ULTRASONIC WAVE PROPAGATION IN A TOOTH PHANTOM

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INTRODUCTION

Although ultrasonic energy has been used widely for general medical diagnostic purposes and as a tool for the nondestructive inspection of engineering components, a lack of understanding of the fundamental wave propagation phenomena in teeth has largely precluded its application to dentistry. This paper studies the physics of ultrasound/teeth interactions. Based on the authors' experience with the detection of defects in engineering materials using ultrasound and the development of finite element computer code to study the coupling of defecting wave propagation, initial studies have already been made on a tooth phantom. Results indicate that this numerical model can indeed be used to study the complex ultrasound interactions with dental anomalies, such as pulpitis and abscesses, and that such results could be used to optimize the design of appropriate transducers and equipment for dentistry applications. The primary reason why this technique is important and should take place is that ultrasonography has a great advantage over the presently available X-ray technology in that it does put any strain on the patient nor does it cause any pain. Most importantly, ultrasonic waves are nonionizing which, when used at low sound intensity levels, do not cause any health risks to the patient nor to the operator.

BRIEF LITERATURE REVIEW

In a first successful ultrasonic experiment performed on a human tooth [1], the transducer/tooth water couplant was replaced with a solid aluminum rod in direct contact with a polished ground area of a tooth. An ultrasonic pulse-echo system was able to detect the dentino-enamel junction and the dentino-pulp interface. Ultrasonic imaging techniques have also been applied to other areas of dentistry, specifically prosthodontics. A B-mode ultrasonic system for determining the thickness of the masticatory mucosa for denture construction has also been developed [2], and the use of the scanning acoustic microscope to image the elastic properties of carious human dental enamel has been described [3]. The need for a high degree of flatness of the specimen, however, makes this technique problematic for measurements in vivo. Acoustic microscopy has been successful for imaging lesions in tooth enamel and detecting the enamel-dentino interface [4]. Ultrasonic measurement methods have also been applied reliably and accurately to orthodontics and odontology as well [5]. Here, the masseter muscle thickness was measured by ultrasonic means to determine the muscle thickness and its relationship to facial morphology. The specific aim of this study was to evaluate ultrasonography as a method for imaging and to assess the feasibility of its future use in measuring growth changes of masticatory muscles. In the
realm of FEM and other simulation techniques as applied to dentistry, a two-dimensional plane-stress FEM simulation model was developed [6]. Finite element simulation of bone resorption phenomena suggested several new ideas in addition to support for current concepts of practice in prosthodontics.

All such studies would benefit from the availability of an appropriate, flexible, modeling tool capable of predicting ultrasonic wave propagation through the complex tooth structure.

The modeling of ultrasonic nondestructive testing (NDT) phenomena is of importance to many engineering industries because of the need to find solutions to the inverse or defect characterization problem. Exact predictions of defect shape from NDT measurements are particularly important to aerospace and nuclear industries, for example, where the cost of catastrophic failure is most high. Although analytical methods based primarily on Kirchhoff and Born approximations [7] have been used to attack simple forward (given the transducer input signal and the defect shape - what is the output transducer signal?) and inverse (given the input and output transducer signals - what is the shape of the defect?) problems (see Fig. 1), only integral (e.g., the boundary element method) and domain (e.g., finite element and finite difference) numerical analysis methods have shown promise for predicting the complex interaction of ultrasound with realistic defect shapes in engineering structures. Finite element (FE) analysis methods [8, 9, 10, 11, 12, 13] have been found to be particularly useful as ultrasonic NDT test-beds. Once validated experimentally, such models can be used to provide useful data for ultrasonic equipment/transducer design as well as providing insight into the physics of just how the ultrasound interacts with materials.

The capability and potential of this modeling technique is probably best illustrated through an example. This paper describes the application of finite element analysis and proposes a tooth phantom for use as a test-bed.

DESCRIPTION OF THE THEORY INVOLVED

The major hypothesis of this paper is that numerical analysis can be used for the study of elastic wave propagation in teeth. The governing equation of motion which describes the propagation of waves in situations where the medium is linear, homogeneous and isotropic [8], is

\[ (\lambda + \mu) \nabla \cdot \mathbf{u} + \mu \nabla^2 \mathbf{u} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} \]  

(1)

where \( \rho \) and \( u \) represent the material density and the displacement field vector respectively; and \( \lambda \) and \( \mu \) are the Lamé constants. In general, the solution to this boundary value problem is a displacement vector \( \mathbf{u} \) which possesses second partial derivatives throughout the domain in question, and satisfies the partial differential equation and boundary conditions. For geometries such as the proposed tooth phantom having axisymmetry about the z-axis, the stresses are independent of the \( \theta \) coordinate. As a result, all derivatives with respect to \( \theta \) are zero, and the displacement component \( u_\theta \) (tangent to the \( \theta \) direction), the shear strains \( e_\theta \), \( e_\phi \), and the shear stresses \( \sigma_{\theta \phi}, \sigma_{\theta \phi} \) also vanish. Usually, a classical or analytical approach to the solution is not possible if the domain does not hold a particular regular shape such as a rectangle or a circle. In this situation, a numerical approach, such as the finite element method, is appropriate.

The combination of the first and the third terms in Eq. 1 represents a stress tensor, \( T \), in the elastic medium. If Eq. 1 is written in terms of stress, the following equation results:

\[ T_{ij} = \rho \ddot{u}_j \]  

(2)

For the case where small displacement values are considered, as in most NDT situations, a linear stress-strain relationship (Hooke’s law) can be established as

\[ T_{ij} = C_{ijkl} S_{kl} \]  

(3)

\( C_{ijkl} \) represents the fourth rank material tensor while \( S_{kl} \) symbolizes the strain-displacement relations given by
where the commas denote partial differentiation. Substituting Eq. 4 into Eq. 3 leads to the following expression

$$T_{ij} = C_{ijkl} \frac{(u_{k,I} + u_{l,k})}{2}. \quad (5)$$

The indices \(i, j, k, l\) can take the values 1, 2 or 3 to conform with the \(x, y\) and \(z\) spatial directions convention. It is also assumed that the summation convention (\(\Sigma\)) over the indices will take place throughout.

The energy functional is given by

$$\int_{E_t} \delta u_{ij} C_{ijkl} u_{kl} \, dv - \int_{E_t} u_{ij} t_{ij} \, ds + \int_{E_t} \delta u_{ij} p \, dv = 0. \quad (6)$$

The minimization of this energy functional is carried out with respect to the nodal variables. Evaluating Eq. 6 for all elements in the domain and assembling all elemental computations, gives rise to a global matrix differential equation

$$\{K\} \{u\} + \{M\} \{\dot{u}\} = \{F\} \quad (7)$$

where \([K]\) and \([M]\) are referred to as the global stiffness and mass matrices, respectively. Note that the coefficients of both \([K]\) and \([M]\) depend on the interpolating shape functions and on both the longitudinal and the shear velocities inside the material [8, 9, 14]. Approximating \([\dot{u}]\) by the explicit central difference formula

$$\dot{u}_i = \frac{1}{\Delta t^2} (u_{i,\Delta t} - 2u_i + u_{i,-\Delta t}) \quad (8)$$

yields the iterative form

$$\frac{1}{\Delta t^2} [M]\{u\}_{\Delta t} = \{F\}_{\Delta t} - \frac{2}{\Delta t^2} [M]\{u\} - \frac{1}{\Delta t^2} [M]\{\dot{u}\}_{\Delta t} \quad (9)$$

Eq. 9 constitutes the scheme followed for a direct computer code implementation for the vector \([u]\), which contains all the nodal displacement values, to be obtained at each time step \(\Delta t\).

TESTS AND RESULTS

Four cases have been studied (Fig. 1). For the purpose of describing the overall picture, the modeled tooth geometry is presumed to be that of a simplistic second molar of a human. All four model cases are described as follows:

CASE I (Fig. 1(a)): Three-layer (3L) sound tooth.
CASE II (Fig. 1(b)): Three-layer tooth with a thin Gold (GD) layer simulating a "crown" over the enamel layer.
CASE III (Fig. 1(c)): Three-layer tooth with an Amalgam (MG) layer simulating a "restoration" inserted in the enamel-dentin layers.
CASE IV (Fig. 1(d)): Three-layer tooth with a quadrilateral void (VD) simulating a cavity or a crack at the enamel-dentin interface.
Only results from case I of this example are shown due to the page limit constraint of this paper.

Due to the axisymmetric assumptions (i.e., symmetric with respect to the primary z-axis) on the geometries and boundary conditions, only half of the cross-section containing the axis of symmetry is discretized. In order to fit the curved boundaries, nonuniform four-node quadrilateral elements are adopted throughout.

Note that the degrees of freedom (number of unknowns) are two, representing the radial (R) and the axial (Z) displacements. The material properties (longitudinal and shear velocities and mass density) are listed in Table I.

As mentioned above, the explicit central difference scheme is adopted for solving the finite element equations shown earlier. Since the scheme is conditionally stable, the time step is restricted. A general rule is that the fastest wavefront should propagate less than one element size in one time step. Due to the nonuniform meshes, the smallest size should be used as a criterion for evaluating the time step which is 0.2 ns for this study.

In order to describe the analytical results stemming from the finite element models, the modeled tooth in the four case studies shown in Fig. 1 is presumed to be generally isotropic (i.e., all individual layers, the enamel, the dentin, the pulp, the gold “crown”, and the amalgam “restoration”, are independently isotropic).

Four sets of numerical tests were performed using the finite element code and phantom tooth. The finite aperture transducer was assumed to be moving along the surface of the tooth capturing reflected signals at the six different locations (Fig. 1(a)). The finite element program provided a set of data representing the displacements in the axial (Z) and the radial (R) directions. A-scan signals (Fig. 2) (at the aforementioned six locations) as well as 3-D visualizations (Fig. 4) of the ultrasonic wave propagation resulted from the simulation.

Fig. 2 shows a typical set of data depicting a comparison of A-scan signals in the axial direction measured at location “D” (Fig. 1) for all four cases. All A-scan signatures at all six locations are collected over a time span of 4 μsec. Analyzing the three peaks (E, D, and P) in the A-scan signature of the three-layer model in Fig. 2, one can determine the round-trip time between every two adjacent peaks. Note that the peaks correspond to reflections of the ultrasonic wave off the surface of the enamel layer (E), the surface of the dentin layer (D), and the surface of the pulp cavity (P). Here, the round-trip time between peaks E and D and between peaks D and P is approximately 0.65 μsec and 1.4 μsec, respectively. The enamel-dentin and the dentin-pulp layers are calculated to be approximately 2 mm and 2.66 mm, respectively; which matches closely the distances between these layers determined from Fig. 1(a) for the three-layer model. Similar calculations can be used to determine the thickness of gold, cavity, and amalgam layers from the A-scans of the other tooth models in Figs. 1(b) through 1(d).

Typical 3-D snapshots of the Z-displacement field with the 2 mm diameter aperture transducer are shown in Fig. 4 for case I shown in Fig. 1. All individual snapshots are taken at 1 μsec, 2 μsec, 3 μsec, and 4 μsec. This time span was determined to be sufficient for acquiring full-field information about the conduct of the ultrasonic wave as it propagates from the surface of the tooth down to the pulp cavity. At first glance, when examining these 3-D visualizations of the ultrasonic fields, they may seem too complicated to understand. Actually, deciphering and interpreting the images can be difficult. For this reason, a classification of the wavetypes is useful.

Table I. Material properties of tooth and restoration layers.

<table>
<thead>
<tr>
<th>LAYER</th>
<th>( V_l ) (m/s)</th>
<th>( V_s ) (m/s)</th>
<th>( \rho ) (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 (enamel)</td>
<td>6250</td>
<td>3100</td>
<td>3000</td>
</tr>
<tr>
<td>M2 (dentin)</td>
<td>3800</td>
<td>1900</td>
<td>2000</td>
</tr>
<tr>
<td>M3 (pulp)</td>
<td>1570</td>
<td>800</td>
<td>1000</td>
</tr>
<tr>
<td>GD (gold)</td>
<td>3240</td>
<td>1200</td>
<td>19700</td>
</tr>
<tr>
<td>MG (amalgam)</td>
<td>4350</td>
<td>2260</td>
<td>7750</td>
</tr>
</tbody>
</table>
Figure 1. FEM discretization of the tooth phantom: (a) Three-layer model, (b) Tooth model with Gold “crown” layer, (c) Tooth model with Amalgam “restoration”, (d) Tooth model with “cavity” at enamel-dentin interface.
Figure 2. Comparison of A-scan signals (Z-Displacement) measured on surface of tooth for all four cases shown in Fig. 1.

Figure 3. Descriptive illustration showing all five waves present in the tooth phantom.
Figure 4. 3-D visualization of ultrasonic wave (Z-Displacement) in the three-layer tooth model; (a) Wave after 1 μsec, (b) Wave after 2 μsec, (c) Wave after 3 μsec, (d) Wave after 4 μsec.
In this study, five types of waves emanating from the finite aperture transducer can be identified:

1. Longitudinal wave
2. Edge longitudinal wave
3. Edge shear wave
4. Head wave
5. Surface wave

A descriptive concept of these waves is illustrated in Fig. 3. The longitudinal and the shear waves are considered to be plane waves which travel together with the longitudinal wave always leading the shear wave because $V_L > V_S$. The edge longitudinal and the edge shear waves are generated by and due to the "edge" of the finite aperture transducer. These two waves travel at almost the same speed as the plane longitudinal and plane shear waves, respectively. In fact, these edge waves are sometimes difficult to separate from the plane waves. The surface wave propagates along the surface of the tooth. This wave plays a very important role when surface cracks and defects are present. The head wave usually connects the longitudinal and the shear waves. In our model, this head wave is not as prominent as the other four waves due to the geometry of the tooth. However, it would be more noticeable if the geometry were larger. Transmission, reflection, and refraction of the five waves produces yet another set of waves as they propagate and interact with the internal complex features of the tooth. One major physical aspect responsible for this behavior is mode conversion. Consequently, information-carrying "background noise" in the insonified medium increases due to the collective presence of all these waves.

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REFERENCES