establish continuity of mass and momentum. Also, some fluid will be turned back into the base region \( \dot{V}_d(A) \) due to the adverse pressure gradient produced by the trailing shock waves. Application of the method presented in this paper to the Minuteman nozzle configuration indicated that the net effect in this region \( \dot{V}_b(A) = \dot{V}_j(A) - \dot{V}_d(A) \) was a flow from the base region into the jet mixing zones; that is to say \( \dot{V}_j(A) \) was larger than \( \dot{V}_d(A) \). However, the exterior parts of the jet plumes, indicated as region B in Figure 19, also require an influx of fluid to establish continuity of mass and momentum, and they will also recirculate some fluid due to the adverse pressure gradient produced by the shock waves at the intersection of the exterior parts of the plumes with the free stream. Again, application of the present theory indicated the net weight flow in region B \( \dot{V}_b(B) \) was into the mixing zones. Note, however, that regions A and B are connected aerodynamically via paths between adjacent nozzles. The analysis made at Boeing indicated that \( \dot{V}_b(B) \) was larger than
Since region B requires a larger influx of fluid than region A, fluid will flow from A to B via the paths between adjacent nozzles, which means that the complete configuration should display a net base recirculation in region A, which it does.

The theoretical study for the Minuteman displayed the same trend as the experimental data, although the magnitude was in error by approximately forty percent. At least part of this error may have been due to the fact that:

1. The three-dimensional configuration was simplified to an equivalent two-dimensional model with the same plume area.
2. Real gas effects were ignored.
3. Values for \(\sigma\) were computed from Equation 20 which has not been verified at the temperatures and Mach numbers which exist in the Minuteman jet plumes.

The theoretical model did, however, give correct qualitative results and showed exactly the same trend as the data.

B. Suggestions for Further Studies

There are many avenues of study which have suggested themselves during this study. One of these is an extension of the method presented here for the two-dimensional case to three-dimensional configurations including real gas effects. Once verified, the three-dimensional model could be used to study analytically the base heating problem on such missiles as the Minuteman, Saturn, and Nova and perhaps learn new techniques for reducing base temperatures, such as relocating or reshaping the nozzles, without sacrificing booster performance.
A parallel effort of interest is the determination of values for $\sigma$ at the temperatures and Mach numbers which exist in rocket plumes.

Even for the two-dimensional problem, there are Mach number and pressure ratio ranges which were not verified by experimental observations because of the limitations of the present facility. As more equipment and better instrumentation becomes available, data in these ranges should provide an even better verification of the theoretical model.
V. SELECTED LIST OF REFERENCES

1. Allen, John L. and Wasko, Robert A. Base heat transfer, pressure ratios, and configuration effects obtained on a 1/27 scale Saturn (C-l) model at Mach numbers from 0.1 to 2.0. United States National Aeronautics and Space Administration Technical Note D-1566. 1961.


13. Minuteman base temperature and pressure test, phase II. The Boeing Company (Seattle, Washington) Motion Picture Film No. 937. ca. 1959.


### VI. LIST OF SYMBOLS

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<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>A</td>
<td>area (in²)</td>
</tr>
<tr>
<td>g</td>
<td>acceleration due to gravity (fps²)</td>
</tr>
<tr>
<td>k</td>
<td>flow coefficient</td>
</tr>
<tr>
<td>M</td>
<td>Mach number</td>
</tr>
<tr>
<td>P</td>
<td>pressure (psf)</td>
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<tr>
<td>R</td>
<td>gas constant</td>
</tr>
<tr>
<td>T</td>
<td>temperature (°R)</td>
</tr>
<tr>
<td>U</td>
<td>velocity (fps)</td>
</tr>
<tr>
<td>Vₐ</td>
<td>speed of sound (fps)</td>
</tr>
<tr>
<td>W</td>
<td>specific weight flow (pounds per second)</td>
</tr>
<tr>
<td>x,y</td>
<td>reference coordinate system</td>
</tr>
<tr>
<td>x*,y*</td>
<td>intrinsic coordinate system</td>
</tr>
<tr>
<td>β</td>
<td>shock wave angle (deg)</td>
</tr>
<tr>
<td>γ</td>
<td>specific heat ratio</td>
</tr>
<tr>
<td>Δ</td>
<td>change in a variable</td>
</tr>
<tr>
<td>η</td>
<td>similarity parameter</td>
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<tr>
<td>θ</td>
<td>turning angle of flow (deg)</td>
</tr>
<tr>
<td>ψ</td>
<td>Prandtl-Meyer angle (deg)</td>
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<tr>
<td>ρ</td>
<td>density (slug/ft²)</td>
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<tr>
<td>σ</td>
<td>empirical constant</td>
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**Subscripts**

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<td>0</td>
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<tr>
<td>1</td>
<td>value along edge of non-viscous core</td>
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<tr>
<td>2</td>
<td>value behind trailing shocks</td>
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</table>
a  ambient conditions
b  value at entrance to base region
d  value along discriminating jet boundary
e  nozzle exit conditions
j  value along separating jet boundary
m  denotes shift between reference and intrinsic coordinates
R  denotes edge of mixing zone
VII. ACKNOWLEDGMENTS

The author is indebted to Dr. E. W. Anderson for providing excellent experimental equipment for use in this study and for assisting in the organization of this paper, and to Dr. C. T. Hsu for his help during the theoretical analysis. The author wishes to thank the Boeing Company for providing part of the experimental equipment used for this study as well as part of the computer time required for the theoretical study.

The author also appreciates the time and effort spent by Mrs. Martha Goheen in the preparation of the original draft of this paper and by Mrs. Renette Peterson in the typing of the final draft.
VIII. APPENDIX

The detailed calculations required to perform the theoretical analysis discussed in this paper were carried out on the ISU IBM 7074 and Boeing IBM 7094 digital computers. A listing of the detailed FORTRAN computer program for the 7094 is included in this section along with a sample printout. The 7094 program is identical to the 7074 program with the exception that the input and output tapes for the 7074 are called 1 and 2 respectively instead of 5 and 6.
FORTRAN

C TWO DIMENSIONAL JET MIXING PROGRAM
PA=29.92

1000 FORMAT (35H1TW0 DIMENSIONAL JET MIXING PROGRAM)
1001 FORMAT (1H0*15X*46H TOTAL EXIT AMBIENT 1 2)
1002 FORMAT (16HPRESSURE *5F10.6)
1003 FORMAT (16HTEMPERATURE *5F10.2)
1004 FORMAT (16HDENSITY *5F10.7)
1005 FORMAT (16HMACH'NO *5F10.4)
1006 FORMAT (16HSPEED OF SOUND *5F10.2)
1007 FORMAT (16HVELOCITY *5F10.2)
1008 FORMAT (1H0*5X*1HI*RX*7HTHETA 0*3X*5HGAMMA*5X*1HL*9X#1HX*9X*8HDELT 1A NU)
1009 FORMAT (1H0*5X*5HTHETA*4X*4HBETA*6X*5HPE/PA*PA*5X*5HP2-P1*5X*5HW DOT 13X*10HBETA-THETA)
1010 FORMAT (1H0*5X*1HI*8X*3META*9X*1HU*7X*7HT TOTAL*7X*1HT*5X*7HMACH
1NO*2X*HMOMENTUM*3X*7HP TOTAL*4X*5HW DOT)
1011 FORMAT (1H0*2F10.4*3F10.2*F10.4*F10.4*F10.6)
1012 FORMAT (7F10.0)
1013 FORMAT (1H0*6F10.4)
1014 FORMAT (32H EXIT PRESSURE LESS THAN AMBIENT)
1015 FORMAT (5H M= *5F8.4*7H PO= *5F8.4*8H THO= *5F8.4)
1016 FORMAT (28H THETA LARGER THAN THETA MAX/5011H*))
1017 FORMAT (1H*3F10.4*F10.2*2F10.4)
1018 FORMAT (1H1)
1019 FORMAT (1H )
1022 FORMAT(15H ETA D BOUNDARY)
WRITE OUTPUT TAPE 6*1018
WRITE OUTPUT TAPE 6*1000
305 READ INPUT TAPE 5*1012*PA*TO*TA*R*THO
READ INPUT TAPE 5*1012*XME*PEOPA*DP*END*PRINT
PA=PA*14.69/29.92

C C
C NON VISCOUS FLOW PART
C
301 RU = 0.0
W1=0.0
DELTA=1
EPSLN=.0001
A=1.0+(G-1.0)/2.0*XME**2
TE=(TO+459.688)/A-459.688
P1=PA
PE=PA*PEOPA
PO=PEP(A**G/1G-1.0))
10 RO=PO/
( TO+459.688)*32.17405*R)*144.0
RE=RO/(A**G/1G-1.0))
RA=PA/
(32.17405*R*(TA+459.688))*144.0
XMO=0.0
XMA=0.0
S=(G*32.17405*R)**.5
VAP=S*(TO+459.688)**.5
VAE=S*(TE+459.688)**.5

Program 1. Listing of IBM computer program
SI-TZ(t-l)+RAT*(TZ(I)-TZ(I-1))

YM1=YM(E(I-1)+RAT*(YM(E(I)-YM(E(I-1))

WRITE OUTPUT TAPE 6+1010
WRITE OUTPUT TAPE 6+1022
WRITE OUTPUT TAPE 6+1010
WRITE OUTPUT TAPE 6+1011;YI;ETA;VI;TOE;TEI;YM(E;XMOM;POE;WI

RUSO=-USO*DETA-XMOM(I)/7.

WD=WI

I=1

WTERM=XL*32/17405/144/SIG/V1*USO*2.

WDOTJ=(WD(61)-WTERM)

28 IF(W(I)-WD(T)25.30,27

26 I=I+1

IF(-61)=31,31,30

WRITE OUTPUT TAPE 6+1025

1025 FORMAT(18H I GREATER THAN 61)

GO TO 302

31 CONTINUE

GO TO 28

27 RAT=(WD(J)=W(I-1))/W(I-1)/W(I-1)

POE=POE(I-1)+RAT*(POE(I)-POE(I-1))

XMOM=XMOM(I-1)+RAT*(XMOM(I)-XMOM(I-1))

W=V(I-1)+RAT*(V(I)=V(I-1))

ETA=ETA(I-1)+RAT*(ETA(I)-ETA(I-1))

YI=Y(I-1)+RAT*(Y(I)-Y(I-1))

V=V(I-1)+RAT*(V(I)=V(I-1))

TOE=TOE(I-1)+RAT*(TOE(I)=TOE(I-1))

TEI=TZ(I-1)+RAT*(TZ(I)-TZ(I-1))

YM(E)=YM(E(I-1)+RAT*(YM(E(I)-YM(E(I-1)

WRITE OUTPUT TAPE 6+1019

WRITE OUTPUT TAPE 6+1026

1026 FORMAT(15H ETA J BOUNDARY)

WRITE OUTPUT TAPE 6+1010

WRITE OUTPUT TAPE 6+1011;YI;ETA;VI;TOE;TEI;YM(E;XMOM;POE;WI

WRITE OUTPUT TAPE 6+1019

WDOTJ=WDOTJ=WD

WRITE OUTPUT TAPE 6+1024;WDTR

1074 FORMAT(27H NET WEIGHT FLOW INTO BASE=F10.6,1RH POUNDS PER SECOND)

WRITE OUTPUT TAPE 6+1019

WRITE OUTPUT TAPE 6+1019

IF(PRINT=5)=40=40,41

41 CONTINUE

WRITE OUTPUT TAPE 6+1073

1023 FORMAT(20H JET MIXING PROFILES)

WRITE OUTPUT TAPE 6+1010

DO 4000 I=1,61

WRITE OUTPUT TAPE 6+1011;Y(I);ETA(I);V(I);TOE(I);TZ(I);YM(E(I);XMOM

1(I);POE(I);WI

4000 CONTINUE

40 CONTINUE

WRITE OUTPUT TAPE 6+1018

PFOPA=PFOPA+DP

GO TO 301

303 IF(END)=305,305,302

302 CALL EXIT

Program 1. (Continued)
TWO DIMENSIONAL JET MIXING PROGRAM

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ETA D BOUNDARY

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ETA J BOUNDARY

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NET WEIGHT FLOW INTO BASE = 0.287697 POUNDS PER SECOND

Figure 20. Sample IBM printout
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Figure 20. Continued