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A similitude study of the draft of moldboard plows working submerged soils

Reynaldo Montecer Lantin

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REYNALDO MONTECER LANTIN 
1968
A SIMILITUDE STUDY OF THE DRAFT OF MOLDBOARD PLOWS WORKING SUBMERGED SOILS

by

Reynaldo Montecer Lantin

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of DOCTOR OF PHILOSOPHY

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Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

Iowa State University
Ames, Iowa
1968
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INTRODUCTION

In many areas where rice is grown, tillage of the soil is done when the water table (temporary or natural) is above the soil surface. In this condition the soil is nearly or completely saturated. This method of tillage, known as wet-land tillage, is commonly done in most of the tropical and sub-tropical rice-growing regions of Asia.

Some Reasons for Tillage of Submerged Soils

The tillage of submerged soils is practiced in most rice-growing areas because of the peculiar characteristics of rice itself, climate, nature of the soils, source of power and tradition. Since an abundance of water is a basic requirement for rice growing, farmers attempt to retain as much of it as possible in the rice paddy. This is especially true if irrigation facilities are lacking or inadequate and the water needs are supplied mainly by rainfall. Instead of draining the field and letting it dry to a moisture desirable for conventional tillage, the farmer proceeds with working the soil while it is saturated.

Poor drainage in some areas is also a reason for wet-land tillage. The land remains water-logged for a long time, especially during the monsoon season. In order to avoid unfavorable weather during the critical stages of plant growth or to avoid certain pests and diseases, the land has to be planted at certain dates. Also, because of seasonal characteristics of certain varieties of rice, the land has to be prepared for planting by some specific dates. Consequently, the farmer is forced to work
the soil while it is wet or submerged.

Even in areas where drainage is good and water control and irrigation are adequate, farmers still prefer to till wet soils because they have experienced that less force is needed to pull a plow in flooded soils than in dryer soils. In general, the rice soils are usually clayey and high in strength when very dry and previously puddled. Such soils, however, lose strength when soaked in water until saturated. Thus, tillage requires less energy when the soil is flooded than when it is dry. Since the common source of power in most rice-producing regions is the animal, an invaluable advantage is realized when the soil is worked wet.

One of the methods of weed control is submergence of the soil. Weeds are easily checked when plowed under the soil and covered with water. The paddy must be leveled for efficient flood irrigation. The soil is easily transported in standing water from higher to lower elevations even with the use of animal power. The water level itself is a convenient, all-inclusive datum plane for the leveling operation.

Need for Similitude Study

Although much of the area devoted to rice production is tilled under wet conditions, the development of tools for such process is insignificant compared with that for conventional dry tillage. Most of the current tools, whether designed for use with animal-power or with tractor power, were developed for dry-land tillage. They are being used with little or no modifications suitable for wet-land tillage. The development of an efficient set of tools for wet-land tillage would be a major contribution
to the welfare and progress of tropic and sub-tropic rice farmer.

An important step toward the development of such tillage tools is an understanding of the basic principles involved in soil-tool relationships. In the much-studied "conventional tillage", the knowledge of such principles has not yet been fully attained despite the advancement of several approaches and theories. The basic reason is the difficulty involved whenever soil is a contributing factor in the process. A number of investigators have attempted to relate the basic theory of soil mechanics to the tillage process. Analytical work has been done concerning soil-tool reactions, but the results are unsatisfactory when compared with field data. On the other hand, the approach of similitude theory, a tool in engineering developed and frequently used in fluid mechanics problems has also been used. Such an approach is justifiable in soil tillage research due to the still undeveloped understanding of the basic theory involved based only on purely analytical procedures.

The objective of using the principle of similitude in tillage research studies is to obtain relationships among the variables believed pertinent to the problem. Since draft of a tool is the power-consuming entity in a tillage process, much importance has been devoted to this in previous soil tillage studies.

Identification of the Problem

Although the researchers who have worked on the application of similitude to tillage processes have shown that the theory of models is, indeed, applicable to the phenomenon, several problems are still met in
arriving at conclusive results. The major problems are the identification of the pertinent variables, the measurement of these variables and the control of such variables during the experiment. For the tool, these variables may be relatively easy to point out, but, for the soil they are still a problem.

An approach to the problem of identification of pertinent variables has been the use of the knowledge of some theories in soil mechanics. A major deterrent to this approach is that the theories developed in soil mechanics may not be applicable to soil tillage where the system is dynamic. In spite of the deterrent mentioned, however, the previous researchers have found some evidence as to their applicability but have pointed out that results could yet be improved by development of suitable soil-variable measuring instruments.

An approach by Schafer (34) includes a process whereby the soil itself is worked in such a way that it will fit the desired conditions. These conditions are based upon the variables which have been chosen and the index which has been developed to obtain similarity of soil conditions under which the model and prototype would work. The index of similarity is, in itself, also a matter of choice and depends upon whether a suitable instrument for establishing such an index can be developed.

The difficulties met in past investigations seemed to center on the soil itself. Moisture content was one of the soil conditions or soil properties which was controlled. This was important because other soil properties considered, such as adhesion, cohesion, etc., generally change with moisture content. The proper control of moisture content requires
special techniques because variations are likely to occur even within a soil bin.

Schafer (34, p. 2) summarized the conclusions of previous investigators using similitude in soil tillage studies by saying that:

"1. Their list of pertinent soil variables was not complete,
2. They did not have good control of soil variables, and
3. They were unable to obtain accurate measurements of soil variables."

The problems involved in the use of similitude for prediction of draft of a plow working flooded soils are essentially the above. In this study, these problems were considered.
OBJECTIVES

In the light of the tillage method being considered and the results of previous workers in the field of application of similitude theory to tillage research, the following objectives were pursued:

1. To modify previous similitude techniques for applicability to the tillage of a submerged soil,

2. To develop a technique in similitude approach such that some soil properties are not necessarily evaluated and

3. To test the validity of the said technique in the way of reasonable prediction of the draft of a reduced-scale prototype moldboard plow working a submerged soil.
REVIEW OF LITERATURE

Prediction of the performance or behavior of a designed tool, whether it is a simple tine or one with complicated shape, is the ultimate objective of soil-tool investigations. From the prediction equations concerning the factors contributing to the behavior of the phenomenon, the tool may be redesigned, improved or modified or an entirely new one designed.

Basically, investigators of soil-tool systems have followed either one or a combination of three approaches towards prediction of tool behavior. These approaches included a) field and/or laboratory testing to gather data so that certain generalizations could be established based on repeated tests of similar and controlled conditions together with the analyses of results of such tests and previous experiences, b) use of certain laws and formulas which have been generalized over wide ranges of conditions and established by careful theoretical or analytical procedures and experimentation and c) use of the theory of similarity, that is, prediction of the behavior of the prototype from observations on the model which are known or designed to be governed by the same phenomenon as the prototype.

The first method of approach was used not only by several pioneers in the study of soil-tool relationships but also by recent investigators in the field. Early studies such as those by White (39), Davidson et al. (4), Collins (3), Nichols and Reed (22), Doner and Nichols (5), Reed (32) and Gordon (9) dealt with the relationships between tool geometry or shape, soil conditions and operational variables with the performance of
the plow. Either field or laboratory tests were conducted in order to isolate the factors and to develop generalizations from the results of tests. Attempts were made to define soil parameters, but, due to complex inter-relationships of factors, no definite conclusions were developed from the use of such soil parameters.

Recent studies using the testing and generalizing approach for prediction of performance of plows were done with more refined techniques, but the basic procedure was the same as in earlier studies.

Nichols et al. (23) studied the effects of variations in plow share design on the total draft of the plow. They found that the nature of the cutting edge of the share affected the entire behavior of the plow especially when the plow worked with sticky soils. Scouring was affected by the degree of bevel of the share because soil movement on the moldboard face was also affected. The result was increased pressure on the soil causing increased total draft.

Söhne (37) evaluated the performance of several high-speed plows of different shapes. He set up some plow body design requirements for satisfactory plowing at high speeds. He also proposed that investigation of scratch curves produced by the soil on the moldboard face would be useful in obtaining some insight into the processes taking place on the moldboard. This suggestion was later followed by O'Callaghan and McCoy (25).

Mayauskas (15) dealt with soil-tool reaction by measuring the pressure distribution on the surface of the plow share. While it added to the knowledge of soil-tool reaction, no generalized conclusion was made.

A definite conclusion was made on the fact that speed as an operational variable had an effect on the draft of the plow. This was found
to be true in the earlier studies as well as in the more recent ones like those of Caparrini (2) and Telischi et al. (38). Developed equations relating speed and draft, however, were not universal; moreover, these relationships depended upon soil conditions and other factors.

The second approach in prediction technique, that of the use of laws and formulas and verification by experimental results, was followed somewhat in some of the recent research on soil-tool systems. Such was the case, for example, in the development of theories regarding soil failure when a basic tool such as the tine was used. Attempts were made to apply the fundamental theories and equations used in soil mechanics to the development of such theories.

Payne (28) made a detailed analysis of the mode of failure of soil by narrow tines using a complicated failure pattern. Osman (27) used existing theories of failure for infinitely long retaining walls. He used Coulomb's solution for granular materials and the combination of logarithmic spiral and plane surface failure boundary. A similar approach was followed by Siemens et al. (35) using high speed movies to find the exact mode of failure. Results of the analysis showed that predicted values did not agree with the observed values of loads on the tine.

O'Callaghan and Farrelly (24) modified the failure hypothesis of Payne by considering the lower part of the tine as a "footing". They developed an equation for draft of narrow vertical tines in terms of soil characteristics and tool geometry. Good predictions were obtained as verified by field experiments in cohesive soils. Such favorable prediction, however, were not true for non-cohesive soils.
Sohn (36) investigated the application of soil-mechanics theory to agricultural engineering problems. He derived equations describing forces acting on simple blades.

While seemingly good results were obtained through the use of theories developed in soil mechanics as applied to tines, complications are to be expected when more complex tools are used. A disadvantage of the approach using equations and laws derived from the soil mechanics rests in the limited scope of applicability of such laws or formulas. Thus, Murphy (17, p. 637) comments:

"It seems to be in the nature of things that the available formula will usually not apply in the situation where it is needed."

When equations for soil-machine systems are developed from theoretical analysis, it is important that accurate values of soil and soil-metal parameters be used. The methods for obtaining such values should give consistent results for any one soil condition or type.

Osman (26) investigated the results obtained from a number of methods of evaluating the soil parameters, cohesion and angle of internal friction. The investigation involved use of instruments for making direct, compression and torsional tests in dry sand, wet sand and clay. The testing procedure included the use of the translational shear box, Bevameter, shear vane, weighted sand-coated annulus, sand-coated slider and the direct measurement of angle of repose in the case of sand.

Osman concluded that accurate measurements of values of soil strength parameters, cohesion and angle of internal friction were possible by means of a careful laboratory technique. Also, the values
could be measured independent of the method of testing.

In contrast with Osman's results, Dunlap et al. (7) found variations in values of cohesion and angle of internal friction with the use of several in situ soil measurement devices. Their work showed that soil measurement was influenced by the measuring device and that the intrinsic strength of the soil was not measured.

The devices used were the Desometer (a soil measuring device developed at the National Tillage Laboratory, Auburn, Alabama), the NIAE shear box, the Cohron sheargraph and the cone penetrometer. Five different grousered annuli were used with the Desometer.

Tests were done on Hiwassee sandy loam, Lloyd clay, Davidson clay and Norfolk sandy loam. The following conclusions were drawn by Dunlap et al. (7, p. 900):

"1. The three soil-strength measuring devices - sheargraph, grousered annulus, and NIAE (National Institute of Agricultural Engineering) shear box - did not give the same values of parameters C (cohesion) and Ø (angle of internal friction).

2. Relative to C, the sheargraph gave higher values, the NIAE shear box lower values, and the grousered annulus gave values between the two.

3. Accounting for relative displacement did not produce agreement of stresses measured geometrically with different grousered annuli in a single soil and soil condition.

4. The soil's nonuniformity with depth affected the magnitude of the measured shearing stress."

The third approach in prediction studies involves the use of similitude principles.

The principles of similarity and dimensional analysis are widely
used in many areas of engineering especially in problems involving fluid mechanics, heat transfer, applied mechanics, hydraulics and several others. The theory of similitude and its applications are discussed fully in the books of Langhaar (13) and Murphy (18). Model theory was only recently applied to important problems in agricultural engineering. Murphy (17) discussed some of the possible uses of similitude in agricultural engineering problems. The application of similitude to soil-machine systems were specifically discussed by Young (40). He gave a general guide to the approach to research in the area with the use of models and recognized problems related to such work.

The principles of dimensional analysis were used by Telischi et al. (38) in the study of small tillage tools in a soil bin. The effect of speed, clay content, moisture content and soil packing force on the draft of the moldboard plow, disk and chisel was investigated. A theoretical equation for draft consisting of four dimensionless quantities was derived, but the constants involved could only be determined experimentally. Similitude theory was in no way used as a means of predicting the draft of full-size implements corresponding to the small tillage implements which were used in the experiment.

Reece (31) presented a generalized theory which could be applied to problems involving soil-implement mechanics. The theory was developed through the use of dimensionless groups of parameters involving soil properties, soil-metal relationships and blade geometry in order to simplify the equations for draft of wide blades. The equation for the
draft of the tine was formulated as shown below:

\[ D = \gamma z^2 N_\gamma + c z N_C + c_a z N_A + q z N_Q \]  

Eq. 1

where \( D \) = the draft force per unit width of blade

\[ N_\gamma = \text{Reece draft factor for weight} \]

\[ N_C = \text{Reece draft factor for cohesion} \]

\[ N_A = \text{Reece draft factor for adhesion} \]

\[ N_Q = \text{Reece draft factor for surcharge} \]

\[ \gamma = \text{soil density, lb./in.}^3 \]

\[ z = \text{depth of working tool, in.} \]

\[ c = \text{cohesion, lb./in.}^2 \]

\[ c_a = \text{soil-metal adhesion, lb./in.}^2 \]

\[ q = \text{equivalent surcharge, lb./in.}^2 \]

The quantities \( N_\gamma, N_C, N_A \) and \( N_Q \) are dimensionless numbers representing the boundary conditions of the failure surface, and are functions of angle of shearing resistance, angle of soil-metal friction, blade geometry and failure boundary.

Vehicle model test techniques for submerged soils were developed by Dugoff and Ehrlich (6). Although their study was concerned with drawbar pull, slip and sinkage of a model tracked vehicle in a submerged sand soil and not with tillage, the technique developed in processing the soil, measurement of submerged soil density and method of testing provided

---

1 The symbols used in the Review of Literature are those of the cited authors and meanings may be different from and independent of each other and those subsequently used by this author.
information on the behavior of submerged soil.

Direct and indirect methods of measuring soil density were tried. A method called the "coffee-can" method was considered promising for in situ analysis. The device for measuring density consisted of an open cylinder, one end of which was sharp-edged for sinking into the soil. A bearing plate attached to the cylinder prevented further sinking of the cylinder into the soil beyond a gauged depth. A slit perpendicular to the longitudinal axis of the cylinder provided for insertion of a soil trapping plate while the cylinder was sunk into the soil. The soil had to be excavated from the slotted side of the cylinder in order to insert the trapping plate. The weight of the known volume of soil was taken and density was calculated. This process of measuring density was similar to that used by Larson (14) and Schafer (34). In the method used by these two researchers, however, the cylinder was pulled out of the soil before the soil tube was cut. This was possible in their case because they worked with dry soils instead of submerged soil.

The problem of soil grain segregation due to repeated agitation of submerged sand during testing and processing was considered to be non-significant so as to affect the model test program.

The application of similitude theory to tillage studies was initiated by Bockhop (1) in 1957. Because of the complex and still undefined nature of the soil-tool relationships, the use of similitude was believed to be an appropriate approach in the prediction of tool reaction such as draft. Recognition, definition and evaluation of the variables pertinent to the phenomenon being considered are important in any similitude study.
The variables deemed pertinent by Bockhop for predicting draft of disks are shown in Table 1.

Only eleven of the variables shown in Table 1 were used by Bockhop. The functional relationship among the dimensionless group called pi terms formed from the eleven variables was:

\[ \frac{R}{L^3d} = f\left(\frac{\lambda_j}{L}, \frac{V^2}{Lg}, U, \alpha, B, m, c\right) \quad \text{Eq. 2} \]

The model tools used were a 5-inch and a 10-inch concave disk. The prototype was a 26-inch disk. The soils used were sand, Ida silty loam, Colo silt clay loam and Luton silt clay. These soils provided a clay content range of 1.6 percent to 51.2 percent. Tests were run at different moisture levels which were three in the sand, four in the Ida, three in the Colo and only one in the Luton.

Bockhop concluded that the principles of similitude could be effectively utilized in the determination of the influence of soil and tool variables upon the tillage tool reactions. He added, however, that more work was needed to develop a precise prediction equation for the resultant forces acting upon a disk. The results of tests indicated that, generally, prediction became less accurate as moisture content and clay content increased.

The work of Bockhop was immediately followed by that of McLeod (16) who also used disks. McLeod modified and improved the procedures, instrumentation and pertinent variables, especially those concerning the soil properties. While Bockhop used the colloidal film theory proposed by Nichols (20), McLeod used principles involved in soil mechanics such
Table 1. List of pertinent variables used by Bockhop (1)

<table>
<thead>
<tr>
<th>No.</th>
<th>Variable</th>
<th>Definition</th>
<th>Basic dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$R_D$</td>
<td>Resultant force, draft</td>
<td>$F$</td>
</tr>
<tr>
<td></td>
<td>$R_S$</td>
<td>Resultant force, lateral</td>
<td>$F$</td>
</tr>
<tr>
<td></td>
<td>$R_V$</td>
<td>Resultant force, vertical</td>
<td>$F$</td>
</tr>
<tr>
<td>2.</td>
<td>$g$</td>
<td>Acceleration of gravity</td>
<td>$LT^{-2}$</td>
</tr>
<tr>
<td>3.</td>
<td>$U$</td>
<td>Ratio of coefficient of friction (soil/metal: soil/steel)</td>
<td>$-$</td>
</tr>
</tbody>
</table>

**Primary soil factors**

| 4.  | $d$      | Bulk density | $FL^{-3}$ |
| 5.  | $m$      | Moisture content in percent | $-$ |
| 6.  | $c$      | Clay content in percent | $-$ |
| 7.  | $T_1$    | Temperature of soil tested | $T$ |
| 8.  | $T_2$    | Temperature of the metal tested | $T$ |
| 9.  | $M$      | Organic matter in percent | $-$ |
| 10. | $Q$      | Type of clay, ratio of swelling to exchange capacity | $LF^{-1}$ |

**Design variables**

| 11. | $\alpha$ | Angle of inclination | $-$ |
| 12. | $\beta$  | Disk angle | $LT^{-1}$ |
| 13. | $V$      | Velocity | $LT$ |
| 14. | $L$      | Diameter of disk | $L$ |
| 15. | $\lambda_j$ | All other pertinent lengths | $L$ |

$^a_F, L, T$ and $\theta$ are dimensions of force, length, time and temperature, respectively.
as the application of Coulomb's equation in the evaluation of soil properties. McLeod recognized distortion of the model system and, consequently, calculated a prediction factor. The list of pertinent variables used by McLeod is shown in Table 2.

Table 2. List of pertinent variables used by McLeod (16)

<table>
<thead>
<tr>
<th>No.</th>
<th>Variable</th>
<th>Definition</th>
<th>Basic dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$R_d, R_s, R_v$</td>
<td>Draft, side, vertical soil force</td>
<td>F</td>
</tr>
<tr>
<td>2.</td>
<td>$D$</td>
<td>Disk diameter</td>
<td>L</td>
</tr>
<tr>
<td>3.</td>
<td>$\lambda$</td>
<td>Other pertinent lengths</td>
<td>L</td>
</tr>
<tr>
<td>4.</td>
<td>$V$</td>
<td>Disk velocity</td>
<td>LT$^{-1}$</td>
</tr>
<tr>
<td>5.</td>
<td>$g$</td>
<td>Acceleration due to gravity</td>
<td>LT$^{-2}$</td>
</tr>
<tr>
<td>6.</td>
<td>$\alpha$</td>
<td>Disk horizontal angle-of-approach</td>
<td>-</td>
</tr>
<tr>
<td>7.</td>
<td>$\beta$</td>
<td>Disk vertical-tilt angle</td>
<td>-</td>
</tr>
<tr>
<td>8.</td>
<td>$w$</td>
<td>Soil bulk volume weight, wet basis</td>
<td>FL$^{-3}$</td>
</tr>
<tr>
<td>9.</td>
<td>$C$</td>
<td>Apparent cohesion</td>
<td>FL$^{-2}$</td>
</tr>
<tr>
<td>10.</td>
<td>$\phi$</td>
<td>Angle of shearing resistance</td>
<td>-</td>
</tr>
<tr>
<td>11.</td>
<td>$A$</td>
<td>Apparent adhesion</td>
<td>FL$^{-2}$</td>
</tr>
<tr>
<td>12.</td>
<td>$\mu$</td>
<td>Angle of soil-metal friction</td>
<td>-</td>
</tr>
</tbody>
</table>

The functional relationship among the nine pi terms formed from the twelve variables listed in Table 2 was:

$$\frac{R}{wD^2} = f(\lambda/D, V^2/gD, \alpha, \beta, \phi, \mu, C/wD, A/wD)$$

Eq. 3
The disks used were of three sizes, viz., 3-inch, 6-inch and 12-inch diameter. Predictions factors were established in order to predict force components of the 6-inch prototype from the 3-inch model and those of the 12-inch prototype from the 6-inch model. Tests were run on sand, Ida silt loam, Colo silty clay loam and Luton silty clay soils.

McLeod concluded that the model system developed for the disk plow would give reasonably accurate and reliable prediction of force components. He recommended, however, that techniques of measurement of soil variable should be developed further.

Another similitude study of tillage tools was that performed by Larson (14) who used moldboard plows in four different soils. The plow shape was generated from the logarithmic spiral. The model system was distorted as in the case of McLeod's so that prediction factors had to be calculated. The pertinent variables which Larson used are listed in Table 3.

The functional relationship among the nine pi terms formed from the twelve variables listed in Table 3 was:

\[
\frac{R}{wD^3} = f \left( \frac{\Lambda}{D}, \frac{V^2}{gD}, \theta, n, \frac{C}{wD}, \tan \phi, \frac{A}{wD}, \tan \mu \right) \quad \text{Eq. 4}
\]

Soil strength parameters as described by Coulomb's equation were obtained by means of a torsion device. The model-prototype sets were tested in the same soil which was fitted by means of a uniform procedure in soil processing.

Larson developed a functional relationship between the pi term containing the dependent variable and the pi terms containing the independent
Table 3. List of pertinent variables used by Larson (14)

<table>
<thead>
<tr>
<th>No.</th>
<th>Variable</th>
<th>Definition</th>
<th>Basic dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>R</td>
<td>Draft</td>
<td>F</td>
</tr>
<tr>
<td>2.</td>
<td>D</td>
<td>Width of plow</td>
<td>L</td>
</tr>
<tr>
<td>3.</td>
<td>\lambda</td>
<td>Other pertinent dimensions</td>
<td>L</td>
</tr>
<tr>
<td>4.</td>
<td>V</td>
<td>Speed of plow with respect to soil</td>
<td>LT</td>
</tr>
<tr>
<td>5.</td>
<td>g</td>
<td>Acceleration due to gravity</td>
<td>LT^{-2}</td>
</tr>
<tr>
<td>6.</td>
<td>\theta</td>
<td>Lateral angle of plow surface</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>n</td>
<td>Cotangent of angle at which curve describing vertical section of plow</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>surface cuts a radius from the origin</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>w</td>
<td>Bulk volume weight of the soil</td>
<td>FL^{-3}</td>
</tr>
<tr>
<td>9.</td>
<td>C</td>
<td>Apparent cohesion of the soil</td>
<td>FL^{-2}</td>
</tr>
<tr>
<td>10.</td>
<td>tan \theta</td>
<td>Tangent of angle of shearing resistance of the soil</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>A</td>
<td>Apparent adhesion of the soil</td>
<td>FL^{-2}</td>
</tr>
<tr>
<td>12.</td>
<td>tan \mu</td>
<td>Tangent of angle of soil to metal friction</td>
<td></td>
</tr>
</tbody>
</table>

Variables. Application of the equation to data resulted in a complex relationship between the prediction factor and the distortion factor as applied to the distorted pi term containing cohesion. After some simplification, a generalized relationship between \( \delta \) and \( \alpha \) was established as

\[
\delta = \alpha^{-1.5} \quad \text{Eq. 5}
\]
where $\delta = \text{the prediction factor}$

$\alpha = \text{the distortion factor}$.

Larson concluded that this simplified relationship should be useful in other model tillage studies utilizing cohesive soils. He also concluded that the apparent cohesion and the tangent of the angle of shearing resistance were valid pertinent soil properties. He suggested that more precise measurement of dominant soil properties such as cohesion should be made and that with this, specific effects of adhesion and soil to metal friction might be determined. The findings of similitude research should be related to field conditions with full size equipment.

From the results of previous similitude studies on soil-tool systems, one could generally conclude that the problem of the proper selection and accurate measurement of pertinent variables still existed.

An attempt to solve the problem of the proper choice and evaluation of soil variables or properties was made by Schafer (34) in a model-prototype study of disks. Instead of isolating and measuring pertinent soil properties, he defined such properties, except bulk volume weight and moisture content, as those soil variables having dimensions which combine from the basic dimensions $F$, $L$ and $T$. All soil variables were held constant for each disk used. Since the tests could not be done in one soil bin in order to meet this requirement, three criteria were chosen for control of fitting the soil in a test run. The following were held the same for model and prototype tool tests and served as the similarity criteria: a) soil type, b) moisture content and moisture history and c) average penetrometer reading over the working depth of each tool.
Table 4. List of pertinent variables used by Schafer (34)

<table>
<thead>
<tr>
<th>No.</th>
<th>Variable</th>
<th>Definition</th>
<th>Basic dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( E )</td>
<td>Average draft force</td>
<td>( F )</td>
</tr>
<tr>
<td>2.</td>
<td>( D )</td>
<td>Disk diameter</td>
<td>( L )</td>
</tr>
<tr>
<td>3.</td>
<td>( \lambda_j )</td>
<td>All other pertinent lengths</td>
<td>( L )</td>
</tr>
<tr>
<td>4.</td>
<td>( V )</td>
<td>Disk velocity</td>
<td>( LT^{-1} )</td>
</tr>
<tr>
<td>5.</td>
<td>( \alpha )</td>
<td>Disk horizontal angle of approach</td>
<td>-</td>
</tr>
<tr>
<td>6.</td>
<td>( \beta )</td>
<td>Disk vertical tilt angle</td>
<td>-</td>
</tr>
<tr>
<td>7.</td>
<td>( W )</td>
<td>Soil bulk volume weight</td>
<td>( FL^{-3} )</td>
</tr>
<tr>
<td>8.</td>
<td>( M )</td>
<td>Moisture content, %</td>
<td>-</td>
</tr>
<tr>
<td>9.</td>
<td>( A_i )</td>
<td>Soil variables other than ( W ) and ( M ) whose dimensions combine from ( F ), ( L ) and ( T )</td>
<td>-</td>
</tr>
<tr>
<td>10.</td>
<td>( g )</td>
<td>Acceleration due to gravity</td>
<td>( LT^{-2} )</td>
</tr>
</tbody>
</table>

The list the variables considered pertinent in the similitude study is shown in Table 4.

The functional relationship among the seven pi terms formed from the ten variables listed in Table 4 was:

\[
\frac{E}{WD^3} = f\left(\frac{\lambda_j}{D}, \frac{V^2}{gD}, \alpha, \beta, M, \text{ and combinations of the } A_i \text{ with } W, g, \text{ and } D \text{ to form dimensionless terms}\right)
\]

Eq. 6
Speed tests for model disks which included the 3, 6 and 12-inch disks and for prototype disks which included 12, 18 and 24-inch disks were conducted in Norfolk sandy loam and Decatur silty clay loam soils. Angle tests were also conducted in the Norfolk sandy loam soil. The 12-inch prototype served as an indicator of similar conditions in the model and prototype tests.

Since a distorted model was used, a prediction factor was determined. This prediction factor was considered to be a function of the length scale n. The dependent pi term containing the dependent variable was $\pi_1 = \frac{E}{WD^3}$.

Results of the speed tests showed that the predicted $\pi_1$ values did not agree with the prototype $\pi_1$ values for all $\frac{V^2}{gD}$, an independent pi term. Good agreement, however, resulted from angle tests in the Norfolk soil.

The degree of agreement between the model and prototype results paralleled that between the 12-inch model and the 12-inch prototype. Whenever there was good agreement between the predicted and actual values of $\pi_1$ as in the angle tests, there was also good agreement between results for the 12-inch model and the 12-inch prototype. Furthermore, a parallel situation existed for non-agreement cases.

Schafer concluded that with the technique described in his study, the draft of a tool could be accurately predicted by the use of models. He added, however, that more data would be required to further evaluate the technique.

A study which involved models for predicting the response of
dynamically-loaded structures was performed by Murphy et al. (19). Although tillage of soil was not actually involved, the study used a technique of establishing similarity conditions for a dynamic system which involved cohesive soils. The basic principle involved could be applied to a similar system like tillage of soils.

The models used by Murphy and his co-workers consisted of hollow aluminum cylinders with diameters ranging from 1 inch to 8 inches. Each cylinder was instrumented with a strain gage and an accelerometer and was buried in a naturally occurring, cohesive soil. A drop-weight loader applied dynamic loads. Strain acceleration rise-times were determined at several different depths of burial. Distortion was evidently present in the model system.

A method of handling distorted models to correlate data satisfactorily was discussed and used. The method involved the description of soil properties by combinations of basic dimensions of soil properties. These properties were not evaluated quantitatively. Because of the use of the same soil type for the model and prototype structures, numerical values of such soil properties were not needed in the prediction equations. Although the method involved a constant soil variable approach similar to that used by Schafer (34), a different technique in establishing prediction equations was used.

The work discussed in this thesis involves an adaptation of the techniques used by Schafer and by Murphy et al. in order to find out whether such techniques can be applied satisfactorily to the similitude study involving tillage of flooded soils with moldboard plows. The details of such techniques are discussed in a later section.
According to Murphy (18, p. 57):

"A model is a device which is so related to a physical system that observations on the model may be used to predict accurately the performance of the physical system in the desired respect. The physical system for which the predictions are to be made is called the prototype."

Certain relationships between the model and the prototype must be satisfied in order to utilize the model as defined above. These relationships or design conditions may be established either by the use of equations developed from the knowledge of the behavior of the system or by the use of dimensional analysis. In exploratory work concerning systems where basic equations to describe the phenomenon are unknown, the dimensional analysis method is used in establishing model designs. Such is the case, for example, in the study of the interaction of a tillage tool and soil.

When the dimensional analysis method is used, the nature of the problem must be known so that variables affecting the phenomenon can be written down and defined with basic dimensions such as force, length and time. Then a functional relationship between the dependent variable and the independent variables can be expressed as

$$x_1 = \varnothing (x_2, x_3, x_4, \ldots, x_s)$$

Eq. 7

where $x_1$ is the dependent variable and $x_2, x_3, \ldots, x_s$ are the independent variables.

The $s$ variables considered may be combined into a number of
dimensionless and independent groups known as pi terms, and the functional relationship becomes

\[ \pi_1 = \emptyset (\pi_2, \pi_3, \ldots, \pi_t) \]  

\text{Eq. 8}

where \( t \) is the new number of variables required to describe the phenomenon. The number \( t \) according to the Buckingham Pi Theorem (18) is less than the original number of variables, \( s \), by an amount equal to the number of basic dimensions involved in all the variables or is explicitly expressed as

\[ t = s - b \]  

\text{Eq. 9}

where \( b \) is the number of dimensions involved.

A similar relationship can be written for another system in which the same phenomenon as the first system is involved. This second system is called the model, and the relationship is

\[ \pi_{1m} = \emptyset (\pi_{2m}, \pi_{3m}, \ldots, \pi_{tm}) \]  

\text{Eq. 10}

which is of the same form as that for the prototype system. If the model system is designed and operated such that

\[ \pi_2 = \pi_{2m} \]
\[ \pi_3 = \pi_{3m} \]
\[ \vdots \]
\[ \vdots \]
\[ \pi_t = \pi_{tm} \]  

\text{Eq. 11}

\[ ^1\text{The subscript } m \text{ refers to the model; no subscript } m \text{ refers to the prototype.} \]
it follows that the pi terms involving the dependent variables are equal in the prototype and model. Thus,

\[ \pi_1 = \pi_{1m} \quad \text{Eq. 12} \]

which is called the prediction equation, is also a valid relationship from which the desired quantity to be predicted is taken. In this case the model is said to be a true model because all design conditions in Equation 11 are satisfied.

In a model system where one or more of the design conditions are not satisfied, the model is said to be a distorted model. When this is so, the degree of distortion of a pi term is expressed by a distortion factor. A prediction factor is introduced in the prediction equation to account for distortion. For example, if \( \pi_3 \neq \pi_{3m} \) then the design condition becomes

\[ \alpha \pi_3 = \pi_{3m} \quad \text{Eq. 13} \]

and the prediction equation is

\[ \pi_1 = \delta \pi_{1m} \quad \text{Eq. 14} \]

where \( \alpha \) is the distortion factor and \( \delta \) is the prediction factor.

The handling of distorted models often presents problems because the expression for the prediction factor has to be found. In some cases, the prediction factor is easily known from its relationship with the distortion factor. However, the prediction factor is generally a function of the distortion factor and the other pi terms since
\[ \delta = \frac{\pi}{\pi_{1m}} = \frac{\phi (\pi_2, \pi_3, \ldots, \pi_L)}{\phi (\pi_{2m}, \pi_{3m}, \ldots, \pi_{Lm})} \]
\[ = \frac{\phi (\pi_2, \pi_3, \ldots, \pi_L)}{\phi (\pi_2, \alpha \pi_3, \ldots, \pi_L)} \quad \text{Eq. 15} \]

The selection of the pertinent variables that go with the model design is a most important step in the use of model theory in experimentation. An inherent danger in the use of models is the fact that until results of the model tests are compared with the prototype tests, errors in the selection of variables cannot be detected.

The verification of the validity of a model design can be accomplished by suitable tests with the prototype but this procedure is sometimes impractical. A procedure utilizing a series of tests with different models may be an alternative. In this latter procedure, one size is treated as the model of the other which in turn may be the model of another size and so on. Data obtained from series of tests utilizing these series of models can be correlated on the basis of the model design. If the modeling technique is satisfactory, then the approach may be valid yet may not be a sufficient indication of the validity of extrapolating to the prototype. Such extrapolation may be done only after verification with prototype data. The advantage gained is the reduction of the number of required prototype tests.
MODEL ANALYSIS OF TILLAGE OF FLOODED SOILS

The theory of models is applicable to a wide range of engineering problems. Its use is particularly helpful in cases where the phenomenon involved is complex, where knowledge about the behavior of the phenomenon is limited and where full-scale testing is uneconomical, time-consuming and impractical.

The tillage of friable soils has been one of the new areas of application of model theory in the last few years. The apparent complexity of the phenomenon involved in the tillage of soils has prompted past researchers to apply this theory. Not only does tillage of flooded soils involve a phenomenon equally as complex as tillage of relatively dry soils, but also it suffers from very limited prior investigations.

This section describes the analysis and development of the modeling technique for application to the prediction of draft of moldboard plows used in flooded soils.

Dimensional Analysis

The best test of validity of a model design technique is the comparison of the results from model tests with those results from prototype tests. Testing of full-scale prototype tools may not, however, always be feasible due to lack of adequate size equipment. For example, in the model tillage laboratory where this study was conducted, testing was limited to the use of reduced-scale tools because of the size of the soil bins. Moreover, when field tests are used, proper control of variables
usually becomes a problem, and the results from such tests may be misleading.

When full-scale prototype tools cannot be tested in the laboratory, an alternative approach to establishing model-prototype relationships may be pursued. Basically, the approach involves the testing of a series of models of different sizes. One model size is treated as the prototype, and the remaining sizes as models of this prototype. The basic philosophy behind this technique is that if the similarity requirements adopted for the model-prototype system are fulfilled and the results are analyzed according to the procedure required by the technique, significant trends and distortion, if they are present, can be detected. When prediction equations are established from the model-prototype system, full-scale prototype testing is reduced to a minimum and perhaps is required only to fully establish the validity of the technique used.

The first and most important step in the establishment of a model-prototype system is the establishment of similarity requirements. This is conveniently done by the use of dimensional analysis.

Selection of pertinent variables

The establishment of the conditions for similarity between model and prototype by means of dimensional analysis requires enough knowledge of the nature of the problem to list all variables pertinent to the behavior of the system. The correct variables must be selected in order that valid design conditions between the model and the prototype may be established.

Ideally, when the pertinent variables are identified, it is desirable,
although not required, that they be evaluated numerically. They must be fully defined, however, by the multiplicative combinations of basic dimensions. It will be found that in the case of soil, not enough is known about the nature and accurate measurement of its properties to obtain full identification and evaluation of soil characteristics which are directly related to tillage operations. If some generalized assumptions about the dimensions of soil properties are made, similarity conditions can still be established. Through techniques in the analyses of data, the properties need not be evaluated numerically. It will be found, however, that the method has some certain limitations.

**Dependent variable** The dependent variable considered in this study was the draft component of the resultant force on the moldboard plow.

**Independent variables** The listing of the independent variables proceeds from considering all possible factors which affect the draft of the plow. Experience and contributions from past studies are usually the best sources of knowledge for enumeration of these factors.

The independent variables for a soil-tool system may be classified into three categories, namely,

1. Geometric properties,
2. Soil properties and
3. Operational variables.

The variables considered in this study are shown in Table 5.

**Geometric properties** The variable $\lambda_1$ represented geometric parameters which had dimensions of length. For example, length of curve
Table 5. List of pertinent variables used in the flooded soil tillage study

<table>
<thead>
<tr>
<th>No.</th>
<th>Variable</th>
<th>Definition</th>
<th>Basic dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><strong>Dependent variable</strong></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>$R$</td>
<td>Draft of plow</td>
<td>$F$</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Independent variables</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Geometric properties</strong></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>$D$</td>
<td>Width of the plow</td>
<td>$L$</td>
</tr>
<tr>
<td>3.</td>
<td>$\lambda_i$, $i=1,2,\ldots,p$</td>
<td>Other pertinent lengths</td>
<td>$L$</td>
</tr>
<tr>
<td>4.</td>
<td>$\psi$</td>
<td>Cutting angle of the plow</td>
<td>$-$</td>
</tr>
<tr>
<td>5.</td>
<td>$\psi_j$, $j=1,2,\ldots,q$</td>
<td>Other angles of plow</td>
<td>$-$</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Soil properties</strong></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>$\rho$</td>
<td>Soil mass density</td>
<td>$FL^{-4}T^2$</td>
</tr>
<tr>
<td>7.</td>
<td>$M$</td>
<td>Moisture content</td>
<td>$-$</td>
</tr>
<tr>
<td>8.</td>
<td>$\mu$</td>
<td>Soil-metal friction</td>
<td>$-$</td>
</tr>
<tr>
<td>9.</td>
<td>$\omega_k$, $k=1,2,\ldots,r$</td>
<td>Soil-property</td>
<td>$-$</td>
</tr>
<tr>
<td>10.</td>
<td>$\sigma$</td>
<td>Soil-property</td>
<td>$FL^{-2}$</td>
</tr>
<tr>
<td>11.</td>
<td>$\sigma_s$, $s=1,2,\ldots,s$</td>
<td>Soil-property</td>
<td>$FL^{-2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Operational variables</strong></td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>$V$</td>
<td>Speed of the plow</td>
<td>$LT^{-1}$</td>
</tr>
<tr>
<td>13.</td>
<td>$g$</td>
<td>Acceleration due to gravity</td>
<td>$LT^{-2}$</td>
</tr>
</tbody>
</table>

*aNotation for the limits of subscripts in variables are:
$p = \text{total number of geometric properties with dimensions of length minus one.}$
$q = \text{total number of angles minus one.}$
$r = \text{total number of soil properties with no dimensions minus two.}$
$s = \text{total number of soil properties with dimensions of } FL^{-2} \text{ minus one.}$
on a vertical section of the moldboard, depth of cut of the tool, depth of water on the soil surface, etc., were included in the parameter $\lambda_i$.

Similarly, the variable $\psi_j$ represented other angles of the plow such as the polar angle in radians of a point on the logarithmic spiral which described the vertical section of the plow, the angle which the vertical plane parallel to the sides of the plow made with respect to the line of travel of the plow (angle of attack), the angles which the plane parallel to the flat bottom surface of the plow made with respect to the soil surface (angle of tilt and angle of suction), etc. In all tests, the angles of attack, tilt and suction were zero for all plows.

Soil properties

The soil property $\rho$ represented bulk density in mass units. In the evaluation of this property, soil clod samples were taken from the flooded soil. The samples were drained for two minutes by gravity under high relative humidity and at room temperature. Then the bulk volume and the weight were determined. Density was considered as a pertinent variable to the draft problem because, from experience, it has been found that draft increases with an increase in soil density. This finding was substantiated, for example, by examination of data from Larson's (14) study on model moldboard plows and by review of reports of Randolph and Reed (30).

For the same width and depth of cut of the plow, more mass of soil is moved by the plow if the soil is dense than if it were less dense. An increase in draft results in order to overcome the force due to acceleration of a greater mass of soil.

Moisture content, in the strict sense, is not a soil property; rather it is a soil condition. The dependence of other soil properties
upon moisture content was known from experience and from experiments of other workers such as Gill (8) and Nichols (20).

The reaction of a tool working bins of soil of the same type are more likely similar at the same moisture content than at different moisture contents. Since similarity of soil properties was a requirement in the similitude technique for this study, maintaining equal moisture contents in all soil fittings used was a logical approach. In tillage of flooded soil, equality of moisture contents in all soil fittings was easily attained by soaking the soil for several hours. Soaking also stabilized the moisture condition of the soil.

Another soil property which was considered pertinent to the phenomenon was soil-metal friction. In the strictest sense, this is not a soil property alone since its value depends upon the soil as well as the characteristics of the metal. In this study, only one type of metal was used. The plow surfaces were polished to approximately the same finish to obtain more or less constant values of friction in all tests under similar conditions.

The effect of friction on draft of moldboard plows was notable in past studies of draft. This was evidenced, for example, in the work of O'Callaghan and McCoy (25). From their calculated values of draft, moldboard friction was estimated to account for 25 to 37 percent of the total draft of the plow depending upon the speed which ranged from 2 to 8 miles per hour. The draft due to total friction from the landside, base and moldboard was estimated to be 74 to 80 percent of the total draft.
The soil property μ_k was included in the list of pertinent variables in order to account for dimensionless soil properties other than soil-metal friction. For example, if the angle of internal friction, a dimensionless soil property which was usually considered in previous tillage studies, were pertinent to the draft problem, then the variable μ_k included it.

The variables σ and σ^k represented those soil properties which had dimensions of FL^{-2}. Since the action on the soil by a moldboard plow involved shear, compressive and tensile stresses, the soil had properties which reacted to these stresses. In most tillage studies concerning soil-tool reaction the cohesion and adhesion were commonly considered soil properties. The variables σ and σ^k, however, did not specifically define cohesion or adhesion. If cohesion and adhesion were indeed contributing factors to the draft of the tool, σ or σ^k included them. If not, the inclusion of σ and σ^k were still valid to account for other soil properties having dimensions consonant with those of stress. It is to be noted that an evaluation of such parameters was not needed. All that was required by the procedure for establishing conditions of similarity was that the variables be defined dimensionally.

The fact that σ and σ^k were not specifically defined or evaluated surmounted the question of whether or not the Coulomb soil strength parameters, cohesion and angle of internal friction were applicable soil parameters in soil-tool systems. Although the Coulomb equation for soil shearing strength is commonly used in foundation engineering where static loadings are encountered, it may not be valid for use in soil-machine
systems where dynamic loadings and high rates of strain are encountered.

On the other hand, the Coulomb parameters had been used almost universally and with a fair amount of success in previous similitude and non-similitude studies of tillage. The results obtained from such studies seemed to justify the use of the Coulomb parameters in a dynamic system. No conclusive study, however, has been done yet on whether or not cohesion and angle of internal friction or a combination of both, as in the Coulomb equation, can be used in dynamic systems. In the procedures used in this study, it is immaterial whether this question was resolved or not.

Non-evaluation of soil properties other than density and moisture content gave the advantage of not having to use particular soil measuring devices. In some studies involving comparisons of results obtained from various devices, conclusions were conflicting. For example, Osman (26) in Great Britain concluded from his results that soil properties could be measured independently of the instrument while Dunlap et al. (7) in the United States concluded the exact converse. The methods of soil measurement used by these workers are described in the Review of Literature.

Operational variables

The variable V which is the speed of the plow has been long recognized as a factor which affects draft of a tillage tool. In general, previous studies revealed that draft increased with an increase in speed and the relationship was usually non-linear. Some past studies on the effect of speed on draft are discussed in the Review of Literature.

Acceleration of gravity g was included in the list of variables
because gravity resists the lifting of the furrow slice by the moldboard, indirectly affecting draft. The consideration of both the operational factor $g$ and the soil property $p$ implied that the weight of the soil was important in the draft problem.

**Development of $\pi$ terms**

Table 5 shows that there were $p + q + r + s + 9$ pertinent variables. If these variables were sufficient for describing the draft problem, then the number of $\pi$ terms which described the phenomenon was, from the Buckingham Pi Theorem or with the use of Equation 9,

$$p + q + r + s + 9 \text{ variables minus 3 basic dimensions}$$

$$= p + q + r + s + 6 \quad \text{Eq. 16}$$

For purposes of brevity, a $\pi$ term containing a subscripted variable (the subscript not $m$) was numbered as one $\pi$ term. Therefore, in this new notation there were ten $\pi$ terms which described the phenomenon.

The functional relationship between the dependent and the independent $\pi$ terms were written as follows:

$$\pi_1 = \emptyset \left( \pi_2, \pi_3, \ldots, \pi_{10} \right) \quad \text{Eq. 17}$$

A possible set of $\pi$ terms is as follows:

$$\pi_1 = \frac{R}{\rho V^2 D^2}$$

$$\pi_2 = \frac{\lambda_i}{D}, \ i = 1, 2, \ldots, p$$

$$\pi_3 = \psi$$

$$\pi_4 = \psi_j, \ i = 1, 2, \ldots, q$$
Establishment of similarity requirements

The design conditions for establishing similarity between model and prototype were obtained by equating the prototype pi terms, $\pi_2$ through $\pi_{10}$ to the corresponding model pi terms. The following design conditions resulted on the basis of such pi terms:

1. $\frac{\lambda_{im}}{D_m} = \frac{\lambda_i}{D}, \quad i = 1, 2, \ldots, p$

2. $\psi_m = \psi$

3. $\psi_{jm} = \psi_j, \quad j = 1, 2, \ldots, q$

4. $M_m = M$

5. $\mu_m = \mu$

6. $\mu_{km} = \mu_k, \quad k = 1, 2, \ldots, r$

7. $\frac{\sigma_{im}}{\sigma_{2m}} = \frac{\sigma_i}{\sigma_{2i}}, \quad i = 1, 2, \ldots, s$
Variables with the subscript \( m \) refer to the model and variables without the subscript \( m \) refer to the prototype.

The first three design conditions required that the model and the prototype be geometrically similar. These conditions were satisfied fairly well by careful fabrication of the model tools. The length scale was maintained in all directions and all angles were equal in the model and the prototype. From the first design condition, the length scale was established. Thus,

\[
\lambda_i = \frac{D}{D_m} \lambda_{i,m} = n\lambda_{i,m}, \quad i = 1, 2, \ldots, p \quad \text{Eq. 20}
\]

where the length scale \( n \) is equal to \( D/D_m \).

Design conditions 4, 5, 6, and 7 involved soil properties only. In general, these design conditions may be satisfied by using different soils, evaluating the values of each property for each soil and selecting these soils such that the \( p_i \) terms in the model soil are equal to those in the prototype soil. This is very difficult if not impossible to accomplish. Another way of satisfying the design conditions is to use the same soil for both model and prototype. Since the properties are identical, the four design conditions are automatically satisfied. The method of using the same soil was followed in this study.

Design condition 8 established a velocity scale. If the model and
the prototype plows were tested in the same gravitational field, that is, $g_m = g$,

$$\frac{V_m^2}{g_m D_m} = \frac{V^2}{g D}$$  \hspace{1cm} \text{Eq. 21}

or

$$V_m = \frac{V}{\sqrt{n}}$$  \hspace{1cm} \text{Eq. 22}

Design condition 9 also established the velocity scale in terms of the density scale $\rho$ and the soil property $\sigma$. The density scale and the soil property scale were unity, because the same soil was used in both model and prototype tests. Therefore, the relationship between the speed for the model and that for the prototype was given by

$$\frac{\rho_m V_m^2}{\sigma_m} = \frac{\rho V^2}{\sigma}$$  \hspace{1cm} \text{Eq. 23}

$$V_m^2 = \frac{\rho \sigma_m V^2}{\rho_m \sigma}$$  \hspace{1cm} \text{Eq. 24}

or since $\rho \sigma_m / \rho_m \sigma = 1$, the speed design condition was

$$V_m = V$$  \hspace{1cm} \text{Eq. 25}

Therefore, design condition 9 required that the model and the prototype tests be run at the same velocity, or the velocity scale should be equal to unity.

Unless the length scale was equal to unity, that is, the model and
prototype were of the same size, design condition 9 contradicted design condition 8. Since, the length scale was not unity, the model was distorted.

The test of validity of the similarity requirements is the accuracy of prediction of the prototype behavior with the use of the model designed according to such requirements. The greater the degree of satisfaction of such similarity requirements the more accurate would be the prediction. If, however, the variables selected to form the $\pi$ terms were not correct, the accuracy of prediction would be low, and/or no meaningful trend would be established.

In order to establish an efficient set of design conditions which would lead to a sufficiently accurate prediction equation, the design conditions 1 through 9, which were established in this section, were considered a flexible set. In other words, a few possible sets of design conditions were investigated in order to compare the accuracy of prediction equations evolving from each set and thus to evaluate each set. Two types of models, one an undistorted model and the other a distorted model, were considered. Certain assumptions were made about the variables listed in Table 5 and about the $\pi$ terms developed in order to proceed with the analyses of these two types of models.

Four different sets of similarity requirements were established upon consideration of the eight design conditions. Design conditions 1 through 7 were common for all sets. Only design conditions 8 and 9 varied among the four sets of similarity requirements. Each similarity requirement set is considered in the next sections.
Set 1. Undistorted model, \( \pi_9 = \frac{V^2}{gD} \) neglected

In order to have an adequate model, one or the other of the two \( \pi \) terms involving speed had to be neglected. This was required because the design conditions for speed designated by such \( \pi \) terms were in conflict with each other. The design condition based on \( \pi_{10} = \frac{\rho V^2}{\sigma} \) was

\[
V_m = V
\]

Eq. 25.

Neglecting the \( \pi \) term \( \frac{V^2}{gD} \) meant that the effect of gravity was neglected or that the weight of the soil mass lifted by the plow had insignificant contribution to the draft of the plow compared with other variables.

This was rationalized by the situation that the soil cut by the plow working in submerged soil experienced a buoyant force during a portion of the movement of the soil on the moldboard face. This buoyant force decreased the effective soil weight being encountered by the moldboard. The findings of O'Callaghan and McCoy (25) indicated that the draft indirectly contributed by lifting of the soil was only about four to twelve percent of the total draft.

Since design condition 8 was neglected, design condition 9 was considered to be included with design conditions 1 through 7.

Set 2. Undistorted model, \( \pi_{10} = \frac{\rho V^2}{\sigma} \) neglected

The second possibility of obtaining an adequate model from the given set of \( \pi \) terms was the neglecting of \( \pi \) term \( \frac{\rho V^2}{\sigma} \) and consideration of the \( \pi \) term \( \frac{V^2}{gD} \). In other words the design speed for the model was governed by

\[
V_m = \frac{V}{\sqrt{\pi}}
\]

Eq. 22
This set had opposite design conditions with those of set 1, as far as design conditions 8 and 9 were concerned. In this set, it was assumed that soil properties having dimensions of stress had insignificant contribution to draft of the plow compared with the weight of the soil. Whether this assumption was true or not could only be tested by the results of prediction. Nevertheless, for comparison with the other design conditions, this set of similarity requirements was also considered in the model analysis.

Set 3. Distorted model, \( \pi_9 = V^2/gD \) distorted. A possibility of considering both pi terms \( \pi_9 \) and \( \pi_{10} \) in the design conditions was investigated. As a result, distortion of either pi term was negligible. In similarity requirement set 3, the pi term \( V^2/gD \) was distorted which meant that design condition 8 was not satisfied, that is,

\[
\frac{V_m^2}{g_mD_m} \neq \frac{V^2}{gD}
\]

Instead, design condition 9 was satisfied. Thus,

\[
\frac{\rho_m V_m^2}{\sigma_m} = \frac{\rho V^2}{\sigma}
\]

Eq. 23

or

\[
V_m = V
\]

Eq. 25

Distortion of \( V^2/gD \) instead of \( \rho V^2/\sigma \) implied that the pi term \( V^2/gD \) was of minor importance relative to the pi term \( \rho V^2/\sigma \). The same arguments as presented for similarity requirement set 1 could be presented for this case.
Set 4. Distorted model, $\pi_{10} = \rho \frac{V^2}{\sigma}$ distorted

The last set of similarity requirements considered in the analysis was the case where the pi term $\rho \frac{V^2}{\sigma}$ was distorted and design condition 8 which was based upon the pi term $\frac{V^2}{gD}$ was satisfied. In this set the design speed was given by

$$V_m = V/\sqrt{n}$$

Eq. 22

de that is, the model should be run at a lesser speed than the prototype.

The validity of this similarity requirement set was determined by the results of tests.

Prediction Equations

Corresponding to each similarity requirement set was a prediction equation which was formed by equating prototype pi term containing the dependent variable R to the corresponding model pi term. Whenever the model was considered distorted, a prediction factor $\delta_1$ or $\delta_2$ was included in the prediction equation to account for the influence of distortion.

For similarity requirement set 1, where the effect of gravity g was neglected, the prediction equation was

$$\frac{R}{\rho V^2 D^2} = \frac{R_m}{\rho_m V_m^2 D_m^2}$$

Eq. 26

Since $\rho_m = \rho$, $V_m = V$ and $D/D_m = n$, the prediction equation reduced to

$$R = n^2 R_m$$

Eq. 27
For similarity requirement set 2, where the effect of the soil property $\sigma$ was neglected, the prediction equation was

$$\frac{R}{\rho V^2 D^2} = \frac{R_m}{\rho_m V_m^2 D_m^2}$$

Eq. 28

Since $\rho_m = \rho$, $V = \sqrt{n} V_m$ and $D/D_m = n$, the prediction equation reduced to

$$R = n^3 R_m$$

Eq. 29

For similarity requirement set 3, where the effects of soil property $\sigma$ were considered but the $p_i$ term involving gravity $g$ was distorted, the prediction equation was

$$\frac{R}{\rho V^2 D^2} = \delta_1 \frac{R_m}{\rho_m V_m^2 g^2}$$

Eq. 30

or

$$R = \delta_1 n^2 R_m$$

Eq. 31

where $\delta_1$ = the prediction factor.

For similarity requirement set 4, where the effects of gravity were considered but the $p_i$ term involving the soil property $\sigma$ was distorted, the prediction equation was

$$R = \delta_2 n^3 R_m$$

Eq. 32

where $\delta_2$ = the prediction factor.

In summary, the design conditions and the prediction equations for the four sets of similarity requirements considered in this study are shown in Table 6.
Table 6. Similarity requirement sets considered in the submerged soil tillage study

<table>
<thead>
<tr>
<th>Similarity requirement set</th>
<th>Type of model</th>
<th>Design conditions</th>
<th>Prediction equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Undistorted or adequate, ( V^2/gD ) neglected</td>
<td>( \frac{\rho_m V_m^2}{\sigma_m} = \frac{\rho V^2}{\sigma} )</td>
<td>( \frac{R}{\rho V^2 D^2} = \frac{R_m}{\rho_m V_m^2 D_m^2} )</td>
</tr>
<tr>
<td></td>
<td>or</td>
<td></td>
<td>or ( R = n^2 R_m )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>or ( R = \delta_1 n^2 R_m )</td>
</tr>
<tr>
<td>2</td>
<td>Undistorted or adequate, ( \rho V^2/\sigma ) neglected</td>
<td>( \frac{\rho_m V_m^2}{\sigma_m} = \frac{\rho V^2}{\sigma} )</td>
<td>( \frac{R}{\rho V^2 D^2} = \frac{R_m}{\rho_m V_m^2 D_m^2} )</td>
</tr>
<tr>
<td></td>
<td>or</td>
<td></td>
<td>or ( R = n^2 R_m )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>or ( R = \delta_1 n^2 R_m )</td>
</tr>
<tr>
<td>3</td>
<td>Distorted, ( V^2/gD ) distorted</td>
<td>( \frac{\rho_m V_m^2}{\sigma_m} = \frac{\rho V}{\sigma} )</td>
<td>( \frac{R}{\rho V^2 D^2} = \frac{R_m}{\rho_m V_m^2 D_m^2} )</td>
</tr>
<tr>
<td></td>
<td>or</td>
<td></td>
<td>or ( R = \delta_2 n^3 R_m )</td>
</tr>
<tr>
<td>4</td>
<td>Distorted, ( \rho V^2/\sigma ) distorted</td>
<td>( \frac{V_m^2}{\sigma_m} = \frac{V^2}{\sigma} )</td>
<td>( \frac{R}{\rho V^2 D^2} = \frac{R_m}{\rho_m V_m^2 D_m^2} )</td>
</tr>
<tr>
<td></td>
<td>or</td>
<td></td>
<td>or ( R = \delta_2 n^3 R_m )</td>
</tr>
</tbody>
</table>

\( a \) Design conditions in addition to the following which were common to all similarity requirement sets:

1. \( \frac{\lambda m}{D_m} = \frac{\lambda i}{D} , i=1,2,\ldots, p \)
2. \( \psi_m = \psi \)
3. \( \psi_{jm} = \psi_{j}, j=1,2,\ldots, q \)
4. \( M_m = M \)
5. \( \mu_m = \mu \)
6. \( \mu_{km} = \mu_k, k=1,2,\ldots, r \)
7. \( \sigma_m = \sigma, \lambda=1,2,\ldots, s \)

Subscript \( m \) refers to model; no subscript \( m \) refers to prototype.
Prediction Factor Equations

If the results of neglecting a pi term causing distortion fails to satisfy the prediction equation, that is, the undistorted model analysis is invalid, then the effect of that pi term must be considered. Distortion which results in such a case is assumed to be influenced by the design condition which is not satisfied.

The equation for each of the prediction factors $\delta_1$ and $\delta_2$ for the distorted model analyses is developed in this section.

The prediction factor $\delta_1$

The distorted pi term in similarity requirement set 3 was $\pi_9 = V^2/gD$. With the use of the first of Equations 15, the prediction factor $\delta_1$ for this set was given by

$$\delta_1 = \frac{\pi_1}{\pi_{1m}} = \frac{\Phi(\pi_2, \pi_3, \pi_4, \ldots, \pi_9, \pi_{10})}{\Phi(\pi_{2m}, \pi_{3m}, \pi_{4m}, \ldots, \pi_{9m}, \pi_{10m})}$$ 

Eq. 33

Therefore, an evaluation of the ratio of the function for the prototype $\pi_1$ to that for the model $\pi_1$ was needed to obtain $\delta_1$. A technique used to obtain an expression for $\delta_1$ involved an assumption that the distorted pi term in the functional relationship for $\pi_1$ was separable, that is, $\pi_1$ was expressed as a product of a function of $V^2/gD$ and a function of the other pi terms. The same assumption was also made for $\pi_{1m}$. Thus, Equation 33 was written as

$$\delta_1 = \frac{\pi_1}{\pi_{1m}} = \frac{f(\pi_2, \pi_3, \pi_4, \ldots, \pi_9, \pi_{10})}{f(\pi_{1m}, \pi_{2m}, \pi_{3m}, \pi_{4m}, \ldots, \pi_{9m}, \pi_{10m})}$$ 

Eq. 34
The assumption that the distorted pi term \( V^2/gD \) was separable required justification. Examination of test results and of data by previous investigators or additional experimental evidence would be needed in this case to obtain information on how \( V^2/gD \) influenced \( \pi_1 \). The procedure for testing the resolution of a function into the product of two component functions was outlined by Murphy (18). The justification of the assumption involved in the writing of Equation 34 is given in Appendix A.

If, in a series of tests, \( \pi_2 \) through \( \pi_8 \) and \( \pi_{10} \) were held constant, Equation 34 could be written as

\[
\delta_1 = \frac{\pi_1}{\pi_{1m}} = \frac{\phi_1(\pi_9)}{\phi_1(\pi_{9m})} \quad \text{Eq. 35}
\]

or

\[
\delta_1 = \frac{\phi_1(V^2/gD)}{\phi_1(V_{m}^2/g_{m}D_{m})} \quad \text{Eq. 36}
\]

The pi term \( V^2/gD \) was distorted by an amount \( \alpha_1 \). Therefore,

\[
\frac{V_m^2}{g_mD_m} = \alpha_1 \frac{V^2}{gD} \quad \text{Eq. 37}
\]

or

\[
\alpha_1 = \frac{V_m^2}{V^2} = \frac{gD}{g_mD_mD_m} = n \quad \text{Eq. 38}
\]

since \( g_m = g \), \( D/D_m = n \) and \( V_m = V \) from design condition 9. Therefore, with the use of Equations 37 and 38, Equation 36 was written as

\[
\delta_1 = \phi_2(n, V^2/gD) \quad \text{Eq. 39}
\]

that is, the prediction factor was a function of the length scale \( n \) and the distorted pi term \( V^2/gD \).
In general, the equation for \( \delta_1 \) may be obtained empirically by analysis of plots of \( \delta_1 \) versus \( n \) at constant \( V^2/gD \) and of \( \delta_1 \) versus \( V^2/gD \) at constant \( n \). Actually, one is a cross-plot of the other. If \( V^2/gD \) has no influence on \( \delta_1 \), the plot of \( \delta_1 \) versus \( n \) is a single curve.

The form of the equations of the curves fitted for the above plots suggests the form of equation for \( \delta_1 \). The ease of obtaining the equation for \( \delta_1 \) depends upon the nature of the curves. It is particularly convenient if the points plot as straight lines on logarithmic paper because then the equation of \( \delta_1 \) is expressed in the form

\[
\delta_1 = A \ n \ h(V^2/gD) \tag{Eq. 40}
\]

where \( A = 1 \), since \( \delta_1 = 1 \) at \( n = 1 \)

\( h(V^2/gD) \) = the slope of the line expressed as a function of \( V^2/gD \).

The final development of the equation for \( \delta_1 \) is shown in the Analysis of Data.

The prediction factor \( \delta_2 \)

The prediction factor \( \delta_2 \) was needed in the prediction equation for similarity requirement set 4 in which the design condition based upon the \( \rho V^2/\sigma \) term was not satisfied. The theory involved in the development of the equation for \( \delta_2 \) was similar to that for the development of the equation for \( \delta_1 \).
Based upon similarity requirement set 4,

\[ \pi_1 = \phi' (\pi_2, \pi_3, \pi_4, \ldots, \pi_9, \rho V^2/\sigma) \quad \text{Eq. 41} \]

In Equation 41, the distorted \( \pi_{10} = \rho V^2/\sigma \) was replaced by its square root. Thus,

\[ \pi_{10}' = \sqrt{\pi_{10}} = \sqrt{\rho/\sigma} V = KV \quad \text{Eq. 42} \]

where \( K = \sqrt{\rho/\sigma} \).

The \( \pi \) term \( \pi_{10} = KV \) was assumed to be separable, that is, Equation 41 was expressed as

\[ \pi_1 = f' (KV) f' (\pi_2, \pi_3, \pi_4, \ldots, \pi_9) \quad \text{Eq. 43} \]

The justification of this assumption is given in Appendix A.

If, in a series of tests, the \( \pi \) terms \( \pi_2 \) through \( \pi_9 \) were held constant, Equation 41 would be written as

\[ \pi_1 = \phi_1' (KV) \quad \text{Eq. 44} \]

For the model,

\[ \pi_{1m} = \phi_1' (K_m V_m) \quad \text{Eq. 45} \]

The \( \pi \) term \( KV \) was distorted by an amount \( \alpha_2 \), and the design condition based upon such \( \pi \) term became

\[ \alpha_2 KV = K_m V_m \quad \text{Eq. 46} \]

where \( \alpha_2 = \text{the distortion factor} \). Since \( K = K_m \), if the same soil was used for the model and the prototype tests,

\[ \alpha_2 = \frac{V_m}{V} \quad \text{Eq. 47} \]
The relationship between $V_m$ and $V$ for similarity requirement set 4 was

$$V_m = \frac{V}{\sqrt{n}} = n^{-\frac{1}{2}} V$$

Eq. 22

Therefore, Equation 47 became

$$\alpha_2 = n^{-\frac{1}{2}}$$

Eq. 48

or, in general,

$$\alpha_2 = \phi(n)$$

Eq. 49

With the use of the definition of prediction factor and Equations 44, 45 and 46

$$\delta_2 = \frac{\pi_1}{\pi_{1m}} = \frac{\varphi_1'(KV)}{\varphi_1'(KV)}$$

Eq. 50

$$\delta_2 = \frac{\varphi_1'(KV)}{\varphi_1'(\alpha_2KV)}$$

Eq. 51

$$\delta_2 = \varphi_2'(\alpha_2, KV)$$

Eq. 52

If the same soil were used for the model and the prototype tests, $K$ would be a constant. With the use of Equation 49, Equation 52 was written as

$$\delta_2 = \varphi_3'(n, V)$$

Eq. 53

Therefore, if the same soil were used in the model and the prototype tests and if the speeds of the model and the prototype were governed by the design condition
\[ \frac{v^2_{\text{m}}}{g D m} = \frac{v^2}{g D} \quad \text{Eq. 21} \]

or

\[ v_{\text{m}} = n^{-\frac{1}{2}} v \quad \text{Eq. 22} \]

then the distortion factor was a function of the length scale \( n \) and the plowing speed \( V \).

In general, the explicit expression for \( \delta_2 \) may be found by a method similar to that for finding the equation for \( \delta_1 \). Plots of \( \delta_2 \) versus \( n \) and \( \delta_2 \) versus \( V \) are made, and the equation for \( \delta_2 \) is then developed.

An alternate procedure for determining the equation for \( \delta_2 \) is by considering the relationship between draft and speed. Since \( \delta_2 \) depends upon the relationship between the pi term \( \pi_1 = R / \rho V^2 D^2 \) and the distorted pi term \( \pi_{10} = KV \) the form of the expression for \( \delta_2 \) depends upon the relation between \( R \) and \( V \). The development of this relationship is illustrated by two cases, namely, that when the plot of draft versus speed is considered as a straight line on logarithmic paper and that when it is considered as a straight line on rectangular coordinate paper. In either case the line may be fitted by the method of least squares.

**Case 1. Equation of the form** \( R = AV^2 \)

A straight-line plot of \( R \) versus \( V \) on logarithmic paper implies that the relationship between \( \pi_1 \) and \( KV \) is

\[ \pi_1 = A (KV)^x \quad \text{Eq. 54} \]

For the model, the relationship is

\[ \pi_{1m} = A_m (\kappa_m V_m)^x_m \quad \text{Eq. 55} \]
The prediction factor is

$$\delta_2 = \frac{\pi \eta}{\pi \eta m} = \frac{A (KV)^x}{A_m (K n V_m)^x_m}$$

Eq. 56

where A, A_m, x and x_m are constants. Since \( V_m = n^{-\frac{1}{n}} V \), Equation 56 becomes

$$\delta_2 = \frac{A (KV)^x}{A_m (K n^{-\frac{1}{n}} V_m)^x_m}$$

Eq. 57

or since \( K = K_m \),

$$\delta_2 = C (KV)^{x-x_m} n^{-x_m/2}$$

Eq. 58

where, \( C = A/A_m \).

The values of the constants C, K, x and x_m are unknown. The expression for \( \delta_2 \), however, can be found by considering the equation for the dependent variable R which is of the form

$$R = A_1 V^z$$

Eq. 59

where A_1 and z are constants. With the use of Equation 32 the prediction factor is given by

$$\delta_2 = \frac{A_1 V^z}{n A_1 m V_{m}^{z_m}}$$

Eq. 60

since \( R_m = A_1 V_{m}^{z_m} \).

With the use of Equation 22,

$$\delta_2 = \frac{A_1 V^z}{n A_1 m (n^{-\frac{1}{n}} V_m)^{z_m}}$$

Eq. 61

or

$$\delta_2 = C_1 V^{z-z_m} n^{-\frac{6-z_m}{2}}$$

Eq. 62
where $C_1 = A_1/A_{lm}$.

A disadvantage of the equation for $\delta_2$ (Equation 62) is that $C_1$ and $z$ are not known since the equation for the prototype is unknown. With the use, however, of auxiliary prototypes and models, different combinations of model and prototype may be formed and certain trends in the distortion factor can be established. For example, with three plow sizes, six model-prototype combinations may be formed. It may be reasonably assumed, depending upon the test results, that the prediction factor is unity if the length scale is unity.

From the different model-prototype combinations values of $C_1$, $z$, $z_m$ and $n$ can be obtained, and subsequently $\delta_2$ can be calculated.

The values of $C_1$, $z$, and $z_m$ may be a function of the length scale depending upon the test results. Therefore, Equation 62 can be written as

$$\delta_2 = f_\lambda(n) \frac{g_\lambda(n)}{h_\lambda(n)}$$

or simply

$$\delta_2 = \varphi'(n,V)$$

Case 2. Equation of the form $R = a + bV$ A straight line plot of $R$ versus $V$ on the linear scale implies that the form of the relationship between $\tau_1$ and $KV$ is

$$\tau_1 = a + b (KV)$$

Eq. 65

For the model, the relationship is

$$\tau_{1m} = a_m + b_m (K_m V_m)$$

Eq. 66
By definition, the prediction factor is

\[ \delta'_2 = \frac{\pi_1}{\pi_{1m}} \frac{a + b (KV)}{a_m + b_m (K_{nm})} \]  

Eq. 67

where \( a, b, a_m \) and \( b_m \) are constants. Since \( V_m = n^{-\frac{1}{3}} V \), Equation 67 becomes

\[ \delta'_2 = \frac{a + b(KV)}{a_m + b_m (K_{nm} n^{-\frac{1}{3}})} \]  

Eq. 68

The constants \( a \) and \( b \) are generally not known. The expression for \( \delta'_2 \), however, can be obtained from the relationship of draft and velocity, which, as assumed in this case, is a linear function. Thus,

\[ R = a_1 + b_1 V \]  

Eq. 69

and

\[ R_m = a_{1m} + b_{1m} V_m \]  

Eq. 70

where \( a_1, b_1, a_{1m}, b_{1m} \) are constants.

With the use of Equation 32 the equation for \( \delta'_2 \) is given by

\[ \delta'_2 = \frac{3}{n} \frac{R}{R_m} \]  

Eq. 71

\[ \delta'_2 = \frac{a_1 + b_1 V}{n (a_{1m} + b_{1m} V_m)} \]  

Eq. 72

Since \( V_m = n^{-\frac{1}{3}} V \) from Equation 22,

\[ \delta'_2 = \frac{a_1 + b_1 V}{n \frac{3}{a_{1m} + b_{1m} (n^{-\frac{1}{3}} V)}} \]  

Eq. 73
or

\[ \delta' = \frac{a_1 + b_1 V}{\sqrt{a_{1m}^2 + b_{1m}^2}} \]  

Eq. 74

where \( a_{1m} = a_{1m}^3 \)
\[ b_{1m} = b_{1m}^{5/2} \]

The expressions for \( \delta_2 \) and \( \delta'_2 \) for the experiments are developed in the Analysis of Data.
MATERIALS AND METHODS

The Model Tillage Laboratory

The model tillage laboratory located at the Department of Agricultural Engineering, Iowa State University was described in detail by Larson (14) and Schafer (34). The discussion of some aspects of the laboratory facility will be repeated here for completeness and for pointing out some modifications made to meet the laboratory requirements of this study.

Basically, the facility for model tillage experiments consisted of the following units:

1. Soil bin to contain the soil,
2. Soil bin carriage and drive and control units to move and regulate the motion of the bin,
3. Soil-processing equipment,
4. Dynamometer stand and tool bar for mounting and adjusting tillage tools and
5. Electronic equipment for obtaining and recording data.

Views of the facility are shown in Figure 1 and Figure 2. The basic soil bin unit could be mounted on and removed from the bin carriage by means of a fork lift. This arrangement provided a fast method of changing to a particular type of soil desired since several bins, each containing a certain type of soil, could be used. Since only one soil type was used in this experiment, the basic soil bin unit was not used. Instead, a watertight, deeper and longer bin was fabricated and
Figure 1. The model tillage laboratory viewed from the soil processing equipment end

Figure 2. The model tillage laboratory viewed from the main drive end
placed semi-permanently on the carriage. The empty soil bin is shown in Figure 3.

Watertightness was required because the soil was processed under saturated conditions and testing was done with soil under a layer of water. The soil bin was made deeper than the basic bin for the tillage laboratory in order to minimize or eliminate any bottom boundary effect of the soil bin on the action of the tillage tool. Although such an effect could be negligible, it was believed that the deeper bin would more or less approach the characteristic depth of a real paddy field. The depths of actual paddy fields vary with soil, type, climate, method of culture, but, in general, they are greater than six inches.

A longer bin was required for the experiment because the tillage tool had to work in water and because more space for soil preparation and water drainage was needed. Water had to be contained inside the bin during tests as the bin was accelerated and decelerated.

During the tests, however, the ends of the bin were swung down to prevent damage to the plow and the measuring instruments. In this case a dike of soil was made at either end of the soil test plot to contain the water. The open-end bin with partly plowed soil is shown in Figure 4.

The carriage on which the soil bin was mounted was pulled by a roller chain which ran along a wooden trough midway between and along the tracks. The roller chain ran over a sprocket at either end of the tracks. Either sprocket which was powered by an electric motor through an infinitely-variable speed transmission could be used as a drive
Figure 3. Empty soil bin used for submerged soil tillage study

Figure 4. Soil bin with open ends, after a few test runs
sprocket depending upon the speed desired for the bin. Thus, the drive units consisted of the slow-speed unit which was used for soil processing and the high-speed unit or main drive unit which was used for plow testing.

In the low-speed drive unit shown in Figure 5, power from the motor was transmitted through a splined shaft which was detached from the drive shaft whenever the high-speed drive unit was used. Overload protection was provided by means of a jump clutch.

In the high-speed drive unit, shown in Figure 6, power was transmitted through a four-speed truck transmission connected in series with the infinitely-variable speed transmission. The reverse position of the transmission provided for returning the bin towards the starting position after a test run.

Several control devices facilitated soil preparation and test runs and provided speed regulation and protection from damage to the equipment. These controls consisted of limit switches alongside the tracks which activated a motor off switch, a pneumatically-operated brake unit and a pneumatic bin accelerator. Spring-loaded stops were provided at the ends of the bin in case of failure of the control devices.

The limited length of the tracks required rapid acceleration of the bin for higher speeds. Such acceleration was provided by means of a catapult system consisting of an air cylinder 80 inches long and 4-3/4 inches in diameter. A piston inside the cylinder was connected to the rods projecting from opposite ends of the cylinder. A 3/8-inch wire cable attached to the rod ends made a complete circuit around a pulley at either
Figure 5. The low-speed drive unit

Figure 6. The high-speed drive unit
end of the tracks. When the bin was to be accelerated from starting position, a spring-loaded, slotted arm was swung from the carriage and set on a knob on the cable. This permitted the carriage to be carried along by the cable when compressed air was admitted into one end of the air cylinder by means of a solenoid valve which was energized at the same time as the drive motor was started.

At the end of acceleration, the drive motor continued to drive the bin at a constant speed. Also, a limit switch alongside the track was operated by a cam on the carriage thereby actuating a three-way solenoid valve. The compressed air which was between 70 and 90 pounds per square inch was then exhausted to the atmosphere.

The cylinder and cable system also functioned as a deceleration snubber. The return run of the wire cable caused a second knob on the cable to engage a grommet on the carriage near the end of the tracks. This caused the piston to be pulled back to its starting position, thus decelerating the bin gradually.

The soil-processing equipment in the laboratory consisted of a rototiller and a drum roller as shown in Figure 7 and Figure 8, respectively. In this study, the rototiller was used only during the initial stages of soil preparation when the soil, which was fresh from the field, had to be broken down. At this stage, the soil moisture was satisfactory for rototilling. However, in the final soil preparation for experimental runs, the rototiller was not used because excessive soil adhesion was encountered at higher moisture contents. Instead, the comb harrow shown
Figure 7. Rototiller used for initial soil preparation

Figure 8. Drum roller used for compacting the soil
in Figure 9 was used. The soil was passed back and forth through the harrow several times until visual observation indicated a uniform soil consistency. The standard laboratory equipment for trimming the soil consisted of a blade shown in Figure 10 which was mounted in the same frame used for the harrow.

The 30-inch diameter steel drum roller was coated with "Emralon 320" to minimize sticking of the soil during rolling. The roller was power driven from the slow-speed drive unit and rotated at a peripheral speed equal to the linear speed of the soil bin.

All of the soil-processing equipment could be adjusted by means of electro-pneumatic cylinders which set the particular equipment to the desired position. This position was determined by stops set by hand-operated screws.

The dynamometer stand shown in Figure 11 straddled the tracks and contained the tool-bar for mounting the dynamometer or tool frame. As shown in Figure 12, the tool bar could be swung in one direction by means of electro-pneumatic cylinders so that fast and free adjustment of the tool could be accomplished. The hand-operated vertical and horizontal screws which were connected to the dynamometer frame and the tool bar provided for fast and accurate positioning of the tool.

---

1 "Emralon 320" is a resin-bonded coating material having the low frictional characteristics of tetrafluoroethylene and is a product of Acheson Colloids Company.
Figure 9. Comb harrow used for soil preparation of wet soil

Figure 10. Soil leveling blade attached to the comb harrow mounting frame
Figure 11. The dynamometer stand with dynamometer in normal position for test run.

Figure 12. The dynamometer stand with dynamometer in raised position after test run.
For the measurement of forces acting on the plow, the dynamometer shown in Figure 13 and Figure 14 was used. This dynamometer was essentially the one used by Larson (14).

A T-shaped bar and its framework were mounted on the dynamometer frame of the tool bar. With this set-up the forces from each of the mutually-perpendicular planes could be isolated. Figure 15 shows that draft could be measured at point 1, side forces at points 2, 3 and 4 and vertical forces at points 5 and 6. The T-shaped bar was linked to the rigid frame at each point by means of a two-piece steel ring. Precision ball rods provided connections between the T-shaped bar and the ring and between the ring and the frame. The rings were bonded with strain gages and were used as force transducers at each point.

The design of the force ring is shown in Figure 16. Each curved member of the two-piece ring was made by bending a 3/8 inch wide by 1/8 inch thick cold steel stock in a die using a hydraulic ram. On either face of the curved member was bonded an SR-4 foil strain gage so that four gages were on each ring or link unit. The four gages were connected as a bridge circuit. The recorded total strain, which may be either compressive or tensile, provided a means of measuring forces sensed in a particular point or link. Each force ring was calibrated individually by applying known loads on the ring and attenuating the signal such that an integral ratio of load versus amount of recorder pen deflection was established.
Figure 13. Perspective view of dynamometer

Figure 14. Bottom view of dynamometer
Figure 15. Line sketch of tool mounting bar (Larson, 14)
Figure 16. Strain links used for measuring draft on moldboard plows (Larson, 14)
Model Moldboard Plows

The tillage tool used for this study was the moldboard plow. Four plow sizes, expressed in inches of width dimension, constituted the model and prototype tools. These were 1.5-inch, 2.0-inch, 3.0-inch and 4.0-inch plows.

The choice of shape of the tool was considered as arbitrary. Following the procedure of Larson (14), the author fabricated model moldboard plows, the vertical section of which was described by a logarithmic spiral. Another reason for choosing the logarithmic spiral-generated plow was the feasibility and relative ease of fabricating the model plows. With the method used in fabrication, the geometrical requirements of the different plow sizes were satisfactorily met.

The equation of the logarithmic spiral in polar coordinates is

\[ r = r_0 \theta \cot \alpha \]

Eq. 75

where \( r \) = the radius vector of a point on the spiral
\( r_0 \) = the radius vector when \( \theta \) is zero
\( \theta \) = the angle of the radius vector with respect to the base line, in radians
\( \alpha \) = the constant angle between the radius vector at a point and the tangent to the spiral at that point.

The characteristic length \( r_0 \) was chosen to be equal to 1.0 inch for a 3.0-inch model. Values of \( r_0 \) for other plow sizes were scaled from this value thus generating equations to be used for such plow sizes. Thus, the 1.5-inch plow had \( r_0 = 0.5 \) inch, the 2.0-inch plow had
\( r_0 = 0.67 \text{ inch}, \text{ etc.} \) The value of \( \alpha \) was chosen to be 40 degrees.

Values of \( r \) in the logarithmic spiral equation were computed through the range of \( \theta \) from -40 degrees to + 90 degrees. These values of \( r \) and those of the corresponding \( x \) and \( y \) coordinates are shown in Figure 17 for each plow. The full-size curves describing the vertical section of the plows are shown in Figure 18.

Larson (14) showed that if the length parameters of the logarithmic spiral were scaled, the resulting curve lengths and surface areas developed from the curves would also be scaled to the desired ratios. The values of \( \theta \) and \( \alpha \) were the same at corresponding points in all curves. With these consequences, geometric similarity among the different sizes of plows was satisfied.

Geometric similarity required that thickness of the moldboard also be scaled accordingly. However, this was not considered to be a serious requirement since thickness of the moldboard except at the cutting edge seemed to be non-pertinent to the forces on the plow. Nevertheless, where it was possible, this requirement was considered. For example, the 4-inch plow was made out of 1/8-inch thick plate, the 3-inch plow was made out of 3/32-inch plate, the 2-inch plow out of 1/16-inch plate. However, for reasons of plow rigidity, the thickness of the 1.5-inch plow was not scaled but was made of 1/16 inch plate.

The cutting edge, which was more pertinent to the problem, was made similar by making the same bevel. Exact geometric similarity was difficult to attain as this required careful and exact sharpening of the tool cutting edge. Moreover, even if the cutting edges were initially
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Figure 17. Computed values of r and the corresponding X and Y coordinates for each moldboard plow size characterized by the logarithmic spiral equation, \( r = r_0 e^{\theta \cot \alpha} \).
Figure 18. Logarithmic spiral curves describing the vertical section of the moldboard plows.
similar geometrically, uneven wearing of the blades eventually disrupted this condition. It was assumed that slight deviations from geometric similarity of the cutting edge had a negligible effect from the standpoint of the similarity requirements.

A three-dimensional template similar to that shown in Figure 19 was fabricated for each plow size. The template consisted of 20-gage galvanized steel sheets mounted on two wooden bases. The sheets were basically rectangular with one narrow end cut out to the shape of the logarithmic spiral. Parallel slots one-fourth inch apart were milled on opposite faces of the wooden bases. The sheets were arranged in the slots such that a 40-degree cutting angle (also the value of $\alpha$ in Equation 75) was obtained for the plow.

A thin sheet of paper was laid over the three-dimensional template and the side contours of the plow were outlined on the paper. The plow outline was transferred to a flat steel plate which was then cut a little larger than the outline. The cut plate was then rolled in a sheet roller and fitted to the three-dimensional plate.

The shaped moldboard surfaces was welded to a mounting bracket and the edges were milled to the desired dimensions. The leading edges of the 3.0-inch and the 4.0-inch plows were milled to 1/32 inch thick. Thus the leading edge of all plow sizes were the same. A bevel of approximately 20 degrees was made for each plow by filing the leading edge from the upper plow surface.

The plow surfaces were polished to approximately the same degree. The same method of polishing was used for all plows. Surface roughness
Figure 19. Three-dimensional template used as a guide for forming the 3.0-inch moldboard plow

Figure 20. The four moldboard plow sizes used in submerged soil tillage study
was assumed to be equal as a result of a uniform procedure. A qualitative check was made by eye observation and finger feeling. It was assumed that the coefficient of friction between the plow surface and the soil of identical properties was the same for all plows. The four sizes of moldboard plows, namely, 4.0-inch, 3.0-inch, 2.0-inch and 1.5-inch, are shown in Figure 20.

There were some differences between the plows used by Larson (14) and those used in this study. The constant angle (the equivalent of $\alpha$ in this study) used by Larson for the logarithmic spiral equation was 45 degrees while the one used in this study was 40 degrees. While Larson fitted landsides and coulters to his plows, none were used with the plows in this study. The absence of landsides and coulters was meant to be a simplification because forces contributed by these were removed from those due to the moldboard surface. It was postulated that with this arrangement repeatability and more consistent results would be obtained.

Each plow was mounted on a shaft from a horizontal arm which was fitted to the vertical shaft of the dynamometer as shown in Figure 11 or Figure 12. The plow was mounted such that the shin was parallel to the direction of travel of the bin. The depth of cut, which was scaled, was adjusted by means of the vertical screw. The width of cut, which was always the same as the width of the plow, was adjusted by means of the horizontal screw.
Soil and Initial Soil Processing

Only one type of soil was used in this study. Because of the nature of approach in the similitude analysis, varied soil types would have different results completely independent and different from those of another soil type. The similarity technique, if found applicable to one soil type, should also be applicable to other soils. However, this needs to be verified by experiment. The emphasis of this study was not on soil type but on the applicability of the approach in similitude.

The soil used was Luton silty clay obtained from Western Iowa. Table 7 lists the physical properties of the soil.

Table 7. Physical properties of the soil used for submerged soil tillage study

<table>
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<th>Mechanical analysis</th>
<th>Atterberg limits</th>
<th>Chemical analysis</th>
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<td>Sand - 8.7%</td>
<td>Liquid limit - 66%</td>
<td>pH - 6.8</td>
</tr>
<tr>
<td>Silt - 40.0%</td>
<td>Plastic limit - 43%</td>
<td>Organic matter - 2.9%</td>
</tr>
<tr>
<td>Clay - 51.3%</td>
<td>Plasticity index - 37%</td>
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</table>

The soil was immediately stored inside a warm building after it was taken from the field during late autumn of 1965. Some roots and other plant materials were present in the soil. After a few weeks of storage, the weed seeds germinated then died because of lack of sunlight. The soil was then placed in a watertight bin. The soil clods were broken by
means of the rototiller. The moisture content at the time of rototilling was considered optimum, in that soil balling with the rototiller was not encountered. The soil was cleaned by hand of plant materials. After a uniform soil pulverization, the soil was flooded and stirred with a stirrer device consisting of a vertical 3/4-inch diameter steel rod fitted with horizontal 3/16-inch diameter steel rods. The rods were evenly spaced in holes located along the axis of the vertical rod. The stirrer was driven by a 3/4-inch hand drill mounted on the tool bar. Stirring of the submerged soil brought more plant materials to the top, and these materials were removed by hand. The process of stirring, letting sediment settle and then cleaning was repeated several times until the soil was uniform and free of foreign material. Seeds which germinated during drying time were removed. Cycles of drying and wetting and puddling were applied to the soil to approximately subject it to conditions similar to those in a paddy field.

Measurement of Soil Properties

In this study, only two soil properties were measured. These were moisture content and bulk density. Other soil properties were defined by some combinations of the basic dimensions, force, length and time and were not evaluated numerically. The reason for this procedure was that, except for the two chosen properties, other soil properties are still vaguely defined, and no standardized method of measurement of such properties have been established. Besides, even if existing soil instruments were valid for dry soils, they may not be valid for saturated
or flooded soils. In the case of moisture content and bulk density, the procedures for obtaining values have been standardized or have been developed from approved definitions of the property.

The paraffin method of determination of bulk density has been used by Gill (8), Johnston (11), Johnston and Hill (12) and Perry (29). Basically, a clod of soil was tied with a string and weighed in air; then it was coated with paraffin to make it waterproof and weighed again. Finally, the coated soil, while suspended by a string, was weighed when submerged in distilled water. Soil clod samples were first drained by gravity for two minutes under high relative humidity before this procedure. The bulk density was calculated from

\[
B.D. = \frac{W}{\frac{W - W_a}{d_w} - \frac{W - W_w}{d_p}}
\]

where

- \( B.D. \) = bulk density
- \( W \) = weight of uncoated soil clod in air
- \( W_a \) = weight of paraffin-coated soil in air
- \( W_w \) = weight of coated soil clod in water
- \( d_w \) = density of distilled water
- \( d_p \) = density of paraffin

Bulk density was in units of pounds per cubic foot. Conversion to mass units was made by division of bulk density by the acceleration due to gravity.

For moisture content determination, a soil clod was weighed and oven-dried at 216° F. for 24 hours. A "Mettler Type H6T" balance was
used in weight determinations. Initial and final weights were recorded and the moisture contents, wet basis, were calculated as follows:

$$M = \frac{W_i - W_f}{W_i} \times 100$$  
Eq. 77

where $M =$ moisture content in percent, wet basis

$W_i =$ initial weight of soil

$W_f =$ final weight of soil (after oven drying).

To investigate the relationship between moisture content and bulk density, several soil clod samples were taken at different moisture contents and bulk densities were obtained. The results are shown in a later section.

Standardized Soil-fitting Process

In the section on model theory it was indicated that certain conditions involving dimensionless quantities had to be satisfied in order to make the prediction equation valid. The design conditions developed for the model-prototype system in this study required that the same soil be used in both model and prototype tests. The most ideal method of meeting this requirement would be to test all the model and prototype tools in the same bin. Such method was, however, not feasible because the bin was limited in size. Instead, the normal procedure was to test a tool of a certain size in one bin fitting at several speeds and then to process the soil again for tests of the other tool of different size.

The requirement of similar soil conditions, therefore, rested on how well the soil was processed to obtain the original condition. In order
to duplicate the soil conditions, the method of processing was standard­
ized. Soil conditions were assumed to be similar whenever the same
procedure was followed in fitting the soil after test runs were completed.
Bulk density measurements were made, and equality of values of bulk den­
sity was used as a gage or index of similarity of soil conditions from
one bin fitting to another.

After a test, the soil was distributed evenly by hand throughout
the working area of the soil bin. This was done with about one half-inch
of water on the soil surface. Whenever clods produced by uneven drying
were encountered, they were broken down by hand and mixed thoroughly with
the other soil. Then the soil bin was left to stand overnight or a few
days to let the sediment settle. The clear water was siphoned off
carefully to minimize draining of the soil colloids. The remaining
moisture was then allowed to evaporate. Faster evaporation was attained
by blowing air over the surface and by using heat lamps. Whenever small
cracks appear on the soil surface, the heat lamps and blower were removed.
At times, when a concentration of heat from the lamp caused large, local
cracks, the bin was wetted again and the process was repeated.

A one-row, comb harrow shown in Figure 9 which replaced the leveling
blade in the soil processing equipment stand, was used to cut through the
soil, producing several strips or flat-top ridges of soil. Then the soil
was pressed by the roller which was lowered gradually until the soil
strips fused together again. The purpose of the strips was to attain
uniformity in the soil. It was found that when the soil had the proper
moisture for rolling, adhesion to the comb harrow was slight.
The leveling blade (Figure 10) was then substituted for the comb harrow and was used to level the soil by cutting the surface layer and bulldozing the excess soil towards the ends of the bin. The leveler was also often used on return trips to press the soil by letting it slide at a pressure on the soil surface with the blade edge trailing. This process gave a better smoothing effect especially when a little moisture was added for lubrication. It also made a better distribution of pressure, assuring a more or less uniform bulk density in the soil working area or test plot in the bin.

Once more the soil was wetted, but this time the water was allowed to stand. Water was allowed to evaporate naturally to produce even drying. This took about three or four days, depending upon the atmospheric conditions.

As the soil dried out, small cracks began to appear on the surface. A well distributed or random cracking indicated uniform drying. Moreover, the absence of a tendency of the cracks to occur along the strips indicated that the soil fused together during rolling and that the original paths of the harrow did not form soil failure planes. A little amount of water was sprinkled on the surface, and then the leveling blade was passed again with the original leading edge trailing. This closed the cracks again and at the same time provided a smooth level surface.

Finally, the soil was flooded with water, the depth of which was determined by the size of the plow to be tested. A small dike was built at either end of the soil working area to contain the water. Water was
allowed to stand for an hour before final testing in order to further condition the soil.

Testing Procedure

To obtain uniformity of testing a certain procedure was followed when making test runs for each plow. The soil bin was moved under the dynamometer stand such that the plow mounted on the dynamometer was above the level soil surface. The location of the soil surface with respect to the plow was determined by lowering the plow by means of the vertical screw crank until its leading bottom edge touched the soil surface. The tool bar was then raised by means of the electro-pneumatic cylinders, the soil bin backed up to the starting point, the tool bar lowered and the plow adjusted to the proper depth.

For opening the first furrow, the plow was positioned farthest to the right side of the bin to cut the first ribbon of soil. The first furrow was opened at a low bin speed (about 0.2 feet per second) provided by the low-speed drive unit. The ribbon of soil was removed by hand as soon as the plow cut through the soil. This was done to prevent overloading of the force rings resulting from soil pressing against the wall of the bin. A second cut on the soil was made, this time of lesser width and at a faster speed in order to trim the edge of the unplowed portion of the soil. The test runs were made after a satisfactory furrow was established.

It was shown in the section on model analysis that the design condition involving speed required that the speed of the model plow be the
same as that of the prototype plow or $V_m = V$ for similarity requirement set 3 and $V_m = V/n$ for similarity requirement set 4. The speed range was from 0.692 feet per second to 6.977 feet per second. For a given plow size and for a given combination of settings of the series-connected infinitely-variable transmission and the four-speed transmission the speed was always essentially constant throughout the run. The same setting, however, did not give exactly the same speed when a different plow size was tested. Thus, the speed design conditions from similarity requirement set 3 and similarity requirement set 4 were not exactly satisfied. Although trial and error could lead to fulfillment of such requirement, this process was time-consuming and inefficient. Therefore, with the limitation of the equipment used, it was decided that transmission settings would be such that the range of speed values would be covered at equal intervals.

Values of speed for different combinations of the transmission settings in series were tabulated as test runs were made. This tabulation served as a guide for setting transmission combinations to obtain approximate values of speed desired for other runs.

In order to minimize error due to a possible effect of a run on the previous run, no definite sequence of speed values was followed. In other words, speed values were randomized within a series of test runs for a soil bin fitting. Since more test runs would be made for a small-sized plow than for a large-sized plow for a particular fitted soil plot (the whole soil area in the bin), the data points were of unequal number for all plow sizes. Each fitted soil plot corresponded to a
series of test runs for a particular size of plow. There were 16 test runs for the 1.5-inch plow test series, 12 test runs for the 2.0-inch plow test series, 8 test runs for the 3.0-inch plow test series and 6 test runs for the 4.0-inch test plow series. Replications of series of tests were 2, 4, 3 and 4 for the 1.5-inch, 2.0-inch, 3.0-inch and 4.0-inch plow, respectively. Figure 21 shows the 2.0-inch plow during a test run.

Randomization was used in scheduling each tool size for a soil bin fitting. This was done to minimize error due to a possible residual effect of a previous test run series on the next test run series or to possible differences in soil fittings. Table 8 presents a summary of the test run schedule.

Table 8. Schedule of tests for submerged soil tillage

<table>
<thead>
<tr>
<th>Item</th>
<th>1.5</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of plowing, inch(es)</td>
<td>0.94</td>
<td>1.25</td>
<td>1.88</td>
<td>2.50</td>
</tr>
<tr>
<td>Depth of water, inch</td>
<td>0.20</td>
<td>0.27</td>
<td>0.40</td>
<td>0.55</td>
</tr>
<tr>
<td>Number of runs per test series</td>
<td>16</td>
<td>12</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Number of test series</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>
Figure 21. The 2.0-inch plow during a test run

Figure 22. Oscillograph equipment used to record strains from strain links of dynamometer
For each run, the draft, side force and vertical force reactions on the plow were recorded simultaneously on the eight-channel Offner Type R Dynograph shown in Figure 22. Only the draft force, however, was considered due to several missing data on other directions because of instrument malfunction. Each test run was labeled properly.

Blips or marks indicating one-second intervals and one-foot intervals of soil bin travel were also recorded along the edges of the recording paper. The output from a direct current tachometer generator which was driven from the high-speed drive shaft was traced in a separate channel of the recorder. This trace gave indications of speed.

Processing of Data

The draft of the plow was considered to be the most significant force exerted by the soil on the tillage tool. With the dynamometer set-up used, the sensing of draft was independent of the magnitude of the other forces in other directions. Therefore, the pen recordings resulting from straining the force ring gave direct readings of draft after application of attenuation and calibration factors. A typical recording of draft is shown in Figure 23.

The average draft for a plow size during a certain run was obtained by determining the area under a section of the force trace with the use of planimeter and by dividing this area by the length of the planimetered section. The result, which was the average width of the section was multiplied by the attenuation factor.

Speed for each run was calculated from the distance travelled by
Figure 23. Typical record from oscillograph
the bin and the time of traversing this distance. Spaced marks on the recorder indicated one foot movements of the bin or soil bin position on the tracks. The time of travel between any two marks (the end marks preferred) was calculated from paper speed which was accurate and constant, and which could be checked with the one-second timer marks on the recorder. Evenly spaced position marks together with constant paper speed indicated a constant speed of bin travel.

Actual tillage speeds were calculated from the common ratios of tillage speed to the distance of soil bin travel and of chart speed to the distance between chart blips marking one-foot intervals of soil bin travel. Speed was checked against the speed trace on another channel of the recorder. The tachometer-generator driven from the final drive shaft was calibrated to give an integral ratio of bin speed to the amount of deflection.

All calculations of values of draft and speed from the original records were done using programmed equations for the "Monroe Epic" electronic desk computer.

Subsequent calculations were made with the use of the IBM 360/50 and IBM 360/65 computer at the Iowa State University Computation Center.
RESULTS AND DISCUSSION

Soil Processing Results

In all tests, the similarity requirements imposed by the model theory were satisfactorily met within the limits of the equipment used for the tests. The requirement that the soil properties be equal in model and prototype tests was equivalent to having the soil reprocessed after a test run such that the original condition was restored.

Since a standard method of soil processing was followed and tilling, leveling and compacting of the soil were consistent for all tests, the resulting soil condition was believed to be similar. A section of the soil bin containing a typical, partially-prepared soil is shown in Figure 24. The intermediate phase in soil fitting wherein the soil was leveled, dried and then submerged is shown. As a result of drying, which was under natural conditions, cracks appeared throughout the soil surface. The even distribution of the cracks indicated even drying and/or uniform soil. After saturation by submergence for a few hours, the soil was drained. Any gaps were closed by a roller. After the final leveling and reflooding of the bin to the prescribed depth of water, the soil was ready for a test run. Figure 25 shows a portion of the unplowed soil which was typical of a fitted soil for a test run series.

Although soil mass densities were not exactly equal as required by the design condition, they were believed to be satisfactory for the kind of material and equipment used in this experiment and for practical purposes. The values of soil density expressed in units of pounds per cubic feet are given in Appendix B. Division of weight density by the
Figure 24. Condition of soil after leveling, drying and flooding

Figure 25. Plowed and unplowed portions of soil in bin during a test run
value of gravitational acceleration converts it to mass density. Since acceleration due to gravity is a constant and numerical values of soil mass density were not used directly in the calculations, weight density was used as the index of similarity among soil bin fittings instead of mass density. Weight density values ranged from 100.9 to 103.6 pounds per cubic foot.

The requirement that moisture content was to be equal for all soil bin fittings was easily satisfied because of the flooded condition of the soil. With prolonged soaking, the soil was always in a saturated state. Determinations of soil moisture contents from the same samples used for determining weight density indicated approximately equal moisture conditions for all soil bin fittings. This is shown in Appendix B.

**Relationship between density and moisture content**

From determinations of weight density of the soil during stages in drying, a relationship between weight density and moisture content was obtained. Density determinations were done by the paraffin method. The average weight density of three determinations was plotted versus moisture content as shown in Figure 26. Soil weight density decreased as soil moisture content increased up to the point when the soil was in the submerged state.

The results of this experiment concerning soil density were similar in a certain aspect to those of Gill (8), who conducted investigations of the effect of moisture content on the density of puddled soils of several types. Gill's results indicated two distinct rates of change in density with moisture content. For moisture contents less than about 15 percent,
Figure 26. Relationship between soil bulk density and moisture content, wet basis
the rate of change in density was very small compared to that when moisture content was greater than 15 percent.

The values of moisture content considered in this experiment did not include the lower range which was reported by Gill. There was no apparent change of slope of the density versus moisture content line with moisture content. The important conclusion based on this experiment was that moisture content should be equal in all soil bin fittings in order to maintain equal density values. As was pointed out, this requirement was easily met by submergence of the soil for a few hours.

Model and Prototype Test Results

Description of the plowing action

The trace of the draft of the two-inch plow during a test run is shown in Figure 23. It represented a typical draft reaction of the type of moldboard plows tested under the conditions in the experiment.

Two distinct stages were noted on the draft trace. The first stage was characterized by a peak draft during a few inches of plow travel after initial contact of the plow blade with the soil and by a varying force cycling around a mean draft. The second stage was also characterized by a peak draft and a varying force cycling around a mean draft. This stage gave somewhat higher peak and mean draft values than the first stage.

The variation in draft was apparently caused by the alternate compressive action on and failure of the soil during plowing. This phenomenon resembled a typical action of a tillage tool on the soil like those reported by Nichols and Reed (22) and Larson (14).
In the first stage, the occurrence of the peak force coincided with the point where the soil flow encountered a change in direction on the surface of the plow. The peak force was caused also by the impact force of the plow on the soil. Soil failure then occurred immediately after the peak force and the draft decreased.

Another cause of the peak force was the cutting by the plow of a relatively dense soil stratum at the dikes built at the ends of the test plot to contain the water on the soil. The dikes were made more compact than the test soil to prevent seepage and failure of the dike. Although the upper portion of the dike along which the test plow would run was leveled just before the plow made contact with the soil, some hard soil was encountered by the plow blade because the lower portion of the dike was not removed. It was assumed in subsequent analyses of data that the peak due to this dike did not affect the behavior of the plow after the plow reaction reached steady state conditions. Visual observation of several graphs revealed that this was a reasonable assumption.

Throughout the first stage, the soil tended to build up on the plow surface because of the failure of the soil to scour. While being elevated at a distinctly slower rate than the plow speed, the soil formed a compacted lump exhibiting lines of failure as the plow progressed. At the end of the first stage, the lump of soil dropped to the side of the moldboard leaving thin patches of soil adhering to the plow surface. These patches of soil were believed to be always on the moldboard throughout the test run. It was not known whether the original patches were maintained or whether they gradually moved towards the rear end of the plow as the plowing progressed and were replaced by a newly formed patch or layer of soil.
Nevertheless, at the end of the test run, that is, when the plow had passed through the soil plot, a lump of soil remained on the moldboard surface. When this lump of soil was removed by hand, patches of soil remained on the surface. The soil was compacted and had to be scraped off with a spatula. The moldboard was washed with water before the next test was run.

As the plow entered the soil, a soil ribbon was cut loose from the soil mass by the leading edge of the plow and shin. The soil ribbon was pushed along the moldboard by the succeeding soil. The ribbon tended to adhere to the moldboard until sufficient force was generated to push it over the surface. While the force was being generated by movement of the plow into the soil, the soil ribbon was compressed. The resulting soil ribbon was not continuous; rather, it broke into chunks as it fell away from the moldboard. If for a test run, the plowed ribbon were laid out on a line with the ends of the chunks joined, a comparison of the length of the soil plot could be obtained. Results of a few trials showed that the soil was compressed by approximately one-third of its initial length. Figure 27 shows the partially drained soil bin which was moved back to the starting point of the tracks after the test run. As shown in the figure, the farther end of the soil plot which was the starting point of plowing showed a partial absence of soil ribbon. This was a few inches away from the initial contact point of the plow with the soil. The test run along the right side of the bin was conducted at high speed. The first chunk of soil fell off farther away from the starting point of the plot than at lower speeds. The pool of water in the foreground was due to the
Figure 27. Partially submerged soil after test runs, viewed from direction of plow travel

Figure 28. Ribbons of soil after tests with the 2.0-inch plow
removal of the soil for use in the dike shown in the nearer foreground. This was necessary because at high speeds, the plow cut through the said dike.

The failure of the soil to scour and the occurrence of a changing plow surface due to the sticking of the soil to the plow are probable causes of failure to predict draft by purely analytical means because of the apparent complexity of the phenomenon involved. In a similitude study such as this, such behavior is immaterial in the analysis because, if the similarity requirements are valid and properly satisfied, the prototype and model behave in a similar manner which in essence is all that is required to be able to meet the prediction objective. If similarity of draft trace is one of the criteria of similarity, then it can reasonably be said that the model and prototype plows are similar.

The high peak at the start of the second stage was due to the impact of the plow on the soil against a new soil layer. In the soil compaction by rolling process, the ends of the test plot tended to be less dense than the intermediate portions because no solid barrier was present at the ends of the test plot.

The soil layer which adhered to the plow during the first stage also increased the mean force in this stage. It was observed that an accumulation of soil on the plow surface occurred until a "critical" amount of soil was reached. It was probable that the soil patch which remained on the plow surface formed an integral part of the plow. The succeeding soil ribbon slid over this layer of soil. There were, however, no quantitative measurements made to describe the actual behavior of the soil layer on the plow surface.
Although some soil still adhered to the plow surface at higher speeds, the soil scoured better at higher speeds than at lower speeds. As shown in Figure 25, the ribbon of soil was deflected to the right side of the plow as the plow advanced through the soil. The ribbon of soil was compressed slightly during the plowing action and the ribbon of soil failed in tension as a result of the weight of the overhanging portion towards the right side of the plow.

There was, however, no pulverization of the soil as occurs generally in dryer soils. The soil was firmly held together by cohesion such that breaking up occurred only during separation of the soil ribbon from the moldboard face. The ribbon of soil was turned over about 120 degrees from its original position as shown in Figure 25 and Figure 28. Figure 28 shows a portion of the soil bin out of which the water was partly drained to show the side of the soil ribbon which was originally the bottom of the soil ribbon. This side was in contact with the moldboard face. The roughness of such side indicated that soil particles had adhered to the moldboard and the soil slid along the layer which adhered to the moldboard surface.

**Satisfaction of design conditions for speed**

For the four sets of similarity requirements considered in this study, two design conditions for speed were established.

The design condition for speed was

\[ V_m = V \] \hspace{1cm} \text{Eq. 25}

for the cases where the Pi term \( V^2/gD \) was either neglected or distorted.
The design condition for speed was

\[ V_m = \frac{V}{\sqrt{n}} \quad \text{Eq. 22} \]

for the cases where the pi term \( \rho V^2 / \sigma \) was either neglected or distorted.

Ideally, these two velocity conditions should be satisfied in two separate and independent experiments. The results of the experimental runs, however, indicated that neither of these two requirements could be strictly fulfilled by the equipment used. In other words, there was no precise control of speed. The speed which was set for a test run was not the same as the actual or measured speed.

Since accurate control of speed could not be made, it was decided that data for the above design conditions should be obtained by utilizing a statistical relationship between draft and speed. From such a relationship, the necessary data for analyses were taken. Thus, only one test series was conducted to consider the two design conditions.

**Relationships between draft and speed of plowing**

An increase in speed was generally accompanied by a corresponding increase in draft of the plow. This increase in draft, however, showed a declining rate with a further increase in speed as shown by the data points in Figure 29. The critical speed, which was defined as the speed at which the increase in draft started to taper off, did not seem to be well defined. It was noted, however, that critical speeds were not the same for all plow sizes. In general, the approximate values of critical speeds increased as plow size increased.

The tendency of draft to level off with increase in speed was in contrast with the results of studies of draft and speed of plowing of
Figure 29. Draft-speed data for each plow
dry soil. It was reported by some researchers that draft generally increased non-linearly with speed (33) or sometimes linearly (3 and 4) depending upon the soil conditions and tillage tools used. These researchers, however, worked with non-flooded or dryer soils.

Improved scouring of the soil after the critical speed was a factor in decreasing the rate of rise in draft at lower speeds. It was possible that the effects of soil properties having dimensions of stress (for example, adhesion and cohesion) were greater than the effects of some other factor, perhaps rate of strain (33), in increasing the draft. After some critical speed, however, such initial effects declined because of the relative smoothing effect of soil flow on the moldboard face as a result of scouring.

Due to the limited range of speeds at which the plowing tests were run, no conclusion was made as to the general behavior of the draft of the plows working with flooded soils, that is, whether or not the increase of draft with speed followed an increasing then a decreasing rate pattern. It was entirely possible that draft could increase again at a greater rate at some speeds faster than those considered in this study.

For the purpose of this study, however, the best fit for the data points of each plow test was all that was necessary. The equation of the fitted line was used for all subsequent calculations for the study. In general, the draft of the plows tended to be related with speed by a logarithmic function.
Analysis of Data

Statistical considerations

The establishment of similarity requirements indicated that design conditions for speed of the plow essentially controlled the conduct of experiments. Test results showed that the required speed relationships between the model and the prototype in either the undistorted or distorted model considerations were not exactly met. The values of draft, however, indicated some relationships with speed. These relationships were utilized to satisfy the requirements for speed.

Regression analyses of draft on speed were made in order to estimate from the regression function the mean of a population of draft values corresponding to each speed value. For comparison of results of prediction, linear and logarithmic functions were considered. These types of functions served to demonstrate the technique of model analysis for two forms of equations relating draft and speed of the plow. Davidson, et al., (4) pointed out that both linear and non-linear relationships between draft and speed were reported by some investigators.

Linear relationship The plot of draft versus speed is shown in Figure 30. The regression equations and values of correlation index or coefficient of determination, $r^2$, for each of the plows are as follows:

1.5-inch plow: $R = 12.80 + 1.676 V; r^2 = 0.74$
2.0-inch plow: $R = 18.12 + 2.152 V; r^2 = 0.77$
3.0-inch plow: $R = 29.35 + 2.128 V; r^2 = 0.76$
4.0-inch plow: $R = 47.46 + 3.016 V; r^2 = 0.64 \quad \text{Eq. 75}$
Figure 30. Linear relationship between draft and speed
where $R = \text{draft in pounds}$

$V = \text{speed in feet per second}$.

All regressions gave highly significant correlation coefficients, indicating that there was a linear relation with non-zero slope.

**Non-linear relationship** The slight tendency of draft values to decrease with further increase in speed (Figure 29) suggested a non-linear fit to the points. A linear relationship was assumed to exist between the logarithm of draft and the logarithm of speed. Regression analysis was done for each data set corresponding to each plow size.

The plot of the data points on logarithmic paper is shown in Figure 31. There were indications that the points lie on a straight line. This was regarded as an improvement over the linear plot shown in Figure 30. The regression equations for each plow and the corresponding correlation index are as follows:

1.5-inch plow: $\log R = 1.1376 + 0.2722 \log V; \quad r^2 = 0.84$

2.0-inch plow: $\log R = 1.2831 + 0.2564 \log V; \quad r^2 = 0.85$

3.0-inch plow: $\log R = 1.4805 + 0.1682 \log V; \quad r^2 = 0.74$

4.0-inch plow: $\log R = 1.6831 + 0.1584 \log V; \quad r^2 = 0.76$ Eq. 76

The correlation coefficients were highly significant for all plows and, in general, were slightly higher than those of the linear regression. It was concluded that use of the transformed values gave a better fit than the original non-transformed values of draft and speed.

The regression equations relating the logarithm of draft and the logarithm speed were written in a more convenient form, viz.

$$R = A_1 V^2$$ Eq. 59
Figure 31. Logarithmic relationship between draft and speed
Draft, pounds

Plow speed, Ft/Sec

4.0-inch plow
3.0-inch plow
2.0-inch plow
1.5-inch plow
where $A_1$ = value of $R$ when $V = 1.0$

$V$ = speed of the plow

$z$ = slope of the regression line

**Undistorted model analysis**

Previous model analysis (see page 41) showed that undistorted models would result if either of the pi terms $\rho V^2/\sigma$ and $V^2/gD$ were neglected. Experimental data were used to determine the validity of an undistorted model for the model-prototype system considered in this study.

$V^2/gD$ neglected The speed design condition and the prediction equation for this case were given by

$$V_m = V \quad \text{Eq. 25}$$

and

$$R = n^2 R_m \quad \text{Eq. 27}$$

respectively.

Predicted values of draft for the 4.0-inch prototype were calculated from the actual or observed values of draft for each of the 1.5-inch, 2.0-inch and 3.0-inch models. Since the design condition required that $V_m = V$, the data points were selected such that the model sizes had approximately the same speeds as the 4.0-inch plow. The criterion for selecting the speeds and hence the appropriate draft data was that the speed of the models should not differ from the design speed by more than 0.1 foot per second. The speed value used in subsequent calculations was the average of the selected speeds of the plow sizes.
Table 9 shows the predicted values of the draft of the 4.0-inch plow compared with the actual or observed values of draft at a given speed. The error of prediction was calculated as follows:

\[
\text{Error of prediction} = \frac{\text{Predicted draft} - \text{Observed draft}}{\text{Observed draft}} \times 100
\]

Eq. 77

where the error of prediction is in percent.

The predicted draft values were greater than the actual or observed draft values at all speeds considered. The predicted draft values from the 3.0-inch and the 2.0-inch models, however, were closer to the observed prototype draft values than those from the 1.5-inch model. The error of prediction ranged from 42.51 percent to 170.67 percent. These results suggested that not only was distortion present in the model system considered in this case but also that the degree of distortion believed to be the influence of the \( \pi \) term \( \frac{V^2}{gD} \) varied with the size of the model or the length scale. This distortion was influenced by the neglected \( \pi \) term \( \frac{V^2}{gD} \). Therefore, it was concluded that an undistorted model analysis based upon consideration of the \( \pi \) term \( \frac{\rho V^2}{\sigma} \) and not of the \( \pi \) term \( \frac{V^2}{gD} \) was invalid for the phenomenon considered in this study.

The use of the regression equations in the calculations of predicted values of draft of the 4.0-inch plow was also investigated. The results are shown in Table 10. The "observed" draft values of the 4.0-inch prototype were those calculated from the regression lines for the 4.0-inch plow. The predicted draft values resulted from the use of two forms of draft-speed relationships, namely, the linear function and the logarithmic function. This procedure was essentially a prediction of
Table 9. Comparison of predicted and observed values of draft for the 4.0-inch plow, undistorted model theory, \( \tau_0 = V^2/gD \) neglected; use of observed data

<table>
<thead>
<tr>
<th>Plow speed ft./sec.</th>
<th>Predicted draft, lb., from model</th>
<th>Average predicted draft lb.</th>
<th>Observed draft lb.</th>
<th>Error of prediction %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.068</td>
<td>125.40</td>
<td>125.40</td>
<td>46.33</td>
<td>170.67</td>
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<tr>
<td>1.476</td>
<td>89.84</td>
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<td>51.59</td>
<td>42.51</td>
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<td>1.736</td>
<td>113.80</td>
<td>100.84</td>
<td>54.00</td>
<td>86.74</td>
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<td>114.52</td>
<td>87.64</td>
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<td>5.960</td>
<td>142.25</td>
<td>75.23</td>
<td>66.86</td>
<td>66.63</td>
</tr>
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</table>
Table 10. Comparison of predicted and observed draft for the 4.0-inch plow; undistorted model theory, \( \tau_0 = V^2/gh \) neglected; use of data from draft-speed relationships

<table>
<thead>
<tr>
<th>4.0-inch plow speed ft./sec</th>
<th>Predicted draft, lb., from model 1.5-inch plow</th>
<th>Predicted draft, lb., from model 2.0-inch plow</th>
<th>Predicted draft, lb., from model 3.0-inch plow</th>
<th>Average predicted draft lb.</th>
<th>Observed draft lb.</th>
<th>Error of prediction %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>96.92</td>
<td>77.12</td>
<td>53.74</td>
<td>75.93</td>
<td>48.34</td>
<td>57.07</td>
</tr>
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<td>102.94</td>
<td>81.13</td>
<td>55.96</td>
<td>80.01</td>
<td>50.48</td>
<td>58.50</td>
</tr>
<tr>
<td>2.0</td>
<td>116.50</td>
<td>90.95</td>
<td>60.38</td>
<td>89.28</td>
<td>53.30</td>
<td>67.50</td>
</tr>
<tr>
<td></td>
<td>114.86</td>
<td>89.74</td>
<td>59.78</td>
<td>86.11</td>
<td>53.49</td>
<td>64.72</td>
</tr>
<tr>
<td>3.0</td>
<td>129.74</td>
<td>100.16</td>
<td>64.64</td>
<td>98.18</td>
<td>56.43</td>
<td>73.98</td>
</tr>
<tr>
<td></td>
<td>126.78</td>
<td>96.34</td>
<td>63.53</td>
<td>96.22</td>
<td>56.51</td>
<td>70.27</td>
</tr>
<tr>
<td>4.0</td>
<td>140.03</td>
<td>107.25</td>
<td>67.89</td>
<td>105.14</td>
<td>58.77</td>
<td>78.73</td>
</tr>
<tr>
<td></td>
<td>138.76</td>
<td>106.95</td>
<td>67.31</td>
<td>104.32</td>
<td>59.32</td>
<td>75.27</td>
</tr>
<tr>
<td>5.0</td>
<td>148.57</td>
<td>113.10</td>
<td>70.64</td>
<td>110.70</td>
<td>60.64</td>
<td>82.55</td>
</tr>
<tr>
<td></td>
<td>150.62</td>
<td>113.56</td>
<td>71.09</td>
<td>112.42</td>
<td>62.54</td>
<td>79.76</td>
</tr>
<tr>
<td>6.0</td>
<td>155.94</td>
<td>118.11</td>
<td>72.63</td>
<td>115.56</td>
<td>62.22</td>
<td>85.73</td>
</tr>
<tr>
<td></td>
<td>162.53</td>
<td>126.17</td>
<td>74.83</td>
<td>120.53</td>
<td>65.56</td>
<td>83.85</td>
</tr>
</tbody>
</table>

aUpper value resulted from use of draft-speed relationship, \( R = A_1 V^2 \) while lower value resulted from use of draft-speed relationship, \( R = a_1 + b_1 V \).
the regression line for the 4.0-inch plow using the regression lines for the 1.5-inch, 2.0-inch and 3.0-inch plows.

There was greater variability in the actual or observed values of draft for the 4.0-inch plow than for the smaller plows as shown in Figures 29, 30 and 31. Even if the predicted values were identical with the "observed" values, that is, a zero error of prediction occurred, the predicted values of draft using actual test values would have some error due to the deviation of the actual test values from the estimated values given by the regression line.

As a result of this procedure, the errors of prediction showed a consistently increasing trend as the speed of plowing increased. This was in contrast with the results of the procedure using the actual test values in prediction. The 3.0-inch plow gave the closest predicted draft values among the three plows used as models. The trend was that the closer the value of the length scale to unity, the more accurate was the prediction.

The procedure of using regression lines in predicting values from another regression line gave prediction errors ranging from 57.07 percent to 85.73 percent as shown in Table 10. The linear relationship gave somewhat lower errors of prediction than the logarithmic relationship for speeds greater than or equal to 2.0 feet per second. The magnitude of the errors, however, were large for both relationships. Therefore, the undistorted model analysis was not considered valid.
The prediction equation for the undistorted model resulting from neglecting \( \rho V^2/\sigma \) and considering \( V^2/gD \) was

\[
R = n^3 R_m
\]

Eq. 29

In this case, the speed design condition was

\[
V_m = \frac{V}{\sqrt{n}}
\]

Eq. 22

Calculation of predicted values of draft for the 4.0-inch plow using observed values of draft from any of the three model plows showed very large errors in prediction. This is shown in Table 11. The data points were selected such that the speed of the model satisfied the required model speed (Equation 22) for a given prototype speed. The criterion for the selection of the data point for a model was that the speed of the model should be equal to the designed speed \( \pm 0.1 \) foot per second. Whenever two or more model sizes were used in predicting the draft of the 4.0-inch plow, the average predicted value was calculated.

The error of prediction ranged from 46.45 percent to 522.85 percent. All models gave predicted values of draft which were much higher than the observed values for the 4.0-inch plow. Of the three models used to predict draft for the 4.0-inch plow, the 3.0-inch plow gave the closest value to the observed draft. It was concluded that distortion was present and that this distortion was influenced by the neglected \( \pi \) term and the size of the model or the value of the length scale.

Another approach to the prediction of draft of the 4.0-inch plow was the use of the draft-speed relationships established for each plow. In Table 12, the non-linear (logarithmic) and linear functions were
Table 11. Comparison of predicted and observed values of draft for the 4.0-inch plow, undistorted model theory, $\eta_{10} = \rho V^2 / \alpha$ neglected; use of observed data

<table>
<thead>
<tr>
<th>4.0-inch plow speed (ft./sec.)</th>
<th>Predicted draft, lb., from 3.0-inch plow</th>
<th>Average predicted draft, lb.</th>
<th>Observed draft, lb.</th>
<th>Error of prediction %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.991</td>
<td>----- 137.36</td>
<td>137.36</td>
<td>52.00</td>
<td>164.15</td>
</tr>
<tr>
<td>1.057</td>
<td>----- 80.00</td>
<td>80.00</td>
<td>41.92</td>
<td>90.84</td>
</tr>
<tr>
<td>1.071</td>
<td>----- 152.96</td>
<td>73.80</td>
<td>113.38</td>
<td>43.74</td>
</tr>
<tr>
<td>1.377</td>
<td>264.82</td>
<td>-----</td>
<td>264.82</td>
<td>53.30</td>
</tr>
<tr>
<td>1.507</td>
<td>227.64</td>
<td>-----</td>
<td>185.82</td>
<td>51.59</td>
</tr>
<tr>
<td>1.747</td>
<td>326.85</td>
<td>164.80</td>
<td>56.24</td>
<td>189.30</td>
</tr>
<tr>
<td>1.949</td>
<td>----- 157.04</td>
<td>-----</td>
<td>157.04</td>
<td>56.00</td>
</tr>
<tr>
<td>2.005</td>
<td>----- 189.84</td>
<td>-----</td>
<td>189.84</td>
<td>54.15</td>
</tr>
<tr>
<td>2.218</td>
<td>----- 161.44</td>
<td>80.60</td>
<td>121.02</td>
<td>53.68</td>
</tr>
<tr>
<td>2.860</td>
<td>----- 128.00</td>
<td>83.13</td>
<td>105.57</td>
<td>59.42</td>
</tr>
<tr>
<td>3.030</td>
<td>312.05</td>
<td>-----</td>
<td>312.05</td>
<td>50.10</td>
</tr>
<tr>
<td>3.058</td>
<td>----- 86.45</td>
<td>-----</td>
<td>86.45</td>
<td>59.03</td>
</tr>
<tr>
<td>3.317</td>
<td>326.47</td>
<td>198.88</td>
<td>94.97</td>
<td>206.77</td>
</tr>
<tr>
<td>3.406</td>
<td>----- 210.88</td>
<td>88.96</td>
<td>149.92</td>
<td>63.81</td>
</tr>
</tbody>
</table>

115
Table 11. (Continued)

<table>
<thead>
<tr>
<th>4.0-inch plow speed ft./sec.</th>
<th>Predicted draft, lb., from model plow</th>
<th>Average predicted draft lb.</th>
<th>Observed draft lb.</th>
<th>Error of prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.067</td>
<td>220.00</td>
<td>151.63</td>
<td>56.72</td>
<td>167.33</td>
</tr>
<tr>
<td>4.303</td>
<td>210.48</td>
<td>210.48</td>
<td>56.74</td>
<td>270.96</td>
</tr>
<tr>
<td>5.298</td>
<td>398.37</td>
<td>249.02</td>
<td>49.51</td>
<td>402.97</td>
</tr>
<tr>
<td>5.603</td>
<td>357.01</td>
<td>231.94</td>
<td>60.89</td>
<td>280.92</td>
</tr>
<tr>
<td>5.836</td>
<td>244.08</td>
<td>169.52</td>
<td>66.86</td>
<td>154.05</td>
</tr>
</tbody>
</table>

4.0-inch Predicted draft, lb., from model plow speed 1.5-inch 2.0-inch 3.0-inch plow speed.
Table 12. Comparison of predicted and observed draft for the 4.0-inch plow; undistorted model theory, $\pi_{10} = \rho V^2 / \sigma$ neglected; use of data from draft-speed relationships

<table>
<thead>
<tr>
<th>4.0-inch plow speed ft./sec.</th>
<th>Predicted draft, lb., from model</th>
<th>Average predicted draft lb.</th>
<th>Observed draft lb.</th>
<th>Error of prediction %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>227.83 140.58 69.94</td>
<td>146.12</td>
<td>48.21</td>
<td>203.09</td>
</tr>
<tr>
<td></td>
<td>262.19 157.21 73.94</td>
<td>164.45</td>
<td>50.48</td>
<td>225.77</td>
</tr>
<tr>
<td>2.0</td>
<td>275.13 167.65 78.59</td>
<td>173.79</td>
<td>53.80</td>
<td>223.03</td>
</tr>
<tr>
<td></td>
<td>281.65 169.39 78.31</td>
<td>176.45</td>
<td>53.49</td>
<td>229.87</td>
</tr>
<tr>
<td>3.0</td>
<td>307.24 185.83 84.13</td>
<td>192.40</td>
<td>57.37</td>
<td>235.37</td>
</tr>
<tr>
<td></td>
<td>301.11 181.56 82.68</td>
<td>188.45</td>
<td>56.51</td>
<td>233.48</td>
</tr>
<tr>
<td>4.0</td>
<td>332.26 199.92 88.30</td>
<td>206.83</td>
<td>60.05</td>
<td>244.43</td>
</tr>
<tr>
<td></td>
<td>320.57 193.73 87.04</td>
<td>200.45</td>
<td>59.52</td>
<td>236.78</td>
</tr>
<tr>
<td>5.0</td>
<td>353.07 211.58 91.67</td>
<td>218.77</td>
<td>62.21</td>
<td>251.66</td>
</tr>
<tr>
<td></td>
<td>340.04 205.91 91.41</td>
<td>212.45</td>
<td>62.54</td>
<td>239.70</td>
</tr>
<tr>
<td>6.0</td>
<td>371.03 221.61 94.53</td>
<td>229.06</td>
<td>64.03</td>
<td>257.74</td>
</tr>
<tr>
<td></td>
<td>359.50 218.08 95.78</td>
<td>224.45</td>
<td>65.56</td>
<td>242.56</td>
</tr>
</tbody>
</table>

\(^a\)Upper value resulted from use of draft-speed relationship, $R = A_1 V^2$ while lower value resulted from use of draft-speed relationship, $R = a_1 + b_1 V$. 
considered in the calculations of the predicted values of draft. Again, each of the three plows (1.5-inch, 2.0-inch and 3.0-inch) was paired with the 4.0-inch plow to form a model-prototype combination.

The results of calculations as shown in Table 12 indicated similar trends as in the previous results; that is, there was no indication of a good prediction using undistorted model theory. The error of prediction ranged from 203.09 percent to 257.74 percent. Among the three model plows, the 3.0-inch plow gave predicted values closest to the "observed" values. With this approach, prediction was not improved. It was concluded that the pi term $\rho V^2/\sigma$ should not be neglected.

**Distorted model analysis**

As discussed in Model Analysis of Tillage of Submerged Soils, similarity requirement set 3 provided for distortion of the pi term $\pi_9 = V^2/gD$ while similarity requirement set 4 provided for distortion of the pi term $\pi_{10} = \rho V^2/\sigma$. The experimental data were analyzed according to each case of distortion.

**Distortion of $V^2/gD$**  The prediction equation for this case was given by

$$R = \delta_1 n^2 R_m$$  
Eq. 31

The prediction factor $\delta_1$ was not known and was expressed as the functional relationship

$$\delta_1 = \phi_2(n, V^2/gD)$$  
Eq. 39

The expression for $\delta_1$ depended upon the form of the draft-speed relationship considered. The relationships were the non-linear
(logarithmic) and the linear functions. In either case, the same procedure in determining the general equation for the prediction factor was followed.

In the calculation of $\delta_1$, values of $V$ were needed. At any given $V^2/gD$, which was chosen such that the calculated $V$ or $V_m$ were within the range of the test speeds, the value of $D$ was taken as that for the prototype in a particular model-prototype combination. With the three sizes of plows, six combinations were possible giving six values of $n$.

It was assumed that draft test results could be repeated with the use of the same plow under the same soil and operating conditions. This implied that no distortion was present for $n = 1$ and that differences in draft values under such conditions were due to experimental errors.

Therefore, there was a total of seven $n$ values. The model-prototype combinations are shown in Table 13. Actually, the last three combinations were the reciprocal combinations of the first three. This

Table 13. Possible model-prototype combinations from three sizes of plows

<table>
<thead>
<tr>
<th>Combination number</th>
<th>Model inches</th>
<th>Prototype inches</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
<td>3.0</td>
<td>2.00</td>
</tr>
<tr>
<td>2</td>
<td>2.0</td>
<td>3.0</td>
<td>1.50</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>2.0</td>
<td>1.33</td>
</tr>
<tr>
<td>4</td>
<td>2.0</td>
<td>2.0</td>
<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>3.0</td>
<td>1.5</td>
<td>0.50</td>
</tr>
<tr>
<td>6</td>
<td>3.0</td>
<td>2.0</td>
<td>0.67</td>
</tr>
<tr>
<td>7</td>
<td>2.0</td>
<td>1.5</td>
<td>0.75</td>
</tr>
</tbody>
</table>
arrangement provided information on the manner of the behavior of a curve in which \( n \) was an independent variable. It should be pointed out, however, that the last three combinations were not independent of the first three and therefore, not valid for use in statistical analysis.

Case 1. **Draft-speed relationship of the form** \( R = AV^z \) **Values** of the prediction factor \( \delta_1 \) were calculated from the equation

\[
\delta_1 = \frac{A_1V^z}{(A_{1m}V_m^{z_m})n^2}
\]

Eq. 78

for each combination of model and prototype formed from the 1.5-inch, 2.0-inch and 3.0-inch plows. Values of \( \delta_1 \) at given values of \( V^2/gD \) were plotted versus \( n \) on logarithmic paper. This plot is shown in Figure 32 for \( V^2/gD = 1.0 \). The straight-line plot of \( \delta_1 \) versus \( n \) at given \( V^2/gD \) indicated that the equation for \( \delta_1 \) was of the form

\[
\delta_1 = n^{-x_1}
\]

Eq. 79

where \( x_1 \) is the slope of the line. In this particular case, the slope was negative. Since the slope of this line varied with the value of \( V^2/gD \), the absolute value of \( x_1 \) was expressed by the relationship

\[
|x_1| = h \left( \frac{V^2}{gD} \right)
\]

Eq. 80

The plot of the absolute values of \( x_1 \) versus \( V^2/gD \) as shown in Figure 33 gave the following relationship:

\[
|x_1| = 1.015 \left( \frac{V^2}{gD} \right)^{0.0867}
\]

Eq. 81
Figure 32. Plot of the prediction factor $\delta_1$ versus the length scale $n$ using data from combinations of three model plows, $V^2/gD = 1.0$.

The equation for $\delta_1$ is:

$$\delta_1 = n - 1.015(V^2/gD)^{0.0867}$$
\[ |\dot{x}_1| = 1.015 \left( \frac{V^2}{gD} \right)^{0.0867} \]

Figure 33. Relationship between the slope of \( \delta_1 \) versus \( n \) curves and \( \frac{V^2}{gD} \)
Substitution of Equation 81 into Equation 79 gave

$$\delta_1 = n^{-1.015} \left( \frac{V^2}{gD} \right)^{0.0867}$$

Eq. 82

Therefore, the prediction equation was given by

$$R = n^{-1.015} \left( \frac{V^2}{gD} \right)^{0.0867} n^2 R_m$$

Eq. 83

In order to test the validity of the similarity requirement and the technique used in determining the prediction equation, the observed values of draft of the model plows were used in the comparison of the predicted and observed values of draft of the 4.0-inch plow instead of the points from the fitted line for logarithm of draft versus logarithm of speed. The data sets were examined for certain values of speed which were approximately the same as those of the 4.0-inch prototype. The criterion for selection of speed was that the speed of the model should be equal to the speed of the prototype \( \pm 0.1 \) foot per second.

With the use of Equation 83, the values of \( R \) were calculated for each combination of the 1.5-inch, 2.0-inch and 3.0-inch models with the 4.0-inch prototype. For a given \( V^2/gD \), the value of \( V \) was calculated using the value of \( D \) as that of the 4.0-inch prototype. Whenever two or more models were used in predicting the prototype draft at a certain speed, the average of the predicted values from the models was taken as the predicted value.

The results are shown in Table 14. The error of prediction ranged from 0.58 percent to 22.70 percent. This was considered an improvement
Table 14. Comparison of predicted and observed values of draft\(^a\) for the 4.0-inch plow; distorted model theory, \(\eta_g = \frac{V^2}{gD}\) distorted; draft-speed relationship of the form \(R = A_1V^2\)

<table>
<thead>
<tr>
<th>Plow speed ft./sec.</th>
<th>Predicted draft, lb., from model 1.5-inch plow</th>
<th>Predicted draft, lb., from model 2.0-inch plow</th>
<th>Predicted draft, lb., from model 3.0-inch plow</th>
<th>Average predicted draft lb.</th>
<th>Observed draft lb.</th>
<th>Error of prediction %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.076</td>
<td>53.93</td>
<td>40.31</td>
<td>45.56</td>
<td>46.60</td>
<td>46.33</td>
<td>0.58</td>
</tr>
<tr>
<td>1.476</td>
<td>-----</td>
<td>48.68</td>
<td>44.37</td>
<td>46.52</td>
<td>51.60</td>
<td>9.82</td>
</tr>
<tr>
<td>1.736</td>
<td>46.65</td>
<td>46.80</td>
<td>-----</td>
<td>46.72</td>
<td>54.00</td>
<td>13.48</td>
</tr>
<tr>
<td>1.940</td>
<td>46.13</td>
<td>46.29</td>
<td>46.33</td>
<td>46.25</td>
<td>56.00</td>
<td>17.41</td>
</tr>
<tr>
<td>2.241</td>
<td>45.59</td>
<td>49.64</td>
<td>-----</td>
<td>47.61</td>
<td>53.68</td>
<td>11.30</td>
</tr>
<tr>
<td>3.047</td>
<td>52.21</td>
<td>52.54</td>
<td>48.09</td>
<td>50.94</td>
<td>57.08</td>
<td>10.75</td>
</tr>
<tr>
<td>3.366</td>
<td>52.08</td>
<td>54.25</td>
<td>46.59</td>
<td>50.97</td>
<td>61.51</td>
<td>17.13</td>
</tr>
<tr>
<td>3.983</td>
<td>48.73</td>
<td>54.58</td>
<td>50.54</td>
<td>51.28</td>
<td>56.69</td>
<td>9.54</td>
</tr>
<tr>
<td>5.308</td>
<td>53.01</td>
<td>46.69</td>
<td>48.97</td>
<td>49.555</td>
<td>59.51</td>
<td>16.73</td>
</tr>
<tr>
<td>5.597</td>
<td>52.26</td>
<td>-----</td>
<td>53.38</td>
<td>52.82</td>
<td>60.89</td>
<td>13.25</td>
</tr>
<tr>
<td>5.960</td>
<td>47.14</td>
<td>53.50</td>
<td>54.43</td>
<td>51.69</td>
<td>66.86</td>
<td>22.70</td>
</tr>
</tbody>
</table>

\(^a\)Prediction factor equation: \(\delta = 1 - 1.015 \left(\frac{v^2}{gD}\right)^{0.0867}\)
In prediction over any of the two cases considered in the undistorted model analysis.

In general, the predicted values of draft were lower than the observed values. The departure of the predicted values from the observed values was explained by the differences in slopes of the lines fitted to plots of $\delta_1$ versus $n$ as shown in Figure 34. One plot considered data from three plow sizes and the other considered data from four plow sizes. The first one was essentially a four-point fit and the second was essentially a seven-point fit. The deviation of the four-point fit from the seven-point fit suggested the degree of accuracy of predicted values. More accurate prediction could be obtained with the use of a larger number of auxiliary models.

**Case 2. Draft-speed relationship of the form $R = a + bV$**

Values of the prediction factor $\delta_1$ were calculated from the equation

$$\delta_1' = \frac{a_1 + b_1V}{a_{1m} + b_{1m}V_m}$$  \hspace{1cm} \text{Eq. 84}

for each combination of three plow sizes. The plot of $\delta_1'$ versus $n$ at $V^2/gD = 1.0$ is shown in Figure 35. The points lie approximately in a straight line on logarithmic paper. Thus the equation of the line was represented by

$$\delta_1' = n^{-x_1'}$$  \hspace{1cm} \text{Eq. 85}

where $x_1'$ is the slope of the line.

For other values of $V^2/gD$ the corresponding plots of $\delta_1'$ versus $n$ were also straight lines but were of different slopes which were negative.
Figure 34. Plots of $\delta_1$ versus $n$ obtained by fitting from results of three and four plows, $V^2/gD = 1.0$
Figure 35. Relationship between $\delta_1$ and $n$
Also, by the assumption of no distortion at $n = 1$, the lines passed through $\delta_1' = 1$. The dotted line shows the relationship for $V^2/gD = 3.0$. Therefore, $x_1'$ was a function of $V^2/gD$. The plot of the absolute values of $x_1'$ versus $V^2/gD$ is shown in Figure 36. This result was that the absolute value of $x_1'$ had a straight line relationship with $V^2/gD$ in the logarithmic plot. The equation for the slope was expressed by

$$|x_1'| = 0.944 (V^2/gD)^{0.074}$$

Eq. 86

where $|x_1'|$ = the absolute value of the slope. Therefore, the prediction factor was given by

$$\delta_1' = n^{-0.944} (V^2/gD)^{0.074} \frac{R_m}{R}$$

Eq. 87

With the use of Equation 27 the prediction equation was expressed as

$$R = n^{-2} n^{-0.944} (V^2/gD)^{0.074} \frac{R_m}{R}$$

Eq. 88

Predicted values of draft of the 4.0-inch plow were calculated according to Equation 88. As in Case 1, the observed values of $R_m$ were used in the prediction equation instead of the values taken from the regression line. The speed values used in Case 1 were the same as those used in Case 2. The results are shown in Table 15.

In general, the predicted values of draft of the 4.0-inch plow were lower than the observed values. Again, the error in prediction was explained by differences in slopes of fitted lines using three and four plow sizes in plotting the prediction factor versus the length scale. The trend was similar to that shown in Figure 34 for Case 2.

In conclusion, for the two cases considered here, the prediction
\[ |x_1'| = 0.944 \left( \frac{x_1^2}{gD} \right)^{0.074} \]

Figure 36. Relationship between \( |x_1'| \) and \( \frac{V^2}{gD} \)
Table 15. Comparison of predicted and observed values of draft\textsuperscript{a} for the 4.0-inch plow; distorted model theory, \( \eta_9 = \frac{V^2}{gD} \) distorted; draft-speed relationship of the form \( R = a_1 + b_1V \)

<table>
<thead>
<tr>
<th>Plow speed ft./sec.</th>
<th>Predicted draft, lb. from model</th>
<th>Average predicted draft lb.</th>
<th>Observed draft lb.</th>
<th>Error of prediction %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.076</td>
<td>55.88</td>
<td>46.03</td>
<td>47.75</td>
<td>46.33</td>
</tr>
<tr>
<td>1.476</td>
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<td>44.94</td>
<td>47.58</td>
<td>51.59</td>
</tr>
<tr>
<td>1.736</td>
<td>48.99</td>
<td>-----</td>
<td>48.71</td>
<td>54.00</td>
</tr>
<tr>
<td>1.940</td>
<td>48.61</td>
<td>47.04</td>
<td>47.89</td>
<td>56.00</td>
</tr>
<tr>
<td>2.241</td>
<td>48.25</td>
<td>-----</td>
<td>49.96</td>
<td>53.68</td>
</tr>
<tr>
<td>3.047</td>
<td>55.83</td>
<td>49.04</td>
<td>53.32</td>
<td>57.08</td>
</tr>
<tr>
<td>3.366</td>
<td>55.89</td>
<td>57.03</td>
<td>53.49</td>
<td>61.51</td>
</tr>
<tr>
<td>3.983</td>
<td>52.62</td>
<td>57.62</td>
<td>53.97</td>
<td>56.69</td>
</tr>
<tr>
<td>5.308</td>
<td>57.88</td>
<td>49.68</td>
<td>52.60</td>
<td>59.51</td>
</tr>
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<td>5.597</td>
<td>57.19</td>
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<td>54.81</td>
<td>60.89</td>
</tr>
<tr>
<td>5.960</td>
<td>51.72</td>
<td>57.12</td>
<td>55.93</td>
<td>66.86</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Prediction factor equation: \( \delta_1 = n_{-0.944} \left( \frac{V^2}{gD} \right)^{0.074} \)
equation could be obtained by the use of the technique described. The situation that plots which were used in determining the expression for $\delta_1$ were straight lines on logarithmic paper gave particular convenience in the calculations of predicted values. The technique, however, could be extended to situations which do not behave in a manner similar to the two preceding cases.

**Distortion of $\rho V^2/\sigma$** From the discussion on model analysis, the prediction equation for the case where the $p_i$ term $\rho V^2/\sigma$ was distorted was given by

$$R = \delta_2 n^3 R_m$$  \hspace{2cm} Eq. 32

and the speed design condition was given by

$$V_m = V/\sqrt{n}$$  \hspace{2cm} Eq. 22

**Case 1. Draft-speed relationship of the form $R = AV^z$** For the case where the draft versus speed test data plotted as a straight line on logarithmic paper, the expression for the prediction factor $\delta_2$ was written as

$$\delta_2 = \frac{A_1 V^z}{A_{1m} V_{m}^{z_m n^3}}$$  \hspace{2cm} Eq. 60

or

$$\delta_2 = C_1 V^{z-m} n^2 - \frac{6-z_m}{n}$$  \hspace{2cm} Eq. 62

Equation 62 was written in general as

$$\delta_2 = f_1(n) V^{m} n^{1-h_1(n)}$$  \hspace{2cm} Eq. 63
Data from the 1.5-inch, 2.0-inch and 3.0-inch plows were used to develop the equation for $\delta_2$. As in the case for $\delta_1$, there were six possible model-prototype combinations giving six length scale values plus the value of $n$ equals unity.

For a particular plow size considered as a model of another plow size, the values of $A_1/A_m$, $z-z_m$ and $(6-z_m)/2$ were calculated using the parameters of the regression equations. Here, $A_1/A_m$ or $C_1$ represented the ratio of the value of draft in the prototype to that in the model when $V = 1.0$ foot per second; $z-z_m$ represented the difference between the slopes of the regression lines of the prototype and the model; and $(6-z_m)/2$ represented the exponent of $n$ in the equation for $\delta_2$ (Equation 62).

The relationship between $C_1$ and $n$ is shown in Figure 37. An equation was fitted through the points calculated from different combinations of model and prototype. This equation was

$$C_1 = f_1(n) = n^{1.137}$$

Eq. 89

Similarly, the functional relationships

$$z-z_m = g_1(n)$$

Eq. 90

and

$$\frac{6-z_m}{2} = h_1(n)$$

Eq. 91

were determined from plots shown in Figure 38 and Figure 39, respectively. The relationships are as follows:
Figure 37. Relationship between $C_1$ and $n$

$C_1 = n^{1.137}$

$C_1 = A_1 / A_{1m}$

LENGTH SCALE, $n$
Figure 38. Relationship between $z - z_m$ and $n$

\[
z - z_m = -\frac{\log n}{2.627}
\]
Figure 39. Relationship between $(6-z_m)/2$ and $n$
\[ g_1(n) = -\frac{\log n}{2.627} \quad \text{Eq. 92} \]
\[ h_1(n) = 2.920 - 0.033n \quad \text{Eq. 93} \]

Substitution of the above functions into Equation 62 gave the equation for \( \delta_2 \) as follows:
\[ \delta_2 = n \left( \frac{1.137 - \frac{\log n}{2.627}}{2.627} \right) - (2.920 - 0.033n) \quad \text{Eq. 94} \]

or
\[ \delta_2 = n \left( \frac{-1.783 + 0.033n}{2.627} \right) - \frac{\log n}{2.627} \quad \text{Eq. 95} \]

The prediction equation was written as
\[ R = n \left( -1.783 + 0.033n \right) \left( \frac{\log n}{2.627} \right) + 2.627 \cdot n^3 \cdot R_m \quad \text{Eq. 96} \]

or
\[ R = n \left( 1.217 + 0.033n \right) \left( \frac{\log n}{2.627} \right) \quad \text{Eq. 97} \]

Each of the three model plow sizes was paired with the 4.0-inch plow which was considered as the prototype. The resulting prediction equation for each of the following model plows paired with the 4.0-inch plow are as follows:

- 1.5-inch: \( R = 3.597 \sqrt{\frac{-0.1622}{R_m}} \)
- 2.0-inch: \( R = 2.434 \sqrt{\frac{-0.1146}{R_m}} \)
- 3.0-inch: \( R = 1.437 \sqrt{\frac{-0.0475}{R_m}} \quad \text{Eq. 98} \)

In order to test the validity of the analysis, the observed data rather than the fitted data for the model plows were used in the calculation of the predicted values of draft of the 4.0-inch plow. The
average of the predicted values were compared with the observed draft values of the 4.0-inch plow. Here, the relationship between the speed of the model and that of the prototype must be related as given by

\[ V_m = \frac{V}{\sqrt{n}} \]  

Eq. 22

The required speed of the model was calculated for each observed speed of the 4.0-inch plow, and then the draft value of the model was considered whenever the observed model speed was equal to the said calculated speed \( \pm 0.1 \) foot per second.

The comparison between the predicted and observed values are shown in Table 16.

An alternate method used to determine the relationship for the prediction factor \( \delta \) was by plotting \( \delta \) versus \( n \) at given speeds since

\[ \delta = \delta_3(n, V) \]  

Eq. 53

This procedure was similar to the method of finding the relationship of \( \delta_1 \) with \( n \) and \( V^2/gD \). To distinguish the prediction factor calculated by this method from that calculated from the preceding, \( \delta_2 \) will be called \( \delta_2^* \).

Model-prototype combinations from the 1.5-inch, 2.0-inch and 3.0-inch plows gave values of \( n \) and \( \delta_2^* \) at selected speeds.

The plots of \( \delta_2^* \) versus \( n \) at given values of \( V \) are shown in Figure 40. The points lie on a straight line for each given \( V \) on logarithmic paper. Therefore, the equation of the line at \( V \) equals a constant was

\[ \delta_2^* = n^{x_2} \]  

Eq. 99
Table 16. Comparison of predicted and observed values of draft\(^a\) for the 4.0-inch plow; distorted model theory, \(\eta_0 = \frac{\rho V^2}{\sigma}\) distorted; draft-speed relationships of the form \(R = A_1 V^2\)

<table>
<thead>
<tr>
<th>4.0-inch plow speed ft./sec.</th>
<th>Predicted draft, lb., from model</th>
<th>Average predicted draft lb.</th>
<th>Observed draft lb.</th>
<th>Error of prediction %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.991</td>
<td>-----</td>
<td>41.82</td>
<td>-----</td>
<td>52.00</td>
</tr>
<tr>
<td>1.057</td>
<td>35.65</td>
<td>-----</td>
<td>35.65</td>
<td>41.92</td>
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<td>1.071</td>
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<td>46.16</td>
<td>44.63</td>
<td>43.74</td>
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<td>1.377</td>
<td>47.68</td>
<td>-----</td>
<td>47.68</td>
<td>53.30</td>
</tr>
<tr>
<td>1.507</td>
<td>40.39</td>
<td>41.79</td>
<td>41.09</td>
<td>51.59</td>
</tr>
<tr>
<td>1.747</td>
<td>56.62</td>
<td>47.02</td>
<td>45.04</td>
<td>54.00</td>
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<tr>
<td>1.949</td>
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<td>44.25</td>
<td>44.25</td>
<td>56.00</td>
</tr>
<tr>
<td>2.005</td>
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<td>53.32</td>
<td>53.32</td>
<td>54.15</td>
</tr>
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<td>2.218</td>
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<td>44.82</td>
<td>47.08</td>
<td>53.68</td>
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<tr>
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<td>3.058</td>
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<td>48.17</td>
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<tr>
<td>3.118</td>
<td>49.21</td>
<td>40.18</td>
<td>44.70</td>
<td>64.91</td>
</tr>
</tbody>
</table>

\(^a\)Prediction factor equation: \(\delta_2 = n^{0.1738 + 0.033n + \frac{\log n}{2.627}}\)
Table 16. (Continued)

<table>
<thead>
<tr>
<th>4.0-inch plow speed ft./sec.</th>
<th>Predicted draft, lb., from model</th>
<th>Average predicted draft lb.</th>
<th>Observed draft lb.</th>
<th>Error of prediction %</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.305</td>
<td>-----</td>
<td>49.55</td>
<td>49.55</td>
<td>59.03</td>
</tr>
<tr>
<td>3.317</td>
<td>50.97</td>
<td>54.43</td>
<td>52.71</td>
<td>64.69</td>
</tr>
<tr>
<td>3.406</td>
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<td>50.92</td>
<td>53.33</td>
<td>63.81</td>
</tr>
<tr>
<td>4.067</td>
<td>-----</td>
<td>56.98</td>
<td>52.12</td>
<td>56.72</td>
</tr>
<tr>
<td>4.303</td>
<td>-----</td>
<td>54.16</td>
<td>54.16</td>
<td>56.74</td>
</tr>
<tr>
<td>5.298</td>
<td>57.64</td>
<td>-----</td>
<td>55.86</td>
<td>56.75</td>
</tr>
<tr>
<td>5.603</td>
<td>51.19</td>
<td>60.96</td>
<td>55.04</td>
<td>60.89</td>
</tr>
<tr>
<td>5.836</td>
<td>-----</td>
<td>60.66</td>
<td>52.99</td>
<td>66.86</td>
</tr>
</tbody>
</table>
Figure 40. Relationship between $\delta_2^*$ and $n$
where \( x_2 \) = the slope of the line. If other plots of \( \delta_2^* \) versus \( n \) were made for other values of speed, such plots would be lines passing through \( \delta_2^* = 1 \) at \( n = 1 \) and would have different slopes. This slope was negative. The plot of the absolute values of slope of the line versus speed is shown in Figure 41. The equation for the absolute values of slope was expressed as

\[
|x_2| = 1.779 \sqrt{V}^{0.0775} \tag{Eq. 100}
\]

where \( |x_2| \) = the absolute value of slope.

Substitution of this equation into Equation 99 resulted into the expression for \( \delta_2^* \):

\[
\delta_2^* = n^{-1.779} \sqrt{V}^{0.0775} \tag{Eq. 101}
\]

With \( \delta_2^* \) known, the prediction equations for draft using each of the 1.5-inch, 2.0-inch and 3.0-inch model paired with the 4.0-inch prototype were obtained by substitution of Equation 101 into the following general prediction equation:

\[
\delta_2 = \frac{R}{R_m n^3} \tag{Eq. 32}
\]

Upon simplification, the resulting prediction equation for each model is shown below:

- 1.5-inch model: \( R = R_m 2.667^{-3-1.779V^{0.0775}} \)
- 2.0-inch model: \( R = R_m 2.000^{-3-1.779V^{0.0775}} \)
- 3.0-inch model: \( R = R_m 1.333^{-3-1.779V^{0.0075}} \) \tag{Eq. 102}

Values of draft were selected from observed data from each of the three model plows such that when the particular model was paired with the
Figure 41. Relationship between $|x_2|$ and $V$

$|x_2| = 1.779V^{0.0775}$
4.0-inch plow, the speed of the model and the prototype were related by the equation
\[ V_m = \frac{V}{\sqrt{n}} \]  
Eq. 22

As in the preceding method of analysis, the criterion for selection of speed was allowed a deviation from the designed speed of ± 0.1 foot per second. The corresponding draft values were used in the calculations of \( \frac{\delta}{2} \).

The predicted draft values for the 4.0-inch plow were calculated using Equations 102 and compared with the observed values. The average predicted value was taken whenever two or three models were used to predict draft at a certain speed of the prototype. The results are shown in Table 17.

The error of prediction ranged from 1.07 percent to 34.11 percent. In general, the predicted values were consistently lower than the observed draft values.

**Case 2. Draft-speed relationship of the form** \( a + bV \) **For**

the case where the relationship between draft and speed was considered linear, the analysis proceeded in the same manner as the alternate method for Case 1. The prediction factor \( \frac{\delta}{2} \) was calculated from

\[ \frac{\delta}{2} = \frac{a_1 + b_1V}{(a_{1m} + b_{1m}V)n^3} \]  
Eq. 103

by using data obtained from the regression lines for the 1.5-inch, 2.0-inch, and 3.0-inch plows. The plots of \( \frac{\delta}{2} \) versus \( n \) at \( V = 1.0 \) foot per second is shown in Figure 42. The slope of the line varied with \( V \).
Table 17. Comparison of predicted and observed values of draft\(^a\) for the 4.0-inch plow using distorted model theory; \(\pi_{10} = \rho V^2/\sigma\) distorted, draft-speed relationship of the form \(R = A_1 V^2\).

<table>
<thead>
<tr>
<th>4.0-inch plow speed ft./sec.</th>
<th>Predicted draft, lb., from model plow</th>
<th>Average predicted draft lb.</th>
<th>Observed draft lb.</th>
<th>Error of prediction %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted draft, lb., from model plow</td>
<td>Average predicted draft lb.</td>
<td>Observed draft lb.</td>
<td>Error of prediction %</td>
</tr>
<tr>
<td>0.991</td>
<td>-----</td>
<td>40.06</td>
<td>40.06</td>
<td>52.00</td>
</tr>
<tr>
<td>1.057</td>
<td>32.88</td>
<td>41.92</td>
<td>41.92</td>
<td>53.30</td>
</tr>
<tr>
<td>1.071</td>
<td>-----</td>
<td>44.21</td>
<td>44.21</td>
<td>51.59</td>
</tr>
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<td>47.67</td>
<td>54.00</td>
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<td>45.79</td>
<td>53.68</td>
</tr>
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<td>51.66</td>
<td>59.42</td>
</tr>
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<td>46.79</td>
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<tr>
<td>2.218</td>
<td>------</td>
<td>46.60</td>
<td>46.60</td>
<td>46.91</td>
</tr>
</tbody>
</table>

\(^a\)Prediction factor equation: \(\delta_2 = n - 1.779 V^{0.0775}\)
<table>
<thead>
<tr>
<th>4.0-inch plow speed ft./sec.</th>
<th>Predicted draft, lb., from model</th>
<th>Average predicted draft lb.</th>
<th>Observed draft lb.</th>
<th>Error of prediction %</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.118</td>
<td>46.40</td>
<td>42.77</td>
<td>64.91</td>
<td>34.11</td>
</tr>
<tr>
<td>3.305</td>
<td>-----</td>
<td>49.34</td>
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<td>3.317</td>
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<td>54.34</td>
<td>63.81</td>
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<td>5.603</td>
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</tr>
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<td>5.836</td>
<td>-----</td>
<td>59.37</td>
<td>66.86</td>
<td>16.08</td>
</tr>
</tbody>
</table>
Figure 42. Relationship between $\delta_2$ and $n$
Therefore, from the plot shown in Figure 43, the absolute value of the slope was expressed as

$$|x_2^1| = 1.84 \sqrt[0.0309]$$

Eq. 104

where $|x_2^1|$ = the absolute value of the slope of the line.

With the use of Equation 101 the expression for $\delta_2^1$ was written as

$$\delta_2^1 = n^{-1.84 \sqrt[0.0309]}$$

Eq. 105

Therefore, the prediction equation was given by

$$R = n^3 \sqrt[1.84 \sqrt[0.0309]} R_m$$

Eq. 106

Predicted values of draft were calculated for the 4.0-inch plow using observed data from the 1.5-inch, 2.0-inch and 3.0-inch plows. For each plow size as model paired with the 4.0-inch plow as prototype, the following prediction equations were derived from Equation 106.

1.5-inch model: $R = 19.000 R_m (2.667^{-1.84 \sqrt[0.0309]}$ )

2.0-inch model: $R = 8.000 R_m (2.000^{-1.84 \sqrt[0.0309]}$ )

3.0-inch model: $R = 2.370 R_m (1.333^{-1.84 \sqrt[0.0309]}$ )

Eq. 107

The results of calculations are shown in Table 18. The error of prediction ranged from 1.64 percent to 32.00 percent. In general, the errors of prediction when using the linear relationship were higher than those when using the logarithmic relationship. This is explained by a better fit of data for the latter relationship.
Figure 43. Relationship between $|x_2'|$ and $V$

$|x_2'| = 1.84V^{0.0309}$
Table 18. Comparison of predicted and observed values of draft for the 4.0-inch plow using distorted model theory; $n_{10} = aV^2/c$ distorted, draft-speed relationship of the form $R = a_1 + b_1V$

<table>
<thead>
<tr>
<th>4.0-inch plow speed ft./sec.</th>
<th>Predicted draft, lb., from model</th>
<th>Average predicted draft lb.</th>
<th>Observed draft lb.</th>
<th>Error of prediction %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.057</td>
<td>31.11</td>
<td>31.11</td>
<td>41.92</td>
<td>25.79</td>
</tr>
<tr>
<td>1.071</td>
<td>42.61</td>
<td>43.02</td>
<td>43.74</td>
<td>1.64</td>
</tr>
<tr>
<td>1.377</td>
<td>39.57</td>
<td>42.78</td>
<td>51.59</td>
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SUMMARY AND CONCLUSIONS

The prediction equations for draft of a 4.0-inch moldboard plow working on a submerged clay soil were developed using similitude theory. The model-prototype system consisted of three auxiliary model plows of 1.5-inch, 2.0-inch and 3.0-inch sizes.

The pertinent variables considered in the submerged tillage study were divided into geometric properties of the tools, soil properties and operational variables.

The geometric properties were:
1. Width of the plow, D
2. Other pertinent lengths, \(\lambda_i\), \(i=1,2,\ldots,p\)
3. Cutting angle of the plow, \(\psi\)
4. Other angles of the plow, \(\psi_j\), \(j=1,2,\ldots,q\)

The soil properties were:
1. Mass density, \(\rho\)
2. Moisture content, \(M\)
3. Soil-metal friction, \(\mu\)
4. Dimensionless properties, \(\mu_k\), \(k=1,2,\ldots,r\)
5. Property having dimensions of stress, \(\sigma\)
6. Other properties having dimensions of stress, \(\sigma_l\), \(l=1,2,\ldots,s\)

The dynamic or operational variables were:
1. Speed of plowing, \(V\)
2. Acceleration due to gravity, \(g\)

The dependent variable was the draft of the plow, \(R\).
A functional relationship among the dimensionless pi terms was:

\[ \frac{R}{\rho V^2 D^2} = \phi (\lambda_1 / D, \psi, \psi_1, u, u_\kappa, \sigma / \sigma_\kappa, V^2 / gD, \rho V^2 / \sigma, M) \]  Eq. 108

The design conditions required that the soil to be used for the model and the prototype tests should have identical properties. Such a requirement was approximately satisfied by using a standardized procedure in soil fitting after the test runs in the soil bin. The velocity pi terms \( V^2 / gD \) and \( \rho V^2 / \sigma \) imposed conflicting design conditions for speed. Neither of these conditions was satisfied in the actual tests due to inaccurate control in the bin drive equipment. The problem, however, was encountered by considering the statistical relationships between draft and speed of the plows instead of the individual observed data. This procedure gave the option of considering the speed design requirements imposed by the two speed pi terms.

Both undistorted and distorted model analyses were considered. In the first case either of the pi terms \( V^2 / gD \) or \( \rho V^2 / \sigma \) was neglected and in the second case either of the said pi terms was distorted. The linear and logarithmic functions which described the draft-speed relationships were considered in the two methods of analyses.

The equations for prediction factors were obtained from the results of the 1.5-inch, 2.0-inch and 3.0-inch plows. From the possible model-prototype pairs among the three plows, the trends in distortion were established. It was found that the prediction factor was a function of the length scale and speed of plowing.

The form of the prediction factor equations was logarithmic, that is,

\[ \delta = n f(V) \]
The prediction factor equation could be developed from the results of tests using three auxiliary models. When used with a prediction equation for draft of a 4.0-inch moldboard plow, the expression for
the prediction factor gave a wide range of error of prediction. The large errors of prediction was due to the deviation of an observed data point from the regression equation which was used to establish the prediction factor equation. This was augmented by the fact that the observed values of draft for the prototype had greater variation compared with those for the models.

4. The prediction factor was a function of the length scale and speed. When the pi term $V^2/gD$ was distorted, the distortion factor was established as a function of the length scale and the said distorted pi term.

5. The results of prediction were not accurate enough to be reasonable for use in design work. Based upon the reasons for the errors in prediction as given in 3 above, however, improvements are still possible. Foremost of these is the refinement of testing technique to lessen the variability of the observed draft values for both the models and the prototype. More data is needed to establish a more accurate relationship between draft and speed of the plow when working submerged soils.
SUGGESTIONS FOR FUTURE RESEARCH

The accuracy of prediction in a similitude study depends mostly upon the correctness of the pertinent variables selected to describe the problem. In this study, the soil property was believed to be a possible source of error in the selection of such variables. Since a dynamic system was involved, a soil property other than mass density having the dimension of time should be included. Some investigations may be done, for example, with the inclusion of rate of strain of the soil or of soil properties having dimensions of viscosity.

Another possibility is the inclusion of rheological properties of the soil.

Perhaps the most critical assumption in the testing procedure is that of identical soil conditions among soil bin fittings based upon a standardized soil processing. There was no check on this identity except on the basis of density and moisture content which were not really identical as required. This problem of soil fitting was difficult to solve since the technique did not require numerical evaluation of the soil properties. One way, however, of satisfying the equality of soil conditions for the model and the prototype is to test the models and prototype on the same soil bin fitting. Of course, this needs a much larger plot and the arrangement in which the soil bin moves rather than the tool may have to be changed. Even then, some variability of soil properties is expected within the soil bin. Statistical analysis may be a possible way of dealing with the problem.

In the functional relationship between the pi term containing the
dependent variable and the pi terms containing the dependent variables, a more rigorous test of separability of the distorted pi terms should be done. This is accomplished by the method outlined by Murphy (18). Independent experimental runs in which other variables are held constant, etc., are needed for this test.

Some simplification might result in the method of analysis if the pi terms could be classified into groups and these groups form separable functions. For example, if all pi terms containing soil properties only form into a function which combine multiplicatively with a function consisting of pi terms describing tool geometry only, the prediction factor might be expressed as functions of these groups.

It is recommended that since a soil property having dimensions of stress seemed to be important, an investigation of the nature of such property may be pursued. The possibilities are that such properties would be related to the compressive, tensile and shear stresses.

Since speed of plowing is a controlling factor in tests, refinement in speed control devices must be made. This may be accomplished by a larger power drive unit so that the changes in load would not affect the speed output. Also, tillage tools may be tested at higher speeds to fully establish the relationship between draft and speed of the plow.


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Professor Henry M. Black, Head, Department of Mechanical Engineering,
Dr. Herbert David, Professor of Statistics
Dr. Don Kirkham, Distinguished Professor of Agronomy and Physics and
Dr. Dennis Lawing, Visiting Lecturer, Statistics.

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APPENDIX A

Separability of \( \pi_9 = \frac{V^2}{gD} \) from \( f(\pi_2, \pi_3, \ldots, \pi_8, \pi_{10}) \)
and \( \pi_{10} = \frac{\rho V^2}{\sigma} \) from \( f(\pi_2, \pi_3, \ldots, \pi_9) \)

The justification for writing the functional relationship for \( \pi_1 \) when the pi term \( V^2 / gD \) is distorted is considered first. The test for the resolution of the function \( \pi_1 \) into a product of two components requires establishing \( \pi_1 \) as a function of \( \pi_9 \) for two different values of one of the other pi terms. In this case, the pi term which will assume two different values is \( \pi_{10} = \frac{\rho V^2}{\sigma} \). The other pi terms, \( \pi_2, \pi_3, \ldots, \pi_8 \) will be held constant. In essence, the test requires that the following relationship which is similar to Equation 49 of Murphy (18), be satisfied:

\[
\frac{F(\pi_{10})}{F(\pi_2, \pi_3, \ldots, \pi_8, \pi_9, \pi_{10})} = \frac{F(\pi_{10})}{F(\pi_2, \pi_3, \ldots, \pi_8, \pi_9, \pi_{10})} \quad \text{Eq. 109}
\]

where a bar indicates holding the pi term at one constant and a double indicates holding the pi term at another constant.

If the component equation (an equation for \( \pi_1 \) determined by holding all but one pi term constant) is a logarithmic function, then

\[
\begin{align*}
\langle \pi_{10} \rangle &= B_{\pi_9}^y \quad \text{Eq. 110} \\
\langle \pi_{10} \rangle &= B_{\pi_9}^{y_1} \quad \text{Eq. 111} \\
F(\pi_{10}) &= B \pi_9^y \quad \text{Eq. 112}
\end{align*}
\]

and

\[
F(\pi_{10}) = B_{1}^{y_1} \pi_9 \quad \text{Eq. 113}
\]
Substitution of Equations 110 through 113 into Equation 109 results in

\[
\frac{B_1^{\pi_9} y}{Y} = \frac{B_1^{\pi_9} Y_1}{Y_1}
\]

Eq. 114

A function of the form \( y = cx^n \) plots as a straight line on logarithmic paper. If in Equation 114 the slopes, \( y \) and \( y_1 \) are equal, Equation 114 reduces to an identity and Equation 109 is satisfied.

From the above, it can be deduced that if the plots of \( \pi_1 = F(\pi_2, \pi_3, \ldots, \pi_8, \pi_9, \pi_{10}) \) and \( \pi_1 = F(\pi_2, \pi_3, \ldots, \pi_8, \pi_9, \pi_{10}) \) are parallel straight lines on logarithmic paper, then \( \pi_1 \) could be written as

\[
\pi_1 = f(\pi)g(\pi_2, \pi_3, \ldots, \pi_8, \pi_{10})
\]

Eq. 115

From the data shown in Appendix B, values of \( \pi_9 = V^2/gD \) and \( \pi_{10} = \rho V^2/\sigma \) were calculated. Since the same soil was used for both the model and the prototype tests and since the design conditions involving \( \pi \) terms \( \pi_2 \) to \( \pi_8 \) were essentially satisfied, then all \( \pi \) terms except \( \pi_9 \) were constant. Since \( \rho/\sigma \) was assumed to be constant in all test series, \( \pi_{10} \) could be varied at two constant values by varying the value of speed, \( V \).

In the final analysis, however, the actual values of \( \rho \) as given in Appendix B were used and since \( \sigma \) was not evaluated it was assumed to be a constant equal to 1 for convenience. Therefore, the values of \( \rho V^2/\sigma \) were not really held constant in a strict sense because of lack of proper control of \( \rho \) and \( V \) in the tests. For practical purposes, however, the test for multiplicative combination of \( V^2/gD \) and the function involving other \( \pi \) terms was pursued. While the test as conducted is not rigorous, it gives an insight into the effect of the \( \pi \) term \( V^2/gD \) on the system.
The plots of $\pi_1 = R/pV^2D^2$ versus $\pi_9 = V^2/gD$ at different values of $\pi_{10} = \rho V^2/\sigma$ are shown in Figure 44. Two interpolated lines are drawn through the points. With due recognition of error committed by interpolation, the lines were essentially parallel, thus satisfying the test given by Equation 109. This indicates that the assumption of separability of the $\pi$ term $V^2/gD$ from $f(\pi_2, \pi_3, \ldots, \pi_8, \pi_{10})$ was valid to a certain degree.

The justification for the separability of the $\pi$ term $\pi_{10} = \rho V^2/\sigma$ or $\pi_{10} = KV$ from $f(\pi_2, \pi_3, \ldots, \pi_9)$ proceeds in the same manner as the above. This time $V^2/gD$ is to be held at two different values. The plots of $\pi_1 = R/pV^2D^2$ versus $\pi_{10} = KV$ is shown in Figure 45. Again, the interpolated lines tended to be parallel indicating that the assumption of separability of $\pi_{10} = \rho V^2/\sigma$ or $\pi_{10} = KV$ from $f(\pi_2, \pi_3, \ldots, \pi_9)$ was valid to a certain degree.
Figure 44. Plots of $\pi_1 = \frac{R}{\rho v^2 D^2}$ versus $\pi_0 = \frac{V^2}{gD}$ at different values of $\pi_{10} = KV; \pi_2, \pi_3, \ldots, \pi_8 =$ constant.
Figure 45. Plots of $\pi_1 = \frac{R}{\rho V^2 D^2}$ versus $\pi_{10} = \rho V^2/\sigma$ or $\pi_{10} = KV$ at different values of $\pi_9 = V^2/gD$; $\pi_2, \pi_3, \ldots, \pi_8 = \text{constant}$
APPENDIX B

Experimental Data

Figures 46 through 49 represent the data processed from the oscillo-graph records in the headings of PLOW SPEED and AVERAGE DRAFT and from results of the paraffin method of density measurement in headings of SOIL DENSITY and SOIL MOISTURE CONTENT.

The following explains the coding in the heading RUN NUMBER:

The code number consists of either seven or eight figures depending upon the date the data was taken. The two right most digits denote individual runs in a soil bin; the next two digits denote the size of the plow in inches (the number is ten times the actual size); the next two digits represent the day of the month and the next one or two digits represent the month number.

Thus, in Figure 46, the first run number which is 9211501 represents:

Date - September 21
Size - 1.5 inches
Run - 1

All tests were conducted in 1967 at the Model Tillage Laboratory, Iowa State University.
Figure 46. Experimental data for tests with the 1.5-inch plow
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Figure 47. Experimental data for tests with the 2.0-inch plow
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Figure 47. (Continued)
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Figure 48. Experimental data for tests with the 3.0-inch plow
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Figure 49. Experimental data for tests with the 4.0-inch plow
The calculations of values for the first line of Table 14, page 124, are illustrated in this Appendix.

The draft-speed relationship for each plow obtained by rewriting Equations 76, page 106, in the form \( R = A_1 V^2 \) is as follows:

\[
\begin{align*}
1.5\text{-inch plow:} & \quad R = 13.73 V^{0.2722} \\
2.0\text{-inch plow:} & \quad R = 19.19 V^{0.2564} \\
3.0\text{-inch plow:} & \quad R = 30.23 V^{0.1682} \\
4.0\text{-inch plow:} & \quad R = 48.21 V^{0.1584} \\
\end{align*}
\]

Equation for \( \delta_1 \)

For each model-prototype pair from the 1.5-inch, 2.0-inch and 3.0-inch plows and for \( V^2/gD = 1.0 \),

\[
V = \sqrt{gD(1.0)} \quad \text{Eq. 117}
\]

where \( g = 32.2 \text{ ft./sec.} \)

\( D \) = size of prototype in ft.

Thus, the speed for each pair (since \( V_m = V \)) is as follows:

\[
\begin{align*}
1.5\text{-inch and 3.0-inch plows:} & \quad V = 32.2(3/12)(1.0) = 2.837 \text{ ft./sec.} \\
2.0\text{-inch and 3.0-inch plows:} & \quad V = 32.2(3/12)(1.0) = 2.837 \text{ ft./sec.} \\
1.5\text{-inch and 2.0-inch plows:} & \quad V = 32.2(2/12)(1.0) = 2.317 \text{ ft./sec.} \\
\end{align*}
\]

Henceforth, for brevity in writing model-prototype pairs, they will be designated as, for example, 1.5-3.0 for 1.5-inch plow as the model of
the 3.0-inch plow, etc.

Substitution of Equations 118 and 116 into Equation 78, page 120, the following values of $\delta_1$ were obtained:

- 3.0-inch and 1.5-inch plow: $\delta_1 = 0.494$, $n = 2.00$
- 3.0-inch and 2.0-inch plow: $\delta_1 = 0.640$, $n = 1.50$
- 2.0-inch and 1.5-inch plow: $\delta_1 = 0.774$, $n = 1.33$
- 3.0-inch and 3.0-inch plow: $\delta_1 = 1.000$, $n = 1.00$  Eq. 119

The values of $\delta_1$ and $n$ in Equation 119 were plotted in Figure 32, page 121. The slope of this line was $-1.015$.

In a similar manner as for $V^2/gD = 1.0$, values of $\delta_1$ were calculated for $V^2/gD = 0.02$ to 6.0. Also the slopes were calculated for each plot of $\delta_1$ versus $n$. The absolute values of slope were plotted as shown in Figure 33, page 122 giving the equation

$$|x_1| = 1.015(V^2/gD)^{0.0867}$$  Eq. 81

Therefore, the prediction factor equation was

$$\delta_1 = n^{-1.015(V^2/gD)^{0.0867}}$$  Eq. 82

**Prediction equation**

The prediction equation was, therefore

$$R = n^{-1.015(V^2/gD)^{0.0867}} n^{-2R_m}$$  Eq. 83
**Predicted values of draft**

Predicted values of draft for the 4.0-inch plow were then calculated from each of the models. The value of D used in Equation 83 was 4/12 ft. = 0.33 ft. From Figures 46 to 49, the values of speed for 1.5, 2.0 and 3.0-inch plows considered to be within 0.1 ft./sec. of the 1.057 and 1.071 ft./sec. of the 4.0-inch plow and the corresponding observed draft values were:

<p>| | | |</p>
<table>
<thead>
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<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
</tbody>
</table>

\[
\begin{align*}
1.5-4.0: & \quad V = 1.071, R_m = 17.23 \text{ lb.}, \quad n = 2.667 \\
2.0-4.0: & \quad V = 1.075, R_m = 19.01 \text{ lb.}, \quad V = 1.093, R_m = 17.00 \text{ lb.}, \quad \text{Ave.} = 18.00 \text{ lb.}, \quad n = 2.00 \\
3.0-4.0: & \quad V = 1.087, R_m = 32.61 \text{ lb.}, \quad n = 1.33 \\
4.0: & \quad V = 1.057, R_m = 45.92 \text{ lb.}, \quad V = 1.071, R_m = 46.74 \text{ lb.}, \quad \text{Ave.} = 46.33 \text{ lb.} \\
\end{align*}
\]

Overall average \( V = \frac{1.071 + 1.075 + \ldots + 1.071}{6} = 1.076 \text{ ft./sec.} \)

The predicted draft values calculated by use of Equation 83 were:

\[
\begin{align*}
1.5-4.0: & \quad R = (2.667)^{-1.015} (1.076)^2/(32.2)(0.33) 0.0867(2.667)^2(17.23) \\
& \quad = 2.667^{0.837}(2.667)^2(17.23) = 53.93 \text{ lb.} \\
2.0-4.0: & \quad R = (2.000)^{0.837}(2.000)^2(18.00) = 40.31 \text{ lb.} \\
3.0-4.0: & \quad R = (1.333)^{0.837}(1.333)^2(32.61) = 45.56 \text{ lb.} \\
\end{align*}
\]
Average predicted draft = \( \frac{53.93 + 40.31 + 45.56}{3} \) = 46.60 lb.

Error of prediction = \( \frac{\text{Predicted draft} - \text{Observed draft}}{\text{Observed draft}} \times 100 \)

\[
\frac{46.60 - 46.33}{46.33} = 0.58 \% 
\]