MEASUREMENT OF PHASE AND GROUP VELOCITIES OF DISPERSIVE WAVES IN SOLIDS

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ABSTRACT

The dispersion relation and the propagational speeds of waves in dispersive solids are determined by a newly developed technique in which the phase function of spectral analyzing broadband pulses is determined. The method is simpler than and in agreement with the continuous wave-resonance technique. Application is made to ultrasonic pulses propagating in fiber-reinforced composite materials and other dispersive wave propagation situations.

Theoretical Bases

A pulse \( u(x,t) \) propagating in a linear medium \( x > 0 \) can be Fourier synthesized with harmonic waves \( \exp(i\omega(t-x/v-F)) \) of all angular frequencies \( \omega = \pm 2\pi f \) and phases \( \phi \). Thus,

\[
u(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} A(\xi)e^{i\omega(t-x/\nu-\xi)}d\xi \tag{1}\]

Here \( \nu = \omega/k \) is the phase velocity which is a function of \( \omega \) in a dispersive medium. For an arbitrary excitation pulse \( u(0,t) = F(t) \) at the end \( x = 0 \), it follows from the Fourier theorem that

\[
A(\xi) = F(\xi) \tag{2}\]

When \( \nu = \nu(\omega) \), the input pulse changes its shape as it propagates through the medium. The signal received at \( x = x \) is

\[
u(t,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{i(\omega t-\omega/k)}d\omega \tag{3}\]

where

\[
F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(t)e^{-i\omega t}dt = |F(\omega)|e^{-i\phi_0} \tag{4}\]

and \( \phi_0 = \omega_0 t_0 \), \( t_0 \) being the time at which the excitation pulse enters the specimen at \( x = 0 \). The Fourier transform of the response \( u(t,t) \) is

\[
\tilde{u}(t,\omega) = |F(\omega)|e^{i(\omega t-\omega/k)} = |F(\omega)|e^{-i\phi(\omega)} \tag{5}\]

It is clear that from the measured phase spectrum, \( \phi(\omega) \), of a dispersive pulse, one is able to determine the dispersion relation,

\[
k(\omega) = \frac{1}{v}(\omega) - \phi_0/\omega \tag{6}\]

From this relation, one can easily calculate the phase velocity, \( v \), and group velocity, \( v_g \), as a function of frequency.

Experiments

Method

Figure 1 shows two broadband ultrasonic transducers which are attached to both sides of a polished specimen. One transducer is excited by a voltage pulse, and the signals received by the second transducer are displayed by a sampling oscilloscope. Through an A/D converter interfaced with a PDP 11/40 digital computer, the signal is digitized and then Fourier transformed to yield both the amplitude (magnitude) spectrum and the phase spectrum. From the phase spectrum of the two transducers directly in contact (no specimen is between), we determine precisely the \( \phi_0 \). The spectral analysis of the signal through the specimen of known length then yields \( \phi(\omega) \).

Example: Undispersed 1 Usec Square Pulse

To check the operation of the signal analysis system, a simulated pulse shown in Fig. 2(a) of 1 volt amplitude and 1 usec duration, centered at 2.0 sec was analyzed. In Fig. 2(b) is shown the magnitude of the Fourier transform and in Fig. 2(c), the phase spectrum, \( \phi(\omega) \). This is a linear function of frequency. The dispersion relation, \( k(\omega) \), was computed assuming that the propagation distance was 1 cm; the results being shown in Fig. 2(d). The computed phase velocity, \( v(\omega) \), and group velocity, \( v_g(\omega) \), are shown respectively, in Figs. 2(e) and 2(f). It is clear that \( v \) and \( v_g \) are independent of frequency and both equal the same value of 0.5 cm/µsec.

Applications

Broadband Pulse in 6061-T6 Aluminum

The results of experiments in a non-dispersive medium are shown in the figures. A broadband ultrasonic pulse, comprised mainly of frequency components from 3 to 12 MHz is propagated through a 1.90 cm thick specimen of 6061-T6 aluminum.

In Fig. 3(a) is shown the excitation pulse \( u(\xi,t) \) and in Fig. 3(b) the first received pulse \( u(t,t) \).
In Fig. 3(c) is shown the Fourier amplitude spectrum of $u(t, \xi, \eta)$ and in Fig. 3(d) the dispersion relation. The calculated phase velocity and group velocity of this specimen are shown in Figs. 3(e) and 3(f) respectively. It is clear from Fig. 3(d) that there is little dispersion in this material in the frequency range from 0 to 20 MHz.

The additional data points shown in Fig. 3(f) represent the values of group velocity measured with the continuous-wave, $\pi$-phase technique. For this specimen, no measurements were made below 1.5 MHz. At frequencies above 7 MHz data were recorded only at integer frequency points. The average group velocity measured using the continuous-wave technique was $0.618 \pm 0.009$ cm/\mu s.

For the pulse measurement, the average of 160 points which comprised the data in the frequency interval 0 to 20 MHz, the average was $0.609 \pm 0.005$ cm/\mu s.

Composite Materials: Example—96-ply Boron-Epoxy

The results for a dispersive medium such as composite materials are shown in the figures. The pulse is propagated through a 0.546 cm thick specimen of 96-ply Boron-Epoxy. In this case, the wave propagation direction coincides with the fiber direction and the particle displacement direction is perpendicular to the fibers and tangent to the ply layers as shown in Fig. 4. The broadband shear excitation pulse is shown in Fig. 5(a). In Fig. 5(b) is shown the dispersed pulse detected at the receiving transducer. It is apparent that the high frequency components of the pulse propagate faster than the low frequency components. The relative amplitudes of the various frequency components in the pulse are shown in Fig. 5(c). As expected, the high-frequency components are markedly lower in amplitude than the low-frequency components. The dispersion relation is shown in Fig. 5(d), which is not a linear function of $\omega$ (or $f$). In Figs. 5(e) and 5(f) are shown the results of the phase and group velocity computation for the frequency range 0-10 MHz. Results for the spectral amplitudes beyond 10 MHz are not dependable. The data points on the group velocity curve are those obtained from continuous wave measurements.

Other Materials, Specimen Geometries, Etc.

Foamglas - Figure 6(b) shows the result of phase velocity measurements up to 1 MHz in Foamglas, an insulating material, shown in cross section in Fig. 6(a). Shown is a dependence on pore diameter—the larger the pore size, the lower the wave speed. Similarly, but not as clear is the porosity dependence. The curves are quite distinct from that found for bubble-free glass.

Thin Wires, Tubing - Figure 7(a) shows a longitudinal pulse propagating a 76.4 cm, 1.61 mm (OD) section of Remendur tubing. The received pulse has been delayed 135.7 \mu s. Figure 7(b) is the amplitude spectrum of the pulse in Fig. 7(a), while Fig. 7(c) shows the determined dispersion relation, phase and group velocities.

Conclusions

Ultrasonic Pulse Phase Spectroscopy measurements yield results equivalent to continuous wave measurements of group velocity of elastic wave propagating in dispersive media.

The phase velocity and dispersion relation can be determined unambiguously with this technique.

The measurements are rapid. Only one received signal is required for analysis.

The technique is not restricted to elastic pulses.

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Figure 1. Schematic of experimental setup.

Figure 2. (a) 1-μsec pulse at T(0) = 2.0 μsec; (b) Fourier spectrum of 1-μsec pulse; (c) phase spectrum of 1-μsec pulse; (d) dispersion curve of 1-μsec pulse; (e) phase velocity: 1-μsec pulse (T = 2.0 μsec); (f) group velocity: 1-μsec pulse (T = 2.0 μsec).

Figure 3. (a) Input P-pulse excitation at T = 0 μsec; (b) 6061-T6 aluminum, P-wave T(S) = 2.6 μsec; (c) 6061-T6 aluminum spectral magnitude; (d) 6061-T6 aluminum dispersion relation; (e) 6061-T6 aluminum phase velocity; (f) 6061-T6 aluminum group velocity.
Figure 4. Boron-epoxy specimen.

Figure 5. (a) Input shear pulse at T = 0 usec; (b) 96-ply boron-epoxy <X3/X2> T(S) = 0 usec; (c) 96-ply boron-epoxy <X3/X2> spectrum; (d) 96-ply boron-epoxy <X3/X2> dispersion rel; (e) 96-ply boron-epoxy <X3/X2> phase velocity; (f) 96-ply boron-epoxy <X3/X2> group velocity.
Figure 6. (a) Cross section of foamglas insulating material. (b) Foamglas P-wave echo 1.

Figure 7. (a) 1.61 mm OD remendur tubing (40 dB: 135.7 μs); (b) 1.6 mm OD remendur tubing; (c) 1.61 mm OD remendur tubing.