ULTRASONIC WAVE DISPERSION AND ATTENUATION IN FLUID FILLED POROUS MEDIA

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INTRODUCTION

The study of ultrasonic wave propagation in granular materials can lead to a better understanding of wave interaction with such materials as uncured cement and concrete. The measured parameters can then be used to investigate the curing process in particular the time required for a given mixture to consolidate. The cohesionless granular materials having loose contact between the constituent grains form a matrix that has negligible shear modulus. Sediment, sandy ground and concrete before solidification can be considered as examples of cohesionless granular materials. The shear and rigidity moduli of these materials can differ greatly from the values obtained by effective medium theories. In particular these differences could affect the ultrasonic wave propagation in such a material. In the case of cohesionless granular material the complete description of mechanical properties requires the consideration of discrete nature of the solid frame and the contact areas between the grains. Therefore wave interaction with such a material should also include the above mentioned effects. The goal of this work is to investigate the ultrasonic wave dispersion and attenuation in cohesionless granular materials the results can be to applied to the monitoring of cement and concrete during the curing process.

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From Hertz's contact theory, the radius of the contact area between two identical spherical grains under compressive force P_t is given by (Timoshenko and Goodier, [1], Brettell, [2]),

$$\mathbf{a} = \left(\frac{3}{4} \frac{\mathbf{P_t} \, (1 - \nu_p^2) \, \mathbf{R_s}}{\mathbf{E}} \right)^{\frac{1}{3}} \tag{1}$$

where P_t is the total compressive force $P + P_o$, and P_o is the compressive force due to external pressure, ν_p, is Poisson's ratio, E, is Young modulus and R_s, is the radius of spherical grain. The center of each spherical grain is displaced by [10],

$$\alpha_s = \frac{\mathbf{a}^2}{\mathbf{R_s}} \tag{2}$$

From the stress-strain relations for a fluid-saturated system (Gassmann, [3])

$$\Delta \mathbf{P} = - \sum_{j=1}^{6} \mathbf{c_{ij}} \; \Delta\alpha_s \quad \frac{\mathbf{dP}}{\mathbf{d}\alpha_s} = \frac{2 \, \mathbf{E} \, \mathbf{a}}{1 - \nu_p^2} , \quad \bar{\mathbf{c}} = \frac{1}{6 \, (2\sqrt{2}\mathbf{R_s})} \frac{\mathbf{dP}}{\mathbf{d}\alpha} . \tag{3}$$

ELASTIC WAVE IN A FLUID FILLED COHESIONLESS GRANULAR MATERIAL COMPRESSED UNDER ITS OWN WEIGHT.

Gassmann[3] obtained the value of compressive force, P, on a spherical grain in a hexagonal-close- packed structure (h.c.p). In a h.c.p., structure a given spherical grain is in contact with six other spheres. For each sphere there are three spheres in contact at the top and three in contact at the bottom. The contact forces make an angle $\theta = 54.73^o$, with the horizontal plane, the resultant compressive force on a spherical grain at the n^{th} grain layer below the surface is given by

$$\mathbf{P_n} = \mathbf{P_{n-1}} + \frac{1}{\sqrt{6}} \mathbf{W_s} + \mathbf{P_o} \tag{4}$$

where P_o is the external compressive force on the surface. The weight of one spherical grain, W_s, minus the buoyant force is

$$\mathbf{W_s} = \frac{4\pi}{3} \mathbf{R_s^3} \, (\rho_s - \rho_f) \, \mathbf{g} \tag{5}$$

for $P_o = 0$,

$$\mathbf{P_n} = \frac{\mathbf{n} \; \mathbf{W_s}}{\sqrt{6}} \tag{6}$$

$$\mathbf{P_n} = \frac{\pi}{3} \mathbf{R_s^2} \, (\rho_s - \rho_f) \, \mathbf{g} \, \mathbf{h} \tag{7}$$

The number of sphere layers in a height h of the material is $\mathbf{n} = \dfrac{\mathbf{h}}{(\frac{\sqrt{6}}{3})\, 2\mathbf{R_s}}$.

The distance between two grain layers is $\mathbf{d} = (2\mathbf{R_s})\sqrt{6}\,/\,3$ (8)

For a random packing of spherical grains we have obtained experimentally, [9] the following values for θ and d, $\theta = \arcsin(0.9343) = 69.1^{\circ}$, $d = (0.9343)\, 2R_S$. The contact area has a radius a_h function of depth. If the force due to external pressure is zero, then we can write

$$a_h = \left[\frac{(1-\nu_p^2)\,\pi\,\mathbf{R_s^3}}{4\,\mathbf{E}}(\rho_s-\rho_f)\,\mathbf{g}\,\mathbf{h}\right]^{\frac{1}{3}} \tag{9}$$

$$\bar{\mathbf{c}} = \frac{1}{12\sqrt{2}}\left[\frac{2\pi\,\mathbf{E}^2(\rho_s-\rho_f)\,\mathbf{g}\,\mathbf{h}}{(1-\nu_p^2)^2}\right]^{\frac{1}{3}} \tag{10}$$

For a closed system, and at the limit of $\lambda \gg 2R_S$, Gassmann's vertical and horizontal velocities, [3], are

$$\mathbf{V_{vf}} = \sqrt{\frac{C_3}{\rho_e}},\quad \mathbf{V_{vs}} = \sqrt{\frac{C_4}{\rho_e}},\quad \mathbf{V_{hf}} = \sqrt{\frac{C_1}{\rho_e}},\quad \text{and } \mathbf{V_{hs}} = \mathbf{V_{vs}}. \tag{11}$$

BIOT'S THEORY ADAPTED TO THE RANDOMLY PACKED SPHERICAL GRAINS IN A FLUID

Biot's equation using Biot's notations, [4], is,

$$z^2(\sigma_{11}\sigma_{22}-\sigma_{12}^2) - z(\sigma_{11}\gamma_{22}+\sigma_{22}\gamma_{11}-2\sigma_{12}\gamma_{12}) + (\gamma_{11}\gamma_{22}-\gamma_{12}^2) + \frac{i\,b}{\rho_e\,\omega}(z-1) = 0$$

where $\mathbf{z} = \dfrac{\mathbf{C_w^2}}{\mathbf{C_b^2}}$ (12)

The last term contains the dissipation parameter b, and C_W is Wood's velocity, [5], or the effective medium velocity, valid for $\omega = 0$,

$$\mathbf{C_W} = \frac{1}{\sqrt{\rho_e\left(\dfrac{\varepsilon}{\mathbf{K_f}} + \dfrac{1-\varepsilon}{\mathbf{K_S}}\right)}} \tag{13}$$

where ε is the porosity, K_f and K_S are bulk moduli for fluid and solid constituents, [9], [11]. The effective density for the medium is

$$\rho_e = \varepsilon\,\rho_f + (1-\varepsilon)\,\rho_s\,. \tag{14}$$

A new dissipation factor b is used, derived from Darcy's law (Martin et al., [6], Tavossi, [7]) and given by Eq (18)

$$\mathbf{b} = \frac{9\,\mathbf{k}'\,\eta\,(1-\varepsilon)^2}{\varepsilon\,\mathbf{R}_s^2} \tag{15}$$

Experimentally the range of k' is (4.5 < k' < 8.33), [7]. In Biot's, [4], equation let

$$\beta = \frac{\mathbf{b}\,\mathbf{F_r}(\kappa)}{\omega\,\rho_e} \qquad \psi = \frac{\mathbf{b}\,\mathbf{F_i}(\kappa)}{\omega\,\rho_e} \tag{16}$$

Where $F(\kappa) = F_r(\kappa) + F_i(\kappa)$, is a complex function defined to give a frequency dependent viscosity.

$$\frac{\mathbf{b}}{\omega\,\rho_e} = \frac{9\,\mathbf{k}'\,\eta\,(1-\varepsilon)^2}{\varepsilon\,\mathbf{R}_s^2\,\omega\,\rho_e} \tag{17}$$

$$\mathbf{z} = \frac{\left[(\mathbf{A}+\psi)\,(\frac{\mathbf{A}}{\mathbf{z_o}}+\psi)+\beta^2\right] + \mathbf{i}\left[\beta\,\mathbf{A}\,(1-\frac{1}{\mathbf{z_o}})\right]}{(\frac{\mathbf{A}}{\mathbf{z_o}}+\psi)^2+\beta^2} \tag{18}$$

Finally, in the limit of weak frame (Johnson & Plona, [11]), when only one longitudinal mode is propagating, Biot's velocity becomes

$$C_b = \frac{\sqrt{2}\,C_w}{\sqrt{|\mathbf{a}'| + \mathbf{R}'}} \tag{19}$$

At high-frequency this expression is not in agreement with the experimental results, [8].

WAVE PROPAGATION IN THE ARRAYS OF SPHERICAL GRAINS IN CONTACT: MODIFICATION OF GASSMANN'S AND BIOT'S VELOCITY EQUATIONS

The cutoff angular frequency, [12], ω_o, results in the dispersion relation for the chain of spherical grains in contact

$$\omega = \omega_o \sin\frac{kR_s}{2} \tag{20}$$

The maximum wave velocity is $C_m = \frac{R_s\omega_o}{2}$. (21)

ADAPTATION OF GASSMANN'S CALCULATIONS FOR A H.C.P., TO THREE DIMENSIONAL RANDOM-PACKING OF SPHERICAL GRAINS

The number concentration or number of grains per unit volume is

$$\mathbf{n_s} = \frac{(1-\varepsilon)}{\frac{4}{3}\pi \mathbf{R_s^3}} \tag{22}$$

The distance between two adjacent grain layers in random packing is

$$\mathbf{d_s} = \left[\frac{1}{\mathbf{n_s^{\frac{1}{3}}}}\right] \tag{23}$$

Experimentally we have $d_S = (0.9343)\ 2R_S$. Fast and slow vertical velocities calculated at a depth of 5 cm (h= 5 cm) are $V_{Vf} = 1748$ m/s and $V_{VS} = 287$ m/s. On the surface of the material with h =0, we have V= 1665 m/s, which is the value for Wood's effective medium velocity.

SECOND MODIFICATION OF BIOT'S VELOCITY

We replace $\mathbf{C_w}$ by $\mathbf{V_{vf}}$, in the Biot's velocity, to take into account the effect of contact areas between the grains

$$\mathbf{C_{bc}} = \frac{\sqrt{2}\ \mathbf{V_{vf}}}{\sqrt{|\mathbf{a'}| + \mathbf{R'}}} \tag{24}$$

The cutoff angular frequency with is $\omega_0 = \dfrac{2C_{max}}{R_S}$.

Finally at high ultrasonic frequencies and with $k = \omega/C_{max}$. The wave velocity as a function frequency and grain size is

$$C_\omega = C_{max} \frac{\sin\left(\frac{kR_s}{2}\right)}{\left(\frac{kR_s}{2}\right)} \tag{25}$$

Experimental data for ultrasonic velocity and calculated values, using Equation (25) with experimental values of C_{max}, are compared in Figure - 1.

EXPERIMENTAL RESULTS FOR THE MEDIA OF IDENTICAL GRAIN SIZES 4 MM AND 6 MM

Through-transmission method was used with a C-Scan Immersion System-Imagery. Transmitter and receiver both have a center frequency of 500 kHz. Transmitter diameter is 5.5 cm, receiver diameter is 2.8 cm. Receiver focal length is 5.08 cm. Two cylindrical granular media are investigated. Medium # 1, spherical grain size, 4 mm, depth = 5.24 cm., inside diameter = 8.25 cm. Medium # 2, spherical grain size, 6 mm, depth = 6.35 cm, inside diameter = 6.35 cm. Transmitter- surface separation was 12 cm, and receiver- surface separations were 5.6 cm for the 4 mm grain size material and 3.5 cm for the 6 mm grain size material. From the results we can state the following. The arrival velocity increases with the grain size, see Figure - 2.

$$V_b \propto R_s \tag{26}$$

The frequency f_{max} of the transmitted signal and the amplitude of the signal decreases as the grain size increases, see Figure - 3. These results suggest the expressions (27)

$$f_{max} \propto \frac{4\pi R_s^2}{\frac{4}{3}\pi R_s^3} \qquad \text{or} \qquad f_{max} \propto \frac{1}{R_s} \tag{27}$$

The attenuation coefficient of the granular material is proportional to the spherical grain size

$$a_{att} \propto R_s. \tag{28}$$

THE DATA ON ARRIVAL VELOCITY AND OPTIMAL FREQUENCY

$V_b = 2392$ m/s, f_{max}= 393 kHz, for the 4 mm grain size material, and V_b= 2668 m/s, f_{max}= 295 kHz, for the 6 mm grain size material. These values of velocity correspond to $\frac{kR_s}{2} = 1.03$ for the material made of 4 mm grains and $\frac{kR_s}{2} = 1.04$, for the material made of 6 mm grains. Therefore, on the average, the above results lead to $\frac{kR_s}{2} \propto 1$ or $\frac{\pi d_s}{2} \cong \lambda_b$. The above results satisfied the expression for the optimal frequency, as a function of grain size and arrival velocity, that is $C_{max} = \frac{R_s \, \omega_{max}}{2}$.

CONCLUSION

To investigate the basic ultrasonic characteristics of cement and concrete during curing, the medium model made of identical spherical grains in water is used. We find that at low ultrasonic frequencies the arrival velocity of ultrasonic pulse, in such a material, increases with the grain size. At the high ultrasonic frequencies a decrease of the pulse velocity with frequency and grain size is observed. The cutoff frequency also decreases with the material grain size. These results are then used to modify Biot's theory for the wave velocity calculated for granular media. A relationship is obtained between ultrasonic signal arrival velocity, grain size and optimal frequency, corresponding to maximum amplitude of the transmitted signal.

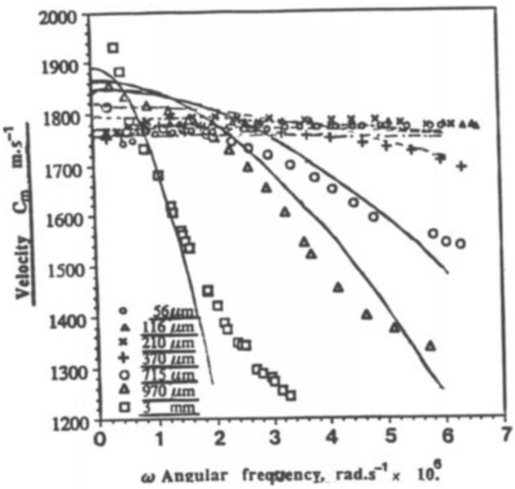

Figure - 1. Ultrasonic signal phase- velocity, in m/s, versus angular frequency in rad./s, calculated by modified Biot's theory (continuous curve), compared to experimental values (points). (The signal is transmitted through granular materials of uniform grain sizes from 56 µm to 3 mm diameter, immersed in water).

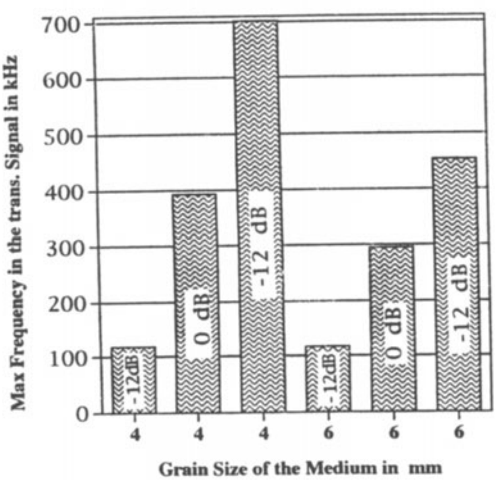

Figure - 2. Histogram of frequency, in kHz, corresponding to ultrasonic signal with maximum amplitude, versus grain size, in mm, at relative intensity levels of - 12 dB, 0 dB (the signal is transmitted throughcompact granular material of uniform grain sizes 4 mm and 6 mm diameter, immersed in water).

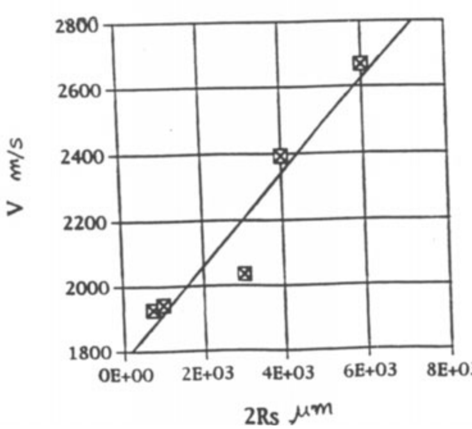

Figure - 3. Ultrasonic signal arrival- velocity, in m/s, versus grain size in µm, when transmitted through compact granular material of uniform grain sizes, from 717 µm to 6 mm diameter, immersed in water.

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