EFFICIENT EDDY CURRENT MODELS FOR EVALUATION OF THIN CONDUCTIVE COATINGS ON FERROMAGNETIC SUBSTRATES

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INTRODUCTION

Eddy current testing is widely used to determine physical characteristics of materials and to detect flaws by measurements of the electrical impedance of an eddy current probe. In this paper two analytical models allowing to determine properties of non-magnetic conductive coatings on ferromagnetic conductive substrates, are reported. Operating at a single frequency, two following quantities can be determined: permeability-to-conductivity ratio of the substrate and thickness-conductivity product of the coating [1, 2]. The method was validated using both long solenoids and air core surface coils, and was applied to the evaluation of zinc coatings on steel wires and sheets. The theoretical solutions given for high arguments are compact, and allow fast inversion, respectively around 400 and 10 ms for a pancake surface coil and for a long encircling solenoid. Two series of samples: Ø2.2 mm low carbon steel electro galvanized wires and 0.75-20 mm thick hot dip galvanized sheets, were inspected. Steel sheet samples with artificial coatings, as aluminum foils glued from both sides, were also examined. Experimental data of the coil electrical impedance were compared to those predicted. Agreement between theory and experiment is excellent. The technique developed has an extremely low sensitivity to the substrate conductivity and permeability variations [2]. A DC magnetic field, significantly diminishing the permeability of the substrate, almost does not influence results of the coating thickness determination. The agreement between measured thickness and that obtained by other methods is excellent. The accuracy of the thickness determination typically about 1 μm is obtained.

TWO-CONDUCTOR CIRCULAR ROD INSIDE A LONG COIL

A long circular conductive rod with diameter \(d\) coated with a thin conductive layer of thickness \(e\) inside a long coil of diameter \(d_c\), is shown in Figure 1. The conductivity of the coating is \(\sigma_1\), and the permeability is \(\mu_1\), the rod core electrical conductivity is \(\sigma_2\) and the permeability is \(\mu_2\). An analytical solution is established from Maxwell’s equations for a linear isotropic medium neglecting displacement currents, combined with the differential Ohm’s law. Specifying standard boundary conditions to determine unknown coefficients, and expressing the result in terms of the normalized electrical impedance of the coil, one obtains

\[
\frac{Z}{Z_{\text{air}}} = 1 - \eta + \frac{2\eta \mu_1}{\alpha \mu_0} \left( \frac{J_1(\alpha) + V' Y(\alpha)}{J_0(\alpha) + V Y(\alpha)} \right),
\]

where \(\eta = (d + 2e)^2 / d_c^2\) is the fill factor, \(J_v\) and \(Y_v\) are respectively Bessel functions of the first and second kind of the \(v\)-order, \(Z_{\text{air}}\) is the electrical impedance of the empty coil, and the quantity \(V\) is

\[
V = -\frac{\delta_2 J_1(\beta_2) J_0(\beta_1) - \delta_1 J_1(\beta_1) J_0(\beta_2)}{\delta_2 J_1(\beta_2) Y_0(\beta_1) - \delta_1 Y_1(\beta_1) J_0(\beta_2)},
\]

where \(\delta_1\) and \(\delta_2\) are defined in the text.
with $\delta_1 = k_1/\sigma_1$, $\delta_2 = k_2/\sigma_2$, and $\alpha_1$, $\beta_1$ and $\beta_2$ given by

$$\alpha = k_1 \left( \frac{1}{2} d + e \right) j^{-1/2}, \quad \beta_1 = \frac{1}{2} k_1 d j^{-1/2}, \quad \beta_2 = \frac{1}{2} k_2 d j^{-1/2}. \quad (3)$$

When parameters $\alpha_1$, $\beta_1$, $\beta_2$ are high, difficulties arise in estimating expressions (1) and (2) due to the Bessel functions calculation. Instead of the Bessel functions, we use their asymptotic expansions:

$$J_n(z) = \frac{1}{\sqrt{2\pi z}} \left( 1 - \frac{2}{\pi} (v + \frac{1}{2}) \right), \quad (4)$$

$$Y_n(z) = \frac{1}{\sqrt{2\pi z}} \sin(z - \frac{\pi}{2} (v + \frac{1}{2})). \quad (5)$$

Substituting expressions (4) and (5) in formula (1) results:

$$\frac{Z}{Z_{\text{air}}} = 1 - \eta + \frac{2 \eta \mu_1}{\alpha \mu_0} \left( \frac{\sin \alpha - \bar{\nu} \cos \alpha}{\cos \alpha + \bar{\nu} \sin \alpha} \right), \quad (6)$$

where $\bar{\nu}$ is

$$\bar{\nu} = \frac{\delta_1 \cos \beta_2 \sin \beta_1 - \delta_2 \cos \beta_1 \sin \beta_2}{\delta_1 \cos \beta_1 \cos \beta_2 + \delta_2 \sin \beta_1 \sin \beta_2}. \quad (7)$$

Formula (6) is valid when the skin depth is insignificant in comparison with the wire diameter. For large $\beta_2$, the following identities hold

$$\cos(\beta_2 j^{-1/2}) = \frac{1}{2} \exp(\beta_2 j^{1/2}), \quad \sin(\beta_2 j^{-1/2}) = -\frac{i}{2} \exp(\beta_2 j^{1/2}). \quad (8)$$

Usually for non magnetic coatings on magnetic substrates $\beta_2 \gg \beta_1$ and $\beta_2 \gg \alpha$, so that we can expand parameter $\beta_2$ only using relations (8). Substituting (8) into (6) and (7) we get

$$\frac{Z}{Z_{\text{air}}} = 1 - \eta + \frac{2 \eta \mu_1}{\alpha \mu_0} \left( \frac{(\delta_1 - \delta_2) - (\delta_1 + \delta_2) \exp(2 k_1 e j^{1/2})}{(\delta_1 - \delta_2) + (\delta_1 + \delta_2) \exp(2 k_1 e j^{1/2})} \right). \quad (9)$$

Let us suppose all geometrical parameters of the problem and the frequency to be known. Then, as can be shown from (9), the electrical impedance of the coil is influenced by the ratio $\mu_2/\sigma_2$. Supposing further $\delta_2 \gg \delta_1$, $k_1 e \ll 1$ and the coating being non-magnetic, it can be easily shown that the impedance depends on the product $\sigma_1 e$. 

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A cross-section of a coated metal sheet between two rectangular surface coils is given in Figure 2. The electrical conductivity of the coating is \( \sigma_1 \), the permeability is \( \mu_1 \), the substrate conductivity is \( \sigma_2 \), and the permeability is \( \mu_2 \). All media are linear, isotropic and homogeneous. Turns are single layered, turns spatial limits in the transversal direction are \( y_1 \) and \( y_2 \), the upper coil liftoff is \( l_1 \), the lower coil liftoff is \( l_2 \), the opening (i.e. spacing between upper and lower coils) is \( l \), the coating thickness are \( e_1 \) and \( e_2 \). Using the two-conductor line approach [3], we will obtain the impedance per unit length for the rectangular coils with a large length-to-width ratio. The vector potential created by the upper coil at \( z=l_1 \) is [3]

\[
A(y, l_1) = \frac{\mu_0 I}{\pi(y_2 - y_1)} \int_0^\infty B_1 \frac{\sin(y_2 \alpha)}{\alpha} \left[ \cos(y_2 \alpha) - \cos(y_1 \alpha) \right] e^{-2\alpha l_1} d\alpha ,
\]

where \( \alpha \) is the spatial frequency. The coefficient \( B_1 \) in the case of a multilayered conductor (Figure 2) can be evaluated using the generalized recurrent formulae of Cheng et al. [4]. However, for thin skin problems, the coefficient \( B_1 \) can be directly approximated by that for the coil placed over a coated conductive half-space:

\[
B_1 = e^{-2\alpha l_1} + \frac{(\alpha + Y_k)(Y_k - Y_2) + (\alpha - Y_k)(Y_k + Y_2)}{(\alpha - Y_k)(Y_k - Y_2) + (\alpha + Y_k)(Y_k + Y_2)} e^{2\alpha l_1} .
\]

where \( Y_k = \alpha + \mu_0 / \mu_k, \alpha = \alpha + j \omega / \mu_k \). To determine the coil impedance, we evaluate the vector potential related to the upper and lower coil, superposing the obtained potentials at \( z=l_1 \) and \( z=l_1 - l \), and then integrating the result over \( y \) in the limits \( [y_1, y_2] \), which results in

\[
Z = \frac{j \omega \mu_0}{\pi(y_2 - y_1)^2} \int_0^\infty \left[ \cos(y_2 \alpha) - \cos(y_1 \alpha) \right]^2 \alpha^3 (B_1 e^{-2\alpha l_1} + B_2 e^{-2\alpha l_2} \pm 2C) d\alpha .
\]

where the sign \( \pm \) relates to the common and opposite connection of the upper and lower coil, and \( B_1, B_2 \) and \( C \) are coefficients depending solely on boundary conditions. Formula (12) can be written in the form

\[
Z = j \omega \mu_0 \int_0^\infty \psi(\alpha)(B_1 e^{-2\alpha l_1} + B_2 e^{-2\alpha l_2} \pm 2C) d\alpha ,
\]

where \( \psi(\alpha) \) is a coil function and represents an effect of the coil spatial filtering and other terms describe the interaction of the plane wave of spatial frequency \( \alpha \) with the conductor. For the coil (Figure 2), \( \psi(\alpha) \) is

\[
\psi(\alpha, y_1, y_2) = \frac{1}{\pi \alpha^3} \left[ \frac{\cos(y_2 \alpha) - \cos(y_1 \alpha)}{y_2 - y_1} \right]^2 .
\]

A similar analysis was done for the circular surface coil using the Fourier-Bessel transform. The impedance is described by (13) where the circular coil function \( \psi(\alpha) \) is

\[
\psi(\alpha, r_1, r_2) = \frac{\pi}{\alpha^4} \left[ \frac{1}{r_2 - r_1} \int_0^{ar_2} x J_1(x) dx \right]^2 ,
\]

where \( r_1 \) and \( r_2 \) are respectively the coil inner and outer radii. For thin skin problems \( C \) in (13) can be neglected. For a sheet centered inside the coil, and coated from both sides with equally thick layers \( e_1 = e_2 = e \), \( l_1 = l_2 \) and so \( B_1 = B_2 \), and the impedance is

\[
Z = 2 j \omega \mu_0 \int_0^\infty \psi(\alpha) B_1 e^{-2\alpha l_1} d\alpha .
\]

For thin skin problems, using (11) to evaluate the coefficient \( B_1 \) and expanding \( \exp(2\alpha l_1) \) in series, we obtain
The exponent was expanded in (17) up to the second order, which ensures good accuracy on relatively thin coatings. The empty coil system impedance is

$$Z_{\text{air}} = 2j\omega \mu_0 \int_0^\infty \psi(\alpha) \left\{ 1 + \frac{\alpha - Y_2 + m(1 + \alpha_2 m)(\alpha - Y_2)(Y_1 + Y_2)}{\alpha + Y_2 + m(1 + \alpha_2 m)(\alpha + Y_2)(Y_1 + Y_2)} e^{-2\alpha_1} \right\} d\alpha.$$  

When inspecting ferromagnetic conductors, the opposite connection of the lower and upper coils (split wound coils), corresponding to $-2C$ in formula (13), yields in a considerable sensitivity gain to the coating and substrate parameters [5].

The coil spatial frequency characteristic $\psi(\alpha)$, has a maximum at $\alpha_{\text{max}}$ which is inversely proportional to the mean coil width $y_1 + y_2$. Spatial frequencies around $\alpha_{\text{max}}$ mostly determine the coil behavior. The coil impedance even can be roughly evaluated taking into account only the main frequency $\alpha_{\text{max}}$. If the term $j\omega \mu_2 \sigma_2 \gg \alpha_{\text{max}}^2$, the following relations hold: $\alpha_{\text{max}}^2 \approx \mu_0 j\omega \mu_2 \sigma_2$, and $Y_2 \approx \mu_0 \sqrt{j\omega (\mu_2 / \sigma_2)}^{-1}$, the $Y_2$ being the only quantity in (17) containing $\mu_2$ and $\sigma_2$. For $Y_1 >> Y_2$ and $Y_1 >> \alpha_{\text{max}}$ as can be shown from (17), the product $\sigma_1 \epsilon$ becomes much larger in comparison with other terms containing $\sigma_1$ or $\epsilon$.

In the normalized electrical impedance diagram (Figure 3), several series of theoretical data for an encircling long solenoid, are given. The series contain points related to varying coating thickness, the coating conductivity being constant within one group. Points with equal values of $\sigma_1 \epsilon$ are almost indistinguishable. In Figure 4 data groups are shown which contain data corresponding to the same substrate conductivity but different permeability. All four series seem to belong to a single curve. These implicit properties make much easier the measurement of the coating thickness on ferromagnetic substrates, for from a single frequency experiment, the product $\sigma_1 \epsilon$ can be determined independently of the substrate properties. Then, supposing the conductivity $\sigma_1$ to be known, the thickness of the coating can be determined or vice versa. Ratio $\mu_2 / \sigma_2$ can be determined as well in this experiment.

EXPERIMENTAL SETUP AND MEASUREMENTS

The measurements were taken at room temperature with a Hewlett Packard HP4549 digital impedance meter. Printed split wound surface coils were used for the measurement on low carbon 0.75-2.0
mm thick hot dip galvanized steel sheets and encircling long coils for \( \Theta 2.2 \) mm low carbon electro galvanized steel wires. The electrical impedance measurements were also performed on three samples of 1.5 mm thick low carbon steel sheets coated from both sides with 15, 30, and 45 \( \mu m \) thick aluminum foils. A numbers of 15 \( \mu m \) aluminum foil were glued to the sheets in order to produce samples with different coating thickness, a technique currently used in eddy currents. The foil thickness was measured using a Heidenhain MT-12 precision mechanical gauge (± 0.5 \( \mu m \)). The foil conductivity was measured with a Sigmatest D at 480 MHz. Coating thickness on hot dip galvanized sheets was determined by the X-rays fluorescent method using a DMC coating weight gauge. On wires the coating thickness was also determined by a chemical method. To verify the performance of the model on substrates with varying permeability, measurements were performed in a uniform DC magnetic field up to 36000 A/m, created using long air core coils.

RESULTS

Theoretical electrical impedance series and experimental data acquired using the \( \Theta 5.5 \) mm 70 mm long solenoid on the set of steel wires, are given in Figure 5. For the inversion, the two-variable Newton-Raphson method has been used. Data were fitted with following parameters for materials: \( \sigma_2 = 6 \text{ MS/m, } \mu_2 = 85, \text{ and } \sigma_1 = 14.5 \text{ MS/m.} \) The impedance was computed using results of thickness determination by a chemical method. The accuracy of such a thickness measurement was around 0.05 \( \mu m \). Values of \( \sigma_1 = 14.5 \text{ MS/m} \) was obtained by fitting Z data and supposing \( e \) equal to the chemical thickness. The value of the \( \mu_2/\sigma_2 \) ratio was also obtained by fitting Z data. The permeability \( \mu_2 \) was calculated assuming the conductivity \( \sigma_2 \) about 6.0±0.3 \( \text{MS/m} \), a value which was measured at 500 Hz on etched wire samples. The low frequency permeability \( \mu_2 \) (500 Hz) around 80±5, is in a good agreement with that measured from \( \mu_2/\sigma_2 \) ratio.

Agreement between the experiment and theory (Figure 5) is excellent for coatings thicker than 12.4 \( \mu m \). Some discrepancies arise for thinner layers. Surface microflaws, such as wire roughness and claws, as well as coating thickness variation, can diminish the conductivity of the coating, and so the \( \sigma_1 e \) value. To take into account these factors, one can introduce an apparent coating conductivity depending on thickness of the coating, carry out a calibration, and then use the obtained dependence \( \sigma_1 (e) \) in the thickness measurements.

Experimentally it was found that the dependence of the actual coating thickness \( e_{\text{act}} \) versus the inferred thickness-conductivity product \( \sigma_1 e \) can be fitted with excellent accuracy by linear function

\[
e_{\text{act}} = \frac{1}{\sigma_1} P + R \quad (19)
\]

![Figure 5. Normalized impedance diagrams for \( \Theta 2.2 \) mm coated wires (Zn 2.7-64.6 \( \mu m \)), theory vs. experiment.](image)

![Figure 6. Actual thickness as a function of thickness-conductivity product \( P \): exact vs. asymptotic formula (electro galvanized wires at 200 kHz).](image)
where $P$ is the measured thickness-conductivity product determined from Z data, and $R$ and $\sigma'$ are constants. The $R$ is usually small, and $\sigma'$ is the apparent conductivity of the coating. To determine $\sigma'$ and $R$, two wires with known parameters are needed. Physically, the validity of (19) means that the coatings tested have rather homogeneous structure, and low thickness discrepancies are due to wire roughness; a tiny layer of the coating on a rough surface acting like an intermediate layer with a reduced conductivity. The $R$ was found to be more or less proportional to the roughness.

A comparison between results of the inversion of Z data using the exact and asymptotic models at 200 kHz is given in Figure 6. The asymptotic model has low accuracy on thin layers, which improves at higher frequencies. With 5 percent errors, the asymptotic model can be used even at 200 kHz for coatings thicker than 12.4 $\mu$m. The simplicity of formula (6) ensures an extremely fast inversion of eddy current data, typically around 10-15 ms. For larger diameters of coated cylindrical products, formula (6) can be applied at lower frequencies.

The performance of the method on substrates with different permeability was shown by carrying out measurements in a uniform DC magnetic field. In Figure 7 experimental and theoretical data obtained on low carbon 1.5 mm thick steel sheets coated from both sides with 15, 30, 45 $\mu$m aluminum layers, are given. Agreement is good. Applied DC magnetic field about 36000 A/m drops the permeability of about a factor of 10. Additional errors due to that decrease do not exceed 1 $\mu$m. Similar experiments carried out with electro galvanized wires as well have shown excellent stability of the inferred thickness in spite of the substrate permeability variations.

Eddy current thickness obtained on series of Ø2.2 mm electro galvanized steel wires and that of 0.75-2.0 mm thick hot dip galvanized steel sheets are given respectively in Tables I and II. Coating thickness on hot dip galvanized sheets was also determined using a DMC X-fluorescent coating gauge. Discrepancies between eddy current thickness and thickness determined by other techniques are typically within one micron.

When measuring thickness on hot dipped coatings, a presence of alloy layers between the coating and base metal implies significant errors. This problem was solved using the same approach as for coated wire, expressed by formula (15) (Figure 8). In fact, the method is sensitive to a non ferromagnetic layer on ferromagnetic substrate. In the case of hot dipped coatings the offset $R$ in formula (19) relates to a quantity of the zinc gone to intermediate layers with relatively high permeability.

### Table I. Coating thickness on Ø2.2 mm wires from eddy current data at 400 kHz.

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<th>Actual thickness $\mu$m</th>
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### Table II. Results of the coating thickness and permeability determination at 100 kHz on hot dip galvanized sheets (the substrate conductivity $\sigma_2=6$ MS/m)

<table>
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CONCLUSION

An eddy current method allowing the determination of parameters of a thin non-magnetic conductive coating on a ferromagnetic conductive substrate is reported. The method was applied to the evaluation of electro galvanized wires and hot dipped metal sheets. Experimental data of the electrical impedance were compared to those predicted. Agreement between theory and experiment is excellent for relatively thick coatings. Despite discrepancies between theory and experiment for very thin layers, arising from various imperfections of the coating and interfaces, the method was applied successfully for thin coatings. To do this, two parameters: a constant apparent conductivity of the coating and a thickness offset, were introduced. The technique developed has an extremely low sensitivity to variations of the ferromagnetic substrate conductivity and magnetic permeability. The agreement between measured thickness and that obtained by other techniques are typically within 1 μm.

REFERENCES