Analysis of coupled core reactors using the natural mode approximation

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ANALYSIS OF COUPLED CORE REACTORS USING
THE NATURAL MODE APPROXIMATION

by

Joaquin Manuel Betancourt

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1. INTRODUCTION

The space independent reactor kinetic equations may be used to describe satisfactorily the dynamic behavior of most chain reacting systems (1). These equations govern the transient response of the system as a whole, assuming that the fundamental mode predominates during the course of the transient.

Several authors (2,3,4,5,6,7) have treated the solution of the space independent kinetic equations by various techniques. Henry (8) has derived the kinetic equations for reactors with stationary fuel without any approximations from the time dependent transport equation. The derivation being exact has yielded precise definitions of reactivity, prompt neutron lifetime, and effective delayed neutron fractions.

In the case of large power reactors one finds that the time dependent flux shape may continue to change significantly throughout the transient when spatially localized disturbances are made (1). This behavior is also true in small thermal reactors when fast local changes are made. It then becomes necessary to consider both flux shape and time dependence of the kinetic equations. The direct numerical integration of the kinetic equations in space and time, though possible, is generally difficult. Thus, a number of approximate methods have been proposed to treat the space-time kinetic behavior of a nuclear reactor.

Three main methods (9) have been used for an approximate solution of the reactor kinetic equations: 1) Adiabatic method, 2) Nodal analysis, and 3) Modal analysis. In these techniques the approach is to try to break the space-time kinetic equations into separate space equations and time
In the adiabatic method the spatial flux distribution is calculated at a particular instant using static criticality calculations based on the properties of the reactor at that instant. The time behavior is obtained by solving the space independent kinetic equations. One assumption of this method is that the flux shape responds instantaneously to changes of the reactor properties. Although this assumption is probably a reasonable one for small reactors, the application of this method to large power reactors is questionable (10,11).

In the nodal analysis the reactor is viewed as divided into regions or nodes. The space part of the problem consists in determining the parameters which govern neutron leakage between nodes, while the time part consists in deriving a set of time dependent equations involving the averages fluxes in each region.

The approach of finite difference techniques (12) has been used as a limiting form of nodal analysis in which the nodes are small and closely spaced. In the past one drawback in this approach was that the computer codes had long running times, thus limiting its use to primarily one dimensional slab geometry and often to only one energy group. The digital computer code WIGLE (13) uses a finite difference approach.

In the modal analysis approximation the unknown space and time dependent neutron fluxes and other dependent variables of the system are approximated by a finite series of products of the unknown time dependent coefficients and the known spatially dependent expansion vectors. The space part of the problem consists in the selection of the modes while the time part involves the determination of the unknown coefficients.
There are several versions (14,15,16,17,18,19) of the modal analysis approximation. The difference in the versions is due to the method of choosing the spatially dependent expansion vectors. Once the expansion vectors are chosen, then the time dependent coefficients are found in a similar manner.

The simplest set of expansion vectors are those which are solutions of the Helmholtz equation (14,15). These modes are orthogonal functions, and are primarily suitable for one dimensional uniform reactor models. For complex geometries many Helmholtz modes are needed, thus it is preferable to use such modes as the lambda and omega modes (16).

The basis functions need not be eigenfunctions of the steady state problem. Dougherty and Shen (17) introduced a set of nonorthogonal modes called Green's function modes. Another set of nonorthogonal modes are the synthesis modes (20,21).

In this investigation a space dependent reactor kinetics approximation, called the Natural Mode Approximation, NMA, (18,19), will be used to study the kinetic behavior of coupled-core nuclear reactors. By a coupled-core reactor is meant that the reactor under consideration will consist of two distinct multiplying regions. These regions are coupled in the sense that neutrons originating in one multiplying region may cause fission in the other multiplying region.

In the natural mode approximation the spatially dependent expansion vectors are the eigenvectors of the linear steady state operator which describes the system at an initial reference condition. These expansion vectors are called the natural modes of the system. Associated with these eigenvectors there is a set of eigenvalues, \( \{ \omega_{mk} \} \), which are called the
steady state parameters of the natural mode approximation.

The perturbation parameters of the natural mode approximation are integrals which involve the natural modes of the system and the changes in the properties of the reactor. Once these parameters are known all the parameters of the NMA will be known and the time dependent coefficients can be found.

Kaplan (18) in using the natural mode approximation introduced the property of "finality" (22). In space-time kinetics the property of "finality" exists whenever the time dependent coefficients may be found independent of each other and of the number of terms retained in the expansion series. If there is no "finality" it is necessary to solve a system of N coupled differential equations with N unknown coefficients.

Foulke (19) has generalized the formulation of the natural mode approximation, and has shown that the NMA may have "finality" in certain special cases.

The present analysis includes time domain and frequency domain investigations. Results of the analysis of space dependent effects due to oscillation tests are compared with those of Carter and Danofsky (23), and Merritt (24). Carter and Danofsky have the only published paper which studies the space dependent effects in the frequency domain for coupled-core nuclear reactors. In their study Carter and Danofsky used an unreflected reactor. Merritt based his work on a one dimensional model of the Iowa State University UTR-10 reactor (25). The basis functions for the investigations by the above authors were the nonorthogonal Green's function modes (17).

In view of the very limited amount of work published and perhaps done
In studying spatially dependent oscillation tests of coupled-core reactors, it is believed that the results of this investigation may be useful to others in future studies.
II. REVIEW OF THE FORMALISM OF THE NMA

In this section a review will be made of the formalism of the Natural Mode Approximation (NMA) as it was suggested by Kaplan (18) and by Foulke (19).

For simplicity it will be assumed that the reference reactor is very low power so that the dependence upon the feedback variables is negligible. Essentially the reactor will be without feedback.

The behavior of a nuclear reactor can always be described by the equation

\[
\frac{\partial \psi}{\partial t} (X, E, \Omega, t) = [H(\psi, X, E, \Omega, t)] \psi(X, E, \Omega, t) + S(X, E, \Omega, t) \tag{1}
\]

where

- \(X, E, \Omega\) represent the independent variables space, energy, and direction;
- \(\psi\) is a K-vector containing the K dependent variables of the system as components. (The K dependent variables are the I delayed neutron precursor densities, \(c^{(i)}(X,t)\), and the density of neutrons, \(N(X,E,\Omega,t)\));
- \([H(\psi, X, E, \Omega, t)]\) is a K x K matrix operator. (The operator is not linear because of its implicit dependence upon \(\psi\), and it consists of all the production and destruction operators);
- \(S(X, E, \Omega, t)\) is a vector containing all external sources.

In order to solve Equation 1 the energy and angular dependence of \(N(X,E,\Omega,t)\) may be eliminated by using multi-group diffusion theory (26,27). In the multigroup approximation the relevant energy range is divided into several discrete intervals. Thus, \(N(X,E,\Omega,t)\) becomes a vector with
components $N^{(1)}(x,\Omega, t), \ldots, N^{(g)}(x,\Omega, t)$.

One of the assumptions in which diffusion theory is based is that the angular distribution of neutron velocity vectors is nearly isotropic. Thus, the total number density is given by

$$N^{(g)}(x, t) = \int_{\Omega} d\Omega N^{(g)}(x,\Omega, t) .$$

The K matrix operator $[H(\psi, x, t)]$ may now be expressed in the following manner

$$[H(\psi, x, t)] = [L_0(\psi)] + [L_1(\psi, x, t)]$$

where

$[L_0]$ = matrix operator at steady state,
$[L_1]$ = perturbation matrix operator.

Equation 1 may be rewritten then as follows

$$\frac{\partial \psi}{\partial t}(x, t) = [L_0]\psi(x, t) + [L_1]\psi(x, t) + \mathcal{S}(x, t) . \quad (2)$$

To solve Equation 2, consider a finite series expansion of the form

$$\psi(x, t) = \sum_{m=1}^{M} \sum_{k=1}^{K} A_{mk}(t) \psi_{mk}(x) . \quad (3)$$

where

$K$ = the number of dependent variables in $\psi$,
$M$ is arbitrary (as $M$ increases it is assumed that the space dependent kinetics approximation improves. In a space independent kinetics approximation $M = 1$).

The $\psi_{mk}$'s are assumed to satisfy the same boundary conditions as $\psi(x, t)$.
The \( \psi_{mk} \)'s are the eigenvectors of the \([L_o] \) operator,

\[
[L_o] \psi_{mk} = \omega_{mk} \psi_{mk}
\]  \hspace{1cm} (4)

The set of eigenvectors defined in Equation 4 are the natural modes of the system.

A convenient set of weighting vectors are the eigenvectors of the adjoint equation

\[
[L_o^*] \psi_{mk}^* = \omega_{mk}^{*} \psi_{mk}^*
\]  \hspace{1cm} (5)

where the linear operator \([L_o^*] \) is the adjoint of the operator \([L_o] \) and is defined by the equation

\[
\langle \psi_{nj}^*, [L_o] \psi_{mk} \rangle = \langle \psi_{mk}^*, [L_o^*] \psi_{nj}^* \rangle \ .
\]  \hspace{1cm} (6)

It is considered that both \( \psi_{mk} \) and \( \psi_{nj}^* \) have the same boundary conditions.

The eigenvectors of the \([L_o] \) operator in combination with the eigenvectors of the adjoint operator have the orthogonality property

\[
\langle \psi_{nj}^*, \psi_{mk} \rangle = 0 \quad \text{for} \quad \omega_{mk} \neq \omega_{nj}^{*} \ .
\]  \hspace{1cm} (7)

This follows from the assumption that the eigenvalues \( \omega_{mk} \) are distinct.

Further it will be assumed that \( \langle \psi_{mk}^*, \psi_{mk} \rangle \neq 0 \), which implies that the set of eigenvalues \( \{\omega_{mk}^*\} \) is the same as the set \( \{\omega_{mk}\} \).

Substitution of Equation 3 in Equation 2, weighting of this equation

\[\text{---}
\]

\[\text{---}
\]

\[\text{---}
\]

\[\text{---}
\]

\[\text{---}
\]

\[\text{---}
\]
by \( \psi^*_n \) and subsequent integration over all space yields the following ordinary differential equation

\[
M \sum_{m=1}^{M} \sum_{k=1}^{K} \frac{dA_{mk}}{dt} \langle \psi^*_n, \psi_{mk} \rangle = 
\]

\[
M \sum_{m=1}^{M} \sum_{k=1}^{K} \left[ A_{mk} \langle \psi^*_n, [L_0] \psi_{mk} \rangle + A_{mk} \left( \psi^*_n, [L_1] \psi_{mk} \right) \right] + \langle \psi^*_n, S \rangle 
\]

(8)

where

\[ n = 1, \ldots, M \]

\[ j = 1, \ldots, K \]

There are \( M \cdot K \) linearly independent weighting vectors, thus, there are \( M \cdot K \) coupled differential equations.

Equation 8 may now be reduced to the following equation

\[
\frac{dA_{mk}}{dt} = \omega_{mk} A_{mk} + \frac{M \sum_{n=1}^{M} \sum_{j=1}^{J} \langle \psi^*_m, [L_1] \psi_{nj} \rangle A_{nj} \langle \psi^*_m, \psi_{mk} \rangle}{\langle \psi^*_m, \psi_{mk} \rangle} + \frac{\langle \psi^*_m, S \rangle}{\langle \psi^*_m, \psi_{mk} \rangle} 
\]

(9)

where there are \( M \cdot K \) expansion coefficients \( A_{mk} \). Letting

\[
S_{mk}(t) = \frac{\langle \psi_{mk}, S(x, t) \rangle}{\langle \psi^*_m, \psi_{mk} \rangle} 
\]

(10)

the following matrix equation is obtained:

\[
\frac{dA}{dt} = \text{diag} \left[ \omega \right] A + [P] A + S 
\]

(11)

where

\[ A = \text{col} \left[ A_{11}(t), \ldots, A_{MK}(t) \right] \]

\[ S = \text{col} \left[ S_{11}(t), \ldots, S_{MK}(t) \right] \]

\[ \text{diag} \left[ \omega \right] \] is an \( M \cdot K \) by \( M \cdot K \) diagonal matrix with elements \( \omega_{mk} \).
\([P]\) = perturbation matrix with elements of the form

\[ p_{\mu\gamma} = \frac{\langle \psi_{mk}^* [L_1] \psi_{n\ell} \rangle}{\langle \psi_{mk}^* \psi_{mk} \rangle} \] (12)

where

\[
\begin{align*}
\mu &= (m-1)K+k \quad \text{for } m = 1, \ldots, M; \quad k = 1, \ldots, K \\
\gamma &= (n-1)K+j \quad \text{for } n = 1, \ldots, M; \quad j = 1, \ldots, K.
\end{align*}
\]

Equation 11 is referred to as the M-th order Natural Mode Approximation. This equation is constructed in terms of variables, \(A_{mk}(t)\), which are the natural mode expansion coefficients and in terms of reference state parameters, \(\omega_{mk}\), which are the natural mode eigenvalues.

It should be pointed out that there is a simpler form of the NMA. Equation 9 can be rewritten in the following form

\[
\frac{dA_{mk}}{dt} = \omega_{mk} A_{mk} + \frac{\langle \psi_{mk}^* [L_1] \psi_o(X,t) \rangle}{\langle \psi_{mk}^* \psi_{mk} \rangle} + S_{mk}(t) \] (13)

If \(\psi(X,t)\) is replaced by \(\psi_o(X) + \delta\psi(X,t)\) and the perturbation theory approximation (product of second order terms small) is made, the following equation is obtained:

\[
\frac{dA_{mk}}{dt} = \omega_{mk} A_{mk} + \frac{\langle \psi_{mk}^* [L_1] \psi_o \rangle}{\langle \psi_{mk}^* \psi_{mk} \rangle} + S_{mk}(t) \] (14)

The set of M*K coupled differential equations has been uncoupled. In this case the NMA has the property of "finality". This means that each \(A_{mk}\) may now be found independently of the other coefficients and of the number, \(M\), of natural modes retained in the expansion. In this special case less effort is required in obtaining the natural mode expansion coefficients.
III. REFERENCE REACTOR FOR ONE-GROUP ANALYSIS

A one-dimensional coupled-core reactor consisting of two semi-infinite multiplying regions joined by a non-multiplying coupling region will serve as the analytical model. The reactor dynamics will be described by one-group diffusion theory, neglecting delayed neutrons. The reactor parameters, given in Table 1, are analogous to those used by Foderaro and Garabedian (15) and Carter (28).

Table 1. Reference reactor critical parameters

<table>
<thead>
<tr>
<th>Region</th>
<th>$\Sigma_a$ (cm$^{-1}$)</th>
<th>$\nu\Sigma_f$ (cm$^{-1}$)</th>
<th>$D$ (cm)</th>
<th>$V_s$ (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq X \leq a$</td>
<td>0.00818</td>
<td>0.0161</td>
<td>0.890</td>
<td>$2.2 \times 10^5$</td>
</tr>
<tr>
<td>$a &lt; X &lt; b$</td>
<td>0.02088</td>
<td>0</td>
<td>0.890</td>
<td></td>
</tr>
<tr>
<td>$b &lt; X \leq c$</td>
<td>0.00818</td>
<td>0.0161</td>
<td>0.890</td>
<td></td>
</tr>
</tbody>
</table>

The kinetics of this reactor is to be described by the one-group diffusion equation

$$D_i \frac{\partial^2 \varphi_i}{\partial x^2} (X,t) - \Sigma_{a_i} \varphi_i (X,t) + \nu \Sigma_{f_i} \varphi_i (X,t) = \frac{1}{V} \frac{\partial \varphi_i}{\partial t} (X,t)$$

where the subscripts denote values for the $i$-th region.

The reactor critical size will now be determined by the technique of solving the criticality determinant. The equations to be solved are

$$D \frac{\partial^2 \varphi_1}{\partial x^2} (X) - \Sigma_{a_1} \varphi_1 (X) + \nu \Sigma_{f_1} \varphi_1 (X) = 0 \quad 0 \leq X \leq a$$

$$D \frac{\partial^2 \varphi_2}{\partial x^2} (X) - \Sigma_{a_2} \varphi_2 (X) = 0 \quad a < X < b$$

$$D \frac{\partial^2 \varphi_3}{\partial x^2} (X) - \Sigma_{a_3} \varphi_3 (X) + \nu \Sigma_{f_3} \varphi_3 (X) = 0 \quad b < X \leq c$$
where
\[ C = \text{reactor critical size (to be determined)}. \]

The boundary conditions are
\[
\begin{align*}
\varphi_1(0) &= 0 ; \quad \varphi_1(a) = \varphi_2(a) ; \quad \nabla \varphi_1 \big|_a = \nabla \varphi_2 \big|_a \\
\varphi_3(c) &= 0 ; \quad \varphi_2(b) = \varphi_3(b) ; \quad \nabla \varphi_2 \big|_b = \nabla \varphi_3 \big|_b 
\end{align*}
\]

The resulting set of solutions are
\[
\begin{align*}
\varphi_1(X) &= T \sin B(1) \cdot X & 0 \leq X \leq a \\
\varphi_2(X) &= A e^{B(2) \cdot X} + F e^{-B(2) \cdot X} & a \leq X \leq b \\
\varphi_3(X) &= G \sin B(3) \cdot X + H \cos B(3) \cdot X & b \leq X \leq c
\end{align*}
\]

where
\[
B^2(1) = \frac{\nu \Sigma f_1 - \Sigma a_1}{D} ; \quad B^2(2) = \frac{\Sigma a_2}{D} ; \quad B^2(3) = \frac{\nu \Sigma f_3 - \Sigma a_3}{D}
\]

From the boundary conditions the following matrix equation is obtained
\[
[B] \gamma = 0 \tag{15}
\]

where
\[
\gamma = \text{col} \begin{bmatrix} T, A, F, G, H \end{bmatrix}
\]

\[
[B] = \begin{bmatrix}
-sinB(1) \cdot a & e^{B(2) \cdot a} & e^{-B(2) \cdot a} & 0 & 0 \\
-B(1) \cos B(1) \cdot a & B(2) e^{B(2) \cdot a} & -B(2) e^{-B(2) \cdot a} & 0 & 0 \\
0 & 0 & 0 & \sin B(3) \cdot c & \cos B(3) \cdot c \\
0 & -e^{B(2) \cdot b} & -e^{-B(2) \cdot b} & \sin B(3) \cdot b & \cos B(3) \cdot b \\
0 & -B(2) e^{B(2) \cdot b} & B(2) e^{-B(2) \cdot b} & B(3) \cos B(3) \cdot b & -B(3) \sin B(3) \cdot b
\end{bmatrix}
\]
Letting
\[ a = 0.5c - 10 \]
\[ b = 0.5c + 10 \]
the value of \( c \) which establishes the determinant of \( B \) as zero is the reactor critical size. This value of \( c \) was found to be 74.0 cm.

The matrix equation \( [B]\gamma = 0 \) may now be solved for the unknowns \( \gamma \). Since the \( |B| = 0 \) the resulting equations are not independent. Thus, one of the unknowns can be chosen to equal unity as it is just a power level factor.

The normalized critical flux distribution is then described by the following equations.

\[ \phi_1(X) = \sin B(1) \cdot X \quad 0 \leq X \leq 27 \]
\[ \phi_2(X) = 0.000400 e^{B(2) \cdot X} + 33.45 e^{-B(2) \cdot X} \quad 27 \leq X \leq 47 \]
\[ \phi_3(X) = -0.766 \sin B(3) \cdot X + 0.642 \cos B(3) \cdot X \quad 47 \leq X \leq 74 \]
Figure 1. Critical flux distribution of reference reactor
IV. STEP RESPONSE OF REFERENCE REACTOR: ONE GROUP ANALYSIS

In this section the response of the system to step reactivity inputs will be analyzed.

The kinetic equations for one-group diffusion theory, neglecting delayed neutrons, can be written in the operator form

\[ L_0 \varphi(X,t) = \varphi(X,t) - L_1 \varphi(X,t) - S(X,t) \quad (16) \]

where

\[ \varphi(X,t) = \sum_{m=1}^{M} a_m(t) \psi_m(X) . \]

It will be assumed that the critical reactor operating at very low power is perturbed by a change in reactor properties through the perturbation operator, \( L_1 \). Thus, \( S(X,t) = 0 \).

The problem now is to determine the space modes that are going to be used in the expansion for the unknown flux. The modes must be continuous functions in the interval \( 0 \leq X \leq L \), satisfying the boundary conditions

\[ \psi_m(0) = 0 \quad m = 1, \ldots, M \]
\[ \psi_m(L) = 0 \quad m = 1, \ldots, M \]

and possessing continuous first derivatives in that interval. If the diffusion coefficient were not uniform throughout the reactor, the condition of continuous first derivatives would be replaced by the conditions

\[ \frac{d\psi_m}{dX} \bigg|_{X=X_j^-} = \frac{d\psi_m}{dX} \bigg|_{X=X_j^+} \quad m = 1, \ldots, M \]
\[ j = 1, \ldots, N-1 \]

where

\( X_j \) denotes region interfaces,
The space modes will be derived as solutions of the following equations

\[ V[D\psi_m(x) - \sum a_i \psi_m(x)] = \omega_m \psi_m(x) \quad 0 \leq x \leq 27 \]
\[ V[D\psi_m(x) - \sum a_2 \psi_m(x)] = \omega_m \psi_m(x) \quad 27 \leq x \leq 47 \]
\[ V[D\psi_m(x) - \sum a_3 \psi_m(x)] = \omega_m \psi_m(x) \quad 47 \leq x \leq 74 \]

In order to find the natural mode eigenvalues, \( \omega_m \), consider the difference approximation (29)

\[ VD\left(\frac{\partial^2}{h^2} + B^2\right) \psi_m(i) = \omega_m \psi_m(i) \quad (17) \]

where \( h \) will be assumed to be unity. The explicit difference equation is

\[ \psi_{m,i+1} = (-B^2 + 2 + \frac{\omega_m}{VD}) \psi_{m,i} - \psi_{m,i-1} \quad (18) \]

where \( i \) ranges from 1 to \( L \).

The value of the function along the boundaries is given by the homogeneous Dirichlet boundary conditions, that is,

\[ \psi_m^0 = \psi_m^{L+1} = 0 \quad . \quad (19) \]

By employing Equation (18) \( \psi_m^{L+1} \) can be determined by a march out process (30,31,32,33) once \( \psi_m^0 \) and \( \psi_m^1 \) are specified. The value of \( \psi_m^{L+1} \) will vanish whenever \( \omega_m \) is an eigenvalue of the equation. The values of \( \omega_m \) found by this technique can then be checked by the determinant technique. The values of \( \omega_m \) which establish the determinant of \( B \) as zero are the natural mode eigenvalues.
It should be pointed out that in using the determinant technique the expressions for $B^2(1)$, $B^2(2)$, and $B^2(3)$ are now given by

$$B^2_m(1) = \frac{\nu \Sigma f_1 - \Sigma a_1}{D} - \frac{w_m}{VD},$$

$$B^2_m(2) = \begin{cases} \frac{\Sigma a_2}{D} + \frac{w_m}{VD} & \text{for } |\omega| < 4593.6 \text{ sec}^{-1} \\ - \frac{\Sigma a_2}{D} - \frac{w_m}{VD} & \text{for } |\omega| > 4593.6 \text{ sec}^{-1} \end{cases},$$

$$B^2_m(3) = \frac{\nu \Sigma f_3 - \Sigma a_3}{D} - \frac{w_m}{VD}.$$

In the non-multiplying coupling region of the reference reactor the natural modes are exponential functions of position for omegas whose absolute values are less than 4593.6 sec$^{-1}$. For omegas whose absolute values are greater than 4593.6 sec$^{-1}$, then the natural modes become sinusoidal functions of position in the coupling region.

The eigenvalue, $\omega_1$, of the first spatial harmonic corresponds to $-\beta/\ell$. In order to find this first eigenvalue the steady state operator, $L_\omega$, in the two multiplying regions is rewritten in the following form

$$L_\omega = V \{ D \nabla^2 - \Sigma_a + \nu \Sigma_f (1-\beta) \}.$$

In finding the natural mode eigenvalues of the other spatial harmonics the factor $(1-\beta)$ is not as significant.

Table 2 lists the natural mode eigenvalues corresponding to the first five spatial harmonics. It should be pointed out that the first eigenvalue was found by using the determinant technique only, as the other technique previously mentioned proved to be unsatisfactory in this region where $|\omega|$
is small.

Table 2. Natural mode eigenvalues of reference reactor

<table>
<thead>
<tr>
<th>Spatial Harmonic Index, m</th>
<th>Eigenvalue (sec(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td>3</td>
<td>4960.5</td>
</tr>
<tr>
<td>4</td>
<td>6758.0</td>
</tr>
<tr>
<td>5</td>
<td>11517.0</td>
</tr>
</tbody>
</table>

The natural modes distribution were then found to be described by the equations:

**Mode 1**

\[
\psi_1(X) = \sin B_1(1) \cdot X
\]

\[
\psi_1(X) = 0.000408e^{33.16e} \quad 0 \leq X \leq 27
\]

\[
\psi_1(X) = -0.000425e^{31.58e} \quad 27 \leq X \leq 47
\]

\[
\psi_1(X) = -0.7687\sin B_1(3) \cdot X + 0.6395\cos B_1(3) \cdot X \quad 47 \leq X \leq 74
\]

**Mode 2**

\[
\psi_2(X) = \sin B_2(1) \cdot X
\]

\[
\psi_2(X) = -0.000425e^{31.58e} \quad 0 \leq X \leq 27
\]

\[
\psi_2(X) = -0.000425e^{31.58e} \quad 27 \leq X \leq 47
\]

\[
\psi_2(X) = 0.6329\sin B_2(3) \cdot X - 0.7748\cos B_2(3) \cdot X \quad 47 \leq X \leq 74
\]

**Mode 3**

\[
\psi_3(X) = \sin B_3(1) \cdot X
\]

\[
\psi_3(X) = -1.093\sin B_3(2) \cdot X + 0.0333\cos B_3(2) \cdot X \quad 0 \leq X \leq 27
\]

\[
\psi_3(X) = -0.9992\sin B_3(3) \cdot X + 0.0280\cos B_3(3) \cdot X \quad 27 \leq X \leq 47
\]

\[
\psi_3(X) = -0.9992\sin B_3(3) \cdot X + 0.0280\cos B_3(3) \cdot X \quad 47 \leq X \leq 74
\]
Mode 4
\[
\psi_4(x) = \sin B_4(x) x \\
\psi_4(x) = -1.044 \sin B_4(x) x - 1.159 \cos B_4(x) x \\
\psi_4(x) = 0.0503 \sin B_4(x) x - 0.9990 \cos B_4(x) x
\]

Mode 5
\[
\psi_5(x) = \sin B_5(x) x \\
\psi_5(x) = -0.9420 \sin B_5(x) x + 1.223 \cos B_5(x) x \\
\psi_5(x) = -0.1521 \sin B_5(x) x - 0.9871 \cos B_5(x) x
\]

It is noted that in the case of one-group diffusion theory \( \psi_m^* \equiv \psi_m \).

Table 3 shows a check on the orthogonality relations for the one-dimensional reactor under consideration.

<table>
<thead>
<tr>
<th>( m )</th>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.670</td>
<td>+</td>
<td>-0.010</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>+</td>
<td>0.639</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>-0.010</td>
<td>+</td>
<td>0.951</td>
<td>+</td>
<td>-0.083</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>0.679</td>
<td>+</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>+</td>
<td>+</td>
<td>-0.083</td>
<td>+</td>
<td>1.0</td>
</tr>
</tbody>
</table>

+ denotes that the magnitude is less than 0.0003

Next it is required to determine the time coefficients for these space modes. At time \( t = 0 \), \( a_1 = 1.0 \) and \( a_m = 0 \) for \( m = 2, 3, 4, 5 \). Thus,
Figure 2. Relative distribution of fundamental (first) natural mode
Figure 3. Relative distribution of second and third natural modes
Figure 4. Relative distribution of fourth and fifth natural modes.
$\varnothing_0(x) \equiv \psi_1(x)$.

As indicated previously the time coefficients are solutions of the equation

$$\dot{a}_k(t) - \omega_k a_k(t) = \frac{\sum_{n=1}^{M} a_n \langle \psi_k^*, L_1 \psi_n \rangle}{\langle \psi_k^*, \psi_k \rangle}.$$ 

This set of $M$ coupled differential equations may now be written as follows

$$\frac{d}{dt} \begin{bmatrix} a_1(t) \\ a_2(t) \\ \vdots \\ a_M(t) \end{bmatrix} = \begin{bmatrix} \omega_1 & 0 & \cdots & 0 \\ 0 & \omega_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \omega_M \end{bmatrix} \begin{bmatrix} a_1(t) \\ a_2(t) \\ \vdots \\ a_M(t) \end{bmatrix} + [P] \begin{bmatrix} a_1(t) \\ a_2(t) \\ \vdots \\ a_M(t) \end{bmatrix}$$

where the elements of the perturbation matrix are of the form

$$p_{\mu\gamma} = \frac{\langle \psi_k^*, L_1 \psi_n \rangle}{\langle \psi_k^*, \psi_k \rangle}$$

where

$$\mu = k \quad \text{for } k = 1, \ldots, M$$

$$\gamma = n \quad \text{for } n = 1, \ldots, M$$

It is noted that for a step response the operator $L_1$ is time independent.

There are several methods for solving such equations as Equation 20. In this investigation the eigenvalue method (34) will be considered. The first step is to assume that solutions of Equation 20 exist in the form

$$a_1(t) = \alpha_1 e^{\lambda t}$$
$$a_2(t) = \alpha_2 e^{\lambda t}$$
$$\vdots$$
$$a_M(t) = \alpha_M e^{\lambda t}.$$
It follows that by substituting the vector $A = \alpha e^{\lambda t}$ into Equation 20 the substitution will result in the eigenvalue problem $\Gamma \alpha = \lambda \alpha$. Since $\Gamma$ is an $M \times M$ matrix, there are $M$ eigenvalues for which nontrivial solutions exist. Corresponding to each eigenvalue there is an eigenvector which represents a solution for the $\alpha$'s. Thus, the solutions to Equation 20 are given by

\[
\begin{align*}
A_1(t) &= b_1 \alpha_{11} e^{\lambda_1 t} + b_2 \alpha_{21} e^{\lambda_2 t} + \ldots + b_M \alpha_{M1} e^{\lambda_M t} \\
A_2(t) &= b_1 \alpha_{12} e^{\lambda_1 t} + b_2 \alpha_{22} e^{\lambda_2 t} + \ldots + b_M \alpha_{M2} e^{\lambda_M t} \\
&\vdots \\
A_M(t) &= b_1 \alpha_{1M} e^{\lambda_1 t} + b_2 \alpha_{2M} e^{\lambda_2 t} + \ldots + b_M \alpha_{MM} e^{\lambda_M t}
\end{align*}
\]

(21)

where

\[
\alpha_j = \begin{bmatrix}
\alpha_{j1} \\
\alpha_{j2} \\
\vdots \\
\alpha_{jM}
\end{bmatrix}
\]

, $j = 1, \ldots, M$ (represent $M$ eigenvectors of $\Gamma$)

The arbitrary constants $b_1$ are then determined by applying the initial conditions.

It is now possible to investigate the dynamic behavior of the reference reactor. Three perturbations are considered, and in each case the perturbation is a step removal of thermal neutron absorber from a localized reactor region.

Perturbation 1 is applied in the coupling region of the reactor. The absorption cross section was reduced in a step wise manner from 0.02088
cm$^{-1}$ to 0.01044 cm$^{-1}$. Perturbation 2 is applied in the region $0 \leq X \leq 27$, and in this case $\delta \Sigma_a = 0.00100$ cm$^{-1}$. The perturbation operator, $L_1$, is equal to $\delta \Sigma_a V$ in the region of perturbation, and zero otherwise.

The dynamic behavior of the reactor is represented in each case by a 5 mode expansion of the flux. The time dependent flux distributions are shown in Figures 5 and 7. Figures 6 and 8 show a graphic comparison of the flux distributions obtained using the 5 mode expansion with WIGLE$^1$. Also plotted in Figure 6 is the flux distribution obtained using 3 spatial harmonics. It should be pointed out that the flux will attain an asymptotic shape after some time $t$. After this time $t$ the flux will rise exponentially, with the asymptotic shape, on a period determined by the positive eigenvalues found in the solutions for $a_k(t)$.

Perturbation 3 is applied in the symmetric region of the reference reactor. The magnitude of the perturbation is 0.00288 cm$^{-1}$. In this case the property of "finality" will also be used in finding the time dependent coefficients.

It should be pointed out that when "finality" is being considered the time dependent coefficients are solutions of the equation

$$\dot{a}_k(t) - w_k a_k(t) = \frac{\langle \psi^\ast_k , L_1 \theta_o \rangle}{\langle \psi^\ast_k , \psi_k \rangle}.$$ 

The solution to this equation for the case $L_1$ is time independent is given by

$$a_k(t) = a_k(0) e^{w_k t} + \frac{P_k}{w_k} (e^{w_k t} - 1) \quad (22)$$

\textsuperscript{1}McFadden, James, Ames, Iowa. Results of computer calculations. Private communication. 1968.
Figure 5. Time dependent flux for $\Sigma_a = 0.01044$ cm$^{-1}$ in region 2
Figure 6. Time dependent flux for $\Sigma_a = 0.01044\ \text{cm}^{-1}$ in region 2.
Figure 7. Time dependent flux for $\Sigma_a = 0.00718$ cm$^{-1}$ in region 1
Figure 8. Time dependent flux for $\Sigma_a = 0.00718 \text{ cm}^{-1}$ in region 1
Where

\[ p_k = \frac{\langle \psi_k^* , L \phi_0 \rangle}{\langle \psi_k^*, \psi_k \rangle} \]

Note that the behavior of the kth natural mode is controlled by its corresponding eigenvalue, \( w_k \).

The dynamic behavior of the reactor is again represented by a 5 mode expansion of the flux. A graphic comparison of the response is then made with WIGLE and with a 5 mode expansion using the property of "finality". This is shown in Figure 9.

From the results obtained in this section one may conclude that the number of natural modes required for a good approximation not only depends on the magnitude of the perturbation, but also on the region of the reactor where the perturbation is applied. This result is in agreement with previous findings (19). Note that the method of solution also depends on the magnitude of the perturbation.

It should be pointed out that for symmetric flux variations no odd harmonics were needed to describe the flux distributions. However, for non-symmetric flux variations all the harmonics contributed in describing the time dependent flux.

At this point it is felt that the agreement of the Natural Mode Approximation with WIGLE serves to verify the validity of the NMA.
Figure 9. Time dependent flux for $\Sigma_a = 0.01800 \, \text{cm}^{-1}$ in region 2
V. STEP RESPONSE OF REFERENCE REACTOR:

ONE GROUP ANALYSIS, ONE GROUP DELAYED NEUTRONS

In a one-group theory model with one group of delayed neutrons, the governing equations are

\[ D_i \frac{\partial^2 \phi_i}{\partial x^2}(x,t) - \sum_{a} \phi_i(x,t) + \nu \Sigma_{\text{f},i}(1-\beta) \phi_i(x,t) + \lambda C_i(x,t) = \frac{1}{\nu} \frac{\partial \phi_i}{\partial t}(x,t) \]

\[ \beta \nu \Sigma_{\text{f},i} \phi_i(x,t) - \lambda C_i(x,t) = \frac{\partial C_i}{\partial t}(x,t) \]

where the subscript \( i = 1, 2, 3 \) denote values for the \( i \)th region.

Observe that at steady state the kinetic equations reduce to the form

\[ D_i \frac{\partial^2 \phi_i}{\partial x^2}(x) - \sum_{a} \phi_i(x) + \nu \Sigma_{\text{f},i}(1-\beta) \phi_i(x) + \lambda C_i(x) = 0 \] (23)

\[ \beta \nu \Sigma_{\text{f},i} \phi_i(x) - \lambda C_i(x) = 0 \] (24)

The solution of the steady state equations is initiated by eliminating \( C \) between Equation 23 and 24 to obtain

\[ D_i \frac{\partial^2 \phi_i}{\partial x^2}(x) - \sum_{a} \phi_i(x) + \nu \Sigma_{\text{f},i} \phi_i(x) = 0 \] .

Thus, the critical flux distribution is described by the equations at the end of Chapter 3. The flux distribution is shown in Figure 1.

Since the precursor concentration in region \( i \) is given by

\[ C_i(x) = \frac{\beta \nu \Sigma_{\text{f},i}}{\lambda} \phi_i(x) \]

then from the equations describing the critical flux distribution one can obtain the precursor concentration distribution in each region of the reference reactor. Note that in region 2 of the reference reactor \( C = 0 \), since region 2 is a non-multiplying medium.

The kinetic equations for one-group diffusion theory can be rewritten
in the form

\[ L_\circ \ddot{\xi}(X,t) = \dot{\xi}(X,t) - L_1 \ddot{\xi}(X,t) \]  \hspace{1cm} (25)

where the operators \( L_\circ \) and \( L_1 \) represent the steady state and perturbation matrix operators respectively. Note that the subscript \( i \) has been omitted for simplicity.

A solution to Equation 25 is obtained by considering the following expansion

\[ \ddot{\xi}(X,t) = \sum_{m=1}^{M} \sum_{k=1}^{2} a_{mk}(t) \psi_{mk}(X) \]

where the vector \( \ddot{\xi}(X,t) \) contains the two dependent variables of the system as components, that is \( \ddot{\xi}(X,t) = \{\xi(X,t), \xi(X,t)\} \).

The \( L_\circ \) operator is a matrix given by

\[
L_\circ = \begin{bmatrix}
V[D^2] - \Sigma_a + \nu \Sigma_f(1-\beta)
& \lambda V \\
\beta \nu \Sigma_f
& -\lambda
\end{bmatrix}
\]

Knowing that the \( \psi_{mk} \)'s are the eigenvectors of the \( L_\circ \) operator, the following equations are obtained

\[ V[D^2] - \Sigma_a + \nu \Sigma_f(1-\beta) \psi_{mk}^{(1)} + \lambda V \psi_{mk}^{(2)} = \omega_{mk} \psi_{mk}^{(1)} \]  \hspace{1cm} (26)

\[ \beta \nu \Sigma_f \psi_{mk}^{(1)} - \lambda \psi_{mk}^{(2)} = \omega_{mk} \psi_{mk}^{(2)} \]  \hspace{1cm} (27)

Eliminating \( \psi_{mk}^{(2)} \) between Equations 26 and 27 yields

\[ \frac{\nabla^2 \psi_{mk}^{(1)}}{V} + \frac{\nu \Sigma_f(1-\beta) - \Sigma_a}{D} \psi_{mk}^{(1)} + \frac{\lambda \beta \nu \Sigma_f}{D(\lambda + \omega_{mk})} \psi_{mk}^{(1)} = \omega_{mk} \psi_{mk}^{(1)} \]  \hspace{1cm} (28)

where
\[ \lambda = \text{decay constant for delayed neutrons} = 0.08 \text{ sec}^{-1} \]
\[ \beta = \text{delayed neutron fraction} = 0.007. \]

The natural modes of a reactor come in clusters of \( K = G + 1 \) where \( G \) represents the neutron groups and \( I \) the precursor groups. The shape of the components of the \( K \) modes of a cluster will be similar, although not necessarily identical. In general, the natural modes become more oscillatory in space as the cluster index \( m \) increases, and it is for this reason that the modes of the \( m \)th cluster are referred to as the modes of the \( m \)th spatial harmonic. The \( I \) eigenvalues associated with the \( m \)th spatial harmonic are called the delayed neutron eigenvalues and the corresponding modes are called the delayed neutron modes. The \( G \) eigenvalues associated with the \( m \)th spatial harmonic have larger magnitudes and are called the prompt neutron eigenvalues. The corresponding modes are called the prompt neutron modes.

As the index \( m \) increases the delayed neutron eigenvalues approach \(-\lambda_i\) for \( i = 1, 2, \ldots, I \), while the prompt neutron eigenvalues approach \(-\infty\) (19).

It then follows that for \(|w_{mk}| \ll |\lambda|\) and for \(|w_{mk}| \gg |\lambda|\) Equation 28 reduces to the following forms respectively

\[ \nabla^2 \psi_{m1}^{(i)} + \frac{\nu \Sigma_f (1-\beta) - \Sigma_{a0}}{D} \psi_{m1}^{(i)} + \frac{\lambda \beta \nu \Sigma_f}{D(\lambda + w_{m1})} \psi_{m1}^{(i)} = 0 \quad (29) \]

\[ \nabla^2 \psi_{m2}^{(i)} + \frac{\nu \Sigma_f (1-\beta) - \Sigma_{a0}}{D} \psi_{m2}^{(i)} = \frac{w_{m2}}{V_D} \psi_{m2}^{(i)}. \quad (30) \]

From Equations 26 and 27 it may be observed that

\[ \psi_{mk}^{(2)} = \frac{\beta \nu \Sigma_f}{\lambda + w_{mk}} \psi_{mk}^{(1)}. \]

The flux component of the delayed neutron modes will then be derived
as solutions of the following equations

\[
D^2 \psi_{m1}^{(1)}(X) - \sum_{a1} \psi_{m1}^{(1)}(X) + \nu \Sigma f_1 (1-\beta) \psi_{m1}^{(1)}(X) + \frac{\lambda \beta \nu \Sigma f_1}{(\lambda + \omega_{m1})} \psi_{m1}^{(1)}(X) = 0 \quad 0 \leq X \leq 27
\]

\[
D^2 \psi_{m1}^{(1)}(X) - \sum_{a2} \psi_{m1}^{(1)}(X) = 0 \quad 27 \leq X \leq 47
\]

\[
D^2 \psi_{m1}^{(1)}(X) - \sum_{a3} \psi_{m1}^{(1)}(X) + \nu \Sigma f_3 (1-\beta) \psi_{m1}^{(1)}(X) + \frac{\lambda \beta \nu \Sigma f_3}{(\lambda + \omega_{m1})} \psi_{m1}^{(1)}(X) = 0 \quad 47 \leq X \leq 74
\]

In order to find the delayed neutron eigenvalues the determinant technique will be used. The values of \( \omega_{m1} \) which establish the determinant of \( B \) as zero are the delayed neutron eigenvalues. The expressions for \( B^2(1), B^2(2), \) and \( B^2(3) \) are now given by

\[
B_{m1}^2(1) = \frac{\nu \Sigma f_1 (1-\beta) - \Sigma_{a1}}{D} + \frac{\lambda \beta \nu \Sigma f_1}{D(\lambda + \omega_{m1})} \quad 0 \leq X \leq 27
\]

\[
B_{m1}^2(2) = \frac{\sum_{a2}}{D} \quad 27 \leq X \leq 47
\]

\[
B_{m1}^2(3) = \frac{\nu \Sigma f_3 (1-\beta) - \Sigma_{a3}}{D} + \frac{\lambda \beta \nu \Sigma f_3}{D(\lambda + \omega_{m1})} \quad 47 \leq X \leq 74
\]

Table 4 lists the delayed neutron eigenvalues corresponding to the first five spatial harmonics.
Table 4. Delayed neutron eigenvalues of reference reactor

<table>
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<th>Spatial Harmonic</th>
<th>Eigenvalue (sec⁻¹)</th>
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<td></td>
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<td>Index, m</td>
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<tr>
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<td>-0.079664</td>
</tr>
<tr>
<td>4</td>
<td>-0.079677</td>
</tr>
<tr>
<td>5</td>
<td>-0.0798812</td>
</tr>
</tbody>
</table>

The flux component of the delayed neutron modes were found to be described by the following equations:

\( m = 1 \)

\[
\psi_{11}^{(1)}(x) = \sin B_{11}(1) \cdot x
\]

\[
\psi_{11}^{(1)}(x) = 0.000400 e^{B_{11}(2) \cdot x} - B_{11}(2) \cdot x
\]

\[
\psi_{11}^{(1)}(x) = -0.7668 \sin B_{11}(3) \cdot x + 0.6426 \cos B_{11}(3) \cdot x
\]

\( 0 \leq x \leq 27 \)

\( 27 \leq x \leq 47 \)

\( 47 \leq x \leq 74 \)

\( m = 2 \)

\[
\psi_{21}^{(1)}(x) = \sin B_{21}(1) \cdot x
\]

\[
\psi_{21}^{(1)}(x) = -0.000392 e^{B_{21}(2) \cdot x} + 32.76 e^{B_{21}(2) \cdot x}
\]

\[
\psi_{21}^{(1)}(x) = 0.6272 \sin B_{21}(3) \cdot x - 0.7798 \cos B_{21}(3) \cdot x
\]

\( 0 \leq x \leq 27 \)

\( 27 \leq x \leq 47 \)

\( 47 \leq x \leq 74 \)

\( m = 3 \)

\[
\psi_{31}^{(1)}(x) = \sin B_{31}(1) \cdot x
\]

\[
\psi_{31}^{(1)}(x) = -0.000571 e^{B_{31}(2) \cdot x} - 48.89 e^{B_{31}(2) \cdot x}
\]

\[
\psi_{31}^{(1)}(x) = 0.4368 \sin B_{31}(3) \cdot x + 0.8769 \cos B_{31}(3) \cdot x
\]

\( 0 \leq x \leq 27 \)

\( 27 \leq x \leq 47 \)

\( 47 \leq x \leq 74 \)
Figure 10. Relative distribution of flux component of first delayed neutron mode
Figure 11. Relative distribution of flux component of second and third delayed neutron modes.
Figure 12. Relative distribution of flux component of fourth and fifth delayed neutron modes
The prompt thermal neutron eigenvalues for the first five spatial harmonics were found in Chapter 4. The flux component of the prompt thermal neutron modes are shown in Figures 2, 3, and 4.

The adjoint of the steady state matrix operator is given by

\[
L_0^* = \begin{bmatrix}
V[Dv^2 - \Sigma_{ao} + (1-\beta) \nu \Sigma_f] & \beta \nu \Sigma_f \\
V\lambda & -\lambda
\end{bmatrix}
\]

Since the \( \psi_{mk}^* \)'s are the eigenvectors of the \( L_0^* \) operator the following equations are obtained

\[
V[Dv^2 - \Sigma_{ao} + \nu \Sigma_f (1-\beta)] \psi_{mk}^{(1)*} + \beta \nu \Sigma_f \psi_{mk}^{(2)*} = w_{mk} \psi_{mk}^{(1)*} \tag{31}
\]

\[
V\lambda \psi_{mk}^{(1)*} - \lambda \psi_{mk}^{(2)*} = w_{mk} \psi_{mk}^{(2)*} \tag{32}
\]

Elimination of \( \psi_{mk}^{(2)*} \) between Equations 31 and 32 yields
Comparing Equations 28 and 33 one finds that \( \psi_{mk}^{(1)} = \psi_{mk}^{(1)\star} \), although
\[ \psi_{mk} \neq \psi_{mk}^{\star} \text{ since } \psi_{mk}^{(2)\star} = \frac{\nu\lambda}{\lambda + \omega_{mk}} \psi_{mk}^{(1)\star}. \]

Next the time coefficients for these space modes may be determined.

From the expansion of the vector \( \tilde{\varphi}(X,t) \) the following equations are yielded

\[ \varphi(X,t) = \sum_{m=1}^{M} \left[ a_{m1}(t) \psi_{m1}^{(1)}(X) + a_{m2}(t) \psi_{m2}^{(1)}(X) \right] \]

At time \( t = 0 \), the initial conditions are

\[ \varphi(X,0) = \varphi_{0}(X) \]

\[ \frac{\partial \varphi(X,t)}{\partial t} \bigg|_{t=0} = 0 \]

Equation 34 at \( t = 0 \) may be rewritten as follows

\[ \varphi_{0}(X) = a_{11}(0) \psi_{11}^{(1)}(X) + a_{12}(0) \psi_{12}^{(1)}(X) + a_{21}(0) \psi_{21}^{(1)}(X) \]

\[ + a_{22}(0) \psi_{22}^{(1)}(X) + \ldots + a_{M1}(0) \psi_{M1}^{(1)}(X) + a_{M2}(0) \psi_{M2}^{(1)}(X) \]

where

\[ \varphi_{0}(X) \equiv \psi_{11}^{(1)}(X) \equiv \psi_{12}^{(1)}(X). \]

Weighting of Equation 36 by \( \psi_{11}^{(1)\star} \) or \( \psi_{12}^{(1)\star} \), subsequent integration over the reactor, and making use of the orthogonality relations yields the following equation
\[ a_{11}(0) + a_{12}(0) = 1 \quad (37) \]

Weighting of Equation 36 by any other \( \psi_{nj}^{(1)} \), subsequent integration over the reactor, and making use of the orthogonality relations yields all other \( a_{mk}(t) \) to be zero at \( t = 0 \).

When the initial conditions are applied to Equation 35 the following equation is obtained

\[ 0 = \sum_{m=1}^{M} \left[ \dot{a}_{m1}(t) \psi_{m1}^{(2)}(X) + \dot{a}_{m2}(t) \psi_{m2}^{(2)}(X) \right] \bigg|_{t=0} \quad (38) \]

where the expression for \( \psi_{mk}^{(2)}(X) \) is given in terms of \( \psi_{mk}^{(1)}(X) \).

Weighting of Equation 38 by \( \psi_{11}^{(1)} \) or \( \psi_{12}^{(1)} \), subsequent integration over the reactor, and making use of the orthogonality relations yields the following equation

\[ 0 = \frac{1}{\lambda + \omega_{11}} \dot{a}_{11}(t) \bigg|_{t=0} + \frac{1}{\lambda + \omega_{12}} \dot{a}_{12}(t) \bigg|_{t=0} \quad (39) \]

If the property of "finality" is assumed the time dependent coefficients are solutions of the equation

\[ \dot{a}_{mk}(t) - \omega_{mk} a_{mk}(t) = P_{mk} \quad (40) \]

where

\[ P_{mk} = \langle \psi_{mk}^{*}, \psi_{mk}^{*} \rangle / \langle \psi_{mk}, \psi_{mk} \rangle \quad . \]

The solutions to the above equations for the case the \( P_{mk} \)'s are time independent, are then given by

\[ a_{11}(t) = a_{11}(0) + P_{11} t \quad (41) \]
Substitution of Equation 40 in Equation 39, and noting that \( w_{11} = 0 \) yields the following equation

\[
a_{12}(0) = -\frac{(\lambda + \omega_{12}) P_{11} + \lambda P_{12}}{\lambda \omega_{12}} \quad (43)
\]

From Equations 37 and 43 one then finds that

\[
a_{11}(0) = \frac{1 + (\lambda + \omega_{12}) P_{11} + \lambda P_{12}}{\lambda \omega_{12}} \quad (44)
\]

If the NMA does not have the property of "finality" this means all time coefficients must be found simultaneously. Since the number of dependent variables of the system, \( K \), is 2, there is a set 2 \( M \) coupled differential equations which may be written as follows

\[
\begin{bmatrix}
an_{11}(t) \\
an_{12}(t) \\
an_{21}(t) \\
an_{22}(t) \\
\vdots \\
an_{M2}(t)
\end{bmatrix}
= \begin{bmatrix}
\omega_{11} & 0 & \cdots & 0 \\
0 & \omega_{12} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & \omega_{M2}
\end{bmatrix}
\begin{bmatrix}
an_{11}(t) \\
an_{12}(t) \\
an_{21}(t) \\
an_{22}(t) \\
\vdots \\
an_{M2}(t)
\end{bmatrix}
+ \begin{bmatrix}
\mu_1 \\
\mu_2 \\
\vdots \\
\mu_M
\end{bmatrix}
\]

\[
\frac{d}{dt}
\]

where the elements of the perturbation matrix are of the form

\[
P_{\mu \gamma} = \frac{\langle \phi_{mk}^* L_i \psi_{n1} \rangle}{\langle \phi_{mk}^* \phi_{mk} \rangle}
\]

where

\[
\mu = (m - 1) \cdot 2 + k \quad \text{for } m = 1, \ldots, M; \quad k = 1, 2
\]
\[ \gamma = (n - 1) \cdot 2 + j \quad \text{for} \ n = 1, \ldots, M; \quad j = 1, 2. \]

For \( t < 0 \) the perturbation matrix, \([P]\), is zero and by restricting \([P]\) to be independent of time for \( t > 0 \), the \( a_{mk}(t) \)'s eventually have a time behavior of \( e^{t/T} \) where \( T \) is called the asymptotic period.

Equation 45 will now be solved using the eigenvalue method introduced in solving Equation 20. Again the arbitrary constants in the solutions will be determined by applying the initial conditions. In this case there will be two more unknowns than equations, since \( a_{11}(0) \) and \( a_{12}(0) \) are unknown. This problem is solved by making use of Equations 37 and 39.

The dynamic behavior of the reactor by a step removal of thermal neutron absorber from a localized reactor region will now be analyzed.

Figure 13 shows the time dependent flux distribution for \( \delta \Sigma_{a} = 0.00148 \) cm\(^{-1} \) in the non-multiplying region of the reference reactor. Note that the perturbation matrix operator, \( L_1 \), is given by

\[
L_1 = \begin{bmatrix}
\delta \Sigma_{a} & 0 \\
0 & 0
\end{bmatrix}
\]

in the region of perturbation, and is equal to the zero matrix otherwise. A 3 mode approximation was used to find the flux distribution.

An analysis of the step response of the reactor was also made assuming the NMA had the property of "finality". Note that in this case 5 spatial harmonics were used to describe the behavior of the reactor. A graphic comparison of this special case is made with the general case of no "finality". This comparison is shown in Figure 14.

For the case the time dependence of \( \Sigma_{a}(X,t) \) was chosen to be a step function in the region \( 0 \leq X \leq 27 \), the property of "finality" failed to
Figure 13. Time dependent flux for $\Sigma_a = 0.01940$ cm$^{-1}$ in region 2
Figure 14. Time dependent flux for $\Sigma_\alpha = 0.01940\ cm^{-1}$ in region 2.
describe the response of the reference reactor with any degree of accuracy. At this time it is not known whether this effect is real or just a numerical difficulty.

It is felt that the following conclusions might be drawn from the results obtained in this section:

1) For small symmetric step removal of thermal neutron absorber the NMA may be formulated to have the property of "finality".

2) If the NMA has the property of "finality" an accurate response of the reactor may only be followed for the first few milliseconds. After these first few milliseconds the flux distribution will lag behind that one obtained when the NMA does not have the property of "finality". This is due to the independence of the time coefficients involved with "finality".

3) The above conclusions help to emphasize how limited the NMA is in analyzing the step response of the reactor when the property of "finality" is used.
VI. FREQUENCY RESPONSE OF REFERENCE REACTOR:
ONE GROUP ANALYSIS, ONE GROUP DELAYED NEUTRONS

In this section the response of the system to an oscillating absorber located in some region of the system will be analyzed.

Assume the reactor to be operating initially at steady state. The reactor is then perturbed by a small thermal neutron absorber which oscillates sinusoidally with a frequency \( \omega \). If sufficient time is allowed for the complete decay of all transient terms, the flux will oscillate with the same frequency but with different phase and magnitude that the absorber.

In order to examine the response of the flux to small oscillations of \( \delta \Sigma_a \) with time, it will be assumed that \( \Sigma_a \) in a region of the reactor can be expressed in the form

\[
\Sigma_a(t) = \Sigma_{a_0} + \delta \Sigma_a e^{j\omega t}
\]

where

\[
j = (1-)^{1/2}
\]

\( \delta \Sigma_a \) = the magnitude of the perturbation

\( \Sigma_{a_0} \) = critical value of \( \Sigma_a \) in the region of oscillation.

The equation which describes the kinetic behavior of the perturbed system is

\[
L_0 \delta(X,t) + L_1 \delta_0(X) e^{j\omega t} = \frac{\delta\delta}{\hbar} (X,t)
\]

where the perturbation matrix operator, \( L_1 \), is given by

\[
L_1 = \begin{bmatrix}
-v\delta \Sigma_a & 0 \\
0 & 0
\end{bmatrix}
\]
Note that the perturbation approximation of replacing \( L_1 \bar{\xi}(x,t) \) by \( L_1 \bar{\xi}_o(x) \) has been made.

The solution vector may be assumed to be expanded in a finite series of the natural modes of \( L_o \) as

\[
\bar{\xi}(x,t) = \bar{\xi}_o(x) + \sum_{m=1}^{M} \sum_{k=1}^{2} \delta a_{mk} e^{j\omega t} \psi_{mk}(x)
\]

where the elements \( \delta a_{mk} \) are the coefficients for the space modes evaluated in the frequency domain. Thus, these elements are complex numbers and may be written in the form

\[
\delta a_{mk} = \alpha_{mk} + j\beta_{mk}
\]

It should be pointed out (19) that after the perturbation is introduced the value of \( \bar{\xi}_o(x) \) will be different from the initial steady state value. Also the operator \( L_o \) will be slightly different from its initial steady state value. The main reason for these differences is that after the perturbation is introduced it must be accompanied by a small change in reactor properties in order to keep the reactor critical.

To solve Equation 46 the usual substitute, adjoint weight, and integrate procedure is followed yielding the following expression for the expansion coefficients, \( \delta a_{mk} \),

\[
\delta a_{mk} = -\frac{p_{mk}}{2w_{mk} + \omega^2} (w_{mk} + j\omega)
\]

where

\[
p_{mk} = \frac{\langle \psi_{mk}^*, L_1 \bar{\xi}_o \rangle}{\langle \psi_{mk}^*, \psi_{mk} \rangle}.
\]

(The value of this constant depends on the size and location of the oscillator).

Note it is assumed that \( L_o \bar{\xi}_o(x) = 0 \).
Since the expansion coefficients are complex numbers, one can separate the \(2M\) uncoupled equations into \(2M\) equations representing the real part and \(2M\) equations representing the imaginary part.

The frequency dependent flux may be expressed then as follows

\[
\Phi(X,\omega) = \sum_{m=1}^{M} \sum_{k=1}^{2} \left[ \alpha_{mk} \psi_{mk}(X) + j\beta_{mk} \psi_{mk}(X) \right].
\]  

(49)

The frequency response of the reference reactor shown in Figure 1 will now be obtained. The magnitude and phase of the response are given by

\[
\text{Magnitude} = \left| \Phi(X,\omega) \right| = \left[ R(X,\omega)^2 + I(X,\omega)^2 \right]^{1/2}
\]

\[
\text{Phase} = \arctan \frac{I(X,\omega)}{R(X,\omega)}
\]

where

\[
R(X,\omega) = \sum_{m=1}^{M} \sum_{k=1}^{2} \alpha_{mk} \psi_{mk}(X)
\]

\[
I(X,\omega) = \sum_{m=1}^{M} \sum_{k=1}^{2} \beta_{mk} \psi_{mk}(X)
\]

Note that the magnitude and phase will be plotted against logarithmic frequency. The magnitude will be expressed in decibels where

\[
\text{db} = 20 \log_{10} \text{magnitude}
\]

Some examples will now be considered. It should be pointed out that the basis for choosing these examples is to be able to compare some of the results with those of Carter (23,28) and Danofsky (23). Carter and Danofsky used calculus of variations and the method of Green's functions in studying the kinetic behavior of coupled-core reactors.

Initially it will be assumed that a plane thermal neutron absorber is oscillated in the center region of the reactor over the frequency range of
0.01 to 10,000 rad/sec. The magnitude of the absorber is assumed to be 0.001 cm$^{-1}$. The magnitude and phase diagrams of the response are shown in Figures 15 a,b, and 16 a,b. Note that the contribution of the delayed group was included in the response, although the delayed neutron region is of no general interest unless feedback is to be studied. The diagrams do indicate spatial dependence of the frequency response, although this effect is primarily a high frequency phenomenon.

As a second example it will be assumed that a plane thermal neutron absorber is oscillated in the 8 cm. center region of the reactor over the frequency range of 10 to 10,000 rad/sec. The magnitude of the absorber will again be assumed to be 0.001 cm$^{-1}$. The magnitude and phase diagrams of the response are shown in Figure 17 and 18. A graphical comparison of the NMA and the method of Green's function modes is shown in this sequence of figures. Note that no graphical comparison was made in the low frequency range because in their example Carter and Danofsky (23) neglected delayed neutrons. It should be pointed out that the X in all the figures refers to the position in the reactor where the magnitude and phase of the response are measured.

The amplitude of oscillation, $\delta \Sigma_a$, used by Carter and Danofsky was 0.0021 cm$^{-1}$. It is interesting to note that having different values of amplitude of oscillation will only shift the magnitude diagram of the frequency response along the ordinate. Thus, the magnitude diagram of the response using the Green's functions modes was shifted by a constant value. The phase diagram of the frequency response will be the same for both $\delta \Sigma_a$'s.

It is also interesting to note that in the magnitude plots the break frequencies occur at approximately $-\lambda$ and $-\frac{\beta}{\epsilon}$ (rad/sec). The slope of the
Figure 15a. Magnitude of frequency response for five mode analysis (oscillator in the center region of the reactor)
Figure 15b. Magnitude of frequency response for five mode analysis (oscillator in the center region of the reactor)
Figure 16a. Phase angle of frequency response for five mode analysis (oscillator in the center region of the reactor)
Figure 16b. Phase angle of frequency response for five mode analysis (oscillator in the center region of the reactor)
Figure 17. Magnitude of frequency response for five mode analysis (oscillator in 8 cm center region of the reactor)
Figure 18. Phase angle of frequency response for five mode analysis (oscillator in 8 cm center region of the reactor)
curve in the vicinity of these break frequencies is about -20 db/dec. This behavior is like that of point kinetics.

From the results as shown in Figures 17 and 18 it seems that both modal techniques used in studying the frequency response of the reference reactor are in good agreement.

It should be pointed out that there is approximately a 5% difference in the value of the absorption cross section in the center region of the reactor as used by Carter and Danofsky (23) and in this investigation, and that different integration schemes were used. In this investigation exact integrals were used.
VII. REFERENCE REACTOR FOR TWO-GROUP ANALYSIS

The reference reactor for the two energy group analysis is the Iowa State University UTR-10 reactor. The University Training Reactor-10 is a thermal, heterogeneous, light water moderated and cooled reactor licensed for operation up to 10-KW. The fuel material is contained in two core tanks separated by a coupling region of approximately 45 cm of nuclear grade graphite. Additional graphite surrounds these regions acting as a reflector.

The UTR-10 reactor is fueled by approximately 3 kg. of highly enriched uranium (greater than 93% U-235) with approximately equal masses of fuel being loaded into each core. Each core tank behaves as a semi-independent, subcritical system and is approximately 15 cm in width as illustrated in Figure 19. Primarily the exchange of thermal neutrons between cores sustains the critical operation of the reactor.

The reactor dynamics will be described by two group diffusion theory, including one group of delayed neutrons. The analysis will be one-dimensional, though the transverse buckling will be used to describe the leakage of the neutrons through the two sides, and top and bottom of the reactor. The transverse buckling was obtained by Merritt (24) by making a horizontal and vertical flux map through a core tank. The fluxes were then extrapolated to zero, and the buckling in each direction was obtained by assuming the flux obeyed an equation of the form \( \phi(z) = C \sin B \cdot z \). The reactor parameters are given in Table 5 (24).
Figure 19. UTR-10 reactor section
Table 5. UTR-10 reactor critical parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>north reflector</th>
<th>north core</th>
<th>coupling region</th>
<th>south core</th>
<th>south reflector</th>
</tr>
</thead>
<tbody>
<tr>
<td>D_p (cm)</td>
<td>1.14</td>
<td>1.30</td>
<td>1.14</td>
<td>1.30</td>
<td>1.14</td>
</tr>
<tr>
<td>D_g (cm)</td>
<td>0.843</td>
<td>0.121</td>
<td>0.843</td>
<td>0.121</td>
<td>0.843</td>
</tr>
<tr>
<td>Σ_R (cm)</td>
<td>0.002961</td>
<td>0.02104</td>
<td>0.002961</td>
<td>0.02104</td>
<td>0.002961</td>
</tr>
<tr>
<td>Σ_a (cm^{-1})</td>
<td>0.000284</td>
<td>0.0684160</td>
<td>0.000284</td>
<td>0.0676418</td>
<td>0.000284</td>
</tr>
<tr>
<td>ψΣ_f (cm^{-1})</td>
<td>0.0</td>
<td>0.110552</td>
<td>0.0</td>
<td>0.110552</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The kinetics of the UTR-10 reactor is to be described by the following equations:

\[
D_{fi} \psi_i f_i - \Sigma_R \psi_i f_i + \nu \Sigma F_i (1-\beta) \psi_s + \lambda c_i = \frac{1}{V_F} \frac{\partial \psi_i F_i}{\partial t}
\]

\[
D_{si} \psi_i s_i - \Sigma_{ei} \psi_i s_i + \Sigma_R \psi_i f_i = \frac{1}{V_S} \frac{\partial \psi_i S_i}{\partial t}
\]

\[
\beta \nu \Sigma F_i \psi_i S_i - \lambda c_i = \frac{\partial c_i}{\partial t}
\]

where

- \psi_f = fast neutron flux
- \psi_s = thermal neutron flux
- \Sigma_R = fast removal cross section
- \Sigma_F = slow fission cross section
- \Sigma_a = slow absorption cross section
- c = precursor concentration
- \beta = delayed neutron fraction = 0.0064
- \lambda = decay constant for delayed neutrons = 0.077 sec^{-1}
- \nu = average neutrons produced for each thermal neutron fission
\[ V_F = 4.36 \times 10^8 \text{ cm/sec} \]
\[ V_S = 2.2 \times 10^5 \text{ cm/sec} \]

*i* denotes values for the *i*th region.

Note it is assumed fast absorption is negligible and that only thermal neutrons cause fissions.

At steady state the kinetic equations reduce to the form

\[
\frac{d^2 \phi_F}{dX^2} - (\Sigma_R + D_F B_T^2) \phi_F + \nu \Sigma_F \phi_S = 0
\]

\[
\frac{d^2 \phi_S}{dX^2} - (\Sigma_a + D_S B_T^2) \phi_S + \Sigma_R \phi_F = 0
\]

(50)

\[
C = \frac{\beta \nu \Sigma_F}{\lambda} \phi_S
\]

where

\[
B_T^2 = B_Y^2 + B_Z^2 = 0.00325 \text{ cm}^{-2}
\]

Note that in the two reflector regions and coupling region of the reactor \( C = 0 \), since these regions are non-multiplying media. Also note that the subscript *i* has been omitted for simplicity.

The two-group critical flux distributions in the reflector regions are found to be

\[
\phi_F = E e^\kappa_1 X - \kappa_1 X + R e^{105}
\]

\[
\phi_S = S_3 \phi_F + P e^{\kappa_2 X} - \kappa_2 X + Y e^{105}
\]

\[
\phi_F = L e^{120} + M e^{165}
\]

\[
\phi_S = S_3 \phi_F + N e^{120} + e^{165}
\]

\[ 0 \leq X \leq 105 \]

\[ 120 \leq X \leq 165 \]
\[ \varphi_F = W \mu X + V e^{-\kappa_1 X} \]

\[ \varphi_S = S_3 \varphi_F + Z e^{-\kappa_2 X} + J e^{-\kappa_2 X} \]

where

\[ \kappa_1 = \frac{\Sigma_R + D_F B_f^2}{D_F} ; \quad \kappa_2 = \frac{\Sigma_a + D_S B_T^2}{D_S} ; \quad S_3 = \frac{\Sigma_R}{D_S} \left( \frac{1}{\kappa_2 - \kappa_1} \right) . \]

In the multiplying regions the steady state equations will be solved by assuming (35) solutions of the form

\[ \frac{d^2 \varphi_F}{dx^2} + B^2 \varphi_F = 0 \]

\[ \frac{d^2 \varphi_S}{dx^2} + B^2 \varphi_S = 0 \]

The two-group critical flux distributions in the multiplying media are then found to be

\[ \varphi_F = A \sin \mu X + B \cos \mu X + C e^{\nu X} + D e^{-\nu X} \]

\[ 105 \leq X \leq 120 \]

\[ \varphi_S = S_1 (A \sin \mu X + B \cos \mu X) + S_2 (C e^{\nu X} + D e^{-\nu X}) \]

\[ \varphi_F = F \sin \mu X + G \cos \mu X + H e^{\nu X} + T e^{-\nu X} \]

\[ 165 \leq X \leq 180 \]

\[ \varphi_S = S_1 (F \sin \mu X + G \cos \mu X) + S_2 (H e^{\nu X} + T e^{-\nu X}) \]

where

\[ S_1 = \frac{\Sigma_R}{D_S \mu^2 + \Sigma_a + D_S B_T^2} = \frac{D_F \mu^2 + \Sigma_R + D_F B_f^2}{\nu \Sigma_f} \]
\[ s_2 = \frac{\Sigma_R}{-D_S \nu^2 + \Sigma_a + D_S B_T^2} = \frac{-D_F \nu^2 + \Sigma_R + D_F B_T^2}{\nu \Sigma_f} \]

\[ \mu^2 = \frac{- (\gamma_1 + \gamma_2) + [(\gamma_1 + \gamma_2)^2 - 4\gamma_3]^{1/2}}{2} \]

\[ \nu^2 = \frac{- (\gamma_1 + \gamma_2) - [(\gamma_1 + \gamma_2)^2 - 4\gamma_3]^{1/2}}{2} \]

\[ \gamma_1 = \frac{D_S B_T^2 + \Sigma_a}{D_S}; \quad \gamma_2 = \frac{D_F B_T^2 + \Sigma_R}{D_F}; \quad \gamma_3 = \gamma_1 \gamma_2 - \frac{\nu \Sigma_f \Sigma_R}{D_F D_S}. \]

The steady state neutron equations, Equation 50, can be written in operator form as \( L_0 \phi = 0 \), where

\[
L_0 = \begin{bmatrix}
\left[ D_F \frac{d^2}{dx^2} - (\Sigma_R + D_F B_T^2) \right] & \nu \Sigma_f (1 - \beta) & \lambda \\
\Sigma_R & \left[ D_S \frac{d^2}{dx^2} - (\Sigma_a + D_S B_T^2) \right] & 0 \\
0 & \beta \nu \Sigma_f & -\lambda
\end{bmatrix}
\]

\( \phi = \text{col} \{ \phi_F, \phi_S, C \} \).

The adjoint steady state operator, \( L_0^* \), is given by

\[
L_0^* = \begin{bmatrix}
\left[ D_F \frac{d^2}{dx^2} - (\Sigma_R + D_F B_T^2) \right] & \Sigma_R & 0 \\
\nu \Sigma_f (1 - \beta) & \left[ D_S \frac{d^2}{dx^2} - (\Sigma_a + D_S B_T^2) \right] & \beta \nu \Sigma_f \\
\lambda & 0 & -\lambda
\end{bmatrix}
\]

Thus, the steady state adjoint equations are
The adjoint steady state equations can now be solved in the same way as the steady state equations. Thus, the adjoint critical flux distributions are

\[ \varphi_F^{\star} = S_3 \varphi_S^{\star} + \kappa_1 X^\prime + \mu_1 X \quad 0 \leq X \leq 105 \]

\[ \varphi_S^{\star} = Ee + Re \]

\[ \varphi_F^{\star} = A \sin \mu X + B \cos \mu X + C e^{\nu X} + D e^{-\nu X} \quad 105 \leq X \leq 120 \]

\[ \varphi_S^{\star} = S_1 (A \sin \mu X + B \cos \mu X) + S_2 (C e^{\nu X} + D e^{-\nu X}) \]

\[ \varphi_F^{\star} = S_3 \varphi_S^{\star} + \kappa_1 X^\prime + \mu_1 X \quad 120 \leq X \leq 165 \]

\[ \varphi_S^{\star} = Ee + Re \]

\[ \varphi_F^{\star} = F \sin \mu X + G \cos \mu X + H e^{\nu X} + T e^{-\nu X} \quad 165 \leq X \leq 180 \]

\[ \varphi_S^{\star} = S_1 (F \sin \mu X + G \cos \mu X) + S_2 (H e^{\nu X} + T e^{-\nu X}) \]

\[ \varphi_F^{\star} = S_3 \varphi_S^{\star} + Ze + Je \quad 180 \leq X \leq 285 \]

\[ \varphi_S^{\star} = W e + Ve \]

where
and \( \chi_1^2, \chi_2^2, \mu^2, \nu^2 \) are as previously defined.

In order to determine the proper values for the solution constants and the adjoint solution constants the following boundary conditions are placed on the solutions:

1) \( \varphi_F(0) = \varphi_F(285) = 0 \); \( \varphi_S(0) = \varphi_S(285) = 0 \)

\( \varphi_F^*(0) = \varphi_F^*(285) = 0 \); \( \varphi_S^*(0) = \varphi_S^*(285) = 0 \)

2) Continuity of neutron fluxes and their currents at each interface.

After applying the boundary conditions to the critical flux distributions a set of 20 coupled homogeneous equations is obtained. This set of equations is reduced to a set of 18 equations which yields the following matrix equation

\[ [B] \chi = 0 \]

where

\([B]\) is an 18 \times 18 matrix

\(\chi\) is a column vector of the unknown solution constants.

The routine GINV2 was used to solve this system of homogeneous equations.

The values for the solution constants are given in Table 6. The critical flux distributions are shown in Figure 20.

The proper values of the adjoint solution constants are determined in the manner previously described. Table 6 lists the values of these constants. The adjoint critical flux distributions are shown in Figures 21 and 22.
<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>R</th>
<th>P</th>
<th>Y</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux</td>
<td>0.00035509</td>
<td>-0.00035509</td>
<td>0.0038832</td>
<td>-0.0038832</td>
<td>0.82902</td>
</tr>
<tr>
<td>Adjoint</td>
<td>0.0022782</td>
<td>-0.0022782</td>
<td>-0.00009467</td>
<td>0.00009467</td>
<td>0.83237</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>L</td>
<td>M</td>
</tr>
<tr>
<td>Flux</td>
<td>1.0994</td>
<td>-0.000000185</td>
<td>-0.0095485</td>
<td>0.047229</td>
<td>1.1122</td>
</tr>
<tr>
<td>Adjoint</td>
<td>1.1039</td>
<td>0.000000135</td>
<td>0.014856</td>
<td>0.11323</td>
<td>1.2624</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>I</td>
<td>F</td>
<td>G</td>
<td>H</td>
</tr>
<tr>
<td>Flux</td>
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<td>2.1001</td>
<td>1.0</td>
<td>1.5303</td>
<td>-0.000000128</td>
</tr>
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<td>-0.014815</td>
<td>-0.36802</td>
<td>1.0</td>
<td>1.5303</td>
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<td>T</td>
<td>W</td>
<td>V</td>
<td>Z</td>
<td>J</td>
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<tr>
<td>Adjoint</td>
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<td>-0.000005689</td>
<td>1.6490</td>
<td>0.000000042</td>
<td>-0.40399</td>
</tr>
</tbody>
</table>
Figure 20. Critical fast flux and thermal flux distribution of the UTR-10 reactor
Figure 21. Adjoint critical fast flux distribution of the UTR-10 reactor
Figure 22. Adjoint critical thermal flux distribution of the UTR-10 reactor
VIII. FREQUENCY RESPONSE OF UTR-10 REACTOR:
TWO GROUP ANALYSIS, ONE GROUP DELAYED NEUTRONS

In this section the response of the Iowa State University UTR-10 reactor to an oscillating absorber located in some region of the reactor will be analyzed.

The UTR-10 reactor is assumed to be operating initially at steady state and low power so that essentially the reactor will be without feedback. The reactor will then be perturbed by a small thermal neutron absorber which oscillates sinusoidally with frequency \( \omega \). The oscillator locations will correspond to possible access points in the UTR-10 reactor. These locations are as follows: 1) in the south reflector 13 cm from the south core, 2) in the center of the south core, and 3) in the center of the coupling region. The oscillator locations are illustrated in Figure 23. Also Figure 23 illustrates the detector locations that are going to be investigated in this study. Detector positions A and E are 20 cm from the nearest fuel region interface and B, C, and D are located in the center of the south core, coupling region, and north core respectively.

In order to investigate the spatially dependent effects of the UTR-10 reactor the natural modes of the system will have to be determined. The multigroup diffusion equations may be written in operator form

\[
L_{D} \tilde{\phi}(X,t) = \tilde{r}(X,t) - L_{1} \tilde{\phi}(X,t)
\]

where

\[ L_{D} = \text{matrix operator at steady state} \]
\[ L_{1} = \text{perturbation matrix operator}. \]

The natural modes of the system are the eigenvectors of the matrix operator.
Figure 23. Schematic diagram of UTR-10 reactor

1, 2, 3 ~ Oscillator positions
A - E ~ Detector positions
at steady state, where $L_0$ is given by

$$L_0 = \begin{bmatrix}
[D_F \frac{d^2}{dx^2} - (D_F \beta_T^2 + \Sigma_R)] V_F & v_0 \Sigma_f(1-\beta) V_F & \lambda V_F \\
\Sigma_R V_S & [D_S \frac{d^2}{dx^2} - (D_S \beta_T^2 + \Sigma_a)] V_S & 0 \\
0 & \beta_0 \Sigma_f & -\lambda
\end{bmatrix}$$

The space modes will thus be derived as solutions to the following equations

$$D_F \frac{d^2 \psi_{mk}^{(1)}}{dx^2} - (D_F \beta_T^2 + \Sigma_R + \frac{\omega_{mk}}{V_F}) \psi_{mk}^{(1)} + v_0 \Sigma_f(1-\beta + \frac{\beta_0}{\lambda + \omega_{mk}}) \psi_{mk}^{(2)} = 0$$

$$D_S \frac{d^2 \psi_{mk}^{(2)}}{dx^2} - (D_S \beta_T^2 + \Sigma_a + \frac{\omega_{mk}}{V_S}) \psi_{mk}^{(2)} + \Sigma_R \psi_{mk}^{(1)} = 0$$

$$\psi_{mk}^{(3)} = \frac{\beta_0 \Sigma_f}{\lambda + \omega_{mk}} \psi_{mk}^{(2)}$$

In the non-multiplying regions of the reactor the set of Equations 52 reduce to the following equations

$$D_F \frac{d^2 \psi_{mk}^{(1)}}{dx^2} - (D_F \beta_T^2 + \Sigma_R + \frac{\omega_{mk}}{V_F}) \psi_{mk}^{(1)} = 0$$

$$D_S \frac{d^2 \psi_{mk}^{(2)}}{dx^2} - (D_S \beta_T^2 + \Sigma_a + \frac{\omega_{mk}}{V_S}) \psi_{mk}^{(2)} + \Sigma_R \psi_{mk}^{(1)} = 0$$

$$\psi_{mk}^{(3)} = 0$$

These equations may now be solved in the manner described in the previous chapter. The distributions of the components of the modes are given by the same type of solutions as previously described, though the expressions for $\kappa_1^2, \kappa_2^2, S_1, S_2, \gamma_1, \gamma_2, \gamma_3$ are now given by
In the non-multiplying regions of the reactor the solutions of the components of the modes are exponential functions of position for omegas whose absolute values are less than 665 sec$^{-1}$. For omegas whose absolute values are greater than 665 sec$^{-1}$, then the general solution of the homogeneous equation of the thermal component of the mode in the reflector regions becomes a sinusoidal function of position. In this case the expressions for $\xi_2$ and $S_3$ are now as follows

$$
\xi_2^2 = \frac{\Sigma_a - D_sB_T^2 - \frac{\omega_{mk}}{V_S}}{D_S}; \quad S_3 = -\frac{\Sigma_R}{D_S} \left( \frac{1}{\xi_1^2 + \xi_2^2} \right).
$$

In order to determine the delayed and prompt neutron eigenvalues the solutions must satisfy the following boundary conditions

1) $\psi_{mk}^{(j)}(0) = \psi_{mk}^{(j)}(285) = 0 \quad j = 1, 2, 3$

2) $m = 1, \ldots, M$

3) $k = 1, 2, 3$
2) Continuity of the component fluxes and their currents at each interface.

After applying the boundary conditions the following matrix equation is yielded

\[ [B] \chi = 0 \]

where

- \([B]\) is an 18 \times 18 matrix
- \(\chi\) is a column vector of the unknown solution constants.

The eigenvalues of the steady state operator are those values of \(\omega_{mk}\) for which the determinant of \(B\) is zero. The matrix equation \([B]\chi = 0\) may then be solved in the usual manner for the unknown solution constants.

Table 7 lists the steady state parameters corresponding to the first five delayed neutron modes and the first nine prompt thermal neutron modes.

<table>
<thead>
<tr>
<th>Spatial Harmonic Index, (m)</th>
<th>Delayed Neutron Eigenvalues, (\omega_{ml}) (sec(^{-1}))</th>
<th>Prompt Thermal Neutron Eigenvalues, (\omega_{m2}) (sec(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>43.7942</td>
</tr>
<tr>
<td>2</td>
<td>-0.066256</td>
<td>343.8615</td>
</tr>
<tr>
<td>3</td>
<td>-0.07675976</td>
<td>814.6350</td>
</tr>
<tr>
<td>4</td>
<td>-0.07677974</td>
<td>881.2175</td>
</tr>
<tr>
<td>5</td>
<td>-0.07694311</td>
<td>1143.015</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>1368.782</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>1652.127</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>1990.517</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>2339.455</td>
</tr>
</tbody>
</table>

It should be pointed out that the first five delayed neutron modes
were considered only for completeness in the study as one is not generally interested in the delayed neutron region unless feedback is to be studied. It is the feeling of the author that a unique answer will be attained for \( m = 5 \) for the delayed neutron modes. By a unique answer is meant that \( m = 5 \) should give an accurate representation of the frequency response of the UTR-10 reactor in the low frequency region.

In this investigation the prompt epithermal neutron modes were not included as it has been shown by Paleocrassas (36) that their contribution is negligible up to oscillating frequencies of approximately \( 10^4 \) rad/sec. Essentially the spatial dependence is carried only by the prompt thermal modes and this phenomenon occurs at about the \( \beta/\lambda \) frequency break.

The fast and the slow components of the first five delayed neutron modes and prompt thermal neutron modes are illustrated in Figures 20 and 24 through 32. Note that the effect of the diffusion coefficient being different in the multiplying and non-multiplying media of the reactor is illustrated very well in this sequence of figures. Even though there is continuity of the slow component of flux and current at the interfaces there is a difference in slope at both sides of the interfaces. This effect is not shown in the natural modes of the reference reactor for the one group analysis as the diffusion coefficient was assumed to be the same in all of the reactor regions. This particular situation illustrates very well the difference between a realistic model of an actual reactor and a non-realistic but simpler model.

Also it should be pointed out that the natural modes of the reference reactor for the one group analysis are very similar to Helmholtz modes. This situation is not the same in the case of the natural modes of the
Figure 24. Relative distribution of the fast and the slow components of the second delayed neutron mode
Figure 25. Relative distribution of the fast and the slow components of the third delayed neutron mode.
Figure 26. Relative distribution of the fast and the slow components of the fourth delayed neutron mode.
Figure 27. Relative distribution of the fast and the slow components of the fifth delayed neutron mode
Figure 28. Relative distribution of the fast and the slow components of the first prompt thermal mode.
Figure 29. Relative distribution of the fast and the slow components of the second prompt thermal mode.
Figure 30. Relative distribution of the fast and the slow components of the third prompt thermal mode.
Figure 31. Relative distribution of the fast and the slow components of the fourth prompt thermal mode.
Figure 32. Relative distribution of the fast and the slow components of the fifth prompt thermal mode
UTR-10 reactor. Looking at the modes in the different sections of the UTR-10 reactor one finds some similarity with the Green's function modes used by Merritt (24). This similarity is expected as both techniques used the same source, that is, multigroup diffusion theory.

Note that in the reference reactor for the one group analysis as well as in the UTR-10 reactor the number of crossings of the components of the modes is equal to the mode number minus one. This is always true except for the fast components of the prompt thermal modes after the second mode. At this time it seems that this behavior is real rather than any numerical difficulty. It seems that the higher harmonics are not being excited and the shape of the fast component beyond the second harmonic depends on whether there are two positive or one positive and one negative source. In the case of two positive sources the shape attained by the higher even fast components is that of the fundamental or first harmonic. For one positive and one negative source the shape attained by the higher odd fast components is that of the second harmonic. The term positive or negative source implies in this case the contribution of the fuel region to that particular mode.

The adjoint steady state operator, $L_{O}^{*}$, is given by

$$L_{O}^{*} = \begin{bmatrix}
[D_F \frac{d^2}{dx^2} - (\Sigma_R + D_F B_{1}^2)] V_F & \Sigma_R V_S & 0 \\
\nu \Sigma_F (1-\beta) V_F & [D_S \frac{d^2}{dx^2} - (\Sigma_a + D_S B_{1}^2)] V_S & \beta \nu \Sigma_f \\
\lambda V_F & 0 & -\lambda
\end{bmatrix}.$$

Thus, the adjoint space modes will be derived in the multiplying and non-multiplying regions of the reactor as solutions of the following equations...
These equations are solved in the manner previously described, and the solutions of the components of the adjoint modes have the same solutions as before. In this case the expressions for $S_1$, $S_2$, and $S_3$ are now given by

$$S_1 = \frac{D_F (\beta_T^2 + \mu^2) + \Sigma_R + \frac{\omega_{mk}}{V_F}}{V_S \Sigma_R}$$

$$S_2 = \frac{D_F (\beta_T^2 - \nu^2) + \Sigma_R + \frac{\omega_{mk}}{V_F}}{V_S \Sigma_R}$$

$$S_3 = \frac{V_S \Sigma_R}{V_F D_F \kappa \left(\frac{1}{\kappa_1^2} - \frac{\kappa_2^2}{\kappa_1^2}\right)}$$

Again in the non-multiplying regions of the reactor the solution for the
thermal component of the adjoint mode becomes a sinusoidal function of position for omegas whose absolute values are greater than 665 sec \(^{-1}\). The expression for \(S_3\) is now given by

\[
S_3 = \frac{V_S}{V_F} \frac{\Sigma R}{D_F} \left( \frac{1}{\kappa_1^2 + \kappa_2^2} \right).
\]

The \(\psi_{mk}\) and the \(\psi_{nj}^*\) have the same boundary conditions. Further since the set of eigenvalues \(\{\psi_{mj}^*\}\) is the same as the set \(\{\psi_{mk}\}\), the unknown solution constants may now be found by solving a matrix equation of the form \([B]\psi = 0\).

The fast and slow components of the adjoint modes are similar in shape to the natural modes of the reactor. In this case the behavior of the fast components of the higher adjoint prompt thermal modes is similar to the slow components, that is, the number of crossings of the components is equal to the mode number minus one.

Table 8 shows a check on the orthogonality relations on some of the natural modes of the UTR-10 reactor. All of the integrals have been normalized to the diagonal elements.

Table 8. Results of a check on some of the natural modes of the UTR-10 reactor

<table>
<thead>
<tr>
<th>(\psi_{m2}(x)), (\psi_{n2}(x))</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>(n)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.0</td>
<td>+</td>
<td>0.0014</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>1.0</td>
<td>- 0.011</td>
<td>- 0.0012</td>
<td>- 0.0015</td>
</tr>
<tr>
<td>3</td>
<td>+</td>
<td>+</td>
<td>1.0</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>- 0.0020</td>
<td>1.0</td>
<td>+</td>
</tr>
<tr>
<td>5</td>
<td>+</td>
<td>+</td>
<td>0.0055</td>
<td>+</td>
<td>1.0</td>
</tr>
</tbody>
</table>

*denotes that the magnitude is less than 0.0009
In order to examine the response of the thermal flux to small oscillations of $\delta \Sigma_a$ with time, it will be assumed that $\Sigma_a$ at a localized point of oscillation, $X_0$, can be expressed in the form

$$\Sigma_a(t) = \Sigma_{ao} + \delta \Sigma_a \delta(X-X_0) e^{jwt}$$

where

$$j = ( -1)^{1/2}$$

$\delta \Sigma_a$ = magnitude of the perturbation

$\Sigma_{ao}$ = critical value of $\Sigma_a$ in the region of oscillation

$\delta(X-X_0)$ = Dirac delta function (37).

Since the absorber in the localized plane oscillator is a thermal one, the perturbation matrix operator, $L_1$, is then given by

$$L_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\nu S \delta \Sigma_a \delta(X-X_0) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The same basic outline used in obtaining the one-group frequency response will be followed in this analysis also.

In order to find the perturbation parameters of the natural mode approximation it is necessary to perform integrations of the type

$$\int_{region} \psi_{nj}^T(X) \psi_{mk}(X) \, dX.$$ 

This integration is performed on the digital computer using a fourth order polynomial in conjunction with the trapezoidal rule for integration (38),

$$\int_a^b Y(X) \, dX = \frac{b-a}{4} \left( 7Y_1 + 32Y_2 + 12Y_3 + 32Y_4 + 7Y_5 \right)$$

$$\int_a^b Y(X) \, dX = \frac{b-a}{2} (Y_1 + Y_2)$$
where \( h = 1 \text{ cm} \).

It should be pointed out that all of the spatially dependent magnitude responses in this study are normalized to 0 db at the lowest frequency plotted on each figure. This is done in order to make the effects caused by changing the oscillator and detector locations more apparent and also for simplicity in graphically comparing results with those of Merritt (24). Note that in the phase response no normalization will be used as both phases should be the same in the spatially dependent frequency range.

Figures 33 through 38 show the spatial dependence of the frequency response of the UTR-10 reactor caused by oscillator location in addition to the dependence on the location of the detector. From this set of figures one can see that the natural mode approximation and the method of Green's function modes are in very good agreement when predicting the behavior of the UTR-10 reactor in the frequency domain.

It is interesting to note from the magnitude plots that the break frequencies occur at approximately \(-\lambda\) and \(-\beta/\xi\) (rad/sec) with the slope of the curve in the vicinity of the break being about \(-20\) db/dec. Also it is interesting to note that the \(\beta/\xi\) break frequency predicted by the three oscillator locations is slightly different when the detector is near the oscillator. This is the reason why in experimental forced oscillation measurements of the \(\beta/\xi\) break the detector is usually far from the oscillator.

The most significant result obtained in this section is the prediction of a sink when the oscillator is in position 2 and the detector is in position E. This sudden decrease in the magnitude of the frequency response followed by a rapid recovery was obtained at an oscillation frequency of
Figure 33. Magnitude of frequency response for oscillator at position 1
Figure 34. Phase of frequency response for oscillator at position 1
Figure 35a. Magnitude of frequency response for oscillator at position 2
Figure 35b. Magnitude of frequency response for oscillator at position 2
Figure 36. Phase of frequency response for oscillator at position 2

- Natural mode approximation
- Green's function modes
Figure 37. Magnitude of frequency response for oscillator at position 3.
Figure 38. Phase of frequency response for oscillator at position 3.
about 1000 rad/sec. Merritt (24) observed the same behavior for the same location of oscillator and detector, although the sink predicted in his analysis occurred at an oscillation frequency of about 2000 rad/sec. Hendrickson (39) observed a similar phenomenon in cross-spectral density measurements of the UTR-10 reactor. A sink frequency of about 700 rad/sec was observed by Hendrickson. It should be pointed out that Kylstra and Uhrig (40) have also observed multiple sinks in experiments dealing with both light-water and heavy-water slab subcritical assemblies.

A physical explanation for the presence of the sink is beyond the scope of this investigation, although the possibility of the actual existence of the sink will now be discussed.

Figure 39 is a plot of the magnitude of the frequency response for the oscillator at position 2 at an oscillation frequency of 1000 rad/sec for the different detector locations. For detector locations A, B, and C it seems that an accurate representation of the frequency response of the reactor up to oscillations frequencies of 1000 rad/sec will be obtained by using a 9-mode expansion. By an accurate representation is meant that when an extra mode is added to the approximation the solution will not change significantly. However, for detector locations D and E this does not seem to be the case.

From the frequency response of the UTR-10 reactor, it appears that convergence of the modes to an accurate solution was generally present up to oscillation frequencies of about 2000 rad/sec. This conclusion arises from the fact that the 9-mode analysis in this investigation was in good agreement with the 5-mode analysis done by Merritt (24). Also it seems the farther the detector is located from the oscillator the greater the
Figure 39. Magnitude of frequency response for oscillator at position 2 (oscillation frequency is 1000 rad/sec)
attenuation and phase shift. Again from Figure 39 one can see that when the oscillator is in position 2 and the detectors are in position D and E no such pattern can be established. At this point one may conclude that for such combination of oscillator and detector locations convergence of the modes does not exist in the sense it was previously described. Thus, a 7-mode analysis for this combination of oscillator and detector locations was used since this combination of modes represented the best agreement with the Green's modes analysis. It should be pointed out that the 9-mode analysis resulted in unrealistic magnitudes for the combination of oscillator and detector locations in question.

From the above analysis it seems that a sink actually exists in the oscillation frequency range of 1000-2000 rad/sec and that it is a characteristic of the model of the UTR-10 reactor. However, the sink observed by Hendrickson (39) is a characteristic of the UTR-10 reactor. The difference in the sink frequency between the model and the reactor probably arises from the approximations made in arriving at the equations which describe the kinetic behavior of the model. For example, in considering the leakage of neutrons through the two sides, and top and bottom of the reactor it is assumed that the transverse buckling is the same for both the fast and the thermal group.
IX. SUMMARY AND CONCLUSIONS

The following conclusions are a result of the study on coupled-core reactors using the natural mode approximation:

1) For symmetric flux variations no odd harmonics are needed to describe the flux distributions, while for non-symmetric flux variations all the harmonics contribute in describing the time dependent flux.

2) For small symmetric step removal of thermal neutron absorber the NMA may be formulated to have the property of "finality".

3) When the oscillator is in the south core convergence of the modes in the usual manner does not exist for detector locations in the north core and north reflector. The behavior of the phase shift is very sensitive in the region of a resonance and this same behavior is present near the sink frequency which is why it is believed the presence of the sink is a reality.

4) For the detector location far from the oscillator location the $\beta/\gamma$ break frequency appears to yield a consistent value for $\beta/\gamma$.

5) A total of nine space modes seems to be sufficient to describe adequately the frequency response of the model of the UTR-10 reactor up to oscillation frequencies of about 2000 rad/sec. A total of five space modes seems to be sufficient for describing the behavior of the reference reactor for the one-group analysis in the frequency domain up to oscillation frequencies of about $10^4$ rad/sec.

6) In the low frequency region the frequency response is independent of spatial effects.

7) Prompt epithermal modes contributions may be safely neglected in
the oscillation frequency range used to study the spatial effects of the model of the UTR-10 reactor in the frequency domain.

8) From the behavior of the reference reactor for the one-group analysis and the model of the UTR-10 reactor in the frequency domain it may be concluded that the number of modes needed for an adequate representation of the frequency response is dependent on the value of the largest prompt thermal eigenvalue used in the expansion. In other words, if oscillation frequencies up to 2000 rad/sec are going to be generated, then at least there must exist one prompt thermal eigenvalue with an absolute value greater than 2000 sec⁻¹.

9) As a final conclusion it may be stated that if a pole-zero representation (41) of the reactor frequency response would be obtained the poles of the system would be the steady state parameters of the NMA. This conclusion seems to be in agreement with the previous conclusion as the zeros of the "transfer function" tend to change the slope in the phase and magnitude of the frequency response for frequencies above the absolute value of the last prompt thermal eigenvalue considered.
X. SUGGESTIONS FOR FURTHER WORK

The following suggestions are made for future investigations:

1) Measure experimentally the spatially dependent frequency response of the UTR-10 reactor in the frequency range investigated in this study to provide a check on the analytical results.

2) Develop a technique to find the prompt epithermal eigenvalues of the system without encountering the difficulty of trying to distinguish these eigenvalues from the higher prompt thermal eigenvalues.

3) Develop a mathematical model which could be used to describe the behavior of a reactor in the frequency domain in the vicinity of a sink.

4) Extend the one-dimensional work done using the NMA to more than one dimension and perhaps consider the inclusion of feedback also.
XI. LITERATURE CITED


XII. ACKNOWLEDGMENTS

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