Characterization of NDE transducers is an important part of current and future programs in quantitative flaw detection. In the work reported here, emphasis has been placed on beam pattern measurements or profiling and circuit modeling of the transducer using electrical network scattering parameters, or S-parameters. The latter topic is relatively new and was developed in the last six to eight months in order to handle the problems of existing commercial transducers whose internal details are unknown. A subset of this goal is to:

(a) explore single crystal or very high quality ceramic materials as the piezoelectric element,
(b) examine the single disk in water as a transducer or reference element, and
(c) look into electrical loading as an alternative to mechanical loading.

Shown in Fig. 1 is a schematic diagram of the setup for measuring or calculating the field pattern of a transducer. The transducer is located in the \((x,y)\) plane and has an assumed extent of \(2a\) for the purposes of field reconstruction. The measurement plane \((u,v)\) is located at distance \(R\) from the source plane in the Fresnel zone or even in the far field. To first order the planes are parallel although a slight non-parallelism is easily detected and accounted for in the analysis. The sampling points are taken as \(\Delta u\) and \(\Delta v\) where usually \(\Delta u = \Delta v\). As described in last year's meeting, the measurement plane is scanned by a computer controlled stepping motor driven table and the measured amplitude and phase are fed to the computer for analysis.

The information used for the field reconstruction is given below.

\[
U(u,v) = \frac{2\pi}{ik} \int \int U(x,y) e^{ik\left[(x-u)^2 + (y-v)^2\right]/2R} \, dx \, dy.
\]

Letting \(\theta = KR\),

\[
R = \frac{U}{\theta} \frac{d\theta}{df}, \quad \text{and}
\]

\[
d\theta = \theta \left[ d\theta/df + dR/R - dV/V \right].
\]

The average distance \(R\) may be determined by measuring the variation of average phase \(\theta\) with frequency. In addition the phase may be measured at several points and the parabolic phase cap evaluated to determine the apparent source-to-field distance. The phase measurement is sensitive to the changes in frequency, average distance, and velocity of sound in water.

\[
U(u,v) = C \int \int U(x,y) e^{ik\left[(x-u)^2 + (y-v)^2\right]/2R} \, dx \, dy.
\]

The average phase was held fixed by controlling frequency in a phased locked loop synthesizer. The natural building vibrations produced \(dR\) variations in the micron range at frequencies up to 100 Hz and resulted in phase fluctuations which could only be eliminated by signal averaging. The velocity of sound in water is quite temperature dependent and the measurement was complicated by the fact that the building air conditioner is shut down at night and turned on again in the morning, causing a long-term gradual shift in phase as the water bath slowly heated during the course of the measurement. More recently, the velocity and distance sensitivity has been circumvented by passing the reference signal of the network analyzer through the water bath as well, and thus greatly reducing the phase fluctuations.
The Fourier transform format is obtained by a normalization by expanding the parabolic phase factor. The results are given below.

\[ \tilde{U}(c', v') = \mathcal{C} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{U}(x', y') e^{ik'(u'x' + v'y')} \, dx' \, dy' \]

where

\[ \tilde{U}(u', v') = U(u', v') e^{-ik'(u'^2 + v'^2)/2} \]

\[ \tilde{U}(x', y') = U(x', y') e^{ik(x'^2 + y'^2)/2} \]

\[ C = 2\pi e^{ikR} \]

\[ x' = x/a, \quad y' = y/a, \quad u' = u/a, \quad v' = v/a \]

\[ k' = ka^2/2R. \]

Once in the Fourier transform format, the inverse fields are readily defined and calculated. One of the research problems was to examine the nature of the field distribution and determine the minimum number of sampling points required to reproduce the field to a given degree of accuracy consistent with experimental accuracy.

Experimental results with computer reconstruction are shown in Figs. 2, 3, and 4. Figure 2 gives the field amplitude and phase of a transducer, at the transducer face, as reconstructed from measurements made in the Fresnel Zone. There is a tilt in the phase due to a slight non-parallelism between the average phase plane of the transducer and the scan plane. The shift in the plots to the right of the assumed origin accounts for the uncertainty in locating the axis of the transducer relative to the scan plane.

\[ -0.565 -0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4 0.5 \]

\[ -0.565 -0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4 0.5 \]

**Figure 2.** Amplitude and phase of the reproduced field of a 1.7 inch diameter transducer. The linear phase contribution is due to axis tilt during the measurement.

Figures 3 and 4 are reconstructions of a transducer whose front face was covered in a small area with a partially absorbing film of rubber cement. The reconstruction is based upon a 17x17 sampling grid taken at intervals of 0.5 inch. The contour plots in Fig. 3 are of the linearly smoothed amplitude at the transducer with the amplitude at the center of the defect normalized to unity. Figure 4 is a grey-scale reconstruction of the same case as in Fig. 3, except that no smoothing has been done to the 17x17 sampling grid. Both power and amplitude are shown in a graphic manner that corresponds to an image of the transducer field.

\[ 1.3 \quad 1.4 \]

\[ 1.2 \quad 1.5 \]

\[ 1.0 \quad 1.7 \quad 1.8 \]

**Figure 3.** Amplitude contours on a 0.7 inch diameter transducer with defect. Dashed circle represents the assumed edge of the transducer.

\[ \text{Figure 4 Grey-scale image of the acoustic field at the transducer surface. Black represents the lowest levels and white the highest levels.} \]

\[ (a) \text{ Field intensity (amplitude squared).} \]
Figure 4. Grey-scale image of the acoustic field at the transducer surface. Black represents the lowest levels and white the highest levels.

(b) Field amplitude.

The methods used to image the transducer as a source can also be used to image a reflecting surface by treating the surface as an apparent source and using the reconstructions described above.

To review briefly, the experimental procedure for beam pattern measurements is to:

1. Locate the apparent beam axis by measuring the "phase cap"
2. Measure $d\theta/df$ and calculate $R$, the average distance between source and receiver
3. Assume a value for the source size and scan the grid accordingly
4. At each measurement point in the field plane, normalize the parabolic phase factor
5. At the end of the scan, calculate the transform of the normalized field, and,
6. Un-normalize the result if both amplitude and phase are desired, and display the source field.

Once having the amplitude distribution at the face of the transducer the acoustic field may be determined at any other location by the transform method. It is of interest in quantitative NDE to relate this acoustic pressure field to the electrical signals at the transducer terminals.

The inverse scattering results reviewed above were used to determine the amplitude and phase variations at the transducer surface based upon measurements made in the Fresnel zone. To complete the transducer characterization, it is necessary to determine the electrical to acoustical transduction efficiency as well. There are numerous procedures which may be used to accomplish this task involving calibrated sources of acousto-optic interactions. The method outlined here involves wave reflections from a known reference surface and can be accomplished with the usual ultrasonic equipment with the slight addition of a coaxial directional coupler. The S-parameters are then derived from measurements taken only at the electrical terminals.

The S-parameters, or scattering parameters, are a convenient means of describing devices involving transmission line type behavior. This concept may be adapted to the NDE transducer through the following set of equations,

$$V_r = S_{ee} V_i + S_{ea} T_i \quad (1a)$$
$$T_r = S_{ae} V_i + S_{aa} T_i \quad (1b)$$

In (1) $V_i$ is the voltage of the wave incident upon the transducer, such as from a generator, $V_r$ is the voltage of the wave reflected from or leaving the electrical terminals, $T_i$ is the acoustic pressure field of a wave incident upon the transducer, and $T_r$ is the acoustic pressure field of the wave reflected or radiated from the transducer. The four subscripted $S$ constants are referred to as the S-parameters.

If the transducer radiates into an empty half space, containing no other sources, then $T_i$ is zero since no acoustic waves can be incident upon the transducer. In this case (1) reduces to

$$V_r = S_{ee} V_i \quad (2a)$$
$$T_r = S_{ae} V_i \quad (2b)$$

Thus $S_{ee}$ is just the electrical reflection coefficient due to an impedance mismatch between the electrical cable and the transducer, and is a readily measured parameter. The parameter $S_{ae}$ relates to the conversion of electrical volts to acoustic stress or pressure and is of direct interest, as will be discussed later here in detail. If the transducer is excited only by an acoustic wave, then $V_i=0$ provided there are no reflections from the receiver input. In this case (1) reduces to

$$V_r = S_{ea} T_i \quad (3a)$$
$$T_r = S_{aa} T_i \quad (3b)$$

For this case $S_{ea}$ is readily identified as the acoustic to electrical conversion factor and $S_{aa}$ as the acoustic reflection coefficient when $V_i=0$.

Before reviewing the techniques that were developed to measure the S-parameters it is useful to first go back and re-evaluate the nature of the parameters $S_{ee}$ and $S_{ae}$ of (1). First of all, note that the parameters are really a hybrid mixture of acoustic and electrical quantities and not dimensionless as might be desired. Essentially $S_{ee}$ and $S_{ae}$ contain a factor which relates the conversion of units that is inherent to the transduction process. To see this in more detail, refer to the Mason equivalent circuit model of a piezoelectric disk, Fig. 5.

$$V_r = S_{ee} V_i + S_{ea} T_i \quad (1a)$$
$$T_r = S_{ae} V_i + S_{aa} T_i \quad (1b)$$
Figure 5. Equivalent circuit of piezoelectric disk including a backing load, $R_g$.

The lumped element parameters have the usual definitions,

\[
\begin{align*}
    n &= 2f_0 k C_t z_0 \\
    Z_1 &= -j z_0 \csc(p) \\
    Z_2 &= j z_0 \tan(p/2) \\
    C_a &= C_t/n \\
    C_t &= \text{disk capacitance} \\
    p &= kd, \text{phase} \\
    k &= \text{wave number} \\
    d &= \text{disk thickness} \\
    z_0 &= \text{disk mechanical impedance} \\
    f_0 &= \text{disk parallel resonant frequency}
\end{align*}
\]

An important concept to be used in the analysis is that the transformer may be "drifted" across the network so long as the circuit elements take on appropriate units. For example, the normal circuit model is drawn with the capacitors on the electrical side of the transformer but are shown here on the acoustic side.

From the network in Fig. 5, it is apparent that the stress $T_e$ is the mechanical equivalent of voltage $V_e$ and is simply related to $V_e$ by the relation,

\[ T_e = n V_e \]

Likewise, $T_0$ is related to $T_e$ through a simple network relation,

\[ T_0 = S T_e \]

The stress $T_0$ may be used to represent either $T_i$ or $T_r$ of (1). Thus the parameter equations may be written as,

\[
\begin{align*}
    V_r &= S_e V_i + S/n T_i \\
    T_r &= n S V_i + S_{aa} T_i
\end{align*}
\]

Now the S-parameters are in a dimensionless form.

A real transducer usually contains an electrical matching network and an acoustical matching network as well, Fig. 6a. However, since the transformer may be "drifted" to the left, the electrical network may now be included with the two acoustical networks to form one single network, Fig. 6b.

In order to characterize the transducer within the scope of this model, it is necessary to establish a measurement procedure. Two kinds of procedures have been adopted and implemented experimentally. The basis for the procedures rests upon establishing a known relationship between $T_i$ and $T_r$, since it is assumed that the acoustic stresses are not to be measured directly. One way to relate $T_i$ and $T_r$ is to use the transducer to scatter sound off of a known target such as a planar surface, Figs. 7 and 8. The relation is then,

\[ T_i = R \, T_r \]  

where $R$ is the reflection coefficient of the surface including the phase factor. Using this relation in (7) an electrical reflection coefficient can be derived.

\[ R_e = V_r/V_i = S_e + R S/(1 - R S_{aa}) \]
Figure 7 Mechanical setup for measuring the S-parameters using air/liquid as the reference scattering surface. In the water bath system the water surface is adjusted for parallelism with the transducer surface. The water level may also be adjusted to move the "short-circuit".

Figure 8 Measuring setup for air-solid reference using a block with parallel surfaces. The bond region is considered part of the transducer network.

By using a directional coupler or bridge in the electrical line, as shown in Fig. 9, $R_e$ may be measured quite easily. If the measurement is in the pulse-echo mode, then the first and second terms may be resolved by time separation,

$$R_e(t) = S_{ee} u(t) + S^2 u(t-t_a) + (S^2)S_{aa} u(t-2t_a) + (S^2)(S_{aa})^2 u(t-3t_a) + \ldots \ldots$$

where $u(t)$ is the unit step function. Thus $RS$ is measured directly if the time resolution is sufficient. The reflection coefficient, $R$, is assumed to be known and thus $S$ is determined.

By using a directional coupler or bridge in the electrical line, as shown in Fig. 9, $R_e$ may be measured quite easily. If the measurement is in the pulse-echo mode, then the first and second terms may be resolved by time separation,

$$R_e(t) = S_{ee} u(t) + S^2 u(t-t_a) + (S^2)S_{aa} u(t-2t_a) + (S^2)(S_{aa})^2 u(t-3t_a) + \ldots \ldots$$

where $u(t)$ is the unit step function. Thus $RS$ is measured directly if the time resolution is sufficient. The reflection coefficient, $R$, is assumed to be known and thus $S$ is determined.

In the steady state mode of operation, all the terms of (9) are determined using well known microwave network measuring techniques. The most simple method is to use a high frequency voltmeter or oscilloscope to measure the magnitude variations of $R_e$. If the phase of the acoustic reflection coefficient, $R$, is changed over several 360 degree intervals, the magnitude of $S$ is seen to go through several maxima and minima, $S^+$ and $S^-$ respectively, corresponding to the interference phenomena taking place between transducer and reflector. From these, $S^2$ and $S_{aa}$ are given by,

$$S^2 = \frac{S^+S^-}{S^a + S^- a} \quad S_{aa} = \frac{S^+ a + S^-}{S^a - S^- a}$$

The phase may be changed either by sweeping the input frequency or by moving the physical position of the reflector. The latter technique was implemented by reflecting off the top surface of a water bath and then raising or lowering the water level to change the distance between the transducer and reflecting surface, Fig. 7. The water level could be lowered to within several wavelengths of the transducer surface before surface tension distorted the reflecting surface. If a solid-air reference is used, then the system shown in Fig. 8 may be used in the swept frequency of pulse-echo mode and the bond considered part of the transducer. Using these techniques $S_{ee}$, $S_{aa}$, and $S$ were measured for several NDE transducers.

The turns ratio $n$ was not measured and cannot be measured without some absolute measure of stress such as by using a transducer or simple disk having a known turns ratio. It should be made clear that the turns ratio need not be known if the same transducer is used for both sending and receiving. Also note that $n$ is a property of the piezoelectric disk itself and not dependent upon electrical or acoustical matching or damping elements.

Two transducers exhibit a composite $S'$ given by

$$S' = S_1 S_2 (n_2/n_1)$$
where $S_1$, $n_1$ belong to the receiving transducer and $S_2$, $n_2$ belong to the sending transducer. If the two transducers are aimed nearby at each other and $S'$ measured via transmission then the ratio of $n$'s is now determined since $S_1$ and $S_2$ may be found separately.

Clearly, if the two transducers have identical disk elements, neither $n$ need be determined since only the $S$'s are required. Generally all that is needed is knowledge of what kind of material is used for the piezoelectric disk since the diameter and disk thickness are usually obvious.

DISCUSSION

PROF. VERNON NEWHOUSE (Purdue University): Are there any questions?

PROF. GORDON KINO (Stanford University): If the term on the $S$-parameter is, as you say, analagous to the wave guide case which postulates no loss, can the theory be extended over into the lossy case?

DR. LAKIN: Oh, yes.

PROF. KINO: Particularly, when you do measure these $S$-parameters, what sort of loss do you see?

DR. LAKIN: Well, the $S$-parameter essentially turns out to be what you would normally think of as the conversion loss of the transducer. The theory is a little more general in that it tells you how to put two transducers together. Say you're doing scattering with two transducers; it tells you how to use two sets of parameters together because you can use the $S$-parameter model for both transducers. I didn't show it.

You can also convert $S$-parameters over to other parameters such as the impedance parameters, the hybrid impedance parameters, the admittance parameters, which ever you like.

MR. ROY SHARPE (Harwell Labs): How in the world do you see this characterization being done? We find that a firm that buys a transducer for, say, $50$, is not going to spend $200$ to get it characterized.

DR. LAKIN: I'm not sure whether you can buy one for $50$, but I understand the point. I think it's not really a matter of what the transducer costs. If it costs you $10$, you still may have need to know what it's characteristics are. So, unfortunately, you may have to spend an hour or so of a man's time and some investment. Equipment investment here is about zero, because if you use your standard pulse echo system, all you have to do is just set up that reflector system and make your measurements right there. Your capital investment is very small, I would say.

DR. EMANUEL PAPADAKIS (Ford Motor Company): I have a comment and a question. In Settig's analysis, he put in a transducer that had two acoustic ports, the front and the back. And you always get losses from the back when you're generating the wave, and when you have an incoming wave, part of it goes out into the electrical system and part of it goes out into the backing system.

DR. LAKIN: That's right.

DR. PAPADAKIS: How are you getting away from that? That's the first question. The second part is how does the entire analysis relate to Settig's work in 1969 in the Microwave Transactions on sonics and ultrasonics?

DR. LAKIN: Okay, the first question. The backing load is included in the formulation. Settig did that, including it as part of the transducer network to make it a two port. As long as there is no active source there in the backing load, then the formulation handles everything. Now, let's say, for example, that your backing load was imperfect and it didn't completely absorb because some sound was reflected back into the transducer. Well, instead of putting the real impedance there you have to put in a second lossy transmission line. So, you will get something coming back from that rear interface. Now, so far as the electrical port is concerned, that's taken into account in the $n_2$ term.

DR. PAPADAKIS: Is the impedance of the backing taken into account in the turns ratio or something--so that you have less power out electrically than you would have because some of the power has gone into the backing?

DR. LAKIN: That is accounted for, but not in the turns ratio. The nice thing about the turns ratio is that it is intrinsic to the piezoelectric disk itself.

DR. PAPADAKIS: Okay.

DR. LAKIN: Not how it's bonded. It's intrinsic to the disk.

DR. PAPADAKIS: Is it some other impedance?
DR. LAKIN: Yes. There was a branch in the circuit called "backing loads." I put it in as a resistor because that's how you would like to have it unless you're trying to do something fancy with the frequency response. You would like to have that as a real absorbing element. If it's not, then it's a section of the transmission line, nevertheless well characterized.

PROF. NEWHOUSE: We're getting short of time. We have time only for one more short question.

PROF. R. E. GREEN (John Hopkins University): With respect to the single disk transducer, I think you'll find, and maybe you already know, that many people who are not in the NDE business or in the medical ultrasonic business use single disk transducers, and to the best of my knowledge the only reason they use the commercially made transducer in a box is because the average NDE guy throws it around in the tool box. It is for protection. The main comment I want to make is why did you ever think of water anyway? I suggest that you could do some optical imaging of the sample, especially with a CW technique, and you can be sure you get the transducer parallel to the surface or whatever, because it's fairly critical in setting up a standing wave in a tank of water and pretty easy to do like the Stearn's technique. If you want to do a more quantitative measurement you can do laser beam probing or something like that. I'm not saying you should do that every day in the shop, but for setting up standards I would suggest some optical technique be used.

DR. LAKIN: Yes. Well, in terms of measuring the S-parameters, you can very easily align the surface of the water parallel with the phase front of the transducer. I don't know whether it's going to be parallel with the geometric surface of the transducer, chances are it won't be. But you can do that just by watching the manifestation of the standing wave, mainly, the fluctuations in the electrical reflection coefficient and then adjust the table for parallelism. In the process of doing that it's hard to keep the slowly varying ripples from going across the surface of the water. It turns out to be a benefit because you know whether you're at the maximum or not.

PROF. NEWHOUSE: Thank you.