THEORETICAL ELEMENTS OF ACOUSTIC EMISSION SPECTRA

J. A. Simmons and R. B. Clough
National Bureau of Standards
Washington, D. C. 20234

I will be describing an acoustic emission program at the National Bureau of Standards sponsored jointly by NBS and the Electric Power Research Institute. This is part of a larger comprehensive NDE program at NBS. The acoustic emission program uses the spectral analysis approach for the characterization of moving defects and, as I will point out, includes the development of an acoustic emission transducer calibration facility as well as the experimental study of crack motion in glass and pressure vessel steels. These require a quantitative theory which serves to guide the experimental design as well as to interpret the results.

I will be speaking about progress in this theory of acoustic emission today. Rather than give you a verbal transcript which is lacking in details, I'd like to submit the text of a paper prepared last March which has been presented by Dr. Simmons at the Eighth World Conference on Nondestructive Testing at Cannes, France:

Frequency spectrum analysis of acoustic emission signals shows great promise as a nondestructive test method. It has potential for discriminating between harmful moving defects and system noise as well as for revealing more of defect characteristics than are currently revealed with threshold counting techniques. However, such an analysis requires a systematic examination of the entire acoustic emission process.

To address this problem, NBS expanded its acoustic emission program last fall through joint sponsorship by the Electric Power Research Institute. This AE spectrum analysis program has the following objectives: (1) develop calibration capability for acoustic emission transducers (2) develop theory of acoustic emission spectra from moving defects (3) measure crack velocities optically and acoustically and correlate with above theory, and (4) explore test methods for measuring crack velocity spectra in structural metals.

In this paper, we will be describing progress in the second task, the theory of acoustic emission. More specifically, we will examine acoustic emission from planar, straight line dislocation segments moving in bursts. For simplicity, we will for the present treat the ideal case of an infinite isotropic body. This case should, however, provide an illustration of the general nature of the acoustic emission spectrum and how it is produced.

Transfer Functions

An acoustic emission system is represented in Fig. 1. Stress waves are emitted from the defect as it moves from A to B. Although we shall discuss a particular type of defect, namely a straight line dislocation, the defect shown in Fig. 1 may in general be a dislocation, a crack, a twin, or any other defect which grows by relieving internal strain energy. The stress waves, following emission, propagate through the body and pass through a coupling interface, where they are detected as acoustic emission by a transducer with associated signal processing equipment. The original acoustic emission signal will be changed by the elasticity and geometry of the body as well as by the transducer and signal processing equipment.

These modifications of the stress wave as it passes through the system are conveniently described by transfer functions [1]. For example, if the input to a system is \( X(\omega) \), where \( \omega \) represents the Fourier frequency, then the output is \( Y(\omega) = T(\omega)X(\omega) \), where \( T(\omega) \) is the transfer function for this input/output set. The transfer function for the entire acoustic emission system can often be separated into the product of the subsystem transfer functions. Then the specimen transfer function and the transducer and signal processing transfer function can be treated separately.

Acoustic Emission from a Moving Dislocation

We shall consider an infinite isotropic body containing a dislocation on a slip plane (Fig. 1). When this defect moves, it emits stress waves which are monitored at some point by a transducer which converts local field conditions into voltages.

In this paper, we employ what we shall call a non-disturbing transducer, that is, one whose interaction with the body does not substantially affect the dynamic elastic equilibrium of the body. The most common types of such transducers are "soft" surface transducers which are essentially strain or displacement sensitive. Another conceivable type of such transducer is an internally embedded unit (Fig. 1) whose elastic constants approximate those of the matrix, and it is this latter type of internally embedded transducer which we shall treat in this paper. To simplify some of the expressions, we shall also assume that the transducer response depends on the displacement \( u \). Similar expressions can be obtained for stress and velocity components [3].
We shall derive the expression for acoustic emission from a moving dislocation by using the simplified configuration shown in Fig. 2. As shown, the slip plane $(x_3=0)$ contains a rectangular dislocation loop of Burgers vector $b$. A line segment of length $L$ of the loop moves along the $x_1$ axis so that its position at time $t$ is given by $x_1(t)$. Following Mura [2], the dynamic displacement components produced by the moving dislocation segment are given by:

$$
u_m(r,t) = -C_{ijk}b_k\delta(x_3(t)) dr dt'$$

where $\nu_m(r,t)$ could be taken as the displacement at the transducer inside the body at point $r$ and time $t$ due to a prior plastic distortion rate $b_k\delta(x_3(t))$ at point $r'$ and time $t'$. $G_{mi,j}(r-r', t-t')$ represents the displacement at point $r$ and time $t$ due to a concentrated force in the $x_j$ direction applied at point $r'$ and time $t'$ and maintained for all times after $t'$. $C_{ijk}$ in Eqn. (1) are the elastic coefficients. With reference to Fig. 2, one can define the plastic distortion rate tensor

$$\delta_{kk'}(r', t') = b_k b_{k'} \delta(x_3(t')) \delta(x_1(t') - x_1(t'))$$

Here, $b_k$ are the components of the Burgers vector of the dislocation, $\delta(x_3)$ are the components (in this example) of the slip plane normal, $\nu_1(t')$ is the dislocation velocity at time $t'$, $\delta(x_3(t'))$ is the Dirac delta function expression for the position of the dislocation segment, $H_2$, $(0, L)$ is the characteristic function for the dislocation line segment:

$$H_2, (0, L) = 1 \text{ if } 0 \leq x_2 \leq L$$

$$= 0 \text{ otherwise},$$

and $\delta(x_3)$ is the Dirac delta function identifying the slip plane $x_3=0$.

Substituting equation (2) into equation (1) gives

$$\nu_m(r,t) = -C_{ijk}b_k\delta(x_3) dr dt'$$

and

$$\times \int_0^L dx_1 \int_0^\infty G_{mi,j}^H(r-x_1(t'), x_2, \omega, t-t') \nu_1(t') dt'.$$

For many applications, it can be shown that the size of the moving dislocation segment and the distance it moves (i.e., the change in the position of $r'$) are both sufficiently small relative to the distance of the transducer from the approximate position of the radiating dislocation segment (i.e., $r-r'$) that their effect on $G^H$ is negligible (i.e., $G^H$ depends only on $r-r'$ and $t$). In that case, $G^H$ becomes independent of $r$ and equation (4) may be rewritten as:

$$\nu_m(r,t) = -C_{ijk}b_k\delta(x_3) \int \int_0^\infty G_{mi,j}^H(r-x_1(t'), x_2, \omega, t-t') \nu_1(t') dt'.$$

Equation (5) contains a convolution integral in time, which Fourier transforms to a product in frequency ($\omega$) space. If $\nu_1(\omega)$ represents the Fourier transform, then

$$\nu_m(r, \omega) = -C_{ijk}b_k\delta(x_3) \int \int G_{mi,j}^H(r-x_1(t'), x_2, \omega, t-t') \nu_1(\omega) dt'.$$

where $L$ is the segment length and $\nu_1(\omega)$ is the velocity. Equation (6) can be recognized as a special case of a transfer function formula for the transmission of acoustic emission by a moving dislocation in an infinite isotropic elastic body:

$$\tilde{u}_m(r, \omega) = \tilde{M}_{mk}(r, \omega) b_k \tilde{S}_3(\omega)$$

where

$$\tilde{M}_{mk}(r, \omega) = -C_{ijk}\delta_{mi,j}^H$$

is the medium transfer tensor, whose components are generally complex numbers, corresponding to the emission source tensor

$$\tilde{S}_3(\omega)$$

and $\delta(x_3)$ is the amount of non-recoverable (i.e., plastic) area swept out by the moving dislocation per unit time on the plane with normal in the "3" direction (e.g., in Eqn. (6), $\delta(x_3 L \nu_1(\omega) = \tilde{S}_3(\omega)$).
It is evident from the above equations that acoustic emission has a tensor nature. This is an important characteristic in that the magnitude of the total signal is not merely the sum of the magnitudes from two dislocation segments near a given point, one with a Burgers vector \( \mathbf{b}_1 \) on a plane with normal \( \hat{n}_1 \) and the second with Burgers vector \( \mathbf{b}_2 \) on a plane with normal \( \hat{n}_2 \). Let their respective velocity histories be \( \mathbf{S}_1(t) \) and \( \mathbf{S}_2(t) \). The total acoustic emission field spectrum at the transducer is the superposition of two separate signals whose magnitude is

\[ V_m(r,\omega) = \mathbf{M}_{mk}(r,\omega)\mathbf{b}_k\tilde{S}_k(\omega). \]

This is not in general the sum of the individual emission spectrum magnitudes:

\[ \tilde{V}_m(r,\omega) = \tilde{V}_m(\omega) \mathbf{b}_k\tilde{S}_k(\omega). \]

Therefore, a precise analysis of acoustic emission due to plastic deformation, which is usually a multislip process, requires careful consideration.

In addition, it is evident from Eqn (7) that acoustic emission due to dislocation segment motion (Fig. 2) is a function only of the Burgers vector and the rate of area change on a given plane. It does not matter what direction the dislocation moves. Thus, dislocation acoustic emission is a function only of the rate of plastic strain volume generated.

**Measurement of the Defect Velocity Spectrum**

In considering applications of this theory, we must examine the voltage output from the transducer. Recall that, for simplicity, the transducer is assumed to be small enough relative to wave lengths in a bandwidth that it may be considered infinitesimal in extent and located at the point \( \Gamma_T \). The voltage output from the acoustic emission system can then be written as

\[ \tilde{V}(t) = \int_{-\infty}^{\infty} T_m(t-t')M_{mk}\tilde{S}_k(t')dt'. \]

with the corresponding voltage spectrum

\[ \tilde{V}(\omega) = \tilde{T}_m(\omega)\tilde{M}_{mk}\tilde{S}_k(\omega). \]

\( \tilde{T}_m(\omega) \) is here the transducer and electronics transfer vector for the acoustic emission signal.

Note that the output of the system is the scalar voltage \( \tilde{V}(\omega) \) while the input is the dislocation motion tensor \( \mathbf{b}_k\tilde{S}_k(\omega) \). It is therefore generally not possible to infer the complete nature of the emitting defect signal from a single acoustic emission signal. In order to completely characterize the source signal, a sufficient number of independent observations must be made, for example, by using strain gage type transducers and/or multiple positioned transducers. To be more specific, the transfer function formalism can be contracted to

\[ R\gamma(\omega) = \tilde{R}_{\gamma}(\omega) \tilde{A}_{\gamma}(\omega) \tilde{D}_\gamma(\omega), \]

where \( \gamma \) is a convenient index representing \( \mathcal{K}_k \) and \( \gamma \) refers to an independent measurement using a transducer of a given type at a given location. Similarly, we refer to \( \tilde{B}_k(\omega) \) as \( \tilde{D}_\beta(\omega) \). This gives the voltage from the \( \gamma \)th measurement as

\[ \tilde{V}(\omega) = \tilde{A}_{\gamma}(\omega) \tilde{B}_{\gamma}(\omega) \tilde{D}_\gamma(\omega). \]

In order to solve for the components of \( \tilde{D}_\gamma \), one must have at least as many measurements (\( \gamma \)) as there are degrees of freedom in \( a \). This requires one \( \gamma \) for each \( a \), that is to say, one \( \gamma \) for each \( \mathcal{K}_k \), and since \( \tilde{M}_{mk}(\omega) \) is symmetric in \( k \), there are six possible components. Thus, unless there are special restrictions on \( \tilde{D}_\gamma \), such as predetermined slip planes, at least 6 independent measurements are required a priori to find \( \tilde{D}_\gamma \). In that case, Eqn. (10c) can be solved for \( \tilde{D}_\gamma \):

\[ \tilde{D}_\gamma(\omega) = [\tilde{A}_{\gamma}(\omega)]^{-1} \tilde{B}_{\gamma}(\omega). \]

**Convolution Smoothing and Stochastic Processes**

Once the defect velocity spectrum \( \tilde{V}_G(\omega) = \mathbf{b}_k\tilde{S}_k(\omega)/L \) is obtained by the above method, its inverse Fourier transform yields the defect velocity history \( \tilde{V}_G(t) \). It may be recalled, however, that in deriving Eqn (5), the frequency range was in effect restricted to frequencies sufficiently low that the finite size of the defect could be removed. We will now be more precise about the maximum frequency cutoff and discuss the effects of such low-pass filtering on smoothing the corresponding time domain signal. This will be applied in particular to dislocation motion treated as a stochastic process.

Introducing a low-pass filter is equivalent to multiplying the "infinite" velocity spectrum \( \tilde{v}_G(\omega) \) which extends over all frequencies by a function \( \tilde{B}_m(\omega) \) which transmits signals only over a restricted frequency range \( |\omega| \leq \omega_m \). The time history after filtering at \( \omega_m \) is

\[ v_m(t) = \frac{1}{2\pi} \int_{-\infty}^{\omega_m} \tilde{v}_G(\omega)e^{i\omega t}d\omega. \]

and since a product in frequency space corresponds to a convolution in the time domain [4], we have

\[ v_m(t) = \int_{-\infty}^{\infty} v_m(\gamma) \tilde{v}_G(t-\gamma)d\gamma. \]

The high frequency components, distorted due to interference effects from the finite size of the moving defect, are filtered out. The equivalent
operation in the time domain is a convolution which, as we shall see, is the same as smoothing. In the context of our theory, this convolution smoothed integral represents the only meaningful information about the time history of a burst of motion.

In real materials one may expect that dislocation motion is generally irregular and dislocation segments may stop and start many times (Fig. 3a) during the course of an emission burst (what may be called the "chitty-chitty-bang-bang" effect). Segment velocity during such a burst may then be thought of as a probabilistic or stochastic process.

Suppose that, as an example of this, the dislocation segment shown in Fig. 2 moves in a burst of motion given by the velocity history in Fig. 3a, where the velocity alternates between $v_0$ and 0. Then, as we have discussed, the signal is (a) recorded, (b) Fourier transformed to frequency space, (c) the system transfer function is eliminated to yield the "infinite" velocity spectrum $\tilde{v}(\omega)$, and finally (d) and (e) this spectrum is passed through the low-pass filter at $|\omega| \leq \omega_m$ and the inverse Fourier transform is taken to the time domain. We wish to know the effects of the low-pass filter or the stochastic characteristics of velocity $v(t)$.

The unfiltered velocity spectrum might appear as shown in Fig. 4, where the real part is plotted (the imaginary component can be treated by a similar procedure). A filter is used, as shown in Fig. 4, that has a Gaussian shape to simplify the example (since the Fourier transform of a Gaussian is also a Gaussian). Let the window function be given by $W(\omega) = \tilde{W}(-\omega)$ and which is chosen so that transmission is "insignificant" for $|\omega| \geq \omega_m$. This window is symmetric and real so that its time transform is also symmetric and real [4] as shown in Fig. 3b. These transform window pairs correspond to multiplication in frequency space (Eqn. 11) and convolution in time (Eqn. 12). The convolved signal is

$$v^c(t_w) = \int_{-\infty}^{\infty} v_0(t_w - \gamma) W(\gamma) \, d\gamma.$$  

Let us imagine, on the other hand, an averaging operation as depicted in Fig. 3a, where the signal is averaged over the unit area element $W(\gamma)$. This produces a smoothing as shown in Fig. 3c, where the relative magnitudes and details of the ripples will depend on the signal and the smoothing operation. The smoothed velocity is given by

$$v^S(t_w) = \int_{-\infty}^{\infty} v_0(t_w + \gamma) W(\gamma) \, d\gamma = \int_{-\infty}^{\infty} v_0(t_w - \gamma) W(-\gamma) \, d\gamma,$$

which, owing to the symmetry of $W$, is the same as the convolution in Eqn. (13). Therefore, we may write that $v^S(t_w)$, that is, time averaging and filtering are equivalent smoothing operations. The exact nature of the smoothing will depend on the signal and filter. We may expect, however, that a smaller defect size will permit the use of a higher cut-off frequency. A broader spectral window, to be somewhat imprecise, will generally correspond to a more narrow smoothing element, permitting greater resolution of the undistorted velocity signal as a function of time. One can generally expect, therefore, (other things being equal) to obtain more details of motion for finer sized defects.

Conclusions

We have attempted to set down a conceptual framework for some of the essential principles needed to make quantitative defect characterizations using acoustic emission spectrum analysis. The simplest possible system was therefore selected - an infinite isotropic body with an essentially one-dimensional moving dislocation segment. We formulated its transfer function and discussed some of the characteristics of the signal and its measurement. There are a number of important problems remaining to be solved: the analysis for cracks and inhomogeneities, the characterization of multiple and/or complex sources, the effects of geometrical boundaries and microstructure, as well as possible non-linear or anharmonic effects.

Figure 3. Dislocation segment velocity burst.

Figure 4. Spectrum of velocity for burst motion of segment.
REFERENCES


3. J. A. Simmons and R. B. Clough, to be published.


DISCUSSION

PROF. JOHN TIEN (Columbia University): Thank you. I guess the obvious question is whether you and Bill Pardee have any large quantum disagreements?

DR. CLOUGH: I haven't had time to examine his work in detail, but it appears that so far he has concentrated on the geometrical effects of the medium, whereas we have more clearly examined the acoustic emission source.

PROF. TIEN: Any comments?

PROF. STEVE CARPENTER (University of Denver): It's interesting that in a lot of materials well into plastic flow where you're making dislocation loops, very great numbers of them, you see no acoustic emission.

DR. CLOUGH: I don't quite understand that. Could you---

PROF. CARPENTER: Well, what I'm saying: in a sample in which you're loading and you get well into plastic flow and the dislocation density is increasing with the plastic strain and you're generating dislocation loops-those loops are moving-you see no acoustic emissions.

DR. CLOUGH: This is not directly concerned with our theory, but let me make a comment. You're getting attenuation with increased total density of the loops. Also, the mobile (emitting) density actually decreases with strain so that one tends to get a maximum emission right at the yield point, right? Then it begins to decrease.

PROF. CARPENTER: Right at the yield point, but is that a motion that you're describing or is that a motion in breaking away from precipitates and that type of thing? I think it's a much different system.

DR. CLOUGH: Oh, yes. I'm not describing such a complex system. As I said, for a multiple dislocation system it's not immediately obvious just how one would obtain the total signal. What I'm talking about is the motion of a single defect in an infinite body.

PROF. TIEN: Otto?

DR. BUCK (Science Center): You distinguished between climb and regular motion of dislocations. Can you speculate what would happen in the case of cross slip?

DR. CLOUGH: Well, yes, we'd be talking about slip and you would have a change in slip systems. Now what I have shown is that the signal is very sensitive to the orientation of the Burgers vector and slip plane normal, so that if there is cross slip, you're going to change the slip plane normal, and the emission field will be correspondingly altered. If you're in a very sensitive position, you might just suddenly get a signal or not get a signal.

I might comment here that those signals that I showed were only along the principal directions and in off-axis directions. There may be other nodes and other maxima and so forth.

DR. BUCK: Does that mean that if you could pick up the dislocation motion, then, that would be a nice method to try to determine the stacking fault energy for the \( r_{III} \) method?

DR. CLOUGH: That's a very interesting idea. I don't think that one could get sufficient signal from a single dislocation. If you had a large number of dislocations, moving in a shear band, let's say, you might be able to make something of that.

PROF. TIEN: Thank you.