DETERMINISTIC AND PROBABILISTIC INVERSION AT LONG WAVELENGTHS*

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ABSTRACT

In contrast with the scalar wave case, the scattering of elastic waves in the long wavelength limit yields data containing a surprising amount of information concerning the nature of the scatterer. We will consider both deterministic and probabilistic versions of the inversion problem pertaining to the above scattering problem. The deterministic version provides theoretical insight into the "blindsight" of an optimal inversion procedure in the hypothetical limit of zero measurement error. The probabilistic version is appropriate for the interpretation of real data containing errors and possible inconsistencies. In the former category our discussion will start with a review of earlier results obtained by Kohn and Rice, Gubernatis, and the author. Some new results dealing with ellipsoidal inclusions will be discussed.

INTRODUCTION

Since the low frequency (long wavelength) limit in the scattering of elastic waves represents a situation in which the limit of resolution is many times the size of the scatterer, one expects to obtain very little information about the nature of the scatterer, which is indeed true in the case of scalar wave scattering in quantum mechanics. However, in the case of elastic wave scattering, a surprising amount of information concerning the quasi-stationary elastic behavior of the scatterer can be deduced from scattering data.

Before considering the detailed results, it is important to ask: What advantages would such an approach have relative to other approaches for defect characterization? The following points can be made in its favor:

1) The theory of the scattering of elastic waves at low frequencies is well established for the case of ellipsoidal inclusions and voids. Thus, the inverse scattering problem for this class of scatterers is quite tractable. At higher frequencies, this is not the case.

2) Low frequency measurements are sensitive only to the overall shape and size of the defect and not to small textural details. This is also the information of importance in fracture (at least in metals).

3) Low frequency scattering measurements are particularly sensitive to cracks compared with other scatterers (e.g., inclusions of the same volume or even the same area). In particular, the scattering measurements are significantly more sensitive to a large crack than to a number of small cracks with the same total area.

4) The elastic processes involved in low frequency scattering are intimately related to those involved in the early stages of the fracture process (at least in most metals) as has been pointed out by Budiansky and Rice. A further advantage is that the relevant stress intensity factor is proportional to the 1/6 power of the scattering amplitude, yielding thereby a substantial reduction of variance in the estimation process, a fact emphasized by Kino.

Thus the low frequency scattering region has a number of attractive features, particular in the context of NDE. The disadvantages of this approach are mainly associated with the extraction of the low frequency scattering amplitude from raw scattering data, a problem that R.K. Elsley will discuss in a later talk at this symposium.

In the present paper, we attempt to give a cursory overview of the inversion problem associated with low frequency elastic scattering with emphasis on both deterministic and probabilistic approaches. A purpose of the deterministic approach is to provide insight into the blindspots that limit what properties can in principle be yielded by an inversion procedure using certain categories of input data, even when these data are assumed to be perfectly accurate and available in any quantity (of course, within the restrictions implied by the definition of each category). In the real world we must deal with noisy data involving incompleteness and near-inconsistencies and here we must use a probabilistic approach. However, in the latter context, the results of the deterministic approach can have substantial value in providing guidance about what kinds of inferences are possible from a given category of data. These considerations provide additional motivation for the talk to be given by Fertig at the end of the present session.

THE DIRECT PROBLEM

We consider a linear, nondissipative elastic medium characterized at each point \( \mathbf{r} = e_{1}x_{1} + e_{2}x_{2} + e_{3}x_{3} \) by a mass density \( \rho + \delta(\mathbf{r}) \) and an elastic constant tensor \( C_{\alpha\beta\gamma\delta}(\mathbf{r}) \) (we use

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Greek subscripts to denote cartesian coordinate directions along with the usual summation convention. We assume that everywhere outside of the scatterer domain \( D_s \) (see Fig. 1) the perturbations \( \delta \rho(\mathbf{r}) \) and \( C_{\alpha\beta\gamma\delta}(\mathbf{r}) \) vanish. Thus, the host material is characterized by the constant density \( \rho \) and elastic constant tensor \( C_{\alpha\beta\gamma\delta} \) where, in accordance with the assumption of isotropy,

\[
C_{\alpha\beta\gamma\delta} = \lambda \delta_{\alpha\beta} \delta_{\gamma\delta} + 2 \mu \delta_{\alpha\gamma} \delta_{\beta\delta}
\]  

(2.1)

in which \( \lambda \) and \( \mu \) are the Lamé constants and

\[
I_{\alpha\beta\gamma\delta} = \frac{1}{2} \left( \delta_{\alpha\gamma} \delta_{\beta\delta} + \delta_{\alpha\delta} \delta_{\beta\gamma} \right).
\]  

(2.2)

the 4th order unit tensor appropriate for elastic processes.

![Fig. 1 Scattering geometry.](image)

At a position \( \mathbf{r} \) and frequency \( \omega \), the displacement field \( \mathbf{u} = \mathbf{u}(\mathbf{r}, \omega) \) can be decomposed into incident and scattered parts in accordance with the relation

\[
\mathbf{u} = \mathbf{u}^i + \mathbf{u}^s.
\]  

(2.3)

The incident part can be written in the form

\[
\mathbf{u}^i = [\mathbf{e}^i \mathbf{e}^i \exp(ik^i \mathbf{r}) + (I - \mathbf{e}^i \mathbf{e}^i) \exp(ik^s \mathbf{r})] \cdot \mathbf{a}
\]  

(2.4)

where \( \mathbf{e}^i \) and \( \mathbf{e}^s \) are the incident and scattered directions, respectively, \( I \) is the second order unit tensor, \( \mathbf{a} \) is the incident polarization and where, finally, \( k^i \) and \( k^s \) are the wave numbers for longitudinal and transverse elastic waves. In the far-field regime, the scattered wave can be written in the form

\[
\mathbf{u}^s_{\text{large}} = \left[ \mathbf{e}^s \mathbf{e}^s \exp(ik^s \mathbf{r}) + (I - \mathbf{e}^s \mathbf{e}^s) \exp(ik^s \mathbf{r}) \right] \cdot \mathbf{a}
\]  

(2.5)

where \( k^s \) is the radial distance from an origin assumed to be placed at a point inside the scatterer domain \( D_s \). As usual, the longitudinal and transverse wave numbers are given by

\[
k^i = \frac{\omega}{c^i}
\]  

\[
k^s = \frac{\omega}{c^s}
\]  

(2.6a)

(2.6b)

where the longitudinal and transverse propagation velocities are given by

\[
c^i = \left[ (\lambda + 2 \mu) / \rho \right]^{1/2}
\]  

(2.7a)

\[
c^s = \left[ \rho / (\lambda + 2 \mu) \right]^{1/2}
\]  

(2.7b)

Since the scatterer is localized, we can expand the scattering amplitudes in a power series in the frequency, namely

\[
\mathbf{K}(\mathbf{e}^i, \mathbf{e}^i; \omega) = \sum_{n=0}^{\infty} \mathbf{K}_n(\mathbf{e}^i, \mathbf{e}^i) \omega^n
\]  

(2.8)

The spatial localization implies that

\[
\mathbf{K}_0 = \mathbf{K}_1 = 0 \text{ for all } \mathbf{e}^i \text{ and } \mathbf{e}^s
\]  

(2.9)

and thus the leading term is \( \mathbf{K}_2 \). Since the quantity in the time domain corresponding to \( \mathbf{K} \) in the frequency domain (i.e., the impulse response function) must be real, it follows that the reality condition

\[
\mathbf{K}^*(\mathbf{e}^i, \mathbf{e}^i; -\omega) = \mathbf{K}(\mathbf{e}^s, \mathbf{e}^s; \omega)
\]  

(2.10)

must hold and hence \( A_n \) is real if \( n \) is even and imaginary if \( n \) is odd.

The higher order terms beyond \( A_2 \) \( \omega^2 \) are negligible if the frequency \( \omega \) is sufficiently low or equivalently the relevant wavelengths are sufficiently long. This is called the Rayleigh (or low frequency) regime which is the sole concern of the present discussion.

The main feature of the results of Gubernatis, et al., is that the coefficient \( A_2 \) is a linear function of the mass excess \( M \) and the D-tensor \( D_{\alpha\beta\gamma\delta} \), collectively representing all of the properties of the scatterer determining the low frequency scattering behavior. If two different scatterers have the same values of \( M \) and \( D_{\alpha\beta\gamma\delta} \), then the low frequency scattering behavior will be the same. The mass excess is given by

\[
M = \int d^3 x \rho(\mathbf{r})
\]  

(2.11)

and the D-tensor by

\[
D_{\alpha\beta\gamma\delta} = \int d^3 x C_{\alpha\beta\gamma\delta}(\mathbf{r}) \mathbf{e}^i \mathbf{e}^j \mathbf{e}^k \mathbf{e}^l
\]  

(2.12)

where \( C_{\alpha\beta\gamma\delta}(\mathbf{r}) \) is the strain proportionality tensor relating the strain \( \mathbf{e}^i \mathbf{e}^j \mathbf{e}^k \mathbf{e}^l \) due to a uniform applied (or incident) strain \( \mathbf{e}^i \mathbf{e}^j \mathbf{e}^k \mathbf{e}^l \) in accordance with the relation

\[
C_{\alpha\beta\gamma\delta}(\mathbf{r}) = \Gamma_{\alpha\beta\gamma\delta}(\mathbf{r}) \mathbf{e}^i \mathbf{e}^j
\]  

(2.13)

It is understood that the above relation is derived in the quasi-static elastic approximation.

It is of interest to consider the particular forms of \( A_2 \) for the various mode-to-mode scattering situations. However, for the sake of brevity we will restrict our discussion to the case of longitudinal-to-longitudinal (i.e.,) scattering described by the scalar scattering amplitude

\[
A_{2,CL} = e^s \mathbf{K}^* e^i
\]  

(2.14)
In deriving the above result we have assumed that \( \hat{a} = \hat{e} \), i.e., the displacement amplitude of the incident wave is a unit vector pointed in the longitudinal direction.

It is useful to break up the scattering amplitude into parts that are even or odd with respect to the reversal of \( e^t \) or \( e^s \). We accordingly define

\[
A_{2,2}^{(1)}(e^s, e^t) = \frac{1}{2} \, A_{2,2}^{(2)}(e^s, e^t) \pm \frac{1}{2} \, A_{2,2}^{(2)}(-e^s, e^t) = \frac{1}{2} \, A_{2,2}^{(1)}(e^s, e^t) \pm \frac{1}{2} \, A_{2,2}^{(1)}(-e^s, e^t)
\]

(2.15)

It is clear from an inspection of Eq. (2.14) that \( A_{2,2}^{(2)} \) depends only on the D-tensor and thus is called the elastic part. On the other hand \( A_{2,2}^{(1)} \) depends only on M and thus is called the inertial part.

We turn finally to a consideration of the properties of \( A_{abys}^{\alpha\beta} \). It is easily seen from Eq. (2.13) that this tensor must be invariant to the interchange of \( a \) and \( b \). Also, Eq. (2.13) implies that it can be assumed, without loss of generality, to be invariant to the interchange of \( y \) and \( s \). It can be proved with relatively complicated arguments that it is also invariant to the interchange of \( a \) and \( y \). Therefore the D-tensor has the same invariance properties as the elastic constant tensor with respect to the interchange of indices.

Thus, the D-tensor has 21 independent elements (i.e., independent as far as the interchange of indices is concerned). Combined with M, this means that there are \( 21 + 1 = 22 \) properties of the scatterer determining low frequency scattering behavior, a fact that has been independently noted by Kohn and Rice and by the author. 7

**THE DETERMINISTIC INVERSE PROBLEM**

There are two kinds of procedures for dealing with the low frequency inverse scattering problem. As shown in scheme below, one procedure is to follow the scheme below, that is,

Scattering \( \rightarrow M, D \)-tensor \( \rightarrow \) Scattered Data \( \rightarrow \) Parameters

start with scattering data and deduce M and the D-tensor which in turn are used as the basis for deducing whatever scatterer properties (or combinations of properties) are accessible. A second and apparently simpler procedure is to deduce the scatterer parameters (more precisely, the accessible combinations) directly from scattering data. We will use the first procedure in dealing with the deterministic inversion problem in the present section and the second procedure for the probabilistic inversion problem in the next section.

To simplify the treatment of the present section, it is expedient to introduce abbreviated notation. We will let a general 4th order tensor \( B_{abys} \) be represented by the bare symbol \( B \), i.e.,

\[
B_{abys} \leftrightarrow B
\]

(3.1)

and the product of two such tensors by the correspondence

\[
B_{abys}^{(1)} B_{ys'd}^{(2)} \leftrightarrow B^{(1)} B^{(2)}
\]

(3.2)

An essential part of our formalism is the trace operation (denoted by the symbol "Tr") defined by

\[
\text{Tr} B = B_{abab}
\]

(3.3)

The 4th order tensors involved in our treatment are assumed invariant to the interchange of the first pair of indices and the interchange of the last pair, i.e., \( B_{abys} = B_{ysab} \), etc. The inverse \( B^{-1} \) corresponding to \( B_{abys} \) is defined by the relation

\[
B B^{-1} = B^{-1} B = I
\]

(3.4)

where

\[
I_{abys} \leftrightarrow I
\]

(3.5)

where, in turn, \( I_{abys} \) is the 4th order unit tensor defined by Eq. (2.2). In actual computation, special provision must be made to limit the above inverse to the "vector" space of 2nd order symmetric tensors. The strain \( e^{ab} \), a typical operand, will be represented by the bare symbol \( e \).

In terms of the abbreviated notation, Eq. (2.13) can be rewritten in the form

\[
D = \int d^3 \vec{r} P(T)
\]

(3.6)

where the correspondences to the previous indicial notation are obvious.

It will be convenient to introduce the compression projection tensor \( P \) defined by the correspondence

\[
\frac{1}{3} \delta_{ab} \delta_{ys} \leftrightarrow P
\]

(3.7)

which projects a general strain \( \epsilon \) into its isotropic or pure compression part, namely

\[
P \epsilon \leftrightarrow \frac{1}{3} \delta_{ab} \delta_{ys} \epsilon_{ys} = \frac{1}{3} \delta_{ab} \epsilon_{yy}
\]

(3.8)

where the scalar quantity \( \epsilon_{yy} \) is clearly the dilatation:

\[
\epsilon_{yy} = \epsilon \cdot \hat{u}
\]

(3.9)

The complementary projection tensor \( \overline{P} \) is, of course, defined by

\[
\overline{P} = 1 - P
\]

(3.10)

and projects a general strain into its traceless or pure shear part. The elastic constant tensor for an isotropic medium can now be written in a simple form, e.g., in the case of the host medium we have

\[
C = 3 P + 2 \mu I
\]

(3.11)
We turn now to the question of how much information concerning $M$ and $D$ can be deduced from various categories of scattering measurements. The determination of the mass excess $M$ is relatively trivial. For example, from Eq. (2.15a), we get
\[
\frac{\delta f}{\delta \lambda} = \frac{1}{\mu\gamma} \frac{\delta S}{\delta \mu} = \frac{1}{\mu\gamma} \frac{\delta \rho}{\delta \mu} \quad (3.12)
\]
and thus, for example, a single pair of $\lambda\mu$ scattering measurements with a single incident direction $\lambda$ and opposite scattering directions $\lambda^\prime = \mu^\prime (|\mu^\prime| = 1, \mu^\prime \times 0)$ will suffice. It has been demonstrated by Kohn and Rice\textsuperscript{5} that a sufficient number of $\lambda\mu$ scattering measurements provides enough information to determine the full $D$-tensor.

We must consider the problem of deducing the values of the scatterer parameters from a knowledge of $M$ and $D$. We will confine (with exceptions as indicated) this discussion to the case of ellipsoidal inclusions (with the void as a special case) in contrast with the immediately previous discussion which was valid for completely general localized inhomogeneities.

Eshelby\textsuperscript{6} has proved that in the case of an ellipsoidal inclusion a uniform applied strain (i.e., a strain field that would be uniform in the absence of an inhomogeneity) produces a uniform strain in the inclusion. Using this peculiar property of the ellipsoidal geometry, one obtains the simple result
\[
D = V(G + \delta C^{-1})^{-1} \quad (3.13)
\]
where $V$ is the volume of the inclusion and $G$ is a constant Green's tensor given by the correspondence
\[
G = \nabla^{-1} \int d^3 \vec{r} \int d^3 \vec{r} G_{\alpha\beta\gamma\delta}(\vec{r} - \vec{r}') \quad (3.14)
\]
where $G_{\alpha\beta\gamma\delta}(\vec{r} - \vec{r}')$ relates in the host medium the strain at $\vec{r}$ due to a stress applied at $\vec{r}'$. $G$ is dependent only on the shape and orientation of the inclusion and on the elastic properties of the host medium. It is independent of the size and material properties of the inclusion. It also possesses remarkable contraction properties, namely that TrGP and TrGP are dependent only on the elastic properties of the host medium. The quantity $\delta C^{-1}$, the inverse of the elastic constant tensor perturbation $\delta C$, is of course a constant and represents the elastic properties of the inclusion.

In the case of general inclusions (ellipsoidal or otherwise) the mass excess is given by the simple expression
\[
M = V\delta \rho \quad (3.14)
\]
where now $\delta \rho$ is the uniform value of density deviation within the inclusion. Thus, from a knowledge of solely the inertial part $\delta \rho$ we can determine only the product $V\delta \rho$ and nothing about the shape and orientation of the inclusion.

In the case of general inclusions in which the elastic property deviations represented by $\delta C$ are small in some suitable sense the $D$-tensor is given by
\[
D = V\delta \mu \quad (3.15)
\]
Here in this case of small $\delta C$ we encounter a situation that is analogous to the one characterizing the mass excess $M$ regardless of the value of $\delta \rho$. From a knowledge of solely the elastic part $\delta \mu \equiv$ we can determine only the produce $V\delta \rho$ and nothing about the shape and orientation of the inclusion. This is the "blind spot" associated with weak inhomogeneities (at least as far as elastic properties are concerned).

However, if we have prior knowledge that the inclusion is a member of a certain finite set of possible inclusions, we can then attempt to match the ratios
\[
\delta C = D \quad (3.16)
\]
to the corresponding ratios for the members of the above set (with suitable searches over orientations of crystallographic axes if the inclusion is not elastically isotropic).

In the category of strong inhomogeneities, we encounter rather different situations. Here we assume that the elastic properties of the inclusion are not all close to those of the host medium. Here we restrict our attention to inclusions with ellipsoidal boundaries. In the present category, the void is an allowable special case, while in the previous category it was not allowable.

Here, Eq. (3.13) is the fundamental tensor equation, which represents a set of at most 21 independent scalar equations. It is then clear that we cannot determine the scatterer parameters if the inclusion has unrestricted elastic properties, since then $\delta C$ then involves 21 parameters by itself, and when this set is combined with the geometrical parameters there are more unknowns than equations. We are thus led to consider inversion problems involving inclusions with greater elastic symmetry.

In the case of an inclusion with isotropic (locally) material and ellipsoidal geometry, the $D$-tensor is given by
\[
V\delta^{-1} = (G + \delta C^{-1}) \quad (3.17)
\]
where
\[
\delta C^{-1} = (3\delta \lambda + 2\delta \mu)^{-1} + (2\delta \mu)^{-1} \quad (3.18)
\]
where, in turn, $\delta \lambda$ and $\delta \mu$ are the perturbations of the Lame' constants $\lambda$ and $\mu$ are the compression and shear projection tensors defined by Eqs. (3.7) and (3.10). Multiplication of Eq. (3.17) successively by $P$ and $P'$ followed by the trace operation yields the relations
\[
(3\delta \lambda + 2\delta \mu)^{-1} = VTrD^{-1}P - TrGP \quad (3.19)
\]
\[
5(2\delta \mu)^{-1} = VTrD^{-1}P - TrGP' \quad (3.20)
\]
Since $D$ is regarded as given and since $\mathbf{TrGP}$ and $\mathbf{TrGP}$ depend only on the elastic properties of the host medium, the above relations give the isotropic elastic properties of the inclusion as a function of the volume $V$, as yet undetermined.

If the inclusion has a spherical boundary, then the $G$ as well as $\mathbf{GC}$ must be isotropic, i.e., it must equal a linear combination of $P$ and $\mathbf{P}$, and therefore according to Eq. (3.13), $D^{-1}$ must have the same property. It then follows that $\mathbf{Tr}^{-1}P$ and $\mathbf{Tr}^{-1}P$ represent the only information contained in $D$ and hence there is no additional information for determining $V$. This is the so-called spherical "blind spot."

We have succeeded in proving that in the case of a nonspherical ellipsoidal inclusion of isotropic material, the inverse problem can be solved, i.e., from $D$ we can deduce the isotropic elastic properties of the inclusion and the relevant geometrical properties. This statement is valid as long as $\mathbf{GC}$ is not too small in some sense. The density deviation $\delta \rho$ can be determined from $M$ via Eq. (3.14) because $V$ is now known.

### PROBABILISTIC INVERSION

As stated earlier, the probabilistic approach to inversion is the appropriate one for dealing with real experimental because of the several reasons we have already discussed. In the probabilistic version we will limit our attention to the parameteric case, i.e., where each possible defect under consideration is defined by a finite dimensional state vector $z$.

Let us model the possible results of the $n$th scattering measurement (assumed in all cases to be longitudinal-to-longitudinal) by the stochastic expression:

$$y_n = f_n(z) + v_n, \quad n = 1, \ldots, N$$

where $y_n$ is a possible measured value and $v_n$ is a possible measurement error. The function $f_n(z)$ is given by

$$f_n(z) = A_{2 \times 4}(z) \cdot \mathbf{e}_n$$

where $A_{2 \times 4}$ is given, except for the inclusion of the vector $z$, by Eq. (2.14). The subscript $n$ added to $e^2$ and $e^4$ denotes the configuration used in the $n$th measurement. In the case in which the included material is known a priori, the vector $z$ represents the geometrical properties of the void. In the spheroidal case, we assumed as shown in Fig. 2 that the semi-axis lengths are denoted by $a$, $b$ and $c$ and that the axis of symmetry is given by

$$\mathbf{e} = e_1 \gamma_1 + e_2 \gamma_2 + e_3 \gamma_3$$

where $e_1$, $e_2$, and $e_3$ are the unit vectors in the $x_1$, $x_2$, and $x_3$ directions and where $\gamma_1$ and $\gamma_2$ are the direction cosines associated with the $x_1$ and $x_2$ directions.

It is to be stressed that the Cartesian coordinates $(x_1, x_2, x_3)$ are defined in the laboratory frame of reference and have no necessary relation to the axis of symmetry of the spheroid.

The definition of the stochastic model is completed by the specification of the a priori statistical properties of $z$ and $\mathbf{v}$, and $\mathbf{v}$ characterized by the probability density (p.d.) $P(z)$. The measurement errors $\mathbf{v}$ are assumed to be Gaussian random variables with the properties

$$E v_n = 0$$

$$E v_n v_n^T = \delta_{v} \delta_{nn}$$

where $E$ is the averaging (or expectation) operator in the a priori sense. We assume that $z$ and the $\mathbf{v}_n$ are statistically independent.

Whatever is chosen for the criterion of performance of the estimation process, we must calculate the observationally conditioned p.d. of $z$, namely $P(z|y)$ where

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix}$$

A convenient estimate $\hat{z}(y)$ is the value of $z$ that maximizes $P(z|y)$.

At the previous meeting of the present symposium series we reported on the application of this inversion technique to spheroidal voids. Estimates based upon theoretical and experimental test data were in excellent agreement with the known properties of the scatterers.

In the later talk to be presented by Fertig, this methodology will be extended to the case of inclusions (with the void regarded as a special case) in which the inclusion type is not known.
In the case of spheroidal geometry this entails replacing the four-dimensional vector $z$ given by (4.4) by a five-dimensional one in which the fifth component is a discrete-valued variable labelling the inclusion types.

REFERENCES


QUESTIONS AND ANSWERS

Mr. De Facio: We have time for a couple of questions. Be sure to identify yourself and your institution.

James Rice: Yes, Rice, Brown University.

John, you talked about some of the weaknesses, the blind spots, but what specifically?

Mr. Richardson: The blind spots are obviously some of the weaknesses. The additional properties that will be discussed by Fertig will compensate for the blind spots. The other weaknesses have to do with experimental problems, i.e., extracting $A_2$ from the raw data. Elsley will discuss that in some detail tomorrow, but I will mention here that there is a problem of signal-to-noise when you get down to the low frequencies. There is also a problem with spurious propagation effects getting in the way. You need a rather big time window to get enough of your signal in there to get an accurate $A_2$ out. Those are some of the difficulties.

Jack Cohen: You also listed the insensitivity to surface structures as a strength of the method. It's also a weakness, if the surface structure is what you're after.

Mr. Richardson: Well, it turns out that the fracture is also somewhat insensitive to textural defects.

Mr. Cohen: In ceramics, you worry about the surface area.

Mr. Richardson: Your point is a good one. In fact, in my talk tomorrow I will talk about one case where the failure model for ceramics is one in which peripheral surface cracks are the cause of failure and in the low-frequency measurements you cannot resolve any of this information - and of course, it's very valuable information to have. So, your point is very well taken. I was thinking of ordinary fracture in metals; in ceramics, you're quite right.

Mr. De Facio: Thank you.
SUMMARY DISCUSSION
(J. Richardson)

James Rice (Brown University): John, you talked about some of the weaknesses, the blind spots, but what specifically --

John Richardson: The weaknesses had not -- the blind spots are obviously some of the weaknesses, and the additional properties that will be discussed by Fertig to compensate for the blind spots, but the other weaknesses have to do with experimental problems, extracting A-2 from the data. Of course Elsley will discuss that in some detail tomorrow, but I will mention there is a problem of signal-to-noise when you get down to the low frequencies. There is also a problem with spurious propagation effects getting in the way. You need a rather big time window to get enough of your signal in there to get a reliable A-2 out. Those are some of the difficulties.

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