Examining elementary pre-service teachers' capacity to use children's mathematical understanding to select and pose mathematical tasks

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Examining elementary pre-service teachers’ capacity to use children’s mathematical understanding to select and pose mathematical tasks

by

Mary N. Gichobi

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirement for the degree of

DOCTOR OF PHILOSOPHY

Major: Education

Program of Study Committee:

Corey Drake, Co-major Professor
Alejandro Andreotti, Co-major Professor
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Iowa State University
Ames, Iowa
2013

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DEDICATION

To

my loving husband,

Josephat G. Njoka,

and our three children:

Joyce, Mercy and Eric;

for your motivation, love,

and support.


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NOMENCLATURE

1. ACT-Advancing Children’s Thinking
2. ACS-Attending to Children’s strategies
3. CGI-Cognitively Guided Instruction
4. CCK-Common Content Knowledge
5. CMU-Children’s Mathematical Understanding
6. ICU-Interpreting Children’s Understanding
7. JCU- Join Change Unknown
8. JRU- Join Result Unknown
9. JSU-Join Separate Unknown
10. KCS-Knowledge of Content and Students
11. MKT- Mathematics Knowledge for Teaching
12. NoTD- Number of the Day
13. ONR- Opening Number Routine
14. PCK-Pedagogical Content Knowledge
15. PST- Pre-service Teachers
16. RBoCMU-Responding Based on Children’s Mathematical Understanding
17. SCK-Specialized Content Knowledge
18. SCU- Separate Change Unknown
19. SRU- Separate Result Unknown
20. TTLP- Thinking Through the Lesson Protocol
21. VAST- Video Analysis Support Tool
AKNOWLEDGMENTS

First, I would like to thank my advisor Dr. Corey Drake. You have served as a wonderful role model for me for the last six years that we have worked together. You have taught me how to think like a scholar, nurtured my desire for research and provided abundant feedback and advice. I sincerely appreciate all of your encouragement and support as I learned what it means to be a researcher and a teacher educator. Thank you for being a wonderful mentor and a friend. My family and I have gone through a lot within the time that we have worked together, and you have stood with us every step along the way. I am sincerely grateful.

I would also like to thank the rest of my dissertation committee: Drs. Alejandro Andreotti, Barbara Dougherty, Heather Bolles, Katherine Bruna. I greatly appreciate your time, guidance and support through not only this study, but also my entire time at Iowa State University. I have special memories for the time I have spent with each of you. You have created a safe environment for me to run to, any time when I needed your help. Alex, thank you for working closely with me, through out the process of writing this dissertation.

To the Pre-service teachers who agreed to be participants in this study, I am sincerely grateful. This work would not have been feasible without your contribution. I am certain, you will positively affect the lives of many children in your journey as teachers.

To my husband and dearest friend, Josephat G. Njoka, thank you for always being very supportive and loving me at all times. Your patience, advice and agreeing to be there for me will always be sincerely appreciated. This work would not have been possible if it were not for your courage and determination. You have helped me achieve my hopes and dreams!
To my children Joyce, Mercy and Eric, this is for you! You have always been there to cheer me up and to make me laugh even when I am tired.

To all my friends and fellow graduate students, thanks for being there for me. Every laughter made a difference as I went through this Journey. Special thanks go to Faith and David Lichoro who have sacrificed their time and gone out of their way to support me in this venture. I am sincerely grateful. Finally and very special thanks to Keith and Linda Cadwell, your friendship and support has positively impacted each one of my family members tremendously. You have been God given gift for us on this journey in this land. May God richly bless you.

And Lastly, I thank my heavenly father for giving me the opportunity to learn the things that I have learned. You provided me with courage, strength, and a strong desire to be persistent. Indeed, God helped me always to remember the lessons learned as I went through the program.
ABSTRACT

The need to design teacher preparation programs to ensure that pre-service teachers (PSTs) are prepared and equipped with knowledge, skills and practices to increase the chances that they will become effective novice mathematics teachers is of prime importance. Teacher educators are facing lingering challenges, since teacher education is contextualized to specific institutions and the field of teacher education still lacks an identified common curriculum (Ball, Sleep, Boerst & Bass, 2009; Grossman & McDonald, 2008). Therefore, little is known about how PSTs acquire the knowledge, skills and practices that they need to become beginning teachers. Specifically, very little is known about how PSTs develop skills and practices needed to attend to children’s strategies, interpret and respond based on children’s mathematical understanding.

This dissertation research addresses this gap by examining the extent to which a group of thirty PSTs enrolled in an elementary mathematics methods course attended to children’s strategies, interpreted and responded based on children’s mathematical understanding as they progressed in their methods course. The PSTs were provided with multiple scaffolds during the course of the semester. The scaffolds were purposefully designed to support PSTs’ understanding of what it means to attend to children’s strategies, interpret and respond based on children’s mathematical understanding. The findings indicate that PSTs’ capacity to attend to children’s strategies and to interpret based on children’s mathematical understanding grew over time. There was a shift from limited evidence that PSTs’ interpretations were based on children’s mathematical understanding to providing robust evidence across two assignments. However, the results also show that PSTs struggled
with the component skill of responding based on children’s mathematical understanding.

The findings also indicate that PSTs’ conceptions of productive tasks and tasks that engage children with high level thinking varied from tasks that *advance children’s understanding*, *extend children’s thinking* to *tasks that are based in real life connections*.

These two findings may have occurred, in part, because PSTs were *purposefully* scaffolded with activities that focused on developing the targeted skills and practices. The findings have theoretical implications for a hypothesized trajectory of professional noticing of children’s mathematical understanding and the design of teacher preparation.
CHAPTER 1. INTRODUCTION

The question of what pre-service teachers (PSTs) can learn in teacher preparation programs to increase the chances that they will become effective mathematics teachers is increasingly drawing national attention in the United States (Allen, 2003; Cochran-Smith & Zeichner, 2005; Grossman & McDonald, 2008; Morris, Hiebert & Spitzer, 2009; National Council of Accreditation of Teacher Education (NCATE), 2010). In part, this national debate has come up as the nation is envisioning how to transform K-12 education and has emphasized the need to prepare students with strong background in science, technology, engineering and mathematics education (STEM). Consequently, there is emphasis (e.g., National Mathematics Advisory Panel (NMAP), 2008) on strengthening the preparation of elementary and middle school teachers as one means of improving teachers’ effectiveness for 21st century classrooms. However, despite this challenge for teacher preparation programs, research that has focused on what PSTs can learn and the most effective strategies for educating and training teachers is still in its infant stage.

Challenges in Studying the Effectiveness of Teacher Preparation Programs

An accumulating body of evidence suggests that lack of a common knowledge base, curriculum or a common pedagogy in teacher preparation programs has made it challenging to study how PSTs acquire the knowledge, practices, and even skills required for them to become effective mathematics teachers (Allen, 2003; Ball, Sleep, Boerst & Bass, 2009; Cochran-Smith & Zeichner, 2005; Grossman & McDonald, 2008; Morris, Hiebert & Spitzer, 2009). For example, Allen (2003) reviewed ninety-two studies that focused on teacher preparation to ascertain the most effective strategies for educating and training teachers.
Specifically, Allen reported that it was unclear from the reviewed studies how PSTs acquire the knowledge and skills required through teacher preparation programs course work or field experiences. Grossman & McDonald (2008, p.3) corroborated this report arguing, “research in teacher education is still in its adolescence, in search of its distinctive identity”. Therefore, a number of reports have recommended that high quality research should be conducted to create a sound base for mathematics preparation of elementary and middle school teachers. For example, NMAP (2008) recommended:

High-quality research must be undertaken to create a sound basis for the mathematics preparation of elementary and middle school teachers within pre-service teacher education, early career support, and ongoing professional development programs. Outcomes of different approaches should be evaluated by using reliable and valid measures of their effects on prospective and current teachers’ instructional techniques and, most importantly, their effects on student achievement. (p. 20)

To address the need of preparing effective mathematics teachers, researchers and the mathematics teacher education community have sought to understand the nature of knowledge that teacher’s need, practices and even skills for them to effectively enact mathematics instruction. The underlying presumption is that understanding the knowledge, practices and skills that teachers need would help teacher educators design activities and experiences in the teacher preparation programs that would develop the required knowledge and skills. For example, the Learning Mathematics for Teaching (LMT) project located in the University of Michigan has provided insights into the nature of mathematical knowledge for teaching required in teaching mathematics (Ball, Hill & Bass, 2005; Hill, Schilling &Ball, 2005; Hill, Ball & Schilling, 2008; Hill, Rowan & Ball, 2005). Other studies have provided insights into how teachers notice what happens in mathematics classroom using videos (Sherin, Linsenmeier & Van Es, 2009; Sherin & Van Es, 2005) and, how teachers use

Informed by the accumulating body of evidence that is identifying the knowledge, skills and practices that teachers need to become effective in teaching mathematics, teacher educators are examining ways they can develop these knowledge, skills, and practices in the teacher education programs to increase the chances that PSTs will graduate with beginning competencies in teaching mathematics. Recent studies have started focusing on structuring the mathematics method courses with a focus of achieving commonalities and shared knowledge among teacher educators that would enhance the quality of teacher preparation. For example, Hiebert & Morris (2009) examined the knowledge building process for K-8 teacher preparation in the University of Delaware. Hiebert & Morris argued that structuring methods courses with common learning goals for teacher preparation involves having the same learning goals for prospective teachers and sharing the activities used to achieve the learning goal with beginning teacher educators. Similarly, Ball et al. (2009) have articulated a preliminary set of criteria that could support teacher educators in identifying “high-leverage practices” (quotations in the original) for beginning teaching of mathematics. As Ball et al. (2009) argued, identifying and focusing on those practices would ensure the methods courses focuses on “practices that are most likely to equip beginners with capabilities for the

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1 Specifically, Ball et al. (2009) identified the criteria of identifying high leverage practices as: “1. Supports work that is central to mathematics; 2. Helps to improve the learning and achievement of all students; 3. Is done frequently when teaching mathematics; and 4. Applies across different approaches to teaching mathematics p. 4.”
fundamental elements of professional work and that are unlikely to be learned on one’s own through experience” (p. 4).

Using the criteria identified by Ball et al. (2009), one would argue that the ability to attend to children’s strategies, interpret and respond based on children’s mathematical understanding is one of the “high leverage practices” (p. 5) that PSTs should learn as they go through the teacher preparation program for various reasons. First, prior research has indicated that effective teaching involves engaging students understanding and misunderstanding and building on that formal or informal students knowledge (Bransford, Brown, & Cocking, 2000; Fuson, Kalchman, & Bransford, 2005; Kilpatrick, Swafford, & Findell, 2001). Second, recent research has shown that the expertise of attending to children’s strategies, interpreting and responding based on children’s mathematical understanding is unlikely to be learned on one’s own through experience (Jacobs, Lamb & Philips, 2010).

Specifically, Jacobs and her colleagues investigated how teachers at different levels of experience with children’s mathematical thinking performed in the three hypothesized component skills of attending to children’s strategies, interpreting and responding based on children’s mathematical understanding. Participants included; 1) prospective teachers, 2) initial participants with teaching experience but no professional development, 3) advancing participants who had teaching experience and two years of professional development and, 4) emerging teacher leaders who had teaching experience, four years of professional development and had been involved in few leadership activities to support other teachers. Jacobs et al. reported that 74% of the initial participants provided no evidence that their response was based on children’s mathematical understanding. In addition, 36% of the advancing participants and 18% of the emerging leaders who had been involved in a two year
professional development did not provide any evidence that their response was based on children’s mathematical understanding. Therefore, since the in-service teachers do not develop these practices on their own, it is most unlikely that the PSTs will develop these component skills without any support.

Third, research has shown that using child’s mathematical understanding helps to improve the learning of all students (Fennema, Franke, Carpenter & Carey, 1993; Franke, Kazemi, & Battey, 2007; Franke & Kazemi, 2001; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Sfard & Kieran, 2001). Fennema et al., 1993, for example, found that at the end of the 4th year of study in Ms J.’s classroom, “there were some consistencies in mathematics thinking of the students across the various ability groups” (p. 24). The children had also developed a strong sense of numbers and they felt comfortable manipulating numbers. Similarly, Jacobs et al. (2007) found that the students from the classrooms where teachers participated in a year-long Cognitively Guided Instruction (CGI) professional development showed “a better understanding of the equal sign and used more strategies as they reflected on relational thinking than children who were in non-participating classes” (p.1). Collectively, these studies suggest that using knowledge of children’s mathematical thinking to inform instructional decision-making is beneficial to students’ learning.

Finally, although some studies (e.g., Bartell, Webel, Bowen & Dyson, 2011; Charalambous et al., 2011; Hiebert & Morris, 2009; Lampert, Beasily, Ghousseini, Kazemi. & Franke, 2010; Morris, 2006) have provided insights on what PSTs can learn in teacher preparation programs, as yet, we have few studies that have shown how PSTs develop the capacity to use children’s mathematical understanding to select and pose worthwhile
mathematical tasks as they progress in the methods course (Jacobs et al., 2010)\(^2\). Jacobs et al. (2010) compared PSTs with practicing and emerging teacher leaders engaged in a professional development. Hence, little is known about the extent to which PSTs develop their capacity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding as they progress in the methods course. As discussed later in Chapter 2, the practices of using children’s mathematical understanding have been found to support work that is central to mathematics teaching and learning and help to improve the learning and achievement of students.

**Purpose of the Study**

Building on Jacobs et al.’s (2010) work, this dissertation explores PSTs’ ability to attend to children’s strategies, interpret and respond based on children’s mathematical understanding at two different times during the methods course. Given the benefits of attending to children’s mathematical understanding in mathematics classrooms and the importance of mathematical tasks in mathematics teaching and learning (Hiebert & Wearne, 1993; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray et al., 1997), this study seeks to understand the extent which PSTs enrolled in an elementary mathematics methods course developed the practices (expertise) of attending to children’s strategies, interpreting children’s mathematical understanding and responding based on children’s mathematical understanding. I hypothesized that using scaffolds and instructional activities in the methods course that could *purposefully* develop the PSTs’ ability to attend to children’s strategies,

\(^2\) Note Jacobs et al. (2010) reported that among the 131 prospective teachers who participated in the study, none provided robust evidence that their decision on how to respond was based on children’s mathematical understanding, 14% had limited evidence 86% of the responses lacked any evidence.
interpret and respond based on children’s mathematical understanding could possibly
develop the expertise of professional noticing of children’s mathematical thinking. This study
examined how the PSTs noticed and attended to children’s strategies, interpreted children’s
mathematical understanding and responded based on children’s mathematical understanding
in the context of scaffolded activities at two different times in their methods course.

There are two significant differences between this study and Jacobs et al.’s (2010)
work. First, in this research, I examine how PSTs perform the three component skills at two
different times in the methods course and compare their performances. These two data
sources enabled me to describe how the PSTs’ capacity to attend to children’s strategies,
interpret and respond based on children’s mathematical understanding changed over the
course of the semester as they learned about the three component skills in the methods
course. This is important because the results provide helpful insights that can support teacher
educators who seek to understand when and how the expertise of professional noticing of
children’s mathematical thinking could be developed in a mathematics methods course.

Second, the participants were purposefully scaffolded with activities designed to
develop their understanding of the three component skills. Hence, these results not only
provide insights to what the PSTs can or cannot do, but the results also enabled me to look
into the activities that could have contributed to the change in their performances. As stated
in the course syllabus:

For each content area discussed in the course, the activities were designed to support PSTs’ learning of a) How children’s thinking typically develops within that content area including common understanding, misconceptions, strategies and errors and b) How to access and assess children’s thinking in the specific content area. (Course Instructor, 2011a, Fall, p. 1)

In other words, the course and its activities were designed to scaffold PSTs in these practices.
Finally, the study explores their intended response during the Inquiry into Student Thinking assignment; meaning that PSTs chose the next task they could pose to children as a response to their analysis on what the children understood or misunderstood but they did not get to pose the tasks to any children in a classroom setting. In the second assignment (tutoring assignment) PSTs stated a learning goal, select or generated tasks and posed the tasks to children in authentic classroom setting. My goal is to explore how the PSTs’ responses changed over time and the extent to which the responses differed across the two assignments. Therefore, I am hopeful that insights gained from this study will be helpful to teacher educators as they seek to develop learning experiences that will develop PSTs’ expertise and practices of using children’s mathematical understanding to choose and pose mathematical tasks.

**Significance of the Study**

**Role of children’s mathematical understanding in authentic practice**

For more than a decade, a lot of emphasis has been given to teachers’ use of children’s mathematical understanding in the classroom. Teachers’ knowledge of children’s mathematical understanding is part of what Shulman (1986) defined as pedagogical content knowledge. Specifically, Shulman defined pedagogical content knowledge “as knowledge of ways of representing and explaining a subject to make it comprehensible, knowledge of students’ thinking, and knowledge of the conceptions, preconceptions and misconceptions students bring to the learning that make it easy or difficult to learn” (p.25). In his description of pedagogical content knowledge, Shulman illuminated the need for teachers to understand children’s thinking, arguing “teachers need knowledge of the strategies most likely to be
fruitful in reorganizing the understanding of learners because those learners are unlikely to appear before them as clean blank slates” (p. 10).

Similarly, Mark (1990, As cited in Carpenter et al., 1996) argued that teachers need knowledge of students’ thinking which includes “teachers’ knowledge of: a) students’ typical understanding, b) students’ learning process, c) what is easy and hard for students, d) the most common errors students make and, e) particular students’ understanding”(p. 12). Carpenter, Fennema and Franke (1996) also argued that teachers’ knowledge of students’ thinking should provide a basis for understanding not only “what problems students can solve but also how they solve them” (p. 12). Carpenter and colleagues further argued that teachers knowledge of students thinking provides a deeper understanding of how students learn for conceptual understanding.

Further research on CGI work has gathered research-based evidence on how to teach elementary mathematics in a way that develops and relates to the benefit of attending to children’s mathematical thinking (Franke, Kazemi, & Battey, 2007; Franke & Kazemi, 2001; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Sfard & Kieran, 2001). For example, Franke and Kazemi (2001) argued that teachers using the Cognitively Guided Instruction (CGI) framework “engaged in sense making around children’s thinking, continually evaluated the children’s understanding, adapted and built on children’s mathematical thinking and figured out how to make use of the children’s thinking in the context of their ongoing practice” (p. 3). Therefore, these results have demonstrated that teachers who have more sophisticated understanding of children’s thinking improve their teaching practices and are more likely to improve students’ learning.
Similar arguments on how teachers respond to children’s mathematical thinking were presented in a research that was conducted by Fraivillig and colleagues (e.g., Fraivillig, Murphy & Fuson 1997; Fraivillig, 2001). In the study, Fraivillig examined how one first grade teacher, Ms. Smith, engaged the children in mathematical thinking and generated mathematical discussions in the classroom. Fravillig argued that what made Ms. Smith’s instruction effective was “her ability to elicit children’s solution methods, her capacity to support children’s conceptual understanding and her skill at extending children’s mathematical thinking” (p. 2). Therefore, it would be reasonable to provide PSTs with opportunities that would develop their capacity to attend to children’s strategies, interpret and use children’s mathematical understanding.

However, although the studies reviewed above suggest that focusing on children’s mathematical thinking is a powerful mechanism for bringing pedagogy, mathematics and students’ understanding together, other research studies have suggested that the expertise of noticing, understanding and using children’s mathematical thinking to inform instructional decisions does not naturally develop (Franke & Kazemi, 2001; Jacobs et al., 2010). Specifically, Franke and Kazemi (2001) argued, “teachers listening to students’ mathematical thinking generally struggled to make sense of the development of their students’ mathematical thinking and how that related to their instructional decisions” (p. 4). Further, Jacobs et al. (2010) developed a hypothetical trajectory, arguing that the expertise of attending to children’s strategies, interpreting and deciding how to respond develop with teaching experience and engaging teachers in a professional development. In summary, the studies discussed in this section illuminate the need to purposefully develop teachers’ (both in-service and PSTs’) ability to use children’s mathematical thinking.
Role of worthwhile mathematical tasks in the mathematics classroom

Over time, a lot of emphasis has been placed on the importance of worthwhile mathematical tasks in mathematics classroom (e.g., Henningsen & Stein, 1997; Hiebert & Wearne, 1993; Hiebert et al, 1997; Houssart, 2002; NCTM, 1991, 2000; Stein, Smith, Henningsen & Siver, 2000; Stein, Groover & Henningsen, 1996; Stein & Lane, 1996; Stein &Smith, 1998; Stylianides & Stylianides, 2008). Inherent in this literature is the fact that mathematical tasks influence what students learn in mathematics classrooms and teachers should select and/or generate meaningful mathematical task. The studies also reveal that there exists a relationship between the nature of students’ thinking required by a mathematical task and the level of students’ understanding of mathematics. For example, Stein, Grover and Henningsen (1996) suggested that their project teachers were successful in selecting and setting up the kind of mathematical tasks that had been viewed as leading to high-level student learning outcomes.

Despite the emphasis given to the use of mathematical tasks in classrooms, multiple research studies have indicated that mathematics teachers (both in-service and pre-service) have challenges when it comes to selecting and posing tasks that would create classroom environments where students will be engaged with high-level thinking (Crespo, 2003; Henningsen & Stein, 1997; Smith, 2004; Stein, Grover & Henningsen, 1996; Stylianides & Stylianides, 2008). Specifically, researchers have argued that teachers generally pose tasks that are of low cognitive demand, meaning that the tasks require memorization or procedures without connection to meaning, and when they choose tasks that would engage students with high level thinking, the level of cognitive demand is often reduced during the task enactment stage (Henningsen & Stein 1997; Stein, Grover & Henningsen, 1996).
In this study, I explored the extent to which activities structured to scaffold PSTs’ learning in teacher preparation programs develop PSTs’ capacity to use children’s mathematical understanding to select and pose worthwhile mathematical tasks. Identifying whether PSTs’ capacity develops in the context of scaffolded activities can support teacher educators in making reasonable and informed conjectures about the nature of experiences and activities to include in a teacher preparation program.

**Definition of Terms**

The following terms were defined for use in this study:

*Selecting and Posing Mathematical Tasks:* The NCTM (1991) suggest that teachers can “choose” tasks from a range of materials like problem booklets, computer software, practice sheets, puzzles, manipulative materials, calculators or textbooks. In addition, the NCTM (1991) suggests that teachers can create or develop tasks for students. Whether they choose or develop the tasks, teachers are responsible for the quality of the mathematical tasks in which the students engage in and should be using tasks that are likely to promote and develop students’ understanding of concepts and procedures in a way that fosters their ability to solve problems, reason and communicate mathematically. Further, Smith et al. (2008) describe the act of choosing and/or developing mathematical tasks as going beyond the act of choosing to a process of selecting and setting mathematical tasks. Smith et al. identified the process to include identifying the mathematical goal of the lesson and purposefully deciding how the task will build on students’ prior knowledge, life experiences, and culture.

In addition, the term problem posing has been used to refer to both the generation of new problems and the reformulation of given problems and as the process by which, on the
basis of concrete situations, meaningful mathematics problem are formulated (Silver, 1994; Silver, Mamona, Leng & Kenny, 1996). In this study, I will use the term “selecting and posing” to refer both to the act of selecting or generating a task, and the process of thinking through the selected or generated task and setting up the task for students.

**Worthwhile Mathematical Tasks:** Mathematical tasks have commonly been defined as the projects, questions, problems, constructions, applications, and exercises which teachers pose to students for them to engage with intellectual contexts for students’ mathematical development (Doyle, 1984; Hiebert & Wearne, 1993; NCTM, 1991). Among other characteristics, the NCTM (1991, p.25) describes worthwhile mathematical tasks as tasks that are based on “sound and significant mathematics as well as knowledge of students’ understanding, interests and experiences”. In addition, the NCTM (1991) describes worthwhile mathematical as tasks “that develop students’ mathematical understanding and skills” (p. 25).

Other research studies have classified tasks as “good” if the tasks have the potential to engage students in high-level thinking (Smith & Stein, 1998; Stein & Smith, 1998). While classifying tasks, Stein and Smith came up with four categories of cognitive demand, namely memorization, procedure without connection to concepts or meaning, procedure with connection to concepts or meaning, and doing mathematics. Further, Smith, Bill and Hughes (2008) described high-level tasks as “tasks that give students opportunities to use reasoning skills, and as “tasks that lack a specific solution path” (p. 2).

In this study, I used the term worthwhile mathematical tasks to refer to tasks that display the components of a mathematical task described by the NCTM (1991) and/or the levels of
cognitive demand described Smith and Stein work (Smith and Stein, 1998; Stein & Smith, 1998), and qualities described by Smith et al., (2008).

**Study Overview and Research Questions**

The study was conducted in the context of a mathematics methods course. The scaffolded activities are two assignments (Inquiry into Student Thinking and tutoring assignment) that were done at two different times as the PSTs progressed through the methods course. In the Inquiry into Student Thinking assignment, the PSTs analyzed the mathematical understanding of one child from a case study of four children. The focus of their analysis was on what the child knew and understood at the end of the case study that they did not understand at the beginning of the case study. Further, the PSTs were prompted to select or generate a task that they would pose to the four students based on the children’s mathematical understanding. Similarly, during the tutoring assignment, PSTs interviewed 2 to 3 children from a nearby elementary school and then planned for a series of four tutoring sessions. I analyzed PSTs’ responses from the two assignments to investigate the following research question and sub-questions:

1. To what extent do PSTs develop practices of using children’s mathematical understanding to select and pose worthwhile mathematical tasks in the context of scaffolded activities?
   a. What happens when PSTs are asked to analyze their own teaching and respond to children’s mathematical understanding as they plan for a series of instructional activities?
b. To what extent is the rationale of the PSTs’ instructional plan based on children’s mathematical understanding?

c. What type of tasks/problems do PSTs pose after assessing children’s mathematical understanding?

d. What are PSTs’ conceptions of a productive task and/or tasks that engage students with high or low level thinking?

The results indicated that PSTs’ ability to attend to children’s strategies, interpret and respond based on children’s mathematical understanding developed in the course of the methods course but to varying degrees. The noted patterns in their responses at the two different times of data collection ranged from: (1) PSTs who had sporadic and inconsistent responses in the three component skills of attending to children’s strategies, interpreting and responding based on children’s mathematical understanding; (2) PSTs who did not notice, interpret or respond in the first assignment but significantly made progress in their performance in the three component skills during the tutoring assignment; (3) PSTs who noticed and interpreted based on children’s mathematical understanding but their choice of tasks were not based on children’s mathematical understanding across the two assignments; and (4) PSTs who noticed, interpreted and responded based on children’s mathematical understanding during the two assignments. Therefore, the results provided insights into hypothetical developmental continuum of PSTs’ ability to attend to children’s strategies, interpret and respond based on children’s mathematical understanding.

In addition, the analysis revealed that PSTs examined in this study had varying conceptions of productive tasks. PSTs’ explanations of a productive task included tasks that advance children’s mathematical understanding, tasks that challenge children to move from
concrete to abstract strategies as well as tasks that are based in real life connections. Analysis also revealed varying PSTs’ conceptions of what it means for a task to engage students in high-level thinking. This study has implications for the design of mathematics methods courses as the study provides evidence that the capacity to attend to children’s strategies, interpret children’s mathematical understanding and respond based on that understanding can be learned in a teacher education context when PSTs are purposefully exposed to learning experiences that are designed to develop this capacity.

**Dissertation Organization**

This dissertation is organized into five chapters, with four chapters following this introductory chapter. Chapter 2 presents a review of literature on conceptions of using children’s mathematical understanding in mathematics classroom and teacher learning. Specifically, I focus on literature that relates to the benefits of attending to children’s mathematical thinking and how teachers, both in-service and PSTs’ learn to notice children’s mathematical understanding. I also explore in-depth one theoretical perspective that conceptualizes the capacity to use children’s mathematical thinking as a set of three interrelated skills – namely, attending to children’s strategies, interpreting children’s understanding and responding based on children’s mathematical understanding. This theoretical framework frames this study. Within this framework, I provide a detailed review on each component of the framework and how mathematics educators develop the component skills. I also describe efforts that teacher educators and professional development providers have made to develop teachers ability to attend to children’s strategies, interpret and respond based on children’s mathematical understanding. The third section of this
Chapter 3 focuses on the literature that relates to the importance of mathematical tasks in the classroom and the challenges that teachers (both in-service and PSTs) face when selecting and/or generating mathematical tasks.

Chapter 3 details the methods used to address the research questions. I describe the instructional context, research participants, data sources and coding and analysis procedures. Chapters 4 and 5 present the results. Specifically, Chapter 4 presents results and findings on how PSTs attended to children’s strategies and interpreted and responded to children’s mathematical understanding. Chapter 5 includes in-depth discussions of these findings and provides insights into a conjectured hypothetical developmental continuum for the development of the three component skills.
CHAPTER 2. LITERATURE REVIEW

Overview

The purpose of this study is to examine the extent to which PSTs develop the capacity to use children’s mathematical understanding to select and pose mathematical tasks in the context of scaffolded activities as they progress with their methods course. As explained in Chapter 1, the capacity to use children’s mathematical understanding incorporates the capacity to attend to children’s strategies, interpret children’s mathematical understanding and respond based on children’s mathematical understanding. This chapter consists of four sections.

In the first section, I start by exploring the historical conceptions of using children’s mathematical thinking in the classroom and developed frameworks that provide insights to the practices of using children’s mathematical thinking in the classroom. Next, the section focuses on developed frameworks, for example, Cognitive Guided Instruction (CGI) and Advancing Children’s Thinking (ACT) framework that relates the benefits of attending to children’s mathematical thinking to teaching and students’ learning. (Carpenter et al., 1989; Carpenter et al., 1996; Fraivillig et al., 1996, Fraivillig, 2001). Finally, I conclude the section by considering the relationship between these frameworks, with a specific focus on the affordances and insights that the two frameworks bring to teaching and learning of mathematics.

In the second section, the chapter reviews literature on the topic of teacher learning (both practicing teachers and PSTs) about using children’s mathematical thinking as a pedagogical tool in mathematics classroom. In particular, I draw on previous studies to
explore the context in which in-service teachers have developed the capacity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding. In doing so, I seek to theoretically justify that the capacity to use children’s mathematical understanding has to be purposefully developed and can be learned. This discussion leads to a justification of the inquiry that this study seeks to explore namely; the extent to which PSTs develop the capacity to use children’s mathematical understanding in the context of scaffolded activities.

Finally, I explore in depth one theoretical perspective that conceptualizes the capacity to use children’s mathematical thinking as a set of three interrelated skills – namely, attending to children’s strategies, interpreting children’s mathematical understanding and responding based on children’s mathematical understanding. I used the framework to theorize the development of PSTs’ capacity to notice children’s mathematical thinking. Specifically, the framework provides a theoretical methodological tool to analyze and make sense of PSTs’ performance in the three component skills and the extent to which the three skills developed over time. In addition, I used the framework to explore the extent to which PSTs’ capacity in the first two component skills impacts their practices of selecting and posing mathematical tasks (responding based on children’s mathematical understanding). Within this framework, I provide a detailed review of each component of the framework and the activities and experiences that mathematics teacher educators have used to help PSTs develop the component skills. Finally, I provide a detailed review of the notion of scaffolding to situate the use of scaffolded activities in the current study.

In the third section of this chapter, I explain the reasons that informed the decision to examine the extent to which PSTs develop the capacity to use children’s mathematical
understanding to select and pose mathematical tasks. Drawing on relevant literature, I explicate the importance of developing PSTs’ capacity to use children’s mathematical understanding to select/generate worthwhile mathematical task. In doing so, I seek to theoretically justify the significance of this study in the current literature and in the field of mathematics education. Figure (2.1) summarizes the context of this study.

**Historical Conceptions of using Children’s Mathematical Thinking in Mathematics Classrooms**

Since Shulman (1986) identified knowledge of students’ thinking as part of pedagogical content knowledge, researchers have focused on conceptualizing and unpacking this knowledge. (e.g., Carpenter, Fennema, Peterson, Chiang & Loef, 1989; Carpenter, Fennema & Franke, 1996; Fraivillig, 2001; Hill, Ball & Schilling, 2008; Mark, 1990). Some researchers (e.g., Hill et al., 2008, p.2) have argued that although teachers’ knowledge of students’ mathematical thinking and learning is important; it is largely underspecified. Further, Hill et al. (2008) conceptualized the teachers’ knowledge of students’ thinking as knowledge of content and students (KCS), defining this construct “as the knowledge of how students think about, know or learn a particular content” (p.4).

Overtime, some research projects have also developed frameworks and models for understanding how teachers use children’s mathematical thinking in mathematics classrooms. For example, the (CGI) researchers (e.g., Carpenter et al., 1989; Carpenter et al., 1996; Franke & Kazemi, 2001) provided teachers with a model (framework) of student’s thinking that teachers could use to assess and understand their own students’ mathematical thinking. In this framework, Carpenter and colleagues characterized critical differences in word
Figure 2.1. Context of the study
problems that are reflected in how students think about and solve those problems. The framework also captures the strategies that children use as they solve word problems ranging from direct modeling and counting up strategies to more sophisticated abstract strategies. Carpenter and colleagues argued that the framework is “useful to teachers as they interpret, transform and reframe their informal or spontaneous knowledge about students’ mathematical thinking” (p. 2). In summary, the CGI model supports teachers to develop deeper knowledge of students’ thinking and relies on teachers to use their general pedagogical knowledge to know how to use it in the classroom.

However, although productive insights about teachers’ knowledge of students’ thinking have been gained, research efforts that have focused on how teacher educators are developing the knowledge, skills and practices that teachers need to access and assess children’s mathematical thinking are limited. In fact, there is a general consensus with teacher educators and researchers (e.g., Allen, 2003; Cochran-Smith & Zeichner, 2005; Grossman & McDonald, 2008; Morris, Hiebert & Spitzer, 2009; NCATE, 2010) that little is known on how teacher preparation programs equip teachers to become effective mathematics teachers. Specifically, Grossman and Mcdonald (2008) argues that research on teaching have not informed the practices and research on teacher education and, suggests that for the research on teacher education to move forward, “researchers in the fields of both teaching and teacher education will need to begin to act as if they were indeed a unified field of inquiry” (p. 16). Similarly, Ball et al. (2009) argues that developing a professional curriculum of preparing PSTs will call for teacher educators to identify an approach to prepare teachers that is focused on practice. Meaning, teacher educators need to identify the practices that are helping in-service teachers enact meaningful mathematics instruction and
make those practices the curriculum in teacher education. This new perspective in teaching and teacher education will entail taking what is known in the field of teaching and using it to inform teacher educators as they plan for activities and experiences that would prepare PSTs. In fact, Ball et al. (2009) argues that teacher educators will have to “make choices” and “focus on practices most likely to equip beginners with capabilities for the fundamental elements of professional work and that are unlikely to be learned on one’s own through experience” (p.4).

Other research studies (e.g., Fraivillig, Murphy & Fuson, 1996; Fraivillig, 2001) developed the Advancing Children’s Thinking (ACT) framework that is composed of three components that characterized one first grade teacher’s (Ms. Smith) classroom practices. The three component skills consisted of:

a) eliciting children’s solution methods, b) supporting children’s conceptual understanding, and c) extending children’s mathematical thinking (p. 2). Specifically, Fraivillig argues that the ACT framework “could help teachers design and implement instruction that makes mathematics personally meaningful for children and establishes a structure for the often-complex interaction that occurs when teachers and students grapple with real mathematical problems. (p. 2)

Further, Fravillig (2001,) explained:

The eliciting component reminded teachers how they might get children’s thinking out in the open for discussion and build instruction on that thinking, the supporting component describes instructional strategies for assisting children at their current level of understanding while the extending component prompts teachers to challenge children’s thinking regardless of the students initial efforts. (p. 6)

The framework suggested by Fraivillig et al. (2001) have provided additional insights to the CGI framework by providing a specific model on how teachers can advance children’s mathematical thinking. While the CGI researchers paid careful attention to the children, the
strategies they use, nature of word problems and how children’s understanding typically develops, Fravillig and the team focused on the teacher. These two frameworks can be necessary resources to develop and assess PSTs’ knowledge and skills as they progress through the teacher preparation program.

As described latter in Chapter 3, the activities and experiences provided to PSTs in this current study focused on developing their capacity to use children’s mathematical thinking. Using the CGI framework and the ACT framework (Carpenter et al., 1989; Fravillig et al., 2001) the PSTs engaged with activities and experiences (watching video clips, analyzing students work and discussions about how children’s thinking develops etc.) as they progressed with their methods course. Further, PSTs’ capacity to use children’s mathematical thinking was assessed at two different times.

The studies discussed above reveals that substantial work have been done to identify and support teachers understanding and practices on how to use children’s mathematical thinking. More recently, Hill et al. (2008) also constructed a domain map for mathematical knowledge for teaching (MKT), indicating that teachers need six types of mathematical knowledge for teaching - common content knowledge (CCK), knowledge at the mathematical horizon, specialized content knowledge (SCK), knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of the curriculum. Specifically, Hill et al. identified knowledge of content and students (KCS) as a subset of the pedagogical content knowledge (PCK), and the pedagogical content knowledge as a subset of mathematical knowledge for teaching (MKT). Finally, Hill et al. (2008) described their efforts to develop measures that can be used to measure the knowledge of content and students.
Researchers and mathematics educators have also gathered research-based evidence on how to teach elementary mathematics in a way that develops and relates to the benefit of attending to children’s mathematical thinking (Franke, Kazemi, & Battey, 2007; Franke & Kazemi, 2001; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Sfard & Kieran, 2001; Silver & Stein, 1996). For example, Franke and Kazemi (2001) argued “teachers using the CGI framework continually evaluated the children’s understanding, adapted and built on children’s mathematical thinking and figured out how to make use of the children’s thinking in the context of their ongoing practice” (p. 3). Therefore, the practices of using children’s mathematical thinking have been found helpful for the practicing teachers and improving students’ learning. Based on the studies discussed above, one can make an informed conjecture that the skill and practices of using children’s mathematical thinking can be one of the high-leverage practices to be addressed in a curriculum of a teacher preparation program since they have been found to improve student learning.

Similarly, Jacobs et al. (2007) focused on how teachers noticed students’ thinking in algebraic ideas and which ideas the students’ found accessible. Jacobs and the team concluded that, “attending to students’ thinking involved more than knowing traditional mathematics content, more than appreciating the existence of multiple strategies; and more than being able to repeat what children said when problem solving” (p. 25). Rather, “teachers needed to be able to differentiate the strategies children used in relation to specific mathematical ideas.” (p. 25). Overall, Jacobs et al. (2007) reported that participating teachers “…generated more strategies than non-participating teachers” and “students in participating classes showed significantly better understanding of the equal sign and used significantly
more strategies in reflecting relational thinking than did students in non-participating classes” (p. 2).

Inherent in research by Jacobs et al. (2007) is the fact that developing teachers’ knowledge on how to use children’s mathematical thinking requires purposeful choices. As Jacob and the team indicates; “the opportunities for teacher and student learning were strongly linked to their (researchers) decision about how to focus and structure the content discussed during the professional development” (p. 27). This speaks a lot to the way teacher educators structure the content of the courses taught in the methods course if we have to improve the quality of teachers graduating from the teacher preparation program (Allen, 2003; NMAP, 2008). The activities need to be purposefully selected and designed to enhance the PSTs’ learning.

In particular, the CGI research and the work done by Jacobs and the team (Franke, Kazemi, & Battey, 2007; Franke & Kazemi, 2001; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Sfard & Kieran, 2001; Silver & Stein, 1996) reveals that the expertise of noticing, understanding and using children’s mathematical thinking to inform teachers’ instructional decisions does not naturally develop. For example, Franke and Kazemi (2001) argued that teachers listening to students’ mathematical thinking generally struggled to make sense of the development of their students’ mathematical thinking and how that related to their instructional decisions” (p. 4). Hence, given the importance of using children’s mathematical thinking in the classroom, researchers have engaged in a variety of work closely related to developing in-service and PSTs’ ability to use children’s mathematical thinking. In the next section, I described the approaches that mathematics teacher educators
have used to develop teachers’ capacity to use children’s mathematical thinking in mathematics classrooms.

**Developing In-service Teachers’ and PSTs’ Understanding and Practices of Using Children’s Mathematical Thinking**

Recent research efforts to address the challenges inherent in developing in-service teachers’ and PSTs’ understanding of children’s mathematical thinking have focused their attention and analysis on the use of multimedia case studies and the use of video and students’ work as resources for teacher learning (Cooper, 2010; Masingila & Doerr, 2002). For example, Masingila and Doerr (2002, p. 1) investigated how multimedia case studies of practice can support PSTs in making meaning of complex classroom experience and in developing strategies and rationales for using student thinking to guide instruction. Masingila and Doerr reported that by using the multimedia case analysis, “the PSTs were able to use their perspective on a common practice to highlight some of the dilemmas and tensions found in teaching” (p. 1). Specifically, the study revealed “the PSTs focused on the difficulties encountered while teachers try to use students thinking and to follow their own mathematical goals in the lesson” (p. 1).

More recently, several studies have focused on developing teachers’ ability to notice characteristics of classroom environment where teachers attend to children’s mathematics thinking using video cases (Jacobs, Lamb, & Philipp, 2010; Sherin, Linsenmeier & Van Es, 2009; Sherin & Van Es, 2005). For example, Sherin and Van Es (2005) examined how videos can be used to help in-service teachers and PSTs learn to notice what happens in mathematics classrooms using data from two related studies. In the first study, four middle school teachers participated in a yearlong series of video club meetings, where they watched
and discussed videos from each other’s classrooms. Specifically, Sherin and Van Es reported that the four middle-school teachers who participated in the study changed their focus on what they noticed over time. At the beginning of the year, “the four teachers focused on the teacher in the video and what the teacher was doing, but “over the course of the video club meetings and discussions, the teachers’ attention shifted from the teachers to the student’s mathematical thinking” (p. 8).

In the second study, six PSTs working towards certification in secondary mathematics or science participated in three hour-long sessions in which they used software Video Analysis Support Tool (VAST) (Sherin & Van Es. 2005). In these sessions, they examined videos of their own and others’ teaching. Specifically, the PSTs were asked to analyze three aspects of their videos: (1) student thinking, (2) teacher’s role, and (3) classroom discourse. Sherin and Van Es (2005) found that the six PSTs changed from the kind of events they noticed. Specifically, at the beginning, the PSTs identified all the events as noteworthy but, over time, they paid more attention to what they noticed and became more discriminating regarding what they noticed. As Sherin and Van Es (2005, p. 10) reported, rather than providing literal descriptions of events as they occurred in the classroom, the PSTs organized their essays around significant aspects of teaching and learning.

Similarly, Sherin et al. (2009) explored the use of videos from a teacher’s own classroom as a resource for investigating students’ mathematical thinking. In the study, Sherin et al. characterized a range of video clips of students’ mathematical thinking in terms of “the extent to which the video clip provides windows into student thinking, the depth of student’s mathematical thinking and the clarity of students’ thinking shown in the video” (p. 4). The results of the study indicate that under certain circumstances, both low and high
depth clips led to productive discussions and conversations of students’ thinking on the part of the teachers.

Likewise, Van Es and Sherin (2008) proposed that the skill of noticing consists of two main aspects. These aspects include: (1) identifying what is important in a teaching situation; and (2) drawing on one’s knowledge of teaching and learning to reason about the situation. Van Es and Sherin argued that the first aspect involves “…the ability to focus one’s attention to what is significant in a complex situation” while the second aspect involves “using knowledge of one’s context to reason about events that occur” (p. 1). Hence, Van Es and colleagues’ framework of learning to notice includes “…(a) identifying noteworthy aspects of a classroom situation, (b) using knowledge about the context to reason about classroom interaction, and (c) making connections between the specific classroom events and broader principles of teaching and learning” (p. 1).

In summary, Sherin and colleagues’ work (e.g., Sherin, Linsenmeier & Van Es; 2009; Sherin & Van Es, 2005, Van Es & Sherin, 2006; 2008) bring an important aspect of teacher learning. First, they argue that teachers can learn how to notice children’s mathematical thinking in the context of video clubs. These studies provide important insights to mathematics teacher educators as they seek to understand how they can develop PSTs’ capacity on how to notice children’s mathematical thinking. Specifically, the studies validate the use of videos as “representations of the practice” (Grossman, Compton, Igra, Ronfeldt, Shahan, & Williamson, 2009, p. 11) because videos can provide PSTs with opportunities to develop ways of noticing important aspects of teaching. Second, inherent in their work is a belief that the ability to notice important aspects of classroom environment can be learned in contexts that are purposefully chosen to develop the ability.
In this study, I partly build on Sherin’s and colleagues work by examining the extent to which PSTs can learn how to notice when exposed to a variety of activities which are purposefully designed to develop their understanding on how to notice children’s mathematical thinking. In addition, this study extends Sherin and colleagues’ work by not only examining what they notice but also the extent to which they use what they notice to respond to children’s mathematical understanding.

Jacobs and Ambrose (2008) discussed how 65 teachers responded to children’s mathematical thinking as they interviewed students using word problems. After analyzing videotaped problem-solving interviews, Jacobs and Ambrose identified eight categories of intentional teacher moves that were productive in advancing mathematical conversations. Four of the moves were “supporting moves” that a teacher used before the children arrived at the correct answer while the other four were “extending moves” that a teacher can use after a child obtains the correct answer. Informed by Jacobs and Ambrose (2008) this current study focuses on how PSTs use what they notice to plan for instructional session. Specifically this study focuses on how PSTs generate or select follow up problems based on children’s mathematical understanding. I briefly elaborate on these teacher moves as follows.

**Teacher moves**

The supporting move includes: (1) Ensuring that a child understands the problem; (2) change the mathematics to match the child’s level of understanding; (3) Explore what the child has already done; and (4) Remind the child to use other strategies. The extending moves include: (1) Promoting reflection on a strategy that is just completed; (2) Encourage the child to explore multiple strategies and their connections; (3) Connect the child’s thinking
to symbolic notation; and (4) Generate follow up problems or select the next problem. These moves had similarities with Fravillig’s (2001) framework of advancing children’s mathematical thinking since the two approaches presented teachers’ actions in a continuum of eliciting children’s solutions, supporting children’s understanding to extending children’s mathematical thinking. This continuum seems an important aspect of teaching practice and might be critical skills that PSTs need to learn as they go through the teacher preparation program.

Building on literature that has focused on teacher noticing (e.g., Sherin & Van Es, 2005; Sherin, Linsenmeier & Van Es; 2009; Van Es & Sherin, 2006; 2008), Jacobs et al. (2010) examined how teachers at different stages in their teaching continuum reported what they noticed in children’s mathematical thinking using written measures. Participants included three groups of practicing K-3 teachers and one group of prospective teachers who were beginning their studies to become elementary teachers. In this study, the practicing teachers engaged by solving mathematical problems, reading research, analyzing videos and written students work in the course of professional development. In addition, between the professional developments, the teachers were asked to pose problems to their students and to bring the written students work to the next sessions. A key product of Jacobs et al. (2010) is the construct of professional noticing of children’s mathematical thinking which is conceptualized as “a set of inter-related skills that include attending to children’s strategies, interpreting children’s understanding and deciding how to respond based on children’s mathematical understanding” (p. 4). These components are similar with the framework of learning to notice that is identified by Van Es and Sherin (2008) Specifically, the two frameworks conceptualize noticing as a skill that is in continuum of identifying important
aspects that are noteworthy, interpreting what one notices and responding based on what you notice. I will briefly elaborate on each component as discussed by Van Es and Sherin’s work and Jacob and Colleagues work.

**Attending to children’s strategies**

Different terms have been used to describe how teachers notice and identify noteworthy aspects of an instructional situation. For example, Sherin and Van Es (2005) analyzed PSTs’ narratives to explore the extent to which PSTs highlighted particular events that occurred, paid attention to specific or general evidence in the video and took a descriptive, evaluative and interpretive stance towards discussing what had occurred in the Video Analysis Support Tool (VAST) study. Using teachers’ work, Van Es and Sherin (2008) proposed that the skill of noticing consists of three main aspects. These aspects include: 1) identifying what is important in a teaching situation and, 2) drawing on one’s knowledge of teaching and learning to reason about the situation. For the component skill of attending to children’s strategies, Jacobs and the team focused in the extent which teachers with different teaching experiences attended to the mathematical details in children’s strategies. Previous research (e.g., Carpenter et al., 1999; Carpenter, Franke, & Levi, 2003; Lester, 2007) had also shown that although children’s strategies are complex, the details given often provide a more nuanced understanding of what the children understand and/or misunderstand. Therefore, it is reasonable to categorize the component skill of attending to children’s strategies with what teachers notice when assessing children’s mathematical understanding.
Interpreting children’s mathematical understanding

Both Sherin and Van Es’ work and Jacobs and colleagues’ work (Jacobs, Lamb & Philip, 2010; Jacobs & Ambrose, 2008; Sherin & Van Es, 2005, 2009) have largely described the component skill of interpreting children’s mathematical understanding. Specifically, Sherin & Van Es (2005) identified three stances (descriptive, evaluative and interpretive) that teachers used as they reported what they noticed. The descriptive stance was taken to be situations where the teachers, specifically, the PSTs involved in the VAST study described each event in the video while the evaluative stance was used to describe situations when the teachers focused on what worked and what they might want to do differently. The interpretive stance was taken to be instances when the teachers focused more on interpreting what occurred than simply describing and evaluating what had happened in teaching and learning. Jacobs et al. (2010) focused on teacher reasoning and described the component of interpreting as “the extent to which the teachers reasoning is consistent with both the details of specific child’s strategies and the research on children’s mathematical understanding” (p.4). In this study, the description of the component skill of interpreting children’s mathematical understanding is consistent with Jacobs et al.’s definition. I take interpreting children’s mathematical understanding to refer to the extent to which PSTs’ reasoning and explanations are consistent to specific children’s strategies.
Responding based on children’s mathematical understanding

As discussed previously, prior research by Jacobs and Ambrose (2008) and Jacobs et al. (2010) has identified different ways (moves) that teachers can build on children’s mathematical understanding. Teachers’ responses can either be in the moment where they support the children to understand the mathematical idea or it can be extending the children’s understanding. The nature of response is determined by the children mathematical understanding.

In summary, the three component skills of professional noticing of children’s mathematical understanding can be taken to be integral to teaching practice and they can be improved through targeted instruction. Notable in finding by Jacobs et al. (2010) was the fact that teaching experience seemed to provide support for individuals to begin developing expertise in attending to children’s strategies, and interpreting children’s mathematical understanding, but there was no similar evidence for expertise in deciding how to respond on the basis of children’s mathematical understanding. Instead, the expertise of deciding how to respond seemed to grow with two years of professional development, coupled with leadership activities.

As other research studies (e.g., Ball et al., 2009; Grossman et al., 2009; Jacobs et al., 2007) have suggested, component skills or practices that teachers only learn in the context of professional development need to be purposefully developed using carefully chosen activities or instructional interventions. Informed by Jacobs et al. (2010) studies, it is reasonable to purposefully choose to develop the component skills of attending to children’s strategies, interpreting and responding based on children’s mathematical thinking because these skills are seen as critical to teaching practice. In fact, Ball et al. (2009) argued that teacher
educators need to identify practices that PSTs cannot learn on their own and make them part of the curriculum in teacher preparation. In Jacobs et al. (2010) study, the prospective teachers were included, as an anchor point for the hypothesized developmental trajectory since it was hypothesized that expertise would develop with experience.

Inherent in the studies reviewed above is a strong assumption that teachers’ capacity to use children’s mathematical understanding is fundamental and critical to effective teaching practice. In addition, the studies have concluded that skills or practices of using children’s mathematical understanding do not naturally develop even with teaching experience. The studies have also identified a developmental trajectory of teacher noticing moving from mostly evaluating the actions of students and teachers to marking and attending to details of students’ thinking, interpreting trends and details and finally using the interpretations to impact practice (deciding how to respond).

Building on this work, I focused my attention on PSTs’ capacity to notice and use children’s mathematical understanding as they progress through their elementary mathematics methods course. I use two activities; one activity turned in for grading after six weeks of instruction and a second activity turned in after ten weeks of instruction. I used the data from the two activities to explore the extent to which PSTs develop practices of using children’s mathematical thinking to select and pose worthwhile mathematical tasks in the context of scaffolded activities. Specifically, I focused my analysis on how PSTs use what they notice in children’s mathematical thinking to select and pose worthwhile mathematical tasks. Figure 2.2 provides a summary of the practices identified to represent the hypothesized developmental trajectory.
The theoretical framework guiding this study is grounded in the notion of professional noticing of children’s mathematical thinking (Jacobs, et al., 2010). Mason (2002) states that what professionals’ notice impacts what they learn with respect to their discipline. Further, research on teachers’ professional noticing has focused on how teachers attend to students’ thinking (noticing), how teachers interpret what they notice with respect to students mathematical understanding, and ultimately how those interpretations impact on teachers instructional practices (Jacobs, et al., 2010; Jacobs & Ambrose, 2008; Sherin & Van Es, 2009; Van Es & Sherin, 2008). Jacobs et al. (2010) conceptualized a hypothetical trajectory of teacher learning how to notice and interpret children’s mathematical understanding.

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3 The component skills and the hypothesized developmental trajectory are adapted from Jacobs et al. (2010) and Van Es & Sherin (2008). The components on the right side of the figure come from Van Es & Sherin (2008), while the ones on the right side of the figure come from Jacobs et al. (2010)
understanding as a set of three interrelated skills namely, attending to children’s strategies, interpreting children’s understanding and, responding based on children’s mathematical understandings.

Besides advancing the notion of professional noticing and mapping the terrain of the knowledge gained from the studies, the analysis of the data collected by the studies described above validates the theoretical underpinning of the construct of professional noticing of children’s mathematical thinking. For example, the studies reveal that the skills of professional noticing of children’s mathematical thinking can be learned when teachers are engaged in a sustained professional development (Jacobs, et al., 2010). What still remains a question in mathematics education is whether the activities used in preparing PSTs can develop beginning competencies in these skills. So, the goal of this study is to explore whether PSTs involved in scaffolded activities can develop this expertise and, if so, to what extent? Since Sherin and the teams work (e.g., Sherin, et al., 2009; Sherin & Van Es, 2005, Van Es and Sherin, 2008) has focused on how PSTs notice children’s mathematical understanding, a major focus in the current study is how PSTs respond based on children’s mathematical understanding as they plan for an instructional session.

As Ball et al. (2009) indicated, “developing an approach to preparing teachers that is focused on practice entails analyzing and naming aspects of the work of teaching and identifying the key demands of that work, including the content knowledge needed” (p. 2). Further, Ball et al. (2009) argued that developing an approach to prepare teachers involves choosing those aspects of the work – high-leverage practices – which when done well give teachers a lot of capacity in their work. In addition, Ball and colleagues argued that these practices include activities of teaching “that are essential to the work and that are used
frequently, ones that have significant power for teachers’ effectiveness with pupils” (p. 4). Grossman et al. (2009) collaborated this argument indicating “part of the work of professional education lies in identifying components that are integral to practice and that can be improved through targeted instruction” (p. 15).

Using the construct of professional noticing of children’s mathematical thinking (Jacobs et al., 2010) and the use of children’s mathematical thinking in the classroom, I make an informed conjecture that the practice of attending to children’s strategies, interpreting and responding based on children’s mathematical understanding is a “high-leverage practice” that PSTs should learn as they go through the teacher education program. In fact, the three component skills identified in the construct of professional noticing of children’s mathematical understanding meet the criteria identified by Ball et al. (2009) “in that teachers’ expertise in the component skills: 1) support work that is central to mathematics, 2) helps to improve learning and students achievements, 3) is done frequently and with teaching mathematics, and 4) can be articulated and taught” (p. 5). Indeed, prior research (e.g., Jacobs et al., 2007; Fraivillig, 2001; Franke & Kazemi, 2001) has indicated that teachers with capacity to use children’s mathematical thinking create classroom environment where students learn.

In this study, I analyzed PSTs’ capacity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding in the context of scaffolded activities as the PSTs progress in a mathematics methods course. The next subsection briefly defines and elaborates how scaffolds have been used in different contexts in teaching and learning. Further, I contextualize the use of scaffolds in this study based on the existing literature.
Scaffolds

Over time, the concept of scaffolding has been conceptualized differently. For example, Wood, Bruner and Ross (1976) considered scaffolds as “a tutor or an adult supporting a child or novice solve a problem, carry out a task or achieve a goal which would be beyond his unassisted efforts” (p. 2). During the scaffolding process, Woods et al. argue, an adult takes control of the elements of the task that are initially beyond the learners independent capacity and allows them to “concentrate upon and work on the elements of the task that are within his range of competence” (p. 2). Scaffolds have also been described as structures; act of teaching, tools and assistance from more knowledgeable others that allow learners to engage in practices beyond their independent capacity (Anghileri, 2006; Holton & Clark, 2006). Specifically, Holton and Clark (2006) defined scaffolds as “an act of teaching that supports the immediate construction of knowledge by the learner and provides the basis for the future immediate independent learning of the individual” (p. 6). These studies present scaffolds as an interaction between a more knowledgeable adult and an individual or small groups of students learning a specific content or doing a specific task or an act of teaching a whole classroom. Scaffolds in this context are very specific directive prompts and interactions between the teacher and the learner[s].

Other studies have considered the idea of using distributed scaffolding (also referred to as multiple or synergistic scaffolds) to support a single learning need (e.g., Puntambekar & Kolodner, 2005; Tabak, 2004; Van Zoest & Stockero, 2008). Inherent in these studies is a conception that learners need to be provided with different tools, agents, and activities; each of which has its own unique affordances that support students’ conceptual understanding of the content or the task. Specifically, Tabak (2004) identified three models of distributed
scaffolding namely; 1) Differentiated scaffolds 2) Redundant scaffolds and, 3) Synergistic scaffolds. Below, I briefly elaborate each of these models of scaffolding.

**Differentiated**

This refers to a scaffolding model where one identifies the range of support needs in a group of learners and identifies the type of agent or material that best supports each need. Therefore, each need is supported by specific individual scaffolds.

**Redundant**

This refers to a scaffolding design that recognizes that not all students will benefit from a particular scaffold since students learn differently and some might require more support than others. As Tabak (2004) argued, “the goal in redundant scaffolds is to provide multiple scaffolds for the same need” (p.14). Therefore, the learners can be provided with different supports at one point or at different points in time in their learning continuum that would increase the chances for students to benefit from the scaffolds. In addition, redundant scaffolds provide learners with access and multiple opportunities to perform the same task or achieve the same goal under guidance.

**Synergistic**

This refers to scaffolding model where the learners are supported with different agents, tools, activities or prompts that augment each other to guide a single performance or a goal. The rationale behind using synergistic multiple scaffolds is that some knowledge, skills and practices are incongruous and would require more than one scaffold to achieve the desired outcome. Sometimes the performance of the task will require different skills that
would support the overall goal. As discussed in Chapter 3, this model provided insights and a framework as I conceptualized the nature and function of the activities used in the methods course.

Scaffolding has been used in teacher preparation programs to support PSTs’ learning to do the work of teaching and has been found to have impact on PSTs’ learning (Kaste, 2004; Sleep & Boerst, 2011; Van Zoest & Stockero, 2008). For example, Kaste (2004) focused on the use of 1st grade cases to scaffold a diverse constructivist perspective in a literacy methods course for PSTs. Specifically, PSTs listened to the audio taped sessions to grasp how Kaste (the course instructor) used particular approaches to teach the children some strategies of decoding words, and how she used instructional materials to facilitate students’ learning. In addition, PSTs were given specific student’s written artifacts to analyze and recommend future instruction. Further, PSTs listened to audio taped sessions of students reading instructional level books and the instructor guiding the students to use cues to figure out the unknown words on his own. These purposeful constructed cases scaffolded PSTs’ understanding on how to use the diverse constructivist approach. Specifically, Kaste used the cases to “bring effective practices from the field into the university courses through case studies. The analysis of videotapes course sessions and PSTs’ assignments revealed that many PSTs showed beginning attempts at articulating a diverse constructivist orientation over time when assignments focused on actual students. (p. 1).

Kaste, a university professor, developed cases with three first grade readers from a classroom where she provided weekly instructional support. She collected the three students work samples, wrote field notes and audio taped her working sessions with the three first graders to create course materials for the methods course. (See detailed description for the development of the cases in Kaste (2004).
Similarly, Van Zoest and Stockero (2008) investigated the role of synergistic scaffolds in supporting PSTs’ knowledge of self – as a teacher. The scaffolds included: (1) General instructors feedback on the initial drafts provided in an electronic course management system; (2) More specific feedback written on students’ initial papers; (3) a criterion based scoring sheet that students were required to complete and submit with their final paper; (4) verbal instruction given by the course instructor; (5) Mathematics education readings, and (6) a group card sort activity focused on evaluating responses from previous students in the course. As Tabak (2004) indicated, all the scaffolds were designed to serve a particular function (and some multiple functions) but they all augmented each other to develop their capacity to write an MTA.

To use the synergistic scaffolds, the Mathematics Teaching Autobiography (MTA) was assigned on the first day of the course and the first draft copy was due at the second meeting. The instructor introduced the assignment, provided instructions verbally and explained the importance of the assignment in terms of understanding one’s current beliefs in order to think about teaching and learning. After submitting the electronic copy, they were given written feedback on the initial drafts and whole group verbal feedback. They were also assigned readings that would support their understanding of their own beliefs. After being exposed to different scaffolds, PSTs revised the MTA assignment and turned in a revised copy. A paired t-test of the initial and revised MTA scores showed that the difference between the two scores was significant, indicating that the students were better able to meet the MTA criteria with the scaffolding. Van Zoest and Stockero (2008) concluded that carefully designed synergistic scaffolds supported PSTs in their exploration of self as they developed their MTA.
More recently, Sleep and Boerst (2011) used a set of tasks that focused on place value to provide interns with scaffolds as they assessed and elicited children’s mathematical thinking. The set of scaffolds included “hard scaffolds”, like questions that the interns used in their work with children and “soft scaffolds”, like instructor feedback that may have supported the interns work in the latter part of the assignment. As Sleep and Boerst (2011) stated:

The scaffolds were designed to provide guidance about the mathematics content to consider with students (conceptual) how to engage with students (procedural) alternative ways of acting or thinking during the interview (strategic) and what to consider as the students work (metacognitive). (p. 3)

The tasks provided interns with procedural scaffolding that supported their understanding as they elicited children’s mathematical thinking and strategic scaffolding in the form of prompts that were used to support interns’ probing of student thinking related to a specific task. The instructors feedback (soft scaffolds) provided helped the interns to be more explicit with their assertions over time. These studies reveal that scaffolds have been used differently in teacher preparation to support PSTs’ learning.

In this study, I conceptualize scaffolding as a combination of “an act of teaching” (Holton & Clark, 2006) and a set of activities designed to provide PSTs with targeted support to develop their capacity to use children’s mathematical understanding. Holton and Clark (2006, p. 6) argued that an act of teaching should: “1) supports the immediate construction of knowledge by the learner, and 2) provide the basis for the future independent learning of the individual. I expected the whole group and small group classroom discussions to support PSTs’ construction of knowledge. The set of activities include sequenced tasks/activities and assignments that are redundant and distributed over time to support the development of
PSTs’ capacity to use professional noticing of children’s mathematical understanding. As discussed in Chapter 3, the scaffolds were in the form of classroom discussions, minor homework activities with specific prompts, viewing targeted video clips, and major assignments with prompts similar to the ones in the minor homework activities to provide PSTs with multiple opportunities to perform the same skill over time. As Tabak (2004) argued, the goal in multiple scaffolds is to provide different supports using different modalities that are sequenced at different times in the curriculum. In this study, the different scaffolds were progressively distributed within the eight weeks of university learning. Based on previous literature (Grossman et al., 2009, p.11), one would argue that the classroom discussions were used as opportunities for “decomposition of practice”, videos were used as “the representation of practice” and, the minor and major assignments were used as “approximations of practice” where PSTs practiced how to respond based on children’s mathematical understanding. Figure 2.3 represents the scaffolding model used in this study.

In the next section, I review a set of studies as a theoretical justification for the inquiry the study pursues. The review that follows places emphasis on the importance of mathematical tasks and the challenges that teachers face when it comes to selecting and posing worthwhile mathematical tasks.
Figure 2.3. A model for scaffolding PSTs’ learning

- Developing PSTs’ capacity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding
- Needs

Multiple Distributed Scaffolds

- Classroom small group and whole group discussion
- Tornado problem activity
- Instructor’s written and verbal feedback
- Inquiry into Student Thinking
- Creating problems with specific learning goals
- Fish bowl problem activity

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Importance of Pre-service Teachers’ Understanding How to use Children’s Mathematical Understanding to Select and Pose Worthwhile Mathematical Tasks

Mathematical tasks influence what students learn in the mathematics classroom

For more than two decades, mathematics education reforms (e.g., NCTM, 1991) have emphasized the importance of using worthwhile mathematical tasks in the classroom, indicating that students need to be exposed to meaningful tasks that are problematic and will engage them in meaningful mathematics learning. These are tasks that would provide students with opportunities to impose meaning and structure, make decisions about what to do and how to do it, and interpret the reasonableness of their actions and solutions. NCTM (1991) has also indicated that “the teacher of mathematics should orchestrate classroom discourse by posing questions and tasks that elicit, engage and challenge each students thinking” (p. 35). The (NCTM, 2000) Teaching Principle emphasizes the fact that effective mathematics teaching involves using well-chosen worthwhile mathematical tasks that should be used to introduce important mathematical ideas and to engage and challenge students intellectually.

Although some researchers in mathematics education have provided insights on how we can develop in-service teachers’ abilities to select and pose worthwhile mathematical tasks (e.g., Arbaugh, Lannin, Jones, & Park-Rogers, 2006; Arbuagh & Brown, 2005, Smith et al, 2008), and have suggested that the Standards-based curriculum materials have tasks of high cognitive demand, (Senk & Thompson, 2003; Stein & Kim, 2009), literature on how we can develop PSTs’ understanding and practices related to using children’s mathematical thinking to select and pose worthwhile mathematical tasks is limited (Crespo, 2003; Routledge & Norton, 2008 being exceptions). Crespo (2003), and Routledge and Norton
(2008) used letter writing as a context of learning; however, as Adler, Ball, Krainer Lin, and Novotna (2005) argued, we lack studies that would help mathematics teacher educators compare how different opportunities and learning experiences would develop PSTs’ pedagogical content knowledge of how to use children’s mathematical thinking to select and pose worthwhile mathematical tasks.

Over time, research studies have also consistently highlighted the relationship between mathematical tasks and student learning (Doyle, 1984, 1988; Henningsen & Stein, 1997; Hiebert & Wearne, 1993; Houssart, 2002; NCTM, 1991, 2000; Stein, Smith, Henningsen & Siver, 2000; Stein & Lane, 1996; Stein & Smith, 1998; Stein, Grover, & Henningsen, 1996; Stylianides & Stylianides, 2008). Inherent in this literature is the fact that mathematical tasks influence what students learn in mathematics classrooms. These literatures also reveal that there exists a relationship between the nature of students’ thinking required by a mathematical task and the level of students’ understanding of mathematics. For example, Stein, Grover, and Henningsen (1996) suggested that the project teachers were successful in selecting and setting up the kind of mathematical tasks that had been viewed as leading to high level student learning outcome. Stein, Grover, and Henningsen (1996) also indicated “with appropriate set up, students were found to actually use multiple solution and multiple representations and to use mathematical justification in majority of the cases” (p. 30).

Other studies (e.g., Hiebert et al., 1997) have highlighted that selecting and designing tasks is one of the many critical responsibilities for a teacher. Additionally, Hiebert et al. argued that the teacher’s role goes beyond choosing good individual tasks, to selecting sequences of tasks so that over time they could build up students’ understanding gradually
and through a variety of experiences. The selection of appropriate tasks therefore includes thinking about how the tasks are related and how they can be chained together to increase the opportunity for students to gradually construct understanding. Therefore, these studies illuminate the need for teacher educators to develop the PSTs’ capacity to choose worthwhile, challenging and accessible tasks in the classrooms.

**Mathematics teachers have challenges in posing and enacting worthwhile mathematical tasks**

Although a lot of emphasis has been placed on the importance of tasks (and in this case mathematical tasks in the classroom) multiple research studies have indicated that mathematics teachers have challenges when it comes to selecting and posing tasks that would create classroom environments where students will be engaged with high-level thinking (Edward & Mercer, 1987; Henningsen & Stein, 1997; Smith, 2004; Stein, Grover, & Henningsen, 1996; Stylianides & Stylianides, 2008) Specifically, researchers have argued that teachers generally pose tasks that are of low cognitive demand, meaning that the tasks require procedures without connection to meaning or memorization, and when they choose tasks that would engage students with high-level thinking, the level of cognitive demand was reduced during the task enactment stage (Henningsen & Stein 1997; Stein, Grover, & Henningsen, 1996). Henningsen and Stein (1997) also argued that students and teachers perceive high-level tasks as ambiguous and complex, and teachers have a tendency to take over the challenging part of the task and perform them for the students. Teachers also have a tendency to emphasize the completeness and accuracy of the answers that quite often weakens the cognitive demand of the task and students cognitive processes.
Other researchers (e.g., Stylianides & Stylianides, 2008) have argued that it is often challenging for teachers to implement high-level mathematical tasks embedded in real life context in ways that exploit their motivational aspect, without overshadowing the mathematics involved. Stylianides argued that their analysis of the teaching episodes of one teacher [Nancy], who was well versed with mathematical knowledge, suggested that strong mathematical knowledge is not enough to ensure fidelity of implementation of tasks in a meaningful way. Rather, they suggested that teachers need to be equipped with the necessary mathematical and pedagogical knowledge that would allow them to understand and appreciate, not only the mathematical affordance of the task but also “the correspondence between these affordances and specific operations required by students to complete the tasks and the idea that the level of cognitive demands associated with these operations is consequential for students’ opportunities to learn mathematics” (p. 14). Similarly, Smith (2004) argued that teaching mathematics in a way that encourages students to make connections is a challenging endeavor and that the dynamic aspects of the classroom make it difficult to provide prescriptive lists of things to do to implement tasks as designed to help students make connections.

In summary, the studies highlighted above illuminate the need to develop teachers’ understanding and practices on how to pose worthwhile mathematical tasks and enact them in a manner that will ensure that students will interact with the necessary mathematical ideas.

**High-level tasks from curriculum materials or any other source are not generally implemented at the intended level**

Research studies have also revealed that supplying teachers with high-level tasks through curriculum material does not ensure that the tasks are implemented at their intended
level (Arbaugh et al, 2006; Tarr et al., 2008). For example, in their study with 26 teachers who were implementing a mathematics textbook series (Core-plus), Arbaugh et al indicated that the teachers’ instructional practices fell along a wide continuum of lesson implementation. Although initially the tasks adapted from the Core-Plus mathematics series involved problematic situations where the students had not developed any prior solution strategy, some teachers made modifications of the tasks and reduced the cognitive demand of the task. Therefore, eleven of the 26 lessons analyzed fell under the low-level quality lessons, where the teachers provided procedures to solve the tasks and did not follow up on unclear or incorrect students’ responses. The 11 teachers also shifted the focus from meaning towards a procedure to the correctness of the answer without discussing the underlying mathematical concepts.

Consider also the case of Ellen, the 1st grade teacher described by Olson and Barret (2004). Ellen was an experienced teacher who had taught for 16 years and had her master’s degree in curriculum and instruction. She had also participated in a professional development that had been designed to support mathematics reforms and had considered her practice as exemplary reform recommendations. However, Olson and Barret described her teaching practices and beliefs as completely traditional where she told the children the procedures to follow, asked them to recall the procedures, required the children to practice until they were successful, and reviewed the procedures by asking questions designed to solicit predictable responses. Although she adapted tasks from Standards-based curriculum materials, she modified the lessons in a way that she “attended to superficial aspects of instruction, which prevented her from attending to substantive mathematics.” Olson and Barrett theorized that Ellen was unable to utilize the rich mathematical tasks to explore mathematical ideas with
children because she focused on modifications instead of the mathematical concepts and needed help to unpack the mathematics in the lesson.

Over all, the studies suggest that providing teachers with curriculum materials that have tasks of high cognitive demand does not mean that they will be implemented at the intended level. Instead, teachers need the pedagogical content knowledge that will support their understanding on how to adapt the task and enact it at the intended level. In addition, teachers need to gain knowledge about students and teaching in order to adapt tasks according to the students’ needs.

Selecting and posing mathematics problems

The idea of problem posing is not new in mathematics education research and has long been recognized as an important pedagogical tool in the teaching of mathematics (Kilpatrick, 1987; Silver et al, 1996, Silver, 1994). For example, Kilpatrick, (1987) and Silver (1994) suggested that the incorporation of problem solving and problem posing situations in mathematics classroom could have a positive impact on students’ learning. Specifically, Kilpatrick (1987) argued, “problem formulation should be viewed not only as a goal for instruction but also as means of instruction” (p. 123). Additionally, Goldenberg and Walter (2003) argued, “problem posing is both a tool for teaching mathematics through problem solving and an integral part of learning in that way” (P.69). Goldenberg and Walter further argued that posing problems and extending them to enrich students learning are central to teaching mathematics through problem solving.

Other studies have also highlighted specific ways in which in-service teachers and PSTs pose problems and the cognitive process that lead to the formulation of tasks or
problems in the classroom (Silver et al., 1996; Smith et al., 2008). For example, Silver et al. (1996) explored how 53 middle school teachers and 28 prospective teachers worked either individually or in pairs to pose mathematical problems associated with a reasonably complex task setting before, during, or after attempting to solve a problem within that task setting. The results indicated that, although some problems were ill posed or poorly stated, the teachers (both in-service and prospective teachers) generated a large number of reasonable problems, suggesting that the teachers and the prospective teachers had some personal capacity for mathematical problem posing.

Further, Smith et al. (2008) provided a framework (Thinking through the Lesson Protocol [TTLP]) of developing lessons that use students’ thinking as a critical ingredient in developing students’ understanding of key mathematical disciplinary ideas. The framework is intended to promote careful and detailed planning that will help teachers anticipate what students will do to enable them to generate questions they might ask to promote students’ learning prior to a lesson being taught (see Smith et al., 2008, for details of the framework). Using the framework, the teachers are supposed to think through the lesson as they select and set up the mathematical tasks, think about what they will do to support students’ exploration of the task, and think about what they will do as they share and discuss the tasks in the classroom. Specifically, Smith et al. argued that the cumulative experience of the teachers who have used the TTLP framework over time suggests that the TTLP can be a useful tool in planning, teaching and reflecting, and can lead to improved teaching.

While there are many studies that have focused on problem posing, there are limited studies that have focused on how we can develop PSTs’ capacity to use children’s mathematical thinking to select and pose problems or tasks (Crespo, 2003; Norton &
Rutledge, 2006; Rutledge & Norton, 2008). Specifically, Crespo (2003) and Rutledge and Norton (2008) investigated how PSTs developed their ability to pose mathematical tasks in the context of a letter writing activity with 4th grade and middle-school algebra classrooms, respectively. Crespo (2003) examined the changes in the problem posing strategies of a group of elementary PSTs as they posed problems to pupils using letter writing and found that the “PSTs problem posing strategies changed from traditional single steps and computational problems to problems that required multiple steps open ended, exploratory and were cognitively more complex” (p. 1).

Similarly, Rutledge and Norton (2008) examined the various social contexts in which the letter writing interactions were situated as they considered the cognitive activities that both the PST and the middle school algebra student (Jacque) engaged in the task from a constructivist perspective. Routledge and Norton inferred that, from the pair’s interaction, the students had constructed ways of using procedures, such as the Pythagoras theorem, that were connected to meaningful concepts. Norton and Rutledge (2006) also emphasized that the letter writing activity demonstrated significant growth in terms of elicited responses.

Crespo (2003) indicated, “the PSTs problem posing strategies were significantly affected by having an authentic audience that interacted and engaged with the task that the PSTs generated” (p. 1), and found “that the introduction and in-class exploration of non-traditional mathematical problems and engaging in collaborative problem posing were two instructional moves that the PSTs thought to have been very helpful” (p. 23). In the current study, I sought to understand how PSTs used children’s mathematical understanding to pose tasks when given an opportunity to instruct elementary school children based on their understanding of students’ thinking.
In summary, the studies discussed in this chapter illuminated the need for PSTs to learn how to pose worthwhile mathematical tasks, specifically, using children’s mathematical understanding. The next section discusses the research context, participants, and data analysis and coding procedures.
CHAPTER 3. RESEARCH METHODOLOGY

Overview

In this study, I examined the extent to which elementary PSTs enrolled in a mathematics methods course developed their capacity to attend to children’s strategies, and interpret their mathematical understanding and respond in the context of two scaffolded activities. To examine the PSTs capacity, I collected and analyzed two classroom assignments: (a) the Inquiry into Student Thinking assignment; and (b) the tutoring assignment, which the PSTs did after six weeks and ten weeks of instruction respectively. The two assignments required PSTs to summarize what they perceived the children knew and understood, and respond based on the children’s mathematical understanding. These two data sources enabled me to describe the PSTs’ capacity to attend to children’s strategies, and interpret and respond based on children’s mathematical understanding at two different times in their methods course, and ascertain how this capacity changed as they progressed in the methods course.

This chapter describes the study setting and the methods used to collect and analyze the data. First, I discuss my role as a researcher in the context of the study. Next, I describe the setting of the study, research context and participants. Then, I provide a detailed discussion of the research design. Finally, I conclude by describing the data collection methods, coding, and analysis procedures.

Role of the Researcher

From fall of 2009 to Fall 2011, I audited an elementary PSTs mathematics methods course in a large university in the mid-western region of the United States. During that time, I
was a graduate research assistant participating in a research study that was developing and refining instructional modules for the mathematics methods course. Specifically, the course activities were designed to develop PSTs’ ability to design and implement mathematics instruction that is both reflective and mathematically significant. Auditing the mathematics methods course allowed me to observe the PSTs interacting with the instructional activities throughout the semester for two consecutive academic years. During this experience and my constant interaction with the instructors, I became interested in exploring the extent to which PSTs’ capacity to notice and use children’s mathematical understanding to select and pose tasks develop in the context of a mathematics methods course.

Prior to working in the aforementioned research study; I examined the influence of in-service teachers’ mathematics curriculum implementation strategies on the nature of instructional tasks, classroom discourse and students’ learning. The results suggested that teachers’ decisions about how to use the curriculum materials impact the nature of instructional tasks, classroom discourse, and student learning. Both of these experiences piqued my interest to better understand whether elementary PSTs’ develop the capacity to observe and utilize children’s mathematical understanding to select and pose tasks as they progress in their mathematics methods course and, if so, to what extent?

During the dissertation study, I audited three sections of the elementary mathematics methods course during the fall 2011 semester and served as the primary researcher for this study. Prior to conducting the study, I applied for and received approval to conduct the study. As the primary researcher, I also obtained consent from the PSTs to use a subset of their course work in my study. I collected and copied the written work from each participant (the original work was returned to the students) with their responses to Inquiry into Student
Thinking assignment, the tutoring assignment and any work related to the course packet (the course text) across the semester. Nevertheless, this study only focused on the analysis of PSTs’ responses to the Inquiry into Student Thinking and the tutoring assignment.

Study Setting and Participants

Setting

This study was conducted within a context of an elementary mathematics methods course required for elementary PSTs enrolled in a two-year teacher education program during the fall of 2011, in a university located in the Midwestern region of the United States. The course took place during the second semester of the elementary education and early childhood majors’ teacher preparation program. An overview of the teacher preparation program is provided in the following paragraphs.

Brief overview of the teacher preparation program

During the four semesters of intensive teacher preparation program, the university offers a sequence of three courses to elementary PSTs in order to fulfill the bulk of their undergraduate mathematics knowledge for teaching requirements. The first two courses, mathematics for elementary education 1 and mathematics for elementary education 2, are offered in the mathematics department, and are generally taken before the PSTs are admitted in the teacher preparation program. The elementary mathematics methods course is offered in the Curriculum and Instruction Department. During fall of 2011, the pre-requisite for the elementary mathematics methods course included mathematics for elementary education 1, mathematics for elementary education 2, and concurrent enrollment in a literacy block course
(The Teaching of Reading and Language Arts in the Primary Grades (K-3). In addition, the PSTs were required to be concurrently enrolled in a mathematics teaching practicum that included observation, application of current methods, and instructional experiences with children in a supervised elementary classroom. Next, I briefly elaborate on the content taught to the PSTs in the two mathematics content courses and the methods course alike.

**Mathematics for Elementary Education 1.** As indicated previously, the program entails strong academic preparation and intensive study as well as teaching in an elementary classroom. Mathematics for elementary education 1 targeted the mathematics subject matter specialization or the specialized content knowledge. As described by Hill et al. (2008, p. 6), “Specialized Content Knowledge (SCK) is the mathematical knowledge that allows teachers to engage in particular teaching tasks including how to accurately represent the mathematical ideas, provide mathematical explanations for common rules and procedures and examine and understand unusual solution methods to problems.” As stated in the course syllabus (see Appendix A-1 for a detailed description of the course goals), Mathematics for elementary education 1 is designed to help PSTs understand the central concepts, tools of inquiry, and structure of mathematics. Specifically, the course “is designed to support PSTs understanding on how to create learning experiences that make the mathematical concepts meaningful for elementary students” (Course Instructor, 2011a, Spring, p. 2).

**Mathematics for Elementary Education 2.** Similarly, Mathematics for Elementary Education 2 targets the mathematics knowledge for teaching for elementary education teachers. This is a content course and students are expected to learn the mathematical terminologies and concepts. The topics covered includes, “elementary statistics and
probability concepts as well as fractions, decimals, integers, percent and geometry” (Course Instructor, 2011b. Spring). As stated in the course syllabus, students are expected to have both content and process knowledge at the end of the course (see Appendix A-2 for detailed descriptions of the course goals). The course is also expected to give PSTs an opportunity to experience what it means “to think mathematically, understand the value of conceptual insights, and appreciate how mathematical knowledge is constructed in an exploratory manner” (p. 2). As shown in our earlier work, mathematics for elementary education 1 & 2 courses significantly contribute to the development of PSTs’ mathematical knowledge for teaching as they go through the teacher preparation program (Gichobi, Andreotti, Drake & Bolles, in preparation).

**Mathematics Methods for Elementary Education/Early Childhood majors.**

During Fall 2011, the elementary mathematics methods course met for three hours and forty minutes each week for a period of eight weeks before going to a field-based experience. The stated primary goal of the course was to develop PSTs’ ability to design and implement mathematics instruction that is both reflective and mathematically significant (see Appendix A-3 for a detailed description of the course goals). Specifically, “the course focuses on developing PSTs’ capacity to “teach mathematics for understanding by developing a practice of using curriculum materials to teach through problem-solving and building on children’s mathematical thinking through skilled questioning” (Course Instructor, 2011a, Fall, p. 1).

The course is organized around four goals for instruction. As stated in the course syllabus, for each content area, the PSTs were expected to learn:

1) How children’s thinking typically develops, including common understanding, misunderstanding, strategies and errors
2) How to access and assess children’s mathematical thinking within different content areas.
3) How to use children’s mathematical thinking to select and pose worthwhile mathematical tasks
4) How to use curriculum materials, family and community resources and, other supports to help the PSTs facilitate the development of children’s mathematical thinking. (Course Instructor, 2011a, Fall, p. 1)

I expected all the four goals for instruction to have played an important role in developing PSTs’ capacity, but the first, second and third goal served as the main focus of this study.

**Intervention**

The mathematics methods course involves a variety of activities and assignments contained in a course packet developed by the mathematics education team in the University (Course Instructors, 2011b, Fall). These activities are classified under seven themes:

1) Elementary Mathematics standards
2) Tools and Frameworks for teaching mathematics
3) Number of the day (NOTD) and other opening number routines
4) Problem posing
5) Making sense of and responding to students work
6) Facilitating whole class discussion of strategies.
7) Pulling it all together. (Course Instructor, 2011b, Fall, p. 1)

The instructor began the course by discussing the elementary mathematics standards, specifically, the Common Core State Standards (CCSS, 2011) and the state standards. PSTs also viewed a video clip of a 2nd grade teacher enacting a mathematics lesson by facilitating a number of the day (NoTD) task, with ½ (see Appendix B for a summary of the course activities). The video and video analysis served as a starting point for future instruction as the PSTs had opportunities to learn how to use children’s mathematical understanding to select and pose mathematical tasks during the methods course. The primary goal for viewing the video and the video analysis was to introduce PSTs to the practices that would be discussed
during the semester. In other words, the teacher in the video used an opening number routine (ONR) that comprises strategically chosen numbers, introduces the problem that children are supposed to solve, gives them an opportunity to interact with the problem and, finally, facilitates the sharing session. These practices observed in the video that is shown during the first session are revisited throughout the eight weeks of instruction in the university classroom, before PSTs start their field-based experience.

During the first two weeks of the semester, the instructor also discusses the levels of cognitive demand framework (Stein & Smith, 1998) as a tool for teaching and a framework for critiquing the nature of tasks. The discussion is followed by an activity where PSTs have an opportunity to select and sort a group of tasks at several cognitive levels. Then, the PSTs explore and discuss the CGI problem type framework and solution strategies (Carpenter et al., 1996). PSTs also prepare their own charts with CGI word problem types and strategies for solving them. In addition, they discuss the tasks that they placed on their own charts and how one would increase the problem’s level of complexity in small as well as large groups.

During the third week of the semester, PSTs are required to read Chapter 1 of *Number Talks* (Parrish, 2010) which describes how number talks can be a purposeful opportunity to support children’s development of “efficient, flexible and accurate computation strategies that build upon the key foundational ideas of mathematics” (p. 2). Next, the instructor introduces the Opening Number Routines (ONR); a routine of purposefully posing well-crafted tasks with selected numbers that are accessible to children, and using those tasks to focus on mathematical relationships that can build mathematical understanding and

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5 To validate the functions of the scaffolds used in the methods course, I prepared Appendix C, and gave the Course Instructor an opportunity to read through the goals and the activity descriptions. In doing this, I got to verify the function of the scaffolds.
knowledge. PSTs also have an opportunity to view a variety of video clips of practicing teachers using the ONR and unpacking and posing CGI problem types. As Sherin et al. (2009) indicated, video clips that are carefully selected can be used to facilitate productive discussions and provide PSTs with opportunities to view practice. The ONR include open number sentences, true or false number sentences and different open number routines from the curriculum materials. After viewing the video clips the PSTs discuss (as a class as well as in small groups) practices of unpacking and posing CGI problem types so that the children can access the mathematical concept.

Although there are other themes, most of the in-class activities purposefully focus on “developing PSTs’ understanding of how children’s thinking typically develops, including common understandings/misunderstandings, strategies and errors, as well as how teachers can respond on the basis of children’s understanding” (Course Instructor, 2011a, Fall, p. 1). Each session begins with a warm-up reflection question to check for understanding, followed by in-class activities and discussions. These activities engage the PSTs in learning about strategies for selecting and posing mathematical problems, and making sense of and responding to students’ work as well as strategies for facilitating whole class discussions. The PSTs are also given a variety of assignments (see Appendix C for a summary of assignments and activities) that focus on understanding and using children’s understanding to select and pose mathematical tasks. Of relevance to this study are three in-class activities (learning goals with CGI problem, Natalie’s Tornado Problem and the fishbowl problem), and two major assignments (Inquiry into Student Thinking and a tutoring assignment), both of which are the focus of this study. Following are brief descriptions of the expectations of these activities.
Creating problems for specific learning goals: In this activity, PSTs are:

1. Provided with the learning goal from a state standard and problem types with number choices, which are strategically chosen to address the specific learning objective. PSTs are expected to read through the problems, number choices, and the rationales for each problem.

2. Expected to choose a grade level and three specific learning objectives either from operations and algebraic thinking, number and operations in base 10 or number and operations and fractions strands in the common core state standards from which to write three problems. The chosen problems will have at least three number choices and a rationale for each number choice.

This activity is particularly relevant to this study because it was done during the 4th week of the semester and involved PSTs selecting or generating tasks based on a specific learning goal. Before doing this activity, PSTs had already discussed the CGI problem types and how to intentionally select tasks that would support children’s understanding of a specific learning goal. This activity was a “hard scaffold” since PSTs were given pre-determined prompts in form of questions to guide their responses. After turning in the assignment for grading, the instructor provided written feedback (as soft scaffolds) and, I found the feedback would have scaffolded PSTs as they selected tasks in the next assignment.

Fish bowl problem: As stated in the course packet (Course Instructors, 2011b, Fall, p. 68), the fish bowl problem activity was chosen to focus and support PSTs’ understanding on how the choice of problem type and number choices support the students in working towards the learning goal. In this activity, PSTs are provided with the following problem:
Sam had __ fish bowls. He had __ in each bowl. How many fish did Sam have?

A                  B                  C                  D
(2, 10)            (4, 20)            (3, 11)            (4, 12)
(5, 10)            (8, 20)            (6, 11)            (8, 12)

The PSTs were expected to do the following:

1. Consider the learning goal that aligns with the problem;
2. Look at the students’ work and identify any evidence that the students are or are not progressing towards the identified learning goal;
3. Write an appropriate problem for the next day along with a rationale that will meet the range of needs of the students’ work they had analyzed and specifically reference how the problem and number choices will meet the needs of at least three of the students.
4. Watch the teacher (who had posed the problem) facilitating the sharing session and consider if the selected problem for the next day was still appropriate.

In this activity, PSTs examined the various children’s solutions and discussed in small groups the strategies that the children used, what the children understood and/or misunderstood, and errors that the children made. In the whole group discussion, PSTs volunteered and shared the way they thought about the children’s strategies and the common understanding and misunderstanding. Next, they individually selected problems with specific number choices that would extend specific students’ thinking and the rationale for their choices. Then, PSTs viewed a video clip of the teacher who had posed the problem facilitating the sharing session, with specific focus on children’s strategies. Finally, PSTs re-examined the tasks they had selected/generated to ascertain whether they were still appropriate problems. I expected that this activity provided an opportunity for PSTs to “decompose the practice”, “approximate the practice” and see a “representation of the practice” (Grossman et al., 2009) of using children’s mathematical understanding.

I thought this activity was productive because PSTs identified the learning goal, just like the first activity, but added the component of analyzing children’s work to determine
whether they understood the learning goal or not. Next, they selected a task with a specific focus on what the students understand and/or do not understand. Unlike the first activity, this fish bowl problem activity was more situated in the classroom, and I expected that it would scaffold PSTs on how to assess specific student and plan for the tasks that they would pose after assessing children’s mathematical understanding. In addition, the fish bowl problem provided a repeated opportunity for PSTs to select or generate a task with the learning goal in mind. Finally, PSTs got an opportunity to view a video clip of the children whose work they had analyzed and the classroom teacher facilitating the strategy sharing session. I expected the viewing of the video clip to scaffold PSTs’ understanding on the strategies that the children used and how to facilitate the sharing session, gradually increasing their opportunity to learn how to use children’s mathematical understanding to select and pose mathematical tasks.

**Tornado Problem:** As stated in the course packet (Course Instructor, 2011b, Fall, p. 68), the Tornado problem activity was supposed to focus PSTs’ understanding on the ways in which a teacher can unpack the problem to ensure that the children understand what the problem is asking and they can access the mathematical idea. Below is the problem that was posed by the teacher:

Last year the national weather services recorded ___ tornados in the United States. They recorded some tornados in other parts of the world. They recorded a total of ___ tornados. How many of the tornados were in other parts of the world?

(18, 28)   (26, 48)   (22, 75)   (39, 81)   (83, 150)
(77, 168)   (95, 194)   (101, 283)   (156, 381)   (274, 475)

In this activity, PSTs generated questions that they would ask the children to make sure that they understand the tornado problem. Finally, they analyzed the work of students
who had done the problems, viewed and analyzed a video of the classroom teacher unpacking the problem. This activity provided PSTs with a repeated opportunity to analyze students’ work, pay careful attention to what the children understands and the common errors that they make before responding based on children’s understanding. In addition, PSTs got an opportunity to see a classroom teacher posing the task to the students. Finally, PSTs turned in the activity for grading and the instructor of the course provided written feedback.

It’s important to note that PSTs did the above described activities before doing the Inquiry into Student Thinking assignment. As previously discussed, the instructor provided individual written feedback for each student’s work and group responses. In addition, the instructor provided verbal feedback when returning the responses to the students. Some common written feedback I identified in the student written feedback included: “you need to identify the problem types”, “provide justification for number choices”, and “provide more details on what the children knew last time”. Although these assignments were not part of the analyzed data\(^6\) for this study, in part, I expected the assignments to have contributed to developing PSTs’ capacity by the time they did the Inquiry into Student Thinking assignment and the tutoring assignment. Table 3.1 outlines the scaffolds and the supported skills or practices as they progressed in their methods course.

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\(^6\) I made a decision not to analyze the three activities described above for this study because PSTs had done some of the work in small groups of 3-4 students. For example, the Tornado problem was turned in as group responses. Therefore, it was difficult to determine the contributions of individual students in the overall group response. However, due to the nature of these activities, I expected them to have scaffolded PSTs’ understanding of how to attend to children’s strategies, interpret and respond based on children’s mathematical understanding.
Table 3.1. Skills and practices supported by scaffolding activities

<table>
<thead>
<tr>
<th>Scaffold</th>
<th>Type</th>
<th>Supported skill/practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Creating problems for specific learning goals</td>
<td>Hard</td>
<td>• Identifying the learning goal from a state or national standards</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Selecting/generating and posing worthwhile mathematical tasks that align with a specific learning</td>
</tr>
<tr>
<td>Fish bowl problem activity</td>
<td>Hard</td>
<td>• Identifying the learning goal from children’s written work</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Attending to children’s strategies</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Assessing children’s understanding and/or misunderstanding from written work</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Selecting/generating and posing worthwhile mathematical tasks based on children’s understanding on children’s mathematical understanding</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Facilitating the sharing of the identified strategies</td>
</tr>
<tr>
<td>Tornado Problem Activity</td>
<td>Hard</td>
<td>• Attending to children’s strategies</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Assessing and interpreting children’s mathematical understanding from students written work</td>
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<tr>
<td></td>
<td></td>
<td>• Selecting/generating and posing problems for next instructional plan based on children’s mathematical understanding</td>
</tr>
<tr>
<td>Instructors written and verbal feedback for all the homework activities</td>
<td>Soft</td>
<td>• Attending to children’s strategies</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Assessing children’s mathematical understanding</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Selecting/generating problems for next instructional plan</td>
</tr>
<tr>
<td>Classroom discussions and readings</td>
<td>Soft</td>
<td>• Attending to children’s strategies</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Assessing children’s mathematical understanding</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Selecting/generating and posing problems for the next instructional plan</td>
</tr>
<tr>
<td>Inquiry into Student Thinking assignment</td>
<td>Hard</td>
<td>• Attending to children’s strategies</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Assessing children’s mathematical understanding</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Selecting/generating problems for next instructional plan</td>
</tr>
<tr>
<td>Tutoring assignment</td>
<td>Hard</td>
<td>• Attending to children’s strategies</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Assessing children’s mathematical understanding</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Selecting/generating and posing worthwhile mathematical problems based on children’s mathematical understanding</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Facilitating sharing of specific strategies</td>
</tr>
</tbody>
</table>

In summary, using Tabak’s patterns of distributed scaffolding (Tabak, 2004), I classified the scaffolds described in Table (3.1) as redundant scaffolds because each scaffold offered PSTs additional opportunities to learn what it means to attend to children’s strategies, interpret and respond based on children’s mathematical understanding.
Research participants and procedures

The participants in this study were PSTs from three sections of an elementary mathematics methods course who consented that I use a set of their course work for research purposes. I informed the PSTs that they could withdraw their consent for participation at any time during the study and that I would maintain confidentiality by using pseudonyms for all the participants’ scanned work. For this reason, the names on PSTs’ written work were replaced with codes to ensure that every scanned document had pseudo-identifiers before scanning. The scanned documents were stored in a computer with a password, to which I was the only person who had access. In total, 56 PSTs consented to participate in the study at the beginning of the semester. However, for the purposes of this study, I only used the course work for 30 participants because I had access to all of their course work for the activities and reflections.

Among the 30 PSTs whose responses were considered for analysis, 4 were early childhood majors while 26 were elementary education majors. In addition, 29 were females whereas only one was a male. Most of the PSTs were in the 2nd half of their junior year and had already met the pre-requisite of completing mathematics for elementary education 1 and mathematics for elementary education 2.

Study Methods

Qualitative interpretive research

This study involved a qualitative interpretive research approach and an interpretive case study (Klein and Meyers, 1999; Golfashni, 2003; Orlikowski, & Baroudi, 1991; Walsham, 1995). The foundational assumption in interpretive research methodologies is that
“knowledge is gained through social constructions such as language, consciousness, and shared meanings, documents, tools, and other artifacts” p.25. Additionally, as Walsham (1995) indicated, interpretive researchers do not report facts. Rather, they report the interpretation and the meaning assigned to specific phenomenon by other people. They focus on analytically disclosing those meaning-making practices, while showing how those practices configure to generate observable outcomes. The sharable results in interpretive research should be in form of development of concepts, generation of theory, drawing specific implications as well as contribution of rich insights. Data collection and analysis are part of iterative cycles.

**Interpretive aspects of the study**

Building upon other studies that have largely focused on the use of children’s mathematical thinking as a pedagogical tool in mathematics classroom (Franke & Kazemi, 2001; Franke, Kazemi, & Battey, 2007), are what and how teachers notice children’s mathematical thinking as well as how they respond to children’s mathematical thinking (Jacobs et al., 2007, 2010; Sfard & Kieran, 2001; Silver & Stein, 1996; Sherin & Van Es, 2005), this study focused on exploring two important aspects. First, the study explored the extent to which PSTs’ developed the capacity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding at two fixed time periods as they go through the methods course. Second, the study focused on developing a more nuanced conception on how PSTs use what they notice to select and pose mathematical tasks in the context of scaffolded instructional activities. I used the hypothesized theoretical trajectory of professional noticing of children’s mathematical thinking as conceptualized by Jacobs et al.
(2010), and Sherin and Van Es (2005) to make sense and meaning of PSTs’ responses. In other words, my primary role as a researcher was to interpret the meaning that PSTs were giving to children’s strategy, and how they were making sense about children’s mathematical understanding.

Specifically, I used PSTs’ written assignments for the scaffolded activities (Inquiry into Student Thinking and the tutoring assignment) as a source of data. However, as discussed previously, there were other in-class activities and minor assignments that could have scaffolded the PSTs’ understanding as they progressed with the methods course and before they did the Inquiry into Student Thinking assignment. As Hannafin, Land and Oliver (1999) indicated, the scaffolds provided guidance to PSTs in recognizing ways of thinking that are helpful as they attended to children’s strategies, interpreted, and responded based on children’s mathematical understanding. Further, these scaffolds were distributed to support the PSTs’ engagement at different points as they progressed in the methods course. After doing a series of activities and homework assignments, the Inquiry into Student Thinking and the tutoring assignment provided PSTs with opportunities to refine their own ideas about attending to children’s strategies, interpreting, and responding based on children’s mathematical understanding.

As Orlikowski and Baroudi (1991) indicated, meaning is made [by participants] as they understand a phenomenon. Therefore, this study was designed to interpret the PSTs’ meaning-making process as they attend to children’s strategies, interpret, and respond based on children’s mathematical understanding. The main goal was to make meaning by drawing inferences from their responses and provide insights on their ability to use children’s mathematical understanding to select and pose mathematical tasks.
One important aspect of this study was the fact that the PSTs used similar scaffolds during the Inquiry into Student Thinking assignment and the tutoring assignment, respectively. During the Inquiry into Student Thinking the PSTs attended to children’s strategies, interpreted and responded based on children’s mathematical understanding even though they did not pose the selected tasks to children. During the tutoring assignment, they had an opportunity to select and/or generate tasks and pose them to children in real classroom settings, an opportunity that provided PSTs with an authentic experience of selecting and posing tasks.

Additionally, this study involved creating multiple case studies that represented emerging patterns from the PSTs’ responses. These case studies were also based on the interpretive approach where I interpreted PSTs’ responses (which was their process of making meaning) of how to use children’s mathematical understanding to select and pose tasks. As Walsham (1995) indicated, the case studies provided tendencies and emerging patterns in PSTs’ responses that are worth being investigated.

Validit y and Reliability

Creswell and Miller (2000) described three lenses for determining the credibility of qualitative research. First is the lens of the researcher, “where the researcher determines how long to remain in the field to collect enough data that is saturated enough to establish good themes and determine whether the analysis of data can evolve into good narratives”(p. 3). This also involves “the researcher going to the field over and over again to see if the constructs, the categories, explanations and interpretations make sense” Patton (as cited in Creswell & Miller, 2000, p. 8). The procedure of validity in qualitative research may include
the strategies used by the researchers to establish the credibility of their study or the researchers’ paradigm assumptions.

Second, the researcher can also use the participants in the study to establish the validity of the study. Using this strategy, the researcher takes the qualitative paradigm that assumes that reality is socially constructed and it is what participants perceive it to be. This lens of validity check suggests the importance of checking how accurately participants’ realities have been represented in the final account by actively involving the participants in assessing how accurately the interpretations represent them (Creswell & Miller, 2000).

The third lens comprises the paradigm assumptions, or the worldview of the researcher described by Guba and Lincoln (as cited in Creswell & Miller, 2000). These paradigm assumptions include “the positivists, constructivist, and critical influence of the researcher”. For example, the positivist researcher assumes that qualitative research consists of rigorous methods and systematic forms of inquiry. Individuals embracing the post-positivist position both recognize and support validity, look for quantitative equivalence of it, and actively employ procedures for establishing validity using specific protocols. Constructivists or interpretivists believe in pluralistic, interpretive, open-ended, and contextualized (e.g., sensitive to place and situation) perspectives. The validity procedures reflected in this thinking present criteria with labels distinct from quantitative approaches, such as trustworthiness (i.e., credibility, transferability, dependability, and confirmability), and authenticity (i.e., fairness, enlarges personal constructions, leads to improved understanding of constructions of others, stimulates action, and empowers action).

Finally, the researchers should uncover the hidden assumptions about how narrative accounts are constructed, read, and interpreted (Creswell & Miller, 2000). Therefore, the
researcher’s orientation to any of the world-views influences their choices of validity procedures.

In this study, I took a number of measures to enhance reliability and validity. First, I developed the coding scheme in iterative cycles. In the first iteration, I developed a coding scheme based on literature review on noticing (Mason, 2002; Sherin & Van Es, 2005; Van Es & Sherin, 2008) and professional noticing of children’s mathematical thinking (Jacobs et al., 2010). After developing the codes, I did peer debriefing with an independent member who was knowledgeable in mathematics education. During the peer debriefing process, we (peer and I) verified the responses that were coded under each category to ensure that the codes represented the description of the coding scheme. This verification process on the coding scheme provided feedback on the coding scheme, emergent patterns, and interpretations thereby increasing the credibility of the assertions in the study. After verification of the coding scheme, I did a second iteration of coding responses. Finally, a second coder who was knowledgeable in mathematics education coded 50% of the responses sampled for this study. The discrepancies were discussed and we reached 90% agreement. Finally, although I developed other codes based on emergent themes, the main coding scheme was based on literature review on teacher noticing children’s mathematical understanding that had been applied by Jacobs et al. (2010).

I used an independent member to check (the 2nd coder) for reliability and an inquiry audit to enhance the dependability of the results. The independent member check helped in examining both the process and the product of the study. In addition, during the times that I audited the course, I got an opportunity to be in the field over a period of time as well as in the classroom during instruction, which provided insights into the problem under
investigation. I used the thick rich descriptions of phenomenon under investigations that provided many details on how the PSTs responded.

**Data Sources**

Two of the course assignments served as data for this study. These assignments included the Inquiry into Student Thinking assignment and the tutoring assignment. I briefly elaborate on the expectations of these course assignments and their purpose.

**Inquiry into student thinking assignment**

For this assignment, the primary source of information was found in a case study of four second graders located in www.edb.utexas.edu/empson. The PSTs read through the case study, and were prompted to analyze the case study and write 2-3 pages of reflection using the following questions:

a. Choose one of the four students from the case study and trace their responses and learning through the study.
   1. Summarize what you think they knew or understood at the end of the study that they did not know or understand in the beginning
   2. Choose 2 examples of this student’s responses as evidence for your claim from part a) and explain how they support your claim about this student’s learning. (You will probably want one example toward the end of the study and one example toward the beginning)

b. Choose 2 tasks or problems that were posed to the students that seemed particularly productive for advancing the thinking of the student’s as a group. For each problem, provide evidence from at least two of the four students to support your claim that
these were productive tasks. Conjecture (i.e., make an educated guess) why this were productive tasks and problems

a. Choose one instance of teacher decision-making or reflection that was particularly interesting or surprising to you. Summarize in a paragraph what made that instance stand out for you and how you might use it to inform your own teaching

b. If you were to teach the next lesson to this group of students, write one problem that you could give them and explain why you think this would be a good problem for this group. (See Appendix C for details of Inquiry into Student Thinking assignment.)

I used the PSTs' responses to the prompts listed above as a data source to interpret and make meaning of PSTs’ capacity to attend to children’s strategies, interpret, and respond based on children’s mathematical understanding. Specifically, this assignment shed light on the extent to which the PSTs used the component skills of professional noticing of children’s mathematical thinking and the nature of tasks they selected after six weeks of instruction. PSTs were expected to use what they had learned in class about problem types, number choices, and students’ solution strategies to support their decision. In this assignment, PSTs got a repeated opportunity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding.

**Tutoring assignment**

The tutoring assignment is a field-based assignment that PSTs did at three local elementary schools. The classrooms ranged from kindergarten to 4th grade, with one school allowing the PSTs to do the tutoring assignment in a 2nd grade classroom only. The thirty
students selected for data analysis ranged from kindergarten to a 4th grade classroom. I decided not to hold the grade level as a variable because the PSTs were elementary and early childhood majors, and their responses were not determined by the grade level of the children. In addition, the unit of analysis was PSTs’ learning capacity, and it was not influenced by the children’s grade level.

In this assignment, the PSTs were supposed to demonstrate their understanding of how they can plan for instruction based upon knowledge of subject matter, students, community, curriculum goals, and state curriculum models. To complete the assignment, the PSTs were required to interview one or two elementary level (K-5) students about their understanding of number and operations and problem solving. After the interview, PSTs were required to write evidence-based claims about what the children seemed to understand or be able to do. Based on what they learned from the interview about children’s mathematical understanding, PSTs were required to plan and implement three tutoring sessions with the students using problems that are challenging and yet accessible to the students. In other words, PSTs were required to select a learning goal and then select tasks that were based on children’s mathematical understanding. During the final session, they interviewed the student(s) again to assess their mathematical understanding after the three tutoring sessions. The activity was done in four 50-minute sessions during a period of one month.

As discussed previously, before the students went for the field based experience sessions, the instructor introduced the Cognitively Guided Instruction framework (CGI) (Carpenter et al., 1999) in the university classroom, and the PSTs had an opportunity to construct tasks using the framework. The rationale for using the CGI framework and
allowing the PSTs to develop tasks and practice them before going to the field based experience was similar to one used by Lampert, Beasley, Ghousseini, Kazemi, and Franke (2010), wherein they provided the PSTs with “routine instructional activities” which they could practice in the university classroom, and thereafter, teach the activities to the elementary children. In addition, the instructor provided PSTs with a task pool for every grade level (see Appendix D for sample tasks) from which to select tasks to use during the interview. As a result, PSTs had tasks for the specific grade level for the children they were tutoring, as well as tasks for a grade level below and above as they assessed children’s mathematical understanding. Further, PSTs were provided with the prompts listed as a guide to their reflection:

1. What does each student know, think and understand about number, operations and problem solving?
2. How will what you learned in the interviews influence how you work with the children during the next sessions?
3. How are these tasks intended to build on what you know about your students’ understanding and misunderstanding?
4. What did you learn about learning, teaching mathematics and students during this experience? How did your original goal for the tutoring session compare with what really happened, how effective were your plans and your teaching?

In their responses, the PSTs provided detailed explanations on how the children interacted with the tasks, the strategies that the children used, what the children understand and/or did not understand. Thus, PSTs were given repeated opportunity to analyze the children’s work and interpret their mathematical understanding, and respond based on the
children’s mathematical understanding. In this assignment, the PSTs were given an opportunity to reflect on their instruction, the children’s strategies as they interacted with the tasks, the children’s mathematical understanding, and the rationale for their choice of tasks based on children’s mathematical understanding.

I selected this activity because the PSTs actually had an opportunity to interact with children using the given tasks as well as the ones they designed repeatedly over the period of one month. PSTs also reflected on their teaching at the end of every session, which provided a window to see their meaning making processes. After every session, PSTs also had group reflection time in the university classroom where they discussed their experience and the challenges they were facing as they interacted with the children. During the group reflection time, the PSTs shared their interaction with the children, and the instructor had opportunities to provide verbal feedback for their next plan and actions.

At the end of the four tutoring sessions, the PSTs wrote a reflection using the prompts listed previously, and described in detail how the children interacted with the tasks in every tutoring session, the strategies which the children used, and what the PSTs interpreted as the children mathematical understanding. In their descriptions, the PSTs also included the problems/tasks that they posed as a response to children’s mathematical understanding and the rationale of choosing and posing those problems. Some of the PSTs also attached the students’ work to support their explanations of how the students interacted with the tasks.

I used the detailed reflections as primary source of data for analysis and to answer the main research questions and the sub-questions. Specifically, this assignment shed light on the extent to which the PSTs attended to children’s strategies, interpreted and responded based on children’s mathematical understanding. Additionally, I sought to understand the nature of
tasks that they selected and posed to children. I also paid careful attention to particular students who were included in specific emerging clusters to gain a more nuanced understanding on the PSTs’ developmental continuum of noticing using children’s mathematical understanding.

Data Analysis and Coding

This section describes the data analysis approaches that I used to address the main research question and the sub questions outlined as follows:

To what extent do PSTs develop practices of using children’s mathematical understanding to select and pose worthwhile mathematical tasks in the context of scaffolding activities?

a. What happens when PSTs analyze their own teaching and respond to children’s mathematical understanding as they plan for a series of instructional activities?

b. To what extent is the rationale for the PSTs’ next instructional plan based on children’s mathematical understanding?

c. What type of tasks/problems do PSTs pose after assessing children’s mathematical understanding?

d. What are PSTs’ conceptions of a productive task and/or tasks that engage students with high or low level thinking?

Table 3.2. outlines a series of questions that guided the data analysis to answer the main research question, hypothesized findings for each question and the data source for each analysis question.

In this section, I describe the data coding and the analysis procedures. Data analysis was done in three stages. In the first stage, I developed a coding scheme based on literature review on noticing (Mason, 2002; Sherin & Van Es, 2005; Van Es & Sherin, 2008) and professional noticing of children’s mathematical thinking (Jacobs et al., 2010). As discussed in Chapter 2, different terms have been used to describe how teachers notice and identify noteworthy aspects of an instructional situation (Jacobs et al., 2010; Sherin & Van Es, 2005; Van Es & Sherin, 2008). Prior research (e.g., Carpenter et al., 1999; Carpenter
<table>
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<tr>
<th>Analysis Question</th>
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<tbody>
<tr>
<td>What happens when we ask PSTs to analyze students’ work and respond to children’s mathematical understanding?</td>
<td>Their responses during the Inquiry into Student Thinking assignment will demonstrate basic understanding of and abilities of analyzing children’s mathematical understanding. Since they analyze students’ work multiple times during the course of the semester, I anticipate that the PSTs’ responses will be more detailed and with robust evidence that their interpretation and choice of task were based on children’s mathematical understanding during the tutoring assignment.</td>
<td>Inquiry into Student Thinking</td>
<td>Choose one of the four students from the case study and trace their responses and learning through the study. Summarize what you think they knew or understood at the end of the study that they did not know or understand in the beginning. Choose 2 examples of this student’s responses as evidence for your claim from part a) and explain how they support your claim about this student’s learning. (You will probably want one example toward the end of the study and one example toward the beginning). What does each student know, think and understand about number, operations and problem solving?</td>
</tr>
<tr>
<td>What type of tasks/problems do PSTs select and pose after assessing children’s mathematical understanding?</td>
<td>The PSTs will initially select tasks that are not of high cognitive demand but over time will select tasks based on what students understand and/or misunderstand and are of high cognitive demand.</td>
<td>Tutoring assignment</td>
<td>How will what you learned in the interviews influence how you work with the children during the next sessions? If you were to teach the next lesson to this group of students, write one problem that you could give them and explain why you think this would be a good problem for this group</td>
</tr>
<tr>
<td>To what extent is the rationale of the PSTs’ next instructional activities based on children’s mathematical understanding?</td>
<td>Only a few of the explanation and reasoning will be based on children’s mathematical understanding</td>
<td>Inquiry into Student Thinking</td>
<td>How are these tasks intended to build on what you know about your students’ understanding and/or misunderstanding?</td>
</tr>
<tr>
<td>What are PSTs’ conceptions of a productive task and/or tasks that engage students with high or low level thinking?</td>
<td></td>
<td>Tutoring assignment</td>
<td>Choose 2 tasks or problems that were particularly posed to the students that seemed particularly productive for advancing the thinking of the student’s as a group. What was your plan for this week? Describe the activities, problems, literature etc that you planned to use during this week’s tutoring session and explain your rationale for this plan. Included in your rationale should be: What makes these tasks high cognitive demand for your students?</td>
</tr>
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</table>
et al., 2003; Lester, 2007) has also shown that, although children’s strategies are complex, the details given often provide a more nuanced understanding on what the children understand and/or misunderstand. In this study, my conceptualization of the component skill of attending to children’s strategies is similar to Jacobs et al. (2010), since I focus on the extent to which PSTs develop the component skills of professional noticing of children’s mathematical thinking in the context of scaffolded activities as they progress with their methods course. To examine participants’ ability to attend to children’s strategies, I used the prompts outlined next, given in the Inquiry into Student Thinking assignment and the tutoring assignment, respectively.

**Inquiry into Student Thinking assignment**

Choose one of the four students from the case study and trace their responses and learning through the study:

1. Summarize what you think they knew or understood at the end of the study that they did not know or understand in the beginning

2. Choose 2 examples of this student’s responses as evidence for your claim from part a) and explain how they support your claim about this student’s learning. (You will probably want one example toward the end of the study and one example toward the beginning)

In this assignment, the PSTs chose one student and tracked their progress in learning from beginning of the case study to the end. That means, PSTs selected one child, described and interpreted what the child understood and/or misunderstood and summarized their interpretations on what the children knew or understood at the end of the study that they did not know at the beginning of the study. Specifically, I focused on prompt b) where the PSTs chose two examples of one student’s responses to support their explanations and claims related to students learning.
In the tutoring assignment, I used the prompt:

*What does each student know, think and understand about number, operations and problem solving?*

Coding responses for the component skill of attending to children’s strategies was done in two stages. For each of the responses in the two assignments (Inquiry into Student Thinking and tutoring assignment), I identified the extent to which the PSTs provided mathematical details while attending to children’s strategies. For example, describing the strategy used with details, providing and describing details of how the child solved a specific task using the strategy was considered as mathematical details. Additionally, comparing two different strategies that the children used was also considered as mathematical details. The responses for both the Inquiry into Student Thinking and tutoring assignment were given one of the three codes: most mathematical details, some mathematical details and lack of mathematical details. The responses that had details that were not mathematical were coded as lack of mathematical details. In the following paragraphs, I provide examples that were considered to have most mathematical details, some mathematical details and lack of any mathematical details.

For attending to children’s strategies, a response demonstrating evidence of “*most mathematical details*” had the PST providing detailed explanation of the strategy that the child used such as how the child counted up, broke the numbers apart, direct modeling using tallies or any other manipulative[s]. In addition, for a response to be coded as most mathematical details, the PST ought to have provided details of the mathematical essence of

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7 The PSTs used more than one task to attend to children’s strategies, interpret children’s mathematical understanding and respond based on children’s mathematical understanding, but I chose to code only one task that corresponds with their choice to respond.
the strategy and details of how the children made sense and/or interacted with the mathematical idea. For example the following responses were coded as responses with most mathematical detail:

The next task was to determine whether number sentence were true or false. When given the number sentence 8+6 = 10+5, Matthew was able to determine that it was false by breaking the six into 2+ 4 and using that 2 to make a 10 (2+ 8 = 10 on the left side) leaving 4 on the left side. He then noticed that there was a ten on each side of the equal sign and was able to compare the four on the left to the five on the right. Since four doesn’t equal to five he knew the number sentence was false.

Jessica’s equation work did not challenge her at all. Because she solved them effortlessly and was able to fully explain her thinking, I gave her the 4th grade problems. For this section, both Jessica and Patty used the breaking apart method and succeeded when using addition. For the problem 249 + 367 = 247 + 369, both girls failed to look at the numbers and see that 247 is two less than but 369 is two more than 367 and the answers would be the same. Instead, they broke the numbers apart by place value 200+300 = 500, 40 + 60 = 100 and 7+9 = 16

The next equation (583-265 = 593–275) was a little bit more difficult for her to solve. She spent a lot of time working this problem out so I asked her questions like “How are you thinking about this problem?” and “Is this one a little harder for you?” but she completely ignored me. She ended up deciding that the equation was true after she figured out that each side of the equation is equal to 318. To find the left side, she took 583- 200 to get 383. Then she got 383 – 60 to get 323. Next she took 325 -5 to get 318. In this case she broke the number 265 apart into 200+60+5 in order to make the subtraction problem easier. She used the same method to solve the right side of the equation. She broke the number into one hundreds, tens and ones. She noticed that the equation is true after all the hard work.

In these responses, the PSTs’ explanations provided details of how the children decomposed the numbers to make them easier to manipulate, how the children added or

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8 The first iteration in data analysis revealed that there were no instances where the participants used incorrect mathematical details to describe a child’s strategy. In response where there were incorrect additions or subtraction of numbers, it was not possible to determine whether the child had made the mistake or it was the PST who had described the response with incorrect mathematical details. In addition, responses that provided details that were not mathematical were coded under lack of mathematical details.
subtracted the numbers and, details of how the children made sense of the mathematical essence of the strategy. In fact, the second participant also provided details of the alternative strategy of using relational thinking that she anticipated the students would use. Therefore, I used the code “most mathematical details” for responses where the PSTs’ responses demonstrated detailed evidence of attention to children’s strategies and described using details how the children used the strategy with substantial details about the mathematically important aspect of that strategy.

I used the code “some mathematical details” for responses where the PSTs only provided some details about the strategy but did not provide substantial mathematical details on how the child solved any specific task or provided a general description with no specific details on how the child solved a specific task. In addition, I coded a response as “some mathematical details” in responses that only had some details on the mathematical essence of the strategy and some details of how the children made sense and/or interacted with the mathematical idea:

Jack did not have a solid understanding of using ten as a unit. He always had to solve a problem counting up, using tally marks to help him count. He constantly used individual ones; however by the end of the study he eliminated a lot of his counting by ones for units of tens. An example of this is from February 22nd at the beginning of the study, “For example, if the problem is something like 20 pennies and how many more pennies to have 45… Jack solved this problem handily although his strategy made no use of tens. He counted up by ones from 22 using tallies to keep track.

Emilio began to understand the meaning and use of 10’s as a unit. In the beginning of the sessions, he solved most of his problems by counting on (sometimes erroneously). This is a good method for students to use to start out with, but for larger number problems a better system is more efficient. Session 7 is really where Emilio’s understanding progressed. After simplifying the problem, Emilio understood that ten 1’s could make one unit of 10! This made solving problems in the future much easier for him.
Oliver was able to see that he just needed to add these two numbers together in order to find the correct answer. He stacked the two numbers vertically on top of each other. He then added the ones and bundled them if needed then added the tens. He was able to find the correct answers every time...I asked him to explain how he saw the problem 89+62. He said I know there are 11 tens so I must bundle 10 and add it to the tens column...

Notice, in the first response the PST identified the strategy (*always had to solve a problem counting up using tally marks*) and described that the child (*constantly used individual ones*) but did not provide substantial details on the mathematical essence of the strategy. In addition, the participant did not provide substantial details on how the child made sense and /or interacted with the mathematical idea. Similarly, the second participant stated the strategy used (*counting up by ones*) but did not provide substantial mathematical details on how the child made sense and interacted with the mathematical idea using the strategy.

I used the code *lack of mathematical details* in responses where the PSTs did not identify the strategy and did not provide any mathematical details on how the children or child solved a specific task. Consider the following response:

We only worked with the first set of numbers that were 4 and 5. I reminded them that they did not have to do all the problems in their heads and if they wanted to use their papers or manipulative[s] that it is perfectly fine. So, Alesha decided to try the cube to help her solve the problem. We went over the problem reading it a few times and making sure that they understood what it was asking. Neither of the children was able to come up with the answer on their own this time and so I decided to use some questioning to see if with a little help they could come up with an answer.

I chose to follow Emilio through this case study. Emilio seemed to be the student that was struggling the most with the base-ten concept. At the beginning of the study Emilio seemed completely off track with the problems that the instructor presented the students. She wanted the students to use direct modeling to help them solve problems, but Emilio did not understand that idea either. There were certain times that Emilio would surprise me, because he would know a fact off the top of his head. To me, that was an indication that maybe he knew more than he was leading on.
In the first response, the PST did not describe how the child did the specific task. Instead, she described what she did to help the child unpack the problem. Therefore, the response did not provide details on any strategy and did not provide any mathematical details explaining how the child made sense of the mathematical task. In the second response, the PST identified that Emilio struggled the most in understanding base 10. She also asserted that Emilio was completely off track at the beginning of the case study, but did not identify how Emilio did the task assigned to them. Therefore, the response was coded to have no mathematical details. Table 3.3 represents examples of responses that were coded under each category, both in the Inquiry into Student Thinking and the tutoring assignments. I used the coding scheme to examine PSTs’ ability to attend to children’s strategies.

**Interpreting children’s mathematical understanding**

Research by Sherin and Van Es (2005, 2008), Jacobs and Ambrose (2008), and Jacobs et al (2010) have largely described the component skill of interpreting children’s mathematical understanding. Specifically, Sherin and Van Es (2005) identified three stances (descriptive, evaluative and interpretive) that teachers used as they reported what they noticed. The descriptive stance was taken to be situations where the teachers, specifically, the PSTs involved in the VAST study described each event in the video while the evaluative stance was used to describe situations when the teachers focused on what worked and what they might want to do differently. Similarly, the interpretive stance was taken to be instances when the teachers focused more on interpreting what occurred than simply describing and evaluating what had happened in teaching and learning. In addition,
Table 3.3. Examples of responses coded under each category

<table>
<thead>
<tr>
<th>Code</th>
<th>Inquiry into Student Thinking</th>
<th>Tutoring Assignment</th>
</tr>
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<tbody>
<tr>
<td>Most mathematical details</td>
<td>He solved it with a direct modeling procedure and drew each individual soccer ball in the designated three bags. Emilio worked on the problem: Dr. E has 4 rolls of candy and 11 loose candies. How many candies does she have altogether? He initially spat out the number 40 and explained on his own that there were candies in each of the four rolls. He had trouble counting up from 40 to 51 for the 11 single candies, but this is an issue he had the first day as well and shows that may be another issue. However, because he knew to count up from 40 by 1 single candies shows that he is able to distinguish groups of 51n from single units which is very significant in using base ten problem-solving strategies.</td>
<td>On the JRU problems, Tyler would use a breaking-the-number-apart strategy. He would like to get the numbers into base 10 so that they would be easier to add together. For example, on the first set of numbers (42, 36) for the apples problem, Tyler told me that the answer was 78. When I asked how he knew that he wrote out that 42+30=72 and then wrote 72+6=78. I was really excited that he knew a shortcut for how to do the problem. He also used this same strategy for the SRU problems.</td>
</tr>
<tr>
<td>Some mathematical details</td>
<td>Jack throughout the case study counted up by ones to find his answer. From the very first day, Jack miscounted the total number of soccer balls because he had the wrong number of soccer balls in one bag, even though all of the bags had simply 10 balls in each. In his first few sessions, Jack tended to write tall marks to keep track of whatever he was counting, no matter how big the number was. Sometimes, because he was counting one by one, he would mess up and that would affect his final answer. For Jack, counting up worked, but it was not the goal nor would that method be very effective in the future.</td>
<td>Apple problem: The student started with the original amount of blocks (3) then found the number of picked apples (12). After this the student started counting the blocks starting with 3, counting up to 12 on, starting with 3, 4, 5, 6, 7, 8, 9, 10… This led me to doing a problem that would include counting since my goal for the lesson was to get my student to be able to count in sequence starting from a given number in the known sequence.</td>
</tr>
<tr>
<td>Lack of any mathematical details</td>
<td>In the beginning, Jack did not recognize ten as a numerical unit. It seemed that, to him 10 was no different than 4 or 9. Because of this, he often counted up to the answer. As the study went on, he began to develop an understanding of ten, first by using a representation of 10 (rather than tally marks or other such one-to-one representations) in session 5 and later by solving number sentences by counting tens rather than counting up by ones( seen in session 9, but also hinted at from session 5 on.</td>
<td>Second, any straightforward problem (e.g., 23+57=__) was not difficult for them. It did not seem to matter whether a task was JSU, SCU, SIU or SRU; those sort of problems were simply too easy for these three students unless the numbers were sufficiently large enough to require them to use paper just to keep track of their carrying…</td>
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</table>

Van Es and Sherin (2008) described this component skill as the ability to use knowledge about the context to reason about classroom interaction. Jacobs et al. (2010) conceptualized this component skill as “the extent to which the teachers reasoning is consistent with both the specific child’s strategy and research on children’s mathematical development” (p. 172).

Due to the nature of the prompts given in the two assignments, the first glimpse of data analysis revealed that most PSTs took an interpretive stance rather than descriptive or
evaluative stance. Therefore, I decided to code the extent which the PSTs’ interpretation of children’s mathematical understanding were consistent with the details of the specific children’s strategies.

Specifically, I used the codes robust evidence, limited evidence, and lack of evidence to examine PSTs’ responses. I coded the response as having robust evidence if the PST made sense of details of children’s strategy and noted how the details reflected children’s understanding and/or misunderstanding. In other words the PST made sense of the strategy details in a variety of ways but all the interpretation were consistent with the strategy details:

I feel that by the end of this case study Emilio understood how to work with groups of ten, and he did not just count on from the first number given. He could also use those groups of ten to grasp that he could add on the single numbers not in tens (for example the 2 soccer balls left out of the bags). During their first meeting, Emilio was asked a question about soccer balls. The question was if you have three bags with ten soccer balls in each bag and two left over, how many balls you have altogether. For this question Emilio did not think in the terms of 10 + 10 +10 + 2. He instead drew pictures of the three bags, and then drew each individual ball, and he counted all of them to get his answer. After the problem was reread to him he did say that he knew 10 + 10 is 20, but there was never anything linking that to the problem he was working on. He never went up to thirty and the 2 extra balls were confusing. Then during the eighth session Emilio appears to understand that groups of ten can be used to solve different problems. In this session he could solve that four rolls of ten candies was 40. He didn’t have to draw a picture because he was thinking as it in terms of groups of ten. He could also add on from those forty. Emilio was asked what he should do with the 11 extra and he responded with the answer would be 52, and that he got that answer by adding that eleven to the forty we already had. These shows a lot of improvement made from the first session when Emilio had to count everyone as an individual.

After going through the first two pages, I was anxious to see how Tyler’s story problem solving abilities would be. On the JRU problems, Tyler would use a breaking the number apart strategy. He would like to get numbers into base 10 so that they would be easier to add together. For example, on the first set of numbers (42,36) for the apples problem, Tyler told me that the answer was 78. When I asked how he knew that he wrote out that 42+30=72 and then wrote 72+6=78. I was really excited that he knew a shortcut for how to do the problem. He also used this same strategy for the SRU problems. For the first set of numbers (87,20) he knew that 6+2=8 so 8-2=6. He figured that it would
work the same with 80 and 20. He subtracted 20 from 80 to get 60 and then added the 7 onto his final answer. Tyler really has demonstrated that he understands base 10 and that he can break apart numbers to add and subtract.

In the first quotation, the PST made an assertion that by the end of this case study Emilio understood how to work with groups of ten. The PST also provided in-depth interpretation of what Emilio understood at the beginning of the study.

*(During their first meeting, Emilio was asked a question about soccer balls. The question was if you have three bags with ten soccer balls in each bag and two left over, how many balls you have altogether. For this question Emilio did not think in the terms of 10 + 10 +10 + 2. He instead drew pictures of the three bags, and then drew each individual ball, and he counted all of them to get his answer) and, compared with what Emilio knew and understood at the end of the case study (Then during the eighth session Emilio appears to understand that groups of ten can be used to solve different problems. In this session he could solve that four rolls of ten candies was 40. He didn’t have to draw a picture because he was thinking as it in terms of groups of ten. He could also add on from those forty). Therefore, I coded the response to have robust evidence because the PST made sense of Emilio’s strategy and the interpretation was consistent with the strategy details.*

Similarly, in the second quotation, the PST provided details of Tyler’s strategy *(Tyler would use a breaking the number apart strategy. He would like to get numbers into base 10 so that they would be easier to add together)* and provided details that were consistent with the child’s strategy *(For example, on the first set of numbers (42,36) for the apples problem, Tyler told me that the answer was 78. When I asked how he knew that he wrote out that 42+30=72 and then wrote 72+6=78. I was really excited that he knew a shortcut for how to*
He also used this same strategy for the separate result unknown (SRU) problems. For the first set of numbers (87,20) he knew that 6+2=8 so 8-2=6. He figured that it would work the same with 80 and 20. He subtracted 20 from 80 to get 60 and then added the 7 onto his final answer. Therefore, I coded the response to have robust evidence that the PST interpreted children’s mathematical understanding.

Likewise, I assigned the code-*limited evidence* to responses that focused on children’s mathematical understanding and/or misunderstanding but their responses did not have in-depth interpretation like the robust evidence. In these responses, the participants interpreted children’s mathematical understanding but with broader undefined terms. Also, connections to children’s strategies were over-generalized and often did not provide specific evidence on how the children did the task. For example, the following are sample responses that provided limited evidence:

Mathew is great at base 10 concepts and understanding the basics behind problems. He was able to solve almost all these problems by relating back to what he knew which usually included some foundational decade. The only problem he got wrong was the filling the blank that he admitted to not understanding what he was supposed to do.

Jack’s response for the soccer ball problem was 28 because he subtracted two instead of adding two to 30. This was an incorrect response due to poor difficult wording of the problem. Jack solved the penny problems by counting up from 22 by ones. This is an ineffective method to solving the problem because it is time consuming. Jack solved the candy problem in the last session by identifying each roll as ten and then as 1 item. This shows his understanding for base ten.

From the first session, Jack appeared to be the most knowledgeable about using tens than any of the other children though this wasn’t exactly saying much. He seemed like he rarely would put the concept of using tens to use because in the beginning he often just counted up. The main strategy he tends to use in the early problems was using the direct modeling method. While he seemed to be getting the right answer most of the time, he wasn’t grasping the concept they were trying to teach, which is the base ten concept. At the half way point, Jack had somewhat of a break through showing that he knew how
to use the base ten and count by tens. As the sessions got closer to the end Jack started going back to using his old direct modeling method by just counting up. At the beginning of the sessions, I believe Jack knew or understood how to use direct modeling method by counting up by ones. He seemed to be one of the stronger students of the four as far as mathematical abilities because he was usually getting the correct answer just not solving the problem the way the teacher was looking for. By the end of the study, I believe he understood how to use base ten the strongest out of any of them.

Notice that, in the first response, the participants made generalization and assertions on the child’s understanding, e.g., “Mathew is great at base 10” but did not provide any specific evidence that shows that Mathew understands base 10. In addition, the participant made an assertion that Mathew was able to solve all the problems by “relating back to what he knew” but the participant did not identify what he knew. In the second example, the participant identified the strategy that Jack used (direct modeling). The participants also made an assertion that, although “Jack seemed to be getting the right answer most of the time, he wasn’t grasping the concept they were trying to teach, which is the base ten concept” but did not provide any evidence using the child’s strategy that he did not understand. At the half way point, Jack had somewhat of a break through showing that he knew how to use the base ten and count by tens” but again did not explain or describe how Jack did the tasks.

Finally, in this category, I used the code lack of any evidence to code the responses that did not provide any evidence of interpreting children’s mathematical understanding and/or misunderstanding even though they had been prompted to do so. For example one participant commented: “Overall I was surprised by how much my students know along with different strategies to solve the problems. I was also surprised about how the students knew how to solve the CGI but when I presented them with the true or false and open number equations, they struggled”. Instead of focusing on the specific strategies that the child used,
the PST focused on the teacher. One could infer from the explanation that the child used strategies to solve the task, but the participant noticed but did not interpret their understanding or misunderstanding. Table 3.4 provides examples of responses that were coded under the specific categories described previously.

Table 3.4. Examples of responses coded under each category

<table>
<thead>
<tr>
<th>Code</th>
<th>Inquiry into Student Thinking – Interpretation</th>
<th>Tutoring Assignment – Interpretation</th>
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<tbody>
<tr>
<td>Robust evidence</td>
<td>I believe that Jack, by the end of the study, had a much better understanding of how to use his knowledge of base ten in solving problems.</td>
<td>Gary understands how to count by 10’s as long as the number is zero. For example, Gary could count, 60, 70, 80, but he could not count 63, 73, 83, …</td>
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<td></td>
<td>One of the problems was this: (JCU) 22 pennies, how many more to have 50. Jack solved this problem by counting up by ones from 22 using tallies to keep track, which solves the problem but shows evidence that he does not fully understand how to use his base tens knowledge to help solve problems.</td>
<td>Brian had a good understanding of base 10 and basic problems. He was able to count very well and almost never stumbles when switching decades, e.g., 97, 98, 99, 100, 101… He also demonstrated that he was capable of counting by 10’s both forward and backward. I was especially appy to see him easily counting backwards, 204, 195, 184, 174, etc. Brian also understand the use of the equal sign …</td>
</tr>
<tr>
<td></td>
<td>In session ten they did a problem that was as follows: 30 pencils, 29 more. This is a somewhat similar problem from the one in example 1. For both he needed to count up by about 30 to get the answer, but this time to solve the problem he drew a picture that represented groups of tens and then ones. This time he did use his knowledge of base ten to help make this problem easier to solve.</td>
<td>I think both Calvin and Karl seemed to have a good understanding of numbers when counting forward and backward by rote memory. They were able to answer all the questions with ease. I even tried using some of the second-grade questions and they were able to answer them without even thinking.</td>
</tr>
<tr>
<td>Limited evidence</td>
<td>10 have now become a unit for Jack instead of just the 1’s unit. His thinking for base 10 is fragile though, and he will need more practice. For the last problem, he couldn’t decide between if 45 beads could make 4 or 5 necklaces.</td>
<td>Overall, I was surprised by how much my students know along different strategies to solve the problems. I was also surprised how the students knew how to solve the CGI but, when I presented them with the true or false and open-number questions, they stuggles</td>
</tr>
<tr>
<td>Lack of any evidence</td>
<td>Jack didn’t have an understanding of base ten at the beginning of the case study but, by the end, he had a concrete understanding of the base ten process. At first Jack got confused with the terminology of loose and thought that he should subtract the balls instead of adding the balls together and he misunderstood the problem type. By the end of the case study, Jack had a better understanding of how to decode problems more properly.</td>
<td></td>
</tr>
</tbody>
</table>
Responding based on children’s mathematical thinking

As discussed in Chapter 2, prior research (e.g., Jacobs & Ambrose, 2008; Jacobs et al., 2010) has identified different ways that teachers can build on children’s mathematical understanding and/or misunderstanding. For this study, I considered the teacher move that involves the teacher generating follow up problems or selecting the next problems based on children’s mathematical understanding and/or misunderstanding. Specifically, I focused on the nature of tasks that the participants generated or selected after as a response to children’s mathematical understanding. To analyze whether or not the tasks were based on children’s mathematical understanding, I used their reasoning and rationale as they selected the next problem in the Inquiry into Student Thinking and tutoring assignments. Specifically in the Inquiry into Student Thinking assignment, the participants were given the following prompt: *If you were to teach the next lesson to this group of students, write one problem that you could give them and explain why you think this would be a good problem for this group.*

Jacobs et al. (2010) also described the skill to respond as the reasoning that teachers use when deciding how to respond. Specifically, Jacobs and colleagues examined the extent to which teachers use what they have learned about children’s understanding from the specific situation and whether their reasoning is consistent with the research on children’s mathematical development. Further, Jacob and colleagues focused on the in-the-moment decision that the teachers need to respond to children’s verbal or written work in the classroom. Jacobs and colleagues argued that this response requires the three component skills: *attending to children’s strategies, interpreting children’s mathematical thinking and responding based on children’s mathematical understanding.*
Similar to Jacobs and the teams’ study, I coded the PSTs’ explanations and rationale of their intended choice of next task in the Inquiry into Student Thinking assignment to either have robust evidence, limited evidence or no evidence that the choice was based on what the PSTs had interpreted as children’s mathematical understanding and/or misunderstanding.

Likewise, I coded the learning goals, explanations and the rationale of selecting the specific tasks in the tutoring assignment after PSTs conducted the initial interview. The responses were coded either to have robust evidence, limited evidence or lack of any evidence of their interpretation of children’s mathematical understanding.

Specifically, I coded a response to have robust evidence if there was evidence that the choice of task was based on children’s mathematical understanding and/or misunderstanding. In other words, the participant considered children’s understanding as reflected in particular strategy used, and how the next task could further the specific children’s understanding. In some cases, the participants anticipated the alternative strategies that the children could use and how the task would build on existing children’s understanding:

**Problem:** I would create Joint-Result-Unknown (JRU) or Separate-Result-Unknown (SRU) problems for the students. A JRU example would be “Sunny has ____ fish, and then she buys ____ more. How many fish does she have now?” Number choices would include {(10, 50) (20, 30) (10, 41) (15, 25)}.

**Rationale:** This type of problem would be good for all of the students. Sunny and Daniella struggle to count by tens past the numbers 20 and 30, and this problem challenges them to do so. Emilio would be challenged to count by tens and keep track of the ‘one’ in 41. Both Jack and Emilio would be challenged by the last number choice, as both understand the concept of counting by tens, but they would have to extend their understanding to non-zero ending numbers.

**Problem:** Student 1 (use their names in the real setting) has ____ race cars. Student 2 gave student 1-____ more racecars. Have many racecars does student 1 have in total?
Rationale and number choices
Student 1 adding to 10_ (5, 5) (7, 3) (4, 6) (15, 5)
Student 2 adding to 100_ (50, 50) (80, 20) (35, 65) (42, 58)

This problem type (Joint Result Unknown) was chosen because both students have a firm grasp of solving this style of a problem in the initial interview. So since they understand how to solve the problem it will help them focus on the task of adding numbers together to form either 10 or 100.

(5,5) These numbers were chosen because doubles are one of the first thing that this students learn, so this problem is accessible to the students and will help the student become familiar with the problem, while still adding up to 10

(7, 3) These numbers were chosen because the student can count up from 7 to 10

(4,6) This set was chosen because it is a larger distance to count up from 4 to 10, and so it's less accessible to the students and therefore forces the students to think more about their prior knowledge

(15,5) This reason for this number set is that it increases the students thinking by making the answer higher than 10 but still applying previous knowledge such as the fact that 5+5 = 10

(50,50) This number set was chosen because it is a higher level of thinking because it adds to a century but it is still a double, which is much easier solved than other number sets.

(80, 20) I chose these numbers because they are adding together hold decades, which is harder than doubles, but easier than adding non-decades.

In the first response, the participant made an assertion that the problem would be good for all students. Further, s/he explained what each child understood and how the selected task would extend specific children’s mathematical understanding. Notable is how the participant took careful consideration into specific students understanding and/or misunderstanding (e.g., Sunny and Daniella still struggle to count by 10, while Emilio and Jack understand how to count by 10 but would need to be challenged with non-zero ending numbers). Similarly, the second response focused on what the children understood and misunderstood. The numbers were specifically chosen to extend the two children’s mathematical understanding and/or misunderstanding.
For *limited evidence*, I used the responses where the participants referenced children’s mathematical understanding and/or misunderstanding but did it in a general way. These responses were also characterized by presumptions that all the children had similar understanding or misunderstanding and therefore need the same next problem to extend their thinking. Below is a sample response that was coded as having limited evidence that it was based on children’s understanding or misunderstanding:

I decided to focus on counting forwards and backwards by ones fives and tens because the students’ struggled a little bit with this when I first interviewed them. I also focused on the true/false and open number sentences because the students seemed to be really confused on how to solve these problems. I will present the students with set of word problems that we will explore together. These number choices will help the student’s progress from counting 1 by 1 to using base 10 strategies to solve problems. The students will start with decade numbers and progress to non-decade number for this JRU problem.

Notice that the participant indicates that the students struggled with counting forward during the interview but did not focus on what specific students struggled with. The participant also decided to pose true or false tasks and open number sentences because the students seemed confused but did not identify what confused them and how the specific task would clarify the confusion. Although the participant chose a problem and number choices that would help the students’ progress from counting by 1 to using base 10, but there was no evidence in the response that this was building on what the children understood and/or misunderstood.

Other responses coded to have limited evidence that the response was based on children mathematical understanding and/or misunderstanding were based on the fact that the children had done familiar problems, or the children already know how to solve the task, but did not identify what the children know. Below are sample responses where the participants
did not consider students understanding and/or misunderstanding. The following is a sample response that I coded to have a limited evidence of children’s mathematical understanding and/or misunderstanding:

I chose this problem because I thought the student would be very familiar with it and would understand that it was just like the opening number routine sentences.

These problems are designed to make the children think about the facts that they already know and apply these strategies when they solve word problems.

Similarly, I used the code *lack of evidence* for responses that provided no evidence of responding based on children’s mathematical understanding. These responses did not reference prior children’s mathematical understanding and/ or misunderstanding or how the task would extend children’s mathematical understanding. In some cases, the responses were characterized by participants who wanted to give children harder tasks, re-evaluate children’s mathematical understanding or continue practicing what they had already learned.

Sometimes the participant did not provide any rationale. Below are some responses that I coded as having no evidence that they were based on children’s mathematical understanding:

**Problem**: 40+60 = -+-

**Rationale**: I know it is not a story problem but I think it would be a good way for me as the teacher to see where they were at, what they are ready to learn, and see what previous strategies they used correctly and what new strategies they came up with.

**Problem**: Emilio had 64 soccer cards. Sunny had 28 soccer cards. How many more soccer cards does Emilio have than sunny?

**Rationale**: I think for the next lesson, I would work with subtraction as one of the students brought it up in the last session. I think subtraction would work good because it still includes the base 10 strategies.

**Problem**: For the main activity the student will do a story problem:
Ben has ___ basketball and his parent’s give him ___ more. Jack has ___ basketballs and his sister gives ___ more. How many basketballs does Ben have in comparison to Jack?
A (15,26) B (58,64) C (27,13) D (18,6)  
(28,15) (62,58) (25,15) (14,10)

**Rationale:** This problem is high cognitive demand because it takes the student beyond just adding both sides. I want the student to notice patterns with the numbers. One pattern from the numbers I chose would be that one number is the same on both sides but the other one is different. Another pattern that the student could notice is the difference between the numbers instead of adding up both sides. I chose this problem because I thought the student would be very familiar with it and would understand that it was just like the opening number routine sentences.

In these responses, the participants did not consider the children’s understanding and/or misunderstanding. They chose good tasks but they were based on reassessing children’s mathematical understanding (1st response), what the participant wanted to work on (2nd response) and, the strategies that the participant wanted the children to use (3rd response).

Table 3.5 provides a summary of the coding scheme for the component skill of responding based on children’s mathematical understanding and misunderstanding.

One difference between this study, and Jacobs and colleagues work is that Jacob's work focused on the in-the-moment intended response. This study extends this work by examining the in-the-moment intended response in the Inquiry into Student Thinking where the PSTs select or generate a task that they would pose to the children based on what they learned about their understanding and/or misunderstanding. The study also examines how PSTs would respond when asked to analyze their own teaching and respond to children’s mathematical thinking as they plan for a series of instructional activities, or the “long term decision making” response (Jacobs et al., 2010, p. 173).

After coding the PSTs’ responses for each component skill, I quantified the data in order to foster more meaningful comparisons and allow patterns to be identified and further
Table 3.5. Examples of responses coded under each category

<table>
<thead>
<tr>
<th>Code</th>
<th>Inquiry into Student Thinking-selected task</th>
<th>Rationale</th>
<th>Tutoring assignment-selected task</th>
<th>Rationale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robust evidence</td>
<td>I would create Joint-Result-Unknown (JRU) or Separate-Result-Unknown (SRU) problems for the students. A JRU example would be “Sunny has ____ fish, and then she buys ____ more. How many fish does she have now?” Number choices would include {(10, 50) (), (20, 30) (), (10,41) (), (15, 25) ().</td>
<td>This type of problem would be good for all of the students. Sunny and Daniella struggle to count by tens past the numbers 20 and 30, and this problem challenges them to do so. Emilio would be challenged to count by tens and keep track of the ‘one’ in 41. Both Jack and Emilio would be challenged by the last number choice, as both understand the concept of counting by tens, but they would have to extend their understanding to non-zero ending numbers.</td>
<td>Conor has ____Wii games in his cupboard. He found ____ more Wii games under his bed. How many Wii games does Conor have. Number Choices: {(7, 3) (), (4,6) (). James has ____Wii games at his house. Conor let James borrow ____ more Wii games. How many Wii games does James have at his house now? Number choices: {(5,5) (), (2,8) ().</td>
<td>After my initial interview with the students, I knew they did not have a clear understanding on how to count on from a number other than one. When I gave each student that question in the interview none could count on from the number I had given them... I chose to do a joint result unknown story problem because I wanted the problems to begin with a number other than 1.</td>
</tr>
<tr>
<td>Limited evidence</td>
<td>If Quinn had 89 pieces of pizza, and 10 pieces of pizza made a whole pizza, how many whole pizzas can Quinn make?</td>
<td>This is a Separate Result Unknown problem. I chose 89 because the students have the concept of base 10 down; they are able to do it with the easy numbers, now I want to challenge them with bigger numbers, hoping they would use the manipulative[s] and not their fingers. I would hope students could lay out the manipulative[s] and see easily that they can make 8 pizzas. If students understand this concept they should have no problem with this problem.</td>
<td>The student will be given visual balance with numbers in blocks. One block on the right side will be blank. My number choices are 6 and 2 on the left sides and a blank and 4 on the right side.</td>
<td>I plan to work on commutative property and relational thinking to help with those problems. Latter I plan to focus on his subtraction skills so that he will be willing to use them in other problems... The purpose of this exercise was to have Mathew begin thinking in terms of something balancing or equaling something else in a horizontal format. The balance scale is meant to be a visual tool to eventually lead to understanding of number sentences.</td>
</tr>
<tr>
<td>Lack of any evidence</td>
<td>If I were to teach the next lesson to these students, one problem I could give them would be a: Jack has 45 crackers. Sunny gives him 10 more. How many crackers does Jack have? I choose this Separate Result Unknown problem because I wanted the students to continue using addition. These are the problems they have been used to and need to keep getting trying to understand. I chose the numbers 45 and 10 because the students need to continue using large numbers so they can’t just count by ones and learn to use going by 5’s or 10’s as a first choice.</td>
<td>The student will be presented with these problems one at a time and they determine whether the problem is true or not. 4+2 = 6 3+3= 6 4+2 = 3+3</td>
<td>These equations allow the student to look at the two different equations and see that although the numbers are different they equal the same thing.</td>
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</tbody>
</table>
explored (Chi, 1997; Miles & Huberman, 1994). I assigned a value of two to responses that were coded to have most mathematical details, a value of one to responses that were coded to have some mathematical details and zero to responses that were coded to have no mathematical details for the component skill of attending to children’s strategies. Similarly, I assigned a value of two to any response with robust evidence, a value of one to any response with limited evidence and a value of zero to any response with lack of any evidence, both for the component skill of interpreting and responding based on children’s mathematical understanding. Table 3.6 presents a summary of the quantified codes. Further, I conducted two-tailed paired t-tests in order to compare the PSTs’ performance in the Inquiry into Student Thinking and tutoring assignments. I used the two-paired t-tests to describe any changes in PSTs’ performance in the component skills over time.

Table 3.6. Scoring rubric for assessing skills of professional awareness of children’s mathematical thinking

<table>
<thead>
<tr>
<th>Component skill</th>
<th>Code</th>
<th>Sub-code</th>
<th>Score</th>
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</thead>
<tbody>
<tr>
<td>Attending to children’s strategies</td>
<td>Considers the extent to which PSTs’ explanation demonstrated evidence of providing mathematical details</td>
<td>Most mathematical details</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Some mathematical details</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Lack of mathematical details</td>
<td>0</td>
</tr>
<tr>
<td>Interpreting children’s mathematical understanding</td>
<td>Considers the extent to which PSTs’ explanations demonstrated evidence that the interpretation was based on children’s mathematical understanding (e.g., explanations being consistent with children’s strategies for them to conclude that they understand or do not understand).</td>
<td>Robust evidence</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Limited evidence</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Lack of evidence</td>
<td>0</td>
</tr>
<tr>
<td>Responding based on children’s mathematical understanding</td>
<td>Considers the extent that PSTs’ rationale demonstrated evidence that it was based on children’s mathematical understanding</td>
<td>Robust evidence</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Limited evidence</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Lack of evidence</td>
<td>0</td>
</tr>
</tbody>
</table>
In the second stage of data analysis, I answered the last two research sub-questions:

c. What type of tasks/problems do PSTs pose after assessing children’s mathematical understanding?

d. What are the PSTs' conceptions of a productive task and/or the levels of cognitive demand?

To examine the nature of tasks that PSTs posed after assessing children’s mathematical understanding, I looked at the tasks that the PSTs selected after analyzing students’ work during the Inquiry into Student Thinking. Further, to examine the PSTs’ conceptions of productive tasks and the conceptions of levels of cognitive demand framework, I used PSTs’ responses on two prompts given in the Inquiry into Student Thinking assignment and the tutoring assignment, respectively:

1. Choose 2 tasks or problems that were particularly posed to the students that seemed particularly productive for advancing the thinking of the student’s as a group. For each problem, provide evidence from at least two of the four students to support your claim that these were productive tasks. Conjecture why this were productive tasks. (Inquiry into student thinking)

2. What was your plan for this week? Describe the activities, problems, literature etc. that you planned to use during this week’s tutoring session and explain your rationale for this plan. Included in your rationale should be: What makes these tasks high cognitive demand for your students? (Tutoring assignment)

In this stage of data analysis, I developed codes based on literature that has largely focused on the nature of mathematical tasks (Stein & Smith, 1998; Smith & Stein, 1998). I used the “task analysis guide” to classify tasks in terms of the levels of cognitive demands. The task analysis focused on the kind of thinking processes entailed in solving the task.

Specifically, the thinking processes that the students engage in are categorized as memorization, use of procedures and algorithms (with or without attention to concepts, understanding, or meaning) and doing mathematics. Table 3.7 expounds on the levels of cognitive demand.
Table 3.7. Characteristics of mathematical tasks at each of the four levels of cognitive demand

<table>
<thead>
<tr>
<th>Level of Cognitive Demand</th>
<th>Memorization Tasks</th>
<th>Procedures with Connection Tasks</th>
<th>Procedures Without Connection Tasks</th>
<th>Doing Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lower</strong></td>
<td>• Involves either reproducing previously learned facts, rules, formulae or definitions to memory</td>
<td>• Focus students attention on the use of procedures for the purpose of developing deeper levels of understanding mathematical concepts and ideas</td>
<td>• Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience or placement of the task</td>
<td>• Requires complex and non-algorithmic thinking (i.e. there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task example or worked out example)</td>
</tr>
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<td></td>
<td>• Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure</td>
<td>• Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas opposed to narrow algorithms that are opaque with respect to underlying concepts</td>
<td>• Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it</td>
<td>• Require students to explore and understand the nature of mathematical concept, processes or relationship</td>
</tr>
<tr>
<td></td>
<td>• Are not ambiguous—such tasks involve reproduction of previous seen materials and what is to be reproduced is clearly and directly stated.</td>
<td>• Usually are represented in multiple ways (e.g., visual diagrams, manipulative[s], symbols and problem situations) making connections among multiple representations helps to develop meaning.</td>
<td>• Have no connection to the concepts or meaning that underlie the facts, rules formulae or definition being learned or reproduced.</td>
<td>• Demand self-monitoring or self-regulations of one’s cognitive process</td>
</tr>
<tr>
<td></td>
<td>• Have no connection to the concepts or meaning that underlie the facts, rules formulae or definition being learned or reproduced.</td>
<td>• Require some degree of cognitive effort. Although general procedures maybe followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding</td>
<td></td>
<td>• Requires students to access relevant knowledge and experiences and make use of them in working through the task</td>
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<td></td>
<td></td>
<td></td>
<td>• Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions</td>
</tr>
</tbody>
</table>

Note: Adapted from Stein & Smith, 1998.

Smith et al. (2008) provided a framework (Thinking Through the Lesson Protocol [TTLP]) of developing lessons that use students’ thinking as a critical ingredient in developing students’ understanding of key mathematical disciplinary ideas. The framework is intended to promote careful and detailed planning that would help teachers anticipate what
students will do, so that they can generate questions that they would ask to promote students’
learning prior to a lesson being taught (See Smith et al., 2008 for details of the framework)⁹.
Using the framework, the teachers think through the lesson as they select and set up the
mathematical tasks, think about what they will do to support students’ exploration of the task,
and think about what they will do as they share and discuss the tasks in the classroom.
Specifically, Smith et al. argued that the cumulative experience of the teachers who have
used the TTLP framework over time suggests that the TTLP can be a useful tool in planning,
teaching and reflecting and can lead to improved teaching.

Further, Smith et al. (2008) described the act of choosing and /or developing
mathematical tasks as going beyond the act of choosing to a process of selecting and setting
mathematical tasks. Smith et al. also identified the process to include identifying the
mathematical goal of the lesson, purposefully deciding how the task will build on students’
prior knowledge, life experiences, and culture.

In the third stage of data analysis, I focused on the nature of tasks selected or
generated by the PSTs in the Inquiry into Student Thinking assignment as an intended
response after they analyzed the students’ work in the case study and, the tutoring
assignment. I coded the tasks selected or generated by the PSTs as low-level cognitive
demand or high-level cognitive demand. With low-level cognitive demand, the children are
expected to produce memorized information or perform procedures without connection to
any meaning. With high-level cognitive demand the children were expected to perform
procedures with connection to meaning or doing of mathematics.

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⁹ The participants in this study had been assigned to read Smith et al. (2008) framework as a
class reading. Therefore, they were conversant with (TTLP) framework as they did the two
assignments.
Further, I examined PSTs’ conceptions of a productive task and/or tasks that engage students with either low or high level thinking (the levels of cognitive demand) using their responses in the Inquiry into Student Thinking and the tutoring assignments respectively. To analyze the PSTs' conceptions of productive tasks, I coded each of the PSTs’ responses to the prompts that were given in the Inquiry into Student Thinking assignment:

Choose 2 tasks or problems that were particularly posed to the students that seemed particularly productive for advancing the thinking of the student’s as a group. For each problem, provide evidence from at least two of the four students to support your claim that these were productive tasks.

Using open coding, I paid close attention to PSTs’ descriptions of a productive task and, identified three themes that characterized the PSTs’ conceptions of a productive task. Specifically, I coded their descriptions of productive tasks into 3 categories namely; advanced children’s mathematical understanding, challenges children’s understanding, and tasks that are based on real life contexts and children can build connections. I also used a code “others” because there are some explanations that could not be categorized under any of the common themes that I identified. Below, I will elaborate on the three coding themes identified above.

**Advanced children’s mathematical understanding**

I coded a rationale to have a conception of advancing children’s mathematical understanding if the explanations demonstrated evidence that the task was meant to support students to advance from a concrete strategy (counting by 1’s) to a more complex strategy (like counting by 10’s). I also used the code to represent responses, which had detailed explanation on how a child would change from a simple strategy to a more complex strategy
like breaking the numbers apart. For example one PST described the following problem as a productive task and provided the rationale stated below:

**Problem:** Sunny has 94 chocolate chips. She needs 10 chocolate chips to make a cookie. How many cookies can Sunny make?

**Rationale:** During this problem set, Jack was able to understand that instead of drawing each cookie inside of each package (which gets tedious), he could just draw a package, knowing that each package represents 10 cookies. This really helped to instill the base 10 concept in Jack’s thinking… Given this problem, Emilio understood the concept of base 10 much better than he ever had before. It seemed that Emilio used base ten knowledge to count up to 40. Although he still struggled to add 11 more onto that, his thinking had greatly improved. When Jack was given this problem, he first direct modeled it and got the correct solution.

In this explanation the PST argued that the task was productive because it provided the students (Jack and Emilio) with an opportunity to start counting by 10’s instead of counting by 1’s. Therefore, I coded the PST’s conception of a productive task to be a task that advances children’s mathematical understanding.

**Challenging children’s mathematical understanding**

Similarly, using open coding, I identified the responses that described a productive task as a task that challenges students’ mathematical thinking and understanding. For example, one of the PST remarked:

**Problem:** In this set, the teacher had made true/false number statements and wanted the students to determine whether or not the statement was correct (22 +10) = 30

**Rationale:** Sunny’s response to the number sentence 22+10=30 shows that she has made progress. For the first time, Sunny used relational thinking to get her response, and did not count by ones to obtain the answer. She not only realized that 22+10=30 is incorrect, therefore making the number sentence correct, but also explained what had to be done to the problem for it to be true. She explained that since 20+10=30, the extra 2 from 20 needed to be taken away from the ten, so to balance the numbers. She decided that 22+8=30
In this response the PST identified the task as productive because the task challenged students understanding of base 10 to using relational thinking to solve the task. Notice the PSTs remark: *For the first time, Sunny used relational thinking to get her response, and did not count by ones to obtain the answer. She not only realized that 22+10=30 is incorrect, therefore making the number sentence correct, but also explained what had to be done to the problem for it to be true.* Therefore, I coded the response as a conception that a productive task challenges students’ mathematical understanding.

**Using tasks that are based on real-life contexts so children can build connections**

I identified and coded response that identified productive tasks as task that are based on real life contexts and provided students with opportunities to make connections. In this category, I identified PSTs’ responses that explained that the task is productive if the students “*had a real life object they could compare their thinking*” or “*This was the first time the students were able to “make connections to real-world problems and understand the concept of base ten numbers”*” Below is a sample response that was coded in this category.

**Problem:** The first task I think really helped with the students thinking was when they were told to think of dimes as also being 10 cents.

**Rationale:** They had a real life object they could compare their thinking to and it got them thinking about numbers grouped together. “Oh you mean a dime is the SAME AS ten cents.” This opened them up to thinking about how one object could represent a number of other objects such as the candy having a number of pieces in each box.

Notable in the above quotation is the fact that the PST’s explanation of a productive task focused on task that would give students an opportunity to interact with real life objects.
Other tasks

I coded any other response under this category. For example, some PSTs thought that a task is productive if the students are given bigger numbers that they cannot manipulate or if the teacher uses the task to evaluate the students. For example, one of the PSTs responded, I think this problem was so productive for the students because it involved larger numbers, which made it tedious and time-consuming to draw out each individual cookie. Table 3.8 provides examples of responses coded under each category.

Table 3.8. Coding scheme for PSTs’ conceptions of a productive task

<table>
<thead>
<tr>
<th>Code</th>
<th>Rationale</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advanced children’s mathematical understanding</td>
<td>This was the problem that encouraged Jack to go from drawing individual components out during direct modeling to writing 10 instead. So this problem allowed him to make this adjustment for the first time...</td>
<td>One problem that seemed to help advance all the students toward the goal of applying base 10 concepts was the problem “Sunny has 11 packages of cookies. Each package has 10 cookies in it. She also has 5 extra cookies. How many cookies does she have in all?”</td>
</tr>
<tr>
<td>Challenges children’s mathematical understanding</td>
<td>I think this task was productive because the teacher chose large numbers (11x10) in an effort to guide students away from using tallies (which worked). This task was also productive because by using a stick to represent ten “cookies” Sunny was able to see the “ten representing one” concept and successfully counted by tens...</td>
<td>Problem set #6. The teacher posed the problem of 11 packages of cookies, each with 10 cookies inside.</td>
</tr>
<tr>
<td>Task based on real life contexts</td>
<td>This was the first time the students were able to make connections to real-world problems and understand the concept of base ten numbers. They did a problem involving money, which they are used to using in real life. For the first time, Jack demonstrated understanding of base ten concepts by drawing five dimes and knew that each dime represented ten cents because he crossed out two dimes when the problem said the person spent twenty cents.</td>
<td>The first task that was posed to the students that was particularly productive for advancing the thinking of the students as a group was the dime problem in session five</td>
</tr>
<tr>
<td>Others e.g., why the instructor posed the task</td>
<td>The instructor felt that Emilio and Jack could use this as a point of reference. She stated, “Emilio and Jack both solved problems in ways today that showed me they are building this understanding.” She was especially pleased to see Jack draw one circle to represent 10 things instead of the 10 separate tally marks that he had been using previously. or This was also an interesting problem because Emilio struggled with the concept. This was a great problem to help the teacher gauge where each student was when it came to this concept.</td>
<td>The first problem that I noticed where the children started being more productive was the “One dime is 10 cents and 10 cents is one dime”</td>
</tr>
</tbody>
</table>
Next, I examined PSTs’ conceptions of a task that would engage students with either high or low level thinking (the levels of cognitive demand). To examine the PSTs’ conceptions of a task that engages students with either low or high level thinking, I used the PSTs’ responses to the prompt:

*What was your plan for this week? Describe the activities, problems, literature etc that you planned to use during this week’s tutoring session and explain your rationale for this plan. Included in your rationale should be: What makes these tasks high cognitive demand for your students?*

I scrutinized the PSTs’ descriptions, comments and rationale of classifying a task in either low or high level of cognitive demand and, identified common themes suggestive of their conceptions. I coded the data using the three themes that I identified namely: considering what the children will do as they engage with the mathematical idea, considering what the teacher will do when enacting the task and, the rationale of choosing the task. Table 3.9 summarizes the coding scheme for PSTs’ conceptions of tasks that engage students with high-level thinking.

In ascertaining what the children will do as they engage with the task the PSTs, I considered responses such as:

This problem is high cognitive demand because it takes the student beyond just adding both sides. I want the student to notice patterns with the numbers. One pattern from the numbers I chose would be that one number is the same on both sides but the other one is different. Another pattern that the student could notice is the difference between the numbers instead of adding up both sides

or

This problem is a high level thinking because the students are asked to analyze the problem and use different strategies to solve it. I believe this lesson will be challenging for students and will force them to think algebraically…
Table 3.9. Coding scheme for PSTs’ conceptions of tasks that engage students with a high level of thinking

<table>
<thead>
<tr>
<th>Category</th>
<th>Examples</th>
</tr>
</thead>
</table>
| Considers what the students will do as they engage with the mathematical idea and the cognitive effort required to engage with the task. | • Problems are designed to make the children think about the facts they already know and apply these strategies when they solve word problems.  
• Students need to engage with conceptual ideas that underlie the problem to complete the task successfully.  
• This problem is high cognitive demand because it takes the student more than just adding both sides. I want the student to notice patterns with the numbers.  
• Because the instruction for each sheet was open ended and allowed students to come up with their ways to solve and complete each problem. |
| Considers what the teacher will do | • Because I am urging Mathew to use relational thinking which places them (problems) in the procedures with connections category.  
• Because I am teaching them the patterns that go along with it. By having the children to do this, they are needed to cognitively be able to ideas I am giving them and apply to the problems I gave them. This can require a lot of thinking to someone who is just learning a new skill. |
| Rationale of the task choice (why did I chose the task) | • Problems are fairly basic but do have a higher amount of cognitive demand because of the numbers chosen based from the struggles Elly is having with the number type.  
• Although general procedures may be followed they cannot be followed mindlessly.  
• Can be presented in multiple ways. Can be solved as a backward multiplication problem or division problem. |

Notable in the two responses quoted previously is the fact that the participants view a task to be of high cognitive demand because of how the students will engage with the task and, the strategies that the teachers expect them to use.

In addition, responses that were coded under “considering what the teacher will do when enacting the task” focused on what the teacher will do and acknowledged that the task will be challenging for the students to assimilate the information being taught. For example, one participant remarked: The task that I am giving the children is high cognitive demand because I am teaching them the patterns that go along with it. By having the children to do this they are needed to cognitively be able to grasp the ideas I am giving them and apply to
the problems I gave them. This can require a lot of thinking to someone who is just learning a new skill.

Other responses focused on the rationale for giving that particular task. For example, one participant remarked: “Although general procedures may be followed they cannot be followed mindlessly,” implying that the task is of high cognitive demand if a student do procedures connected to meaning. Another PST explained that the task is meant to engage students’ with a high level thinking because the problem is fairly basic and based on what one of the students struggled with. This analysis shed light on the common PSTs’ conceptions and/or misconceptions of the nature of tasks, specifically, focusing on the PSTs’ understanding of a productive task and/or tasks that can engage children with either high or low -level thinking.

In the third stage of data analysis, I clustered the participants into four clusters based on four categories of emerging codes. First cluster represented the PSTs who noticed and attended to children’s strategies, interpreted children’s mathematical understanding and responded based on children’s mathematical understanding. The second cluster represented PST’s who noticed and attended to children’s strategies, interpreted children’s mathematical understanding but did not choose tasks based on children’s mathematical understanding. The third cluster represented PSTs who showed progress in the performance of the components skills in the two assignments. In other words, there was a notable change in their performance during the tutoring assignment in comparison with the Inquiry into Student Thinking. Finally, the fourth cluster represented the PSTs whose performance was inconsistent within the two assignments. In other words, their performances could not be categorized in any one of the clusters. I made a decision to highlight the patterns of the PSTs
in each cluster because of the uniqueness of their performances. Therefore, cluster four has all the PSTs who had inconsistent responses while cluster one to three has one PST’s responses that represented that particular cluster. My goal is to create a multiple case analysis that would provide insights into the PSTs’ responses twice as they progressed in their methods course. The multiple cases would also contribute to a deeper understanding and explanation of notable patterns of PSTs’ responses. That is after six and ten weeks of instruction respectively, as they progress through their methods course. Table 3.10 represents a summary of the noticeable patterns and themes.

Table 3.10. Summary of emerging patterns from PSTs’ responses

<table>
<thead>
<tr>
<th>Emerging Clusters</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noticed and used</td>
<td>Considers PSTs who provided most mathematical details as they attended to children’s strategies, provided robust evidence as they interpreted children’s mathematical understanding and there was robust evidence that the choices were based on children’s mathematical understanding to select tasks.</td>
</tr>
<tr>
<td>Noticed and did not use</td>
<td>Considers PSTs who provided most or some mathematical details as they attended to children’s strategies, provided robust or limited evidence as they interpreted children’s mathematical understanding but there was lack of any evidence that the choice of task was based on children’s understanding and/or misunderstanding.</td>
</tr>
<tr>
<td>Did not notice or use during the IST assignment but noticed and used during the tutoring assignment</td>
<td>Considers PSTs who had limited or lack of any mathematical details when attending to children’s strategies, had limited or no evidence of interpreting children’s mathematical understanding and/or misunderstanding had limited or no evidence of basing their choices on children’s mathematical understanding and/or misunderstanding during the IST assignment but made notable progress during the tutoring assignment. That is they noticed and provided most or limited mathematical details, there was robust evidence that they interpreted children’s mathematical understanding and/or misunderstanding and the task selected was based on children’s mathematical understanding and/or misunderstanding.</td>
</tr>
<tr>
<td>Responses had no consistent pattern in either the Inquiry into Student Thinking or tutoring assignments</td>
<td>Considers PSTs who had no consistent pattern in either the Inquiry into Student Thinking or tutoring assignments. In this cluster, the PSTs’ responses seemed sporadic as they responded to the prompts. In other words, some of the responses had most mathematical details in the component skill of attending to children’s strategies, and sometimes no evidence or limited evidence that they interpreted children’s mathematical understanding, and robust, limited, or no evidence that their responses were based on children’s mathematical understanding in the Inquiry into Student Thinking and/or the tutoring assignment.</td>
</tr>
</tbody>
</table>
Conclusions

This chapter described the research context and participants and the methodological approaches used in the study to address my main research question and four sub-questions. This study used interpretive approach to examine PSTs’ capacity to use children’s mathematical understanding and/or misunderstanding to select and pose worthwhile mathematical tasks. The interpretive analysis approach enabled me to describe the PSTs’ ability to notice and attend to children’s strategies, interpret children’s mathematical understanding and respond based on children’s mathematical understanding at two different times as they progress through their methods course. I drew upon two assignments, the Inquiry into Student Thinking (turned in after 6 weeks of instruction) and the tutoring assignments (turned in for grading after 8 weeks of instruction).

For analyzing the data, I developed coding schemes based on the literature review on how teachers notice and attend to children’s strategies, how they interpret children’s mathematical understanding and how they use what they notice to select and/or generate tasks for their long term response. In addition, I developed and iteratively revised the coding scheme to account for emergent codes. Further, I quantified some of the codes in order to foster more meaningful comparisons of the data and conduct statistical tests. Eventually, I identified emergent themes in the coded and quantified data in order to describe the PSTs’ capacity to use children’s mathematical understanding to select and pose mathematical tasks. In addition, I developed a multiple case analysis of four emergent clusters that represented the notable patterns in PSTs' responses. The next chapter present the results of my analysis, organized around the main research questions as well as the four sub-questions.
CHAPTER 4. RESULTS AND FINDINGS

This chapter is organized based on the main research question as well as the four sub-questions, listed as follows:

To what extent do PSTs develop practices of using children’s mathematical understanding to select and pose worthwhile mathematical tasks in the context of scaffolded activities?

a. What happens when PSTs analyze their own teaching and respond to children’s mathematical understanding as they plan for a series of instructional activities?
b. To what extent is the rationale for the PSTs’ next instructional plan based on children’s mathematical understanding?
c. What type of tasks/problems do PSTs pose after assessing children’s mathematical understanding?
d. What are PSTs’ conceptions of a productive task and/or tasks that engage students with high or low level thinking?

The first section presents findings on how the PSTs attended to children’s strategies, interpreted children’s mathematical understanding and responded based on children’s mathematical understanding. The second section describes the nature of tasks that the PSTs selected or generated as well as the PSTs’ conceptions of a productive task and/or tasks that engage students with high or low-level thinking. In the third section, I describe four cases that were selected based on emerging patterns of PSTs’ performance across the two assignments to provide more detailed explanations of notable patterns of attending, interpreting, and responding based on children’s mathematical understanding. In each case study, I discuss the nature of PSTs’ responses on the three component skills under consideration - attending, interpreting and responding based on children’s mathematical understanding. I conclude the discussion of each case study with a summary of the case, specifically focusing on the specific cluster.
Pre-service Teachers’ Capacity to Attend to Children’s Strategies-Interpret and Respond based on Children’s Mathematical Understanding

The results presented in this first section inform the main research question: To what extent do PSTs use children’s mathematical understanding to select and pose tasks in the context of scaffolded activities? This section describes the extent to which PSTs attended to children’s strategies, interpreted children’s mathematical understanding and responded based on children’s mathematical understanding, both in the Inquiry into Student Thinking and the tutoring assignment. As discussed in Chapter 3, I used the construct of professional noticing of children’s mathematical thinking (Jacobs et al., 2010) to make sense of the PSTs’ responses as they attended to children’s strategies, interpreted and responded based on children’s mathematical understanding.

Specifically, the section presents notable trends of PSTs’ performance across the three component skills as well as patterns showing the PSTs’ capacity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding in the two assignments. For the component skill of responding based on children’s mathematical understanding, I describe the extent to which PSTs used what they noticed to select the next task in the Inquiry into Student Thinking assignment as an intended response and, how the PSTs defined the learning goal and, selected and posed worthwhile mathematical tasks as a long term response in the tutoring assignment.  

I provided more details while analyzing the component skill of responding based on children’s mathematical understanding because prior research have focused on how PSTs attend to children’s strategies and interpret children’s mathematical understanding (Jacobs et al., 2010; Masingila & Doerr, 2002; Sherin & Van, Es 2005; Sleep & Boerst, 2011), but as yet the component skill of responding based on children’s mathematical understanding have been understudied.
Overview of Results for the Inquiry into Student Thinking Assignment

This section addresses the analysis of the PSTs’ responses to the following prompts in the Inquiry into Student Thinking assignment:

1. Summarize what you think they (children) knew or understood at the end of the study that they did not know or understand in the beginning

2. Choose 2 examples of the student’s responses as evidence for your claim from part a) and explain how they support your claim about this student’s learning.

3. If you were to teach the next lesson to this group of students, write one problem that you could give them and explain why you think this would be a good problem for this group.

I analyzed the responses to the prompts in terms of the extent to which the PSTs attended to children’s strategies, interpreted children’s mathematical understanding and responded based on children’s mathematical understanding. Specifically, I explored the extent to which PSTs provided mathematical details (most, some or lack of mathematical details) as they attended to children’s strategies. Likewise, I explored the extent to which the PSTs’ interpretation of children’s mathematical understanding had evidence (robust, limited, or lack of evidence) that they were consistent with the details of the specific children’s strategies. In addition, I explored the extent to which the PSTs’ responses had evidence (robust, limited or lack of evidence) that they were based on children’s mathematical understanding.
Attending to children’s strategies

Table (4.1) summarizes PSTs’ responses during the Inquiry into Student Thinking assignment. As shown in Table 4.1, 40% of the responses demonstrated evidence that PSTs provided mathematical details as they attended to children’s strategies, 56.7% provided some mathematical details and only 3.3% of the participants provided no mathematical details. These results show that most of the PSTs had some ability to attend to children’s strategies by the time they did the Inquiry into Student Thinking assignment but they differed in the level of mathematical details that they provided. Some provided most mathematical details while others provided just some mathematical details.

Table 4.1. Summary of PSTs’ responses to the Inquiry into Student Thinking assignment

<table>
<thead>
<tr>
<th>Component Skill</th>
<th>Category</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attending to Children’s Strategies</td>
<td>2-Most mathematical details</td>
<td>12</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>1-Some mathematical details</td>
<td>17</td>
<td>56.7</td>
</tr>
<tr>
<td></td>
<td>0- lack of mathematical details</td>
<td>1</td>
<td>3.3</td>
</tr>
<tr>
<td>Interpreting children’s Mathematical Understanding</td>
<td>2-Robust evidence</td>
<td>18</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>1-Limited evidence</td>
<td>10</td>
<td>33.3</td>
</tr>
<tr>
<td></td>
<td>0-Lack of evidence</td>
<td>2</td>
<td>6.7</td>
</tr>
<tr>
<td>Responding based on Children’s Mathematical Understanding</td>
<td>2-Robust evidence</td>
<td>4</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>1-Limited evidence</td>
<td>18</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>0-Lack of evidence</td>
<td>8</td>
<td>26.7</td>
</tr>
</tbody>
</table>

It was interesting to note that, after six weeks of instruction, some PSTs’ responses demonstrated evidence of providing detailed explanation of the children’s strategy and details of how the children made sense of the mathematical idea in the problems posed. Also, the some responses demonstrated evidence that PSTs had the ability to provide some mathematical details as they noted children’s strategies, but did not provide substantial
mathematical details. I speculated that some PSTs could have made progress in understanding how to access the strategies that the children used and, understood the mathematical significance of the strategies, but might have struggled with articulating how the children interacted with the mathematical idea. These results were not surprising because the class discussion during the second week of the semester (see Appendix B for a summary of course activities) focused on PSTs’ learning the CGI problem types and the strategies that children typically use to solve word problems. As discussed in Chapter 3, PSTs had also been scaffolded using a number of homework’s and in-class activities that required them to attend to children’s strategies and select tasks based on children’s mathematical understanding. The instructor had also scaffolded them with written and verbal feedback in some of the activities done individually and in small groups and, the feedback could also have supported their understanding as they attended to children’s strategies.

Interpreting children’s mathematical understanding

For the component skill of interpreting children’s mathematical understanding, 60% of the PSTs’ responses demonstrated robust evidence that they interpreted children’s mathematical understanding. In other words, their responses demonstrated evidence that they made sense of details of children’s strategies and noted how the details reflected children’s mathematical understanding. The results also show that 33.3% demonstrated limited evidence that they had in-depth interpretation of children’s mathematical understanding. Only 6.7% of participants’ responses had no evidence of interpreting children’s mathematical understanding. These results reveal that the participants had the potential to interpret
children’s mathematical understanding, with 93.3% responses having provided robust or limited evidence of interpretation of children’s mathematical understanding.

A closer look at the individual PSTs’ performance revealed interesting patterns. For example, the number of PSTs (18 of 30) whose responses demonstrated robust evidence that they interpreted children’s mathematical understanding was larger than the number of PSTs (12 of 30) whose responses had most mathematical details. Specifically, nine of the 18 PSTs had provided most mathematical details and there was robust evidence in their responses that their interpretation was based on children’s mathematical understanding. That means, their performance in the component skill of attending to children’s strategies and interpreting children’s mathematical understandings were consistently high. The remaining nine participants had provided some mathematical details in the component skill of attending to children’s strategies but their responses had robust evidence that their interpretation was based on children’s mathematical understanding.

Similarly, the ten responses that had limited evidence that the PSTs’ interpretation was based on children’s mathematical understanding varied in their performance in the component skill of attending to children’s strategies. Three of them had provided most mathematical details in the component skill of attending to children’s strategy but their responses had limited evidence that their interpretation was based on children’s mathematical understanding. Seven of the participants only provided some mathematical details in the component skill of attending to children’s strategies but their responses had limited evidence that their interpretation was based on children’s mathematical understanding. Finally, two responses had no evidence that the PSTs provided any evidence of interpretation based on children’s mathematical understanding. Between the two responses, one had not provided
any mathematical details in the component skill of attending to children’s strategies, but one had provided some mathematical details.

These results contradict earlier studies that have presented the three component skills as interrelated (Jacobs et al., 2010), and suggested that a participant has to attend to children’s strategies for them to interpret children’s mathematical understanding. In this current study, the PSTs’ responses demonstrated robust evidence that their responses were based on children’s mathematical understanding even when they had not provided substantial mathematical details (most mathematical details) while attending to children’s strategies. Also, some PSTs provided most mathematical details but their interpretation was not based on children’s mathematical understanding. …

**Responding based on children’s mathematical understanding**

For the component skill of responding based on children’s mathematical understanding, 13.3% of the participants’ responses had robust evidence that the responses were based on children’s mathematical understanding, 60% had limited evidence and 26.7% of participants responses lacked any evidence that the PSTs’ responses were based on children’s mathematical understanding. Table (4.1) provides a summary of the PSTs’ responses in the three component skills during the Inquiry into Student Thinking assignment.

A closer look at the component skill of responding based on children’s mathematical understanding revealed that four of 30 PSTs had robust evidence that their response had been based on children’s mathematical understanding. Among the four PSTs who provided robust evidence that the responses were based on children’s mathematical understanding, two had provided most mathematical details and had robust evidence in their interpretation of
children’s mathematical understanding. This results shows that it’s only two participants who provided most mathematical details while attending to children’s strategies, their responses had robust evidence that their interpretation was based on children’s mathematical understanding and their responses were based on children’s mathematical understanding during the Inquiry into Student Thinking assignment. The remaining two PSTs’ had provided some mathematical details and their responses had robust evidence of interpreting children’s mathematical understanding. Over all, these four participants seemed to have some understanding of the three component skills by the time they did the Inquiry into Student Thinking assignment.

The results also revealed that most of the PSTs’ responses (18 of 30) had limited evidence that their responses were based on children’s mathematical understanding. Among the 18 participants, six had provided most mathematical details in the component skill of attending to children’s strategies and robust evidence in the component skill of interpreting children’s mathematical understanding, but only limited evidence that their responses were based on children’s mathematical understanding. Four participants had provided some mathematical details in the component skill of attending to children’s strategies and had limited evidence that their interpretation was based on children’s mathematical understanding. For the remaining eight participants, three had some mathematical details in the component skill of attending to children’s strategies but robust evidence in the component skill of interpretation based on children’s mathematical understanding. In summary, all the 18 participants seemed to be in a developmental continuum of attending to children’s strategies, interpreting children’s mathematical understanding and, to some extent responding based on children’s mathematical understanding.
The remaining 8 of 30 participants responses had no evidence that their responses were based on children’s mathematical understanding. Their responses in the component skill of attending to children’s strategies and the component skill of interpreting children’s mathematical understanding also differed. Specifically, one participant had provided most mathematical details while attending to children’s strategies, and the responses had robust evidence that the interpretation was based on children’s mathematical understanding. However, there was no evidence that the PST responses were based on children’s mathematical understanding.

For the other seven participants whose responses had no evidence of responding based on children’s mathematical understanding, two had provided most mathematical details in the component skill of attending to children’s strategies, but only limited evidence that their interpretation was based on children’s mathematical understanding and, three had some mathematical details and robust evidence that their interpretation was based on children’s mathematical understanding. Finally, two had some mathematical details but no evidence that their interpretation is based on children’s mathematical understanding. These results suggest that most of the PSTs struggled with the component skill of responding based on children’s mathematical understanding. As discussed later in section 3 of this chapter, most of the PSTs thought about the next task for the whole group, without necessarily considering the individual children’s understanding.

Next, I report the analysis of the PSTs’ responses to the following prompts in the tutoring assignment:

1. What does each student know, think and understand about number, operations and problem solving?
2. How will what you learned in the interviews influence how you work with the children during the next sessions?

3. How are these tasks intended to build on what you know about your students’ understanding and misunderstanding?

**Overview of Results for the Tutoring Assignment**

**Attending to children’s strategies**

As shown in table 4.2, 73.3% of PSTs’ responses demonstrated evidence that they provided detailed explanations of the children’s strategy, including details of the mathematical essence of the strategy and details of how the children made sense of and interacted with the mathematical idea. By mathematical essence, I am referring to the mathematical significance of the strategy that the children used. The results also reveal that 23% of the participants only provided some details about the strategy and/or provided a general description with no specific details of how the child interacted with the given tasks. Some responses also had some details on the mathematical essence of the strategy. In total, 96.6% of the participants either provided some or most mathematical details. Therefore, only one participant who did not provide any detailed explanations of the children’s strategy or any details of the mathematical significance of the strategy.

**Interpreting children’s mathematical understanding**

For the component skill of interpreting children’s mathematical understanding, 70% of the responses had robust evidence that the participants made sense of details of children’s strategies as they interpreted children’s mathematical understanding and noted how the
Table 4.2. Summary of PSTs’ responses in the tutoring assignment

<table>
<thead>
<tr>
<th>Component Skill</th>
<th>Category</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attending to Children’s Strategies</td>
<td>2-Most mathematical details</td>
<td>22</td>
<td>73.3</td>
</tr>
<tr>
<td></td>
<td>1-Some mathematical details</td>
<td>7</td>
<td>23.3</td>
</tr>
<tr>
<td></td>
<td>0-Lack of mathematical details</td>
<td>1</td>
<td>3.3</td>
</tr>
<tr>
<td>Interpreting children’s Mathematical Understanding or Misunderstanding</td>
<td>2-Robust evidence</td>
<td>21</td>
<td>70.0</td>
</tr>
<tr>
<td></td>
<td>1-Limited evidence</td>
<td>7</td>
<td>23.3</td>
</tr>
<tr>
<td></td>
<td>0-Lack of evidence</td>
<td>2</td>
<td>6.7</td>
</tr>
<tr>
<td>Responding based on children’s mathematical understanding</td>
<td>2-Robust evidence</td>
<td>11</td>
<td>36.7</td>
</tr>
<tr>
<td></td>
<td>1-Limited evidence</td>
<td>10</td>
<td>33.3</td>
</tr>
<tr>
<td></td>
<td>0-Lack of evidence</td>
<td>9</td>
<td>30.0</td>
</tr>
</tbody>
</table>

responses showed limited evidence that the participants made sense of details of children’s strategies as they interpreted children’s mathematical understanding. In total 93.3% of the participants’ responses had some evidence that they made sense of details of children’s strategies. Therefore, only 6.7% of the responses that had no evidence that the participants made sense of children’s strategies.

Further scrutiny of component skill of interpreting children’s mathematical understanding revealed interesting patterns. For example, among the 21 PSTs whose responses had robust evidence that their interpretation was based on children’s mathematical understanding 18 had also provided most mathematical details while attending to children’s strategies. The remaining three participants had provided some mathematical details when attending to children’s strategies. For the remaining participants, seven provided some mathematical details while two did no provide any mathematical details. These results show that some PSTs had the ability to interpret children’s mathematical understanding by the time they did the tutoring assignment.
Responding based on children’s mathematical understanding

For the component skill of responding based on children’s mathematical understanding, 36.7% of the responses had robust evidence that participants’ responses were based on children’s mathematical understanding. In addition, 33.3% of the responses had limited evidence that the participants’ responses were based on children’s mathematical understanding. However, 30.0% of the responses had no evidence that the PSTs’ responses were based on children’s mathematical understanding. Table (4.2) summarizes PSTs’ performance in the three component skills in the tutoring assignment.

Comparison of PSTs’ Responses Across the Two Assignments

Component Skill 1: Attending to children’s strategies

These results show that the number of PSTs (22/30) who provided detailed explanation of children’s strategies, including details of the mathematical significance of the strategies the children used and details of how the children made sense of the mathematical idea in the task, as they attended to children’s strategies increased during the tutoring assignment in comparison with the Inquiry into Student Thinking assignment (12/30). Of the twenty-two participants who provided most mathematical details in their responses during the tutoring assignment, nine had previously provided most mathematical details during the Inquiry into Student Thinking assignment. That means their responses were consistent across the two assignments. These results were encouraging because the nine PSTs had noticed and attended to children’s strategies in meaningful ways at two different times. The remaining eleven participants had provided some mathematical details in the Inquiry into Student Thinking assignment but their responses during the tutoring assignment had evidence that
they provided most mathematical details. These results indicate that the eleven PSTs’
performance shifted from general descriptions of children’s strategies to providing details of
children’s strategies and including details on how the children interacted with the
mathematical ideas.

The increase with the number of PSTs who provided most mathematical details
reduced the number of PSTs who provided some mathematical details during the tutoring
assignment. That is, only 7 of 30 PSTs provided some details as they attended to children’s
strategies in comparison to the numbers (17 of 30) in the Inquiry into Student Thinking
assignment. Of the seven PSTs, three had provided most mathematical details as they
attended to children’s strategies during the Inquiry into Student Thinking but provided some
mathematical details during the tutoring assignment. Therefore, their performance shifted
from noticing and paying careful attention to children’s strategies and how the children
interacted with the tasks to including just general descriptions. This was specifically worth
noting because one would have expected them to provide more details during the tutoring
assignment because they were interacting with children in a real classroom setting. The
remaining four PSTs had provided some mathematical details during the Inquiry into Student
Thinking assignment and their performance did not change during the tutoring assignment.
Finally, there was only one participant who did not provide any mathematical details, in both
the Inquiry into Student Thinking assignment and the tutoring assignment.

These results reflect that some PSTs had the potential of attending to children’s
strategies by the time they did the Inquiry into Student Thinking assignment. In addition, the
number of PSTs who provided most mathematical details increased during the tutoring
assignment while the number of PSTs who provided some mathematical details reduced.
Therefore, I inferred that the PSTs were developing their capacity to attend to children’s strategies as they progressed through their methods course.

**Component Skill 2: Interpreting children’s mathematical understanding**

**Robust evidence**

With regards to the component skill of interpreting children’s mathematical understanding there was robust evidence that 18 of 30 PSTs made sense of details of children’s strategies and noted how the details reflected children’s understanding in the Inquiry into Student Thinking assignment. This number increased slightly during the tutoring assignment with (21/30) PSTs’ responses providing robust evidence that their interpretation was based on children’s mathematical understanding. However, there were intriguing emerging patterns from the PSTs’ responses. Next, I briefly elaborate on the individual patterns.

*Responses had robust evidence in the two assignments.* Among the 21 participants whose responses had robust evidence during the tutoring assignment, 13 had also provided robust evidence during the Inquiry into Student Thinking assignment. These results suggest a consistent performance by some participants while it came to interpreting children’s mathematical understanding. These results also suggest that a number of participants (9 of 30) may have been comfortable interpreting children’s mathematical understanding during the tutoring assignment than the Inquiry into Student Thinking assignment.

*Responses had robust evidence in tutoring assignment but limited evidence in the Inquiry into Students Thinking Assignment.* Among the 21 participants whose responses had robust evidence during the tutoring assignment, six had limited evidence during the
Inquiry into Student Thinking assignment. This was encouraging to notice since the change from providing limited evidence to robust evidence might imply that PSTs were gradually developing the capacity to interpret based on children’s mathematical understanding as they progressed in the methods course. These results would also be encouraging if PSTs’ responses had evidence of interpreting children’s mathematical understanding during the tutoring assignment, because the assignment gave them an opportunity to interact with children in a real classroom setting.

**Responses had robust evidence in tutoring assignment but no evidence in the Inquiry into Student Thinking.** Two participants had robust evidence during the tutoring assignment but no evidence during the Inquiry into Student Thinking assignment. These two participants responses are worth further investigations in order to explore the big shift from having no evidence to having robust evidence in their responses.

**Limited evidence**

Further scrutiny revealed that 7 of 30 participants’ responses had limited evidence that their interpretation was based on children’s mathematical understanding. This number of participants was lower than the participants who provided limited evidence (10 of 30) in the Inquiry into Student Thinking assignment. Further scrutiny of the responses revealed different patterns. Next, I briefly elaborate on the patterns.

**Responses had limited evidence in tutoring assignment but robust evidence in Inquiry into Student Thinking assignment.** Of the seven participants whose responses had limited evidence, four of their responses had robust evidence in the Inquiry into Student Thinking but limited evidence in the tutoring assignments. Like some of the other component
skills, these results were surprising because one would have expected the participants’ responses to have robust evidence in the tutoring assignment.

Responses had limited evidence both in tutoring and Inquiry into Student Thinking

Among the seven participants who provided limited evidence, two participant responses had limited evidence in the Inquiry into Student Thinking. That means the two participants did not improve the performance in the component skill of interpreting children’s mathematical understanding during the two assignments. The responses of these participants are worth further investigation to understand what would have contributed to these participants having only limited evidence in the two assignments.

Responses had limited evidence in tutoring assignment but no evidence in the Inquiry into Students’ Thinking. Among the seven participants, only one participant responses that had limited evidence in the component skill of interpreting children’s mathematical understanding during the tutoring assignment but no evidence during the Inquiry into Student Thinking assignment. That means most of the participants as discussed earlier either provided robust or limited evidence.

Lack of any evidence

Among 30 participants, only two participants’ responses had no evidence that their interpretation was based on children’s mathematical understanding. The responses of one participant during the Inquiry into Student Thinking assignment had robust evidence while the other had limited evidence, but the two participants’ responses had no evidence that their interpretation was based on children’s mathematical understanding.
Component Skill 3: Responses based on children’s mathematical understanding

Robust evidence

With the component skill of responding based on children’s mathematical understanding, only 4 of 30 participant responses had robust evidence that the responses were based on children’s mathematical understanding during the Inquiry into Student Thinking assignment. This number of participants slightly increased during the tutoring assignment, with 10 of 30 responses having robust evidence that their choices were based on children’s mathematical understanding. Indeed, a closer look at the responses revealed different patterns that represented the PSTs’ performance at the two static points.

Responses had robust evidence in the two assignments. Among the 10 responses that had robust evidence during the tutoring assignment, there were only two PSTs’ responses that demonstrated consistent robust evidence that their responses were based on children’s mathematical understanding in the two assignments. In other words, there were only two participants who had consistently provided robust evidence within the two time periods. As discussed below, the other participants either had limited or no evidence during the Inquiry into Student Thinking assignment but their performance demonstrated having robust evidence during the tutoring assignment. These results were encouraging because it shows that more PSTs demonstrated robust evidence that their responses were based on children’s mathematical understanding as they progressed in their methods course. It would also be interesting to investigate the two participants to further understand what could have contributed to their performance across the two assignments.
Responses had robust evidence in tutoring assignment but limited evidence in the Inquiry into Student Thinking assignment. From the 10 participants whose responses had robust evidence in the tutoring assignment, only five participants’ narratives demonstrated limited evidence during the Inquiry into Student Thinking assignment. This was not a surprise because I anticipated that the PSTs’ performance would shift within the course of the methods course.

Responses had robust evidence in tutoring assignment but no evidence in Inquiry into Student Thinking assignment. Finally, three participants’ narratives had no evidence that their responses were based on children’s mathematical understanding during the Inquiry into Student Thinking assignment but there was robust evidence during the tutoring assignment. Specifically, this was a big shift for the three participants and a case worth further investigations.

Limited evidence

Further, the number of participant responses that had limited evidence during the Inquiry into Student Thinking and the tutoring assignment were 18 of 30 and 10 of 30 respectively. According to this analysis, it’s evident that a number of PSTs’ responses had shifted from providing limited evidence to providing robust evidence or to no evidence, hence, the lower number during the tutoring assignment. The performance of some of the PSTs could not be easily explained using the available data. I briefly explain the different notable patterns here below.

Responses had limited evidence in tutoring assignment but robust evidence in the Inquiry into Student thinking assignment. For example, two of the 10 PSTs whose
responses demonstrated limited evidence, provided robust evidence during the Inquiry into Student Thinking assignment. I anticipated that the PSTs would improve their performance by the time they did the tutoring assignment, and this was not the case with the two participants. This was an interesting case where the participants’ responses were against the anticipated trajectory as they PSTs progressed with their methods course.

**Responses had limited evidence in both tutoring and the Inquiry into Student Thinking assignment.** The analysis revealed that 5 of the 10 participants’ responses demonstrated limited evidence during the tutoring assignment and the Inquiry into Student Thinking assignment. With these five participants, there was indication of growth or shift from providing limited evidence to providing robust evidence

**Responses had limited evidence in the tutoring but no evidence during the Inquiry into Student Thinking assignment.** Three of the nine participants whose responses had limited evidence during the tutoring assignment had provided no evidence during the Inquiry into Student Thinking assignment. The performance of the three participants was encouraging since there was a shift from having no evidence to having limited evidence.

**No evidence**

In the Inquiry into Student Thinking assignment, 8 of 30 responses demonstrated no evidence that PSTs based their responses on children’s mathematical understanding. This number was slightly higher during the tutoring assignment with 10 of 30 participants responses having no evidence that the responses were based on children’s mathematical understanding. Of these 10 participants, two participants’ responses had no evidence during the Inquiry into Student Thinking assignment and seven participants had provided limited
evidence during the Inquiry into Student Thinking. Therefore, there was a shift in two ways. Some participants provided evidence during tutoring assignment and some responses had no evidence.

In total, more than two-thirds of the participants either had limited or no evidence that their responses were based on children’s mathematical understanding during the Inquiry into Student Thinking assignment. Similarly, two-thirds of the participants had limited or no evidence that their responses were based on children’s mathematical understanding during the tutoring assignment. This implies that, on average, PSTs struggled more with the component skill of responding during the two assignments than with the component skill of attending to children’s strategies and interpreting children’s mathematical understanding. Table 4.3 provides a summary of the PSTs’ responses during the two assignments.

**Outcome of statistical significance in the PSTs’ performance in the two assignments**

Finally, I conducted a paired sample t-test to compare the PSTs’ capacity to attend to children’s strategies, interpret children’s mathematical understanding, and respond based on children’s mathematical understanding as they progressed in their methods course. A paired sample t-test showed that there was a significant difference in PSTs’ capacity to attend to children’s strategies during the Inquiry into Student Thinking assignment (ACS) (M= 1.37, S.D = 0.556) compared with their responses during the tutoring assignment (ACS-Tutoring). 

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11 In this study, I recognize the limitations of written assignments in that the PSTs may have had differing abilities to articulate their ideas in writing. I also consider using written assignment as a limitation because it’s not easy to verify whether what they described is what they noticed.
Table 4.3. Summary of PSTs’ responses in the two assignments

<table>
<thead>
<tr>
<th>Component skill</th>
<th>Student thinking assignment</th>
<th>Tutoring assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attending to children’s strategies</td>
<td>12  Most mathematical details</td>
<td>22  Most mathematical details</td>
</tr>
<tr>
<td></td>
<td>17  Some mathematical details</td>
<td>7   Some mathematical details</td>
</tr>
<tr>
<td></td>
<td>1   Lack of any mathematically details</td>
<td>1   Lack of any mathematical details</td>
</tr>
<tr>
<td>Interpreting children’s mathematical understanding and/or misunderstanding</td>
<td>18  Robust evidence</td>
<td>21  Robust evidence</td>
</tr>
<tr>
<td></td>
<td>10  Limited evidence</td>
<td>7   Limited evidence</td>
</tr>
<tr>
<td></td>
<td>2   Lack of evidence</td>
<td>2   Lack of evidence</td>
</tr>
<tr>
<td>Responding based on children’s mathematical understanding</td>
<td>4   Robust evidence</td>
<td>10  Robust evidence</td>
</tr>
<tr>
<td></td>
<td>18  Limited evidence</td>
<td>9   Limited evidence</td>
</tr>
<tr>
<td></td>
<td>8   Lack of evidence</td>
<td>11  Lack of evidence</td>
</tr>
</tbody>
</table>

(M= 1.70, S.D= 0.535) with t (29) = -2.763, p= 0.010. For the component skill of interpreting children’s understanding (ICU), a paired sample t-test show that there was no significant difference in PSTs’ capacity during the Inquiry into Student Thinking assignment (M=1.53,S.D= 0.629) compared with their responses during the tutoring assignment (M=1.63, S.D=0.615) with t (29) = -0.619 p= 0.541. Similarly, for the component skill of responding based on children’s mathematical understanding, a pair wise comparison revealed that there was no significant difference between PSTs’ capacity to respond during the Inquiry into Student Thinking (M= 0.87 S.D= 0.629) compared with their responses during the tutoring assignment (M=1.07 S.D= 0.828) with t (29) = -1.099, p= 0.281.

These results show that the PSTs’ capacity to attend to children’s strategies had significantly improved by the end of the tutoring assignment in comparison with the performance during the Inquiry into Student Thinking assignment. However, there was no significant change in the PSTs’ capacity to interpret children’s mathematical understanding and respond based on children’s mathematical understanding.
Summary

One of my goals in this study was to explore the PSTs’ capacity to use children’s mathematical understanding to select and pose tasks in the context of scaffolded activities. Overall, the PSTs demonstrated that they had some capacity to attend to children’s strategies during the Inquiry into Student Thinking assignment and a significantly increased capacity to attend to children’s strategies during the tutoring assignment. The number of PSTs’ responses that demonstrated robust evidence of interpretation based on children’s mathematical understanding slightly increased during the tutoring assignment in comparison with the Inquiry into Student Thinking assignment, even though the change was not significant. Finally, the PSTs seemed to have struggled most in both assignments with the component skill of responding based on children’s mathematical understanding.

These results are congruent to other studies (e.g., Jacobs et al., 2010; Sherin & Van Es, 2009) that have reported that the capacity to attend to children’s strategies and interpret children’s mathematical understanding can be learned. Specifically, Sherin and Van Es (2007) found that PSTs who participated in video club meetings where they used the Video Analysis Support Tools (VAST) changed the kind of things they paid careful attention to in a classroom instructional environment over a period of one year. Consistent with Sherin & Van Es findings, this study shows that some PSTs started providing more mathematical details as they attended to children’s strategies and their responses demonstrated robust evidence that their interpretation was based on children’s mathematical understanding.

In addition, the results demonstrated evidence that some PSTs were developing their capacity to respond based on children’s mathematical understanding. These results were contrary to the findings in (Jacobs et al., 2010) study, where the prospective teachers
struggled with all three-component skills. In summary, this study reveals that expertise in professional noticing can be learned in the context of a methods course where the PSTs are exposed to activities and experiences that will purposefully develop the capacity. In the next section, I will describe results informed by the 2nd stage of data analysis.

A Closer Look at the Component Skill of Responding based on Children’s Mathematical Understanding

The results presented in this section inform the third and fourth sub-questions:

c. What type of tasks/problems do PSTs pose after assessing children’s mathematical understanding?
d. What are PSTs’ conceptions of productive tasks and/or tasks that engage students with high or low level thinking?

First, this section explores the nature of instructional tasks selected by PSTs, both in the Inquiry into Student Thinking assignment and the tutoring assignment. Second, the section explores the PSTs’ reasoning and rationale as they respond based on children’s mathematical understanding, both in the Inquiry into Student Thinking assignment and tutoring assignment. In the first section, I used the PSTs’ responses to the following prompts in the Inquiry into Student Thinking and tutoring assignment:

1. If you were to teach the next lesson to this group of students, write one problem that you could give them and explain why you think this would be a good problem for this group (Inquiry into Student Thinking assignment)
2. What was your plan for this week? Describe the activities, problems, literature etc. that you planned to use during this week’s tutoring session and explain your rationale for this plan. Included in your rationale should be: What makes these tasks high cognitive demand for your students? (Tutoring assignment)

Inquiry into Student Thinking assignment

Nature of mathematical tasks
When prompted to select the next problems in the Inquiry into Student Thinking assignment, 28/30 PSTs selected word problems, one participant selected a relational thinking number sentence \((40+60 = -+\cdot)\) and one participant selected true or false sentences. It seemed that most of the PSTs used the CGI problem-type framework that was created by Carpenter et al. (1999) for addition/subtraction word problems. As discussed in Chapter 2, the CGI problem type framework groups addition and subtraction word problems into four basic classes, involving: (a) joining action, (b) separating action, (c) part-part whole relations, and (d) comparison situations. The PSTs had developed word problems using the framework in class prior to doing the assignment, but some PSTs paid specific attention to the number choices; focusing on specific students’ understanding.

The analysis of the problems that had multiple number choices showed that some of the PSTs seemed to be cognizant of number choices that would develop students’ understanding. For example, one participant selected the word problem written, as follows, and provided the rationale:

**Problem:** I would create Joint-Result-Unknown (JRU) or Separate-Result-Unknown (SRU) problems for the students. A JRU example would be “Sunny has ____ fish, and then she buys ____ more. How many fish does she have now?” Number choices would include \{(10, 50), (20, 30), (10,41), (15, 25)\}.

**Rationale of the number choices:** This type of problem would be good for all of the students. Sunny and Daniella struggle to count by tens past the numbers
Table 4.4. Nature of tasks selected during the Inquiry into Student Thinking assignment

<table>
<thead>
<tr>
<th>Type</th>
<th>N</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Join Change Unknown</td>
<td>5</td>
<td>Joshua collects rocks. He likes to keep them in bags of ten. Last week he had a total of __ bags. After finding some more this week, he now has __ bags. How many rocks did Joshua find this week? (5,6) (5,8) (8,12) (9,15)</td>
</tr>
<tr>
<td>Join Result Unknown</td>
<td>8</td>
<td>Ms. Jennifer has ___ skittles. Then, she gets ___ more skittles from her friend. How many skittles does she have in total?”</td>
</tr>
<tr>
<td>Separate Result unknown</td>
<td>4</td>
<td>Emilio has ___ soccer balls. ___ Soccer balls roll away. How many Soccer balls are left? (15, 5) (26, 16) (57, 27) (81, 51)</td>
</tr>
<tr>
<td>Group size unknown</td>
<td>2</td>
<td>Jack has 65 pencils. He wants his 5 classmates to each have an equal amount of pencils. How many would each classmate get? ”</td>
</tr>
<tr>
<td>Number of Groups unknown</td>
<td>3</td>
<td>Sunny has 94 chocolate chips. She needs 10 chocolate chips to make a cookie. How many cookies can Sunny make?</td>
</tr>
<tr>
<td>Both product unknown &amp; Join result unknown</td>
<td>4</td>
<td>Sunny has 8 rolls of candy. Each package has 11 candies in it. She also has 12 extra candies. How many candies does she have in all?</td>
</tr>
<tr>
<td>Product unknown and compare result unknown</td>
<td>1</td>
<td>Mary had ___ bags of cookies with ___ cookies in each bag and Amy had ___ bags of cookies with ___ cookies in each bag. Which one had more cookies?</td>
</tr>
<tr>
<td>Relational thinking problems</td>
<td>1</td>
<td>40+60 = -+-</td>
</tr>
<tr>
<td>True /false sentences</td>
<td>1</td>
<td>10+2= 6. 10+5= 5+10 10+10+10+9= 20+19.</td>
</tr>
</tbody>
</table>

Note: N=number of problems.

20 and 30, and this problem challenges them to do so. Emilio would be challenged to count by tens and keep track of the ‘one’ in 41. Both Jack and Emilio would be challenged by the last number choice, as both understand the concept of counting by tens, but they would have to extend their understanding to non-zero ending numbers.

This response indicates that the PST selected and used number choices that would advance the children’s mathematical understanding. The PST focused on how the task would support Sunny and Daniella’s understanding because “they struggled to count by 10” and both Jack and Emilio would be challenged by the last number choice because they “understand the concept of counting by 10”. This type of reasoning seemed to be a tendency for a number of PSTs even when they did not select multiple number choices. For example, one PST selected the task, “Danielle has 55 beads. She wants to make as many necklaces as she can, but she must have 10 beads on each necklace. How many complete necklaces can she make? How
many beads will she have left over?” Explaining why she selected those numbers, she remarked:

I chose 55 because the students already demonstrated knowledge of knowing 50 is 5 groups of 10 and 5 is an easy number to work with as a remainder. I chose 10 because the goal is to get the students to develop and use the idea of ten as one and use it to problem solve.

These examples suggest that some PSTs had started paying careful attention to the mathematical potential of the task that they intended to pose to the children as they did the Inquiry into Student Thinking assignment. The examples also provide persuasive evidence that the PSTs had some capacity to choose tasks based on children’s mathematical understanding by the time they did the Inquiry into Students’ Thinking assignment.

**Tutoring assignment**

To examine the nature of tasks that the PSTs selected in the tutoring assignment, I used the PSTs’ lesson plans for the first tutoring sessions. To prepare the lesson plan, they had been scaffolded to start with an opening number routine followed by the main activity, which they would use to tutor the children based on the interpretation of children’s mathematical understanding in the interview that they had conducted. In general, the results revealed that all the PSTs selected tasks that were similar to the interview questions (which was not surprising, because some children could not do the tasks and PSTs might not have been familiar with other types of tasks). Specifically, 17/30 PSTs selected CGI word problems, 3/30 PSTs selected true or false number sentences, 3/30 selected number sentences

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12 Although I used PSTs’ responses for the interview and the first tutoring session, I would like to notify the reader that PSTs had an opportunity to tutor the same child or children for three 50-minute sessions. Therefore, they got an opportunity to interpret children mathematical understanding and select and pose tasks for three times.
and equations, 5/30 selected counting tasks and 2/30 selected tasks that required the children to identify the place value. Given the nature of activities that they had done in class (e.g., exploring the CGI word problem chart and generating their own tasks), it was reasonable to find that majority of the PSTs selected the CGI word problems. The other 13 problem types were likely due to the interview tasks that had been provided to them with varying types of problems. Table (4.5) illustrates the nature of tasks selected and/or generated during the tutoring assignment.

Further analysis revealed emerging patterns in how the PSTs paid attention to the multiple number choices when selecting and/or generating problems. Using the emerging patterns, the rationale for the number choices was classified into: 1) Selected number choices that started with easier numbers to more challenging numbers; 2) Considered the strategy that the children would use to solve the task; 3) Considered children’s understanding or how the children will make sense of the problem; and 4) No rationale to the number choices. I will briefly explain each pattern below. ¹³

*Selected number choices that started with easier numbers to more challenging numbers*

Twelve of the participants selected number choices that started with easier numbers to more challenging numbers. In this approach, the PSTs’ rationales revealed their tendency to start with easier numbers that the children could manipulate with ease, followed by larger numbers that the children could not manipulate using mental strategies alone. The PSTs either referred to those first numbers as “easier numbers” or “familiar numbers”. From their

¹³ Note that some of the PSTs’ reasoning with number choices was characterized in more than one of the outlined categories.
Table 4.5. Nature of tasks selected and/or generated during the Tutoring assignment

<table>
<thead>
<tr>
<th>Type</th>
<th>N</th>
<th>Example</th>
<th>Number Choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>CGI-word problems</td>
<td>17</td>
<td>Student 1 (use their names in the real setting) has ___ race cars. Student 2 gave student 1 ___ more racecars. How many racecars does student 1 have in total?</td>
<td>Student 1 adding to 10 (5,5) 7,3)(4,6)(15,5) student 2 adding to 100 (50,50) (80,20)(35,65)(42,58)</td>
</tr>
<tr>
<td>CGI-word problems</td>
<td>17</td>
<td>Student 1 (use their names in the real setting) has ___ race cars. Student 2 gave student 1 ___ more racecars. How many racecars does student 1 have in total?</td>
<td>Student 1 adding to 10 (5,5) 7,3)(4,6)(15,5) student 2 adding to 100 (50,50) (80,20)(35,65)(42,58)</td>
</tr>
<tr>
<td>True or false sentences</td>
<td>3</td>
<td>The student will be presented with these problems one at a time and they determine whether the problem is true or not.</td>
<td>Those numbers are chosen because they are within the range of 1-10 and they are familiar 10's fact for the student. The values of the equations are slightly higher but the sum allows for more differing equations to be used. I chose this equations and numbers using low numbers in value while presenting anew concept in to make her more comfortable and confident in the use of those numbers...</td>
</tr>
<tr>
<td>Number sentences and/or</td>
<td>3</td>
<td>5+8 = 8+5 4+3 = hmm + 2</td>
<td>5+8 = 8+5 - I chose this number because I want to see if my students understand that the number to the right is the same with the number to the left 4+3 = hmm + 2 - I chose this number sentence because I wanted to see if the students understand that the equal sign means the same as and that both sides of the equation should add up to the same number.</td>
</tr>
<tr>
<td>equations</td>
<td></td>
<td></td>
<td>No number choices</td>
</tr>
<tr>
<td>Counting</td>
<td>5</td>
<td>After the students complete the number 10 worksheet the teacher will pass out the dot-to-dot worksheet. Students will need to complete both worksheets by drawing lines from the numbers 1-30 and 5-500 first by counting up by ones and then by 5's. This will give the students the bases for counting so that they will be able to count the &quot;how many objects&quot; worksheet.</td>
<td></td>
</tr>
<tr>
<td>Place value</td>
<td>2</td>
<td>I will start by writing a two digit number on my scratch paper for both students to see e.g., 6.1 will ask them to say the number and then I will point at the different digits and then ask them what this number represents (prompting students to point out the place). After discussing the two-digit number, I will add to digit to the nd of the number making it a 3 digit three digit number.</td>
<td>I chose some two digit and three digits because at this grade level students know three digit numbers and breaking them into place values is a good task, but I also choose two digits so that they can see the difference...</td>
</tr>
</tbody>
</table>

Note: N=number of problems.
reasoning, the easier or familiar numbers could make the children more comfortable with the concept before working with more challenging numbers. For example, one PST selected the following task for kindergarten children:

**Problem:** Conor has ___Wii games in his cupboard. He found more Wii games under his bed. How many Wii games does Conor have?

**Number Choices:** (7,3) (4,6)

James has ___Wii games at his house. Conor let James borrow ___more Wii games. How many Wii games does James have at his house now?

**Number choices:** (5,5) (2,8)

**Rationale for number choices:** I chose\(^{14}\) (7,3) because it should be easier for the students to start at 7 and only count 3 up to 10. Then, I choose 4 and 6 because counting from 4 is further from 10. I chose 5 and 5 for the second problem because I want to see if students will use their strategies or if they just know that 5+5 = 10. Finally I chose 2,8 because it is more difficult than the other three choices. The students will have to count up 8 times to get to 10.

From the aforementioned rationale, the PST was cognizant of the number patterns and the connections that she wanted the children to make. The results also revealed that one PST considered numbers that were easier to challenging but paid specific attention to the way the numbers would advance children’s mathematical understanding. Intentionally, she chose the following task for two kindergarten children:

**Student 1 (use their names in the real setting) has ___racecars. Student 2 gave student 1 ___more racecars. How racecars does student 1 have in total?**

**Number choices:**

- Student 1 adding to 10 (5,5) (7,3) (4,6) (15,5)
- Student 2 adding to 100 (50,50) (80,20) (35,65) (42,58)

To justify her number choices she explained:

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\(^{14}\) I noted that a number of PSTs were using the word “choose” instead of “chose” when referring to tasks that they had selected. I made a decision to edit the error in the document to avoid readers’ distraction.
(5,5) This set of numbers was chosen because doubles are one of the first things that students learn, so this problem is accessible to the students and will help the students become familiar with the problem while adding up to 10.

(7,3) This set of numbers was chosen because the students can count up from 7 by 3 to get to ten. So the problem is still accessible to the students but not with numbers that are easily mentally calculated like the previous (5,5) set.

(4,6) This set was chosen because it is a larger distance for students to count up from four to 10, so it is less accessible to the students and forces the students to think more about the prior knowledge and what they know about making the number 10.

(15,5) The reason for this number set is it increases the students’ thinking by making the answer higher than 10, but still applying previous knowledge such as the fact that 5+5 = 10.

(80,20) I chose this numbers because their adding together sums up to a decade, which is harder than doubles but easier than adding non-decades.

(35,65) These numbers were chosen because they are non-decade numbers but are still fairly accessible to the students because the students can use their knowledge of counting by 5’s.

(35,68) These number set was chosen because the numbers are not decades or fives and the child will be forced to use high level thinking because the answer will be less accessible to the child and the child will need to use their knowledge of adding up to 100 to find the answer.

The aforementioned quotation revealed that the numbers that this PST selected would probably progress individual students’ thinking. Specifically, she selected two different sets of numbers because the two children were not at the same level of mathematical understanding. She also considered different numbers that would sum up to 10 and 100 respectively. She considered the strategies that they might use, especially adding by doubles, counting up and even adding by 5’s. Although the multiple number choices could be characterized from easier to more challenging, the PST did not select the numbers randomly. Instead she paid careful attention to each number.
Considered the strategy that the children would use when solving the task

One PST selected a task with number choices that would support the children to identify a pattern and use doubling strategy. Below is an example of the task she selected:

**Problem:** Sara has ___ bags of candy bars with ___ in each bag. How many candy bars does Sara have?

Number choices:
- (2,5) (2,6) (2,3) (4,5)
- (2,10) 2,12) (4,3) (4,10)

To justify her choice of numbers she explained:

**Rationale:** The numbers that I chose in the main activity start by doubling one number then only one of the numbers in the next set would double. I was looking to see if Hilary would see that when one number is doubled in the number set then the answer for the sets would double.

It’s notable in the above explanation that the PST paid attention to the number choices, specifically, focusing on the strategy that the students would use (keeping one number constant while the second number is doubled), having anticipation that the children will recognize the pattern and use it. As discussed above, it’s probable that the PST modeled the tasks that had been provided to them in the course packet. Therefore, it’s premature to argue that the PST independently developed the task.

Although the PST discussed above paid attention to the number choices, it was concerning to note that the first two number sets and the last number set [(2,5)(2,10) (2,6) (2,12)(4,5) (4,10)] had the first number constant and the second one doubled while in the third set [(2,3) (4,3)] she changed to the 2\textsuperscript{nd} number remaining constant while the first number doubled. This would be tricky to a second grader because the first one implies that the number of groups remains the same while the size per group changes while the 3\textsuperscript{rd} set means the number of groups change but the size per group remained the same. It was not clear whether this was intentional or the PST did not consider the strategies that the child
would use. Prior research (e.g., Crespo, 2003) has also revealed that PSTs have a tendency of posing problems to children when they have not solved them or considered the different ways that the children might interact with the mathematical idea. So, there may be a possibility that the PST did not consider the way the children will interact with the mathematical idea.

Even if PSTs modeled the tasks that they had been provided with in the course packet, it’s still worth noting that their reasoning was clear and articulate. This results are encouraging and provide evidence that PSTs can put into consideration the strategies that the children will use as they choose a task.

**Considered children’s understanding or how the children will make sense of the problem**

Three participants considered children’s understanding and/or how the children would make sense of the problem. For example, one participant selected the following problem:

**Problem:** You had ___ cookies. Your brother ate ___ of your cookies. Now how many cookies do you have?
(20,4) (30,12)(15,9)(37,11)(21,49)(33,24)

To justify her number choices she argued:

**Rationale:** I chose the numbers that I did, because the girls could do decade-to-decade problems but couldn’t do non-decade-to-non-decade all of the time and Betty couldn’t do all the decade to non-decade problems.

Again, it is notable in the above explanation that the PST put into consideration what the children knew as she selected the numbers (doing decade-decade) and wanted to extend that to non-decade numbers. However, one of the number choices (21,49) could not be solved in the context of the problem that the PST selected. In other words, change result unknown, and one would not subtract a larger number from a smaller number. Again, it was
not clear the reasoning of the above quoted response when it came to that specific number choice.

In addition to considering students’ understanding, the other two participants also put into consideration how the children would make sense of the mathematical idea. For example, one of the PSTs selected the following word problem and provided the following explanation:

**Problem:** Mary has ___grapes and ___oranges. How many grapes and oranges does she have altogether?  
(3,1) (3,3) (3,5)

**Rationale:** Started with the same number of grapes every time so that they see that they don't have to start all over every time.  
(3,1) - Started off adding one since these are different problem types than in the opening number routine  
(3,3) - Stayed with the same first number to see if they can use prior knowledge from first problem to help them solve this problem. Also working on doubles.  
(3,5) - Want to see how to start the problem

This PST chose numbers that would support the children to build connections and make sense of the mathematical idea. Specifically, this numbers would have supported the children’s understanding that you always don’t have to start from as a counting strategy.

Finally, fourteen PSTs did not provide an explanation or the reasoning behind their number choices. This group provided different number choices but there was no written rationale why they chose the numbers. However, that does not mean that they did not have a rationale for the number choices.

**Summary**

These analyses reveal that the PSTs attempted to respond based on children’s mathematical understanding and they paid attention to the number choices. Even if there was evidence that some number choices were not thoughtfully considered, it was encouraging to
note that they did not just choose the tasks, but they thought about the connections that the
students would make with specific numbers. In other words, their rationale was based on
children’s mathematical understanding. However, it is not clear why some PSTs did not
provide the rationale for their number choices even though it was required in one of the
prompts. Probably, explaining the number choices was challenging since one need to
critically consider why you are selecting the numbers and how the children will make sense
of the numbers and/or the mathematical idea. In the next section, I examine the PSTs’
conceptions of productive tasks and/or tasks that engage children in high-level thinking.

**PSTs’ Conceptions of Productive Tasks and/or Tasks that Engage Students
in High-Level Thinking**

To explore the PSTs’ conceptions of productive tasks and/or tasks that engage
students’ in high-level thinking, I used the following prompts from the Inquiry into Student
Thinking assignment and the tutoring assignment:

1. Choose 2 tasks or problems that were particularly posed to the students that seemed
   particularly productive for advancing the thinking of the students as a group. (*Inquiry
   into Student Thinking assignment*)
2. Describe the activities, problems, literature etc. that you planned to use during this
   week’s tutoring session and explain your rationale for this plan. Included in your
   rationale should be: What makes these tasks high cognitive demand? (*Tutoring
   assignment*)

First, I will discuss the PSTs’ conceptions of productive tasks followed by the PSTs’
conceptions of tasks that engage students in high-level thinking in the tutoring assignment.

**PSTs’ conceptions of productive tasks**

To analyze the PSTs’ conceptions of a productive task, I coded their rationales into
advancing children’s understanding, extending students’ thinking, tasks that were based in
real-life connections so children could build connections, and others. The results show that
11 of 30 PSTs described a productive task as a task that advances children’s mathematical understanding, 5 of 30 described productive tasks as tasks that extend children’s thinking, while seven of 30 PSTs described productive tasks as tasks that were based in real life connections and children can build connections. The remaining participants’ (7 of 30 PSTs) descriptions of productive tasks varied and all the explanations were coded as others. I will briefly explain each of the characterizations below.

**Advancing children’s understanding**

The responses that were characterized to have described a productive task as a task that advances children’s mathematical understanding identified how the task helped the students understand a mathematical idea that they had not understood before. Further, I looked at explanations that tended to clarify how the task-supported student[s] to grasp the main mathematical idea, like counting by10. These include responses such as:

> It is in solving this problem that Jack and Emilio first represent 10 of something with a picture that does not involve making 10 individual marks or using 10 manipulative[s].

> It was then (referring to the moment that Sunny grasped the mathematical idea) Sunny grouped the tally marks into groups of tens, and when asked to solve another problem using the unifix cubes, they did so easily.

> The problem that asks the students to find how much money is left after the student spent 20 cents of his five dimes. Jack seemed to grasp the idea when he drew out five dimes, and circled two of them to take away. He understood that each dime was worth 10 cents, and the item he bought cost 20 cents.

Inherent in the above quotations is the PSTs’ idea that a productive task supports children’s understanding as they make sense and interact with the mathematical idea. Their conceptions tended to emphasize what the students did and how the interaction with the idea provided opportunities for learning. This conception was particularly interesting because the
PSTs based their explanation on the mathematical content, the students and the ways in which the students learned the mathematics. The conception is also worth further investigation to understand whether their conceptions influence the tasks that they select based on children’s mathematical understanding.

**Extending children’s thinking**

Responses that were coded to have described a productive task as extending students’ thinking identified tasks that had large numbers, which made it tedious and time consuming for the children to keep counting. This meant that the task made the children to start thinking of alternative ways to solve the task, eventually extending their thinking in a variety of ways. In some cases, the PSTs realized that the teacher posed the task to steer the children away from counting by one while other tasks made the children realize that it is easier to count by 10 rather than one by one. The following examples represent some typical explanations of the PSTs who claimed that a productive task should extend students’ mathematical thinking:

The use of the dime is a great way to continue thinking about 10s.
I also liked how money encourages different ways to see the same problem such as $10+10+2=22$ is equivalent to $10+5+5+2=22$.
I think this task was productive because the teacher chose large numbers (11x10) in an effort to guide students away from using tallies (which worked). This task was also productive because by using a stick to represent ten “cookies” Sunny was able to see the “ten representing one” concept and successfully counted by tens...

The examples reveal the PSTs’ understanding that a productive task should help the students explore the task and the mathematical idea in ways that they had not done before.
**Tasks that are based in real-life connections so children can build connections**

Although the descriptions differed, seven PSTs described a productive task to be a task that is based on real-life connections and, to some extent, can help children build connections. To explain why the task was productive, one PST argued:

They had a real life object they could compare their thinking to and it got them thinking about numbers grouped together. “Oh you mean a dime is the same as ten cents.” This opened them up to thinking about how one object could represent a number of other objects such as the candy having a number of pieces in each box.

This explanation reveals the PST’s acknowledgement that a task with real-life connections helps the students interact with mathematics in a more meaningful way. Specifically, in the above quotation the PST argued that the task got the children thinking about how one object could represent a number of other objects. Similarly, other PSTs explained that a productive task is a task that gives children an opportunity to “visualize the objects” and/or “make connections to real-world problems and understand the concept of base ten numbers”. Specifically, one PST explained “For the first time, Jack demonstrated understanding of base ten concepts by drawing five dimes and knew that each dime represented ten cents because he crossed out two dimes when the problem said the person spent twenty cents.”

Inherent in the above quotations is the PSTs’ reasoning that a task based on real setting helps the students understand mathematics better. In other words, they recognize the importance of choosing tasks based on real life setting and the affordances thereof that this type of tasks would offer as students learn mathematics. It would be interesting to explore how this knowledge impacts on their choice of tasks.
Other

Finally, there are some responses that could not be classified under any one of the above categories. For example, one PST described a task as productive because it helped the teacher realize what the problem was and he changed the problem to a problem that the children could access. It was not clear from the explanation whether the task, which confused the child, was productive or whether the reformulated task was productive. In other responses, the PSTs argued that the task was productive because of what the teacher did to support students understanding of the task. For example in her explanation, one PST remarked:

The teacher really wanted the students to see each other’s way of thinking instead of working things out on their own. The teacher made 30 tallies for the problem, which represented Jack’s way of thinking. When it was laid out, Sunny noticed that the 30 tallies could be easily grouped into 3 sets of 10. This helped Sunny because at the end of the study she notices that 3 groups of 10 would be equal to 30 and can use this to solve other problems. I think this helped Jack see that 10 is a base unit because after this problem, he used strips to represent 10 instead of counting by 1’s.

Again in this explanation the PST focused on what the teacher did with the task. Other responses that were coded under this category included responses that argued that the task was productive because 1) the students got the right answer; 2) it was a great problem to help the teacher assess the students; and/or 3) the task has the potential to engage students and progress their understanding to more complicated strategies.

Overall, these results reveal that PSTs’ conceptions of a productive task varied. Particularly, it was interesting to note that two-thirds of the participants’ conceptions revolved around mathematical content and the affordances that tasks will give students as they engage with the mathematics. These ideas seem to be good first steps towards
developing their understanding of how to choose a worthwhile mathematical task. In other words, the results are suggestive that the PSTs were gaining insights that would probably help them when selecting mathematical tasks in their own classrooms. However, since we had not given them a pre-instruction assessment, it would be premature to determine the effect of the course activities on their understanding of productive and/or unproductive tasks.

**PSTs’ Conceptions of Tasks that Engage Students in High-Level Thinking**

To analyze the PSTs’ conceptions of tasks that engage students in high-level thinking, I used the responses to the last part of the following prompt:

> Describe the activities, problems, or literature that you planned to use during this week’s tutoring session and explain your rationale for this plan. Included in your rationale should be: What makes these tasks high cognitive demand?

In this prompt, the PSTs were supposed to provide explanations of why they think that their choice of task is of high cognitive demand. Specifically, Smith and Stein (1998) and Stein and Smith (1998) describe tasks of high cognitive demand to be tasks that engage students with procedures with connection to meaning or involve students in “doing mathematics” (see Table 3.6).

I coded the PSTs’ responses into four categories: (1) Considers what the students will do as they engage with the mathematical idea and the cognitive effort required to engage with the task; (2) Considers what the teacher will do as they instruct the students; (3) Rationale of the task choice (why did I choose the task), and (4) No rationale.\(^\text{15}\)

\(^{15}\) I would like to notify the reader that the analysis at this stage was not examining whether the PSTs’ conceptions were right or wrong, but it was meant to provide insights into their conceptions and understanding of criteria for high cognitive demand tasks. Whether the conceptions were right or wrong is not within the scope of this study.
The results show that 12/30 participants explained that the tasks they selected and/or generated were of high cognitive demand because of what the students will do as they engage with the task. Specifically, the PSTs considered the task high demand if the children are going beyond adding numbers and noticing patterns with the selected numbers. In other instances, the PSTs explained that the task is of high demand because “students were required to explain their answers.”.

These explanations were particularly interesting because the responses were suggestive that the PSTs’ understanding of a task of high demand included the thinking process that the children will engage in as they solve the task. Although prior research has shown that selecting a task of high demand does not necessarily mean that the students will access those opportunities (Stein, Grover & Henningsen, 1996) one’s conception of task that involves high level thinking might influence the nature of task that they would select. Therefore, these results are encouraging to see that approximately one-third of the participants thought about a high level task as a task that will challenge the children’s thinking.

The results also show that 3 of 30 participants considered selected tasks to be of high cognitive demand because of what the teacher will do when instructing the students. For example, one participant argued that the selected tasks are of high cognitive demand because “I am teaching them the patterns that go along with it. By having the children to do this they are needed to cognitively be able to grasp the ideas”. Similarly, the other two participants argued that the task is of high cognitive demand because they are urging the students “to use relational thinking” and “I would like to work on with this two students developing their knowledge of multiplication and division as well as strengthening their knowledge and
application of base 10”. These conceptions can be concerning, bearing in mind that these PSTs did the tutoring assignment after nine weeks of instruction. Specifically, they had used the levels of cognitive demand task sort framework individually and in small group activities.

Further, 6 of 30 participants considered the selected task to be of high cognitive demand because of the nature of the task. Specifically, one participant in this category argued that the task is of high cognitive demand if “it can be presented in multiple ways and can be solved as a backward multiplication problem or division problem”. Other explanations included “By making that mathematical connection between the activity and the equation, the lesson requires high level thinking” and “Because the instruction for each sheet was open ended and allowed the students to come up with their ways to solve and complete each worksheet”.

Finally, 9 of 30 PSTs did not explain what made their tasks to be of high cognitive demand even though they had been prompted to do so. Because of the limitations of written responses, it is not possible to make conclusions about why they did not provide any explanations.

In summary, these results show that the PSTs had varied conceptions of what makes a task productive and/or what makes a task a high-level or low level of cognitive demand. It was also interesting to note that none of the PSTs included statements in their rationales suggesting that a productive task involves more than one of the conceptions identified. In other words, the PSTs provided only one reason why the task was either productive or of high-level thinking. It would be interesting to explore how their conceptions impacted on their decisions on how to respond.
In the next section, I further elaborate on the PSTs’ performance on the three component skills using emerging clusters that are based on the patterns identified in section one. I use specific PSTs’ responses that are included in the emergent clusters as illustrative examples of PSTs’ performance as they progress in the methods course. In other words, I use multiple-case analysis of four emerging patterns that include PSTs’ responses that had clear manifestation of these clusters. My goal in this case analysis is to provide more detailed explanations and understanding of the PSTs’ responses in the three component skills as they progress in their methods course.

**Multiple-Case Analysis of Four Emerging Clusters of PSTs’ Responses**

This section is organized in two parts. In the first part, I introduce the four cases that I selected for further analysis and explain the criteria that informed their selection. The second part discusses the cases under investigation. Each case under investigation shares the same structure. In each case, I discuss how the PSTs included in the cluster attended to children’s strategies, interpreted children’s mathematical understanding and responded based on children’s mathematical understanding in the Inquiry into Student Thinking assignment and the tutoring assignment.

**Selection of the four cases**

As discussed in Chapter 3, the four cases were selected based on the emerging clusters that represented various manifestations of the PSTs’ capacity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding. Based on the coding scheme, I clustered the PSTs’ responses into four emerging clusters, which were
representative of PSTs’ responses in the Inquiry into Student Thinking and tutoring assignments, respectively (see Table 3.10).

First Cluster: Noticed and used children’s mathematical understanding during the Inquiry into Student Thinking and Tutoring assignment

This cluster includes PSTs who attended to children’s strategies, interpreted and responded based on children’s mathematical understanding, categorized as “noticed and used”. In other words, the PSTs in this cluster provided most mathematical details as they attended to children’s strategies, provided robust evidence as they interpreted children’s mathematical understanding and gave robust or limited evidence that their response was based on children’s mathematical understanding, both in the Inquiry into Student Thinking and the tutoring assignment. In total, there were eight participants in this cluster.

Second Cluster: Noticed and did not use both in Inquiry into Student Thinking and Tutoring assignment

The second cluster includes PSTs who noticed and attended to children’s strategies, interpreted children’s mathematical understanding but did not choose tasks based on children’s mathematical understanding, categorized as “noticed and did not use”. In other words, the cluster includes PSTs who provided most or some mathematical details as they attended to children’s strategies, provided robust or limited evidence as they interpreted children’s mathematical understanding but there was lack of any evidence that the decision on how to respond was based on children’s mathematical understanding in both the Inquiry into Student Thinking and the tutoring assignment. In total there were five participants in this cluster.
**Third Cluster: Did not notice or use during the Inquiry into Student Thinking assignment but noticed and used during the Tutoring assignment**

The third cluster includes PSTs who did not attend to children’s strategies, did not provide any evidence of interpreting children’s mathematical understanding, and did not provide any evidence of deciding how to respond based on children’s mathematical understanding. This cluster was categorized as “did not notice or use” during the Inquiry into Student Thinking assignment but “noticed and used” during the tutoring assignment. Specifically, the PSTs considered in this cluster had limited or lack of any mathematical details when attending to children’s strategies, had limited or no evidence of interpreting children’s mathematical understanding and, had limited or no evidence of basing their choices on children’s mathematical understanding during the Inquiry into Student Thinking assignment but made notable progress during the tutoring assignment. That is, they noticed and provided most mathematical details, there was robust evidence that they interpreted children’s mathematical understanding and/or misunderstanding and the task selected was based on children’s mathematical understanding during the tutoring assignment. In total there were eight participants in this cluster.

**Fourth Cluster: Includes PSTs who had no consistent pattern in either the Inquiry into Student Thinking or Tutoring assignments**

The PSTs’ responses seemed sporadic as they responded to the prompts. In other words, some of the responses had most mathematical details the component skill of attending to children’s strategies, and sometimes no evidence or limited evidence that they interpreted children’s mathematical understanding, and robust, limited, or no evidence that their responses were based on children’s mathematical understanding in the Inquiry into Student
Figure 4.1 illustrates the emerging four clusters, and a summary of the prompts that provided evidence is provided in Table 4.6. In total, there were nine participants in this cluster. My goal was to create a multiple-case analysis (Baxter & Jack, 2008; Stake, 2000) that would provide explanations of identified patterns of PSTs’ responses at two time periods during their methods courses; that is, in the Inquiry into Student thinking assignment that was done after six weeks of instruction and the tutoring assignment, which the PSTs turned in after ten weeks of instruction as they progressed through their methods course.

Figure 4.1. Summary of emerging clusters in the PSTs’ responses
Table 4.6. Summary of the prompts used to provide evidence

<table>
<thead>
<tr>
<th>Component Skill</th>
<th>Inquiry into Students’ Thinking</th>
<th>Tutoring Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attending to children’s strategy</td>
<td>Choose one of the four students from the case study and trace their responses and learning through the study.</td>
<td>What does each student know, think and understand about number, operations and problem solving?</td>
</tr>
<tr>
<td></td>
<td>- Summarize what you think they knew or understood at the end of the study that they did not know or understand in the beginning</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Choose 2 examples of this student’s responses as evidence for your claim from part and explain how they support your claim about this student’s learning. (You will probably want one example toward the end of the study and one example toward the beginning)</td>
<td></td>
</tr>
<tr>
<td>Interpreting children’s mathematical understanding</td>
<td>Summarize what you think they knew or understood at the end of the study that they did not know or understand in the beginning.</td>
<td>How will what you learned in the interviews influence how you work with the children during the next sessions?</td>
</tr>
<tr>
<td></td>
<td>Choose 2 examples of this student’s responses as evidence for your claim from part and explain how they support your claim about this student’s learning. (You will probably want one example toward the end of the study and one example toward the beginning)</td>
<td></td>
</tr>
<tr>
<td>Deciding how to respond based on children’s mathematical understanding</td>
<td>If you were to teach the next lesson to this group of students, write one problem that you could give them and explain why you think this would be a good problem for this group</td>
<td>How will what you learned in the interviews influence how you work with the children during the next sessions?</td>
</tr>
<tr>
<td></td>
<td>How are these tasks intended to build on what you know about your students’ understanding and misunderstanding?</td>
<td></td>
</tr>
</tbody>
</table>

1st Cluster: Noticed and Used Children’s Mathematical Understanding in the Inquiry into Student Thinking Assignment and the Tutoring Assignment

**Jodie**

Jodie, the case considered in this cluster, performed high in the three component skills in the Inquiry into Student Thinking assignment and the tutoring assignment. For the component skill of attending to children’s strategies, she provided most mathematical details. For the component skill of interpreting children’s mathematical understanding, her responses had robust evidence that she made sense of details of children’s strategy and noted how the details reflected children’s understanding. For the component skill of responding based on
children’s mathematical understanding, her responses were coded to have robust evidence
showing that she considered children’s understanding as reflected in particular strategy used, 
and how the next task could further the specific children’s understanding. In the next part, I highlight her responses during the Inquiry into Student Thinking assignment and tutoring assignment respectively.

Performance in the Component Skills of Professional Noticing of Children’s Mathematical Understanding in the Inquiry into Students’ Thinking
Attending to Children’s Strategies

As indicated previously, Jodie’s responses for the first prompt in the Inquiry into Students’ Thinking assignment were coded to have most mathematical details for the component skill of attending to children’s mathematical understanding. For example, in analyzing Jack’s mathematical understanding she provided the following details:

At the end of the study, Jack not only understood place value, but more importantly Jack understood the idea that ten is a unit. He is also able to create number sentences as a strategy for solving problems. At the beginning of the study, Jack used direct modeling and counted by ones to get his answers. Towards the end of the study, it was clear that Jack was beginning to understand subtraction using the basic algorithm, as he used his solution to a problem and attempted to use the basic algorithm.

As this quotation suggests, Jodie identified the different strategies that Jack had used to solve the task (direct modeling and counting by ones). She seemed to understand the different strategies that the child [Jack] used at different times in the case study and articulated the child’s learning progression of the different strategies. Specifically, she referenced the problem regarding three bags of soccer balls that reflected Jack’s understanding of base 10 at the beginning of the case study. In her explanation she remarked:

Jack direct modeled the problem by drawing all of the balls individually. Though he drew the soccer balls into three groups of ten, he did not count by tens to obtain his answer. He instead counted each ball individually and
miscounted, causing him to get the answer incorrect. This example shows that Jack did not understand or use the base-10 principle that ten is a unit.

This explanation identified the strategy that Jack used and provided detailed explanation of the strategy. In addition, Jodie explained that Jack showed progress in his understanding of the base 10 concepts in the case study. In her explanation she commented:

A little bit into the study, Jack begins to show progress. In a problem about packages of cookies, Jack began the problem by direct modeling and representing each individual cookie with a circle. However, when he counted these cookies, he counted each package as ten; he no longer had to count each cookie in every package.

The quotation suggests that Jodie was cognizant of the progress that Jack was making in his understanding of base 10 concepts and the progression of the strategies from direct modeling to grouping the items by 10. In addition, using an example, she provided details describing how Jack solved the task at the end of the case study as evidence of his mathematical understanding. For instance, to support her argument that Jack understood the concept of base 10 at the end of the study Jodie explained:

By the end of the study, Jack had a good grasp of the idea that ten is a unit, as shown in the ‘candy roll’ problem given on his last day. This time, Jack draws out 8 rolls of candy, and writes out ‘10’ above each roll. He not only counts by ten to obtain the answer, but also is able to create a number sentence. Here, Jack also used a number sentence as a way to solve the problem. Using this strategy is noteworthy for Jack because it shows he no longer needs to represent each individual unit to make a ten; therefore, he understands that ten is a unit.

Notable in the this quotation is the fact that Jodie made an assertion that Jack had a good grasp of ten as a unit by the end of the study. She also provided details of how Jack solved the problem (This time, Jack draws out 8 rolls of candy, and writes out ‘10’ above each roll. He not only counts by ten to obtain the answer, but is also able to create a number
sentence). Eventually, she concluded that the strategy that Jack used shows that “he no longer needs to represent each individual unit.”

Therefore, I characterized her response to have details of the mathematical essence of the strategies used and details of how Jack made sense of and interacted with the mathematical idea. Specifically, she noted what Jack understood at the beginning of the case study, the point at which he showed some progress in developing the concept of 10 as a unit and his understanding of base 10 at the end of the case study.

**Interpreting children’s mathematical understanding**

Jodie’s interpretation of children’s mathematical understanding and misunderstanding during the Inquiry into Student Thinking assignment had robust evidence of interpretation of children’s understanding. In other words, Jodie made sense of details of Jack’s strategies, and noted how the details reflected his understanding. As discussed above, Jodie provided examples with vivid details of the strategies that Jack used, and interpreted his understanding by providing details of what he understands and what he does not understand. While explaining Jack’s strategy she remarked:

In the particular problem regarding three bags of soccer balls, Jack directly modeled the problem by drawing all of the balls individually. Though he drew the soccer balls into three groups of ten, he did not count by tens to obtain his answer. He instead counted each ball individually and miscounted, causing him to get the answer incorrect. This example shows that Jack did not understand or use the base-10 principle that ten is a unit...

By the end of the study, Jack has a good grasp of the idea that ten is a unit, as shown in the ‘candy roll’ problem given on his last day. This time, Jack draws out 8 rolls of candy, and writes out ‘10’ above each roll. He not only counts by ten to obtain the answer, but also is able to create a number sentence. Here, Jack also used a number sentence as a way to solve the problem. Using this strategy is noteworthy for Jack because it shows he no longer needs to represent each individual unit to make a ten; therefore, he understands that ten is a unit.
In the preceding quotation, Jodie contrasted Jack’s understanding at the beginning of the case and the end of the case study. Further, she made sense of details of the strategies. A good example is her remark that “Though he [Jack] drew the soccer balls into three groups of ten, he did not count by tens to obtain his answer”. Likewise, she noted how the details reflected his understanding, arguing that “Using this strategy is noteworthy for Jack because it shows he no longer needs to represent each individual unit to make a ten; therefore, he understands that ten is a unit”

**Responding based on children’s mathematical understanding**

Jodie’s conception of a productive task is a task that advances children’s mathematical understanding. When prompted to select one task she thinks is a productive task in the Inquiry into Student Thinking assignment, Jodie selected the task Problem Set #9. In this set, the teacher had made true/false number statements (22 + 10= 30 and wanted the students to determine whether or not the statement was correct Jodie argued that the task was productive because the children advanced their thinking using the task. In her explanation she remarked:

Sunny’s response to the number sentence 22+10=30 shows that she has made progress. For the first time, Sunny used relational thinking to get her response, and did not count by ones to obtain the answer. She not only realized that 20+10=30, therefore making the number sentence correct, but also explained what had to be done to the problem for it to be true. She explained that since 20+10=30, the extra 2 from 20 needed to be taken away from the ten, so to balance the numbers. She decided that 22+8=30.

The explanation above shows that Jodie was cognizant of tasks that would advance children’s understanding. She identified the task from the case study and provided an explanation why she thought the task was productive. It was interesting to note that her
conception of a productive task was related to students’ learning. In other words, she thought about specific students’ learning and how the task advanced their learning. In addition, Jodie argued that a task is of high cognitive demand if the task will engage students with high cognitive thinking. This explanation was also interesting because she focused on students learning as she thought of a task engaging children in high-level thinking.

For the component skill of responding based on children’s mathematical understanding, Jodie responded to the following prompt in the Inquiry into Student Thinking assignment: *If you were to teach the next lesson to this group of students, write one problem that you could give them and explain why you think this would be a good problem for this group.* Jodie selected a join result unknown (Carpenter et al., 1999) word problem:

Problem: Sunny has ____ fish, and then she buys ____ more. How many fish does she have now?” Number choices would include {(10, 50) (20, 30) (10, 41) (15, 25)}. Rationale: This type of problem would be good for all of the students. Sunny and Daniella struggle to count by tens past the numbers 20 and 30, and this problem challenges them to do so. Emilio would be challenged to count by tens and keep track of the ‘one’ in 41. Both Jack and Emilio would be challenged by the last number choice, as both understand the concept of counting by tens, but they would have to extend their understanding to non-zero ending numbers.

Jodie’s reasoning and rationale showed that her decision on how to respond was based on children’s mathematical understanding. Notable in her explanation is the fact that she considered how the task would advance individual students’ understanding. In other words, she considered how the task would advance Sunny and Daniella’s understanding arguing that “Sunny and Daniella struggle to count by tens past the numbers 20 and 30, and this problem challenges them to do so”. She also considered how the task would challenge Jack and Emilio explaining; “Emilio would be challenged to count by tens and keep track of the ‘one’ in 41. Both Jack and Emilio would be challenged by the last number choice, as both
understand the concept of counting by tens, but they would have to extend their understanding to non-zero ending numbers”. Notice in this quotation, that Jodie intended to use the task to extend and challenge students’ thinking after she had evaluated what they knew and what they did not know.

Therefore, I coded her responses to have demonstrated robust evidence that she based her choice of task on students understanding. Further, she gave careful consideration to the specific number choices. Specifically, she chose numbers that extended individual students’ understanding and responsive to students thinking.

In summary, Jodie’s responses represents the PSTs who provided evidence that they paid careful attention to children’s strategies and interpreted children’s mathematical understanding during the Inquiry into Students’ Thinking assignment. This cluster of PSTs also used what they noticed as they decided how to respond during the Inquiry into Students’ Thinking. The results are intriguing because the component skill of responding based on children’s mathematical understanding is particularly challenging even to practicing teachers. In the next part, I will discuss Jodie’s responses in the tutoring assignment.

Tutoring assignment

Attending to children’s strategies

During the tutoring assignment, Jodie had an opportunity to interview and to teach two children [Jessica and Patty] who are in 3rd and 4th grade respectively. After the interview, Jodie responded to the following prompts: What does each student know, think and understand about number, operations and problem solving? In the next section, I will use her
analysis of the interview questions to reflect on her ability to notice and attend to children’s mathematical thinking.

Jodie’s analysis of the two students’ solutions had mathematical details and she made assertions about each student’s [and sometimes both students’] understanding and/or misunderstanding. For example, in her analysis of Jessica and Patty’s ability and understanding of equations she remarked:

Both Jessica and Patty used the breaking apart method and succeeded when adding up numbers. For the problem 249 + 367 = 247 + 369, both girls failed to look and see that because 247 is two less than 249 but 369 is two more than 367, the answers would be the same. Instead they broke the numbers apart by place value 200 + 300 = 500, 60 + 40 = 100 and 7 + 9 = 16.

In this quotation, Jodie provides details of the strategy that the children used and how they made sense of the mathematical idea. Using the children’s work she asserted, that both children are at the same level of mathematical thinking and reasoning because they both demonstrated fluency in counting and they did not take a lot of time to think about what will come next.

In addition to making assertions, Jodie also identified what the children did not understand. She argued that the children [Jessica and Patty] had a well-developed understanding of subtraction of numbers, but when presented with a problem where the subtrahend has larger unit than the minuend, the children solved the problem incorrectly. Consider the following explanation of Jessica’s strategy:

In the problem 583–265 = 593–275. I asked that she explain her work and this is what she said 500-200= 300, 80-60 is 20. So 3-5 is -2. So then I added them together to get 322. She had the same reasoning for 593 – 275 and again said 3-5 was -2 but added 2 instead of subtracting 2 from the total…Patty made similar type of error though she changed the one unit from number to number.”
This quotation shows that Jodie captured the mathematical essence of the strategy that Jessica used (breaking apart strategy) and provided a detailed explanation of how the child made sense of the strategy; “I asked that she explain her work and this is what she said 500-200= 300, 80-60 is 20. So 3-5 is -2”. In addition, she made observations about what she did not understand or the error that she made commenting, “She had the same reasoning for 593 – 275 and again said 3-5 was -2 but added 2 instead of subtracting 2 from the total…Patty made similar type of error though she changed the one unit from number to number”.

Likewise, Jodie observed that Jessica and Patty succeeded in using the breaking apart strategy of solving equations when adding up but not when they were subtracting numbers. The explanation from the children’s work was detailed and specific on what the students did. In addition, she looked at other alternative strategies that she anticipated the children could use, and analyzed what the children failed to notice. For example, her analysis on how the children solved the equation read as follows.

For this section both Jessica and Patty used the breaking a part method and succeeded when using addition. For the problem 249+367 = 247 +369, both girls failed to look at the numbers and see that because 247 is two less than 249 but 369 is two more than 367, the answers would be the same. Instead they broke the numbers apart by place value 200+300 = 500, 40+60 = 100 and 7 + 9 = 16.

In conclusion, these results indicated that Jodie provided detailed explanations of the strategy that the children used, and again captured the mathematical essence of the strategy in the component skill of noticing and attending to children’s strategies after ten weeks of instruction in the methods course. She became more specific and articulate as she responded to the prompts during the tutoring assignment. In general, there was evidence that she provided mathematical details during the two assignments
Interpreting children’s mathematical understanding

Comparing the Inquiry into Student Thinking and the tutoring assignment, Jodie’s ability to interpret children’s mathematical understanding was almost the same, bearing in mind that there was robust evidence that she paid attention to the children’s strategies in both assignments. During the tutoring assignment, she focused on individual students’ strategies and their corrective understanding and misunderstanding. For example, while explaining Patty’s strategy of solving the relational thinking problem, Jodie remarked:

In the problem $82 - \_ = 83 - 48$, I was excited by her relational thinking. She said that because $83$ is one more than $82$, the answer must be $47$ because $47$ is one less than $48$. When I asked her to double-check her thinking and actually solve the problem by writing out the answer, I was surprised that she said the following (for $83-48$) $80-40 = 40$ and $8-3 = 5$. Then I add $40$ and $5$ together to get $45$. I asked Patty why this answer was different with her previous one and she decided that her previous one was wrong. In other problems of this type Patty makes the same type of errors and always subtract the smaller ones unit from the larger ones…

Considering Patty’s strategy, Jodie was concerned whether Patty really understood the strategy she was using. She acknowledged the two strategies and puts forward a clear distinction of the misconception that Patty had with relational thinking. Further, she made an assertion that the two children [Jessica and Patty] made similar errors when it comes to subtracting numbers explaining that: “In subtraction problems of this kind, Jessica continued to break the numbers apart and have negatives but always added them to the total, thus causing her to get the wrong answer”. Analyzing Patty’s strategy, Jodie concluded that in other problems of this type [relational thinking problems] “Patty makes the same type of errors and always subtract the smaller unit from the larger ones regardless of which unit belongs to which number”.

Similar to the component skill of attending to children’s strategies, Jodie’s responses had robust evidence that she made sense of details of children’s strategies and noted how the strategies reflected individual children’s understanding. Specifically, she paid careful attention to the mathematical details of the task, identified children’s understanding and paid attention to the errors that the students made.

**Responses based on children’s mathematical understanding**

This section addresses Jodie’s response to the following prompt:

1). How will what you learned in the interviews influence how you work with the children during the next sessions?

2). How are these tasks intended to build on what you know about your students’ understanding and misunderstanding?

As stated in her lesson plan, Jodie’s main goal was to present the students with problems in which the subtrahend has some units larger than the minuend. She anticipated that the students would use relational thinking to develop their understanding of standard algorithm. While interpreting children’s mathematical understanding, Jodie had explained that Jessica and Patty understood the breaking apart method when adding up numbers but always got the answer wrong when they used the same strategy for subtraction. During the interpretation of children’s mathematical understanding, Jodie made the following remark and provided an example that supported her analysis:

Both girls have a well-developed understanding of subtraction but when presented with a problem where the subtrahend has some larger units than those in the minuend, the girls solved the problem incorrectly. For example, in the problem 583 – 265 = 593 – 275, Jessica declared it was true. I asked her to explain her work and this is what she said; 500-200 = 300; 80-60 = 20 and 3-5 = -2. So then I added together to get 322.

To respond to children’s mathematical understanding, Jodie selected three problems listed as follows:
Opening number routine

250-130
250-131
250-135
252-135

Main Activity

1). Patty made ___ bracelets. She gave away ___ to Jessica and other friends. How many bracelets does Patty have left?

Number choices: A (250,135) B (100, 76) C (1000,458) D (1781,897)

Justification

250, 135) connected the opening number routine to the main activity and urges students to continue thinking in the mind set that was established early in the lesson
100, 76) challenges students because both the tens and the ones places in the subtrahend are larger than the units in the minuend
(1000, 458) challenges the students because the hundreds tens and ones places in the subtrahend are greater than the responsible units in the minuend; here the students must think about all the three place values and not only ones.
(1781, 897) challenges the students because the minuend does not have any zeros in it and all three digits of the subtrahend are larger than those in the minuend

2). Joan has __ brownies to share with ___ friends. How many brownies does each friend get? How many brownies does each friend get? Joan is not going to eat any.
Number Choices: A (16, 4) B (32, 4) C (13, 4) D (100,4)
16, 4) will allow me to see if the students can do basic division without any reminders
32, 4) will allow me to see few things: if the students can see basic division, know their division basic facts and if they can use relational thinking to see that 32 is 2 times more than 16 and so the answer for b will be twice the answer for A. Will they start from the beginning or add to the previous answer?
13, 4) will allow me to see whether the children can solve the remainders and whether they will keep it as a reminder, fraction or decimal
100, 4) will allow me to see if the students can use any strategy or methods to solve the problem for larger numbers besides direct modeling the problem.
Jodie’s rationale for choosing the three problems focused on children’s understanding and/or misunderstanding. In her rationale she explained:

Both my opening number routines and separate result unknown problem build on what the students already know and their misunderstanding. My students used the breaking apart method frequently but struggle to get the correct answer when subtracting. Both activities have number choices in which the subtrahend has some units larger than the minuend, which is when the mistake typically occurs for the students… I hope to see thinking that is not limited to the break a part method and those other strategies are used. If students use that method, I hope to see them using it correctly.

As already explained, Jodie’s response was cognizant of children’s mathematical understanding. As she remarked, “I chose to give a separate result unknown problem type because in the initial interview, the students did not get this problem correct”. In addition, her explanation of the number choices showed that she paid careful consideration on the errors that Jessica and Patty made when subtracting numbers. She remarked that all the number choices would extend and challenge the two children’s mathematical understanding because “they increased the level of complexity”. In other words, the tasks progressively had more units in the subtrahend than those in the minuend and Jodie intended to use the tasks to support students’ understanding while subtracting numbers with regrouping.

Summary

Jodie’s ability to notice and attend to children’s strategies, interpret children’s mathematical understanding, and respond based on children’s mathematical understanding provides an example of the PSTs’ ability to notice and use children’s mathematical understanding to select mathematical tasks. In this section, I have described how Jodie was able to closely interpret children’s mathematical understanding or misunderstanding and make sense of the children’s solutions in order to identify the errors and misconceptions.
Further, she made assertions about children’s understanding and/or misunderstanding and based her selection of the next problems and tasks on what the children understood.

It was also notable that Jodie’s responses militate around the course content. Before she completed the tutoring assignment, the PSTs had already discussed the different types of word problems and the strategies that children commonly use. Purposefully, the instructor had introduced the students to this nature of tasks and had given the PSTs an opportunity to analyze students’ work and to view a clip of a teacher discussing the children’s strategies. Therefore, I inferred that the course content had impacted on the way Jodie responded to children’s mathematical understanding.

Further scrutiny of the choice of tasks and rationale reveals that Jodie closely mirrored the nature of tasks that were included in the course packet. This observation may have two different implications. First, Jodie may have understood the nature of tasks that she would pose to advance children’s understanding using the scaffolds given in the course packet. That would be quite encouraging to note that she had the potential to attend to children’s strategies, interpret and respond to children’s mathematical understanding at least by the time they did the tutoring assignment. Second, this observation may also mean that Jodie used the tasks that PSTs were given for the interview session to come up with new tasks for the children. Therefore, it would be premature to suggest that Jodie selected and generated the tasks on her own.

In this study, Jodie’s case is worth noting because she represents the PSTs who noticed and attended to children’s strategies, interpreted children’s mathematical understanding and provided robust evidence that her response was based on children’s mathematical understanding and/or misunderstanding consistently, both in the Inquiry into
Students’ Thinking assignment and the tutoring assignment. In total, there were 8 of 30 PSTs from the sample who consistently exhibited their potential to notice and attend to children’s strategies, interpret children’s mathematical understanding, and respond based on their interpretation of children’s mathematical understanding during the two assignments. As discussed later in Chapter 5, the performance of the PSTs included in this cluster contrast Jacobs et al.’s (2010) findings, where none of the prospective teachers provided robust evidence of interpreting children’s mathematical understanding and/or deciding how to respond based on children’s mathematical understanding. This study shows that some PSTs had the capacity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding.

2nd Cluster: Noticed but did not use Children’s Mathematical Understanding while Responding

Katherine

The PSTs considered in this cluster (5/30 PSTs), performed at high levels in two of the component skills in the Inquiry into Student Thinking assignment and tutoring assignment. In other words, for the component skill of attending to children’s strategies, they provided most mathematical details, meaning that their responses were coded to have detailed explanation of the strategy that the child used, details of the mathematical essence of the strategy and/or details of how the children made sense and/or interacted with the mathematical idea. For the component skill of interpreting children’s mathematical understanding PSTs’ in this cluster demonstrated robust evidence that they made sense of details of children’s strategy and noted how the details reflected children’s understanding. However, their responses demonstrated no evidence that they considered children’s
understanding as reflected in a particular strategy used, and/or how the next task could further the specific children’s understanding. In the next part, I highlight Katherine’s responses during the Inquiry into Student Thinking assignment and tutoring assignment respectively.

**Performance in the Three Component Skills during Inquiry into Student Thinking**

When prompted to choose one task that was particularly productive in advancing the children’s mathematical thinking, Katherine chose the following problem:

**Problem:** Sunny has 11 packages of cookies. Each package has 10 cookies in it. She also has 5 extra cookies. How many cookies does she have in all?”

**Rationale:** I think this problem was so productive for the students because it involved larger numbers, which made it tedious and time-consuming to draw out each individual cookie.

Using this rationale, I described her conception of productive tasks as tasks that challenge children’s mathematical thinking. Notice that in the quotation above she argued; “the problem was productive because it involved large numbers”. In the tutoring assignment, Katherine argued that the task she selected was of high cognitive demand because it involved some cognitive effort and although general procedures may be followed they cannot be followed mindlessly. Therefore her understanding of tasks that engage students with high-level thinking was coded under “considering what the students will do”. In other words she considered how the children could interact with the mathematical idea.

**Attending to children’s strategies**

To respond to the first prompt in the Inquiry into Student Thinking, Katherine chose to evaluate Jack’s mathematical understanding. In her explanations, Katherine made two assertions about Jack’s understanding and misunderstanding. First, she explained that at the
beginning of the study, Jack had a hard time counting by anything but ones. Further, she supported her argument by providing an example that illustrated Jack’s strategy at the beginning of the study. In her explanation she remarked:

The first example I will use for evidence came from problem set three. The problem was Join Change Unknown, and he needed to get to fifty pennies from twenty-two pennies. Although he solved the problem handily, he did not solve it in the most efficient way. He counted up by ones from twenty-two using tallies to keep track. He made no use of tens.

I coded her explanation to have most mathematical details because in her explanations there was evidence that she provided details that reflected that she paid attention to the specific strategies that Jack used (he counted up by ones from twenty two using tallies to keep track). She also argued that Jack did not solve the task in the most efficient way. Finally, she made an assertion that Jack made no use of tens at the beginning of the study.

Further, Katherine described Jack’s mathematical understanding at the end of the case study, concluding that Jack understood number concepts and processes. In her explanations, she identified examples with detailed explanations and representation of Jack’s strategies at the end of the case study. For example, in her explanation she commented:

Jack used one unit (rectangle) to represent a group of ten candies. As shown in figure 4.2, he initially drew eight rolls (rectangles) and drew the extra twelve candies individually. As he was counting them, he self-corrected himself and saw that he could make another group (roll) of ten from the twelve candies he drew individually. He then counted his answer, by tens, until he got to the individual candies and came up with the answer of ninety-two.

Katherine also used the illustration shown in Figure 4.2 to show what Jack could do at the end of the case study. Her explanation shows that she noticed Jack’s strategy at the beginning of the case study and at the end of the case study. Using the written work, she was able to identify what Jack could do and what he could not do. Her explanation
Figure 4.2. Digrammatic representation of Jack’s strategy

shows that she provided detailed explanations and provided details of the mathematical essence of the strategy. In addition, she observed and provided explanations and illustrations of how Jack made sense and interacted with the mathematical idea.

*Interpreting children’s mathematical understanding*

When asked to summarize what one child knew or understood at the end of the study that s/he did not know or understand in the beginning, Katherine remarked:

> In the beginning of the study, Jack had a hard time counting by anything but one’s. Throughout the study, Jack made improvement during almost every session. The instructor’s prompts helped him to think outside the box of tallies and counting by ones. At the end of the study, Jack seemed to have an understanding of base ten number concepts and processes.

Further, Katherine used the example provided above as evidence that Jack understood base 10 number concepts and processes. In addition, she identified an example of a task that the teacher in the case study used in order to develop students’ thinking. In her explanation she commented:

> One task that I thought was particularly productive for advancing the thinking of the students as a group was the 3rd task in problem set six (11 packages of 10 cookies each). I think this problem was so productive for the students because it involved larger numbers, which made it tedious and time-consuming to draw out each individual cookie.
In this new quotation, Katherine selected the task (11 packages of 10 cookies each) because it provided opportunities and challenged students to move from counting by ones to counting by 10’s. It was notable that Katherine focused not only on Jack’s strategy of solving the task but also on how the task advanced Sunny’s mathematical understanding. In her explanation she commented:

As you can see in the picture shown in Figure 4.3, Sunny used cubes in groups of ten to represent a package of cookies. She then counted the total “beautifully”, as the instructor of the study described it, by tens.

![Image](image.png)

Figure 4.3. Sunny’s colorful illustration of her mathematical understanding of tens

In summary, Katherine’s work in noticing and analyzing students work reveals that she made sense of details of children’s strategies and noted how the details reflected children’s mathematical understanding. Based on this analysis, I characterized her response to have robust evidence that she interpreted children’s mathematical understanding. In other words, she noted and interpreted students’ understanding and identified possible tasks that could have supported children’s understanding of the concept of base 10. In the next section,
I will discuss Katherine’s ability to respond based on her interpretation of children’s mathematical understanding.

**Responding based on children’s mathematical understanding**

For the component skill of responding based on children’s mathematical understanding, Katherine responded to the following prompt:

*If you were to teach the next lesson to this group of students, write one problem that you could give them and explain why you think this would be a good problem for this group.*

In her response Katherine chose a word problem listed below:

*There are 50 balls in the gym. 34 of them are basketballs. How many of the balls are volleyballs?*

However, the task was not based on what the students knew. In her explanation and reasoning on how she chose the task, Katherine remarked:

*This is a part-part-whole problem with a missing addend. I think this would be a good problem for this group of students because they only really worked with joining, separating, and comparing problems until the very last day (when it was one of their options as a strategy to use to solve). I chose the numbers because I wanted them to keep focusing on counting by tens, and then ones once they can’t by tens anymore.*

Notice in the above quotation, Katherine focused on the nature of tasks that the students had done in the case study and not on what the students understood and/or misunderstood. In addition, she never focused on how the task would develop individual children’s understanding. Instead, she focused on the students doing a task that differed with what they had done during the case study. Finally, her number choices were not focused on developing any particular child’s understanding but were aimed at exposing the children to another type of task to provide more practice with base 10 numbers. In summary, Katherine
noticed and attended to children’s strategies and interpreted children’s mathematical understanding. In addition, she selected tasks that were worthwhile but her rationale of the choice of tasks was not based on what the children understood and/or misunderstood during the Inquiry into Student Thinking assignment.

**Tutoring assignment**

During the tutoring sessions, Katherine had an opportunity to interview and tutor two 3rd grade children [Alex and Macy] who attended an elementary school where teachers have participated in (CGI) professional development workshops (Carpenter et al., 1999). After the interview, she responded to the following prompt: *What does each student know, think and understand about number, operations and problem solving?* In the next section, I will use her description of children’s mathematical understanding to reflect on her ability to attend to children’s mathematical understanding.

**Attending to children’s strategies.** During the tutoring assignment, Katherine noticed and paid attention to Alex and Macy’s strategies in counting, solving word problems and solving mathematical equations. She concluded that both Alex and Macy have a clear understanding of numbers and counting since they [children] did not need to use any manipulative[s]. In her explanation, Katherine provided detailed explanations of the strategies that the children used. For example, while describing Alex’s strategies, Katherine provided the following explanation:

I wrote the problem $28 + 46 = 45 + 28$ at the top of Alex’s paper and asked him if he thought it was true or false. Without even picking up his pencil he responded false. I asked him to write down false and how he knew so quickly that it was false. He explained: well both sides have 28 [as he crossed out 28
on both sides] and it says 46 = 45 which isn’t true because 46 is one more than 45. I was impressed with this explanation. Katherine also provided vivid details of Macy’s strategies to support her claim that Macy understands counting and solving equations. In her explanations she commented:

I wrote the first equation on 28+46 = 45 + 28 on Macy’s paper and asked her if it was true or false. When she told me false I asked her how she thought about it. She told me she took 2 from 46 and added it to 28 to get 30. Then she added 30 +44 to get 74. She used the same strategy for the other side of the equation, this time taking 5 from 28 to get 50 and then adding 50 + 23 to get 73. Then she stated that 74 and 73 are not equal.

In addition, the explanation shows that Katherine paid attention to the strategies that the individual children used. In addition, she provided details of the mathematical essence of the strategy and details of how the children made sense of and interacted with the mathematical idea. It was also notable that Katherine paid attention to the misconceptions and errors that the children made. Specifically, she selected from the pool of tasks provided by the instructor the task 44 + 13 = ___ + 14 for the two children. In her explanation Katherine noted that Alex struggled with the task but Macy did not. She also compared Macy and Alex’s strategies and remarked:

Alex first added 13 to 44 and got 57. He then seemed to be stuck about what to do next. I asked him if he saw any patterns in the number or anything he could cross out and to make it easier to look at (since he used that strategy before). No, he exclaimed…

Macy didn’t seem to have as much struggle with 44 +13 = ___ + 14 as Alex did. When she got 43 for her answer, I asked her how she solved for it. She showed me how she changed the 14 to a 13 on the right side and put 44 in the blank…She then added the one back to make it 14 again and subtracted one from 44 to get the answer of 43…

This explanation show that Katherine noticed and attended to specific children’s strategies and provided mathematical details for the understanding and misunderstanding that existed between the children.
Interpreting children’s mathematical understanding. For the component skill of interpreting children’s mathematical understanding, Katherine made two assertions. First, she noticed and interpreted that Alex and Macy had clear understanding of numbers and counting because they did not use manipulative[s] when doing the mathematical problems. This assertion was consistent with her reasoning and descriptions of the specific strategies. For example, in her explanations of the children’s counting skills, Katherine remarked:

Alex did an excellent job with this portion of the interview. When showing me 134 and 256 with the base 10 cubes he used hundreds tens and ones. When I asked him how many cubes he would have if we added ten to the pile of 256, he grabbed a group 10 cubes and responded 266…

Macy also did a great job with this portion of the interview. She showed me 134 and 256 using hundreds tens and ones. When I asked her how many cubes she would have added if we added a group of ten to the pile of 256 she immediately responded “266” without touching the base 10 blocks to find her answer.

Second, Katherine asserted that both Macy and Alex are capable of solving addition and subtraction word problems and have a good understanding of place value. In her explanation, she commented:

As far as problem solving goes, Alex and Macy seem to have an understanding of adding place value as they displayed in the joint result unknown problem [referring to the problem: Annie had ____ apples. She picked ____ more apples. How many apples does Annie have in all? (42,36)(23,48)(89,62)]. For the first set of numbers choices (42,36) both Macy and Alex solved this by adding 2+6 to get 8, and 40 +30 to get 70 and then added 70 + 8 to get 78…

For the separate result unknown problems [referring to the problem: There were ____ leaves on the tree. A wind blew ____ off. How many leaves are left on the tree? (87,20) (92,12) (140,60)] again, both Alex and Macy displayed great understanding of place value. For the last set of numbers (140, 60), Macy took 14-6 to get 8 and added 0 to the 8 to get 80 leaves. I asked her how she thought about this equation and she replied. Since they both end in zeros you can cross them out. This told me that Macy has a good understanding of place value…
In this explanation, Katherine showed clearly that she made sense of Alex and Macy’s strategies and understanding in problem solving. She also noted how the details reflected the two children’s understanding in the specific task as she explained, “For the last set of numbers (140, 160), Macy took 14-6 to get 8 and added 0 to the 8 to get 80 leaves. I asked her how she thought about this equation and she replied. Since they both end in zeros you can cross them out. This told me that Macy has a good understanding of place value…”

Her explanation was informed and guided by what Alex and Macy did in the tasks that she assigned to them. Also, this explanation characterized her subsequent explanation and interpretations of children’s mathematical understanding.

In addition, Katherine supported her explanations and interpretation with the children’s verbal responses. For example, when it came to the third number set, Katherine clearly differentiated Macy’s strategy and Alex’s strategy concluding:

Macy used the same strategy for the 3rd set of number choices. However Alex rounded to the nearest 10 so he made 89 into 90 and 62 into 60. He told me he knew 9+6 = 15, so 90 + 60 = 150. Next he subtracted one from 150 to get 149.

To summarize, in both explanations considered above, Katherine appear to have made sense of details of children’s strategies, and noted how the details reflected the specific children’s understanding on the component skill of noticing and interpreting children’s mathematical understanding. She was able to provide explanations that were consistent with every child’s strategy and well unpacked the child’s strategies to understand the child’s mathematical understanding and/or misunderstanding.

In developing these explanations, her interpretation also appears to have been informed by the in–the-moment interaction with the child as they did the task. She asked the children probing questions that clarified her analysis of what the children understood and/or
misunderstood. For example, while interpreting Alex mathematical understanding she remarked:

I wrote down the next number sentence $74 = 10 + 60 + 4$ and asked whether he believed this to be true or false. This time he picked up his pencil and wrote $70 + 4 = 74$. Then he wrote down a capital T for true. While I asked him how he came with the answer he said “I added 6+1 and got 7 and since the one and 6 has a zero…”

The above quotation shows that she also used probing questions the children for them to clarify the strategies they were using and their understanding. In the next section, I describe how Katherine responded to what she knew and understood about children’s mathematical understanding.

**Responding based on children’s mathematical understanding.** For the component skill of responding based on children’s mathematical understanding, Katherine responded to the following prompt after the first interview with Alex and Macy.

1). *How will what you learned in the interviews influence how you work with the children during the next sessions?*
2). *How are these tasks intended to build on what you know about your students’ understanding and misunderstanding*

In response to the first prompt, Katherine explained that she would use what she learned from the interview to influence how she worked with Alex and Macy. She explained that she would take their understanding of place value and expand on it beyond the decimal and also by incorporating games into the lesson as often as possible arguing; “they both told me they liked playing any kind of game”. The objective of her first tutoring session was that the students will develop an understanding for the decimal and the tenths place and recognize that a digit in the ones place value represents ten times as much as it would represent in the tenths place value and vice versa.
Katherine selected two mathematical tasks. One was an opening number routine, which she used to introduce her lesson to the students, followed by the main activity. In the opening number routine, Katherine gave the students four numbers; 41, 83 and 628. For the first number she had the students write down number 41 and explain how they can write 4 and 1/10 using the same digits. She also anticipated asking the same question using 83 and 628. As noted in her lesson plan, the purpose of the opening routine was to prepare the students for the main activity by getting them to think about the number at the right of the decimal.

In the main activity, each child was given sticky notes with numbers 0-9 on them and a board with two lines before the decimal and one line after the decimal. She gave the children three numbers at a time and guided them into making the greatest and the smallest number. In addition, they were expected to label each place value and provide a verbal explanation why they labeled the numbers in any particular order.

Although Katherine claimed that the task did build on children’s mathematical understanding and misunderstanding, it was not clear from her explanation how the task connected with what she learned about the children during the interview. In addition, it was not clear how she was connecting the place value with putting the decimal to the right and the left. When she introduced the task to them Katherine remarked: “I handed my students a sheet of paper and asked them how they could make that number into 4 and one tenth. They both had a confused look on their face and glanced over at each other. I used this as a clue that they needed further explanation. I elaborated by telling them that they were going to need to insert a symbol somewhere in the number…”

Her explanations reflected that the children immediately understood how to put the decimal point in any number. Specifically, she explained that the students were adding zeros
to represent any place value. When Katherine noticed what the students were doing she decided to instruct the students not to use the zero, explaining that the zero does not change the value of a number if it is the last number on the right side of the decimal but it would change the value if it was in between two numbers on the left or right side of the decimal.

Although the goal of Katherine’s task was to develop students understanding of decimals and the tenths, she mainly focused on students’ identifying the place value and making numbers out of given digits. For example, in the main activity, she provided them with a board, which had blanks so that they could fill them out with numbers. In her explanation she remarked:

> When I passed out their boards marked __ __. ___; I had them write the place value name under each line. Both Alex and Macy wrote from left to right; tens ones and tenths. I described to them how the activity was going to work. First, I would read them three numbers. Then they would use the sticky notes with the three numbers I gave them and place them on the board, creating the biggest number they can with the three numbers. Then using those same three numbers create the smallest numbers that they can.

Furthermore, Katherine considered the tasks to be of high level of cognitive demand because it requires some degree of cognitive effort. She also argued that “although general procedures maybe followed they cannot be followed mindlessly” prompting the instructor to urge her to explain the procedures that the students will be following. She also argued that the activity was based on what the students already knew about place value “because the tens and ones place is still included”. In addition, she argued that she chose to do the activity because the students seemed to have a good understanding of place value to the left of the decimal and she wanted to extend that understanding to show the students the same concept of place value apply and occur to the right of the decimal.
Katherine’s decision on this situation is worth noting in many aspects. Although she interpreted that the students had a good understanding of place value, it was not clear how her response was based on children’s understanding. In fact, it appeared like her responses did not have anything to do with her assessment of children’s understanding. Hence, I argued that descriptions of how she responded to children’s mathematical understanding were not consistent with her interpretation. Katherine’s responses represent an example of a case where the PST noticed and interpreted children’s mathematical understanding but faced challenges when it came to responding based on children’s mathematical understanding and/or misunderstanding.

Second, Katherine consistently attended to children’s strategies, interpreted children’s mathematical understanding but her responses were not based on children’s mathematical understanding, both in the Inquiry into Student Thinking assignment and the tutoring assignment. Notable in this response is the fact that the Inquiry into Student Thinking assignment required the PSTs to have an intended response while the tutoring assignment required an instructional response to students in a classroom. Nevertheless, in the two assignments, there was a gap between the first two component skills of professional noticing (attending to children’s strategies and interpreting children’s mathematical understanding) and the third component skill of responding based on children’s mathematical understanding.

Although this case will be discussed further in Chapter 5, Katherine’s case raises fundamental questions on the “interrelated relationship” between the component skill of noticing and attending to children’s strategies, interpreting children’s mathematical understanding and respond based on children’s mathematical understanding as discussed in Jacobs et al.,(2010). Specifically, PSTs included in this cluster only demonstrated evidence to
attend to children’s strategies and interpret. Specifically, it would be informative to explore how this component skill develops with the same group of teachers over time and whether developing one component skill necessarily means the PSTs can develop the other skills.

3rd Cluster: Did not Notice or use Children’s mathematical Understanding during the Inquiry into Students’ Thinking but did use Children’s Mathematical Understanding during the tutoring Assignment

Hannah

In the Inquiry into Student Thinking assignment, the PSTs considered in this cluster (8/30 PSTs) had provided some or no mathematical details as they attended to children’s strategies. Also, their responses reflected limited or no evidence that they made sense of children’s strategies as they interpreted children’s mathematical understanding and there was limited or no evidence that their response was based on children’s mathematical understanding.

During the tutoring assignment the PSTs included in this cluster showed remarkable progress in their performance in the three component skills. In other words, they provide most details as they attended to children’s strategies and there was evidence that they made sense of the children’s strategies as they interpreted children’s mathematical understanding. Finally, there was evidence that they used children’s mathematical understanding as they select. mathematical tasks to pose to the children. In the following paragraphs I discuss Hannah’s responses as an example of the responses exhibited in this cluster.

Performance in the three Component Skills during the Inquiry into Students Thinking Attending to Children’s Strategies

When asked to summarize what she thinks one child knew or understood at the end of the study that they did not know or understand in the beginning, Hannah chose to follow
Emilio’s learning through the case study. She explained that Emilio did not understand the idea of base 10 and was not able to use it in an easier way. Further, Hannah explained how Emilio solved the problem: “There are 4 rolls of 10 candies, how many pieces of candy are there?”

While attending to Emilio’s strategy, Hannah noted that Emilio added 10 and 4 to get 14 because she did not recognize that the 4 rolls had 10 candies in each roll. She further explained that after the teacher worked with her “a few more times she started to understand the idea of having 4 rolls and each roll had 10 candies”. In addition, Hannah explained that when Emilio started to understand that 10 plus 10 was twenty she was able to apply that to this problem to come up with 40 as the answer.

Notable in the above explanation is the fact that Hannah did not provide mathematical details on how Emilio solved the task. Instead, she provided over generalized statements like “So when Emilio started to understand that 10 plus 10 was twenty she was able to apply that to this problem to come up with 40 as the answer”. However, she did not provide detailed explanation of the strategy that Emilio used and she did not provide details of the mathematical essence of the strategy. Her response only had some details.

**Interpreting Children’s Mathematical Understanding**

For the component skill of interpreting children’s mathematical understanding, Hannah made three assertions. First, she argued that “at the beginning she [Emilio] was simply counting by ones and at the end of the first day she understood that ten plus ten was twenty. Second, she argued that Emilio eventually got much better at “this” [referring to the base 10 concept] and after a few days of working with this idea she was able to fully understand that counting by tens was much easier and faster than counting by ones. However,
this response was over generalized and did not provide any specific evidence on how Emilio did the task. Specifically, Hannah did not make sense of details of children’s strategies and did not note how the details reflected children’s understanding. Third, Hannah argues that Emilio was able to use the unifix cubes to make groups of tens then have single cubes to have the ones represented, but again there was no evidence in her response that Emilio used any unifix cubes to solve any task.

**Responding Based on Children’s Mathematical Understanding**

For the component skill of responding on the basis of children’s mathematical understanding, I considered whether there was evidence that the choice of task was based on children’s mathematical understanding as reflected in the specific strategy that the child used and, whether the choice of the next task could further the specific child’s understanding. In addition, I considered responses where the PST’s response would build on existing children’s understanding.

In her response Hannah chose the task shown below:

Problem: If I was to teach the students another lesson the next day, the first thing I would try to have them do is to do the problems without using manipulative. The problem I would give the students would be one such as 30 + 20=_____ then I would give 30 + 45=_____.

Rationale: The reason I would give them these problems is because they are very similar to the problems that was given to them the day before but this time I would like to have them try it without manipulative[s] and use different strategies to solve them.

As noted in the quotation above, Hannah chose a join result unknown problem, but there was no evidence in her rationale that it was based on children’s understanding. Instead, she selected the task because they were “very similar to the problem that was given to them the day before…” In the interpretation of children’s mathematical understanding, she made
assertions that Emilio understood how to add in groups of 10 by the end of the case study. So it was not clear in her explanation how solving the problems without manipulative[s] would advance her understanding.

Although Hannah attempted to give meaning and justify the tasks that she selected, again her explanation did not have evidence that the response was based on children’s understanding. Specifically, it was not clear how the task would advance Emilio’s understanding. In her explanation, she commented:

I would walk around and see the different ways they tried to solve the problems since they didn’t have actual objects in front of them. Since I am having them do it without manipulative[s] though, I would not want to give them new problems or ones they didn’t know how to do which is why I would use similar problems as the day before. I would then base the next day off the different strategies they used today and if any students are really struggling then I would let them use manipulative[s] so they could eventually get the problem without getting too frustrated.

From this respect, I characterized her decision to respond as having no evidence that it was based on children’s mathematical understanding. In summary, Hannah’s responses provided few mathematical details in the component skill of attending to children’s strategies, had limited evidence that she made sense of children’s strategies, noted how the details reflected children’s understanding or made sense of the strategy details in a variety of ways. In other words, there was no in-depth interpretation of children’s mathematical understanding. In addition, there was no evidence that her response was based on children’s mathematical understanding and/or misunderstanding during the Inquiry into Student Thinking assignment. In the next part of this section, I discuss Hannah’s response during the tutoring assignment.
**Performance in the three Component Skills during the Tutoring Assignment**

During the tutoring sessions, Hannah got an opportunity to interview and tutor three 1st grade children [Tommy, Jack and Sara] who attended an elementary school located within the University neighborhood. After the interview, she responded to the following prompt: *What does each student know, think and understand about number, operations and problem solving?* In the next section, I will use her description of children’s mathematical understanding to reflect on her ability to notice and attend to children’s mathematical understanding

**Attending to children’s strategies**

For the component skill of noticing and attending to children’s strategies, I explored the level of mathematical details that the PSTs provided as they described what they think that the children knew and understand. Hannah used the 1st grade problems that the instructor had provided to them. The tasks assess the children’s understanding on counting skills, the meaning of the equal sign and problem solving. Like Jodie and Katherine, Hannah provided details on how the three children interacted with the task.

For example, Hannah explained that Jack was good in counting skills but he counted the base 10 cubes one by one. She describes Jack as a student who did not seem to have the skill of counting from another number or counting by 10. Explaining Jacks ability to count, Hannah commented:

> After he had his 24 cubes separated, I asked him, Can you show me 34? Jack then pushed his pile of 24 back in with the others and started over from 0 to get to 34 counting again one by one. When I asked Jack the last question of this series of; “If I add a group of 10 to this pile, how many cubes would I have. Jack was unsure what to do or maybe did not understand the question…
In considering Jack’s work, Hannah noticed his ability to count by one, and his challenge of counting from any other number. Hannah also explained that Jack did not understand how to form groups of ten or counting by 10. Similarly, Hannah explained how Jack solved equation work and noted that Jack had a problem with solving equations. For example, when Jack was asked whether the equation $6+2 = 5+2$ is true or false, Hannah noted that he did not understand how to solve the problem. In her explanation she remarked:

I wrote down on a piece of paper the following equation $6+2 = 5+2$, then asked Jack if he thought it was correct. Jack looked at the problem and tried to read it around. As he did he said “6 plus 2 minus 2” then hesitated and looked very confused at the rest of the problem. I asked him to try to re-read the problem and again he read the equal sign as a minus…I then gave him the problem: $5+\_ =7$. When I gave Jack the paper he filled in the blank with a 6. When I asked him why he filled a 6, he shrugged again and said because it goes 5, 6, and 7 again having a question in his mouth as he said it.

Similarly, in considering Tommy’s work, Hannah explained that he only counted by one and did not understand the base 10 concepts. She observed that Tommy counted 24 cubes starting from one. In her explanation she remarked: “Tommy grabbed the longs and started to try to use them, but then He realized that he did not know how to use them and kept them down and started to count by ones”. For the skill of solving equations, Hannah noted that Tommy did not understand and could not attempt the problem. A good example was the problem $5+\_ = 7$ where Tommy was asked to fill in the blank so the equation can be true.

Explaining how he attempted the task, Hannah remarked:

When Tommy saw the problem, he looked confused but was still looking at the paper so I gave him few seconds. After a bit he said, “I am not sure what to do”. So I said 5 plus what equals to 7?” He then said, “Oh”! Then looked a few more seconds and said, “I am not sure”

She concluded that Tommy did not understand how to add by ten and/or solve equations.
Finally, Hannah quickly noticed that Sara, the third student that she worked with did a little better than Tommy and Jack. She observed that when asked to count 24 cubes, she quickly counted by 10. To explain Sara’s counting skills, Hannah remarked:

She instantly grabbed 2 longs and said it loud but to herself; 10, 20. Then she grabbed 4 more unit cubes and then showed her results. I asked her how she got that and she said, “Well 2 longs plus 4 more is 20 and 4 so 24”. I then asked her to show me what 34 would be and she took her 2 longs she already had and added one more. Then looked at those and grabbed the 4 units she had before. I again asked her how she got that answer and she said: Because 3 longs and 4 more so that’s 34!

Notable in the above quotation, Hannah explained her observations on Sara’s strategies and provided details on how she interacted with the mathematical idea. In addition, she provided details on the questions that she asked to probe her thinking and understanding. Overall, she provided details on how the three students counted up, and in some cases how the students used the manipulative[s]. Hannah also provided details of the mathematical essence of the strategy.

**Interpreting children’s mathematical understanding**

For the component of skill of interpreting children’s mathematical understanding, Hannah talked about the three students. In considering children’s mathematical understanding, Hannah made three assertions. First, she explained that each one of the three students had their own way of solving the problems given to them but that their strategies were similar in some way. She argued that Jack started counting by ones; Tommy attempted to use the longs and then turned to counting by ones while Sara counted by 10 using the manipulative[s]. In addition, Hannah concluded that Sara was the only one who could do the equation work. In her explanation she commented; “she knew what she needed to do but was
not able to transfer knowing that you can flip an equation around and it will be the same thing.”

Taking Hannah’s explanations and considerations of three students’ strategies and her interpretation of children’s mathematical understanding, one could argue that she made sense of details of children’s strategies and noted how the details reflected children’s mathematical understanding.

**Responding based on children’s mathematical understanding**

Like Jodie and Katherine, Hannah also responded to the following prompts:

1) *How will what you learned in the interviews influence how you work with the children during the next sessions?*
2) *How are these tasks intended to build on what you know about your students’ understanding and misunderstanding*

For the first prompt, Hannah explained that her plan for the first week was to introduce the students to the idea of counting by tens. Her lesson objective was that students would be able to correctly add or subtract by 10 to 100. The general standard addressed in this lesson was: Given a two digit number, mentally find 10 more or less than the number without having to count and explain the reasoning used. Therefore Hannah’s lesson was within the standard that the children were supposed to be covering in first grade.

Considering her lesson objective and the general goals, she selected two tasks; one that she used as an opening number routine and the next one was the main activity. Below are the problems that Hannah selected.

**Selected Problems:**
For my opening number routine, I will write on the board the following problems
10, 20, 30, ___ ___
First activity- counting forward
   i.  40, 50, 60 __ __ __
   ii. 45, 55, 65, ____
   iii. 13, 23, 33, 43, ____
   iv. 47, 57, 67, ____

Questions asked
   i. How many are you counting up each time?
   ii. How do you know we are going up each time?
   iii. What patterns do you recognize?

Second activity counting backwards
   i. First have the students write down the numbers 20 to 26
   ii. Then count with them backwards; 26, 25, 24, 23, 22, 21, 20
   iii. Then, I will write the number
   iv. 70, 60, 50, 40, 30, 20, 10

Rationale: I wanted to show the kids how to count both forward and backward by 10 using a pattern to help then realize what come next without having to count. The pattern I will show them is that when counting by tens the first number in the series will always go up or down by one depending on whether we are counting forward or backward.

Although it was not clear whether the strategy that Hannah was explaining would support the students’ conceptual understanding of base 10, there was some evidence that her choices were based on her interpretation of children’s mathematical understanding. Notice that in the interpretation of children’s mathematical understanding, she argued that Jack and Tommy could only count by one. So, one would argue that it was reasonable for her to decide to use tasks that would support students’ understanding of how to count by 10.

In addition, Hannah explained the questions that she anticipated to use to help the students interact with the mathematical idea. Specifically she talked about how she would ask the students to explain their thinking using the questions “How much are we going to add each time? How will you know how much you add? What patterns did you recognize? In her explanation she argued: “By asking this questions, it will help me to get into their brains to see how they look at the story problems”
Therefore, regarding the practice of selecting a task based on children’s mathematical understanding, Hannah selected tasks where the students will do procedures with connection to meaning, and intended to use them to help Tommy, Jack and Sara understand the meaning of adding by tens instead of by ones. Specifically, it was interesting that Hannah pre-planned the probing questions that would help the students explain their understanding because she wanted to ensure that they understand what they were writing down and not just memorizing. Therefore, Hannah’s response was characterized to have robust evidence that her lesson goal and the tasks selected were based on children’s mathematical understanding.

Overall, Hannah was seen closely attending to children’s strategies, and interpreting children’s mathematical understanding during the tutoring assignment. She mainly explored whether the students understood the base 10 concept and, her analysis of Jack, Tommy and Sara’s strategies reveal her capacity to attend to children’s strategies, interpret children’s mathematical understanding and respond based on children’s understanding.

Hannah’s responses are worth noting in this multiple-case analysis, because it represents the cluster of PSTs (8/30) whose responses during the Inquiry of Student Thinking assignment had limited or no evidence that they provided most mathematical details. In addition, Hannah represents the cluster of PSTs whose responses had no evidence that they interpreted children’s mathematical understanding or responded based on children’s mathematical understanding during the Inquiry into Student Thinking but they made significant progress during the tutoring assignment.

Hannah’s responses were also important to note because she represented a shift from responses that provided general strategy descriptions during the Inquiry into Student Thinking to responses that provided detailed explanations of the children’s strategy during
the tutoring assignment. Her responses also represent a shift from general interpretation of students’ understanding during the Inquiry into Student Thinking to comments that have robust evidence that she made sense of details of children’s strategies.

PSTs’ responses included in this cluster are congruent with other studies that have suggested that PSTs can learn how to recognize evidence of students’ understanding during the methods course (Bartell, Webel, Bowen & Dyson, 2012), and that PSTs can learn how to provide instructional explanations as they interpret children’s mathematical understanding (Charalambous, Hill and Ball, 2011). As will be discussed later in Chapter 5, this case extends Bartell and colleagues’ as well as Charalambous and colleagues’ work by showing that PSTs can also respond based on the evidence that they recognize and their mathematical explanations.

4th Cluster: Had no Consistent Pattern on their Responses Either in the Inquiry into Student Thinking or Tutoring Assignment

Alexa

This cluster includes PSTs (9/30) who had no consistent pattern in their responses either in the Inquiry into Student Thinking assignment or the tutoring assignment. By no consistent pattern, I refer to the PSTs’ who had sporadic responses in the two assignments. In some instances the PSTs provided mathematical details as they attended to children’s strategies during the Inquiry into Student Thinking assignment but provided some or no mathematical details during the tutoring assignment. In other cases, PSTs improved their performance in only one component skill like interpreting children’s mathematical understanding. I also decided to include the PSTs who did not improve in any of the component skill in this cluster. Unlike other clusters described above, I made a decision to
summarize the performance of all the participants in this cluster since none of the students could best represent the others. Table 4.7 represents the performance of the nine participants.

**Performance in the Component skills During the Two Assignments**

**Attending Children’s Strategies.** For the component skill of attending to children’s strategies, three PSTs in this cluster provided most mathematical details, five provided some mathematical details, and one did not provide any mathematical details during the Inquiry into Student Thinking assignment. In the tutoring assignment four provided most mathematical details. However, two of the ones who had provided most mathematical details during the Inquiry into Student Thinking still provided most mathematical details in the tutoring assignment. The other two had provided some mathematical details during the Inquiry into Student Thinking.

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<th>RBoCMU</th>
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Key: ACS—Attending to children’s strategies; ICU- Interpreting children’s mathematical understanding; RBoCMU- Responding based on children’s mathematical understanding.
In summary, for the component skill of attending to children’s strategies, two participants consistently provided most mathematical details in the two assignments, three consistently provide some mathematical details, one consistently did not provide any mathematical details while the other three changed from most mathematical details to some mathematical details and vice versa.

**Interpreting Children’s Mathematical Understanding**

For the component skill of interpreting children’s mathematical understanding, three responses had robust evidence during the Inquiry into Student Thinking assignment and during the tutoring assignment. However, among the three participants, it’s only one who consistently provided robust evidence in the two assignments. The other six responses had limited evidence of interpreting children’s mathematical understanding during the Inquiry into Student Thinking. However, one PST’s responses provided no evidence during the tutoring assignment. Over all, most of the participants in this cluster either provided robust or limited evidence in the two assignments. Even though the responses were not consistent, it’s encouraging to note that all the nine PST had the potential to interpret children’s mathematical understanding.

**Responding based on Children’s Mathematical Understanding**

Among the nine participants, two participants’ reflections had robust evidence that they responded based on children’s mathematical understanding, six had limited evidence while one had no evidence during the Inquiry into Student Thinking assignment. During the tutoring assignment, six had no evidence that they responded based on children’s mathematical understanding. This observation reveals that the cluster of PSTs who had no
consistent pattern in their responses struggled more with the component skill of responding based on children’s mathematical understanding during the tutoring assignment. For the other three participants, two had limited evidence while the other one had no evidence that her responses were based on children’s mathematical understanding.

However, I will use Alexa’s responses as an example because her responses were not only inconsistent but the inconsistencies also had concerning responses especially in her selection of number choices.

**Performance in the three Component Skills during the Inquiry into Student Thinking**

*Attending to Children’s Strategies*

When asked to summarize what she thinks one child knew or understood at the end of the study that they did not know or understand in the beginning, Alexa chose to follow Jack’s mathematical understanding in the case study. First, she asserted that Jack counted by one throughout the case study to find his answers. In attending to Jack’s strategy, she argued that from the very first day, Jack miscounted the total number of soccer balls because he had the wrong number of soccer balls in one bag, even though all of the bags had simply 10 balls in each. This explanation was not clear because she argued that “Jack had wrong number of soccer balls in one bag” and “all the bags had simply 10 balls in each”. However, she went ahead and concluded that Jack did not understand the concept of grouping by 10.

In the example quoted above, Alexa’s explanations were inconsistent, and she did not provide substantial details of how the child solved the task. Instead, she generalized Jack’s misconception and concluded that he “miscounted the total number of soccer balls because he had the wrong number of soccer balls in one bag”. I noted that instead of paying closer
attention to how Jack counted, Alexa concluded that he miscounted the total number of soccer balls.

Although she first argued that Jack counted by one throughout the case study, she latter noted that “Jack showed significant progress throughout the study and showed a particular understanding of the unifix cubes and with story problems involving money” In her explanation she remarked: “Jack solved the JCU problem involving pennies by counting up using tallies. The strategy was indisputably successful as he achieved the right answer, but his thought process showed no evidence of using tens”.

Lastly, Alexa argued that when doing problem set #10 (referring to Sunny has 8 rolls of candy. Each package has 10 candies in it. She also has 12 extra candies. How many candies does she have in all), Jack drew out the first number in groups of ten (8 groups) and then added on another group of ten and two ones. Using this justification, she concluded “we see Jack using single units in the beginning, but after a few sessions he is still using modeling but he is able to abstractly group units to represent ten”.

**Interpreting children’s mathematical understanding**

For the component skill of interpreting children’s mathematical understanding, Alexa made two assertions and provided justification. First, she asserted that Jack did not understand the concept of grouping by 10 at the beginning of the case study. To support her argument, Alexa provided one example (problem set 3: Sunny has 22 pennies. How many more pennies does she need, to have 50 pennies to buy a book? (Join Change Unknown) that was done at the beginning of the case study and made the following remark:
Jack solved the JCU problem involving pennies by counting up using tallies. The strategy was indisputably successful as he achieved the right answer, but his thought process showed no evidence of using tens.

In the above quotation, I noted that Alexa provided details of Jack’s strategy and made an assertion that the strategy was indisputably successful because Jack achieved the right answer. Further, Alexa used problem set #10 to justify her claim that Jack had made progress in understanding 10 as a unit. In her responses, she remarked:

Jack was asked how many candies were left after a Join Result Unknown problem. Jack drew out the first number in groups of ten (8 groups) and then added on another group of ten and two ones. These examples support my claim as we see Jack using single units in the beginning, but after a few sessions he is still using modeling but he is able to abstractly group together units to represent ten.

In the quotation above, it’s notable in Alexa’s explanation that she provided details of how Jack grouped items by 10’s while solving the problem set 10, arguing “Jack drew out the first number in groups of ten (8 groups) and then added one group of ten and two ones”. Eventually Alexa concluded; “the examples support my claim as we see Jack using single units in the beginning, but after a few sessions he is still using modeling but he is able to abstractly group together units to represent ten.”

**Responding based on children’s mathematical understanding**

In responding, Alexa argued that she would start off by extending the work that the teacher in the case study had done in the past 10 sessions. In addition, she remarked that she would probably use a join result unknown or join change unknown with specific number choices like (10,30) (25, 20) or (40,64)\(^{16}\), but she did not identify specifically which ones she

\(^{16}\) I noted that the number choices that Alexa intended to use would vary depending on whether she is having a JCU or JRU. The second set (25,20) would only work for a join
would use for the join change unknown or join result unknown. After a review of basic problems that involve making 10’s, she would move on and work with the students in adding tens and then adding or taking away ones to get an answer. In her responses, Alexa explained:

Problem: I would start by using unifix cubes and show them what happens to the stack when you take away one (it makes 9 cubes) or add one (it makes 11 cubes). I would give problems like this: Sunny has 9 m&m’s. She gets 1 more m&m’s. How many m&m’s does Sunny have now?

Rationale: I used these numbers because in the case study the students had difficulty seeing the problem as adding two tens and just taking one away. For instance, Sunny didn’t know if she should add or subtract the two extra soccer balls when adding a total of twelve soccer balls to the stack. I would encourage using direct modeling for this problem, and I think unifix cubes were successful in the past and definitely be a valuable tool for this concept. I would use increasingly harder numbers like 19 and 11 (they can also solve this by adding a group of ten to the previous answer), 21 and 29.

Notable in the above quote is the fact that Alexa’s choice of task was based on children’s mathematical misunderstanding, arguing, “in the case study the students had difficulty seeing the problem as adding two tens and just taking one away”. In addition, she gave an example of Sunny’s response and explained; “Sunny didn’t know if she should add or subtract the two extra soccer balls when adding a total of twelve soccer balls to the stack”. Further, Alexa explained that she would increase the level of difficulty in the task, arguing; “I would use increasingly harder numbers like 19 and 11 (they can also solve this by adding a group of ten to the previous answer)”. Finally, she anticipated the strategies that the students might use arguing, “I would encourage using direct modeling for this problem, and I think unifix cubes were successful in the past and definitely be a valuable tool for this concept”. I coded the response to have robust evidence because Alexa reflected on the strategy that result unknown. I presumed that she intended to use it for a join result unknown because she had not provided any other information.
particular students could use, and how the next task could further specific children’s understanding.

In addition, Alexa is seen being careful as she selected the task. Her choice of task and reasoning with the numbers suggests that she was cognizant of children’s understanding and misunderstanding. In summary, Alexa’s responses were seen to have some mathematical details, had robust evidence to have interpreted children’s mathematical understanding and had robust evidence that her decision on how to respond was based on children’s mathematical understanding in the Inquiry into students’ Thinking assignment. In the next part, I explore her responses in the tutoring assignment.

**Performance in the three component skills during the Tutoring assignment**

During the tutoring assignment, Alexa had an opportunity to tutor two 4th grade children (Maddie and Makeyle) in a nearby elementary school. She used the interview tasks that the instructor had provided; which had three different types of tasks. First she needed to evaluate children’s understanding on counting forward and backward using their knowledge of grouping by 10 and place value. Second, the tasks focused on children’s ability to add and subtract numbers and, work on relational thinking. Finally, the last section of tasks evaluated children’s problem solving skills.

In noticing and attending to children’s strategies, Alexa identified the strategies that the two children used, collectively and individually. For example, Alexa noted that both children were able to count forward but they were confused when they counted by 10 from 965 to 975. However, she did not explain or provide details on how they counted and what confused them. Instead, she concluded that students were intimidated by the large numbers;
arguing that it was just as if hearing that they were counting by 900 intimidated them”. In problem set two where the students added and subtracted, Alexa noticed that the two children added or subtracted by place value and did not use the standard algorithm. She also noted that Makayla used the doubling strategy in one equation when explaining her answer. In her explanation she remarked:

When explaining her answer, she said that she “knew that 42 is half of 84, so the answer must be doubled to be equal… She also explained that Maddie got the equation right but went about it going from right to left on the equation without looking at the number relationships.

This response suggests that Alexa was cognizant of different strategies that the children could use to interact with the mathematical idea and differentiated what they used and what they did not use. For example she noticed that the children could have used the standard algorithm but they chose to use number relationships, specifically using the doubling to solve the problem.

Finally, when it came to the problem solving skills task, Alexa noted that both girls knew what to write down and added and subtracted as needed. In this problem set, Alexa provided an example of how Makayla solved one of the problems arguing; “Makayla solved her problem of 1001 minus 420 by rounding. She made it into 1000 – 400 then 20. But instead of adding the extra one she subtracted.”

Although she argued that Makayla rounded off the numbers as a strategy, it’s notable that Makayla was using place value understanding to make the numbers more convenient to subtract using mental strategy. In addition, one would only make a presumption that Alexa meant “subtracting 20” when she said “then 20”. This presumption would be made because she never provided Makayla’s work that would provide more details on how she had
interacted with the mathematical idea. From this response, I realized that it’s possible for a PST to notice children’s strategy differently so long as the strategy provides the right answer.

Overall, Alexa’s responses were coded to have some mathematical details because she only provided some details about the strategies that the two children used but did not provide substantial mathematical details on how the child solved the task (e.g., how they counted up or down). The responses also had some details on how the children made sense and interacted with the mathematical idea but one would need to make presumptions of her reasoning and to infer how the child interacted with the task.

**Interpreting children’s mathematical understanding**

For the component skill of interpreting children’s mathematical understanding, Alexa made three assertions about the two children’s mathematical understanding. Specifically, she argued that the two students know about the basic numbers and operations, know how to count and skip count as demonstrated by the students counting by 2, 5’s and 10’s and, and have solid concept of numbers over 100 and can make the jump from 900 to 1000 fairly easily as demonstrated in the interview. This assertion is worth noting because Alexa had argued that the children had a problem when counting numbers within hundreds 10 while attending to children’s strategies. When it came to interpreting children’s mathematical understanding, she argued that they had a “solid concept of numbers over 100”

Further, Alexa explained that the two children understood the concept of 10 and place value and demonstrated this by using base 10 blocks and using numbers and manipulating

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17 Note that Alexa had previously noted that the children did not know how to add by 10 at the 100th level. It was therefore not clear whether it was a misinterpretation or she did not make the right evaluation
them in base 10. In addition, she concluded the students struggled with making equations true or false, could balance simple equations but had difficulties making sense of the numbers on either side of the equal sign in an algebraic equation. However, she did not use any specific tasks that the students had done to elaborate or support her argument. Finally, she argued that the two children could add and subtract but neither came up with the right answer when subtracting problems that required regrouping. For this assertion she explained “they wrote an answer 32-2 instead of 322”. Again, it was not clear what Alexa was referring to.

Notable in the above explanation is the fact that Alexa did not make sense of details of children’s strategies and did not note how the details in a particular strategy reflected children’s mathematical understanding. In addition, she did not make sense of the children’s strategy details and some of the interpretations were not consistent with the strategy details. Instead, her interpretation focused on children’s mathematical understanding but the responses did not have in-depth interpretation like the robust evidence. Overall, connections to children’s strategies were overgeneralized and did not provide specific evidence on how the children solved the task. In situations where she provided evidence, it did not align with what she had noticed. Therefore, I coded her responses to have limited evidence that the interpretation was based on children’s mathematical understanding.

**Responding based on children’s mathematical understanding**

In the first tutoring session, Alexa’s teaching goal was to have the children compare the relationships of numbers and make equations true by analyzing patterns and number relationships using specific strategies. She provided the students with an opening number routine (*an opening activity designed to make the students start thinking about math and*
analyze the relationship between numbers). With this opening number routine, she provided these number choices  (15-19) (75-74) (37-37) (108-180) (5734- 5756) and 781- 781) to get the students thinking about math and to analyze the relationship between numbers; arguing that after completing this opening activity, students will begin to see the use of the equal sign in making mathematical equations true and not just signaling an answer. Further, Alexa explained that the students will also be asked to explain how they can make both sides of the equation true and explain how they know.

Alexa’s reasoning and rationale was unclear. First the numbers that she chose and the directions she gave the children were not so clear what the children will do with the opening number routine. The explanation was suggestive that she wanted the children to compare the numbers using less than, greater than and equal to sign, yet, she argued that she wanted “the children to begin to see the use of the equal sign” and to make both sides of the equation true but yet the numbers had no equal sign”. It was also unclear whether she understood what she was asking the children to do with the task. Although it’s not within the scope of this study, the inconsistency was worth noting because it raised questions of her understanding of the equal sign.

In the main activity, Alexa decided to present equations to the children and they make them true. To justify why she gave the main activity, she remarked: “I decided to do this concept because the girls I worked with last week struggled with making equations true and I believe it’s a crucial first step to algebra.” For each of the problems, Alexa provided a rationale of her choice of the problems and an explanation of how she will use the task to extend the children’s understanding. Below are the problems she chose to give the children as a main activity and the rationale of the number choices:
Problem set #1:

24 - 12 = 24 - ___

Rationale: A problem similar to this confused the students last week and would be a good first step for the lesson. To unpack the problem, I would ask them to think about the opening number routine and how they determined the answer to that problem. They would be encouraged to look at both sides and see what the other sides need in relation to the other.

24 - 12 = 25 - ___

Rationale: This would be getting to the core of the lesson, as it requires the students to decide if she should add or subtract in relation to the previous answer with similar numbers.

Problem set #2:

24 - 12 = 34 - ___

Rationale: Not only makes them know about how they need to subtract but also encourages base 10 strategy

Problem set #3

1. 87 - 54 = ___ - 54
2. 87 - 54 = ___ - 53
3. 87 - 54 = ___ - 65

Rationale: These numbers are significantly larger than the previous problem set and also further apart which can be further for the students to mentally grasp right away. They build a progression similar to the previous problem set with the first problem being equal set …This problem also have a different order for the blanks as it is a joint start unknown rather than a join change unknown problem.

Notable in the above quotation is the fact that Alexa considered students’ misunderstanding arguing “a problem like this confused the students last week”. In addition, she considered how the task would progress students’ understanding. For example, Alexa argued that the second problem set “Not only makes them know about how they need to subtract but also encourages base 10 strategy”. For the third problem set, Alexa argued: “The
numbers are significantly larger than the previous problem set and also further apart which can be further for the students to mentally grasp right away. They build a progression similar to the previous problem set with the first problem being equal set …” Further, Alexa explained that the 3rd problem set “have a different order for the blanks as it is a joint start unknown rather than a join change unknown problem.”

Alexa’s responses were characterized to have robust evidence that she based her responses on children’s mathematical misunderstanding. In other words, Alexa considered the challenges that the two children faced and chose tasks that would support Maddie and Makayla’s understanding. In addition, she considered how the tasks would progress the students understanding. Specifically, she increased the level of difficulty in the problems so that she would continuously challenge the students.

Overall, Alexa’s responses were worth noting because of the inconsistencies in her responses. For example, there was evidence that she only provided some mathematical details as she attended to children’s strategies both in the Inquiry into Student Thinking assignment and the tutoring assignment. However, in the component skill of interpreting children’s mathematical understanding, there was robust evidence that she made sense of details of children’s understanding and the interpretation was consistent with the strategy during the Inquiry into Student Thinking assignment but not during the tutoring assignment.

These results were also surprising and unpredicted. For example I did not anticipate that the PST could respond based on children’s mathematical understanding without robust evidence that interpretation was based on children’s mathematical understanding. During the tutoring assignment, there was limited evidence that she made sense of the details in children’s strategies and in some cases her interpretation was inconsistent with the children’s
strategies. However, there was evidence that her response was based on children’s mathematical understanding.

Hypothetically, I anticipated that she would make sense of children’s strategies and her interpretation would be more consistent with details in the children’s strategy during the tutoring assignment in comparison with Inquiry into Student Thinking assignment since they had discussed children’s strategies in the classroom by the time she did the tutoring assignment. In addition, her choice of task was consistent with the evidence that she provided in the Inquiry into Student Thinking assignment and, to some extent with the limited evidence that she provided during the tutoring assignment. However, there were many examples both in the Inquiry into Student Thinking and the tutoring assignment, where her number choices primarily raised questions regarding how the children would interact with the mathematical idea and, specifically whether she had considered the affordances and constraint there off. In summary, Alexa’s responses were classified to have an inconsistent pattern on interpreting children’s mathematical understanding and responding based on children’s mathematical understanding. In the next few paragraphs, I will summarize the lessons learned and insights from the multi-case case studies.

**Summary**

The responses discussed in the first three clusters were representative examples of how that group of PSTs performed in the three component skills across the two assignments. Concerning the component skill of attending to children’s strategies there was evidence that PSTs in the first and second cluster seemed to have consistently provided most mathematical details in the two assignments. These results were encouraging because the component skill
of attending to children’s strategies is a foundational skill to interpreting and responding based on children’s mathematical understanding. Therefore, the fact that the PSTs’ responses explored in the two clusters provided persuasive evidence that they had the capacity to attend to children’s strategies repeatedly across the two assignments was worth noticing.

With regards to the 3rd cluster, there was no evidence that the PSTs provided substantial mathematical details. Instead, they provided some or no mathematical details during the Inquiry into Student Thinking assignment but there was evidence that they provided most mathematical details during the tutoring assignment. These results also provided evidence that the PSTs ability to attend to children’s strategies had improved across the two assignments. Therefore, it seemed reasonable to argue that the course content had some impact on their ability to attend to children’s strategies by the time they did the tutoring assignment.

The fourth cluster was unique because of the unpredictability of the PSTs’ responses and the sporadic nature of their responses. For example, in the case of Alexa, there was no evidence that she improved in the component skill of attending to children’s strategies across the two assignments. One would have anticipated that she would perform better in the tutoring assignment because the course content specifically focused on paying attention to children’s strategies but that was not the case. Instead of improving in the component skill of interpreting children’s mathematical understanding, her response had limited evidence that she paid attention to children’s strategies as she interpreted children’s mathematical understanding during the tutoring assignment.

For the component skill of interpreting children’s mathematical understanding, the PSTs responded differently across the clusters. For example, there was evidence that the
PSTs included in the first and second cluster interpreted children’s mathematical understanding, consistently providing robust evidence of paying attention to children’s strategies during the two assignments. But in the 3rd cluster, there was evidence that they only provided robust evidence as they interpreted children’s mathematical understanding only in the tutoring assignment. This performance was reasonable since I either expected the PSTs to have the ability to interpret children’s mathematical understanding by the time they did the Inquiry into Students’ Thinking or to develop the ability as they progressed in their methods course. However, there was no reasonable explanation for the sporadic responses in the 4th cluster.

With regards to the component skill of responding based on children’s mathematical understanding, PSTs in the 1st and 3rd cluster seemed to have demonstrated evidence of responding based on children’s mathematical understanding. However, PSTs in the 1st cluster seemed to have provided evidence of their ability to respond based on children’s mathematical understanding earlier in the Inquiry into Student Thinking assignment than cluster 3. By the time they did the Inquiry into Student Thinking assignment, there was evidence that PSTs in cluster one responded based on children’s mathematical understanding. However, since I did not do a pre-course assessment, it was particularly unclear at what point the PSTs in cluster one had started noticing, interpreting and responding based on children’s mathematical understanding.

In this analysis, the PSTs decision on how to respond based on children’s mathematical understanding across the two assignments was of particular interest. As Jacobs and Ambrose (2008) argued, “by carefully sequencing problems, a teacher can create unique opportunities for mathematical discussions,” (p. 265). Yet, prior research (e.g., Crespo, 2003;
Nicol, 1998) has reported that PSTs have challenges in responding to children’s mathematical understanding. Interestingly, in this study, some of the PSTs seemed to consider children’s mathematical understanding as they responded.

Over all, the PSTs’ performance seemed to be in a developmental continuum from inconsistent and sporadic responses to those who noticed and used children’s mathematical understanding to choose and pose tasks in the two assignments. In this developmental continuum some of the PSTs responded in a sporadic manner in all the component skills in the two assignments. In fact, some PSTs’ responses that were included in cluster 4 had demonstrated evidence of interpreting and responding in the Inquiry into student thinking but in some cases they did not perform as well in the tutoring assignment in the same component skill. Instead they performed well in other component skill or in neither of the component skill. It was therefore not easy to generalize their performance. Figure 4.1 represents the hypothesized developmental learning continuum of the four clusters.
Figure 4.6. Hypothetical developmental continuum of PSTs’ learning the component skills of Professional Noticing of Children’s Mathematical Understanding.
CHAPTER 5. SUMMARY AND CONCLUSIONS

The need to design the content taught in teacher preparation programs in such a way that PSTs are prepared and equipped with knowledge, skills and practices to increase the chances that they will become effective novice mathematics teachers is of prime importance. Although researchers and mathematics education community have highlighted lingering challenges in teacher preparation programs (Ball et al., 2009; Cochran-Smith & Zeichner, 2005; Grossman & McDonald, 2008; Morris, Hiebert & Spitzer, 2009) the need for high quality teachers in K-12 classrooms has been and still remains imperative. Despite the challenges, there is an urgent call for teacher educators to change the way they prepare PSTs (NMAP, 2008). In fact, the National Mathematics Advisory Panel (2008) suggested that the “outcomes of different approaches should be evaluated by using reliable and valid measures of their effects on prospective and current teachers’ instructional techniques and, most importantly, their effects on student achievement” (p.20). Similarly, other teacher educators (e.g., Ball et al., 2009) have suggested that teacher educators need to identify “high-leverage practice which when done well by a teacher is likely to lead to comparatively large advances in student learning and make those practices the curriculum in teacher preparation” (p.4). Informed by the above-discussed studies, I conjectured that it is important to understand how PSTs develop the capacity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding.

This dissertation research examined PSTs’ capacity to use children’s mathematical understanding to select and pose worthwhile mathematical tasks in the context of scaffolded activities as they progress in their mathematics methods course. Considering the benefits of
using children’s mathematical understanding in mathematics classrooms (e.g., Franke & Kazemi, 2001; Franke, Kazemi, & Battey) and the importance of worthwhile mathematical tasks in students’ learning (Henningsen & Stein, 1997; Hiebert & Wearne, 1993; Hiebert et al., 1997), I chose to focus on PSTs’ skills and practices of using children’s mathematical understanding as they select and pose worthwhile mathematical tasks. This chapter begins with a discussion of the findings from this study. Specifically, I discuss the extent to which the PSTs developed the capacity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding. I conclude by describing theoretical insights and implications of this work for future research directions and teacher preparation.

**Using Scaffolded Activities to Understand PSTs’ Capacity to Apply Children’s Mathematical Understanding to Select and Pose Mathematical Tasks**

During the mathematics methods course, the instructor *purposefully* structured the activities and discussions so that PSTs had the opportunity to learn about:

1) How children’s thinking typically develops, including common understanding, misunderstanding, strategies and errors
2) How to access and assess children’s mathematical thinking within different content areas.
3) How to use children’s mathematical thinking to select and pose worthwhile mathematical tasks. (Course Instructor, 2011a, Fall)

To provide PSTs with opportunities to learn how children’s thinking typically develops, the instructor had a whole class discussion on the strategies that children typically use while adding and subtracting numbers. These strategies included direct modeling, counting up and using derived facts (Carpenter et al, 1999). Further, the PSTs had multiple opportunities to analyze children’s work and identify the strategies that children use as they progressed through the methods course As shown in Table 3.1, the opportunities were in the
forms of small group discussions, whole class discussions, viewing and analyzing video clips, analyzing students’ written work to assess children’s mathematical understanding, instructor’s written and verbal feedback, minor homework assignments and major homework assignments which had specific prompts for PSTs to use as they analyzed children’s mathematical understanding. In addition, PSTs were assigned readings that specifically focused on how to select tasks based on children’s mathematical understanding and how to assess children’s mathematical thinking as a teacher facilitates classroom discussions. (Parish, 2010; Smith et al., 2008). These activities were classified in six themes as stated in the course packet, with 4th, 5th and 6th theme focusing on problem posing, making sense of and responding to students’ work and facilitating whole class discussion of strategies that children use in the mathematics classroom. Although there were other themes in the course of the semester, the activities in the three selected themes purposefully focused on developing PSTs’ capacity to use children’s mathematical understanding to select and pose worthwhile mathematical tasks.

After learning about the strategies children commonly use and how children’s thinking typically develops, the PSTs analyzed one child’s response from a case study and they were specifically provided with scaffolds that would support them attend to children’s strategies, interpret children’s mathematical understanding and respond based on children’s mathematical understanding during the Inquiry into Student Thinking. The findings reveal that during the Inquiry into Student Thinking, an assignment that the PSTs did after six weeks of instruction, there was evidence that some PSTs had the capacity to attend to children’s strategies. Specifically, the findings reveal that 40% of the participants provided
most mathematical details and 56.7% provided some mathematical details. Further, only 3.3% of the participants’ responses did not have any mathematical details.

Additionally, when prompted to interpret children’s mathematical understanding, the PSTs’ responses demonstrate evidence that they interpreted children’s mathematical understanding. Specifically, 60% of the participants’ responses demonstrated robust evidence that their interpretation was based on children’s mathematical understanding, 33.3% of the responses had limited evidence that their interpretation was based on children’s mathematical understanding and 6.7% of the responses had no evidence that their responses was based on children mathematical understanding. Finally, the PSTs exhibited a potential to respond based on children’s mathematical understanding with 13.3% of the participants’ responses having robust that their responses were based on children’s mathematical understanding limited evidence, 60% having limited evidence and 26.7% having no evidence that their responses were based on children’s mathematical understanding.

These findings are noteworthy and encouraging in light of other studies (e.g., Jacobs et al., 2010) which reported that prospective teachers narratives mostly had limited or no evidence of attending to children’s strategies, limited or no evidence of interpreting children’s mathematical understanding, and has limited or no evidence that the response was based on children’s mathematical understanding. Table 5.1 provides a detailed comparative analysis.

In this study, the PSTs exhibited a capacity in the three component skills, with majority of them providing limited evidence or some mathematical details. If we take the findings by Jacobs et al. (2010) as a beginning point, PSTs in this study had already started
Table 5.1. Comparison of the PSTs’ performance in the component skills after six weeks of instruction to findings by Jacob et al. (2010)

<table>
<thead>
<tr>
<th>Component skills</th>
<th>Findings by Jacob et al. (2010)-</th>
<th>Current study-results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attending to Children’s Strategies</td>
<td>Provided evidence: 42%</td>
<td>Most mathematical detail: 40%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Some mathematical details: 7.7%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Limited mathematical details: 3.3%</td>
</tr>
<tr>
<td>Interpreting Children’s Mathematical Understanding</td>
<td>Robust Evidence: 0%</td>
<td>Robust Evidence: 60%</td>
</tr>
<tr>
<td></td>
<td>Limited Evidence: 47%</td>
<td>Limited Evidence: 33%</td>
</tr>
<tr>
<td></td>
<td>Lack of evidence: 53%</td>
<td>Lack of evidence: 6.7%</td>
</tr>
<tr>
<td>Responding based on Children Mathematical Understanding</td>
<td>Robust evidence: 0%</td>
<td>Robust evidence: 13.3%</td>
</tr>
<tr>
<td></td>
<td>Limited evidence: 14%</td>
<td>Limited evidence: 60%</td>
</tr>
<tr>
<td></td>
<td>Lack of any evidence: 86%</td>
<td>Lack of any evidence: 26.7%</td>
</tr>
</tbody>
</table>

developing the capacity in the skills and practices of attending to children’s strategies and interpreting based on children’s mathematical understanding. Although they were provided with scaffolds it was quite encouraging seeing some PSTs could provide robust evidence after six weeks of instruction.

Comparing the performance of the component skills in the two assignments

During the tutoring assignment, the PSTs were provided with an opportunity to interview children and analyze children’s mathematical understanding. The findings indicate that PSTs experienced positive changes in the three component skills. For the component skill of attending to children’s strategies, 73.3% of the participants responses had most mathematical details, 23.3% had some mathematical details and 3.3% had no mathematical details. For the component skill of interpreting children’s mathematical understanding, 70% of the responses demonstrated robust evidence that the interpretation was based on children’s mathematical understanding, 23.3% demonstrated limited evidence and 6.7% had no evidence that their interpretation was based on children’s mathematical understanding. For
the component skill of responding based on children’s mathematical understanding, 36.7% demonstrated robust evidence that their responses were based on children’s mathematical understanding, 33.3% demonstrated limited evidence and 30% of the responses had no evidence that their responses were based on children’s mathematical understanding. Table 5.2 compares the performance at two different times. These findings reveal that PSTs are able to improve the skills to notice and attend to children’s strategy when provided with an opportunity to learn.

Table 5.2. Comparison of PSTs’ performance in the component skills after 6 and 10 weeks to findings by Jacob et al. (2010)

<table>
<thead>
<tr>
<th>Component skills</th>
<th>Beginning of study (Jacob et al. (2020))</th>
<th>After 6 weeks Thinking assignment</th>
<th>After 10 weeks Tutoring assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attending to children’s strategies</td>
<td>Provided evidence: 42%</td>
<td>Most mathematical detail: 40%</td>
<td>Most mathematical detail: 73.3%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Some mathematical details: 57.7%</td>
<td>Some mathematical detail: 23.3%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Limited mathematical details: 3.3%</td>
<td>Limited mathematical detail: 6.7%</td>
</tr>
<tr>
<td>Interpreting children’s mathematical understanding</td>
<td>Robust evidence: 0%</td>
<td>Robust evidence: 60%</td>
<td>Robust evidence: 70%</td>
</tr>
<tr>
<td></td>
<td>Limited evidence: 47%</td>
<td>Limited evidence: 33%</td>
<td>Limited evidence: 23.3%</td>
</tr>
<tr>
<td></td>
<td>Lack of evidence: 53%</td>
<td>Lack of evidence: 6.7%</td>
<td>Lack of evidence: 6.7%</td>
</tr>
<tr>
<td>Responding based on children’s mathematical understanding</td>
<td>Robust evidence: 0%</td>
<td>Robust evidence: 13.3%</td>
<td>Robust evidence: 36.7%</td>
</tr>
<tr>
<td></td>
<td>Limited evidence: 14%</td>
<td>Limited evidence: 60%</td>
<td>Limited evidence: 33.3%</td>
</tr>
<tr>
<td></td>
<td>Lack of any evidence: 86%</td>
<td>Lack of any evidence: 26.7%</td>
<td>Lack of any evidence: 30%</td>
</tr>
</tbody>
</table>

These findings are encouraging and worth noting since they reflect a shift from providing some mathematical details to most mathematical details in the component skill of attending to children’s strategies during the tutoring assignment. In other words, the percentage of PSTs who provided limited mathematical details during the Inquiry into Student Thinking reduced in comparison with the tutoring assignment. Similarly, there was a shift from providing limited evidence to robust evidence during the tutoring assignment for
the component skill of interpreting children’s mathematical understanding. Although the performance in the component skill of responding based on children’s mathematical understanding was generally low, the change was still positive, an indication that the component skill of responding based on children’s mathematical understanding can be learned in a methods course. At least in this study, 36.7% of the participants provided robust evidence and 33.3% provided limited evidence that their responses were based on children’s mathematical understanding. Overall, there was evidence that PSTs’ capacity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding seemed to have improved.

These findings are congruent with other studies (e.g., Jacobs et al., 2010; Sherin & Van Es. 2005, 2009) that have reported that expertise in attending to children’s strategies, interpreting children’s mathematical understanding, and responding based on children’s mathematical understanding can be learned. Specifically, Jacobs et al. (2010) reported that the expertise in attending to children’s strategies “grew with teaching experience and continued to grow with two years of professional development” (p.14). This current study extends Jacobs et al.’s (2010) findings and provides evidence that the expertise of attending to children’s strategies, interpreting children’s mathematical understanding and responding based on children’s mathematical understanding can also develop in the context of scaffolded activities as the PSTs progress with the methods course. Although it maybe premature to argue that PSTs can independently provide robust evidence based on children’s mathematical understanding without scaffolds, the findings provide evidence for the idea that PSTs are able to enhance their capacity when provided with multiple opportunities to practice. Indeed, some PSTs consistently provided robust evidence across the two assignments.
One probable reason for these findings was the fact that PSTs were provided with an opportunity to develop knowledge, skills and practices of attending to children’s strategies, interpreting and responding based on children’s mathematical understanding using different scaffolds. The findings support the argument that multiple distributed scaffolds can enable students who are not able to learn from one scaffold to benefit from another scaffold. In fact, using the same prompts in different assignments may have enabled PSTs to develop an improved understanding over the course of the semester and, ultimately develop a better understanding by the time they did the tutoring assignment in comparison to the Inquiry into Student Thinking assignment.

These findings extend what we know about the design and the decisions that are made about the focus and the content to be taught in a teacher preparation program. Previous research has shown that the decisions made either in professional development or in teacher preparation program about the focus and the content to be taught can determine what the PSTs and/or in-service teachers will learn (Jacobs et al., 2007). Similarly, the decisions that were made on the content to be taught and the scaffolding activities to be used in this methods course may have significantly determined what the PSTs learned. Therefore, this current study shows that after eight weeks of instruction that purposefully focused on providing opportunities for PSTs’ learning about children’s mathematical understanding, some PSTs’ responses demonstrated evidence of attending to children’s strategies, interpreting children’s mathematical understanding and responding based on children’s mathematical understanding. In fact, the tutoring assignment provided PSTs with an opportunity to practice what they had learned in the university classroom with children in an authentic setting.
Hypothetical developmental continuum of PSTs’ learning

As shown in figure (4.1), the findings in this study also provide additional insights to a hypothetical developmental continuum of how PSTs develop the component skills of professional noticing of children’s mathematical thinking. Some PSTs’ responses had very inconsistent evidence of how they developed the three component skills. Meaning that some of them provided most mathematical details in the Inquiry into Student Thinking, but did not provide any mathematical details during the tutoring assignment. The same case applied with component skill of interpreting children’s mathematical understanding and responding based on children’s mathematical understanding. These sporadic and inconsistent responses might have been caused by some PSTs feeling comfortable with one component skill and not the other.

I speculated that one probable cause of the sporadic responses is the fact that PSTs could have noticed some aspects of classroom interaction but they did not write them down in their responses. Therefore, analyzing written responses could not have fully exhibited PSTs’ capacity in all the component skills. It’s also possible that PSTs included in this cluster struggled with different component skills as they progressed with the methods course.

The second recognized pattern included PSTs who did not attend to children’s strategies, interpret children’s mathematical understanding or base their responses on children’s mathematical understanding during the Inquiry into Student Thinking assignment. However, there was evidence that they attended to children’s strategies, interpreted children’s mathematical understanding and responded based on children’s mathematical understanding during the tutoring assignment. This pattern was intriguing and worth further investigation because the Inquiry into Student Thinking assignment involved analysis of
students’ written work in an online case study and could mean that PSTs might be more comfortable with accessing and assessing students’ mathematical understanding in a real authentic setting. Therefore, if we only use the students’ work to determine the PSTs’ ability to attend to children’s mathematical understanding, interpret and respond based on children’s mathematical understanding, then we underestimate their potential. Like any other learning, some PSTs might understand better what it means to access and analyze children’s mathematical understanding in a real authentic setting.

Additionally, it might be possible that PSTs had not fully understood the component skills of professional noticing of children’s mathematical understanding by the time they did the Inquiry into Student Thinking, but the assignment provided them with an opportunity to learn, which could have contributed to a shift from providing limited or just some mathematical details to robust evidence.

The third recognized pattern included those responses that demonstrated evidence that PSTs attended to children’s strategies and interpreted children’s mathematical understanding in the two assignments, but did not respond based on children’s mathematical understanding. This pattern is congruent with the reported findings in Jacobs et al. (2010), showing that the component skill of responding based on children’s mathematical understanding takes a long time to develop. In their study, Jacobs et al. reported that this component skill developed with “teaching experience and with more than two years of professional development” (p. 23). However, although this is the case, it is still important to find ways and strategies to support PSTs’ understanding of this component skill before they become involved with professional development in the schools where they start their teaching profession.
The last recognized pattern includes the PSTs’ responses that had evidence (either most or just some mathematical details) of attending to children’s strategies, interpreting and responding based on children’s mathematical understanding in the two assignments. Although the PSTs exhibited this potential in a scaffolded activity, it is worth noting and encouraging, knowing that some PSTs capacity to use children’s mathematical understanding to select and pose tasks developed within the eight weeks of instruction. Although this claim should be taken with caution, this group of PSTs provided some evidence that PSTs are able to attend to children’s strategies, interpret and respond based on children’s mathematical understanding when provided with multiple scaffolds that would purposefully develop the three component skills. Overall, performance in the three component skills was in a continuum of providing inconsistent evidence to those PSTs who had the capacity to attend to children’s strategies, interpret and respond based on children’s mathematical thinking during the two assignment.

Further scrutiny of the component skill of responding based on children’s mathematical understanding revealed that the PSTs paid careful attention to number choices during the Inquiry into Student Thinking assignment and tutoring assignment respectively. Although the tasks selected mirrored the tasks in the course packet, it’s encouraging to note that the PSTs were cognizant of the number choices as they selected instructional tasks.

In addition, PSTs had started developing an understanding of what it means to have a productive task. When prompted to identify a task that was productive to advance children’s mathematical understanding and provide a rationale for the choice, most PSTs described productive tasks as a task that advanced children’s mathematical thinking, extended children’s thinking and tasks that are based on real life connections. However, there was no
notable connection between the tasks that they selected and their understanding of a productive task.

Similarly, while prompted to provide a rationale explaining why their choice of task is of high cognitive demand, the results indicates that PSTs’ understanding of the levels of cognitive demand varied. Some PSTs explained that the task is of high cognitive demand because of how the children will engage with the task, while some explanations indicated that the task is of high cognitive demand because of how the teachers will use the task to instruct students. Surprisingly, none of the PSTs related a productive task with the levels of cognitive demand or vice versa.

These results provide insights to PSTs’ understanding of the nature of instructional tasks. Some PSTs described a task as productive if the children were engaged and the task supports their understanding. Other PSTs focused on how the teacher will use the task during instruction. Since most of the PSTs tended to stick closely to the tasks that were provided to them in the course packet and/or the task pool provided during the interview session, it would be worth investigating whether their conceptions of productive tasks and/or tasks of high cognitive demand limited their ability to respond based on children mathematical understanding.

Overall, these findings suggest that providing PSTs with scaffolded activities that supported their learning on how children’s thinking typically develops, how to access and assess children’s mathematical understanding and how to select and pose mathematical tasks was beneficial to their learning.

Further scrutiny of PSTs’ responses indicates that most of the participants selected tasks with careful attention to the number choices. Some PSTs selected multiple number
choices that would provide the children with opportunities to extend and challenge their thinking while others chose numbers that would advance children’s mathematical understanding in both the Inquiry into Student Thinking and tutoring assignment. During the Inquiry into Student Thinking assignment most PSTs modeled the CGI word problem framework. This format changed during the tutoring assignment when some PSTs selected word problems with multiple number choices. However, it was particularly concerning to note that some number choices were not consistent with the PSTs’ rationale. In other words, some number choices did not match with the nature of task selected and could not actually be solved with the selected number choices.

In addition, PSTs’ interaction with children in a classroom setting provided them with an authentic experience with the children. As Van Zoest and Stockero (2008) argue, using “carefully designed synergistic scaffolds” (p.1) could possibly provide the PSTs with more opportunities to learn than what they could have learned from one scaffold. This finding extends what we know about PSTs problem posing strategies. Prior research (e.g., Crespo, 2003; Norton & Rutledge, 2006; Rutledge & Norton, 2008) has shown that PSTs can develop their ability to pose mathematical tasks in the context of letter writing. Specifically, Crespo (2003) reported PSTs’ problem-posing strategies changed “from traditional single steps and computational problems to problems that required multiple steps open ended, exploratory and were cognitively more complex” (p.1). In this study, PSTs were provided with the CGI framework of word problems, strategies that children typically use, discussions on how to access and assess children’s mathematical understanding, as well as multiple opportunities to select and or generate tasks before planning for the tutoring sessions. Although they may have mirrored the tasks provided in the course packet, these
findings indicate that PSTs were able to pay careful attention to number choices and specifically focused on tasks that would challenge or advance students’ understanding.

However, PSTs exhibited a limited understanding of what it means for a task to engage children in high-level thinking or the levels of cognitive demand as described by Stein & Smith (1998). Although the PSTs had discussed the levels of cognitive demand in the university classroom and had actually done a sorting activity (see Appendix B for details of assignments and activities in the course) before planning for the tutoring assignment, their conceptions of the levels of cognitive demand varied in focus from “what the teachers will do when enacting the task, what the children will do when enacting the task, the nature of the task and some did not provide a rationale why the selected tasks were of high cognitive demand. The findings are noteworthy because PSTs’ understanding of a high level task might determine the nature of tasks they select, generate and pose to the students.

**Summary**

After having an opportunity to learn about how children’s thinking typically develops, strategies that children commonly use, common errors and typical misunderstanding and how to select and pose a task based on children’s mathematical understanding during the mathematics methods course, the PSTs exhibited a capacity to attend to children’s strategies, interpret children’s mathematical understanding and to some extent some PSTs responded based on children’s mathematical understanding. Their responses to the Inquiry into Student Thinking and tutoring assignment provided evidence (either limited or robust) that they attended to children’s strategies, interpreted and responded
based on children’s mathematical understanding especially when they had the opportunity to practice during the tutoring assignment.

However, despite the positive outcome in the first two component skills, the PSTs struggled with the component skill of responding based on children’s mathematical understanding with very few PSTs providing any robust evidence that their responses were based on children’s mathematical understanding. One possible reason for this struggle is the fact that the course took a short time before the PSTs went for the practicum and the fact they did not have prior experience with children. In addition, the component skill of responding based on children’s mathematical understanding required the PSTs to make an instructional judgment on their own after the first interview. Although they had reflections in the university classroom after every tutoring session, the PSTs were expected to select or generate task for their individual students. Further, the PSTs struggled with identifying the levels of cognitive demand of the tasks they had selected even though it was one of the prompts they were supposed to respond to.

**Implications of the Study**

**Theoretical Implications**

The findings from this study shed light on the PSTs’ performance of the component skills of professional noticing of children’s mathematical understanding. This study also provides insights into PSTs’ practices of selecting and/or generating tasks that would engage students in high-level thinking after assessing children’s mathematical understanding.
Professional Noticing of Children’s Mathematical Thinking

Sherin and colleagues’ (e.g., Sherin, Linsenmeier & Van Es; 2009; Sherin & Van Es, 2005; Van Es & Sherin, 2007) work has largely focused on developing teachers’ ability to notice and attend to children’s strategies in mathematics classroom. Jacobs and colleagues’ (e.g., Jacobs, Lamb & Philip, 2010; Jacobs & Ambrose, 2008) work has extended the work of noticing by identifying three component skills of professional noticing of children’s mathematical thinking by describing the expertise of noticing as three components of interrelated skills of attending to children’s strategies, interpreting children’s mathematical understanding and deciding how to respond based on children’s mathematical understanding (See figure (2.1)). For the component skill of deciding how to respond, Jacobs and Ambrose (2008) have outlined four extending moves that a teacher can use after a child obtains the right answer. Drawing the analysis from two assignments, Inquiry into Student Thinking and the Tutoring assignment, this study builds on this work by highlighting PSTs’ capacity in the three component skills of attending to children’s strategies, interpreting children’s mathematical understanding and responding based on children’s mathematical understanding, at two different times as they progressed through the methods course. The study provides additional insights by revealing PSTs’ practices in the three component skills as they go through a methods course.

Additionally, this study sheds light on the nature of tasks that PSTs selected after accessing and assessing children’s mathematical understanding, their selection of number choices, PSTs’ conceptions of productive tasks and/or tasks of high level thinking.
Both Sherin and colleagues’ (e.g., Sherin, Linsenmeier & Van Es; 2009; Sherin & Van Es, 2005; Van Es & Sherin, 2007) work and Jacobs and colleagues’ (Jacobs, Lamb & Philip, 2010; Jacobs & Ambrose, 2008) work have argued that accessing and assessing children’s mathematical understanding involves noticing and attending to noteworthy aspects of classroom environment, interpreting children’s mathematical understanding, using knowledge about the context to reason and using what you have learned in a particular situation to respond based on children’s mathematical understanding. Jacobs et al. (2010) also proposed a hypothesized developmental trajectory of professional noticing of children’s mathematical thinking with three interrelated component skills of attending to children’s strategies, interpreting and responding based on children’s mathematical understanding.

Further, Jacobs et al. (2010) argued that the component skills can be learned in a professional development context that is purposefully designed to develop the component skills. This study’s findings provide additional insights on how PSTs performed in the component skills at two different times as they progress in their methods course. Also, this study reveals that although prior research (e.g., Jacobs et al., 2010) has suggested that the component skills develop with teaching experience and in the context of a professional development, our analysis revealed persuasive evidence that the component skills can be developed in the context of a mathematics methods course. Figure 5.1 presents a theoretical model for PSTs’ learning in context of scaffolded activities.
This study also provides important insights to the hypothesized theoretical trajectory of professional noticing of children’s mathematical understanding. Jacobs et al. (2010) theorized the three component skills as interrelated skills where, attending to children’s strategies would or should influence the PSTs’ interpretation of children’s understanding and eventually influence the way the participant responds based on children’s mathematical understanding, as shown in Figure 5.2.
However, in this study, some participants’ performance did not fall within the hypothesized trajectory. For example, some participants interpreted children’s mathematical understanding even when they had not provided any mathematical details. The findings in this study suggest that it’s possible for PSTs to interpret children’s mathematical understanding even if one has not attended to children’s strategies\textsuperscript{18}. In addition, some participant responses had robust evidence that their responses were based on children mathematical understanding but yet they had not provided any mathematical details as they

\textsuperscript{18} I notify the reader that the conceptualization of responding based on children’s mathematical understanding in this study does not include supporting children’s exploration of the task. It refers to the selection of the task that PST is going to pose to the students based on what they understand. Whether the teachers successively support the exploration of the task is not within the scope of this study
attended to children’s strategies and their interpretation was not based on children’s mathematical understanding. Therefore, the findings in this study suggest that it’s possible for teachers to develop capacity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding independently. As shown in Figure 5.3, I conceptualize the model for PSTs’ learning to be more independent in the three component skills.

Therefore, the findings in this study suggest that it’s possible for teachers to develop capacity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding independently. Figure 5.3 depicts my conceptualized model for PSTs’ learning to be more independent in the three component skills.

![Figure 5.3. Model for PSTs’ learning of the component skill: Professional Noticing of Children’s Mathematical Understanding](image)

One possible explanation for these findings is that PSTs may have noticed children’s strategies but did not necessarily write down what they noticed. Such probability would make participants use what they noticed as they interpreted children’s mathematical understanding even if it’s not necessarily written in the narrative. Therefore, these findings challenge the hypothesized trajectory of professional noticing of children mathematical thinking and
conceptualize the trajectory as less interrelated as suggested. Figure (5.3) presents the modified hypothesized trajectory. Therefore, the findings in this study suggest that it’s possible for teachers to develop capacity in one of the component skills independently.

**Implications on teacher preparation programs**

The findings from this study have implications for teacher preparation programs. PSTs need opportunities to experience and learn how to use children’s mathematical understanding as they plan for instruction. Specifically, they need to learn how to respond based on children’s mathematical understanding, a component skill that most teachers only acquire after a sustained period of professional development. Based on the benefits of using children’s mathematical thinking in mathematics classroom (Carpenter et al., 1989; Franke, Kazemi, & Battey, 2007; Franke & Kazemi, 2001; Jacobs et al, 2007; Sfard & Kieran, 2001; Fraivillig, 2001) PSTs’ learning the component skills will support their practices of using children’s mathematical understanding as beginning teachers. Therefore teacher educators should structure the activities and learning experiences that would purposefully develop the three component skills. In the next few paragraphs, I outline a few suggestions.

This study is congruent with other studies that have reported that due to lack of teaching experience, PSTs have limited ability to notice important aspects in the classroom (Cooper, 2010; Jacobs et al., 2010; Sherin & Van Es, 2005; Van Es & Sherin, 2007). Teacher educators need to be aware of the PSTs’ abilities in order to structure methods courses with activities that can develop this ability. I suggest that teacher educators can do a pre-assessment as the PSTs start their methods course to determine the individual support and scaffolds that each PST might need as they progress in the program. The scaffolds should be
distributed within the semester in such a way that they develop individual skills and eventually support PSTs’ ability to see the connection between the three component skills. Specifically, some scaffolds should focus on the individual skills while some should focus on all the three component skills as PSTs progress with the methods course. The activities that the instructor provided to the participants in this study can be productive first steps towards looking for scaffolds that would be beneficial to PSTs learning. However, I would propose that teacher educators provide feedback and interventions after every tutoring session that would support the PSTs to move from attending and interpreting to responding based on children’s mathematical understanding.

As discussed at the beginning of this chapter, the activities provided by the instructor in the context of this study can serve as beginning next steps of building up activities that can support PSTs ‘learning how to use children’s mathematical understanding as they select and pose mathematical tasks. Specific suggestions might also include providing more opportunities for PSTs to interact with children in real authentic settings. Even though the tutoring experience is only a snapshot of real classroom experience, the opportunity would be beneficial for PSTs’ learning. Using these activities supported the PSTs to shift their responses from limited evidence to robust evidence in the 1st two component skills or from lack of evidence to limited or robust evidence.

The findings in this study illuminate the practices of attending to children’s strategies, interpret and respond based on children’s mathematical understanding as one of the “high-leverage practice” (Ball et al., 2009) that teacher educators should consider as they structure content and the practices to teach in the mathematics methods courses. I propose that teacher educators need to purposefully choose what to teach, how to teach and the activities to use as
they develop PSTs in the three component skills. As Jacobs, et al. (2007) indicates, the decisions that are made either in professional development or in teacher preparation program about the focus and the content to be taught can determine what the PSTs and/or in-service teachers will learn. As noted earlier, the PSTs were provided with multiple scaffolds and opportunities to support their understanding. In other words, for every content area, they had an opportunity to interpret children’s mathematical understanding using written work, think about the task they would pose to the children after interpreting their mathematical understanding and, view a video clip of a teacher posing the next instructional task as they respond to children’s mathematical understanding. Therefore, teacher educators should purposefully choose intervention activities that would facilitate the PSTs’ learning.

Finally, more research is needed to determine activities and experiences that would develop the component skill of responding based on children’s mathematical understanding in the context of a methods course because the activities provided in this study seems to have been insufficient to significantly develop the component skill. Although previous research have suggested that developing the component skill of responding takes long to develop, teacher educators cannot underestimate the need to equip PSTs and beginning teachers with beginning competencies if they have to become well started off teachers.
Limitations

This study was conducted with the following limitations that should be taken into account when interpreting the findings:

First, I used the PSTs’ reflections to examine and make meaning of their performance of the three component skills. I acknowledge the limitation of using written work because PSTs may not have written down what they noticed due to limitations in language use. Interpreting their written work may also mean that I have made different inferences than the original intent. Therefore, the claims made in these study should be taken with caution. More data in the form of interviews or surveys would be needed to back up the written narratives.

Second, PSTs did these two assignments for the purpose of a grade for their methods course. It is not very clear how much of their understanding would translate into classroom practices when they become beginning teachers and especially bearing in mind that using children’s mathematical understanding does not naturally develop. However, since other studies (e.g., Franke et al., 2001) have suggested that using children’s mathematical understanding is generative, I am hopeful that starting teachers in the methods course provides opportunities that will support them as they begin their teaching.

Conclusion

This dissertation research contributes to the field’s understanding of PSTs’ capacity to attend to children’s strategies, interpret and respond based on children’s mathematical understanding as they progress in the methods course. In addition, the study contributes to the body of knowledge that is seeking to understand what PSTs can learn in teacher preparation programs in order to become effective mathematics teachers. Specifically, the
study suggests that PSTs can learn how to attend to children’s strategies, interpret and respond based on children’s mathematical understanding in the context of scaffolded activities. Specifically, the course packet that was used for this course had tasks that had been purposefully chosen to develop their ability. Finally, although the scaffolds used in this course seem to have developed the PSTs’ capacity to use children’s mathematical understanding, the scaffolds did not support them to the same degree in developing the capacity to respond based on children’s mathematical understanding. Therefore, it’s important to continue modifying the scaffolds to provide PSTs with opportunities to develop all the component skills.
APPENDIX A. COURSE SYLLABUS


Course Materials: Bring these to class beginning with the first class meeting.
Supplemental Packet: All students must purchase the Math for Elementary Education 1 packet from the University Bookstore. (This must be new.)

Other:
1. A calculator (a basic one is fine.) Your cell phone cannot be your calculator.
2. 3 × 5 Index cards (for use on quizzes – see below)
3. Loose leaf paper for text homework assignments

Highlights of the course components (see details on next pages):

1. There will be reading assignments from the text.
2. Practice exercises from the text are suggested and homework Problems from the text are assigned. Homework assignments go on lined loose-leaf paper unless otherwise stated in the schedule or by your instructor. DO NOT HAND IN TORN PAPERS FROM A SPIRAL NOTEBOOK.
3. The homework problems are generally collected and graded. No late homework will be accepted. The lowest HW score is dropped. The practice exercises are suggested but not collected and graded.
4. Activities from the Activity Manual will be done in class and may be assigned as homework.
5. Supplemental material out of the Bookstore Supplemental Packet will be done in class and may be assigned as homework.
6. Quizzes are given generally once a week. There are no make-up or early quizzes. The two lowest quiz scores are dropped.
7. A computation quiz will be given about ½ way through the semester
8. 3 exams and a comprehensive final are given.
9. Attendance is required.
10. If you miss a class, you are responsible for turning in all assignments on time and for all work missed, including activities from the Activity Manual or Bookstore Supplemental Packet. It is your responsibility to contact a classmate to get class notes, up-to-date homework assignments, etc. At the discretion of your instructor, late Activity Manuals checks may be accepted with a % or point penalty.
4. You are encouraged to have a ruler, protractor, compass, scissors, and colored pencils. Some will be provided during class, but you may prefer to use your own and these items may be needed for homework.

Course Goals: This course targets the mathematics subject matter specialization standard of the Iowa State Teacher Education Standards. It is designed to help you understand the central concepts, tools of inquiry, and structure of mathematics and prepare you to create learning experiences that make these aspects meaningful for elementary students. At the end of this course, you will have both content and process knowledge. You will have experienced what it means to think mathematically, understand the value of conceptual insight, and appreciate how mathematical knowledge is constructed in an exploratory manner.

Course Prerequisites: Satisfactory performance on the placement exam, 2 years of high school algebra, 1 year of high school geometry, enrollment in an early childhood or elementary education program.

Websites:
Textbook Companion Website: http://wps.aw.com/aw_beckmann_mathelem_3/

Content Covered: The course content focuses on systems of whole numbers, numeration, algorithms and interpretations for whole number computation, topics from number theory, algebra, geometric shapes and measurement, congruence, and transformations. We will cover portions of chapters 1, 3, 4, 6, 8, 9, 10, 11, 13, & 14 of the text and accompanying activities from the activities manual. You are responsible for all content covered in class and in the assigned readings.

Homework: Homework Problems will be assigned daily. To complete the assignments:

- Begin each assignment on a new page (lined loose leaf paper) and clearly identify the assignment by day (see schedule), corresponding pages, and problem numbers. Each problem should be numbered, completed in order, and include all the work for each problem next to the number. The problems should all begin on the left side of the page (i.e., do not complete HW problems in columns on your paper). Staple all pages of an assignment together and label the upper right hand corner with first and last name, course and section. At the top of the first page write your table number (big). See the sample HW on WebCT.

1. If you do not “get” a problem, then be sure to leave a trail of your attempts and thinking: did you draw a picture? Did you try some numbers? Did you attempt an algebraic solution? Make a serious attempt on all problems and document your efforts (show all work).

2. On most days when you enter class, begin discussing your HW solutions with the person(s) next to you. For problems on which you disagree about the solution, explain your process (es) and answer(s) to each other until you agree on the solution. For problems on which you do not reach an agreement, indicate on the board, which problem(s) need to be discussed as a class. It is ok to make changes on your homework when it is discussed in class if you have gained some new insight into the problem. Note that we may not have time on all class days to discuss all homework questions thus you should try to get questions answered before coming to class.

3. Homework will be collected and scored based on 10 points. For each collected set of homework, a portion of the points will be based on completeness of the problems. I will be checking to see if each problem was completed and included an explanation or supporting work. The remaining points will be awarded based on “spot checks” (i.e., 1-4 problems will be check thoroughly for thoughtful work and explanations). These problems will not be indicated prior to collection. Practice exercises will not be collected.

4. No late Homework is accepted. Homework may be turned in early or by a classmate.

5. Your lowest homework score will be dropped at the end of the semester.
Activity Manuals:
- Answers to activities from the Activities Manual are an important component of the course.
- You are to record your answers in the manual or on a sheet of colored paper from the back of the Bookstore Supplemental Packet, as directed by your instructor.
- For the occasions for which answers are to be recorded on the colored paper, you are to label the upper right hand corner of the page with first and last name, course # and section. At the top of the page clearly write the class activity number. Clearly indicate the question numbers down the left side. Do NOT put your answers in two columns on the page, unless your instructor directs you.
- Your responses to the activities will be collected periodically and graded.

Quizzes:
- The content will include topics from assigned HW, readings, class discussions, and work from the activity manual. The content will be indicated on the schedule.
- For each quiz, you may use ONE 3 × 5 index card with information recorded from the text, Activity Manual, and your notes. You cannot share prepared index cards with others in the class (es).
- No early or make-up quizzes will be administered.
- Each quiz is worth 8 points.
- Your lowest two quiz scores will be dropped at the end of the semester.

Attendance: This course emphasizes active participation, small group work, the processes of exploration and discovery, and communication of mathematical ideas. You are expected to attend class daily, arrive on time, attend the entire class session, participate in and complete all activities, and share in the discussion of assigned problems, readings, and activities. Due to the laboratory nature of this course, the use of manipulative[s] in activities, and the communication component of class, daily attendance is extremely important and will be taken every class session. More than 2 absences are considered excessive and will result in a deduction of the final grade by .5% for EACH absence beyond two.

Absences due to university activities or athletics count toward these two excused absences. If you must miss more than two class periods due to team travel or other university related activity, these absences will not affect your grade. You are expected, however, to attend all other class sessions.

Illness Attendance Policy: We wish to minimize the spread of the flu and other illnesses that might be spread to other students and request that students not come to class when they are ill. Therefore if you have the flu or another contagious health issue, a continuous set of days missed will count as a single absence if you turn in a completed “flu attendance form” found on the course WebCT page within one week of returning to class. If you have a serious illness please talk with your instructor about how this may affect your grade.

Cell Phones and other distractions: In order for you to fully participate in class, cell phones should be turned off and put away while you are in the classroom. Any use of your phone including texting or checking to see missed calls during class will result in a recorded absence for that class session. If you are in a situation where you need to have your phone on during class, please inform the instructor before class begins.

Similarly, all newspapers and other materials not related to the class are to be put away once class begins. Failure to do so will result in a recorded absence for that class session.

Exams: Three evening exams and a comprehensive final will be administered during the term. The three evening exams (8:00-9:30 PM) are scheduled as follows:
- Exam I: Tuesday, Feb. 8
- Exam II: Thursday, March 10
- Exam III: Thursday, April 14
• If you are unable to take an evening examination at the scheduled time because of a course conflict or other reason, you must notify me in advance to arrange for an alternate time.
• Due to the three evening exams, three regular class meetings will be omitted on dates to be announced.
• The $3 \times 5$ index cards used as aids on the quizzes may NOT be used on the exams.

**Computation Quiz:**
• There will be a no-calculator quiz administered during class, covering order of operations, multi-digit addition, subtraction, multiplication, and long division computations.
• Fifteen of the total points on the final may be earned on this no-calculator quiz.
• If you receive 90% or above on the quiz, then you will get the full 15 points on the final.
• If you do not score at least 90%, you will be given an opportunity to retake the quiz after you have made arrangements to strengthen your skills.
• There will be a maximum of 3 retakes on the quiz.
• A sample computation quiz is available on WebCT.
• The Computation Quiz takes place about ½ way through the semester as announced.

**Grading:** The components of the course are weighted, and grades will be assigned as follows:

<table>
<thead>
<tr>
<th>Component</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework</td>
<td>10%</td>
</tr>
<tr>
<td>Activity Manuals</td>
<td>10%</td>
</tr>
<tr>
<td>Quizzes</td>
<td>10%</td>
</tr>
<tr>
<td>Exams (3 evening)</td>
<td>50% (total)</td>
</tr>
<tr>
<td>Cumulative Final (including computation quiz)</td>
<td>20%</td>
</tr>
</tbody>
</table>

**Course grading scale:**
A 90.00-100%
B 80.00-89.99%
C 70.00-79.99%
D 60.00-69.99%
F 0.00-59.99%

Pluses and minuses may be given.

**Note:** A grade of C- or better is a prerequisite to enrolling in Mathematics for elementary education 2 and the mathematics methods class. Incompletes will be given rarely and in accordance with university guidelines.

**Cheating:** Any student who is caught cheating on an assignment, quiz, or exam will earn a zero for that assignment, quiz, or exam and will be reported to the dean of students for academic dishonesty.

**Special Needs:** Please address any special needs or special accommodations with me at the beginning of the semester or as soon as you become aware of your needs. Those seeking accommodations based on disabilities should obtain a Student Academic Accommodation Request (SAAR) form from the Disability Resources (DR)

Course Materials: Bring these to class beginning with the first class meeting.


Supplemental Packet: All students must purchase the Mathematics for Elementary Education 2 course packet from the University Bookstore.

Other:
5. A calculator (a basic one is fine.) Your cell phone cannot be your calculator.
6. 3 x 5 Index cards (for use on quizzes – see below)
7. Loose leaf paper for text homework assignments
8. You are encouraged to have a ruler, protractor, compass, scissors, and colored pencils. Some will be provided during class, but you may prefer to use your own and these items may be needed for homework.

Course Goals: This course targets the mathematics subject matter specialization standard of the Iowa State Teacher Education Standards. It is designed to help you understand
- the central concepts
- tools of Inquiry
- Structure of Mathematics

and to prepare you to create learning experiences that make these aspects meaningful for elementary students. At the end of the course you will have both content and process knowledge. You will have experienced what it means to think mathematically, understand the value of conceptual insight, and appreciate how mathematical knowledge is constructed in an exploratory manner.

Therefore, in this course an emphasis will be placed on the active participation of students, working in small group settings in the process of discovering and communicating mathematical ideas at the same time, this is a content course and students are expected to learn the mathematical terminology and concepts. Topics covered will include elementary statistics and probability concepts as well as fractions, decimals, integers, percents and geometry.

Prerequisite: Completion of Mathematics for Elementary Education 1 or its equivalent with a grade of C- or better. Only elementary education or early childhood majors are permitted to take Math 196.

Websites:

Textbook Companion Website: http://wps.aw.com/aw_beckmann_mathelem_3/

Homework: Homework Problems will be assigned daily. To complete the assignments:

6. If you do not “get” a problem, then be sure to leave a trail of your attempts and thinking: did you draw a picture? Did you try some numbers? Did you attempt an algebraic solution? Make a serious attempt on all problems and document your efforts (show all work).

7. On most days when you enter class, begin discussing your HW solutions with the person(s) next to you. For problems on which you disagree about the solution, explain your process (es) and answer(s) to each other until you agree on the solution. For problems on which you do not reach an agreement, indicate on
the board, which problem(s) need to be discussed as a class. It is ok to make changes on your homework when it is discussed in class if you have gained some new insight into the problem. Note that we may not have time on all class days to discuss all homework questions thus you should try to get questions answered before coming to class.

8. Homework will be collected and scored based on 10 points. For each collected set of homework, a portion of the points will be based on completeness of the problems. I will be checking to see if each problem was completed and included an explanation or supporting work. The remaining points will be awarded based on “spot checks” (i.e., 1-4 problems will be check thoroughly for thoughtful work and explanations). These problems will not be indicated prior to collection. Practice exercises will not be collected.

9. No late Homework is accepted. Homework may be turned in early or by a classmate.

10. Your lowest homework score will be dropped at the end of the semester.

Quizzes:
- The content will include topics from assigned HW, readings, class discussions, and work from the activity manual. The content will be indicated on the schedule.
- For each quiz, you may use ONE 3 × 5 index card with information recorded from the text, Activity Manual, and your notes. You cannot share prepared index cards with others in the class.
- No early or make-up quizzes will be administered.
- Each quiz is worth 8 points.
- Your lowest two quiz scores will be dropped at the end of the semester.

Attendance: Due to the laboratory nature of the course, the use of manipulative[s] in the explorations and the emphasis on oral and written communication, daily attendance is important. Students must come to class on time, with proper materials.

It is our experience that at times student may need to miss class for a good reason: a student may become seriously ill, may need to attend a funeral, may have a doctors appointment that cannot be scheduled at a different time, another class may require students attend a special event during our regular class period or may encounter weather or car problems. Therefore we will allow two excused absences during the semester. Emergencies happen—hoard those excused absences. Of course you are still responsible for obtaining from a classmate any changes in assignments and class notes, and for turning in assignments on time, missed work, and explorations. Any absences beyond two will result in a lowering of the final grade by .5% for EACH absence.

Absences due to university activities or athletics count toward these two excused absences. If you must miss more than two class periods due to team travel or other university related activity, these absences will not affect your grade. You are expected, however, to attend all other class sessions.

Illness Attendance Policy: We wish to minimize the spread of the flu and other illnesses that might be spread to other students and request that students not come to class when they are ill. Therefore if you have the flu or another contagious health issue, a continuous set of days missed will count as a single absence if you turn in a completed “flu attendance form” found on the course WebCT page within one week of returning to class. If you have a serious illness please talk with your instructor about how this may affect your grade."
Cell Phones and other distractions: In order for you to fully participate in class, cell phones should be turned off and put away while you are in the classroom. Any use of your phone including texting or checking to see missed calls during class will result in a recorded absence for that class session. If you are in a situation where you need to have your phone on during class, please inform the instructor before class begins. Similarly, all newspapers and other materials not related to the class are to be put away once class begins. Failure to do so will result in a recorded absence for that class session.

Exams Three NIGHT exams (8:00-9:30 PM) are scheduled as listed below and a comprehensive final will be administered.

- Tuesday, Feb. 8
- Thursday, March 19
- Thursday, April 14

- If the scheduled evening exam poses a problem for you, notify me in advance to arrange for an alternate time.
- Due to the three evening exams, three regular class meetings will be omitted on dates to be announced.
- The 3 × 5 index cards used as aids on the quizzes may NOT be used on the exams.

Computation Skills: While a calculator is a valuable tool, it is important that elementary teachers be able to perform simple calculations without a calculator and model for their students appropriate calculator use. Therefore there will be three “no calculator” quizzes administered during the semester—one on integers, fractions, and decimals.

- There will be 15 points on the final reserved for computation quizzes: 5 points for Integers, 5 points for fractions, 5 points for decimals. If you earn 90% or above on a quiz, then you earn the full five points allotted for that quiz. If you earn 70-89% on a given quiz, then you earn 2 of the 5 points on that topic. If you earn less than 70%, then you earn 0. You may retake a quiz up to three times. Your highest score counts. All quizzes must be completed by April 22. Assume it will take three days for the grader to grade the computation quizzes. Therefore, do not plan to take a quiz April 21, and if you do not pass, to retake it on April 22. PLAN AHEAD.

Grading:
- 10% Quizzes
- 10% Collected HW
- 10% Activity manuals
- 50% Exams, 100 points each
- 20% Comprehensive Final (150 points: 135 on final, 15 on Computation Quizzes)

Course grading scale:
- A 90.00-100%
- B 80.00-89.99%
- C 70.00-79.99%
- D 60.00-69.99%
- F 0.00-59.99%

Pluses and minuses may be given.

Help: Instructor: Office hours. If you need help, please do not hesitate to ask. I am also available at times other than office hours, and am easy to reach electronically. The help room is a valuable, but underutilized service. It is staffed during certain hours with student experts. These students can help with HW, help answer questions, and review class material.

Academic dishonesty: Anyone cheating on an assignment/exams will get a 0 on that assignment and will be referred to the Dean of Students for academic dishonesty.

Special Needs: Please address any special needs or special accommodations with me at the beginning of the semester or as soon as you become aware of your needs. Those seeking accommodations based on disabilities should obtain a Student Academic Accommodation Request (SAAR) form from the Disability Resources (DR)
A-3. Syllabus: Teaching Children Mathematics, Fall 2011

Course Goals:
The primary goal of this course is for you to develop your ability to design and implement mathematics instruction that is both reflective and mathematically significant. In particular, we will focus on learning to teach mathematics for understanding. This means developing a practice of using curriculum materials to teach through problem-solving building on children’s mathematical thinking through skilled questioning. For each content area we discuss, you should expect to learn:

- How children’s thinking “typically” develops within that content area, including common understandings, misconceptions, strategies, and errors
- How to access and assess children’s thinking within that content area
- How to use curriculum materials, family and community resources, and other supports to help you facilitate the development of children’s thinking

Please Note: You will NOT leave this course feeling completely prepared to teach mathematics. There is always more to learn, and a large part of that learning will occur in your own classroom. The goal is to feel prepared to begin teaching mathematics by developing:

1) A vision of what it means to teach for understanding through problem-solving; 2) key practices that will allow you to enact this vision in your classroom; and 3) an analytic approach to teaching that will allow you to continue to learn from children and develop your instructional practices over time.

Course Texts: A course packet is available through the course website.

Important Web Sites: · National Council of Teachers of Mathematics (NCTM):
http://www.nctm.org ·


Every Student Counts: http://www.iowa.gov/educate/esc/

Assignments

Math Story: A 2-page version of your mathematics story is due Monday, August 29. We will begin this assignment during the first day of class and then you will have two days to complete your story. There is no way for math stories to be “correct” or “incorrect” in their content as long as they relate to mathematics learning and/or teaching and include the major sections. Instead, math stories will be evaluated based on the level of thoughtfulness and detail evidenced in the writing.

Homework: Four times during the course you will be asked to complete or extend an in-class activity with a homework activity. Letters “HW” on the course outline identifies these activities and, they are required in addition to course readings and other assignments. Opening Number Routine Presentation: Each small group will lead one Opening Number Routine (also called Number of the Day or NOTD) presentation during the course. You will work with your assigned group (2-3 people) to plan and present this activity to the whole class.
Major Assignments:

**Inquiry into Student Thinking:** You will conduct a detailed inquiry into children’s mathematical thinking using a Web-based case study of children’s problem solving. The case study can be found at http://www.edb.utexas.edu/empson. Detailed questions to structure this inquiry will be provided in class.

**Tutoring Assignment:** You will interview two students in your tutoring classroom using CGI problems and other questions developed in class. You will then design and implement two activities – one based on number routines and one focused on problem-solving -for the students based on what you learned during the interview. You will go through 3 cycles of tutoring. Details for writing up this experience will be provided in class. The Tutoring Assignment is the Standard Assessment for this course.

**Practicum Assignments:** During your practicum, you will complete two assignments for this course. The first is a student case study that will involve conducting a CGI and number interview with a student, shadowing that student through an entire school day, and developing an understanding of the student’s family and community mathematical resources. The second assignment involves conducting a whole-class *Number of the Day* activity. This activity will be informed by your observations of the class (including your case study student) and your knowledge of the teacher’s mathematical goal for the day. More information about these activities will be provided in class.

**Final Exam:** The final will be completed in groups of 1 to 4 students. The focus of the exam will be responding to scenarios describing student strategies and student errors on problems involving operations with whole numbers and fractions, as well as the use of curriculum materials and teaching for understanding.

Grading:

**Math Story:** 3 points  
**Inquiry into Student Thinking:** 12 points  
**Tutoring Assignment:** 24 points  
**NOTD Class Presentation:** 3 points  
**Practicum Assignment:** 20 points (10 points each)  
**Homework:** 12 points (3 points each)  
**Final Exam:** 20 points  

**Class Attendance and Participation (including responses to warm-up questions in-class reflections, as well as consistently thoughtful contributions to class discussion):** 6 points (1 point will be automatically be deducted for each occurrence of being engaged in social media, e.g., facebook or texting, during class time.) Smaller assignments and most questions in the larger assignments are worth 3 points. These 3-point items (either whole assignments like the Math Story, or single questions in longer assignments) will be graded according to the following rubric:

- 0-Not turned-in (or turned-in late) 1-Minimal effort 2-Average work (this will be the most common grade) 3-Extraordinary work.

Total Points for all assignments = 100 points

**Minimal percent for each grade**

- A= 92.5 points  
- A-= 89.5 points  
- B+= 86.5 points  
- B= 82.5 points  
- B-= 79.5 points  
- C+= 76.5 points  
- C= 72.5 points  
- C-= 69.5 points  
- D= 62.5 points  
- F= 0 points
Class Policies

**Accommodations:** If you have a documented disability and anticipate needing accommodation in this course, please make arrangements to meet with me soon. Please request that the Disability Resources staff send a SAAR form verifying your disability and specifying the accommodation you will need.

**Attendance:** Regular attendance AND participation are key to your success in this class. We will have daily attendance quizzes that will help set the tone for the day's activities. I will not grade the quizzes (though I will read each one), but I will use them to check for attendance and tardiness. If you will not be able to come to class, it is your responsibility to let me know at least a day in advance. Active and appropriate participation is also required – I may talk with you individually if I feel this is not happening.

**High-Quality Work:** Every assignment you turn in should be “high quality work.” This means that it should be typed and double-spaced. Care should be taken to avoid typographical, spelling, and grammatical errors. Consistent errors of these types may result in loss of points.

**Late Work:** Late assignments will be accepted up to one week after the due date. Five percentage points will be deducted for each day an assignment is late.

**Plagiarism:** Plagiarism is the use of another person’s words or ideas without crediting that person. Plagiarism and cheating will not be tolerated and will lead to failure on any assignment containing plagiarized material.

**Information about Standards Assessment**

The Department of Curriculum and Instruction demonstrates to the Iowa Department of Education that each student recommended for initial teacher licensure has an understanding of the following 12 standards: (1) Content/Subject Matter Specialization; (2) Student Learning; (3) Diverse Learners; (4) Instructional Planning; (5) Instructional Strategies; (6) Learning Environment/Classroom Management; (7) Communication; (8) Assessment; (9) Foundations, Reflective Practice, and Professional Development; (10) Collaboration, Ethics, and Relationships; (11) Technology Related to Instruction; and (12) Methods of Teaching.

Mathematics methods course introduces or reinforces concepts and issues related to Standard Four, which states, “The candidate plans instruction based upon knowledge of subject matter, students, the community, curriculum goals, and state curriculum models.” You will demonstrate your understanding of Standard Four with the Practicum Assignment. Your understanding of Standard Four will be graded by the following scoring criteria: Using the attached rubric, you must score at least 70% (17.5 points out of 25) to pass this assignment and demonstrate understanding of Standard Four.

For initial licensure, students need to demonstrate an understanding of the 12 standards listed above with two artifacts for each standard (for a total of 24 artifacts). From this course, you may choose the Interview-Teach-Interview assignment as an artifact for Standard Four. The artifact must be uploaded into your e-portfolio to meet the licensure requirement.
### APPENDIX B. MATHEMATICS COURSE TOPICS AND ACTIVITIES

#### Fall 2011

<table>
<thead>
<tr>
<th>Date</th>
<th>Topic</th>
<th>Goals</th>
<th>Activity Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>8/22</td>
<td>Introduction to Class – Math Stories, Natalie’s Fraction Video and Video Analysis</td>
<td>Introduction to all practices to be highlighted in the course</td>
<td>Watch a video (Natalie’s fraction video) and analyze the video. The video clip has most of the practices to be discussed in the course.</td>
</tr>
<tr>
<td>8/24</td>
<td>MKT Surveys Common Core Standards Read Introduction (pp. 2-7) and pp.19-22</td>
<td>Familiarize PSTs with mathematics standards so that they can be used as a tool for teaching and a framework for critique.</td>
<td>An exploration and discussion of the common Core State Standards and the Iowa Core Curriculum.</td>
</tr>
<tr>
<td>8/29</td>
<td>Cognitive Demand Framework Task Sort Addition Starter Sentences (Part 1) Math stories due today. Read pp. 24-25</td>
<td>Familiarize PSTs with tasks and cognitive demand levels so which can be used as tools for teaching and a framework for critique.</td>
<td>PSTs sort a group of tasks into categories specified by their level of cognitive demand.</td>
</tr>
<tr>
<td>8/31</td>
<td>CGI Problem Types and Solution Strategies Read pp. 25-37. Make your own chart for the Levels of Cognitive Demand to keep in your notebook. Make sure it is clear to you what each level means. We will complete the chart of CGI problem</td>
<td>PSTs learn the CGI problem types and the strategies that students might use Problem types and # choices Strategies of solving and different levels of problems with increase of complexity levels</td>
<td>Introduction to the Cognitive guided instruction framework (CGI). An exploration of and discussion about the problem type framework.</td>
</tr>
<tr>
<td>9/7</td>
<td>Opening Number Routines: Number of the Day and Counting Sequences Read pp. 44-50 CGI Matching Problem Types to Learning Objectives (HW)</td>
<td>PST learning how to use opening number routines (ONR) flexibly to adopt common core state standards goals</td>
<td>Introduction to opening number routine practices.</td>
</tr>
<tr>
<td>9/12</td>
<td>Opening Number Routines: Open Number Sentences, True/False Number Sentences Number Talks Opening Routines in Curriculum Materials Read Number Talk pdf Learning goals with CGI problems due today. (HW)</td>
<td>PSTs learn practices related to using opening number routines in details</td>
<td>An exploration of and discussion about the different strategies of using opening number routines in the classroom.</td>
</tr>
<tr>
<td>9/19</td>
<td>NOTD Group 2 Natalie’s Tornado Problem Next fishbowl problem due today. (HW)</td>
<td>PSTs learn practices of posing a problem unpacking the CGI problem types with the children so that they can access the mathematical idea</td>
<td>In this activity the PST view a video clip and analyze how the teacher (Natalie) unpacks the problem. Using literature connections.</td>
</tr>
<tr>
<td>9/21</td>
<td>Tutor</td>
<td>PST learn how to access and assess children’s mathematical understanding and misunderstanding</td>
<td>Activity involves interview a group of children in an elementary classroom. Use the problem types developed by the instructor to interview children in number and operations and base 10.</td>
</tr>
<tr>
<td>9/26</td>
<td>NOTD Group 3 Stickers Lesson (Part 1) and Problem Types Across a Unit Inquiry into Student Thinking due today</td>
<td>PSTs learn how to use and examine a curriculum material PSTs answer a series of questions that help summarize the overall goals of the unit</td>
<td>Curriculum use and curriculum analysis Examine a unit that focuses on base -10 in a curriculum material and identify the number choices, problem type(s) across the unit, problem context and the level of cognitive demand.</td>
</tr>
<tr>
<td>Date</td>
<td>Group</td>
<td>Activity</td>
<td>Details</td>
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</tr>
<tr>
<td>9/28</td>
<td>Tutor</td>
<td>PSTs practice how to pose problems, unpack the mathematical details and support and extend children’s mathematical understanding</td>
<td>Activity involves teaching children for 50 minutes with a lesson that the PST have developed as a response to children understanding and/or misunderstanding</td>
</tr>
<tr>
<td>10/3</td>
<td>NOTD Group 4</td>
<td>PST learn how to analyze students work and how to support and extend students understanding using questioning e.g., asking specific high level questions, asking “what if” questions to support and extend children’s thinking, and using questioning to help children make connections</td>
<td>This activity focuses on the choice of problem types specifically in base – 10 number and operations and how the number choices support the students in working towards the learning goal. Also considers how a teacher makes sense and responds to students work. Using given students work, PSTs choose two students to support and extend their thinking. The generate supporting or extending questions and provide a rationale for their choice of question(s).</td>
</tr>
<tr>
<td>10/5</td>
<td>Tutor</td>
<td>PSTs practice how to pose problems, unpack the mathematical details and support and extend children’s mathematical understanding</td>
<td>Activity involves teaching children for 50 minutes with a lesson that the PST have developed as a response to children understanding and/or misunderstanding</td>
</tr>
<tr>
<td>10/10</td>
<td>NOTD Group 5</td>
<td>PSTs learn different models of representing fractions and analyzing students work</td>
<td>Activities focus on models of representing fractions - Area model, length and set model. Activities also focus on how to analyze students work using fraction and making decision after the analysis.</td>
</tr>
<tr>
<td>10/12</td>
<td>Tutor</td>
<td>PSTs practice how to pose problems, attend to children’s strategies and support and extend children’s mathematical understanding</td>
<td>Activity involves teaching children for 50 minutes with a lesson that the PST have developed as a response to children understanding and/or misunderstanding</td>
</tr>
<tr>
<td>10/17</td>
<td>NOTD Group 6</td>
<td>PSTs practice how to plan for an instructional session based on children’s understanding or misunderstanding.</td>
<td>Activity involves using the frameworks and assessment discussed in class to put together the next lesson. Activity also focuses on curriculum analysis and fractions. PSTs use a curriculum material to adapt and outline a lesson to teach a group of children who they had analyzed their students work and watched a video of a teacher enacting a lesson.</td>
</tr>
<tr>
<td>10/24</td>
<td>NO CLASS – Practicum</td>
<td>Field experience</td>
<td>Field experience</td>
</tr>
<tr>
<td>11/28</td>
<td>Practicum</td>
<td>PSTs learn how to use the van Hiele levels of geometric thinking in teaching geometry</td>
<td>Problem types, posing problems and analyzing students work using geometry as a context</td>
</tr>
<tr>
<td>12/5</td>
<td>Practicum</td>
<td>Applying the Lesson Structure in Geometry</td>
<td></td>
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APPENDIX C. COURSE ASSIGNMENTS

Inquiry into Student Thinking

For this assignment, your primary source of information will be a case study of four second-graders found at: www.edb.utexas.edu/empson (Click on “Case Study: Four Second Graders under “Categories”).

Read through the entire case study first to get an idea what it is about. Then, answer the following questions in a total of 2-3 double-spaced pages:

1) (3 pts.) Choose one of the four students from the case study and trace their responses and learning through the study.
   a) Summarize what you think they knew or understood at the end of the study that they did not know or understand in the beginning.
   b) Choose 2 examples of this student’s responses as evidence for your claim from part 1.a and explain how they support your claim about this student’s learning. (You will probably want one example toward the end of the study and one example toward the beginning).

2) (3 pts.) Choose 2 tasks or problems that were posed to the students that seemed particularly productive for advancing the thinking of the students as a group. For each problem, provide evidence from at least two of the four students to support your claim that these were productive tasks. Conjecture (i.e., make an educated guess) why these were productive tasks or problems.

3) (3 pts.) Choose one instance of teacher decision-making or reflection that was particularly interesting or surprising to you. Summarize in a paragraph what made that instance stand out for you and how you might use it to inform your own tutoring experience.

4) (3 pts.) If you were to teach the next lesson to this group of students, write one problem that you could give them and explain why you think this would be a good problem for this group. Use what you have learned in class about problem types, number choices, and students’ solution strategies to support your decision.

Tutoring Assignment
(Standard Assessment for Mathematics Methods Course)

For this assignment you will first interview up to three students about their understanding of numbers and operations. You will be provided with grade-appropriate questionnaires for this interview.

Even though you might work with more than one student, and you will be teaching all of them, you will focus on just one student of your choice for this assignment. However, you will use the other students’ data for comparison purposes, so make sure to take notes on all students you are working with. You might even change the student you are focusing on after the last tutoring session (or the student you chose to work with at first might be absent for some of the later tutoring sessions).

Based (at least in part) on what you learn in the interview, you will plan and implement two tutoring sessions with these students.

You will use Opening Number Routines and CGI story problems during your tutoring sessions. These will be tailored to target the specific goals you devised for the students.

You will then analyze the student work and your own teaching. You must use the frameworks learned in the course as tools in analyzing your work with the students. In particular, you must use Levels of Cognitive Demand, Common Core Standards, CGI problem types and CGI student solutions strategies.

Answer each question below separately. I will need a printed copy of this. Don’t staple it. Use either a paper clip or a folder to return this to me. Include any student work that you collected at the end. (This is so you can scan the document to upload to your e-portfolio)
Finally, let me quote a great teacher: “Please be informed that I draw no distinction between the quality of one’s ideas and the quality of those ideas’ verbal expression, and I will not accept sloppy, rough-‘draftish’, or semiliterate college writing.” So, write nice.

**Part I: First Interview**

This is a unique opportunity. You won’t get many chances to get such an in-depth look at student thinking when you are teaching daily in the future. Take advantage of it and think about how you can use these interviewing practices to help you understand and access children’s thinking when you are working with a whole class.

**Writing a Summary/Analysis of the Initial Interviews**

1. Briefly describe the setting of the interview and the interviewed students. Remember to use PSEUDONYMS instead of students’ actual names!

2. For each of the questions on the provided interview, describe what your chosen student’s responses looked like and sounded like. Identify the CGI strategy he/she used or it seems liked she/he tried to use (modeling, counting, derived facts, invented algorithms). Remember that you might see variations on strategies as well as combinations from other things they have learned.

3. **Use the interview data as evidence** in answering these questions:

   a) What does your chosen student know/think/understand about number, operations, and problem solving? Think about what strategies or problem types they DO know and which they are on the verge of learning but have not yet learned. Be **VERY specific**. You must back up your claims with **SPECIFIC EVIDENCE** of children’s words/actions.

   b) Based on your answers to part a, what specific mathematics will you focus on with these students? How will what you learned in the interviews influence how you work with these children during the next sessions?

**Part II: Tutoring**

**Preparing to Tutor**

You will tutor your student(s) two times after the first interview. The tutoring sessions must include an opening number routine activity and CGI-type problems. **Use the lesson plan format described in our course.** The second plan will probably be a modification (sometimes slight, sometimes not, depending on your assessment of the first tutoring session) of the first lesson plan. Check your plans with your colleagues and your instructor.

**Tutoring**

There is a lot to think about while you are teaching/tutoring. First, you will be thinking about what you are doing and saying – you want to follow your lesson plan, but also be flexible based on your formative assessments of what your students are and are not understanding. During each session, you will also want to pay close attention to your students’ thinking – their strategies, responses (correct and incorrect), questions, etc.

Remember that your students should be doing the bulk of the talking. They are the ones doing the mathematics. You will be unpacking the problem, asking questions, planning the sharing of solutions, and doing some rephrasing. If you find yourself explaining most of the time, you are doing it wrong.
Writing Up Your Tutoring Sessions

After each tutoring session, complete a write-up addressing the following:
1. What was your plan for this week?

Include your Lesson Plan and explain:
a) What makes these tasks high cognitive demand? Use the criteria from the packet to justify your answer.
b) How are these tasks intended to build on what you already know about your student’s understandings and misunderstandings? Here justify your Opening Number Routine, story problems types, and number choices. Why did you choose the numbers/problems/activities you did?

2. What actually happened during your tutoring session this week?

Describe IN DETAIL what happened during your lesson – what you said and did, what your focus student said and did, what CGI strategies you observed students using, what questions students had. Do not just say, “I followed the lesson plan.” Instead, provide SPECIFIC descriptions of what you and the students said or did – especially the strategies your student used and how you responded to those strategies. Include major decisions you made while teaching. What types of mathematical issues arose that were unexpected? What range of strategies did students use? How did you organize and assist their explanations of their strategies?

3. How effective were your plan and your teaching?

What did your assessment plan tell you about your students’ growth, and what didn’t they tell you about your students? Provide specific evidence of what the students said and did to support your answer to this question.

4. What do you still wonder about your students and their mathematical thinking?

5. What will you focus on during the next session, and why?

Remember that growth in understanding is gradual. Don’t try to make dramatic jumps. Build on little successes and take small steps.

Part III: Reflecting On Your Experience

Finally, spend some time reflecting on this experience. Use data from your interview and tutoring session to justify your answers. There should be a lot of sentences that look like “I know this because the student...” Also, focus on the mathematics, rather than behavior or other issues.

This reflecting piece should be used as part of your e-portfolio synthesis of evidence. In particular, address the following questions:

1. What did you learn about learning, teaching, mathematics, and students during this experience? Address here how this assignment helped you with Standard Four, which states, “The candidate plans instruction based upon knowledge of subject matter, students, the community, curriculum goals, and state curriculum models.”

2. How do your original goals for the tutoring sessions compare with what really happened?

3. Overall, how do you feel your interviews and tutoring sessions went?

4. What would you do next with these students?

5. How can you use this process in your own teaching?
### Grading Rubric

<table>
<thead>
<tr>
<th>First Interview (1+1+3+3=8 points)</th>
<th>Description/Analysis of Tutoring Sessions (4x3=12 points)</th>
<th>Reflection (1+3=4 points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. All questions are answered, <strong>pseudonyms are used (and noted)</strong>, and the writing is acceptable.</td>
<td>1. All questions answered. A lesson plan is included and has the correct format. Writing is acceptable.</td>
<td>1. All questions are answered. Writing is acceptable.</td>
</tr>
<tr>
<td>2. Student’s responses are <strong>very detailed</strong>.</td>
<td>2. All choices (ONR, story problems, and number choices and progressions) are solidly based on students’ work and frameworks, and the level of cognitive demand is appropriately justified.</td>
<td>2. Answers are thoughtfully supported by specific examples. evidence from interviews, observations, analyses, feedback, etc. Focus is on Mathematics.</td>
</tr>
<tr>
<td>3. Strategies used are correctly identified. <strong>CGI framework is used</strong>.</td>
<td>3. Multiple strategies that students might have used and actually did use are described in detail (include examples of student work as appropriate).</td>
<td></td>
</tr>
<tr>
<td>4. Conclusions about student thinking are supported by <strong>specific evidence</strong> from interviews, student work, using the frameworks discussed in the course.</td>
<td>4. Any claims made about strategies, students’ thinking, or the effects of your teaching are backed up with <strong>specific evidence</strong> of what people said and did.</td>
<td></td>
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</tbody>
</table>
APPENDIX D. SAMPLE TASKS

Sample Tasks Used by PSTs during the Tutoring Session

Third Grade Math Interview
Name ___________________ Date __________

<table>
<thead>
<tr>
<th>Counting Skills (have base ten cubes including hundreds, tens and ones)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can you show me 134 cubes? (Notice if child uses groups of 100’s, 10’s and/or ones)</td>
</tr>
<tr>
<td>Can you show me 256 cubes? (leave group of 134 out)</td>
</tr>
<tr>
<td>If I add a group of 10 to this pile. How many cubes would I have?</td>
</tr>
<tr>
<td>If I add 2 groups of 100 to this pile how many would I have?</td>
</tr>
<tr>
<td>Can you count by 10’s out loud for me starting at ___ and I’ll tell you when to stop?</td>
</tr>
<tr>
<td>Level 1: 17,87</td>
</tr>
<tr>
<td>Level 2: 96,136</td>
</tr>
<tr>
<td>Level 3: 597,647</td>
</tr>
<tr>
<td>Can you count backward starting at ___ and I’ll tell you when to stop.</td>
</tr>
<tr>
<td>Level 1: 62,49</td>
</tr>
<tr>
<td>Level 2: 108,89</td>
</tr>
<tr>
<td>Level 3: 212,996</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write number sentences one at a time on paper. Ask child if the number sentence is true or false. Ask why they think that. Ask child if they can rewrite the number sentence to make it true.</td>
</tr>
<tr>
<td>28+46= 45+28</td>
</tr>
<tr>
<td>74= 10 +60 +4</td>
</tr>
<tr>
<td>12 + 5 =17 + 2</td>
</tr>
<tr>
<td>Ask child to put a number in the blank to make the number sentence true. Ask how they solved this equation.</td>
</tr>
<tr>
<td>35+12-12 = ___</td>
</tr>
<tr>
<td>44+13 = ___ + 14</td>
</tr>
<tr>
<td>8+4= ___ +5</td>
</tr>
</tbody>
</table>
**Problem Solving**

Read the problem to the child starting with the first number choice. Continue with additional number choice if child has demonstrated proficiency. Have base ten blocks available.

**Join Result Unknown (JRU)**
Annie had ___ apples. She picked ___ more apples. How many apples does Annie have in all?
- 42,36
- 23,48
- 89,62

**Separate Result Unknown (SRU)**
There were ___ leaves on the tree. A wind blew ___ off. How many leaves are left on the tree?
- 87,20
- 92,12
- 140,60

**Multiplication**
Joey has ___ bags of cookies. Each bag has ___ cookies in it. How many cookies does Joey have?
- 8,4
- 18,10
- 12,5

**Join Change Unknown (JCU)**
Jack has ___ blocks. How many more blocks does he need to get to have ___ blocks altogether?
- 38,70
- 65,100
- 8,24

**Measurement Division (MD)**
Alice has ___ candies. ___ candies fit in a bag. How many bags does Alice need?
- 24,4
- 100,20
- 35,5
REFERENCES


Course Instructor. (2011a, Fall). Course syllabus: Teaching Children Mathematics. Department of Curriculum & Instruction, Midwestern University, Ames, IA.

Course Instructor. (2011b, Fall). “Course packet” Teaching Children’s Mathematics. Department of Curriculum & Instruction, Midwestern University, Ames, IA.

Course Instructor. (2011a, Spring). “Course syllabus: Mathematics for elementary education 1.” Department of Mathematics, Midwestern University, Ames, IA.

Course Instructor. (2011b, Spring) “Course syllabus: Mathematics for elementary education 11.” Department of Mathematics, Midwestern University, Ames, IA.


