ADHESIVE JOINT EVALUATION USING LAMB WAVE MODES WITH APPROPRIATE
DISPLACEMENT, STRESS, AND ENERGY DISTRIBUTION PROFILES

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INTRODUCTION

One of the most elusive yet critical problem in adhesive joints
characterization is that of 'kissing bond' wherein good contact exists
among the adherend and the adhesive, however with no acceptable levels of
adhesion. To date, the kissing bond is difficult to be detected reliably
by any of the methods including conventional ultrasound and thermal
waves. Kissing bond which is a manufacturing defect/anomaly will
substantially compromise the load bearing capability of the adhesive
joint by initiating adhesive failure (in contrast to cohesive failure
wherein the failure occurs within the thickness of the adhesive layer
instead of a failure at the interface). Attempts to develop methods of
detection of kissing bonds have been unsuccessful to date.

BACKGROUND INFORMATION

Lamb waves or plate waves are generated by an obliquely incident
transducer transmitting ultrasonic waves to interrogate layered
structures such as adhesive joints, composites, etc. The plate waves
travel along the layered structure in many modes of propagation. Each
mode generated in the structure is dependent on the combination of
wavelength to thickness ratio and the interfacial conditions. The
experimental setup usually consists of two transducers arranged in a
pitch-catch arrangement.

The null zone method of detection of Lamb modes has been
traditionally used by many researchers to monitor the existence of Lamb
waves in the layered structure [1]. Another method of implementation
would be positioning the receiver in the leaky region that exists beyond
the null zone and the specular reflection regions. In either
implementation, the Lamb wave generation and propagation are sensitive to
the overall layered structure geometry as well as the boundary conditions
at the interface among the adherends and the adhesive layers. Therefore,
many researchers have reported successful implementation as discussed in
the next few paragraphs.

Several researchers have studied the use of Lamb waves for the
characterization of adhesive joints. The results of these research
efforts have been documented by Nagy, Rokhlin, Mal, Rose, Pilarski, and
others in many journal and conference publications. Nagy and Adler [2]
outlined nondestructive evaluation of adhesive joints by guided waves.
They proposed interface waves traveling into the adhesive layer as well
as the interfaces as an alternative approach to adhesive joint characterization. Rokhlin et al., [3, 4] have applied the use of Lamb waves through measurement of phase delay and transmission losses. They proposed the mode selection and optimization on an adherend outside the adhesive joint to optimize the modes in the adhesive joint itself. Mal et al. and Xu et al. [5, 6] studied the correlation between the dispersion of Lamb waves and the interfacial properties of Lamb waves. They reported that the correlation was ‘quite strong’ and was identifiable in at least the laboratory specimens. They further outlined an inversion scheme based on an iterative least-squares procedure to determine the cohesive properties of the adhesive bonds from the Lamb wave dispersion data.

Rose et al. and Pilarski et al. have conducted several studies involving various ultrasonic techniques to assess adhesive joints. They have reported [7-9] their research wherein the Lamb wave modes were propagated through the adhesive structure. They applied feature extraction and mapping techniques to extract information from Lamb wave amplitude, velocity, and frequency and correlated these features to the bond quality. Rose et al. [10, 11] further proposed an approach to guided wave mode selection for inspection of laminated plates wherein they discussed concepts of mode selection based on field distributions of each mode as a function of thickness of the plates. They discussed possibilities of using energy distribution, stress distribution, and normal as well as in-plane displacement distributions for the evaluation.

LAMB WAVE MODE SELECTION BASED ON FIELD DISTRIBUTION IN ADHESIVE JOINTS - ANALYSIS

Development of NDE methods for characterizing the stiffness and ‘quality’ of adhesive joints is critical to maintainability for aging systems applications. However there are significant technological challenges for the development of the specialized NDE methods because of the need for extreme sensitivity to subtle changes in localized properties of adhesives and the adhesive joints. For example, the ability to detect ‘poor’ adhesion of adhesive bonds cannot be achieved without a method of assessing small changes in bond interfacial behavior as well as changes in adhesive mechanical properties.

Many researchers have used Lamb waves in the past for the evaluation of adhesive joints. However, limited work has been done to date regarding the selection of Lamb waves based on the stress profile and/or displacement profile of various Lamb Wave modes for maximum sensitivity to the interfacial conditions. It is logical that when Lamb waves are selected for the inspection, it will be critical to select a mode which has amplitude/stress/energy profile suitable for the intended evaluation. Such a selection is possible only with a thorough Lamb wave dispersion and modal analysis.

Modeling of Lamb Waves for Mode Selection Using Field Distributions as the Criteria

Rose et al. and Pilarski et al. [10, 11] have provided the steps necessary for modeling with welded as well as smooth boundary conditions for cases using either Lamb waves or surface waves (surface waves, interfacial waves, and Lamb waves will exist when one of the adherends is a semi infinite plate compared to the wavelength as in the case of a thin composite patch on an underlying thick aluminum plate). However, in the present study, since only metal-to-metal samples are considered with similar plate thicknesses and with a geometry of two adherends with one layer of adhesive in-between, the modeling to be presented herein will be specific for this geometry.

The configuration of interest in this paper consists of three homogeneous isotropic, linearly elastic layers perfectly bonded along their two common interfaces. Only in-plane particle motions are considered; a condition which is permitted since the horizontal shear modes will not couple to the Lamb-type modes in such a geometry. In addition, because of the great numerical simplification which it affords, the composite structure is treated as ‘traction free’ at its uppermost and lowermost surfaces, even though the experimental results to be presented later were taken on metal plates immersed in water.

In each of the layers, the particle displacement field is assumed to satisfy the governing differential equations,
where the superscript \( n \in \{1, 2, 3\} \) is used here and throughout to denote the fields within, or properties of, layer \( n \). In writing Eqn (1), it has been assumed that the displacement field (and consequently all other fields) vary with time as \( e^{i\omega t} \).

Equation (1), which represents a pair of coupled equations for the two non-vanishing displacement components, is most conveniently solved by making use of the Helmholtz decomposition, \( u'(x) = \nabla \phi'(x) + \nabla \times (\nabla \times \phi'(x)) \), where the two potentials, \( \phi(x) \) and \( \psi(x) \) are yet to be determined, and \( \delta \) represents a unit vector in the \( x \) (perpendicular to propagation) direction. The particular form chosen for the vector potential is consistent with the requirement that \( u_y = 0 \) for the modes of interest. This representation for \( u \), when substituted into Eqn (1), results in separate, uncoupled wave equations for the scalar and vector potentials.

Because of the assumed homogeneity of the structure, the coefficients in these equations are constants, and plane wave solutions are possible. Thus, the solutions for the potentials in each layer can be expressed in the form:

\[
\begin{align*}
\phi^n(y,z) &= C^n_1 e^{i(k_y z + k_z z)} + C^n_2 e^{i(k_y z - k_z z)} \\
\psi^n(y,z) &= C^n_3 e^{i(k_y z + k_z z)} + C^n_4 e^{i(k_y z - k_z z)}
\end{align*}
\]  

(2)

where, \( (k^n_y)^2 = \left( \frac{\omega}{\nu^n_y} \right)^2 - k^2 \) and \( (k^n_z)^2 = \left( \frac{\omega}{\nu^n_z} \right)^2 - k^2 \)

(3)

and \( C^n_1, C^n_2, C^n_3, \) and \( C^n_4 \) are at this point, 4x3 - 12 arbitrary complex constants. \( \nu^n_y = [4 \mu^n / \rho^n]^{1/2} \), \( \nu^n_z = (\mu^n / \rho^n)^{1/2} \) represent the bulk longitudinal and shear wave velocities, respectively, in layer \( n \). Note that the "z" component of wavenumber, \( k_z \), is equal for each term in (2), although this is not necessary for (2) to satisfy (1). It will, however, be required later when trying to satisfy boundary conditions, and it is therefore assumed at the outset.

Having expressions for the potentials, one can calculate all other field components. In particular, the rectangular Cartesian components of the displacement vector and stress tensor can be expressed in the form:

\[
\begin{align*}
u^n_i &= \frac{\partial \phi^n}{\partial x_i} + \epsilon_{imn} \frac{\partial \psi^n}{\partial x_m} \\
T_{ij} &= \left\{ \lambda^n \delta_{ij} \frac{\partial^2}{\partial x_i \partial x_j} + 2 \mu^n \frac{\partial^2}{\partial x_i \partial x_j} \right\} \phi^n + \mu^n (\epsilon_{imn} \delta_{ij} + \epsilon_{imn} \delta_{ji}) \frac{\partial^2 \psi^n}{\partial x_m \partial x_n}
\end{align*}
\]

(4)  

(5)

where \( \epsilon_{imn} \) is the Levi-Civita (alternating) tensor, \( \delta_{ij} \) represents the Kronecker delta, and summation is implied over all repeated subscripts. In deriving Eqn. (5), use has been made of the fact that only the \( x \) (or \( x \)) component of the vector potential \( \Psi \) is non-zero, and this component has been abbreviated simply as \( \Psi \).

As can be seen in Eqns (4-5), the displacements and stress components are linear combinations of the potentials and their derivatives, and hence, referring to Eqn. (2), it will be seen that the constants \( C^n_1 \) through \( C^n_4 \) will also appear linearly in the resulting expressions for the displacement and stress components. It can also be noted that the only non-vanishing components of Eqns (4)-(5) will be \( u_y, u_z, T_{xx}, T_{yy}, T_{zz}, \) and \( T_{yz} \).

To complete the specification of the problem, boundary and interface conditions must be imposed on the fields. In the problem of an unloaded, multilayered plate, the upper and lowermost surfaces must be traction free, i.e.,
where \( y = a \) denotes the upper surface of the layer stack and \( y = H \) represents the lower surface. In addition to the boundary conditions, the requirement of continuity of traction and displacement at each interface within the layer stack is imposed, i.e.,

\[
\begin{align*}
    u_1(h^m) &= u_1(h^m) \\
    u_z(h^m) &= u_z(h^m) \\
    T_{yy}(h^m) &= T_{yy}(h^m) \\
    T_{yz}(h^m) &= T_{yz}(h^m)
\end{align*}
\]  

where \( h^m \) represents the thickness of layer \( m \), and the plus and minus signs are used to symbolically indicate fields just above and just below the interface \( y = h_m \). For an \( N \)-layered medium, there would thus be \( 4xN \) unknowns and a corresponding \( 4xN \) equations, with \( 4x(N-1) \) of the equations coming from the interfaces, and the remaining \( 4 \) equations coming from the upper and lower surfaces.

For the three-layered problem considered herein, imposition of the conditions (6) through (7) results in a set of 12 equations in 12 unknowns which can be written symbolically in the form,

\[
\begin{bmatrix}
    A_{1,1} & A_{1,2} & \cdots & A_{1,12} \\
    A_{2,1} & A_{2,2} & \cdots & A_{2,12} \\
    \vdots & \vdots & \ddots & \vdots \\
    A_{12,1} & A_{12,2} & \cdots & A_{12,12}
\end{bmatrix}
\begin{bmatrix}
    C_1 \\
    C_2 \\
    \vdots \\
    C_4
\end{bmatrix} =
\begin{bmatrix}
    0 \\
    0 \\
    \vdots \\
    0
\end{bmatrix}
\]  

(8)

with each of the coefficients, \( A_{i,j} \), in general, being complex-valued functions of frequency \( \omega \), wavenumber \( \kappa \), and the geometric \( (h) \) and material \( (\mu, \lambda) \) properties of each layer.

For non-trivial solutions to exist, it is required that the determinant of the coefficient matrix vanish, i.e.,

\[
\text{det}[A_{i,j}(\omega, \kappa, h^m, \mu, \lambda)] = 0.
\]  

(9)

For a given choice of frequency, there will be an infinite number of wavenumbers which satisfy (9), a finite number of which will be real, and the rest imaginary/complex. It should be noted that the complex wavenumbers do not correspond to modes which loose energy as they propagate, since no losses have been modeled into the problem. For these modes, it can be shown [12] that the particle velocity and stress fields are such that the net power flow in the propagation direction is everywhere zero.

In general, for each of the frequency-wavenumber pairs, \( (\omega, \kappa) \), which satisfy Eqn (9), one can determine 11 of the 12 unknown \( C \)'s, in terms of the twelfth arbitrarily chosen constant. This results from the fact that, by definition, permissible \( \omega-\kappa \) pairs cause the determinant of the coefficient matrix to vanish. Because of the homogeneous nature of the problem (i.e., no loads have been prescribed) and the resulting homogeneous set of equations, only the relative values of the field variables can be determined, not absolute values.

**Field Distributions**

Having determined the unknown constants, \( C \)'s, in Eqn (9), one can proceed to calculate the modal fields of the Lamb waves for any permissible \( \omega-\kappa \) pair. Explicit expressions for the displacements and stresses can be found by carrying out the operations indicated in Eqns (4) and (5), followed by substitution of the \( C \)'s obtained from solution of (8). Again, it must be kept in mind that the fields so obtained can only be compared to each other at the given - and \( k \), since they all
contain an arbitrary, frequency-dependent, multiplier. Because of this, in all of the plots to follow, the fields were normalized to unity maximum. Since only the longitudinal (i.e., the component along the propagation direction) component of displacement is plotted, it was independently normalized to unity maximum.

The stresses in each of the stress graphs were normalized by dividing all components appearing in that graph by the same normalizing factor so that the maximum value of any one of the stresses was unity. Of course, the other components would then, in general, have maxima less than one, depending on the nature of the Lamb wave mode. This normalization was done independently for each mode in the figures to be presented later in this paper.

Finally, the projection of the time-averaged Poynting (power flux) vector in the direction of propagation was also calculated [12]:

\[ P_{\text{d}}(y) = -\Re \left[ \frac{v(y) \cdot \mathbf{T}(y) \cdot \mathbf{e}^*}{2} \right] \quad (10) \]

where \( v \) represents the particle velocity field (i.e., \( v = \partial \mathbf{u}/\partial t \)), \( \mathbf{T} \), the stress field, and \( \mathbf{e} \), a unit vector in the \( z \) (along the plate) coordinate direction. \( \Re \) indicates the real part, and the asterisk denotes complex conjugation. These functions have the property that for propagating modes,

\[ P = \int_{0}^{\infty} P_{\text{d}}(y) dy \quad (11) \]

equals the power transported by the mode along the layer per unit length (perpendicular to the propagation direction) and per unit time [12].

A physical basis for expecting these functions to be indicators of how strongly a mode would be scattered is based on the assumption that the amount of energy scattered from a defect will be proportional to the amount of energy incident on it. Therefore, a mode with large energy flux at a specific location would be sensitive to a defect at that location and a mode with zero energy flux at a specific location should not be sensitive to defects located there. In a very specific instance, this assertion is discussed and proven in literature [12]. Note that if either the modal displacement or stress field is identically zero at a specific location, the energy flux will be zero there as well.

RESULTS AND DISCUSSION

Two types of samples, one each of traditional aluminum-to-aluminum and copper-to-aluminum joints, were used for the experiments (Tables 1 and 2).

Table 1 Geometrical dimensions and material properties for the aluminum-epoxy-aluminum sample used for the experiments.

<table>
<thead>
<tr>
<th>Layer No.</th>
<th>Material</th>
<th>Longitudinal Velocity (m/s)</th>
<th>Shear Velocity (m/s)</th>
<th>Density (kg/m²)</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Aluminum</td>
<td>6348</td>
<td>3133</td>
<td>2700</td>
<td>1.6764</td>
</tr>
<tr>
<td>2</td>
<td>Epoxy</td>
<td>2610</td>
<td>1100</td>
<td>1170</td>
<td>0.1092</td>
</tr>
<tr>
<td>3</td>
<td>Aluminum</td>
<td>6348</td>
<td>3133</td>
<td>2700</td>
<td>1.5621</td>
</tr>
</tbody>
</table>

Table 2 Geometrical dimensions and material properties for the copper-epoxy-aluminum sample used for the experiments.

<table>
<thead>
<tr>
<th>Layer No.</th>
<th>Material</th>
<th>Longitudinal Velocity (m/s)</th>
<th>Shear Velocity (m/s)</th>
<th>Density (kg/m²)</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Copper</td>
<td>4857</td>
<td>2361</td>
<td>8537</td>
<td>1.7780</td>
</tr>
<tr>
<td>2</td>
<td>Epoxy</td>
<td>2610</td>
<td>1100</td>
<td>1170</td>
<td>0.0762</td>
</tr>
<tr>
<td>3</td>
<td>Aluminum</td>
<td>6348</td>
<td>3133</td>
<td>2700</td>
<td>0.5080</td>
</tr>
</tbody>
</table>
Figure 1 (a) Lamb wave image of the Al-epoxy-Al structure using asymmetric mode at 0.8773 MHz (Vph = 2.598 mm/μs). (b) Corresponding longitudinal displacement and power flux profiles. (c) Corresponding stress profiles.
Figure 2 (a) Lamb wave image of the Cu-epoxy-Al structure using Mode 2 at 1.023 MHz (Vph = 6.377 mm/μs). (b) Corresponding longitudinal displacement and power flux profiles. (c) Corresponding stress profiles.
Figure 3 (a) Lamb wave image of the Cu-epoxy-Al structure using symmetric mode at 0.8803 MHz (Vph = 3.063 mm/μs). (b) Corresponding longitudinal displacement and power flux profiles. (c) Corresponding stress profiles.

Sample Geometry, Properties and Dispersion Curves

Table 1 shows the geometry and properties of the aluminum-epoxy-aluminum joint used in the experiments. Since the plate thicknesses are not identical, the structure is asymmetric in nature. There are 12 modes of Lamb waves propagating in the structure in the frequency range up to 3.5 MHz. Table 2 shows the geometry and properties of the copper-epoxy-aluminum joint used in the experiments. Since the plate thicknesses and materials are not identical, the structure is asymmetric in nature. For the sake of simplicity, the structure is assumed to be nonattenuative. There are over 14 modes of Lamb waves propagating in the structure in the frequency range to 5 MHz.

Experimental Results - Ultrasonic Images

Dispersion curves for Lamb waves in the adhesive joints described in Tables 1 and 2 respectively were generated. Since the shape of the
distribution profiles for each mode (in-plane displacement amplitudes, stresses, and energy in specific) are variable from mode-to-mode as well as from frequency to frequency for a given mode, it is imperative that a mode at a specific frequency should be selected such that the amplitude/stress/energy profiles provide maximum sensitivity at the two metal-to-adhesive interfaces in the joint.

The first set of images/distribution profiles in Figures 1a through 1c are for the Al-epoxy-Al sample. The second set of images in Figures 2 and 3 are from the Cu-epoxy-Al sample. The samples were scanned with various modes generated by varying either the frequency, angle, or both. The modes were selected based on the magnitude of amplitude/stress/energy at the two interfaces.

Discussions

The image in 1a has been obtained from the Al-epoxy-Al joint using the asymmetric mode with displacement amplitude, stress, and energy distribution profiles wherein there is similar sensitivity at both the interfaces (Figs. 1b and 1c). This fact has been further demonstrated by images using other modes possessing different distribution profiles. The image in 2a has been obtained by using a mode with close to zero displacement at one of the interfaces while there is a relatively large amplitude at the other thereby providing no sensitivity to one interface and high sensitivity to the other interface (Fig. 2b). Similar inference can be drawn from the power-flux profile where in the energy distribution is concentrated entirely within the lower layer (Fig. 2c). This is, therefore, a localized mode in the lower layer and this adds support to the statement that the lower interface was imaged. Figure 3a shows the same Cu-epoxy-Al sample imaged using the symmetric mode at a frequency and phase velocity such that there is some sensitivity at both the interfaces as seen from the various distribution profiles. Comparison of images 2a and 3a clearly show additional features in 3a which were not visible in Figure 2a. Since space limitations here makes it impossible to present all the experimental results, please refer to literature for further details [13].

SUMMARY AND CONCLUSIONS

Lamb wave mode selection has been based on displacement, stress, and energy distribution profiles across the thickness of the layered structure. Two adhesively bonded samples have been used to demonstrate the feasibility of the methodology. The results obtained in this study are indicative of the robustness, sensitivity, and hence the feasibility of the approach. It has been shown that by proper mode selection based on these distribution profiles, interfacial features from individual interfaces can be imaged.

ACKNOWLEDGEMENTS

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