I would like to present some of our results on angular and frequency
dependence of sound scattering from flaws which are two-dimensional and
have sharp edges, and specifically on our study of the end of the flat
bottom hole, which may be considered as a circular aperture or disk in
an infinite medium.

We have used the frequency analysis system shown in Fig. 1. We have
a commercial pulser which shock excites the transmitter. The broad band
signal hits the target and the scattered signal is picked up by a receiver.
This signal is amplified and gated out and spectrum analyzed. We are not
using a digital system; we have used the actual spectrum in our study.

Basically, we are interested in studying the response from the flat
bottom hole, and we're simulating these in our experiments by using a flat­
ended rod and studying the scattering from it. The actual setup is shown
in Fig. 2. The goniometer system is also shown. We can adjust the angle
between the two transducers and thereby study angular dependence.

Figure 3 shows typical spectra of a large diameter rod at various
angles. Angular dependence of the frequency content is seen even at small
angles. In Fig. 4 you see similar behavior for another size rod. Thus, we
have concluded that the amplitude distribution or power distribution depends
on the frequency, angle and the size of these reflectors.

The problem was to find a relationship between the amplitude dis­
tribution or the intensity distribution and the frequency and angular behavior
of the scattered energy. We searched through the literature, and the problem
of the angular and frequency dependence of scattering from a disk or a
 circular aperture is not treated in detail, at least not for the elastic wave
case. We have found a treatment of the geometric theory of diffraction,
introduced by Keller, which I think was mentioned in a previous paper. Keller,
about 15 years ago, developed a geometrical theory of diffraction for an
electromagnetic case and, basically, he considered the following. I'm just
going to review briefly Keller's theory and extend its relevancy to our work.

If you treat geometrical optics or geometric acoustics, ordinarily you
consider incident, reflected and refracted rays. Keller introduced diffracted
rays. These diffracted rays are produced every time a ray interacts with an
edge or vertex or grazes an interface or a flaw. The advantage of this theory

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Fig. 1. Schematic Diagram of the Experimental System.
Fig. 2. Photograph of the Experimental System.
Fig. 3. Frequency Spectrum of the Scattered Energy from a 0.281 in. diam. Reflector.
Fig. 4. Frequency Spectrum of the Scattered Energy from a 0.063 in. diam. Reflector.
is not only that Keller introduced these diffracted rays from a qualitative view, but he also assigned amplitude and phase, actually a field value, to each of these rays. So, one can calculate the field value of the scattered energy at a given point by simply summing the field due to all the diffracted rays at that point.

Without going into too much detail, for an arbitrary shaped aperture, the amplitude of the scattered or the diffracted field at a given point due to all the diffracted rays is expressed as:

\[
U(P) = - \frac{iA(k)}{2(2\pi k)^{1/2} \sin \beta} \left( e^{i(k(\beta + s))} + \frac{k}{4} \right) \times \\
\left[ \sec \frac{1}{2} (\theta - \alpha) \cdot \csc \frac{1}{2} (\theta + \alpha) \right] \times \left[ s(1 + \frac{s(\cos \delta - a \delta \sin \beta)}{a \sin^2 \beta}) - \frac{1}{2} \right].
\]

Here \(A(k)\) is the amplitude distribution of the incident rays, \(k\) is the wave number and \(\beta\) is the phase. The distance from the origin to the point of the ray is represented by \(s\). The angle of incidence is represented by \(\alpha\), and the angle of diffracted ray is expressed by \(\beta\). The radius of the curvature of the aperture is \(a\), and \(\beta\) is the angle between the incident ray and the positive tangent to the aperture, and this parameter will determine what type of aperture we have. The derivative of \(\beta\) with respect to the arc length is \(\dot{\beta}\). Basically, for a given type of aperture, the parameters \(\beta\), \(\dot{\beta}\) and \(\alpha\) are important. Now, how many of these diffracted rays one takes into account in the summation depends on the type of geometry one has. It is also important to mention that each of these diffracted rays, when they interact with the edge, diffract again so one can consider multiple diffraction also.

We have worked out the problem for the circular case shown in Fig. 5. We have an aperture of radius \(a\), and we consider two incident rays coming in with angles of incidence \(\alpha_1\) and \(\alpha_2\). (We also worked out the problem for four incident rays, but the result happens to be the same. Thus, it appears that two rays are sufficient to take into account for circular aperture. For other shaped apertures one needs to have more than two rays in the summation.) The diffracted rays are shown also. For the far field approximation, we came up with an expression in terms of simple trigonometric functions:
Fig. 5. Diffraction of "Rays" for a Circular Aperture.
Here $r$ represents actual distance between the center of the scatterer to the receiver, and $A(k)$ is the amplitude distribution of incident wave which we get directly by frequency analyzing the transducer. The incident and the scattered angle are represented by $\phi_1$ and $\theta$, respectively. That gives all the parameters there.

We took our measurements directly from the spectrum analyzer. The frequency dependence is shown on the spectrum analyzer, and the angular dependence of our data was obtained by taking a given frequency from the spectrum analyzer and varying the angle between the transmitter and the receiver.

Figure 6 is the spectrum of the output of our transmitter, and, unfortunately the band is not very wide. We don't have a better transducer, so we have to use whatever we have. But in our expression which was given in Eq. (2), we used this function for $A(k)$. We carried out the experiments both by measuring the scattered energy as a function of frequency and angle from the end of these rods, as well as from flat bottom holes in the aluminum sample.

Figure 7 shows the frequency dependence of the intensity from a 245 mil diameter rod for a 23 degree scattering angle. The incident was normal in this case. The solid curve is the theoretical curve which was calculated from Keller's theory, and the dots are the experimental points. At first glance the agreement is not too bad. We have the same number of peaks and some of them are overlapping. The agreement is better for the 1/8 inch rod.
Fig. 6. Intensity Distribution of the Incident Wave vs Frequency.
Fig. 7. Intensity Distribution of the Diffracted Wave vs Frequency.
(Fig 8). Here we have plotted the intensity as a function of frequency. Unfortunately, the frequency range is rather limited. We are dealing with frequencies between approximately 1 to 6 MHz. The frequency spectrum is not as wide as given in some of the previous papers, but at least we can get some kind of comparison for the angular dependence here with 15° axis angle. In Fig. 9 we vary the angle for a given frequency, and here again, you have to use your imagination to match up some of the points. In the theoretical curve the frequency was 5.5 MHz and the angle varied up to 40°. In Fig. 10 the agreement is better for another type of reflector. All these experiments are for scattering from the end of a flattened brass rod, so, it's like from a disk in water. Since the agreement between theory and experiment is reasonably good, one could begin to believe that there may be some merit in using this geometrical theory of diffraction.

After carrying out the experiments on scattering sound from the end of the rod, we decided to look at some metal samples and study the scattering from flat bottom holes, from the end of these holes. We used two different types of samples. Since one of the problems which came up in this investigation is the effect of the surface of the sample on the scattering results, we used two types of samples. In one case we have a cylindrical surface and we drilled a flat bottom hole into it, such that the end of the hole is at the center of the curvature. In this case all the scattered waves are not going to refract when they come out at the bottom. The other sample furnished by the Science Center, has a flat top. In both of these samples we used identical sized flat bottomed holes in order to compare the results. Both samples are shown in Fig. 11.

In Fig. 12 we have plotted the theoretical curve with the experimental points for the hole in the aluminum sample. I guess I didn't mention it previously, but it is obvious that we used a scalar theory. Thus, one has to use some reservation as far as agreement is concerned. At any rate, the dots here are the experimental points and they are just connected so one can see the behavior of the experiment. But here the diameter of the flat bottomed hole was quite large (1/2 inch), and the off axis angle is 29°. We have, again, some kind of agreement between theory and experiment. The agreement is much better for a 1/4 inch hole, as shown in Fig. 13. There are some details in the theory which one may not be able to resolve experimentally, but there are two distinct peaks and the experiments seem to follow the theoretical predictions.

Both of these results were for normal incidence, and the off axis angle in this case, was 25°. We used a curved cylindrical surface sample. In Fig. 14 we show data for an angle of incidence of 18° and a scattered angle of 4°. Here again we have reasonable agreement -- the peaks are shifted to the right, but we have 3 peaks, and the agreement is not too bad considering that we are dealing with the scalar theory.

I guess I should mention at this point that we are using short pulses, and can separate the longitudinal and the shear scattered waves experimentally.

In Fig. 15 we show a comparison between the two samples we used and the theoretical curve. In both samples there is a 1/8 inch hole, and
Fig. 8. Intensity Distribution of the Diffracted Wave vs Frequency.
**Fig. 9.** Intensity Distribution of the Diffracted Wave vs Angle.
Fig. 10. Intensity Distribution of the Diffracted Wave vs Angle.
Fig. 11. Photograph of the Mechanical System. (a) Cylindrical Sample; (b) Flat Sample.
Fig. 12. Intensity Distribution of the Diffracted Wave in Aluminum vs Frequency.
Fig. 13. Intensity Distribution of the Diffracted Wave in Aluminum vs Frequency.
Fig. 14. Intensity Distribution of the Diffracted Wave in Aluminum vs Frequency.
PATTERN AS A FUNCTION OF FREQ
DIAMETER IN MILS = 125.0
OFF AXIS ANGLE = 27.1
VELOCITY IN MILS/MSEC IS 250.0

Fig. 15. Comparison of the Intensity Distribution vs Frequency of the Diffracted Wave. (a) Flat aluminum sample, dots; (b) Cylindrical aluminum sample, crosses; (c) Theoretical line is solid curve.
the measurement was made at the same scattered angle. These dots are for the sample with the flat surface and the crosses are for the curved surface sample. It seems to be that above about 3 MHz or so both experiments agree reasonably well with the theory; and at the lower frequencies, both of them deviate from the theoretical curve. Thus it is difficult to draw a conclusion as to what the effect of the surface is but it is most likely that at low frequencies we have problems with both of them. So, I guess we would have to carry out the investigation to a higher frequency range. One way to study the effect of the surface of the sample on the spectrum is by carrying out experiments on both samples and comparing with the theory. But, since the theory is not perfect, I don't know if we are comparing the right thing or not.

We thought it would be useful, since we are interested in angular and frequency dependence of the scattered energy, to make a three-dimensional plot in order to study some of the fine features of the spectrum. In Fig. 16 the frequency and the scattered angle is plotted vs. the scattered intensity. We can get some idea of what's going on in the various regions, and we tried to use this information in our experiments. We can also program the distance dependence or the flaw size dependence as a function of frequency. In Fig. 17 we learn by plotting the distance dependence as the function of frequency that after about an inch or so the structure of the scattered energy is unchanged. This is a plot of the results that came from Keller's theory. We learned that up to about an inch or so from the scatterer, there is not much distance dependence as far as the structure is concerned. But if you are going in the near field within one inch, then you will have some problems.

We are interested in going in the direction of studying other than circular shaped or circular types of flaws, and we are simulating some other types of flaws. At the moment we don't have a theoretical comparison for these, but we are trying to solve the problem of the elliptical aperture and also some rectangular apertures using Keller's theory. I would like to mention an approximation of this theory which we also are using in order to determine the size of various nonsymmetrical types of reflectors.

In Fig. 18 is shown a whole set of reflectors which are used in our experiments. We used elliptical, rectangular, and some irregularly shaped reflectors. We looked at the frequency spectrum scattered from these reflectors. In the long run we will correlate the results with diffraction theory from geometrical diffraction theory. But for the moment we make the assumption that the contribution to the spectrum is coming from wavelets at the opposite end of these edges. Thus we take waves coming from two points. We are not considering any amplitude, but we say, "Well, they are either going to meet in or out of phase." If they meet in phase, they form maximum amplitude; if they meet out of phase, they are going to be at a minimum amplitude. On the spectrum we are looking for the formation of these peaks that were shown before.

What we are doing is correlating the positions of these frequency maxima to the size by these two approximate equations:
X IS FREQ FROM 0.5 TO 5.5 MHZ
Y IS OFF AXIS ANGLE FROM 10.0 TO 40.0 DEGREES
DIAMETER IN MILS = 245.0
VELOCITY IN 10^4 MILS/SEC IS 57.1
FIGURE IS ROTATED FROM XZ PLANE BY - 300.0 DEGREES
FROM XY PLANE BY - 60.0 DEGREES

Fig. 16. 3D Plot of Intensity Distribution vs Frequency and Angle.
\[ \Delta f_1 = \frac{v}{d[\sin \theta + (\sin \theta \pm \alpha_1)]} \]  
\[ \Delta f_2 = \frac{v}{d[\sin \theta + (\sin \theta \pm \alpha_2)]} \]

Here \( d \) is the size of the reflector. By placing the receiver at two different positions, we come up with the frequency spectrum where the separation of the frequency maximum is given as \( \Delta f_1 \) and \( \Delta f_2 \).

The orientation of this random-shaped reflector we designate \( \theta \) in the direction where the plane of the transmitter and receiver is. The parameters \( \alpha_1 \) and \( \alpha_2 \) are the angles of the receiver with respect to the transmitter. Solving these two equations simultaneously, we obtain the dimension \( d \) in terms of \( \Delta f_1 \) and \( \Delta f_2 \), which we measure from the spectrum analyzer, and from \( \alpha_1 \) and \( \alpha_2 \) which we get from the goniometer.

In Table I is a list of the nine or so reflectors that are shown in Fig. 18. Here we compare some of the dimensions which we measured with the actual sizes for which there is reasonable agreement. The actual dimensions and the measured dimensions, using the approximate evaluation, agree pretty well as you can see. We came within 10\% most of the time. Thus we can get some indication of these dimensions, and we are carrying out similar experiments in solid bodies.

One of the experiments which we are doing in solids is to measure the depth of notches using this same technique. This is an immersed system. We have an aluminum plate with notches of various depths in it. The incident ray is set at such an angle that only a shear wave is propagated into the aluminum, as shown in Fig. 19. Then we studied the scattering from the notches, picked up from a receiver directly above it, and displayed the signal of the spectrum analyzer. From the spacing, we get the measurement of the depth. Actually what we are hoping to do is treat this problem as a diffraction problem, also using Keller's theory, where we have an incident wave, incident shear wave, and a diffracted wave at the slit, and then come up with some analytic expression which can be compared with experiment.

In conclusion, I would just like to say that we are in the process of evaluating Keller's theory for various two-dimensional type of flaws, and comparing the results with the experiments. In the meantime, we are using some approximate theory to evaluate the dimensions of odd shaped objects.
Table I. Comparison of Actual and Measured Sizes for Simulated "Real Flaws".

<table>
<thead>
<tr>
<th>Flaw Type</th>
<th>Dimensions</th>
<th>Flaw Size in Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Actual</td>
</tr>
<tr>
<td>1. Rectangular</td>
<td>Small Side</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>Long Side</td>
<td>0.43</td>
</tr>
<tr>
<td>2. Ellipse</td>
<td>Small Diameter</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>Long Diameter</td>
<td>0.55</td>
</tr>
<tr>
<td>3. Ellipse</td>
<td>Small Diameter</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>Long Diameter</td>
<td>0.85</td>
</tr>
<tr>
<td>4. Irregular</td>
<td>Smallest</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>Largest</td>
<td>0.81</td>
</tr>
<tr>
<td>5. Irregular</td>
<td>Smallest</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>Largest</td>
<td>0.29</td>
</tr>
<tr>
<td>6. Rectangular</td>
<td>Small Side</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>Long Side</td>
<td>0.63</td>
</tr>
<tr>
<td>7. Square</td>
<td>Side</td>
<td>0.48</td>
</tr>
<tr>
<td>8. Irregular</td>
<td>Smaller</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>Largest</td>
<td>0.69</td>
</tr>
<tr>
<td>9. Circle</td>
<td>Diameter</td>
<td>0.52</td>
</tr>
</tbody>
</table>
Fig. 19. Schematic Diagram of a Shear Wave Diffracted at a notch in Aluminum.
PROF. KRUMHANSL (Cornell University): A very interesting point that I think you went over very quickly may be of considerable use to the theorists. That had to do with, I thought I heard you say, that you were able to separate in your transducer and your signal processing, perhaps, the longitudinal and transverse components?

DR. ADLER: Well, if you have a very short signal, about a micron or so, then there is a possibility of separating the shear and the longitudinal waves. From time measurement you can tell whether the signal is coming with a shear velocity or a longitudinal velocity.

PROF. KRUMHANSL: I see. So, was this spectrum analysis, for example, of one component, a longitudinal component?

DR. ADLER: Right.

PROF. KRUMHANSL: Oh, I think it may well be that a form of Keller's theory can be developed for the elastic case. One of the principal problems is sort of a power cross section in that both longitudinal and mode converted transfers are there. Now, if in fact you can separate this, I believe that, as a guide, this kind of semi-geometrical theory would really apply much more easily to one component than to the total of two longitudinal and transverse components.

DR. ADLER: Yes. Actually Keller, in one of his papers, claims that you can treat mode conversion by considering both longitudinal and shear waves as you just mentioned. The problem, I guess, is the coupling.

PROF. KRUMHANSL: The coupling, right.

DR. ADLER: That's what we are trying to do maybe, experimentally come up with some sort of a functional relationship between the different components and incorporate it into the theory.

DR. HENRY BERTONI (Polytechnical Institute of New York): Keller's theory has been applied to classic propagation in an anistropic medium where you can have an electromagnetic. In anisotropic media where you have a mode conversion problem, all the ideas carry across. In this case of an edge, you must solve, in an elastic medium, the semi-infinite edge problem and then that gives you a coefficient, the diffraction coefficient of Keller's theory. Does anyone know if that has been solved?

PROF. KRUMHANSL: That's what Hanson does with electromagnetics. It has been done.

DR. BERTONI: I'm not aware of it for the elastic case.