A SELF-COMPENSATING TECHNIQUE FOR THE CHARACTERIZATION OF A LAYERED STRUCTURE

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INTRODUCTION

The characterization of a layered structure has been considered by many investigators. A variety of techniques has been introduced. Among these normal incidence longitudinal waves can be applied in the low frequency regime, and the transmitted and/or reflected signals can be used for the evaluation of a layered medium [1, 2]. In the technique discussed in Ref. [1], a reference signal has to be introduced in order to calibrate the evaluation. In recent work by the present authors, a self-compensating technique has been proposed to evaluate a layered structure, in that a ratio of transmission and reflection coefficients has been used [3]. Applications using a self-calibrating or self-compensating technique for the measurement of surface waves reflected or transmitted by a surface-breaking crack have been reported in Refs. [4, 5]. The major advantage of the technique is that the evaluation is independent of the characteristics of transmission and reception of ultrasound by the transducers, the attenuation in the couplant and the surface condition of the specimen. The technique is not only applicable for a contact configuration but an immersion configuration as well.

In order to effectively deduce the parameters for a layered structure, the sensitivity of the ratio of transmission and reflection coefficients of a layered structure to variation of the parameters and the distribution of minima in an error surface have been studied. The study finds that for parameters containing a strongly coupled pair, for example, the thickness and phase velocity from the same layer, the true minimum is located in a valley, in an insensitive direction in which the sensitivity of the ratio function to parameter change vanishes. To simplify the inverse problem two kinds of constraint conditions have been proposed for the determination of the unknown parameters. These constraints reduce the search of the true minimum to the insensitive direction only. The global minimum is then obtained with greater ease and accuracy.

SELF-COMPENSATING TECHNIQUE

A schematic of the self-compensating configuration is shown in Fig. 1. A layered plate is immersed in water and a transducer is placed at each side of the plate. In the procedure that is followed, transducer 1 is fired first. In the frequency domain, the voltage signal received at 2 can be expressed as

\[ V_{12} = A_1 D_{10} T_{1n} D_{02} S_2, \]  

(1)
where $A_1$ is the response function for transmission of transducer 1, $D_{10}$ is the attenuation over the distance from 1 to 0, $T_{in}$ is the transmission coefficient of the plate from 0 to $0'$, $S_2$ is the response function for reception of transducer 2, and $D_{02}$ is defined analogously to $D_{10}$. The reflected signal received by transducer 1 may similarly be expressed as

$$V_{11} = A_1 D_{10} R_{11} D_{01} S_1,$$

where $R_{11}$ is the reflection coefficient of the plate, $D_{01}$ and $S_1$ are defined analogously to $D_{10}$ and $S_2$. When transducer 2 is fired, the signals received by transducers 1 and 2 can be expressed in analogous forms.

Now, considering $V_{12} V_{21} / V_{11} V_{22}$, a ratio function of the transmission and reflection coefficients can be obtained as

$$R(\omega, r) = \frac{T_{in} T_{n1}}{R_{11} R_{nn}} = \frac{V_{12} V_{21}}{V_{11} V_{22}},$$

where $r$ is a vector in the parameter space of the layered medium and $\omega$ is the circular frequency.

The transmission and reflection coefficients of a layered medium, when the incident source is placed at side 1, have been derived in Ref. [3] as

$$T_{in} = \frac{2i \omega (M_{12} M_{22} - M_{12} M_{21})}{(M_{21} - 2z_0^2 \omega^2 M_{12}) + i \omega (M_{11} + M_{22})},$$

$$R_{11} = -\frac{(M_{21} + z_0^2 \omega^2 M_{12}) + i \omega (M_{11} - M_{22})}{(M_{21} - z_0^2 \omega^2 M_{12}) + i \omega (M_{11} + M_{22})},$$

where $z_0$ is the acoustic impedance of the couplant and $M_{ij}$ are the components of the total stress-displacement transfer matrix of the layered medium which may be derived as

$$[M] = M_n M_{n-1} ... M_j M_{j-1} ... M_3 M_2 M_1,$$

where

$$M_j = \begin{bmatrix} \cos(k_j h_j) & \frac{1}{z_j \omega} \sin(k_j h_j) \\ -z_j \omega \sin(k_j h_j) & \cos(k_j h_j) \end{bmatrix}.$$
Here \( z_j, k_j \) and \( h_j \) are the acoustic impedance, the circular wave number and the thickness of layer \( j \).

The ratio function is a complex function containing both amplitude and phase information. Since it depends on the parameters of the layers and the ultrasonic signal only, the relation between these parameters using Eq. (3) is independent of the characteristics of transmission and reception of ultrasound by the transducers and the condition of the couplant. Hence Eq. (3) is called self-compensating.

**EXPERIMENT**

The experiment setup is shown in Fig. 2. A matched pair of Panametrics broadband piezoelectric transducers with center frequency 7.5 MHz was used as transmitter and receiver. For the ratio function \( R \) of the layered medium, both transmitted and received signals from each side of the medium have to be collected as shown in Eq. (3). Therefore, the alignment of the two transducers is very important. In this experiment, a specially designed fixture was used to keep the two transducers perfectly aligned. The position of the line between the transducer pair can be kept perpendicular to the specimen by adjusting a rotation and tilt stage. A 3-layered plate of aluminum/epoxy/aluminum with thickness 0.470/0.471/0.757 mm was constructed as a specimen. The plate was constructed by using two aluminum plates separated by a feeler gauge along with a specially designed fixture, and epoxy was placed into the gap in-between the two aluminum plates. The area of the aluminum plates is 89 \( \times \) 127 mm\(^2\). The phase velocity and the density of aluminum plates and epoxy were measured as 6.335 (km/sec), 2.573 (km/sec) and 2.712 (g/cm\(^3\)), 1.2 (g/cm\(^3\)), respectively.

The data acquisition procedure is as follows: a short duration pulse, generated by a Panametrics Ultrasonic Analyzer mode 5600 and attenuated 10 dB by an attenuator, is distributed by a switch box to one of the transducers. The reflected and transmitted signals from the specimen are received by both transducers, and amplified 34 dB by a Panametrics Preamplifier one after another through the switch box. The signals are then sent to a Tektronix TDS 540 oscilloscope where they are digitized into 5000 points with a sampling interval of 4 ns. The digitized signals are then acquired by a personal computer through a GPIB interface. The reflected and transmitted signals from the other side of the specimen are obtained in the same manner. The four time domain signals are transferred into the frequency domain by using the FFT to obtain the spectrum of the experimental ratio function in both amplitude and phase. A comparison of the measured and theoretical ratio function is shown in Fig. 3. The peaks in the amplitude spectrum correspond to the layer resonance frequencies. Away from resonance the comparison is excellent. The comparison of the phase curves is good, the error is less than \( \pm 0.15 \) radius (slightly more at resonance).

![Fig. 2 A schematic of the experiment setup.](image)
A peak in the amplitude spectrum generally corresponds to the resonance frequency of a single layer, even though it may be shifted a little from the value for single layer plate due to mode coupling.

**INVERSE PROBLEM**

**Sensitivity Analysis**

Let $\mathbf{r} = (x_1, x_2, x_3, \ldots, x_{n-1}, x_n)$ represent a vector in the parameter space of the ratio function $R$. The sensitivity $S_r$ of $R$ can then be written as the derivative along the direction of an incremental change $\Delta \mathbf{r}$ as

$$S_r = \lim_{\Delta \mathbf{r} \to 0} \frac{\Delta R/R}{|\Delta \mathbf{r}|} = \sum_{i=1}^{n} \left( \frac{1}{R} \frac{\partial R}{\partial x_i} \right) \frac{\partial R}{\partial x_i} \frac{\partial \mathbf{r}}{\partial \mathbf{r}},$$

where $\partial r/\partial x_i$ is the cosine of the angle $\theta$ between $\Delta \mathbf{r}$ and $\Delta x_i$. The sensitivity of the ratio function can be studied with respect to one parameter as well as several or all parameters.

The sensitivity of $R$ versus frequency and direction angle $\theta$ is shown in Fig. 4. The parameters are the thickness and the phase velocity of layer 2. Figure 4 shows that the sensitivity is high at resonance of the layer and low at other frequencies. The sensitivity with respect to the thickness only ($\theta=0$) at resonance is orders higher than that with respect to the phase velocity only ($\theta=90$). The sensitivity peak decreases when the angle changes from the thickness toward the phase velocity. At $\theta=79.6$, the sensitivity vanishes. This angle defines the direction that the tangent of $\theta$ is equal to the ratio of the two parameters, i.e., $\tan(\theta) = c_2/h_2$. Since this ratio is equal to twice the fundamental resonance frequency of the single layer, i.e., $f_0 = c_2/(2h_2)$, there is no appreciable change of the resonance frequency when the two parameters change in this direction. There always exists a direction in which the sensitivity vanishes when the two relevant parameters are the thickness and the phase velocity of the same layer. We call the direction an insensitive direction and the pair of parameters a strongly coupled pair.

Sensitivities with respect to parameters containing no strongly coupled pairs have also been calculated. The difference is that there is no zero sensitivity point even though the magnitude of the peaks decreases significantly at some points. In the example shown in Fig. 5 the thicknesses of layer 1 and layer 2 are taken as variables. In general, an insensitive
Fig. 4 The sensitivity of $R$ versus frequency and angle $\theta$, where the parameters are the thickness and the phase velocity of layer 2.

Fig. 5 The sensitivity of $R$ versus frequency and angle $\theta$, where the parameters are the thicknesses of layer 1 and layer 2.

direction occurs only when the parameters contain a strongly coupled pair. For the case of more than one strongly coupled pair, the insensitive directions cannot be shown visually. However, it is possible to make a numerical analysis to determine those directions.

**Error Surface**

The objective of an inverse algorithm generally is to locate the true minimum in an error surface in the parameter space of a layered plate. An expression for the error is

$$E(r) = \frac{1}{N} \sum_{i=1}^{N} \left| R_E(\omega_i) - R(\omega_i, r) \right|^2,$$

where $R_E$ is the amplitude of the measured ratio function and $R$ is the amplitude of the theoretical ratio function. To study the distribution of minima, the error surface was calculated by taking the parameters in the theoretical ratio function to differ 20% from the true values. To contrast the true minimum, $1/E$ is plotted. Hence, the maximum in a displayed surface corresponds to a minimum in an error surface.

Figure 6 shows a case of a strongly coupled pair, where the thickness and the phase velocity in layer 2 are the parameters of interest. A valley exists in the insensitive direction in the error surface. Since it is positioned in the valley, the true minimum may not be easily located. Figure 7 shows a case when no strongly coupled pair is present, the thickness of
layer 1 and the velocity of layer 2 are the parameters. There are two minima in the error surface. The true minimum can be obtained by choosing the values with smaller error.

In summary, for a case with no strongly coupled pair the minimum can be located easily. For parameters containing a strongly coupled pair, however, a search for the true minimum may be difficult due to the presence of the valley in the insensitive direction. The reason is that changes of phase velocity and thickness in the insensitive direction do not significantly affect the resonance frequency of the layer. However, the peak values of the amplitude spectrum at resonance may depend on the velocity. Therefore, even though the resonance frequency keeps the same value when the velocity and thickness change in the insensitive direction, the magnitude of the peak will change (the change is not as significant as when the velocity and thickness are changed in other directions). Only the values that make both $R_E$ and $R$ match not only the frequency but the amplitude as well are true values. This argument was confirmed by the search of minima along the insensitive direction shown in the next section.

**Constraint Equation**

A time constraint, which relates the time difference of first-arrival of the transmitted and reflected signals from a plate to the time of flight in the plate, may be stated as

$$\Delta t = (t_{1}^{f} + t_{2}^{r}) - (t_{1}^{t} + t_{2}^{r}) = 2 \sum_{j=1}^{n} \frac{h_{j}}{c_{j}}, \quad \text{(10)}$$
where \( t \) is the arrival time, the superscripts "t" and "r" represent the transmitted and reflected signals, and the first and second subscript number represents the transmitting and the receiving transducer. The inverse process has been conducted by using both the error equation and the constraint, i.e., all parameters evaluated in an error surface have to satisfy the constraint equation. For instance, when the thickness and the phase velocity of layer 2 are variables, for a given thickness, the velocity has to satisfy Eq. (10). Therefore, the introduction of a constraint for the error equation reduces the number of unknown parameters in the search.

A resonance constraint can also be introduced to simplify the search for the true minimum along the insensitive direction. In order to use the resonance frequency, the first and second derivative of \( R \) with respect to frequency at the resonance frequency have been taken as a constraint to search for a strongly coupled pair in the insensitive direction, i.e.,

\[
\frac{\partial}{\partial f} R(f_0, t) = 0 \quad \text{and} \quad \frac{\partial^2}{\partial f^2} R(f_0, t) < 0,
\]

where \( f_0 \) is the measured resonance frequency of the layer. Equation (11) is the condition for a maximum at the resonance frequency. For a given unknown parameter, such as the thickness, another parameter, such as the phase velocity, can be obtained as the solution of Eq. (11).

For the previous case with a strongly coupled pair the error along the insensitive direction has been recalculated but with the time and resonance constraints, respectively. The error curve in the insensitive direction and the error curves obtained with the two constraints in the error surface are shown in Fig. 8. The results show that the error converges well around the true value for both constraints.

Results

The Downhill Simplex method for multi-dimensional minimization was applied in the search for the minimum in an error surface. The determination of two variables has visually been presented. Determination of three parameters has also been performed. All results and the corresponding relative errors are listed in Table I. The determined unknown parameters for the plate agree well with the true values. The maximum error is less than 2% for the determined thicknesses and velocities.

![Fig. 8 The error curve in the insensitive direction and the error curves obtained by two constraints.](image-url)
Table I The determined values and the relative error

<table>
<thead>
<tr>
<th>Plate</th>
<th>$h_1$ (error%)</th>
<th>$c_1$ (error%)</th>
<th>$h_2$ (error%)</th>
<th>$c_2$ (error%)</th>
<th>$h_3$ (error%)</th>
<th>$c_3$ (error%)</th>
</tr>
</thead>
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<tr>
<td>$h_1,h_2$</td>
<td>0.466 (.83)</td>
<td>0.473 (.64)</td>
<td>2.55 (.78)</td>
<td>2.55 (.78)</td>
<td>6.387 (.82)</td>
<td>0.473 (.64)</td>
</tr>
<tr>
<td>$h_1,c_2$</td>
<td>0.466 (.83)</td>
<td>0.474 (.85)</td>
<td>2.54 (.12)</td>
<td>6.37 (.55)</td>
<td>6.374 (.62)</td>
<td>0.474 (.85)</td>
</tr>
<tr>
<td>$c_1,h_2$</td>
<td>2.55 (.78)</td>
<td>0.755 (.2)</td>
<td>6.347 (.2)</td>
<td>6.347 (.2)</td>
<td>6.37 (.55)</td>
<td>6.374 (.62)</td>
</tr>
</tbody>
</table>

CONCLUSION

A self-calibrating technique for the characterization of a layered medium has been developed by using the ratio of the reflection and transmission coefficients. The ratio function is self-compensated, in that it is independent of the characteristics for transmission and reception of ultrasound by the transducers and the condition of the couplant. Unknown parameters have been obtained by an inverse method. Using the sensitivity equation, the sensitivity of the ratio function has been studied. The insensitive direction has been defined as the direction in which the sensitivity of the ratio function to parameter changes vanishes when the parameters form a strongly coupled pair. A valley along the insensitive direction in an error surface contains the true minimum. The introduction of a constraint equation reduces the search for the true minimum to the insensitive direction only. The global minimum is then obtained with greater ease and accuracy. The results show that the method can be effectively applied for the characterization of a layered structure.

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REFERENCES