Computer assisted instruction in college general education mathematics

Milton Monroe Underkoffler

Iowa State University

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COMPUTER ASSISTED INSTRUCTION IN COLLEGE
GENERAL EDUCATION MATHEMATICS.

Iowa State University, Ph.D., 1969
Education, general

University Microfilms, Inc., Ann Arbor, Michigan
COMPUTER ASSISTED INSTRUCTION IN COLLEGE
GENERAL EDUCATION MATHEMATICS

by

Milton Monroe Underkoffler

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Education

Approved:
Signature was redacted for privacy.

In Charge of Major Work
Signature was redacted for privacy.

Head of Major Area
Signature was redacted for privacy.

Head of Graduate College

Iowa State University
Of Science and Technology
Ames, Iowa
1969
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INTRODUCTION

Mathematics is in many respects an entirely different discipline today than it was at the beginning of the century. The changes in mathematics have been so extensive, so far reaching in their implications, and so profound that they can be described only as a revolution.

G. Baley Price (1) stated that there were three causes of this revolution. The first was the advance made in mathematical research. In the second place, the revolution was caused by the automation revolution. The introduction of the high speed, automatic digital computer was the third cause of the revolution in mathematics.

One of the results of the research in mathematics has been the creation of new subject matter. Such fields as mathematical logic, probability and statistical inference, topology, and modern abstract algebra are largely or wholly the product of recent mathematical research. Mathematical logic, for example, was little known to most mathematicians a generation ago. Similarly, the field of probability and statistical inference has been extensively developed in recent years.

In addition to the new uses to which the older mathematics is now put, there are also rapidly expanding fields of application for contemporary mathematics. Mathematics is no longer reserved for the use of engineers and physical
scientists, even though a great many of its application are still in their hands.

During this past decade the relationship of mathematics to computing and to computing machines has changed significantly (2, p. 1). In the beginning, the digital computer was thought of as a device for solving complicated but essentially routine problems in numerical mathematics. However, the advent of the stored program as well as the striking improvements in components and circuitry, have together brought about a radical change in the depth and variety of the problems that can be treated. Computers are now being used for such non-numerical tasks as the simulation of various types of complex systems, the performance of complicated logical operations, and the design of new computers and computer systems.

There has emerged a field of study called Computer Science, embracing such topics as numerical analysis, theory of programming, theory of automata, switching theory, etc. (2, p. 1). Many of the problems arising from the use of computers are intimately associated with questions in combinatorial mathematics, abstract algebra, and symbolic logic. But an even more fundamental relationship also exists. The structure of a computer forces the computer specialist to strive for the type of generality, abstraction, and close attention to logical detail that is characteristic
of mathematical arguments. Workers in Computer Science must have a knowledge of the spirit and techniques of mathematics.

The changing role of mathematics has created new challenges for curriculum development in mathematics. The leadership in meeting this challenge has been provided by the Mathematical Association of America. In 1959 the Committee on the Undergraduate Program in Mathematics (hereafter called CUPM), was formed as a successor to the Mathematical Association of America Committee on the Undergraduate Program, and charged with finding ways to improve mathematics curricula and instruction (3, p. 1). Since 1960 these activities have been supported by the National Science Foundation.

Until 1965, CUPM's activities were concentrated in the work of four panels, two ad hoc subcommittees, the CUPM Consultants Bureau, and the Advisory Group on Communication. The names of the panels and subcommittees indicated the scope of their curricular interest: (3, p. 2)

1. Panel on Teacher Training
2. Panel on Pregraduate Training
3. Panel on Mathematics for the Physical Sciences and Engineering
4. Panel on Mathematics for the Biological, Management, and Social Sciences
5. Ad hoc Subcommittee on a General Curriculum in Mathematics

Recently there has been a major reorganization of CUPM's activities. Of the panels and subcommittees listed above, only the Panel on Teacher Training continues on an active basis. The two subcommittees published curricular recommendations and were discharged. In the fall of 1966, CUPM created the Advisory Group on the Application of Mathematics to survey the whole area of applications (3, p. 6). In addition to this advisory group three new panels were formed devoted to applications:

1. Panel on Mathematics for the Life Science
2. Panel on Statistics

These changes represent a reorganization of CUPM's activities in this area, based on a reassessment of the relative urgency of the curricular needs among the many fields where mathematics has been found useful.

Within the past three years major new panels have been created to consider the problem of Junior Colleges and of college teaching in general. There are two new panels and three new subpanels: (3, p. 8)

1. Panel on College Teacher Preparation
2. Panel on Mathematics in Two Year Colleges
   A. Subpanel on Mathematics for University Parallel Students
B. Subpanel on Mathematics for Technical and Occupational Education

C. Subpanel on Mathematics for General Education in Two Year Colleges.

These new CUPM activities reflect some changes now taking place in undergraduate education in this country. The leadership provided by CUPM through the reports of the panels has had a profound effect on the development of mathematics in higher education.

The availability of computers has created new challenges for curriculum development in mathematics. Computers have changed the nature of the solution process in mathematics. In addition to existence proofs, it has now become very important to provide constructive methods for obtaining solutions. Many traditional special techniques have been rendered obsolete while the need for an algorithmic approach to mathematical thinking has been strongly reinforced. At the same time, the use of computing methods provides the student with a greater understanding of the concepts involved as well as with greater problem solving ability.

The computer can help the teacher to explore systems much too complicated for analytical solutions or, at least, which have analytical solution whose written expression gives no simple insight into what is going on. These solutions can be graphed and explored on a computer quite quickly if proper programs and equipment are available.
Computer connected display devices, when used in the classroom can provide the instructor with a means for illustrating sophisticated mathematical concepts graphically in such a way as to allow for almost instantaneous response to changes in input that might be suggested by the students. Display devices in the classroom make it possible to integrate the theoretical and computational aspects of mathematics in a way which was not possible before the advent of the computer. That is examples illustrating theoretical and numerical concepts can be worked out more quickly and modified more rapidly with the aid of the computer than can be done by hand at a blackboard. The use of computers can strengthen and stimulate a student's interest in mathematical problems arising in other disciplines. If the basic undergraduate mathematics courses are not appropriately modified to reflect the new points of view which are associated with computer applications in mathematics these courses will lose much of their relevance.

In the past, the mathematics instructor has been severely limited with respect to classroom demonstration with student participation. In most cases, the time required for realistic problem solving and graphical representation is simply not available. Potentially the most promising applications of computers to the teaching of mathematics lie in the transformation of mathematics teaching from a completely passive subject to a partially active subject
with cooperation on the part of the student.

Need for the Study

A conference was held on December 8-9, 1967, at the University of Maryland, to explore and summarize current thinking about the role of the computer for the undergraduate curricula in the physical and mathematical sciences. The focal question of the conference was; (4, p. vii)

What goals of the existing undergraduate curriculum might better be realized utilizing the computer, and how extensively would the present curriculum have to be revised to realize these potential benefits.

Twenty-four individuals representing the four fields of mathematics, physics, statistics and chemistry met to exchange views on the impact they foresaw computers making in their own undergraduate programs, and to present a summary of their views and recommendations to the National Science Foundation.

The mathematics panel made the following four recommendations: (4)

1. Computing activities and relevant areas of numerical analysis should be integrated into the first college mathematics course and in subsequent courses when appropriate and natural.

2. Wider experimentation in designing and introducing new courses in applied mathematics, in which computers play a central role, either in classroom instruction or in individual work on the part of the student, is needed.
3. A conference of interested persons be arranged to develop a detailed outline for such a programmed learning system for calculus including both curriculum content and software.

4. That experimentation with the development of computer connected display devices and appropriate software for classroom instruction in mathematics be undertaken.

This conference supported the fact that computer assisted instruction offers unexplored opportunities for improving mathematics instruction and that the real problem is the generation of appropriate learning situation coupled to a computer with well defined educational objectives in view.

In the present, relatively undeveloped, state of the art it seems that many drill aspects could be handled by means of programmed learning techniques in which the computer gives directions to the students and monitors their responses. Such a procedure would require the availability of a computer and appropriate software. There is a pressing need for the preparation and evaluation of a set of suitable computer-oriented exercises to be used in such an approach. The development of an effective method of using computer assisted instruction would be extremely important in initiating a wide-scale introduction of computer assisted instruction into the mathematics curriculum.

**Purpose of the Study**

The purpose of this study was to evaluate the
effectiveness of a programmed learning system, in which the computer played a central role, in a college general education mathematics course. This required the development of a set of computer-oriented exercises and the supporting software, and the designing of an experiment to compare the computer oriented procedure with a more traditional procedure. The computer oriented procedure was designated as the experimental procedure and the traditional procedure as the control procedure.

The experiment was designed to test the following null hypothesis.

1. There is no difference between the mean achievement of the experimental group and the control group.

2. There is no difference between the mean achievement of the upper ability level subgroups, of the experimental group and the control group.

3. There is no difference between the mean achievement of the lower ability level subgroups, of the experimental group and the control group.

4. There is no difference between the mean achievement of the upper background level subgroups, of the experimental group and the control group.

5. There is no difference between the mean achievement of the lower background level subgroups, of the experimental group and the control group.

6. There is no difference between the mean achievement of the upper ability level subgroup and the mean achievement of the lower ability level subgroup.
7. There is no interaction between ability level and method of instruction.

8. There is no difference between the mean achievement of the upper background level subgroup and the mean achievement of the lower background level subgroup.

9. There is no interaction between background level and method of instruction.

Delimitations of the Study

This study was restricted to one programmed learning system devised by the investigator and its effects on a sample drawn from the fall and winter quarter general education classes at Winona State College during the 1968-69 school year.

Definitions

For the purpose of this study the following definitions were used:

"Computer assisted instruction" was used to refer to the interaction of a student with a computer used for information retrieval and information transfer processes. In these uses the responses of the student are often compared with the prerecorded responses which are to be expected, or the student is given bits of information which he requests from the files, which have been previously stored in the computer.

"Experimental group" was used to designate those students whose instruction involved the use of computer-
oriented exercises.

"Control group" was used to designate those students whose instruction involve a more traditional method of handling the exercises.

"Upper ability level subgroup" was made up of those students whose composite ACT score was twenty or above.

"Lower ability level subgroup" was made up of those students whose composite ACT score was below twenty.

"Upper background level subgroup" was made up of those students with three or four years of high school mathematics.

"Lower background level subgroup" was made up of those students with one or two years of high school mathematics.

"ACT" was used to designate the American College Testing program.
REVIEW OF LITERATURE

The purpose of this review was to make a critical evaluation of selected studies related to the teaching of college level mathematics. A review of published studies for the past ten years revealed a dearth of significant research in college level mathematics during this period. Very little definite research was reported in the literature examined. Much of the current research deals with the nature of views held by students, teachers and "experts." Studies reported during the past ten years were examined and included if they seemed significant. Studies which reported to be experimental were included if the following criteria were met:

1. Provision was made in the design for use of a control group.

2. A statistical test of significance of the hypothesis under test was used and reported.

Studies Related to Teaching Techniques

Weiner (5) compared the effectiveness of two methods of teaching mathematics on the functional competence of college students in a first semester course in mathematics.

The method designated as the control started with an explanation of the general concept and then proceeded to illustrations and exercises. The experimental procedure was an attempt to lead up to the concept through the use of
a problematic situation. It was hypothesized that the latter approach would lead to a better understanding of the concepts and their interaction, thereby increasing their functional competence in mathematics.

Both the control and experimental groups were given three preliminary tests, namely the Schools and Colleges Achievement Test, the Lankton Algebra Test, and the Davis Test for Functional Competence in Mathematics. An analysis of variance of the scores of the two groups showed no significant difference between the groups on these preliminary tests.

At the end of one semester of mathematics each group was given the Davis Test for Functional Competence in Mathematics. The results were analyzed for three levels of ability using single analysis of variance and for the group as a whole using single analysis of covariance.

The results of the analysis indicated that:

1. The experimental method produced significantly better results than the control method with students rating average and below average on the Davis Test for Functional Competence in Mathematics.

2. There was no significant difference in the results of the above average groups.

3. The experimental method produced a significantly better result than the control method on the group taken as a whole.

This study has the characteristics of good experimental design and used modern and acceptable statistical
treatment of the data. The claim for randomization was based on the fact that the samples of students were not influenced by the investigator in the selection of their classes. This claim was rendered more plausible by the results of the analysis of the means and variance of the two groups.

Plachy (6) attempted to determine which one of three approaches would be most effective in teaching the introductory college mathematics course. The three approaches investigated were the conventional approach, the vector approach, and the set theory approach. A group of seventy-two students at Central State College, Edmond, Oklahoma was subdivided into three subgroups. Each group studied the subject matter using one of the three approaches. Statistical procedures were used to test for significant differences between the three groups which might be attributed to the approach used.

The t-test was used to determine whether there were any differences between the mean scores of the three groups at the beginning of the experiment with regard to scores attained on the American College Testing Program, the Cooperative Algebra Test, the Rasmussen Trigonometry Test, and the Sequential Test of Educational Progress in Mathematics. No significant differences were found at the five percent level of significance. After eighteen weeks of instruction, similar forms of the Cooperative Algebra Test,
the Rassmussen Trigonometry Test, and the Sequential Test of Educational Progress in Mathematics were administered. Using the difference between the pre-test and post-test scores as the dependent variable, the analysis of covariance was used to test the difference between the three groups. The groups showed no significant difference at the five percent level.

This study used modern statistical techniques in testing the hypothesis, but it should be noted that large differences would have been required between means to have been significant with classes of twenty-four students.

Brian (7) carried out an experiment to test the effect which a knowledge of the behavior associated with four processes of mathematics would have on problem solving behavior in mathematics. The processes were defined as:

1. The process of constructing mathematical models.
2. The process of conjecturing.
3. The process of settling conjectures, as being either true or false.
4. The process of using known or given axioms and theorems or algorithms on problems where they clearly apply.

Seventeen students who were enrolled in an upper division course in mathematical problem solving at San Jose State College were used in the study.

Three measures were taken:

1. A standardized spatial relation measure.
2. An initial measure dealing with the processes of mathematics behavior.

3. A final measure dealing with the processes of mathematics behavior.

In the initial measure and in the final measure, the first three processes of mathematics were covered by oral tests and the fourth process was covered by a written section.

The experiment tested two hypothesis:

1. Among upper division mathematics students, spatial relation ability is not a predictor of ability to exhibit the processes of mathematics behaviors in the solution of mathematical problems.

2. Among upper division college mathematics students, a two weeks course designed to help the student acquire the behaviors of the processes of mathematics, does not improve the ability of these students to exhibit these process behaviors in solving mathematical problems.

Hypothesis 1 was evaluated by comparing results of the spatial relations measure against each part of the initial and final measures dealing with the processes of mathematics. A non-parametric test involving the computation of exact probabilities was used. No significant relationship was found between spatial relations ability and the problem solving behavior involving the processes of mathematics and hence there was no reason to reject hypothesis 1.

Hypothesis 2 was evaluated by comparing the results of each section of the initial measure with each comparable portion of the final measure. The Wilcoxon sign-rank
test was used as the statistical test. No significant change in behavior was identified in processes one and two and four, but the hypothesis of no improvement in behavior identified with process number three, the process of settling conjectures, was rejected at the one percent level of significance.

This study did succeed in giving a definition of the processes of mathematics, but the following criticism seems to be in order:

1. Large differences would have been required to have been significant with a class of seventeen students.

2. The assumption was made that a standard measure of spatial relations ability was a valid measure of problem solving ability.

3. The measure dealing with the processes of mathematics behavior were largely oral test with questionable validity and reliability.

Bassler (8) reported on a study carried out at the University of Maryland to compare the effects of two types of exercises. The two types of exercises were designated as "circle" those framed in a mathematical setting, and "triangle" those framed in an applied setting. The material to be learned consisted of the first four units in a geometry course designed exclusively for elementary education majors. The chapter titles for this mathematical content were "A Review of Some Basis Ideas," "A System of Sets," "Sets of Points," and "Additional Topics in Logic."
The experimental subjects consisted of all students enrolled in Mathematics 30, Elements of Geometry, during the spring semester, 1963. There were six sections ranging in size from 21 to 26 students per section, a total of 135 students.

The instructional period consisted of twenty-four class periods, each of fifty minutes in length. One class received instruction supplemented with only "triangle" exercises; one class received instruction supplemented with only "circle" exercises; two classes received instruction supplemented with a combination of both types of exercises; and two classes, designated as controls, received instruction without emphasis on either type of exercises. There were eleven treatment days when the supplementary exercises were provided. It usually required about ten minutes for the students to complete the exercise and to check their work.

During the two class periods immediately following instruction all classes were given a post-test. Two forms of a test were developed, and both were used to evaluate the relative effects of the different treatments. Each form consisted of forty-five short answer type questions sampling the instruction materials used during the study. The two forms were administered on successive days. The forty-five exercises on each form of the test were of three distinct types which were designated as "rote" (simple recall or recognition), "circle," and "triangle."
Approximately ten weeks after the completion of instruction a 75-item multiple choice retention test was administered. Responses were again required for the three distinct types of items designated as "rote," "circle," and "triangle."

Two factors were identified which could possibly affect the criterion measures. These were Mathematics 30 final grades which were available for all the subjects and percentile ranks on the Cooperative Mathematics Pre-test for College Students which were available for approximately 85 percent of the subjects. Two single classification analysis of variance were conducted and the results led to the conclusion that the six treatment groups were comparable with regard to Mathematics 30 grades and with regard to percentile ranks on the Mathematics Pre-test.

The scores that each individual obtained on the two forms of the post-test were added to obtain the post-test score, "rote" subscore, "circle" subscore, and "triangle" subscore.

A single classification analysis of variance was conducted to test each of the following four hypothesis at the one percent level of significance:

1. The post-test means for the six treatment groups are equal.

2. The post-test "rote" subscore means for the six treatment groups are equal.
3. The post-test "circle" subscore means for the six treatment groups are equal.

4. The post-test "triangle" subscore means for the six treatment groups are equal.

All four of the hypotheses stated above were accepted.

The retention test score was used to test each of the following four hypotheses at the one percent level of significance:

1. The retention test means for the six treatment groups are equal.

2. The retention "rote" subscore means for the six treatment groups are equal.

3. The retention "circle" subscore means for the six treatment groups are equal.

4. The retention "triangle" subscore means for the six treatment groups are equal.

The hypotheses (1 and 3 above) were rejected indicating that the means for the six treatment groups on the retention test and the retention "circle" subscore were not equal. Hypotheses (2 and 4 above) were accepted.

In each case where the F-ratios were significant, the differences among the means were tested using a t-test. In each of these instances, the t-test indicated that the mean for the second control group was significantly larger than the mean for any of the other treatment groups at the one percent level of significance.
A reliability coefficient for the post-test and the retention test was computed following the administration of the tests using the split halves method and corrected by the Spearman-Brown Formula. The reliability coefficient for the post-test was .84, and for the retention test, was .86.

The investigator drew several conclusions for the specified population. One such conclusion was that the methods of instruction involving the experimental variables were equally effective when achievement of the instructional objectives was measured by the post-test and the retention test. A second conclusion was that the conventional method of instruction without emphasis on exercises was as effective as instruction involving the experimental variables when achievement was measured by the post-test and possibly more effective when achievement was measured by the retention test.

Although this experiment had some characteristics of good experimental design the following should be noted:

1. An analysis of covariance would have provided a more sensitive test of the hypothesis.

2. Since the time spent on the exercises was so short it would be misleading to interpret the results as showing that instruction without emphasis on exercises was as effective as instruction involving the use of exercises.

Mason (9) sought to determine if required homework
results in greater achievement by students in a course in College Algebra. The required homework group consisted of 241 students in nine classes and the non-homework group was composed of 191 students in nine classes taught by the same teachers. The analysis of variance techniques, Mann-Whitney U Test and Covariance Analysis were the statistical procedures used to analyze the data. Mason found that there was no significant difference in results obtained by requiring homework or not requiring homework when averaged over all teachers. However, there was a high degree of teacher by level interaction which led Mason to the conclusion that each teacher should determine which of the two methods to use in order to be most effective as a teacher.

This study had great merit in the description of the method used and the teaching content. Unfortunately there were shortcomings in the statistical treatment.

The groups were pooled without ascertaining whether the conditions for doing so had been met. In addition, it is necessary to point out the inadequate bases for assuming equivalence of experimental and control groups.

Gasaway (10) conducted an experiment with tests during the fall quarter of 1960-61 at Tennessee Agricultural and Industrial State University. The purpose was to determine the effectiveness of frequent short tests as a teaching technique in algebra classes with freshmen of varying pro-
ficiencies, and, to guide the students into recognizing the valuable use of tests in a learning situation.

Data collected on each of the 576 subjects included scores for the Otis-Quick Scoring Mental Ability Test, the Cooperative Mathematics Pre-Test for College Students, Form Y, the Cooperative School and College Ability Test, Parts 2 and 4, the number of units earned in high school mathematics and the grades received in these courses. At the end of the quarter, the Cooperative Mathematics Pre-Test for College Students, Form X, or the Cooperative School and College Ability Test, Parts 2 and 4 was administered to determine the gain in score for each individual.

Nine of the eighteen sections were randomly selected as experimental groups and the remaining nine sections served as control groups. Each of the ten members of the mathematics staff was instructor of one or more of the sections.

The method used in teaching the experimental group differed from that used with the control group in one respect, namely, the administering of short tests during each class period for which an hour examination was not scheduled. The scored papers were returned to the students.

Fifteen hypothesis concerning the effectiveness of the testing techniques were investigated statistically with the t-test, the analysis of covariance, or multiple regression analysis.
When the subjects were retested at the end of the quarter, significant differences between the experimental and control groups were found in (a) the mathematical achievement, (b) the gain score. Both differences were in favor of the experimental groups.

The mean gain score of the three ability level subgroups of the experimental group did not show significant differences.

The students evaluation of the technique indicated a change of attitude toward tests and a recognition of their valuable uses in learning situation.

Gasaway concluded that the testing technique seems to be one which any instructor of freshman mathematics may use profitably to supplement his teaching methods.

Further research is needed to determine the combination of frequent testing and other teaching techniques that are effective.

This experiment was well designed and executed and the large number of subjects used resulted in conclusions which are valuable to those interested in improving mathematics instruction.

Tulligan (11) studied the effect of student constructed assignments on achievement in the algebraic content of an introduction to mathematics course and the effect which these assignments have on the retention of algebra content, algebraic problem solving ability, and critical thinking by
freshmen liberal arts college students.

Participating in this study were twenty-two students of the 1956-57 freshman class at Ladycliff College, Highland Falls, New York; acting as the experimental group and nineteen students of the 1956-57 class of Good Council College, White Plains, New York acting as the control group.

At the beginning of the term, the American Council on Education Psychological Examination for College Freshman, and the Cooperative General Achievement Test, Mathematics, were administered at both institutions. The achievement level and mathematical proficiency level were determined from the total scores of both groups. The mean difference when tested was found to be insignificant.

Both classes were taught by the lecture-demonstration method. The variable factor, the student-constructed assignments numbering twenty-one, was used in the first semester with only the experimental group.

At the close of each unit every pupil in the experimental group composed an assignment on the material suggested by the instructor. One student compiled the assignment, a second worked it, while a third checked and evaluated it according to check lists compiled by the student and the instructor.

A standardized test on algebra content and problem solving was administered to both groups to test achievement in subject matter and ability in problem solving. The Watson-
Glasen Critical Thinking Appraisal Test was also given at this time to both groups. The same tests in algebra content, problem solving and critical thinking were re-administered in May to both groups to test for retention four months after instruction in algebra had been completed.

The investigator listed the following conclusions:

1. The students in this study do not differ significantly initially in any category under discussion and are representative of all United States female freshmen students in four-year colleges for women. Their mathematical achievement in particular is representative of the freshmen in other colleges.

2. At the end of the first semester there was a statistically significant difference in algebraic subject matter and problem solving ability in favor of the experimental group. After four months elapsed time the experimental group also showed greater retention of these abilities.

3. Although both groups showed a highly significant increase in critical thinking over their initial ability, there was no appreciable difference between them at the end of the first semester. The experimental group, however, sustained a greater loss in retention over a four month period.

This study was well planned and executed and used a modern and acceptable statistical treatment of data. The first conclusion stated would seem to imply that the results could be generalized beyond the classes used in the study but such an interpretation does not seem justified since the groups were not randomly selected from a larger population.
Simmons (12) conducted a study at the University of Wichita to study the effect of class size on achievement in intermediate algebra. The control group was selected from students enrolled in intermediate algebra for the fall semester of 1956. These students were in classes whose average size was 21.4 students and they were taught by standard lecture discussion methods. The experimental group was selected from students enrolled in intermediate algebra for the fall semester of 1957. These students were assigned to classes whose average size was 84.6 students. These classes were taught by a formal lecture method with many illustrative examples worked out on the chalkboard. For the students in the large classes, six hours for conference or help sessions were provided each week.

A sample of 200 students was selected for the control group and another 200 were selected for the experimental group. These students were so selected that 50 students graduated from each of four groups of high schools. These groups were: group I - Wichita High School - East, group II - Wichita High Schools other than East, group III - high schools located outside Wichita with an enrollment of over 250 students, and group IV - high schools located outside Wichita whose enrollment was less than 250 students. The criterion of achievement chosen for this study was the
grade in intermediate algebra.

An analysis of covariance was used to test the effect of class size, high school group, and the interaction of class size and high school group on algebra achievement. This analysis was made holding constant prior achievement as measured by the high school grade average and student aptitude as measured by the total score on the American Council on Education Psychological Examination and the mathematics achievement score on the Cooperative General Culture Test.

The covariance analysis revealed a highly significant difference in favor of the students in the small sized classes over students in the larger lecture sections. The differences found among the four high school groups were not significant. Likewise the interaction between class size and high school group was not statistically significant.

This experiment was well designed and executed and used modern statistical methods. It should be noted that the small class procedure was superior even though extra help was provided for members of the large class group. An analysis comparing the effect on different ability levels would have provided interesting additional information.

Kerce (13) studied some aspects of the problem of teaching larger numbers of students in general mathematics courses at the freshman college level.
A total of 201 students enrolled in Fundamentals of Mathematics at David Lipscomb College during the spring quarter, 1965 were used in the main study. Three teachers each taught a small section and a large section by one of three methods. The discussion method, lecture-laboratory method, and lecture method were the three methods used. The three small classes each had 13 students and the large classes each had 54 students. The final test was the criterion variable.

The data gathered from the basic design were analyzed by an analysis of covariance with intelligence and mathematical ability as the two control variables.

An additional class of thirteen students was taught by a fourth teacher using varying teaching methods, and the performance of this group was compared with the performance of the composite group in the basic design. The method of orthogonal comparisons was used to assess the difference in the mean achievement of the two groups.

There were no significant differences among the adjusted means of the six treatment subgroups in the basic design when these means were corrected for class sizes and teaching methods differences. Furthermore, there were no significant differences in the means of the treatment subgroups with respect to different class sizes or different teaching methods. There was no significant difference
between the mean of the group taught by varying the teaching method and the mean of the composite group taught by teaching methods which were held constant.

Although the experiment did show the characteristics of a good design, the use of two teaching methods rather than three probably would have been better. With the number of students available the use of two methods would have increased the likelihood of establishing one as being superior. The large classes in this experiment each had 54 students. If the number of students had been increased there would have been a greater contrast between the two sizes of classes.

Studies Related to the Status of General Education

Cornish (14) made an analysis of the role played by college mathematics in general education. The project consisted of three parts. The first part surveyed the professional literature relevant to the general education movement. The second part of the study surveyed the literature related to the question of determining the value of mathematics in general education. The third part of the project introduced a course which included four major topics.

The general education movement was characterized chiefly as a quest for unity, as a counterbalance to overspecial-
ization, as that education suitable for universal education, as that education directed toward improving communicative skills, and as that education oriented toward everyday activities of men. The literature emphasizes that many educators are of the opinion that some core of knowledge should be prescribed for all students.

Mathematics considered as a required course in general education has had an uneven history during the past thirty years. A number of colleges and universities have in recent years added mathematics to the list of general education requirements. Essentially three types of content have been incorporated in general mathematics courses. Traditionally, topics from college algebra and trigonometry have been used; however, a number of educators are of the belief that historical cultural aspects of the subject should be emphasized, while still others believe that the nature of mathematics should form the background of such a course.

Cornish concluded that opinions concerning what content should be taught in general mathematics, and what methods of presentation are best for such a course are so varied that it seemed that still other types of courses might be designed.

Cornish designed a course in which students were required to make mathematical calculations, but in a setting which emphasized their application to life, and also presented a historical-cultural perspective. The course included four
major topics: Non-Euclidean geometry, group theory, the derivative, and the integral. The course was taught to two groups of college freshmen who were taking mathematics solely to satisfy graduation requirements.

This study was primarily concerned with a survey of the professional literature dealing with general education mathematics. This part was very well done and served as a source of valuable knowledge concerning general education mathematics. The course presented is worthy of consideration as a type of general education course. There was no formal comparison made between students' achievement in this course and student achievement in any other course.

Lefstad (15) attempted to analyze the mathematics courses offered by junior colleges in the United States and Canada for general education. The criteria for the rating were developed from the literature pertinent to the study.

The study was restricted to those mathematics courses, offered by junior colleges, which were identified by personnel of the junior colleges as being non-traditional and non-vocational in scope and purpose. The data for the study were gathered by means of questionnaires. The criteria, based on statements in the literature, were used to rate the courses examined.

The review of the literature revealed fourteen objectives on which there was substantial agreement. These objectives
may be stated as follows: a course in mathematics designed for general education provides the student the opportunity to increase and/or develop his

1. Powers of critical and logical thinking.

2. Knowledge of the understanding of the terminology and basic ideas of mathematics.

3. Appreciation of the importance of mathematics in today's world.


5. Knowledge of the history of mathematics and understanding of the contributions mathematics has made to the development of civilization.

6. Ability to analyze and formulate problems; use the scientific approach.

7. Ability to communicate by increasing skill and understanding in the use of mathematical symbols.

8. Understanding of the nature of proof, including ideas of truth and validity.

9. Knowledge of the role of mathematics in cultural activities.

10. Understanding of the function concept.

11. Understanding of the principles of probability and statistics.

12. Mathematical background as an aid to further study.

13. Appreciation of the utility and power of mathematics.

14. Power to do independent thinking.

Lefstad reported that approximately 24 per cent of the junior colleges of the United States and Canada offered
such courses and that approximately 19 per cent of the junior colleges offered courses that met the criteria of the study. He also reported that there was little agreement on the topics covered in the courses offered. Only four topics appear in 70 per cent, or more, of the courses examined. These topics were: concept of number, fundamental operation, bases other than ten, and the laws of algebra.

This study was not on the same research level as the other studies reviewed but did bring together information in a useful way, and the use of the questionnaire did result in interesting information not otherwise available.

Summary

Weiner (5) found that a procedure which led up to a concept through the use of a problematic situation was superior to a procedure which started with an explanation of the general concept and then proceeded to illustrations and exercises. He found the use of the problematic situation to be better with the average and below average student and also with the entire group.

Plachy (6) compared the conventional approach with the vector approach, and the set theory approach in the introductory college mathematics course. He did not find a significant difference between the approaches.

Drian (7) tested the effect which a knowledge of the behavior associated with four processes of mathematics
would have on problem solving behavior in mathematics and found a significant change in the process of settling conjectures, which was one of the four behaviors.

The results of these three studies support the idea that procedures which invite student participation are superior teaching procedures. Such procedures can transform mathematics teaching from a completely passive subject to an active subject.

Bassler (8) concluded that a conventional method of instruction without emphasis on exercises was as effective as instruction involving the experimental variables when achievement was measured by a post-test and possibly more effective when achievement was measured by a retention test.

Mason (9) found that there was no significant difference in results obtained by requiring homework or not requiring homework when averaged over all teachers.

Gasaway (10) determined that the use of frequent short tests was an effective teaching technique in algebra classes with freshmen of varying proficiencies.

Mulligan (11) showed that the retention of algebra content, and algebraic problem solving ability was improved by making use of student constructed assignments in a freshman liberal arts mathematics course.

The studies on the use of exercises and drill suggest that repetition alone is not sufficient to insure learning.
If on the other hand, repetition is paired with other variables, such as a high amount of guidance, then interaction of the variables may produce a beneficial effect upon learning.

Cornish (14) made an analysis of the role played by college mathematics in general education and found that a number of colleges and universities have recently added mathematics to the list of general education requirements. He concluded that essentially three types of content have been incorporated in general education mathematics courses but that still other types of courses might be designed.

Lefstad (15) analyzed the mathematics courses offered by the junior colleges in the United States and Canada for general education. He found fourteen objectives on which there was substantial agreement. He reported that approximately 24 per cent of the junior colleges of the United States and Canada offered such courses and that approximately 19 per cent of the junior colleges offered courses that met the criteria of the study.

These studies suggest that general education mathematics plays an important role in the total mathematics offering in colleges and junior colleges.

Simmons (12) conducted a study at the University of Wichita to study the effect of class size on achievement in intermediate algebra and found a highly significant
difference in favor of the students in the small sized classes over students in the larger lecture sections.

Kerce (13) compared the effectiveness of three methods of teaching college general mathematics and did not find any significant difference between them. He also studied the effect of small sections and large sections on each of the three teaching methods and did not find a significant difference between small or large classes.

Such studies emphasize the need for teaching techniques which are effective with large classes.

It is essential to recognize that results such as those just presented may be seriously in error because all of these studies were inadequate in one or more of the following ways:

1. There was inadequate description of the tests and testing techniques used.
2. Random assignment to groups was not practiced.
3. Various experimental and control groups were pooled when the assumptions basic to pooling were not tested.
4. There was a tendency to go beyond the generalizability of the experimental evidence.

With the possibility that exercises handled by means of programmed learning techniques would enhance the learning of mathematical concepts, a teaching scheme was developed in which the computer monitors the students responses and gives them directions.
METHOD OF PROCEDURE

An experimental study was designed at Winona State College to investigate the effects of an instructional variable in mathematics learning. Specifically the instructional variable was two distinct procedures for handling the exercises: (1) the experimental procedure involved submitting the exercises to a computer for correcting and (2) the control procedure involved handing in the exercises for correction by the instructor.

The choice of the computer oriented procedure stemmed from the belief that this method would enhance the learning of the mathematical concepts by providing the student with either an immediate confirmation of his response to each exercise item or provide him with instructions for restudying the concept presented in that item. The method also provided the teacher with an analysis of the students responses and thus provided for an early reteaching of concepts found to be troublesome.

Treatment Materials

The material to be learned consisted of six units in a mathematics course designed as a general education course for students at Winona State College. The six units were based on material selected from six chapters of the text used in the course. The test used was "Essentials of
Mathematics" by Meserve (16). The chapter titles for this mathematical content were "Sets and Numbers," "Sentences in One Variable," "Sentence in Two Variables," "Mathematical Structures," "Probability and Statistics," and "Functions and Relations."

A rigid schedule indicating the concepts to be taught each day was prepared for the presentation of the learning materials and this schedule was used in developing sets of exercises to cover the work of the course.

Nine sets of exercises were constructed to cover the material of the course. Each set of exercises consisted of 40 objective questions. The objective types used were true-false and multiple choice question. Each of the nine exercises covered the material of about three lectures.

The nine sets of exercises were duplicated in two different formats. One format was with directions to the student on the method of indicating replies on a separate answer sheet (see Appendix A). The second format was with directions to the student on the method of indicating the replies on a hand punched IBM card (see Appendix B).

A second group of nine exercises was constructed which paralleled the first group (see Appendix C). The second group was duplicated only in the format with directions to the student for indicating the replies on a hand punched IBM
Computer programs were also written by the investigator to process the student replies submitted on hand punched IBM cards.

Subjects

The experimental subjects consisted of students enrolled in Mathematics 112, Fundamentals of Mathematics, at Winona State College during the 1968-69 school year.

Winona State College is one of six state colleges in Minnesota which are centers of teacher education for the public schools of the state. About 80 per cent of its recent graduates have entered the field of public teaching. The non-teacher group is growing percentage-wise, and is made up principally of pre-professional students for medicine, engineering, agriculture, etc., together with liberal arts and business students who seek the B.A. degree. A great many teachers in eastern Minnesota and western Wisconsin look to Winona State for improvement in their professional standing by attending summer sessions at the college. Enrollment in the college has doubled in the last five years, and during the 1967-68 school year it was about 3,300 students. The first summer session enrolls about 1,600 students and the second about 850. Degrees offered are the B.S. and M.S for those in teacher preparation and the B.A. and M.A. for those in liberal arts.
Mathematics 112 was designed as a general education course. It is a terminal course which meets for three fifty minutes periods each week for three quarter hours of credit. It is required of all students in the B.S. and B.A. program unless they replace it by a higher level mathematics course.

Twenty two sections of Mathematics 112 were scheduled during the 1968-69 school year. Eight sections were taught fall quarter, seven sections were taught winter quarter and seven sections were taught spring quarter.

In the fall quarter a section of Mathematics 112 was scheduled each of the eight periods. Two of these sections were randomly assigned to the investigator by drawing two numbers from a box containing the numbers one through eight. The Mathematics 112 sections assigned were those taught the period corresponding to the numbers drawn.

In the winter quarter a section of Mathematics 112 was scheduled each of the first seven periods. Two of the sections were assigned to the investigator by drawing two numbers from a box containing the numbers one through seven and two more sections were assigned to another teacher by drawing two additional numbers from the box.

The two sections selected in the fall quarter for the study were assigned the status of control section and experimental section by the flip of a coin. In the
experimental section the computer was used for correcting the exercises while in the control section the exercises were corrected by the instructor. The investigator's two winter sections were given their designation so that the teaching of the experimental section would proceed the teaching of the control section. This procedure was followed in order to reverse the order of teaching which occurred during the fall quarter. The other teacher's two sections were also assigned the status of control group and experimental group by flipping a coin.

During the fall quarter the control group met on Monday, Wednesday and Friday second period and the experimental group met on Monday, Wednesday and Friday during the fifth period. The enrollment was 34 students in the experimental section and 35 in the control section. During the winter quarter the investigator's groups met on Monday, Wednesday and Thursday third period and the control group met of Monday, Wednesday and Thursday fourth period. The enrollment was 38 students in each of the sections. The control group taught by the other instructor winter quarter met on Monday, Wednesday and Thursday first period and the experimental group met on Monday, Wednesday and Friday fifth period. There were 31 students in each of these sections.

Hereafter the fall quarter experimental and control groups will be designated as E-1 and C-1 respectively. E-2 and C-2 designates the investigator's winter quarter
sections and E-3 and C-3 refers to the sections taught by the other instructor.

The investigator has had considerable experience in teaching Mathematics 112. His schedule for the past ten years has normally included at least two sections of Mathematics 112 each year.

The other teacher has taught one or two sections of Mathematics 112 each quarter during the past three years since joining the faculty at Winona State College and volunteered to participate in the study. This offer was accepted since it would give greater depth to the study and the other teacher was known to be a very superior teacher with a general interest in improving mathematics instruction and a specific interest in developing effective methods of teaching mathematics to the non-mathematics major.

Procedure

The first day of class was used for orientation and for beginning the instruction. The text for the course was introduced and the students were informed that the material in the text would form the basis for the course. The content of the course was indicated by listing the titles of the six chapters to be covered in the course. The chapter titles for this mathematical content were "Sets of Numbers," "Sentences in One Variable," "Sentences in Two Variables,"
"Mathematical Structures," "Probability and Statistics," and "Functions and Relations." It was pointed out that three chapters would be omitted. The sections to be omitted from the listed chapters were also pointed out.

The class was told that the learning activities would include lectures by the instructor to introduce the topics of the course, reading assignments from the text over these topics, discussion of the reading assignments from the book, weekly exercises to be handed in for checking, two one hour examinations and a two hour final examination. The contribution of each of these factors to the final grade was as follows:

1. Two one hour examinations - 200 points
2. Ten exercises - 100 points
3. Final examination - 200 points.

They were told that the first exercise would be a survey of their knowledge of the material covered in the course and that this exercise would be completed in class and handed in. They were informed that the other nine exercises would be handed out on a weekly basis and would be returned to the instructor for checking.

A biographical questionnaire was also distributed and completed during the first day (see Appendix D). The biographical questionnaire was a 10 item questionnaire and was used to obtain some background information from the subjects.
Questions number nine and 10 dealt with the number of years of mathematics taken in grades nine through 12 and this information was used in the analysis of data.

The introduction to counting numbers and whole numbers was also included in the first day's work.

The second period was largely devoted to administering the pretest (see Appendix E). The pretest was formed by selecting 40 of the 100 items making up the post-test. Since each of the exercises had been constructed to cover the work of approximately three class periods the 40 items were distributed about equally among the contents of the nine exercises. Four of the items were allotted to the content of each of the first five exercises and five of the items were allotted to the content of each of the last four exercises. The number of correct responses was taken as the raw score for the pretest and the raw score was used in the analysis.

Exercise 1 was distributed during the second period and the procedure for handling it was also described during this period.

The instructors of the six sections adhered to the following procedure. Three class periods were spent in covering the material included in each exercise. The class time in both the control and experimental sections was spent in covering the same material. The material covered by
the teachers was correlated through the strict adherence to the prepared outline and by weekly conferences. Each exercise was distributed on the first of the three days devoted to the material covered by that exercise and the students were told that the assignment was due one week later. The difference in the treatment given the control sections and the treatment given the experimental sections was restricted to the manner in which the exercises were handled.

In the control sections each exercise was handed out and the students were told that the results were to be handed in one week later. The results were submitted on a separate answer sheet with the students retaining the set of questions. The replies were checked by the instructor and the papers were handed back the next period. The instructor answered any questions raised by the students when the papers were returned. No analysis of the replies was made and hence any reteaching was as a result of the students questions.

In the experimental sections the students were told that they could turn in each exercise as soon as they had completed it but that they must turn it in by one week after it was handed out. The results were submitted by handing in a hand punched IBM card. The IBM cards were collected at the beginning of each class period and the computer
corrected results were returned to the students at the close of the period. The print-out indicated whether each reply was correct or incorrect and if the reply was incorrect the student was referred to a page in the textbook for further reading and also assigned a substitute question to be answered. This procedure provided the students with an early evaluation of their mastery of the material and also provided them with information useful in overcoming any difficulty they may have encountered. The students received a set of substitute questions with the print-out of the analysis of his original replies. He then submitted a second IBM card at the beginning of the next period and the results were again available at the end of the class period. The print-out for the second card indicated whether the reply was correct or incorrect and if incorrect instructed the student to see the instructor for help in clarifying the concept. This procedure encouraged the student who really needed help to seek out the instructor and gave the instructor the opportunity to seek the basis of the lack of understanding of the concept.

After each due date the computer furnished a summary of the students replies. This summary was used to provide a record of each student's work on the exercise and also to determine which concepts needed to be retaught during the following period. The procedure for reteaching the
concepts was carefully planned and carried out at the beginning of the next class period. Students questions on the exercise were answered at that time. The practice of having the computer correct the students replied not only relieved the teacher of the task of correcting the exercises and recording the results but also provided the results in a form that revealed the concepts that needed reteaching.

Two 50 minute tests were given during the quarter. The first test was over the work covered by the first three exercises and the second test was over the work covered by the next three exercises. The work covered by the last three exercises was not tested prior to the final examination.

The post-test was administered to all classes as their final examination. The testing days were on the days designated by the final examination schedule. The students were allowed two hours to answer the question. The test packet consisted of an answer sheet, a blank sheet of paper stapled to the answer sheet and the test questions.

Prior to handing out the test the students were told to clear their desk of everything except their pencils or pens. Prior to taking the test the students were instructed to answer all of the questions, to guess if they were not sure of the correct response, and to ask no questions.
during the examination period. They were instructed to turn their answer sheet over when finished and to leave all the material supplied on their desk when they left the examination room.

Evaluation of Post-Test

The post-test developed was a 100 item multiple choice test. Each of the items had three distractors in addition to the correct response. The number of questions devoted to each unit was in proportion to the class time spent on them (see Appendix F). The number of correct responses was taken as the raw score and the raw score was used in the analysis as the criterion measure.

The significance of research depends on the effectiveness of the measuring instruments. In order to make a sound judgment concerning the results of any research we must be able to evaluate the measures used.

There are several characteristics that are important in determining the effectiveness of measuring instruments in education. These characteristics are objectivity, validity and reliability.

Objectivity, as it refers to measurement in education, is determined largely by the degree to which the measure is uninfluenced or undistorted by the beliefs or prejudices of the individual using the instrument. Objectivity is an extremely important factor in evaluating the effectiveness
of measuring instruments in education. The decision to use a post-test consisting of 100 multiple choice items assured a high degree of objectivity.

Validity is generally defined as the degree to which a test measures what it claims to measure. Consideration of validity was restricted to content validity. (Content validity is the degree to which the sample of test items represents the content that the test is designed to measure.) It was felt that a local developed achievement test would have a higher content validity than a standardized test used nationally. It was decided that the concepts covered in the course should be emphasized in proportion to the class time spent on them. The exercises served as the basis for this measurement since each exercise had been constructed to cover the work of approximately three class periods.

Eleven test questions were allotted to the content of the first eight exercises and twelve test questions were allotted to the content of the ninth exercise. Within a particular exercise the questions were formed so that all of the major concepts were covered and as many of the specific concepts as possible.

To determine the merit of any test item, test results must be subjected to an item analysis. An item analysis was performed using the results from the fall quarter control and experimental groups. As a result of this item
analysis, three kinds of information were obtained concerning each item: (1) the difficulty of the item, (2) the discrimination index of the item, and (3) the effectiveness of the distractors. The first of these, the difficulty of the item, is the proportion of individuals who answer the item correctly. The second, the discrimination index, is a measure of how well the item separates two groups. The purpose of most tests is the "spread out" the individuals taking it. The item which separates good students from poor ones is said to discriminate. The third result, the effectiveness of the distractors, determines how the incorrect responses in the multiple-choice item are working.

The difficulty of an item (p) is the proportion of individuals who answer an item correctly. Since the post-test was a power test, that is, a test in which everyone had a chance to read every item, the calculation of the item difficulty offered no problem.

There are two general ways of demonstrating item discrimination: (1) a test of the significance between two proportions, and (2) correlation techniques. The correlation approach to item analysis was used. In the correlation approach to item analysis, a correlation coefficient is computed that shows the relationship of the response to the total test score. In other words, it indicates how well the item is doing what the test itself is doing.
To demonstrate item discrimination the biserial $r$ was used to show the relationship of the response to the total score.

The formula for biserial $r$ is: $r_b = \frac{\bar{X}_p - \bar{X}_t}{S_t \sqrt{p(1-p)}}$

where $\bar{X}_p$ = the mean score of those answering the item correctly.
$\bar{X}_t$ = the mean of the total test scores
$S_t$ = the standard deviation of the test
$p$ = the proportion of the total group answering the item correctly
$y$ is the ordinate cutting off an area equal to $p$ above it on the probability table.

Table 32 Appendix F lists each item number with the corresponding difficulty level and discrimination index.

The method used in studying the distractors was to obtain the mean score of the individuals who responded to each of the distractors as well as to the correct answer. An analysis of the distractors was made to determine:

1. those distractors which were selected by no one, and
2. those distractors which were selected by individuals with a mean score higher than the mean score of those who selected the correct answer. Distractors used by no one contributes nothing to the test and should be revised in an attempt to make them useful. When a distractor is found which is selected more frequently by the good student than the poor student the test maker may revise the distractor
to make it less appealing to the better student or he may conclude that the concept has been poorly taught.

As a result of studying the distractor analysis data sheet as presented in the Table 33 Appendix F it was determined that the items numbered 5, 7, 8, 9, 21, 22, 23, 24, 29, 79, 83 should be revised. The revised test was used with the groups during the winter quarter.

Reliability, as applied to educational measurement may be defined as the level of consistency of the measuring device. In general, this consistency reflects the degree to which the test may be considered stable or may be depended upon to yield similar test results under similar circumstances. Reliability is expressed as a coefficient and can be obtained by several different approaches. Reliability is an extremely important characteristic of measurement devices and must be considered carefully in selecting measures for research purposes. The lower the reliability of the measuring instrument, the greater will be the chance fluctuations we can expect in the scores of our subjects.

The method of rational equivalence was the technique selected for calculating reliability. The method of rational equivalence is the only widely used technique for calculating reliability that does not require the calculation of a correlation coefficient. This method gets at the
internal consistency of the test through an analysis of the individual test items.

Kuder-Richardson formula number 20 was used to compute the reliability (17, p. 220). The Kuder-Richardson formula is easily applied to the data obtained by an item analysis.

The formula for Kuder-Richardson number 20 is:

$$\rho_{tt} = \frac{k}{k-1} \left( 1 - \frac{\sum pq}{S^2_t} \right)$$

where

- $k$ = number of items on the test
- $S^2_t$ = the variance of the test
- $p$ = the proportion of those responding to an item who answer it correctly
- $q = 1 - p$

The reliability coefficient was computed using the item analysis results from the fall quarter control and experimental groups. The analysis yielded a reliability coefficient of .93.

An item analysis was performed using the results from the winter quarter control and experimental group. The difficulty level and discrimination index is presented for each item in Table34 Appendix F. The results of the analysis of distractors is presented in Table35 Appendix F.

The reliability coefficient was again computed using the item analysis results from the winter quarter control and experimental groups. The analysis yielded a reliability coefficient of .92.
FINDINGS

The experiment was conducted using six sections of mathematics 112 during the fall and winter quarters of the 1968-69 school year. In the three control sections the exercises were handled by the instructor and in the three experimental sections the exercises were processed by the computer. Of the 220 students originally enrolled in these sections 207 completed the course. Four measures were recorded for each student completing the course (see Appendix G Table 36). The number of years of high school mathematics, the pretest score, the ACT composite score, and the post-test score were the four measures recorded.

Analysis of Pre-Study Data

It is the investigator's job to assure himself of the original comparability of the treatment groups in any study; hence any condition or variables which enter into the experimental setting and which are not under investigation should be controlled by the investigator. Randomization is one means by which these extraneous variables may be controlled in an effort to eliminate any biases by distributing these variables at random within the treatment groups. However, if the investigator identifies any variables which might produce results not necessarily attributable to the treatments, he should ascertain whether
the treatment groups are comparable with regard to these variables.

Two factors, in addition to the treatment variable, which could possibly effect the criterion measure were considered in the present study. These were the academic ability of the students and the previous knowledge of the particular topics covered in the course. The ACT composite score provided a measure of the academic ability of the student and the pretest provided a measure of the previous knowledge of the material of the course.

Table 1 gives the means and variances for each section on these two prestudy measures. E-1 designates the experimental section taught fall quarter and C-1 designates the control section taught fall quarter. The other sections were taught winter quarter with the investigator teaching E-2 and C-2.

Table 1

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Number in Treatment group</th>
<th>Composite ACT Scores Mean</th>
<th>Composite ACT Scores Variance</th>
<th>Pretest Mean</th>
<th>Pretest Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td>34</td>
<td>21.000</td>
<td>15.272</td>
<td>16.941</td>
<td>13.632</td>
</tr>
<tr>
<td>E-3</td>
<td>31</td>
<td>19.258</td>
<td>9.664</td>
<td>16.225</td>
<td>15.780</td>
</tr>
<tr>
<td>C-3</td>
<td>31</td>
<td>19.129</td>
<td>15.649</td>
<td>16.510</td>
<td>12.991</td>
</tr>
</tbody>
</table>

Two single classification analysis of variance were
used to test the hypotheses:

1. The mean ACT scores for each of the six treatment groups are equal.

2. The mean pretest scores for each of the six treatment groups are equal.

A major assumption of analysis of variance is that the variances within the subgroups are homogeneous. The assumption of subgroup homogeneity was tested by dividing the smallest variance into the largest variance as suggested by Popham (18, p. 181). The resulting F ratios were 1.62 for the ACT scores and 1.21 for the pretest scores. The tabulated values (19, p. 248) were 1.84 at the 0.05 level and 2.38 at the 0.01 level. Since the calculated values were below the five and one percent values homogeneous variances for each measure was concluded.

Table 2 is the analysis of variance for the ACT scores and Table 3 is the analysis of variance for the pretest scores. Note that $F(5,201) = \frac{0.05(4.38)}{0.01(9.07)}$ is used to indicate that the tabulated F value for 5 and 201 degrees of freedom is 4.38 at the 0.05 level and 9.07 at the 0.01 level.

Since the calculated values were below the five and one percent table values it was assumed that the six treatment groups were comparable with regard to ACT scores and pretest scores.
Table 2

Analysis of Variance of Composite ACT Scores for the Six Sections

<table>
<thead>
<tr>
<th>Source</th>
<th>Degree of freedom</th>
<th>Sum of squares</th>
<th>Mean square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>5</td>
<td>97.033</td>
<td>19.406</td>
<td>1.495</td>
</tr>
<tr>
<td>Within</td>
<td>201.</td>
<td>2607.996</td>
<td>12.975</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>206</td>
<td>2705.029</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ F(5,201) = \frac{0.05(4.38)}{0.01(9.07)} \]

Table 3

Analysis of Variance of Pretest Scores for the Six Sections

<table>
<thead>
<tr>
<th>Source</th>
<th>Degree of freedom</th>
<th>Sum of squares</th>
<th>Mean square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>5</td>
<td>11.819</td>
<td>2.363</td>
<td>0.169</td>
</tr>
<tr>
<td>Within</td>
<td>201.</td>
<td>2809.930</td>
<td>13.975</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>206</td>
<td>2820.850</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Two single classification analysis of variance were used to test the hypotheses:

1. The mean ACT scores of the pooled experimental group and pooled control group are equal.

2. The mean pretest scores of the pooled experimental group and pooled control group are equal.

Table 4 is the mean and variance of the pooled groups on these two prestudy measures.

The ratio of the variance of the experimental group to the variance of the control group on the ACT scores was 1.16 and the ratio of the variance of the experimental
group to the variance of the control group of the pretest was 1.08. The test of significance of these ratios at the five and one percent levels resulted in the conclusion of homogeneous variance for each measure.

Table 4
Means and Variances of ACT Scores and Pretest Scores for Pooled Groups

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Number in Group</th>
<th>Composite ACT Scores</th>
<th>Pretest</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Variance</td>
<td>Mean</td>
<td>Variance</td>
</tr>
<tr>
<td>Experiment</td>
<td>103</td>
<td>19.951</td>
<td>16.533</td>
<td>14.249</td>
</tr>
<tr>
<td>Control</td>
<td>104</td>
<td>19.461</td>
<td>16.682</td>
<td>13.267</td>
</tr>
</tbody>
</table>

Table 5 is the analysis of variance for the ACT scores and Table 6 is the analysis of variance for the pretest scores.

Table 5
Analysis of Variance of ACT Scores for the Pooled Groups

<table>
<thead>
<tr>
<th>Source</th>
<th>Degree of Freedom</th>
<th>Sum of squares</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>1.</td>
<td>12.425</td>
<td>12.325</td>
<td>0.945</td>
</tr>
<tr>
<td>Within</td>
<td>205.</td>
<td>2692.604</td>
<td>13.134</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>206.</td>
<td>2705.029</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
F(1,205) = \frac{0.05(3.89)}{0.01(6.76)}
\]

Since the calculated values were below the five and one percent values it was assumed that the pooled groups were comparable with regard to ACT scores and pretest scores.
Table 6
Analysis of Variance of Pretest Scores for the Pooled Groups

<table>
<thead>
<tr>
<th>Source</th>
<th>Degree of freedom</th>
<th>Sum of squares</th>
<th>Mean square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>1</td>
<td>0.865</td>
<td>0.865</td>
<td>0.062</td>
</tr>
<tr>
<td>Within</td>
<td>205.</td>
<td>2819.985</td>
<td>13.756</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>206.</td>
<td>2820.850</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ F(1,205) = \frac{0.05(3.89)}{0.01(6.76)} \]

Analysis of Post-Study Data

The experiment is the ultimate form of research design. The design employed in this experiment was a form of the classic single-variable design. All single-variable experiments involve the manipulation of one variable followed by observing the effects of this manipulation on a second variable. The variable manipulated in this experiment was the experimental treatments. The variable that was measured to determine the effect of the experimental treatment was the achievement on the post-test.

In single-variable experiments, different groups are exposed to different experimental treatments while an attempt is made to hold constant all variables except the experimental treatments.

The design used involved an experimental group and a control group both given a pretest and a post-test, but
in which the control group and the experimental group do not have pre-experimental sampling equivalence. Campbell (20, p. 47) calls this design, "The Nonequivalent Control Group Design." The control group made it possible to measure the effect of external factors upon the post-test of the dependent variable. Thus, only the post-test change of the experimental group that is over and above the change that occurred in the control group can be attributed to the experimental treatment.

It was necessary to replicate the basic design in order to obtain a sufficient number of cases that had received each treatment. The three control sections and the three experimental sections were pooled to form a single control group and a single experimental group. This pooling was justified on the basis of the analysis of prestudy data.

The analysis of data was carried out using covariance analysis (18, p. 221). This technique, an extension of the analysis of variance model combined with certain features of regression analysis, provides a useful statistical device for educational investigators. The most useful application of this analysis is in experiments where the investigator must deal with classroom groups that are already established. In effect the covariance analysis reduces the effects of initial group differences statistically by making compensating adjustments to the final means on the dependent variable.
Two variables were statistically adjusted so that they did not confound the analysis of the independent-dependent variable relationship. These variables were the academic ability as measured by the composite ACT score and the previous knowledge of the material covered in the course as measured by the pretest. These variables are referred to as control variables.

The first analysis of the data was made through the use of single classification analysis of covariance. This analysis was used to compare the two treatments and to test the null hypothesis: There is no difference between the mean achievement of the experimental group and the control group.

The first step in the analysis was to determine the sums and means of the criterion and control variables as presented in Table 7.

<table>
<thead>
<tr>
<th>Instruction Method</th>
<th>Criterion</th>
<th>Controls</th>
<th>Pretest Achievement</th>
<th>ACT Composite Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n</td>
<td>(\Sigma Y)</td>
<td>(\overline{Y})</td>
<td>(\Sigma X_1)</td>
</tr>
<tr>
<td>Experimental</td>
<td>103</td>
<td>7999</td>
<td>77.660</td>
<td>1705</td>
</tr>
<tr>
<td>Control</td>
<td>104</td>
<td>7517</td>
<td>72.278</td>
<td>1735</td>
</tr>
<tr>
<td>Total</td>
<td>207</td>
<td>15516</td>
<td>74.956</td>
<td>3440</td>
</tr>
</tbody>
</table>

The next step in the analysis was to compute the sums of squares for the raw scores and the various crossproducts.
These quantities are summarized in Table 8.

Table 8

Summary of Sums of Squares for the Raw Scores and Crossproducts of 207 Students

<table>
<thead>
<tr>
<th>Measure</th>
<th>Symbols</th>
<th>Total for Entire Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criterion achievement</td>
<td>( \Sigma Y^2 )</td>
<td>1208152</td>
</tr>
<tr>
<td>Pretest achievement</td>
<td>( \Sigma X_1^2 )</td>
<td>59988</td>
</tr>
<tr>
<td>ACT composite Score</td>
<td>( \Sigma X_2^2 )</td>
<td>83083</td>
</tr>
<tr>
<td>Crossproducts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pretest X criterion</td>
<td>( \Sigma X_1 Y )</td>
<td>263694</td>
</tr>
<tr>
<td>ACT X criterion</td>
<td>( \Sigma X_2 Y )</td>
<td>311529</td>
</tr>
<tr>
<td>Pretest X ACT</td>
<td>( \Sigma X_1 X_2 )</td>
<td>69133</td>
</tr>
</tbody>
</table>

The data from Table 7 and Table 8 were used to compute, in deviation form, the various sums of squares and cross-products associated with:

1. The total variation in the sample.
2. The amount of variation within the two subgroups.

These values are computed as follows:

\[
\begin{align*}
\sum x_1 y_{\text{total}} &= \frac{(\sum x_1)(\sum y)}{n} \\
\sum x_1 y_{\text{within}} &= \frac{(\sum x_1^c)(\sum y_c)}{n_c} + \frac{(\sum x_1^e)(\sum y_e)}{n_e}
\end{align*}
\]

Table 9 is the resulting sums of squares and Table 10 is the crossproducts.
Table 9
Deviation Value for Sum of Squares

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>$\sum y^2$</th>
<th>$\sum x_1^2$</th>
<th>$\sum x_2^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Within</td>
<td>43628.054</td>
<td>2819.986</td>
<td>2692.604</td>
</tr>
<tr>
<td>Total</td>
<td>45126.695</td>
<td>2820.850</td>
<td>2705.029</td>
</tr>
</tbody>
</table>

Table 10
Deviation Values of Crossproducts

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>$\sum x_1 y$</th>
<th>$\sum x_2 y$</th>
<th>$\sum x_1 x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Within</td>
<td>5879.618</td>
<td>5644.928</td>
<td>1350.004</td>
</tr>
<tr>
<td>Total</td>
<td>5843.566</td>
<td>5781.368</td>
<td>1346.719</td>
</tr>
</tbody>
</table>

The next step was to compute the regression coefficients. The two linear equations needed to find the values of $b_1$ and $b_2$ are the following:

$$\sum x_1 y = b_1 \sum x_1^2 + b_2 \sum x_1 x_2$$

$$\sum x_2 y = b_1 \sum x_1 x_2 + b_2 \sum x_2^2$$

These two equations were solved simultaneously, first with the total sums of squares and crossproducts, then with the within sums of squares and crossproducts.

The values of $b_1$ and $b_2$ for both total and within are given in Table 11.

The sum of squares of residuals for both total and within were computed using the equation:
Sum of Squares of Residuals $= \sum y^2 - (b_1 \sum x_1 y + b_2 \sum x_2 y)$

Table 11
Regression Coefficients

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$b_1$</th>
<th>$b_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Within</td>
<td>1.42287</td>
<td>1.38306</td>
</tr>
<tr>
<td>Total</td>
<td>1.37895</td>
<td>1.45074</td>
</tr>
</tbody>
</table>

In computing the total residual sum of squares, the values for the equations are the total regression coefficients and sums. The within residual sum of squares is found by using within values in the equation. The between sum of squares residual was computed by subtracting the within sum of squares of residuals from the total sum of squares residuals.

The degrees of freedom were calculated by the following scheme:

- **Source of variation** | **Degrees of freedom**
- Total                  | $df = n - (1 + \text{number of control variables})$
- Between                | $df = \text{number of groups} - 1$
- Within                 | $df = df \text{ for total} - df \text{ for between}$

The mean square residuals were computed for between and within by dividing the corresponding sum of square residuals by its degrees of freedom.

The F value was obtained by dividing the between residual mean square by the within residual mean square. The analysis
of covariance is summarized in Table 12.

Table 12
Single Classification Analysis of Covariance for Instructional Method

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Residuals</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Sum of squares</td>
<td>Mean square</td>
</tr>
<tr>
<td>Between</td>
<td>1</td>
<td>1226.586</td>
<td>1226.586</td>
</tr>
<tr>
<td>Within</td>
<td>203</td>
<td>27454.816</td>
<td>135.245</td>
</tr>
<tr>
<td>Total</td>
<td>204</td>
<td>28681.402</td>
<td></td>
</tr>
</tbody>
</table>

\[ F(1, 203) = \frac{0.05(3.89)}{0.01(6.75)} \]

The F test showed that the value of 9.07 significant at the 0.01 level. Note that (*) indicates significance at the 0.05 level and (**) indicates significance at the 0.01 level in all tables.

On the basis of this F test the null hypothesis was rejected and it was concluded that there was a significant difference between the experimental group and the control group.

A research project can contribute additional worthwhile knowledge if the major groups are divided into subgroups and further comparisons made. Such analysis often provides information useful in developing a theoretical framework that would fit the results but can only be carried out if the original groups are large enough so that, after such divisions are made, the subgroups still have
sufficient numbers of cases to permit a statistical analysis.

Four additional comparisons of the two treatments were made through the use of single classification analysis of covariance on subgroups of the original group. The four subgroups were the upper background level subgroup, the lower background level subgroup, the upper ability level subgroup, the lower ability level subgroup.

The upper background level subgroup was made up of those students with three or four years of high school mathematics and the lower background level subgroup was made up of those students with one or two years of high school mathematics.

The upper ability level subgroup was made up of those students whose composite ACT score was twenty or above and the lower ability level subgroup was made up of students with composite ACT scores below twenty.

The following four null hypothesis were tested by these comparisons:

1. There is no difference between the mean achievement of the upper ability level subgroups, of the experimental group and the control group.

2. There is no difference between the mean achievement of the lower ability level subgroups of the experimental group and the control group.

3. There is no difference between the mean achievement of the upper background level subgroups, of the experimental group and the control group.

4. There is no difference between the mean achievement of the lower background level subgroups, of the experimental group and the control group.
The steps in the analysis of covariance for the four subgroups were similar to the steps in the first analysis. Table 13 is the sums and means for the criterion variable and the control variables for the experimental group and control group of the upper ability level subgroup. Table 14 is the analysis of covariance carried out on the upper ability level subgroup. Table 15 is the sums and means of the lower ability level subgroup and Table 16 is the analysis of covariance that was carried out on the lower ability level subgroup.

Table 13

Sums and Means for the Criterion Variable and the Control Variable by Instructional Method on the Upper Ability Level Subgroup

<table>
<thead>
<tr>
<th>Instruction Method</th>
<th>Criterion Achievement</th>
<th>Controls Achievement</th>
<th>ACT Composite Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n</td>
<td>$\sum Y$</td>
<td>$\bar{Y}$</td>
</tr>
<tr>
<td>Experimental</td>
<td>56</td>
<td>4763</td>
<td>85.053</td>
</tr>
<tr>
<td>Control</td>
<td>51</td>
<td>3902</td>
<td>76.509</td>
</tr>
<tr>
<td>Total</td>
<td>107</td>
<td>8665</td>
<td>80.981</td>
</tr>
</tbody>
</table>

Table 14

Single Classification Analysis of Covariance for Instructional Method on the Upper Ability Level Subgroup

<table>
<thead>
<tr>
<th>Source</th>
<th>Degrees of freedom</th>
<th>Sum of squares</th>
<th>Mean square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>1</td>
<td>1467.270</td>
<td>1467.270</td>
<td>13.44**</td>
</tr>
<tr>
<td>Within</td>
<td>103</td>
<td>11244.240</td>
<td>109.167</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>104</td>
<td>12711.510</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ F(1,103) = \frac{0.05(3.94)}{0.01(6.90)} \]

Table 15

Sums and Means for the Criterion Variable and the Control Variable by Instructional Method on the Lower Ability Level Subgroup

<table>
<thead>
<tr>
<th>Instruction Method</th>
<th>Criterion</th>
<th>Controls</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pretest Achievement</td>
<td>ACT Composite Scores</td>
</tr>
<tr>
<td></td>
<td>( \bar{X}_1 )</td>
<td>( \bar{X}_2 )</td>
</tr>
<tr>
<td>Experimental</td>
<td>57 3236 68.851 682 14.510 774 16.468</td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td>53 3615 68.207 821 15.490 889 16.773</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>110 6851 68.510 1503 15.030 1663 16.630</td>
<td></td>
</tr>
</tbody>
</table>

Table 16

Single Classification Analysis of Covariance for Instructional Method on the Lower Ability Level Subgroup

<table>
<thead>
<tr>
<th>Source</th>
<th>Degrees of freedom</th>
<th>Sum of squares</th>
<th>Mean square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>1</td>
<td>190.436</td>
<td>190.436</td>
<td>1.17</td>
</tr>
<tr>
<td>Within</td>
<td>96</td>
<td>15576.842</td>
<td>162.257</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>97</td>
<td>15767.277</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ F(1,96) = \frac{0.05(3.94)}{0.01(6.91)} \]

In Table 17 is presented the sums and means for the criterion variable and the control variables for the experimental group and control group of the upper background level subgroup. Table 18 is the analysis of covariance carried out on the upper background level subgroup. The
corresponding sums and means and analysis of covariance for the lower background level subgroup is presented in Table 19 and Table 20.

Table 17

Sums and Means for the Criterion Variable and the Control Variable by Instructional Method on the Upper Background Level Subgroup

<table>
<thead>
<tr>
<th>Instruction Method</th>
<th>Criterion Achievement</th>
<th>Controls Pretest Achievement</th>
<th>ACT Composite Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n  ( \sum Y ) ( \bar{Y} ) ( \sum X_1 ) ( \bar{X}_1 ) ( \sum X_2 ) ( \bar{X}_2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental</td>
<td>57  4825  84.649  1024  17.964  1202  21.087</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td>47  3534  75.191  841  17.893  971  20.659</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>104 8359  80.375  1865  17.932 2173  20.894</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 18

Single Classification Analysis of Covariance for Instructional Method on the Upper Background Level Subgroup

<table>
<thead>
<tr>
<th>Source</th>
<th>Degrees of freedom</th>
<th>Residuals Sum of squares</th>
<th>Mean square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>1</td>
<td>2012.379</td>
<td>2012.379</td>
<td>14.96**</td>
</tr>
<tr>
<td>Within</td>
<td>100</td>
<td>13455.381</td>
<td>134.554</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>101</td>
<td>15467.760</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ F(1,100) = \frac{0.05(3.94)}{0.01(6.90)} \]

The F test, for the upper ability level subgroup, showed that the value of 13.441 was significant at the 0.01 level. On the basis of the F test the null hypothesis was rejected and it was concluded that there was a significant difference
between the mean achievement of the upper ability level subgroups, of the experimental group and the control group. However, the F test for the lower ability subgroup, showed that the value of 1.17 was not significant at the 0.05 level. On the basis of the F test there was insufficient evidence to reject the null hypothesis.

Table 19

<table>
<thead>
<tr>
<th>Instruction Method</th>
<th>Criterion Variable</th>
<th>Control Variable</th>
<th>ACT Composite Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pretest Achievement</td>
<td>Achievement</td>
<td></td>
</tr>
<tr>
<td></td>
<td>n</td>
<td>$\Sigma Y$</td>
<td>$\bar{Y}$</td>
</tr>
<tr>
<td>Experimental</td>
<td>46</td>
<td>3174</td>
<td>09.000</td>
</tr>
<tr>
<td>Control</td>
<td>57</td>
<td>3983</td>
<td>09.877</td>
</tr>
<tr>
<td>Total</td>
<td>103</td>
<td>7157</td>
<td>09.485</td>
</tr>
</tbody>
</table>

Table 20

Single Classification Analysis of Covariance for Instructional Method on the Lower Background Level Subgroup

<table>
<thead>
<tr>
<th>Source</th>
<th>Degrees of freedom</th>
<th>Sum of squares</th>
<th>Mean square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>1</td>
<td>6.650</td>
<td>6.650</td>
<td>0.06</td>
</tr>
<tr>
<td>Within</td>
<td>99</td>
<td>11880.727</td>
<td>120.007</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>11887.377</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The F test, for the upper background level subgroups
showed that the value of 14.96 was significant at the 0.01 level. On the basis of the F test the null hypothesis was rejected and it was concluded that there was a significant difference between the mean achievement of the upper background level subgroups, of the experimental group and the control group. The F test for the lower background level subgroup showed that the value of 0.06 was not significant at the 0.05 level. On the basis of the F test there was insufficient evidence to reject the null hypothesis.

The basic logic of the single classification procedure was extended to a multiple classification scheme, where the relationship between the criterion variable and two independent variables was studied. This provided a test for group differences on the independent or main effect variables, as well as for interaction between the independent variables.

The combination of the computer centered instruction and the upper ability level student might have resulted in learning differences partially due to the ability level of the student, partially due to the method of instruction, and partially due to the interaction between the background level of the student and the method of instruction. These considerations led to the following null hypothesis:

1. There is no difference between the mean achievement of the experimental group and the control group.
2. There is no difference between the mean achievement of the upper ability level subgroup and the mean achievement of the lower ability level subgroup.

3. There is no interaction between ability level and method of instruction.

In order to test the null hypotheses a multiple classification analysis of covariance was carried out in which the independent variables were methods of instruction and ability levels. The dependent variable was the post-test achievement; and the control variables were pretest mathematics achievement and academic ability.

The first step was to compute the sums and means of the four subgroups. These values are presented in Table 21.

Table 21
Sums and Means of Experimental Group and Control Group Classified According to Ability Level

<table>
<thead>
<tr>
<th>Group</th>
<th>Criterion Achievement</th>
<th>Pretest Achievement</th>
<th>Control ACT Composite Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n</td>
<td>Y</td>
<td>$\sum Y$</td>
</tr>
<tr>
<td>Experimental</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper ability 56</td>
<td>4763</td>
<td>85.053</td>
<td>1023</td>
</tr>
<tr>
<td>Lower ability 47</td>
<td>3236</td>
<td>68.851</td>
<td>682</td>
</tr>
<tr>
<td>Subtotal 103</td>
<td>7999</td>
<td>77.660</td>
<td>1705</td>
</tr>
<tr>
<td>Control</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper ability 51</td>
<td>3902</td>
<td>79.509</td>
<td>914</td>
</tr>
<tr>
<td>Lower ability 53</td>
<td>3615</td>
<td>68.207</td>
<td>821</td>
</tr>
<tr>
<td>Subtotal 104</td>
<td>7517</td>
<td>72.278</td>
<td>1735</td>
</tr>
<tr>
<td>Total 207</td>
<td>15516</td>
<td>74.956</td>
<td>3440</td>
</tr>
</tbody>
</table>
The next step was to compute the sums of squares and crossproducts for the total sample. These results are given in Table 22.

Table 22
Summary of Raw Score Sums of Squares and Crossproducts for Criterion and Control Variable

<table>
<thead>
<tr>
<th>Measure</th>
<th>Symbol</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Post-test</td>
<td>$\sum Y^2$</td>
<td>1208152</td>
</tr>
<tr>
<td>Pretest</td>
<td>$\sum x_1^2$</td>
<td>59988</td>
</tr>
<tr>
<td>ACT Composite</td>
<td>$\sum x_2^2$</td>
<td>83083</td>
</tr>
<tr>
<td>Crossproducts</td>
<td>$\sum x_1y$</td>
<td>263094</td>
</tr>
<tr>
<td></td>
<td>$\sum x_2y$</td>
<td>311529</td>
</tr>
<tr>
<td></td>
<td>$\sum x_1x_2$</td>
<td>69133</td>
</tr>
</tbody>
</table>

The deviation sums of squares and crossproducts were computed for total, instructional method, ability level, interaction, and within.

The computation of deviation crossproducts is illustrated by that between the post-test achievement and pretest scores.

For total:
$$\sum x_1y = \sum x_1y - \frac{(\sum x_1)(\sum y)}{n}$$

For instruction method:
$$\sum x_1y = \frac{(\sum x_{1E})(\sum y_{E})}{n_E} + \frac{(\sum x_{1C})(\sum y_{C})}{n_C} - \frac{(\sum x_1)(\sum y)}{n}$$
For ability levels:
\[
\Sigma x_1y = \frac{(\Sigma x_{1U})(\Sigma Y_U)}{n_U} + \frac{(\Sigma x_{1L})(\Sigma Y_L)}{n_L} - \frac{(\Sigma x_1)(\Sigma Y)}{n}
\]

For interaction:
\[
\Sigma x_1y = \frac{(\Sigma x_{1EU})(\Sigma Y_{EU})}{n_{EU}} + \frac{(\Sigma x_{1EL})(\Sigma Y_{EL})}{n_{EL}}
\]
\[
+ \frac{(\Sigma x_{1CU})(\Sigma Y_{CU})}{n_{CU}} + \frac{(\Sigma x_{1CL})(\Sigma Y_{CL})}{n_{CL}} - \frac{(\Sigma x_1)(\Sigma Y)}{n}
\]

For within:
\[
\Sigma x_1y = \Sigma x_1y - \left[ \frac{(\Sigma x_{1EU})(\Sigma Y_{EU})}{n_{EU}} + \frac{(\Sigma x_{1EL})(\Sigma Y_{EL})}{n_{EL}}
\right.
\]
\[
+ \frac{(\Sigma x_{1CU})(\Sigma Y_{CU})}{n_{CU}} + \frac{(\Sigma x_{1CL})(\Sigma Y_{CL})}{n_{CL}} \right]
\]

The deviation values for sums of squares are presented in Table 23 and the deviation values for sums of squares of crossproducts are presented in Table 24.

Table 23
Deviation Values for Sums of Squares

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>(\Sigma y^2)</th>
<th>(\Sigma x_1^2)</th>
<th>(\Sigma x_2^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructional method</td>
<td>1498.437</td>
<td>0.858</td>
<td>12.418</td>
</tr>
<tr>
<td>Ability levels</td>
<td>8039.750</td>
<td>488.070</td>
<td>1829.640</td>
</tr>
<tr>
<td>Interaction</td>
<td>460.250</td>
<td>26.261</td>
<td>0.171</td>
</tr>
<tr>
<td>Within</td>
<td>25458.847</td>
<td>1806.926</td>
<td>278.019</td>
</tr>
<tr>
<td>Total</td>
<td>45126.695</td>
<td>2820.850</td>
<td>2705.029</td>
</tr>
</tbody>
</table>

The within value of each sum of squares and crossproduct was added to each main effect and interaction value. The sums of squares combined with within are summarized in
Table 25 and the sums of crossproducts combined with within are summarized in Table 26.

### Table 24

**Deviation Values for Sums of Crossproducts**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>$\sum x_1y$</th>
<th>$\sum x_2y$</th>
<th>$\sum x_1x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructional method</td>
<td>-36.059</td>
<td>136.429</td>
<td>-3.295</td>
</tr>
<tr>
<td>Ability level</td>
<td>1980.875</td>
<td>3625.312</td>
<td>944.976</td>
</tr>
<tr>
<td>Interaction</td>
<td>99.343</td>
<td>0.124</td>
<td>16.484</td>
</tr>
<tr>
<td>Within</td>
<td>1603.406</td>
<td>-568.406</td>
<td>-151.492</td>
</tr>
<tr>
<td>Total</td>
<td>5843.566</td>
<td>5781.368</td>
<td>1346.719</td>
</tr>
</tbody>
</table>

### Table 25

**Deviation Values for Sums of Squares Combined With Within**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>$\sum y^2$</th>
<th>$\sum x_1^2$</th>
<th>$\sum x_2^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructional method</td>
<td>26957.285</td>
<td>1807.794</td>
<td>290.439</td>
</tr>
<tr>
<td>Ability level</td>
<td>33498.601</td>
<td>2294.996</td>
<td>2107.660</td>
</tr>
<tr>
<td>Interaction</td>
<td>25919.067</td>
<td>1833.187</td>
<td>278.191</td>
</tr>
</tbody>
</table>

### Table 26

**Deviation Values for Sums of Crossproducts Combined With Within**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>$\sum x_1y$</th>
<th>$\sum x_2y$</th>
<th>$\sum x_1x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructional method</td>
<td>1567.346</td>
<td>-431.970</td>
<td>-154.787</td>
</tr>
<tr>
<td>Ability level</td>
<td>3584.281</td>
<td>3266.906</td>
<td>793.484</td>
</tr>
<tr>
<td>Interaction</td>
<td>1702.750</td>
<td>-568.281</td>
<td>-135.007</td>
</tr>
</tbody>
</table>

The regression coefficients associated with each control
variable, considered separately for each source of variation was then computed. The equations used were of the form

\[ \sum x_1 y = b_1 \sum x_1^2 + b_2 \sum x_1 x_2 \]
\[ \sum x_2 y = b_1 \sum x_1 x_2 + b_2 \sum x_2^2 \]

The values for within plus method of instruction were used in the above equations to determine the values of \( b_1 \) and \( b_2 \) for this source of variation. The within plus ability level and within plus interaction values were substituted in the equation to obtain the regression coefficients for these two sources of variation. The values for the regression coefficients are summarized in Table 27.

Table 27

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Within plus instruction method</td>
<td>0.7701</td>
<td>-1.07428</td>
</tr>
<tr>
<td>Within plus ability level</td>
<td>1.17938</td>
<td>1.10600</td>
</tr>
<tr>
<td>Within plus interaction</td>
<td>0.80725</td>
<td>-1.65109</td>
</tr>
<tr>
<td>Within</td>
<td>0.75023</td>
<td>-1.63568</td>
</tr>
</tbody>
</table>

The "within plus" residual sum of squares was computed for each of the four sources of variation. The residual sums of squares were called adjusted sums of squares. In each case the following equation was used to obtain the adjusted sums of squares.

Adjusted sum of squares = \( \sum y^2 - (b_1 \sum x_1 y + b_2 \sum x_2 y) \)

The resulting values are given in Table 28.
Table 28

Within Plus Residual Sums of Squares

<table>
<thead>
<tr>
<th>Source</th>
<th>Residual Sum of Square Plus Within</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructional method</td>
<td>25278.500</td>
</tr>
<tr>
<td>Ability level</td>
<td>25658.133</td>
</tr>
<tr>
<td>Interaction</td>
<td>23606.305</td>
</tr>
<tr>
<td>Within</td>
<td>23326.188</td>
</tr>
</tbody>
</table>

The residual sums of squares were obtained by subtracting the within adjusted sum of squares from each of the three other adjusted sum of squares.

The degrees of freedom were calculated by the following scheme:

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>$df = n - (1 + \text{number of control variables})$</td>
</tr>
<tr>
<td>Treatments</td>
<td>$df = \text{number of treatments} - 1$</td>
</tr>
<tr>
<td>Levels</td>
<td>$df = \text{number of levels} - 1$</td>
</tr>
<tr>
<td>Interaction</td>
<td>$df = df \text{ treatment} \times \text{df levels}$</td>
</tr>
<tr>
<td>Within</td>
<td>$df = df \text{ total} - [df \text{ treatment} + \text{df levels} + df \text{ interaction}]$</td>
</tr>
</tbody>
</table>

The mean square residuals were computed by dividing the corresponding sum of square residuals by its degrees of freedom.

The F values for each main effect and for interaction were computed by dividing the corresponding mean square by the within mean square. The multiple classification analysis of covariance is summarized in Table 29.
Table 29

Analysis of Covariance Significance Tests

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Degrees of Freedom</th>
<th>Residuals</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Instruction method</td>
<td>1</td>
<td>1952.313</td>
<td>1952.313</td>
<td>16.82**</td>
<td></td>
</tr>
<tr>
<td>Ability level</td>
<td>1</td>
<td>2331.946</td>
<td>2331.946</td>
<td>20.09**</td>
<td></td>
</tr>
<tr>
<td>Interaction</td>
<td>1</td>
<td>280.117</td>
<td>280.117</td>
<td>2.41</td>
<td></td>
</tr>
<tr>
<td>Within</td>
<td>201</td>
<td>12216.188</td>
<td>116.051</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
F(1,201) = \frac{0.05(3.89)}{0.01(6.76)}
\]

The F test showed that the values 16.82 and 20.09 were significant at the 0.01 level. On the basis of the F test the first two null hypothesis were rejected and it was concluded that there was a highly significant difference between the mean achievement of the experimental group and the control group, and there was a highly significant difference between the mean achievement of the upper ability level subgroups of the experimental group and control group. However, the F test for interaction showed that the value 2.41 was not significant at the 0.05 level. On the basis of the F test there was insufficient evidence to reject the third null hypothesis.

A second multiple classification analysis of covariance was carried out in which the independent variables were methods of instruction and background levels. The dependent variable was the post-test achievement; and the control
variables were pretest mathematics achievement and academic ability. This analysis was used to test the following hypothesis:

1. There is no difference between the mean achievement of the experimental group and the control group.

2. There is no difference between the mean achievement of the upper background level subgroup and the mean achievement of the lower background level subgroup.

3. There is no interaction between background level and method of instruction.

The sums and means of four subgroups are given in Table 30.

<table>
<thead>
<tr>
<th>Table 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sums and Means of Experimental Group and Control Group Classified According to Background Level</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group</th>
<th>Criterion</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Achievement</td>
<td>Pretest ACT Composite</td>
</tr>
<tr>
<td></td>
<td>n  ΣY  Y  Σx₁  x₁  Σx₂  x₂</td>
<td></td>
</tr>
<tr>
<td>Experimental</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper background</td>
<td>57  4825  84.649  1024 17.964  1202 21.087</td>
<td></td>
</tr>
<tr>
<td>Lower background</td>
<td>46  3174  69.000  681 14.804  853 18.543</td>
<td></td>
</tr>
<tr>
<td>Subtotal</td>
<td>103  7999  77.660 1705 16.553 2055 19.951</td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper background</td>
<td>47  3524  75.191  841 17.893  971 20.659</td>
<td></td>
</tr>
<tr>
<td>Lower background</td>
<td>57  3983  69.877  894 15.684 1053 18.473</td>
<td></td>
</tr>
<tr>
<td>Subtotal</td>
<td>104  7517  72.278 1735 16.682 2024 19.461</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>207 15516 74.956 3440 16.618 4079 19.705</td>
<td></td>
</tr>
</tbody>
</table>
The summary of the results of the second multiple classification analysis of covariance is summarized in Table 31.

**Table 31**

**Analysis of Covariance Significance Tests**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Degrees of Freedom</th>
<th>Residuals</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Sum of Squares</td>
<td>Mean Square</td>
<td>F</td>
</tr>
<tr>
<td>Instructional method</td>
<td>1</td>
<td>901.969</td>
<td>901.969</td>
<td>5.36*</td>
</tr>
<tr>
<td>Background level</td>
<td>1</td>
<td>484.922</td>
<td>484.922</td>
<td>2.88</td>
</tr>
<tr>
<td>Interaction</td>
<td>1</td>
<td>756.031</td>
<td>756.031</td>
<td>4.49*</td>
</tr>
<tr>
<td>Within</td>
<td>201</td>
<td>33835.672</td>
<td>168.337</td>
<td></td>
</tr>
</tbody>
</table>

The F test showed that the values 5.36 and 4.49 were significant at the 0.05 level. On the basis of the F test the first and third null hypothesis were rejected and it was concluded that there was a significant difference between the mean achievement of the experimental group and the control group, and there was a significant interaction, between method of instruction and background level. However, the F test for interaction showed that the value of 2.88 was not significant at the 0.05 level. On the basis of the F test there was insufficient evidence to reject the second null hypothesis.
DISCUSSION

Some of the results of the previous chapter will be discussed and related to the findings of previous research.

In this study it was found that the computer centered procedure for handling the exercises was superior to the instructor centered procedure. The superiority held for the entire group as well as for the upper ability level subgroup and the upper background level subgroup. With the lower ability level subgroup and with the lower background level subgroup the two methods were about equally effective.

It is felt that the superiority of the computer centered procedure was largely due to the fact that this approach was more student centered. The student was able to do the exercises when he felt they would be most beneficial and he was provided either with an immediate confirmation of his response to each exercise item or with instructions for restudying the concept presented in that item. The method also provided the teacher with an analysis of the students responses and thus provided for an early reteaching of those concepts which were found to be troublesome.

Using punched cards to present the students replies to the computer made possible the development of a computer centered procedure using only the basic computer equipment. By using the punched card procedure it was possible to handle large groups of students at the same time and at a
very low cost per student. However, this procedure placed in the hands of the students who submitted their exercises early a list of correct answers which could be passed on to other students. This might account for the fact that the computer centered procedure was not more effective with the lower ability level subgroup and the lower background level subgroup. We might expect that students in the upper ability level subgroup or in the upper background level subgroup would take advantage of the learning opportunities offered by the computer centered procedure and this could account for the superiority of the computer centered approach with these students.

Bassler (8) concluded that a method of instruction without emphasis on exercises was as effective as instruction emphasizing exercises and Mason (9) found no significant difference in results obtained by requiring homework or not requiring homework when averaged over all teachers. However, Gasaway (10) found that the use of frequent short tests was an effective teaching technique in algebra classes with freshmen of varying proficiency and Mulligan (11) showed that the retention of algebra content, and algebraic problem solving ability was improved by making use of student constructed assignments. The results of these studies emphasized the fact that while repetition alone is not sufficient to insure learning, when repetition is paired with other variables then beneficial effects may occur.
In developing the computer centered procedure an attempt was made to generate appropriate learning situations coupled to a computer with well-defined educational objectives in view. There is no reason why a computer must stand alone, isolated from texts, laboratories, specially prepared manuals, audio-visual materials, or from the instructor himself. The effective use of the computer must integrate many of these learning devices. The results of the present study would indicate that such an integration is possible.

Weiner (5) found that a procedure which led up to a problematic situation was superior to a procedure which started with an explanation of the general concept and then proceeded to illustrations and exercises. He found the use of the problematic situation to be better with the average and below average student and also with the entire group. The results of Weiner's study indicated that the superior student can handle a higher level of abstraction than the below average student but the results of the present study suggest that the computer centered procedure is effective in forming the generalization.

Studies by Cornish (14) and Lefstad (15) indicated that general education mathematics is and will continue to play an important role in the total mathematics offerings of the colleges and universities. The trend toward large classes in the general education courses requires that methods which will promote effective teaching in large classes be sought
While Kerce (13) did not find any significant difference in the effect of small sections and large sections on each of three methods studied Simmons (12) found a highly significant difference in favor of the students in the small sized classes over students in the large lecture sections. It appears that the loss of teaching effectiveness as the class size is increased is determined by the degree to which the student involvement in the learning process is restricted by the class size. The computer centered procedure investigated in the present study promises to be very helpful in keeping the instruction student centered.

The present study was designed to give particular attention to the possibility of handling some of the drill aspects of mathematics by programmed learning techniques making use of the computer. The availability of computers for instructional use would carry some additional benefits. The instructor could use the equipment and programming assistance to aid in compiling and making out grades, to study and analyze the results. This should not be automatic grading, but a means of permitting the instructor to better analyze, relative to a student's background, why the student missed certain questions. There are many imaginative uses to which the computer could be put in addition to relieving some of the drudgery of the
instructor's paper work.

The **multiple classification analysis of covariance** in which the independent variables were method of instruction and ability level revealed that there was a highly significant difference in achievement between the experimental group and the control group, and a highly significant difference in achievement between the upper ability level subgroup and the lower ability level subgroup, but no significant interaction between ability level and method of instruction. These are the results we would expect since the basis for the ability level classification was the ACT composite score.

The second **multiple classification analysis of covariance** in which the independent variables were method of instruction and background level revealed that there was no significant difference in achievement between the upper background level subgroup and the lower background level subgroup, but there was a significant difference in achievement between the experimental group and the control group, and a significant interaction between method of instruction and background level.

The fact that there was no significant difference between the achievement of the upper background level subgroup and the lower background level subgroup was somewhat surprising. This result is probably due to the fact that the course emphasized fundamental concepts and did not
assume any previous knowledge of the topics covered. The interaction of method of instruction and background level probably reflects the tendency of students to be less concerned with fundamental concepts after they have achieved some degree of manipulative skill in the area.

This study resulted from the belief that computer assisted instruction offers unexplored opportunities for improving mathematics instruction and that the real problem is the generation of appropriate learning situations coupled to a computer with well defined educational objectives and adequate means of evaluation. The purpose of this study was to evaluate the effectiveness of a programmed learning system, in which the computer played a central role, for use in a college general education mathematics course. The results of the analysis indicates that the programmed system is more effective than the more traditional procedure.
SUMMARY

Purpose

The purpose of this study was to evaluate the effectiveness of a programmed learning system, in which the computer played a central role, for use in a college general education mathematics course.

The choice of the computer oriented procedure stemmed from the belief that this method would enhance the learning of the mathematical concepts by providing the students with either an immediate confirmation of his response to each exercise item or provide him with instructions for restudying the concept presented in that item. The method also provided the teacher with an analysis of the students responses and thus provided for an early reteaching of concepts found to be troublesome.

Method of Procedure

The study was conducted with six sections of college students enrolled in Mathematics 112, a general education mathematics course, at Winona State College. The sections were selected from the sections taught fall and winter quarter of the 1968-69 school year. An experimental and a control section were taught fall quarter by the investigator. During the winter quarter the investigator taught an experimental and a control section and an experimental and
control section were also taught by another teacher.

Nine sets of exercises were constructed to cover the material of the course. Each set of exercises consisted of 40 objectives questions and covered the material of about 3 lectures. The nine sets of exercises were duplicated in two different formats. One format was with directions to the student on the method of indicating replies on a separate answer sheet. The second format was with directions to the student on the method of indicating the replies on a hand punched IBM card.

A second group of nine exercises were constructed which paralleled the first group. The second group was duplicated only in the format with directions to the student for indicating the replies on a hand punched IBM card.

The first day of class was a day of orientation and beginning instruction. During this period a biographical questionnaire was filled in by the student. The second period was used to administer a pretest covering the work of the course.

The instructors of the six sections adhered to the following procedures. The class time in both the control and experimental sections was spent in developmental activities. The difference in treatments consisted of the manner in which the exercises were handled.

In the experimental sections the exercises were handed
out and the students were told that they could be turned in as soon as they were complete but that they must be turned in by one week later to receive full credit. The results were submitted by handing in a hand punched IBM card. The IBM cards were collected at the beginning of each class period and the computer corrected results were returned to the students at the close of the period. This print-out indicated whether the reply to each exercise was correct or incorrect and if incorrect referred the student to a page in the textbook for further reading and also assigned a substitute problem to be submitted. A second IBM card was submitted by the student containing the replied to the substitute problem. The print-out for the second card indicated whether the reply was correct or incorrect and if incorrect instructed the student to see the instructor for help in clarifying the concept.

After each due date the computer was asked to furnish a summary of the students replies. This summary was used for determining which concepts needed to be retaught during the following period. Any student questions about the exercise were answered at that time.

In the control section the exercises were handed out and the students were told that the assignment was to be handed in one week later. When the assignments were handed in they were graded and handed back the next class period. Questions about the answers to the assignment were answered
when the assignment was handed back. No analysis of the replies was made and hence any reteaching was as a result of students questions.

A 100 item multiple choice examination was developed to serve as a post-test. The post-test was administered to all classes as their final examination.

Analysis of Data

The three experimental sections were pooled to form the pooled experimental group and the three control sections were pooled to form the pooled control group. The pooling was justified on the basis of an analysis of variance on pretest data for the six sections.

The students ACT composite score was used to form two subgroups of the experimental group and two subgroups of the control group. The students with ACT composite scores of 20 or above formed the upper ability level subgroup and the students with scores below 20 formed the lower ability level subgroup.

Two subgroups of the experimental group and the control group were also formed on the basis of the number of years of high school mathematics. The students with three or four years of high school mathematics formed the upper background level subgroup and students with one or two years of high school mathematics formed the lower background
level subgroup.

Single classification analysis of covariance was used to compare the achievement of: the experimental group and the control group; the upper ability level subgroups of the experimental group and the control group; the lower ability level subgroups of the experimental group and the control group; the upper background level subgroups of the experimental group and the control group; the lower background level subgroups of the experimental group and the control group.

Two multiple classification analysis of covariance were also carried out on the data. In the first analysis the main effects were methods of instruction and ability levels and in the second analysis the main effects were methods of instruction and background levels.

The control variables in each analysis of covariance was the pretest score and the ACT composite score.

Findings

The single classification analysis of covariance resulted in the following findings:

1. There was a highly significant difference between the mean achievement of the experimental group and the control group.

2. There was a highly significant difference between the mean achievement of the upper ability level subgroups of the experimental group and the control group.
3. There was no significant difference between the lower ability level subgroups of the experimental group and the control group.

4. There was a highly significant difference between the mean achievement of the upper background level subgroups of the experimental group and the control group.

5. There was no significant difference between the lower background level subgroups of the experimental group and the control group.

The first multiple classification analysis of covariance resulted in the findings:

1. There was a highly significant difference between the mean achievement of the experimental group and the control group.

2. There was a highly significant difference between the mean achievement of the upper ability level subgroup and the mean achievement of the lower ability level subgroup.

3. There was no significant interaction between ability level and method of instruction.

The second multiple classification analysis of covariance revealed that:

1. There was a significant difference between the mean achievement of the experimental group and the control group.

2. There was no significant difference between the mean achievement of the upper background level subgroup and the mean achievement of the lower background level subgroup.

3. There was a significant interaction between method of instruction and background level.

The major conclusion of the study, at least in this
experiment, was that the computer oriented procedure was superior to the instructor centered procedure as a method of instruction.

Recommendations for Further Study

The real problem in making effective use of the computer for instructional purposes is the generation of appropriate learning situations coupled to a computer. These learning situations must be developed with well-defined educational objectives in mind and adequate means of evaluation available.

In the programmed learning system developed by the investigator, by using the punched card procedure it was possible to handle large groups of students at the same time and at a very low cost per student. It is evident that much more additional research of this nature needs to be attempted at other colleges or in other courses. A comparison of the results of such studies would also be interesting.

Computer connected display devices when used in the classroom can provide the instructor with a means for illustrating mathematical concepts graphically in such a way as to allow for almost instantaneous responses to changes suggest by students. There is a need for the development and evaluation of the appropriate software for the classroom use of computer connected display devices.
It seems that instruction in many manipulative drill aspects of mathematics could be handled by means of programmed learning techniques in which the computer gives directions to students and monitors their responses. This will require the availability of suitable consoles and appropriate software. It is recommended that detailed outlines for such programmed learning systems, including both curriculum content and software, be developed and evaluated.

Computer aided instruction offers many opportunities to those individuals who have the interest and ability to make imaginative and fruitful progress in research and development in computer assisted learning.
LITERATURE CITED


APPENDIX A
Mathematics 112

Exercise 1

I. Indicate whether the following statements are true or false by placing an X in the correct box after the number on the answer sheet corresponding to the question. Place the X in the first box to indicate true and place the X in the second box to indicate false.

1. Every whole number is a counting number.
2. Every whole number is an integer.
3. Every rational number is an integer.
4. Every real number is an integer.
5. Every rational number is a real number.
6. Every irrational number is a real number.
7. \(-2 + -5 = 3\)
8. \(-2 + 5 = 3\)
9. \((-2)(5) = -10\)
10. \((-2)(-5) = -10\)
11. The counting numbers may be considered as the union of the whole numbers and zero.
12. Every integer is either positive or negative.
13. Every real number is either a rational number or an irrational number.
14. The set of whole numbers is closed under subtraction.
15. The set of rational numbers is closed under division.
16. The graph of the set of integers between \(-3\) and 2 contains four points.
17. The set of rational numbers form a field.

18. The set of counting numbers form a group.

19. There is a one-to-one correspondence between the elements of the set of real numbers and the set of points on the number line.

20. There is an integer replacement for n in the statement $2n = 9$.

21. There is a whole number replacement for n in the statement $n + 5 = 2$

22. The set of real numbers is dense.

23. Real numbers that are irrational numbers are represented by nonterminating, nonrepeating decimals.

24. The number 0 is not a rational number.

25. The set of integers is the same as the set of the opposites of the integers.

II. Indicate your answer to the following by placing an X in the correct box after the number on the answer sheet corresponding to the question.

26. The property illustrated by the statement $\ (x + y) + z = x + (y + z)$ is
   a. distributive
   b. associative
   c. commutative
   d. inverse

27. The property illustrated by the statement $\ x(y + z) = xy + xz$ is
   a. distributive
   b. associative
   c. commutative
   d. inverse
28. The property illustrated by the statement \( x + y = y + x \) is
   a. identity
   b. associative
   c. commutative
   d. inverse

29. The property illustrated by the statement \( n + (-n) = (-n) + n = 0 \) is
   a. identity
   b. associative
   c. commutative
   d. inverse

30. The property illustrated by the statement \( \frac{a}{b} \times \frac{b}{a} = 1 \) is
   a. identity
   b. associative
   c. commutative
   d. inverse

31. The property illustrated by the statement \( 1 \times n = n \times 1 = n \) is
   a. identity
   b. associative
   c. commutative
   d. inverse

32. The number set that does not contain a replacement for \( n \) in the sentence \( 5 + n = 2 \) is
   a. whole numbers
   b. integers
   c. rational numbers
   d. real numbers

33. The set of numbers that contains a solution to the equation \( n^2 + 1 = 3 \) is
   a. whole numbers
   b. integers
   c. rational numbers
   d. irrational numbers
34. The set of numbers that contains a solution to the equation $5n = 3$ is
   a. whole numbers
   b. integers
   c. rational numbers
   d. irrational numbers

35. The number $3/4$ can be represented as
   a. a terminating decimal
   b. a repeating decimal
   c. a nonterminating, nonrepeating decimal
   d. none of the above

36. The number $5/12$ can be written as
   a. a terminating decimal
   b. a repeating decimal
   c. a nonterminating, nonrepeating decimal
   d. none of the above

37. The number $\sqrt{3}$ can be written as
   a. a terminating decimal
   b. a repeating decimal
   c. a nonterminating, nonrepeating decimal
   d. none of the above

38. The correct graph of real numbers between 2 and 5 is
   a. 
   b. 
   c. 
   d. 

39. The correct graph for the rational numbers between 2 and 5 is (see question 38 for the replies)

40. The set of negatives of the whole numbers is
   a. the negative integers
   b. the negative rational numbers
   c. the non positive integers
   d. the negative whole numbers
Mathematics 112

Exercise 2

I. Indicate whether the following statements are true or false by placing an X in the correct box after the number on the answer sheet corresponding to the question. Place the X in the first box to indicate true and place the X in the second box to indicate false.

1. A statement of equality must be true in order to be called an equation.

2. An open sentence cannot be classified as true or false.

3. A placeholder for a member of the replacement set is called a variable.

4. \(a > b\) if and only if there is a positive number \(c\) such that \(a + c = b\).

5. When a variable is replaced by a member of the replacement set we always get a true statement.

6. An equation must be true for all members of the replacement set in order to be called an identity.

7. An inequality cannot be classified as an identity.

8. The equation \(x + 2 = 5\) is an identity.

9. The solution set of the equation \(x + 2 = x\) is the empty set.

10. The solution set for \(x + 5 = 2\) is the empty set when the replacement set is the whole numbers.

11. If \(A = \{1, 2, 3\}\) and \(B = \{2, 4, 6\}\) then \(A \cup B = \{2\}\).

12. If \(A = \{1, 2, 3, 4\}\) and \(B = \{2, 4, 6, 8\}\) then \(A \cap B = \{2, 4\}\).

13. The integer solution set of the statement \(x \geq 1\) and \(x + 1 \leq 5\) is \(\{1, 2, 3, 4, 5\}\).

14. The integer solution set of the statement \(x \geq 1\) or \(x + 1 \leq 5\) is \(\{1, 2, 3, 4, 5\}\).
15. \( 7 > 2 \) and \( 2 < 5 \).

16. \(-2 > 2\) or \(2 < 3\).

17. \(2 + 5 \neq 8\) and \(-2 > -1\).

18. \(2 - 5 = 5 - 2\) or \(-3 > -4\).

19. There is only one number whose absolute value is 3, namely \(-3\).

20. The absolute value of any non zero real number is positive.

21. The symbol \(|-3|\) represents the distance of \(-3\) from the origin.

22. The solution set of \(|x| = 3\) consists of the numbers whose distance from the origin is 3.

23. The solution set of \(|x| \leq 3\) consists of the numbers whose distance from the origin is equal to or greater than 3.

24. The solution set of \(|x - 1| = 3\) consists of the numbers whose distance from the number 1 is 3.

25. The solution set of \(|x| = -3\) is the empty set.

II. Indicate your answer to the following by placing an X in the correct box after the number on the answer sheet corresponding to the question.

26. \(3 - 7 < 7 - 3\) is
   a. a true statement of equality.
   b. a false statement of equality.
   c. a true statement of inequality.
   d. a false statement of inequality.

27. \(-23/25 > -24/25\) is
   a. a true statement of equality.
   b. a false statement of equality.
   c. a true statement of inequality.
   d. a false statement of inequality.
28. $13^2 = 5^2 + 12^2$ is
   a. a true statement of equality.
   b. a false statement of equality.
   c. a true statement of inequality.
   d. a false statement of inequality.

29. The truth set of the sentence $x + 2 \neq x$ is
   a. a single number
   b. the empty set
   c. the entire replacement set
   d. none of the above

30. The truth set of the sentence $x \geq x + 3$ is
   a. a single number
   b. the empty set
   c. the entire replacement set
   d. none of the above

31. The truth set of the sentence $x + 1 \leq x + 2$ is
   a. a single number
   b. the empty set
   c. the entire replacement set
   d. none of the above

32. The graph of the solution set of the sentence $x - 2 \geq 7$ is
   a. a point
   b. a half-line
   c. a ray
   d. a line

33. The graph of the solution set of the sentence $x \leq x + 3$ is
   a. a point
   b. a half-line
   c. a ray
   d. a line
34. The graph of the solution set of the sentence \( x - 2 = 7 \) is
   a. a point
   b. a half-line
   c. a ray
   d. a line segment

35. The graph of the solution set of the sentence \( -3 \leq x \leq 0 \) is
   a. a point
   b. a half-line
   c. a ray
   d. a line segment

36. The graph of the solution set of the compound statement \( x > 1 \) or \( x < 5 \) when the replacement set is the real numbers is
   a. two points
   b. a line segment
   c. union of two rays
   d. a line

37. The graph of \( |x| \leq 7 \) is
   a. two points
   b. a line segment
   c. union of two rays
   d. a line

38. The graph of \( |x - 2| = 5 \) is
   a. two points
   b. a line segment
   c. unions of two rays
   d. a line

39. The graph of \( x \geq 1 \) is
   a. two points
   b. a line segment
   c. union of two rays
   d. a line
40. The graph of $x + 3 \leq 1$ is
   a. two points
   b. a line segment
   c. union of two rays
   d. a line
Mathematics 112

Exercise 3

I. Indicate whether the following statements are true or false by placing an X in the correct box after the number on the answer sheet corresponding to the question. Place the X in the first box to indicate true and place the X in the second box to indicate false.

1. If we add the same number to both members of an equation we obtain an equivalent sentence.

2. If $a = b$ and $c = d$, then $a + c = b + d$.

3. If $a = b$ and $c = d$, then $ac = bd$.

4. If $a < b$ and $c = d$, then $a + c < b + d$.

5. If $a < b$ then $ac < bc$ if $c < 0$.

6. If both members of an inequality are multiplied by a negative number, it is necessary to reverse the sense of the inequality to obtain an equivalent sentence.

7. A pair of replacements is needed for an open sentence in two variables before we can determine whether it is true or false for these replacements.

8. The cartesian product of $U \times U$ is the set of all ordered pairs with the first element from $U$ and the second element from $U$.

9. The cartesian product $A \times B$ is the set of all ordered pairs with the first element from $A$ or $B$ and the second element from $A$ or $B$.

10. The graph of the cartesian product $U \times U$ for $U = \{1,2,3,4\}$ consists of 16 points.

II. Consider the steps in the solutions of $2x + 3 = 8$. Select the number on the answer sheet corresponding to the step and place an X in the correct box to indicate your choice from the four possible reasons.

11. \((2x + 3) + -3 = 8 + -3\)
   a. addition, =
   b. addition
   c. associative, +
   d. zero, +

12. \(2x + (3 + -3) = 8 + -3\)
   a. addition, =
   b. addition
   c. associative, +
   d. zero, +

13. \(2x + 0 = 5\)
   a. addition, =
   b. addition
   c. associative, +
   d. zero, +

14. \(2x = 5\)
   a. addition, =
   b. addition
   c. associative, +
   d. zero, +

15. \(\frac{1}{2}(2x) = \frac{1}{2} \cdot 5\)
   a. multiplication, =
   b. multiplication
   c. associative, x
   d. one, x

16. \((\frac{1}{2} \cdot 2)x = \frac{1}{2} \cdot 5\)
   a. multiplication, =
   b. multiplication
   c. associative, x
   d. one, x

17. \(1x = 5/2\)
   a. multiplication, =
   b. multiplication
   c. associative, x
   d. one, x
18. \( x = \frac{5}{2} \)
   a. multiplication, =
   b. multiplication
   c. associative, \( \times \)
   d. one, \( x \)

III. Consider the steps in the solution of \( 2x + 3 > 8 \).
Select the number on the answer sheet corresponding to the step and place an X in the correct box to indicate your choice from the four possible reasons.

19. \((2x + 3) + -3 > 8 + -3\)
   a. addition, >
   b. addition
   c. associative, +
   d. zero, +

20. \(2x + (3 + -3) > 8 + -3\)
   a. addition, >
   b. addition
   c. associative, +
   d. zero, +

21. \(2x + 0 > 5\)
   a. addition, >
   b. addition
   c. associative, +
   d. zero, +

22. \(2x > 5\)
   a. addition, >
   b. addition
   c. associative, +
   d. zero, +

23. \(\frac{1}{2}(2x) > \frac{1}{2} \cdot 5\)
   a. multiplication, >
   b. multiplication
   c. associative, \( x \)
   d. one, \( x \)
24. \((\frac{1}{2} \cdot 2)x \geq \frac{1}{2} \cdot 5\)
   a. multiplication, $\geq$
   b. multiplication
   c. associative, $x$
   d. one, $x$

25. \(1x \geq 5/2\)
   a. multiplication, $\geq$
   b. multiplication
   c. associative, $x$
   d. one, $x$

26. \(x \geq 5/2\)
   a. multiplication, $\geq$
   b. multiplication
   c. associative, $x$
   d. one, $x$

IV. Indicate your answer to the following by placing an $\times$ in the correct box after the number on the answer sheet corresponding to the question.

27. The solution for the open sentence \(x + 4 = 2\) is
   a. -6
   b. 6
   c. 2
   d. -2

28. The solution for the open sentence \(x - 4 = 2\) is
   a. -6
   b. 6
   c. 2
   d. -2

29. The solution for the open sentence \(3x + 1 = 7\) is
   a. 2
   b. 8/3
   c. 6
   d. 8
30. The solution for the open sentence $3x - 1 = 7$ is
   a. 2
   b. $8/3$
   c. 6
   d. 8

31. The solution for the open sentence $-3x - 1 = 8$ is
   a. 3
   b. -3
   c. $7/3$
   d. $-7/3$

32. If $3x + 1 < 2$ then
   a. $x < 1/3$
   b. $x < 1$
   c. $x > 1/3$
   d. $x > 1$

33. If $-3x + 1 < 2$ then
   a. $x < -1/3$
   b. $x < 1$
   c. $x > -1/3$
   d. $x > 1$

34. The solution for the open sentence $2/3x - 2 = 4$
   a. 9
   b. 4
   c. $4/3$
   d. 3

35. The solution for the open sentence $2/3x + 2 = 4$
   a. 9
   b. 4
   c. $4/3$
   d. 3
36. If $U$ is \{1, 2\} then $U \times U$ is
   a. \{1, 2\}
   b. \{(1,2), (2,1)\}
   c. \{(1,2)\}
   d. \{(1,1), (2,2), (1,2), (2,1)\}

37. If $U$ is \{1, 2, 3\} then the solution set for \(x + y = 3\)
   a. \{(1,2)\}
   b. \{(1,2), (2,1)\}
   c. \{3\}
   d. empty set

38. If $U = \{1, 2, 3\}$ then the solution set for \(x = y\) is
   a. \{1\}
   b. \{1, 2, 3\}
   c. \{(1,1), (2,2), (3,3)\}
   d. empty set

39. If $U = \{1, 2, 3\}$ then the solution set for \(x < y\) is
   a. \{(1,2), (1,3), (2,3)\}
   b. \{(1,2), (2,1), (2,3), (3,2)\}
   c. \{1, 2, 3\}
   d. \{(1,1), (2,2), (3,3)\}

40. If $A = \{2\}$ and $B = \{3\}$ then $A \times B$ is
   a. \{(2,3)\}
   b. \{(2,3), (3,2)\}
   c. \{(2,2), (3,3)\}
   d. \{2, 3\}
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Exercise 4

I. Indicate whether the following statements are true or false by placing an X in the correct box after the number on the answer sheet corresponding to the question. Place the X in the first box to indicate true and place the ☑ in the second box to indicate false.

1. If $U$ is the set of integers then the graph of $U \times U$ consists of the entire plane.

2. The numbers of each ordered pair of real numbers are the coordinates of a point of the plane.

3. Each point of a plane can be represented by an ordered pair of integers.

4. Each ordered pair of integers identify a unique point of the plane.

5. We refer to the point with coordinates $(x,y)$ as the point $(x,y)$.

6. If $U = \{1,2,3\}$ then the graph of $x + y = 3$ is a straight line.

7. Any equation that can be expressed in the form $ax + by + c = 0$ where $a$ and $b$ are not both zero has a parabola as its graph.

8. Any equation that can be expressed in the form $ax + by + c = 0$ where $a$ and $b$ are not both zero has a parabola as its graph.

9. In order to graph a linear equation we must determine three members of the solution set.

10. The graph of $y = 2x + 3$ has a $y$ intercept of 3.

11. The graph of $2x - 3y + 3 = 0$ has an $x$ intercept of 1.

12. The point $(-3,-1)$ is on the graph of $2x - 3y + 3 = 0$. 
13. The graph of \( y \geq 2x + 3 \) consists of all of the points in the half plane below the line \( y = 2x + 3 \).

14. The point \( (2,5) \) is in the solution set of \( y \leq 2x + 3 \).

15. To solve a set of two simultaneous linear equations we find the set of ordered pairs that are solutions of both equations.

16. When the graph of two linear equations intersect in a point the solution set of the system consists of only one element.

17. When the graph of two linear equations are parallel the solution set of the system is the empty set.

18. To solve the system of linear equation
\[
\begin{align*}
x + y - 5 &= 0 \\
x - y - 1 &= 0
\end{align*}
\]
is to find the solution set of the compound sentence \( x + 5 - 5 = 0 \) or \( x - y - 1 = 0 \).

19. When we find the solution set of a system of two linear equations we find the intersection of the solution sets of the two linear equations.

20. The solution set of the sentence
\[(x - y - 1)(x - y - 2) = 0\]
is equivalent to the solution set of the statement \( x - y - 1 = 0 \) and \( x - y - 2 = 0 \).

21. The graph of two linear equations represents the union of the solution sets of the two linear equations.

22. The solution set of the sentence
\[(x - y - 1)(x - y + 2) = 0\]
is the union of the solution set of \( x - y - 1 = 0 \) with the solution set of \( x - y + 2 = 0 \).

23. A sentence that may be expressed in the form \( y = ax^2 + bx + c \) is called a linear equation.

24. The graph of \( y = x^2 \) is a parabola.
25. The $y^2$ axis is the axis of symmetry for the graph $y = x^2$.

26. The point $(2,4)$ is the vertex of the graph of $y = x^2$.

27. The point $(1,8)$ is on the graph of $y = (x + 2)^2 + 1$.

28. The axis of symmetry for the graph of $y = (x + 2)^2$ is $x = -2$.

29. The graph of $y = (x + 2)^2 + 1$ can be obtained from the graph of $y = (x + 2)^2$ by translating the graph of $y = (x + 2)^2$ one unit up.

30. The graph of $y = (x + 2)^2$ has the same shape as the graph of $y = (x - 2)^2$.

II. Indicate your answer to the following by placing an X in the correct box after the number on the answer sheet corresponding to the question.

31. The x intercept of the graph of $3x - 2y = 6$ is
   a. -3
   b. 0
   c. 2
   d. -2

32. The y intercept of the graph of $3x - 2y = 6$ is
   a. -3
   b. 0
   c. 2
   d. -2

33. The x intercept of the graph of $x + 2y - 5 = 0$ is
   a. 0
   b. $5/2$
   c. $-5/2$
   d. 5
34. The $y$ intercept of the graph of $x + 2y - 5 = 0$ is
   a. 0
   b. $5/2$
   c. $-5/2$
   d. 5

35. When the sentence $2x + y = 7$ is solved for $y$ we get
   a. $y = 7$
   b. $y = 7 - 2x$
   c. $y = 7 + 2x$
   d. $y = 7/2$

36. When the sentence $4x - 2y = 8$ is solved for $y$ we get
   a. $y = 8 - 4x$
   b. $y = 4 - 2x$
   c. $y = 2x - 4$
   d. $y = 4x - 8$

37. When the sentence $x + 2y \leq 4$ is solved for $y$ we get
   a. $y \leq 4 - x$
   b. $y \leq 2 - x$
   c. $y \geq 2 - \frac{1}{2}x$
   d. $y \leq 2 - \frac{3}{2}x$

38. When the sentence $2x + y = 7$ is solved for $y$ we get
   a. $y = 7 - 2x$
   b. $y = 7 + 2x$
   c. $y = 7$
   d. $y = 7/2$

39. The axis of symmetry for $y = (x - 2)^2 + 3$ is
   a. $y$ axis
   b. $y = 2$
   c. $x = 2$
   d. $x = 3$

40. The vertex of the graph of $y = (x - 2)^2 + 3$ is
   a. $(0,0)$
   b. $(-2,3)$
   c. $(2, -3)$
   d. $(2,3)$
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Exercise 5

I. Indicate whether the following statements are true or false by placing an x in the correct box after the number on the answer sheet corresponding to the question. Place the x in the first box to indicate true and place the x in the second box to indicate false.

Question 1 - 12 refer to the mathematical system represented by the table:

<table>
<thead>
<tr>
<th>+</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>b</td>
<td>b</td>
<td>c</td>
<td>a</td>
</tr>
<tr>
<td>c</td>
<td>c</td>
<td>a</td>
<td>b</td>
</tr>
</tbody>
</table>

1. The system is closed under addition.
2. \(a + b = a\).
3. Addition is commutative.
4. The additive identity is \(c\).
5. The additive identity is \(b\).
6. The additive identity is \(a\).
7. \(a + (b + c) = b\).
8. \(a + (b + c) = (a + b) + c\).
9. The additive inverse of \(b\) is \(a\).
10. The additive inverse of \(c\) is \(b\).
11. The additive inverse of \(a\) is \(a\).
12. The system forms a commutative group.
13. The statement \(\neg p\) is false when \(p\) is false.
14. The statement \(p\) is the premise of the statement \(p \rightarrow q\).
15. Any statement that is always false is a tautology.
16. A conditional statement and its contrapositive always have the same truth values.

17. Logic is concerned with relations among statements.

18. The statement \( p \land q \) is true when both \( p \) and \( q \) are true and is false otherwise.

19. The compound statement \( p \lor q \) is true when only one of the statements is true and is false otherwise.

20. The statement \( p \rightarrow q \) is defined to be true unless \( p \) is true and \( q \) is false.

21. The statement \( p \leftrightarrow q \) is true when \( p \) and \( q \) are true and is false otherwise.

22. The statement \( p \lor (\neg p) \) is a tautology.

23. If a conditional statement is true then its converse must be true.

24. If a conditional statement is true then its inverse must be true.

25. If a conditional statement is true then the contrapositive must be true.

II. Indicate your answer to the following by placing an x in the correct box after the number on the answer sheet corresponding to the question.

26. The compound statement \( p \land q \) is true when
   a. \( p \) is true and \( q \) is true
   b. \( p \) is true and \( q \) is false
   c. \( p \) is false and \( q \) is true
   d. \( p \) is false and \( q \) is false

27. The compound statement \( p \lor q \) is false when
   a. \( p \) is true and \( q \) is true
   b. \( p \) is true and \( q \) is false
   c. \( p \) is false and \( q \) is true
   d. \( p \) is false and \( q \) is false
28. The compound statement \( p \rightarrow q \) is false when
   a. \( p \) is true and \( q \) is true
   b. \( p \) is true and \( q \) is false
   c. \( p \) is false and \( q \) is true
   d. \( p \) is false and \( q \) is false

29. The compound statement \( p \land (\sim q) \) will be false when
   a. \( p \) is true and \( q \) is true
   b. \( p \) is true and \( q \) is false
   c. \( p \) is false and \( q \) is true
   d. \( p \) is false and \( q \) is false

30. The compound statement \( (\sim p) \lor q \) will be false when
   a. \( p \) is true and \( q \) is true
   b. \( p \) is true and \( q \) is false
   c. \( p \) is false and \( q \) is true
   d. \( p \) is false and \( q \) is false

31. The compound statement \( p \land (p \rightarrow q) \) is true when
   a. \( p \) is true and \( q \) is true
   b. \( p \) is true and \( q \) is false
   c. \( p \) is false and \( q \) is true
   d. \( p \) is false and \( q \) is false

32. The converse of the statement, if \( x \leq 0 \) then \( x^2 \geq 0 \) is
   a. if \( x \leq 0 \) then \( x^2 \geq 0 \)
   b. if \( x^2 \geq 0 \) then \( x \leq 0 \)
   c. if \( x \leq 0 \) then \( x^2 \geq 0 \)
   d. if \( x^2 \geq 0 \) then \( x \leq 0 \)

33. The inverse of the statement, if \( x \leq 0 \) then \( x^2 \geq 0 \) is
   a. if \( x \leq 0 \) then \( x^2 \geq 0 \)
   b. if \( x^2 \geq 0 \) then \( x \leq 0 \)
   c. if \( x \leq 0 \) then \( x^2 \geq 0 \)
   d. if \( x^2 \geq 0 \) then \( x \leq 0 \)

34. The contrapositive of the statement, if \( x \leq 0 \) the \( x^2 \geq 0 \) is
   a. if \( x \leq 0 \) then \( x^2 \geq 0 \)
   b. if \( x^2 \geq 0 \) then \( x \leq 0 \)
   c. if \( x \leq 0 \) then \( x^2 \geq 0 \)
   d. if \( x^2 \geq 0 \) then \( x \leq 0 \)
35. The name given to the compound statement \( x \leq 0 \) or \( x^2 > 0 \) is
   a. conjunction  
b. conditional  
c. disjunction  
d. biconditional

36. The name given to the compound statement if \( x > 0 \) then \( x^2 \geq 0 \) is
   a. conjunction  
b. conditional  
c. disjunction  
d. biconditional

37. The name given to the statement \( x \leq 0 \) and \( x^2 > 0 \) is
   a. conjunction  
b. conditional  
c. disjunction  
d. biconditional

38. The name given to the compound statement \( x > 0 \) if and only if \( x > 0 \) is
   a. conjunction  
b. conditional  
c. disjunction  
d. biconditional

39. The tautology among the following statements is
   a. \((p \rightarrow q) \leftrightarrow (q \rightarrow p)\)  
b. \([p \land (p \rightarrow q)] \rightarrow q\)  
c. \(p \land (p \rightarrow q)\)  
d. \((p \rightarrow q) \land (q \rightarrow r)\)

40. The property that is not a property of all groups is
   a. closure  
b. identity element  
c. commutativity  
d. associativity
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Exercise 6

I. Indicate whether the following statements are true or false by placing an x in the correct box after the number on the answer sheet corresponding to the question. Place the x in the first box to indicate true and place the x in the second box to indicate false.

1. The results (1,2,3,4,5,6) obtained by rolling a normal die is usually considered to be a set of equally likely events.
   
2. By probability of success we mean the ratio of the total number of events to the number of ways a success can occur.
   
3. By probability of failure we mean the ratio of the number of ways a failure can occur to the total number of events.
   
4. The probability of any outcome is between 0 and \( \frac{1}{2} \).
   
5. When success is certain the probability is 1.
   
6. When an outcome is impossible the probability is 0.
   
7. The probability of getting a head or a tail on a single toss of a coin is 0.
   
8. The probability of tossing a 7 with a single toss of a normal die is 1.
   
9. You will obtain exactly 4 heads in 8 tosses of an honest coin.
   
10. The longer you continue to toss a coin the closer you expect to have 50% of the tosses produce a head.
   
11. If the probability of an outcome is \( \frac{1}{3} \) then the probability of the outcome not occurring is \( \frac{1}{3} \).
   
12. A sample space must contain all possible outcomes.
   
13. A sample space may consist of a set of ordered pairs.
14. If two coins are tossed the sample space has 8 elements.

15. If two dice are thrown the sample space has 36 elements.

II. Indicate your answer to the following by placing an x in the correct box after the number on the answer sheet corresponding to the question.

16. The probability of tossing a 4 on one throw of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 5/6

17. The probability of not tossing a 4 on one throw of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 5/6

18. The probability of tossing a 3 or a 4 on one throw of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 4/6

19. The probability of not tossing either a 3 or a 4 on one throw of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 4/6

20. The probability of tossing a number less than 4 on one throw of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 4/6
21. The probability of tossing a number greater than 4 on one throw of a single die is  
   a. 1/6  
   b. 2/6  
   c. 3/6  
   d. 4/6  

22. The probability of tossing at least a 4 on one throw of a single die is  
   a. 1/6  
   b. 2/6  
   c. 3/6  
   d. 4/6  

23. The probability of tossing an odd number on one toss of a single die is  
   a. 1/6  
   b. 2/6  
   c. 3/6  
   d. 4/6  

24. The probability of not tossing an odd number on one toss of a single die is  
   a. 1/6  
   b. 2/6  
   c. 3/6  
   d. 4/6  

25. The probability of tossing a number that is odd and greater than 4 on one toss of a single die is  
   a. 1/6  
   b. 2/6  
   c. 3/6  
   d. 4/6  

26. The probability of tossing a number that is odd or even on one toss of a single die is  
   a. 0  
   b. 1/6  
   c. 3/6  
   d. 1
27. The probability of tossing a number that is odd and also even on one toss of a single die is
   a. 0
   b. 1/6
   c. 3/6
   d. 1

28. The probability of tossing a number that is odd or greater than 4 on one toss of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 4/6

29. The probability of tossing a number that is odd or less than 4 on one toss of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 4/6

30. The probability of tossing a number that is odd and less than 4 on one toss of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 4/6

31. The probability of tossing a 5 on one toss of two dice is
   a. 2/36
   b. 4/36
   c. 6/36
   d. 8/36

32. The probability of tossing a 7 on one toss of two dice is
   a. 2/36
   b. 4/36
   c. 6/36
   d. 8/36
33. The probability of tossing an 11 on one toss of two dice is
   a. \(\frac{2}{36}\)
   b. \(\frac{4}{36}\)
   c. \(\frac{6}{36}\)
   d. \(\frac{8}{36}\)

34. The probability of tossing a 7 or an 11 on one toss of two dice is
   a. \(\frac{2}{36}\)
   b. \(\frac{4}{36}\)
   c. \(\frac{6}{36}\)
   d. \(\frac{8}{36}\)

35. The probability of getting exactly 3 heads on three tosses of a coin is
   a. \(\frac{4}{8}\)
   b. \(\frac{3}{8}\)
   c. \(\frac{2}{8}\)
   d. \(\frac{1}{8}\)

36. The probability of getting exactly 2 heads on three tosses of a coin is
   a. \(\frac{4}{8}\)
   b. \(\frac{3}{8}\)
   c. \(\frac{2}{8}\)
   d. \(\frac{1}{8}\)

37. The probability of getting more heads than tails on three tosses of a coin is
   a. \(\frac{4}{8}\)
   b. \(\frac{3}{8}\)
   c. \(\frac{2}{8}\)
   d. \(\frac{1}{8}\)

38. The probability of not getting more heads than tails on three tosses of a coin is
   a. \(\frac{4}{8}\)
   b. \(\frac{3}{8}\)
   c. \(\frac{2}{8}\)
   d. \(\frac{1}{8}\)
39. A box contains one red and two white balls. Two balls are drawn in succession without replacement. The probability of drawing two white balls is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 4/6

40. A box contains one red and two white balls. Two balls are drawn in succession without replacement. The probability of drawing one white ball and one red ball is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 4/6
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Exercise 7

I. Indicate whether the following statements are true or false by placing an x in the correct box after the number on the answer sheet corresponding to the question. Place the x in the first box to indicate true and place the x in the second box to indicate false.

1. A line graph is more appropriate than a histogram for presenting discrete data.

2. Line graphs appear to associate values with all points on a continuous interval rather than just with points with integers as coordinates.

3. The number which divides the data so that half of the scores are above this number and half are below it is called the mean of the distribution.

4. The number computed by finding the sum of the scores and dividing by the number of scores is called the median of the distribution.

5. The number which appears most frequently in a distribution is called the mode of the distribution.

6. The mode, mean and median are all called measures of dispersion.

7. The set of scores 5, 6, 7, 8 has no mode.

8. The set 2, 3, 3, 4, 5, 5 is bimodal.

9. The median, is the only one of the three averages that is affected by one low score.

10. The mean should be used as a representative of a set of data when extreme scores should be reflected in the average.

11. The median is the appropriate measure of central tendency to describe the average salary of the workers in a factory that employs 100 people.

12. The median is the appropriate measure to describe the average salary in a shop staffed by the owner and five employees.
13. A measure of central tendency provides information on the variability of a set of data.

14. The range is the difference between the largest and the smallest number in a set.

15. The mean, the median, and the mode all have the same value in any continuous distribution.

16. The standard deviation is a measure of dispersion.

17. If an interval of 2 standard deviations from the mean is considered approximately all of the data is included.

18. An interval of 1 standard deviation about the mean includes approximately 68% of the data in a normal distribution.

19. A disadvantage of the range as a measure of dispersion is that it relies on only two extreme scores.

20. The standard deviation can only be computed for normal distributions.

II. Indicate your answer to the following by placing an x in the correct box after the number on the answer sheet corresponding to the question.

21. The probability that a person who is 15 years old will die before the age of 40 is
   a. .02  
   b. .03  
   c. .04  
   d. .05

22. The probability that a person who is 30 years old will die before the age of 50 is
   a. .07  
   b. .08  
   c. .09  
   d. .10
23. The one of the following that is not a measure of central tendency is
   a. mean
   b. median
   c. range
   d. mode

24. The one of the following that is a measure of dispersion is
   a. mean
   b. standard deviation
   c. median
   d. mode

25. The set of scores 71, 79, 79, 81, 87, 89, 95 has a mode of
   a. 79
   b. 81
   c. 83
   d. 87

26. The set of scores 71, 79, 79, 81, 87, 89, 95 has a median of
   a. 79
   b. 81
   c. 83
   d. 87

27. The set of scores 71, 79, 79, 81, 87, 89, 95 has a mean of
   a. 79
   b. 81
   c. 83
   d. 87

28. The set of scores 28, 78, 78, 80, 86, 88, 94 has a mode of
   a. 76
   b. 78
   c. 80
   d. 86
29. The set of scores 28, 78, 78, 80, 86, 88, 94 has a median of
a. 76
b. 78
c. 80
d. 86

30. The set of scores 28, 78, 78, 80, 86, 88, 94 has a mean of
a. 76
b. 78
c. 80
d. 86

31. The set of scores 6, 7, 8, 9, 10, 12, 15, 15, 20, 28 has a mode of
a. 11
b. 12
c. 13
d. 15

32. The set of scores 6, 7, 8, 9, 10, 12, 15, 15, 20, 28 has a median of
a. 11
b. 12
c. 13
d. 15

33. The set of scores 6, 7, 8, 10, 12, 15, 15, 20, 28 has a mean of
a. 11
b. 12
c. 13
d. 15
In exercise 34 through 39, 100 coins are tossed repeatedly, the distribution of the number of heads is a normal one with a mean of 50 and a standard deviation of 5.

34. The percent of the number of heads between 45 and 55 is
   a. 50
   b. 68
   c. 84
   d. 95

35. The percent of the number of heads less than 50 is
   a. 50
   b. 68
   c. 84
   d. 95

36. The percent of the number of heads above 40 is
   a. 84
   b. 95
   c. 97.5
   d. 99.7

37. The percent of the number of heads between 40 and 60 is
   a. 84
   b. 95
   c. 97.5
   d. 99.7

38. The percent of the number of heads below 60 is
   a. 84
   b. 95
   c. 97.5
   d. 99.7
39. The percent of the number of heads between 35 and 65 is
   a. 84  
   b. 95  
   c. 97.5  
   d. 99.7

40. The standard deviation of the set of scores 6, 9, 5, 3, 6, 7 is
   a. 2  
   b. 6  
   c. √3.33  
   d. √20
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Exercise 8

1. Indicate whether the following statements are true or false by placing an X in the correct box after the number on the answer sheet corresponding to the question. Place the X in the first box to indicate true and place the X in the second box to indicate false.

In exercise 1 through 8 we consider n trials of an experiment in which the probability of a success is p and the probability of a failure is q.

1. The expected mean number of successes (X) on such an experiment is found by the formula X = pq.

2. The standard deviation of the number of successes is given by the formula \( \sigma = \sqrt{pq} \).

3. When n is large the normal distribution can be used to determine the limits within which the data will fall.

4. For large n the number of successes will be between \( \mu - 2 \sigma \) and \( \mu + 2 \sigma \) almost 100% of the time.

5. For large n the number of successes will be between \( \mu - \sigma \) and \( \mu + \sigma \) 95% of the time.

6. We say that we are almost 100% confident that the number of successes will be between \( \mu - 3 \sigma \) and \( \mu + 3 \sigma \).

7. \( \mu - 2 \sigma \) and \( \mu + 2 \sigma \) are called the 95% confidence limits.

8. In industry a control chart is used to tell when a process is in control.

9. As long as most of the samples produce data which falls below the lower limit on the control chart the process is in control.

10. The upper control limit of a control chart represents two standard deviations added to the mean.
11. The correspondence of each positive integer \( n \) with a unique number \( f(n) \) is called a function.

12. A correspondence may be presented as a set of ordered pairs of numbers.

13. The set \( \{(2,1),(2,2),(1,2)\} \) is a relation.

14. The set \( \{(1,1),(1,2),(3,1)\} \) is a function.

15. The domain of \( \{(1,1),(2,2),(1,3)\} \) is \( \{1,2,3\} \).

16. The range of \( \{(1,1),(2,2),(1,3)\} \) is \( \{1,2\} \).

17. A relation is a special kind of function.

18. Any relation such that no two ordered pairs have the same second element and different first elements is called a function.

19. The set consisting of the first elements of the set of ordered pairs in a relation is called the domain of the relation.

20. The set consisting of the second elements of the set of ordered pairs in a relation is called the range of the relation.

II. Indicate your answer to the following by placing an X in the correct box after the number on the answer sheet corresponding to the question.

In exercise 20 through 24 we consider the special lunch served in a cafeteria. Experience shows that \( \frac{1}{3} \) of the customers choose the special lunch. On those days when \( 450 \) customers are served;

21. The mean number of lunches to be expected is
   
   a. 50  
   b. 100  
   c. 150  
   d. 200  

22. The standard deviation of the distribution of the number of lunches ordered is
   
   a. 10  
   b. 5  
   c. 3  
   d. 1
23. We can say with almost 100% confidence that the number of lunches ordered will be between
   a. 130 and 170
   b. 125 and 175
   c. 120 and 180
   d. 115 and 185

24. We can say with approximately 95% confidence that the number of lunches will be between
   a. 135 and 165
   b. 130 and 170
   c. 125 and 175
   d. 120 and 180

In exercise 25 through 30 we consider the production of a new kind of light bulb. Experience has shown that 1/10 of the light bulbs are defective. We check each batch of 100 light bulbs.

25. The mean number of defective light bulbs to be expected is
   a. 50
   b. 3
   c. 5
   d. 10

26. The standard deviation of the distribution of the number of defective bulbs is
   a. 3
   b. 5
   c. 10
   d. 50

27. The upper control limit for the control chart is
   a. 10
   b. 13
   c. 16
   d. 19

28. The lower control limit for the control chart is
   a. 1
   b. 3
   c. 4
   d. 7
29. We say that the process is in control if the number of defectives is between

a. 1 and 19
b. 3 and 16
c. 4 and 13
d. 7 and 10

30. We would suspect that something was wrong if the number of defectives was greater than

a. 1
b. 10
c. 19
d. 29

31. The functional rule representing the mapping $1 \rightarrow 2$ is

$$
\begin{align*}
2 & \rightarrow 5 \\
3 & \rightarrow 8 \\
4 & \rightarrow 11 \\
\ldots & 
\end{align*}
$$

a. $n \rightarrow \frac{1}{3}(n + 1)$
b. $n \rightarrow 3n - 1$
c. $n \rightarrow n + 7$
d. $n \rightarrow 2n$

32. The functional rule representing the correspondence $1 \rightarrow 1$ is

$$
\begin{align*}
2 & \rightarrow 4 \\
3 & \rightarrow 9 \\
4 & \rightarrow 16 \\
\ldots & 
\end{align*}
$$

a. $n \rightarrow n$
b. $n \rightarrow 2n$
c. $n \rightarrow 3n^2$
d. $n \rightarrow n$

33. The set of ordered pairs representing the mapping $1 \rightarrow 2$ is

$$
\begin{align*}
1 & \rightarrow 2 \\
2 & \rightarrow 5 \\
3 & \rightarrow 8 \\
4 & \rightarrow 11 \\
\ldots & 
\end{align*}
$$

a. $\{(2,1),(5,2),(8,3),(11,4)\ldots\}$
b. $\{(1,1),(2,2),(3,3),(4,4)\ldots\}$
c. $\{(1,2),(2,5),(3,8),(4,11)\ldots\}$
d. $\{(1,2),(2,3),(3,4),(4,5)\ldots\}$
34. If \( A = \{1, 3\} \) and \( B = \{2, 4\} \) then \( A \times B \) is
   a. \( \{(1, 2), (1, 4), (3, 4), (3, 2)\} \)
   b. \( \{(2, 1), (4, 1), (4, 3), (2, 3)\} \)
   c. \( \{(1, 2), (3, 4)\} \)
   d. \( \{1, 2, 3, 4\} \)

35. The domain of the relation \( \{(10, 1), (9, 2), (5, 5)\} \) is
   a. \( \{10, 9, 5\} \)
   b. \( \{1, 2, 5\} \)
   c. \( \{(1, 10), (2, 9), (5, 5)\} \)
   d. \( \{10, 1, 9, 2, 5\} \)

36. The range of the relation \( \{(10, 1), (9, 2), (5, 5)\} \) is
   a. \( \{10, 9, 5\} \)
   b. \( \{1, 2, 5\} \)
   c. \( \{(1, 10), (2, 9), (5, 5)\} \)
   d. \( \{1, 2, 5, 9, 10\} \)

37. When \( f(x) = x + 3 \) then \( f(2) \) is
   a. 2
   b. 5
   c. 3
   d. -1

38. When \( f(x) = 3x + 4 \) then \( f(3) \) is
   a. 3
   b. 4
   c. 9
   d. 13

39. When \( f(x) = 2x^2 + 4 \) then \( f(3) \) is
   a. 40
   b. 22
   c. 18
   d. 4

40. For positive integers \( x \) and the relation expressed by the equation \( y = 2x + 3 \) if \( x \) is increased by 1 then \( y \) is increased by
   a. 1
   b. 2
   c. 3
   d. 4
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Exercise 9

I. Indicate whether the following statements are true or false by placing an x in the correct box after the number on the answer sheet corresponding to the question. Place the x in the first box to indicate true and place the x in the second box to indicate false.

1. If a relation is a function then each vertical line intersects the graph of the relation in at most one point.

2. The change in the value of x as y is increased by 1 is designated by $\Delta y$.

3. If y is a linear function of x then $\Delta y$ is a constant.

4. If $y = 2x + 3$ then $\Delta y$ is a constant.

5. If $y = ax + b$ then $b = \Delta y$.

6. A function like $y = 2x^2 + 3$ is called a polynomial function.

7. If $y = x^2 - 2$ then $\Delta y$ is a constant.

8. If $y = 2x^2 - 3$ then $\Delta^2 y$ is a constant.

9. If $\Delta^3 y$ is a constant then $f(x)$ may be expressed in the form $f(x) = ax^2 + bx + c$.

10. If a relation is a function then its inverse is a function.

11. If a relation is not a function then its inverse is not a function.

12. If we interchange the x and the y in an equation we have the equation for the inverse of the original function.

13. An expression in which the x occurs as an exponent is called a logarithmic function.
14. The inverse of the logarithmic function is the exponential function.

15. The logarithm of a number is the power to which the base is raised to obtain the number.

16. \( \log (a \times b) = \log a + \log b \)

17. \( \log \frac{a}{b} = \frac{\log a}{\log b} \)

18. \( \log a^n = n \log a \)

19. \( \log \sqrt[n]{a} = \frac{\log a}{n} \)

20. \( \log (a - b) = \log a - \log b \)

II. Indicate your answer to the following by placing an X in the correct box after the number on the answer sheet corresponding to the question.

21. If \( y = 5x - 3 \) then \( \Delta y \) is
   
   a. 1
   b. 2
   c. 3
   d. 5

22. If \( y = 3x - 2 \) then \( \Delta y \) is
   
   a. 1
   b. 2
   c. 3
   d. 5

23. If \( y = ax + b \) and \( \Delta y = 2 \) and \((2,5)\) is an ordered pair of the function then the value of \( b \) is
   
   a. 1
   b. 2
   c. 4
   d. 5

24. If \( f(x) = x^2 - 2 \) then \( f(2) \) is
   
   a. 0
   b. 2
   c. 4
   d. 5
25. If \( f(x) = 2x^2 - x + 5 \) then \( f(3) \) is

   a. 3
   b. 15
   c. 18
   d. 20

26. If \( f(x) = x^2 - 3x + 2 \) then \( f(x + 1) \) is

   a. \( x^2 + 1 \)
   b. \( x^2 - 3x + 2 \)
   c. \( x^2 - x \)
   d. \( x^2 - 3x \)

27. If \( f(x) = x^2 - 3x + 2 \) then \( \Delta y \) is

   a. 2
   b. \( 2x - 2 \)
   c. \( 3x \)
   d. \( x^2 \)

28. If \( f(x) = x^2 - 3x + 2 \) then \( \Delta^2 y \) is

   a. 1
   b. 2
   c. 3
   d. 4

29. If \( f(x) = 2x^2 + 1 \) then \( \Delta^2 y \) is

   a. 1
   b. 2
   c. 3
   d. 4

30. The inverse of the relation \( \{(1,2),(2,1),(3,4)\} \) is

   a. \( \{(3,4),(2,1),(1,2)\} \)
   b. \( \{(1,2),(2,1),(4,3)\} \)
   c. \( \{(2,1),(1,2),(3,4)\} \)
   d. \( \{(1,2),(2,1),(3,4)\} \)

31. The relation which is also a function is

   a. \( \{(1,1),(2,2),(3,3)\} \)
   b. \( \{(1,1),(1,2),(1,3)\} \)
   c. \( \{(1,2),(2,1),(2,3)\} \)
   d. \( \{(1,2),(2,3),(2,1)\} \)
32. The relation whose inverse is a function is
   a. \{(1,1),(1,2),(1,3)\}
   b. \{(1,1),(2,1),(3,1)\}
   c. \{(1,1),(1,2),(2,2)\}
   d. \{(2,3),(3,2),(3,3)\}

33. The inverse of the relation determined by the equation \( y = x + 1 \) is
   a. \( y = x + 1 \)
   b. \( y = 1 + x \)
   c. \( x + 1 = y \)
   d. \( y = x - 1 \)

34. The inverse of the relation determined by the equation \( y = 2x + 3 \) is
   a. \( y = 3 + 2x \)
   b. \( y = -3/2 + 1/2x \)
   c. \( y = 2x + 3 \)
   d. \( 2x + 3 = y \)

35. If \( y = 3^x \) then for \( x = 4 \) the value of \( y \) is
   a. 12
   b. 64
   c. 81
   d. 144

36. If \( \log_2 128 = x \) then \( 2^x \) is
   a. 2
   b. 7
   c. 128
   d. 256

37. The value of \( \log_2 256 \) is
   a. 8
   b. 16
   c. 256
   d. 512
38. The value of \( \log_2 1/16 \) is
   a. -16
   b. -4
   c. 1/4
   d. 1/2

39. \( \log_2 (16 \times 32) \) is
   a. 2
   b. 4
   c. 5
   d. 9

40. \( \log_2 (128^5) \) is
   a. 8
   b. 16
   c. 35
   d. 64
APPENDIX B
Mathematics 112
Exercise 1-A

I. Indicate whether the following statements are true or false by placing a punch in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

1. (2) Every whole number is a counting number.
2. (4) Every whole number is an integer.
3. (6) Every rational number is an integer.
4. (8) Every real number is an integer.
5. (10) Every rational number is a real number.
6. (12) Every irrational number is a real number.
7. (14) \(-2 + -5 = 3\)
8. (16) \(-2 + 5 = 3\)
9. (18) \((-2)(5) = -10\)
10. (20) \((-2)(-5) = -10\)
11. (22) The counting numbers may be considered as the union of the whole numbers and zero.
12. (24) Every integer is either positive or negative.
13. (26) Every real number is either a rational number or an irrational number.
14. (28) The set of whole numbers is closed under substraction.
15. (30) The set of rational numbers is closed under division.
16. (32) The graph of the set of integers between \(-3\) and \(2\) contains four points.
17. (34) The set of rational numbers form a field.
18. (36) The set of counting numbers form a group.

19. (38) There is a one-to-one correspondence between the elements of the set of real numbers and the set of points on the number line.

20. (40) There is an integer replacement for \( n \) in the statement \( 2n = 9 \).

21. (42) There is a whole number replacement for \( n \) in the statement \( n + 5 = 2 \).

22. (44) The set of real numbers is dense.

23. (46) Real numbers that are irrational numbers are represented by nonterminating, nonrepeating decimals.

24. (48) The number 0 is not a rational number.

25. (50) The set of integers is the same as the set of the opposites of the integers.

II. Indicate the answer to the following by placing a punch in the column corresponding to the question.
Punch the number 1, 2, 3, 4 to indicate your choice from the four possible replies.

26. (52) The property illustrated by the statement \((x + y) + z = x + (y + z)\) is

   1. distributive
   2. associative
   3. commutative
   4. inverse

27. (54) The property illustrated by the statement \(x(y + z) = xy + xz\) is

   1. distributive
   2. associative
   3. commutative
   4. inverse
28. (56) The property illustrated by the statement $x + y = y + x$ is

1. identity
2. associative
3. commutative
4. inverse

29. (58) The property illustrated by the statement $n + (-n) = (-n) + n = 0$ is

1. identity
2. associative
3. commutative
4. inverse

30. (60) The property illustrated by the statement $\frac{a}{b} \times \frac{b}{a} = 1$ is

1. identity
2. associative
3. commutative
4. inverse

31. (62) The property illustrated by the statement $1 \times n = n \times 1 = n$ is

1. identity
2. associative
3. commutative
4. inverse

32. (64) The number set that does not contain a replacement for $n$ in the sentence $5 + n = 2$ is

1. whole numbers
2. integers
3. rational numbers
4. real numbers

33. (66) The set of numbers that contains a solution to the equation $n^2 + 1 = 3$ is

1. whole numbers
2. integers
3. rational numbers
4. irrational numbers
34. (68) The set of numbers that contains a solution to the equation $5n = 3$ is

1. whole numbers
2. integers
3. rational numbers
4. irrational numbers

35. (70) The number $\frac{3}{4}$ can be represented as

1. a terminating decimal
2. a repeating decimal
3. a nonterminating, nonrepeating decimal
4. none of the above

36. (72) The number $\frac{5}{12}$ can be written as

1. a terminating decimal
2. a repeating decimal
3. a nonterminating, nonrepeating decimal
4. none of the above

37. (74) The number $\sqrt{3}$ can be written as

1. a terminating decimal
2. a repeating decimal
3. a nonterminating, nonrepeating decimal
4. none of the above

38. (76) The correct graph of real numbers between 2 and 5 is

1. ![Graph 1]
2. ![Graph 2]
3. ![Graph 3]
4. ![Graph 4]

39. (78) The correct graph for the rational numbers between 2 and 5 is (see question 38 for the replies).
40. (80) The set of negatives of the whole numbers is

1. the negative integers
2. the negative rational numbers
3. the non positive integers
4. the negative whole numbers
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Exercise 2-A

I. Indicate whether the following statements are true or false by placing a punch in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

1. (2) A statement of equality must be true in order to be called an equation.

2. (4) An open sentence cannot be classified as true or false.

3. (6) A placeholder for a member of the replacement set is called a variable.

4. (8) \( a > b \) if and only if there is a positive number \( c \) such that \( a + c = b \).

5. (10) When a variable is replaced by a member of the replacement set we always get a true statement.

6. (12) An equation must be true for all members of the replacement set in order to be called an identity.

7. (14) An inequality cannot be classified as an identity.

8. (16) The equation \( x + 2 = 5 \) is an identity.

9. (18) The solution set of the equation \( x + 2 = x \) is the empty set.

10. (20) The solution set for \( x + 5 = 2 \) is the empty set when the replacement set is the whole numbers.

11. (22) If \( A = \{1, 2, 3\} \) and \( B = \{2, 4, 6\} \) then \( A \cup B = \{2\} \).

12. (24) If \( A = \{1, 2, 3, 4\} \) and \( B = \{2, 4, 6, 8\} \) then \( A \cap B = \{2, 4\} \).

13. (26) The integer solution set of the statement \( x \geq 1 \) and \( x + 1 \leq 5 \) is \( \{1, 2, 3, 4, 5\} \).
14. (28) The integer solution set of the statement $x \geq 1$ or $x + 1 \leq 5$ is \{1, 2, 3, 4, 5\}.

15. (30) $7 > 2$ and $2 < 5$.

16. (32) $-2 > 2$ or $2 < 3$.

17. (34) $2 + 5 \neq 8$ and $-2 > -1$.

18. (36) $2 - 5 = 5 - 2$ or $-3 > -4$.

19. (38) There is only one number whose absolute value is 3, namely -3.

20. (40) The absolute value of any non-zero real number is positive.

21. (42) The symbol $|-3|$ represents the distance of -3 from the origin.

22. (44) The solution set of $x = 3$ consists of the numbers whose distance from the origin is 3.

23. (46) The solution set of $|x| \leq 3$ consists of the numbers whose distance from the origin is equal to or greater than 3.

24. (48) The solution set of $|x + 1| = 3$ consists of the numbers whose distance from the number 1 is 3.

25. (50) The solution set of $|x| = -3$ is the empty set.

II. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

26. (52) $3 - 7 \not\leq 7 - 3$ is
   1. a true statement of equality.
   2. a false statement of equality.
   3. a true statement of inequality.
   4. a false statement of inequality.
27. (54) \(-23/25 \geq -24/25\) is
1. a true statement of equality.
2. a false statement of equality.
3. a true statement of inequality.
4. a false statement of inequality.

28. (56) \(13^2 = 5^2 + 12^2\) is
1. a true statement of equality.
2. a false statement of equality.
3. a true statement of inequality.
4. a false statement of inequality.

29. (58) The truth set of the sentence \(x + 2 \neq x\) is
1. a single number
2. the empty set
3. the entire replacement set
4. none of the above

30. (60) The truth set of the sentence \(x > x + 3\) is
1. a single number
2. the empty set
3. the entire replacement set
4. none of the above

31. (62) The truth set of the sentence \(x + 1 < x + 2\) is
1. a single number
2. the empty set
3. the entire replacement set
4. none of the above

32. (64) The graph of the solution set of the sentence \(x - 2 \geq 7\) is
1. a point
2. a half-line
3. a ray
4. a line

33. (66) The graph of the solution set of the sentence \(x < x + 3\) is
1. a point
2. a half-line
3. a ray
4. a line
34. (68) The graph of the solution set of the sentence 
\( x - 2 = 7 \) is

1. a point
2. a half-line
3. a ray
4. a line segment

35. (70) The graph of the solution set of the sentence
\( -3 \leq x \leq 0 \) is

1. a point
2. a half-line
3. a ray
4. a line segment

36. (72) The graph of the solution set of the compound statement \( x > 1 \) or \( x < 5 \) when the replacement set is the real numbers is

1. two points
2. a line segment
3. union of two rays
4. a line

37. (74) The graph of \( x \leq 7 \) is

1. two points
2. a line segment
3. union of two rays
4. a line

38. (76) The graph of \( |x - 2| = 5 \) is

1. two points
2. a line segment
3. union of two rays
4. a line

39. (78) The graph of \( x \geq 1 \) is

1. two points
2. a line segment
3. union of two rays
4. a line
40. (80) The graph of \( x + 3 \leq 1 \) is

1. two points
2. a line segment
3. union of two rays
4. a line
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Exercise 3-A

I. Indicate whether the following statements are true or false by placing a punch in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

1. (2) If we add the same number to both members of an equation we obtain an equivalent sentence.

2. (4) If \( a = b \) and \( c = d \), then \( a + c = b + d \).

3. (6) If \( a = b \) and \( c = d \), then \( ac = bd \).

4. (8) If \( a < b \) and \( c = d \), then \( a + c < b + d \).

5. (10) If \( a < b \) then \( ac < bc \) if \( c < 0 \).

6. (12) If both members of an inequality are multiplied by a negative number, it is necessary to reverse the sense of the inequality to obtain an equivalent sentence.

7. (14) A pair of replacements is needed for an open sentence in two variables before we can determine whether it is true or false for these replacements.

8. (16) The cartesian product of \( U \times U \) is the set of all ordered pairs with the first element from \( U \) and the second element from \( U \).

9. (18) The cartesian product \( A \times B \) is the set of all ordered pairs with the first element from \( A \) or \( B \) and the second element from \( A \) or \( B \).

10. (20) The graph of the cartesian product \( U \times U \) for \( U = \{1, 2, 3, 4\} \) consists of 16 points.

II. Consider the steps in the solutions of \( 2x + 3 = 8 \). Select the column on the answer card corresponding to the step and punch the number 1, 2, 3 or 4 to indicate your choice from the four possible reasons.
11. (22) \((2x + 3) + -3 = 8 + -3\)
   1. addition, =
   2. addition
   3. associative, +
   4. zero, +

12. (24) \(2x + (3 + -3) = 8 + -3\)
   1. addition, =
   2. addition
   3. associative, +
   4. zero, +

13. (26) \(2x + 0 = 5\)
   1. addition, =
   2. addition
   3. associative, +
   4. zero, +

14. (28) \(2x = 5\)
   1. addition, =
   2. addition
   3. associative, +
   4. zero, +

15. (30) \(\frac{1}{2}(2x) = \frac{1}{2} \cdot 5\)
   1. multiplication, =
   2. multiplication
   3. associative, x
   4. one, x

16. (32) \((\frac{1}{2} \cdot 2)x = \frac{1}{2} \cdot 5\)
   1. multiplication, =
   2. multiplication
   3. association, x
   4. one, x

17. (34) \(1x = 5/2\)
   1. multiplication, =
   2. multiplication
   3. associative, x
   4. one, x
18. (36) \( x = \frac{5}{2} \)

1. multiplication, =
2. multiplication
3. associative, x
4. one, x

III. Consider the steps in the solution of \( 2x + 3 > 8 \). Select the column on the answer card corresponding to the step and punch the number 1, 2, 3 or 4 to indicate your choice from the four possible reasons.

19. (38) \((2x + 3) + -3 > 8 + -3\)

1. addition, >
2. addition
3. associative, +
4. zero, +

20. (40) \(2x + (3 + -3) > 8 + -3\)

1. addition, >
2. addition
3. associative, +
4. zero, +

21. (42) \(2x + 0 > 5\)

1. addition, >
2. addition
3. associative, +
4. zero, +

22. (44) \(2x > 5\)

1. addition, >
2. addition
3. associative, +
4. zero, +

23. (46) \(\frac{1}{2}(2x) > \frac{1}{2} \cdot 5\)

1. multiplication, >
2. multiplication
3. associative, x
4. one, x
24. (48) \( \left( \frac{1}{2} \cdot 2 \right) \times \frac{1}{2} \cdot 5 \)
   1. multiplication,
   2. multiplication
   3. associative, \( x \)
   4. one, \( x \)

25. (50) \( 1x > \frac{5}{2} \)
   1. multiplication,
   2. multiplication
   3. associative, \( x \)
   4. one, \( x \)

26. (52) \( x > \frac{5}{2} \)
   1. multiplication,
   2. multiplication
   3. associative, \( x \)
   4. one, \( x \)

IV. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

27. (54) The solution for the open sentence \( x + 4 = 2 \) is
   1. -6
   2. 6
   3. 2
   4. -2

28. (56) The solution for the open sentence \( x - 4 = 2 \) is
   1. -6
   2. 6
   3. 2
   4. -2

29. (58) The solution for the open sentence \( 3x + 1 = 7 \) is
   1. 2
   2. \( \frac{8}{3} \)
   3. 6
   4. 8
30. (60) The solution for the open sentence $3x - 1 = 7$ is

1. 2
2. $8/3$
3. 6
4. 8

31. (62) The solution for the open sentence $-3x - 1 = 8$ is

1. 3
2. -3
3. $7/3$
4. $-7/3$

32. (64) If $3x + 1 < 2$ then

1. $x < 1/3$
2. $x < 1$
3. $x > 1/3$
4. $x > 1$

33. (66) If $-3x + 1 < 2$ then

1. $x < -1/3$
2. $x < 1$
3. $x > -1/3$
4. $x > 1$

34. (68) The solution for the open sentence $2/3 x - 2 = 4$ is

1. 9
2. 4
3. $4/3$
4. 3

35. (70) The solution for the open sentence $2/3x + 2 = 4$ is

1. 9
2. 4
3. $4/3$
4. 3
36. (72) If $U$ is $\{1,2\}$ then $U \times U$ is
   1. $\{1,2\}$
   2. $\{(1,2)\}$
   3. $\{(1,2),(2,1)\}$
   4. $\{(1,1),(2,2),(1,2),(2,1)\}$

37. (74) If $U = \{1,2,3\}$ then the solution set for $x + y = 3$ is
   1. $\{(1,2)\}$
   2. $\{(1,2),(2,1)\}$
   3. $\{3\}$
   4. empty set

38. (76) If $U = \{1,2,3\}$ then the solution set for $x = y$ is
   1. $\{1\}$
   2. $\{1,2,3\}$
   3. $\{(1,1),(2,2),(3,3)\}$
   4. empty set

39. (78) If $U = \{1,2,3\}$ then the solution set for $x < y$ is
   1. $\{(1,2),(1,3),(2,3)\}$
   2. $\{(1,2),(2,1),(2,3),(3,2)\}$
   3. $\{1,2,3\}$
   4. $\{(1,1),(2,2),(3,3)\}$

40. (80) If $A = \{2\}$ and $B = \{3\}$ then $A \times B$ is
   1. $\{(2,3)\}$
   2. $\{(2,3),(3,2)\}$
   3. $\{(2,2),(3,3)\}$
   4. $\{2,3\}$
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Exercise 4-A

I. Indicate whether the following statements are true or false by placing a punch, in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

1. (2) If \( U \) is the set of integers then the graph of \( U \times U \) consists of the entire plane.

2. (4) The numbers of each ordered pair of real numbers are the coordinates of a point of the plane.

3. (6) Each point of a plane can be represented by an ordered pair of integers.

4. (8) Each ordered pair of integers identify a unique point of the plane.

5. (10) We refer to the point with coordinates \((x,y)\) as the point \((x,y)\).

6. (12) If \( U = \{1,2,3\} \) then the graph of \( x + y = 3 \) is a straight line.

7. (14) Any equation that can be expressed in the form \( ax + by + c = 0 \) where \( a \) and \( b \) are not both zero is said to be a linear equation.

8. (16) Any equation that can be expressed in the form \( ax + by + c = 0 \) where \( a \) and \( b \) are not both zero has a parabola as its graph.

9. (18) In order to graph a linear equation we must determine three members of the solution set.

10. (20) The graph of \( y = 2x + 3 \) has a \( y \) intercept of 3.

11. (22) The graph of \( 2x - 3y + 3 = 0 \) has an \( x \) intercept of 1.

12. (24) The point \((-3,-1)\) is on the graph of \( 2x - 3y + 3 = 0 \).
13. (26) The graph of $y > 2x + 3$ consists of all of the points in the half plane below the line $y = 2x + 3$.

14. (28) The point $(2,5)$ is in the solution set of $y \leq 2x + 3$.

15. (30) To solve a set of two simultaneous linear equations we find the set of ordered pairs that are solutions of both equations.

16. (32) When the graph of two linear equations intersect in a point the solution set of the system consists of only one element.

17. (34) When the graph of two linear equations are parallel the solution set of the system is the empty set.

18. (36) To solve the system of linear equation

\[
\begin{align*}
x + y - 5 &= 0 \\
x - y - 1 &= 0
\end{align*}
\]

is to find the solution set of the compound sentence $x + y - 5 = 0$ or $x - y - 1 = 0$.

19. (38) When we find the solution set of a system of two linear equations we find the intersection of the solution sets of the two linear equations.

20. (40) The solution set of the sentence $(x - y - 1)(x - y - 2) = 0$ is equivalent to the solution set of the statement $x - y - 1 = 0$ and $x - y - 2 = 0$.

21. (42) The graph of two linear equations represents the union of the solution sets of the two linear equations.

22. (44) The solution set of the sentence $(x - y - 1)(x - y + 2) = 0$ is the union of the solution set of $x - y - 1 = 0$ with the solution set of $x - y + 2 = 0$.

23. (46) A sentence that may be expressed in the form $y = ax^2 + bx + c$ is called a linear equation.

24. (48) The graph of $y = x^2$ is a parabola.
25. (50) The y axis is the axis of symmetry for the graph \( y = x^2. \)

26. (52) The point \((2,4)\) is the vertex of the graph of \( y = x^2. \)

27. (54) The point \((1,8)\) is on the graph of \( y = (x + 2)^2 + 1. \)

28. (56) The axis of symmetry for the graph of \( y = (x + 2)^2 \) is \( x = -2. \)

29. (58) The graph of \( y = (x + 2)^2 + 1 \) can be obtained from the graph of \( y = (x + 2)^2 \) by translating the graph of \( y = (x + 2)^2 \) one unit up.

30. (60) The graph of \( y = (x + 2)^2 \) has the same shape as the graph of \( y = (x - 2)^2. \)

II. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

31. (62) The x intercept of the graph of \( 3x - 2y = 6 \) is

1. -3
2. 0
3. 2
4. -2

32. (64) The y intercept of the graph of \( 3x - 2y = 6 \) is

1. -3
2. 0
3. 2
4. -2

33. (66) The x intercept of the graph of \( x + 2y - 5 = 0 \) is

1. 0
2. 5/2
3. -5/2
4. 5
34. (68) The y intercept of the graph of
\[ x + 2y - 5 = 0 \]
is
1. 0
2. \( \frac{5}{2} \)
3. \( -\frac{5}{2} \)
4. 5

35. (70) When the sentence \( 2x + y = 7 \) is solved for \( y \) we get
1. \( y = 7 \)
2. \( y = 7 - 2x \)
3. \( y = 7 + 2x \)
4. \( y = \frac{7}{2} \)

36. (72) When the sentence \( 4x - 2y = 8 \) is solved for \( y \) we get
1. \( y = 8 - 4x \)
2. \( y = 4 - 2x \)
3. \( y = 2x - 4 \)
4. \( y = 4x - 8 \)

37. (74) When the sentence \( x + 2y \leq 4 \) is solved for \( y \) we get
1. \( y \leq 4 - x \)
2. \( y \leq 2 - \frac{x}{2} \)
3. \( y \geq 2 - \frac{x}{2} \)
4. \( y \leq 2 - \frac{x}{2} \)

38. (76) When the sentence \( 2x + y = 7 \) is solved for \( y \) we get
1. \( y = 7 - 2x \)
2. \( y = 7 + 2x \)
3. \( y = 7 \)
4. \( y = \frac{7}{2} \)

39. (78) The axis of symmetry for \( y = (x - 2)^2 + 3 \) is
1. y axis
2. y = 2
3. x = 2
4. x = 3
40. (80) The vertex of the graph of \( y = (x - 2)^2 + 3 \) is

1. \((0,0)\)
2. \((-2,3)\)
3. \((2,-3)\)
4. \((2,3)\)
Indicate whether the following statements are true or false by placing a punch, in the column corresponding to the question. Punch 1 if the answer is true and punch a 2 if the answer is false.

Question 1 - 12 refer to the mathematical system represented by the table

<table>
<thead>
<tr>
<th>+</th>
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</table>

1. (2) The system is closed under addition.
2. (4) $a + b = a$.
3. (6) Addition is commutative.
4. (8) The additive identity is c.
5. (10) The additive identity is b.
6. (12) The additive identity is a.
7. (14) $a + (b + c) = b$.
8. (16) $a + (b + c) = (a + b) + c$.
9. (18) The additive inverse of b is a.
10. (20) The additive inverse of c is b.
11. (22) The additive inverse of a is a.
12. (24) The system forms a commutative group.
13. (26) The statement $\neg p$ is false when $p$ is false.
14. (28) The statement $p$ is the premise of the statement $p \Rightarrow q$.
15. (30) Any statement that is always false is a tautology.
16. (32) A conditional statement and its contrapositive always have the same truth values.

17. (34) Logic is concerned with relations among statements.

18. (36) The statement \( p \land q \) is true when both \( p \) and \( q \) are true and is false otherwise.

19. (38) The compound statement \( p \lor q \) is true when only one of the statements is true and is false otherwise.

20. (40) The statement \( p \rightarrow q \) is defined to be true unless \( p \) is true and \( q \) is false.

21. (42) The statement \( p \leftrightarrow q \) is true when \( p \) and \( q \) are true and is false otherwise.

22. (44) The statement \( p \lor (\sim p) \) is a tautology.

23. (46) If a conditional statement is true then its converse must be true.

24. (48) If a conditional statement is true then its inverse must be true.

25. (50) If a conditional statement is true then the contrapositive must be true.

II. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

26. (52) The compound statement \( p \land q \) is true when
   1. \( p \) is true and \( q \) is true
   2. \( p \) is true and \( q \) is false
   3. \( p \) is false and \( q \) is true
   4. \( p \) is false and \( q \) is false

27. (54) The compound statement \( p \lor q \) is false when
   1. \( p \) is true and \( q \) is true
   2. \( p \) is true and \( q \) is false
   3. \( p \) is false and \( q \) is true
   4. \( p \) is false and \( q \) is false
28. (56) The compound statement $p \rightarrow q$ is false when
   1. $p$ is true and $q$ is true
   2. $p$ is true and $q$ is false
   3. $p$ is false and $q$ is true
   4. $p$ is false and $q$ is false

29. (58) The compound statement $p \land (\sim q)$ will be true when
   1. $p$ is true and $q$ is true
   2. $p$ is true and $q$ is false
   3. $p$ is false and $q$ is true
   4. $p$ is false and $q$ is false

30. (60) The compound statement $(\sim p) \lor q$ will be false when
   1. $p$ is true and $q$ is true
   2. $p$ is true and $q$ is false
   3. $p$ is false and $q$ is true
   4. $p$ is false and $q$ is false

31. (62) The compound statement $p \land (p \lor q)$ is true when
   1. $p$ is true and $q$ is true
   2. $p$ is true and $q$ is false
   3. $p$ is false and $q$ is true
   4. $p$ is false and $q$ is false

32. (64) The converse of the statement, if $x < 0$ then $x > 0$ is
   1. if $x < 0$ then $x > 0$
   2. if $x > 0$ then $x < 0$
   3. if $x > 0$ then $x > 0$
   4. if $x < 0$ then $x < 0$

33. (66) The inverse of the statement, if $x < 0$ then $x > 0$ is
   1. if $x < 0$ then $x > 0$
   2. if $x > 0$ then $x < 0$
   3. if $x > 0$ then $x > 0$
   4. if $x < 0$ then $x < 0$

34. (68) The contrapositive of the statement, if $x < 0$ then $x > 0$ is
   1. if $x < 0$ then $x > 0$
   2. if $x > 0$ then $x < 0$
   3. if $x > 0$ then $x > 0$
   4. if $x < 0$ then $x < 0$
35. (70) The name given to the compound statement \( x < 0 \text{ or } x > 0 \) is
   1. conjunction
   2. conditional
   3. disjunction
   4. biconditional

36. (72) The name given to the compound statement if \( x > 0 \) then \( x > 0 \) is
   1. conjunction
   2. conditional
   3. disjunction
   4. biconditional

37. (74) The name given to the statement \( x < 0 \text{ and } x > 0 \) is
   1. conjunction
   2. conditional
   3. disjunction
   4. biconditional

38. (76) The name given to the compound statement \( x > 0 \) if and only if \( x > 0 \) is
   1. conjunction
   2. conditional
   3. disjunction
   4. biconditional

39. (78) The tautology among the following statements is
   1. \( (p \rightarrow q) \leftrightarrow (q \rightarrow p) \)
   2. \( p \land (p \rightarrow q) \rightarrow q \)
   3. \( \sim q \land (p \rightarrow q) \)
   4. \( (p \rightarrow q) \land (q \rightarrow r) \)

40. (80) The property that is not a property of all groups is
   1. closure
   2. identity element
   3. commutativity
   4. associativity
I. Indicate whether the following statements are true or false by placing a punch, in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

1. (2) The results (1,2,3,4,5,6) obtained by rolling a normal die is usually considered to be a set of equally likely events.

2. (4) By probability of success we mean the ratio of the total number of events to the number of ways a success can occur.

3. (6) By probability of failure we mean the ratio of the number of ways a failure can occur to the total number of events.

4. (8) The probability of any outcome is between 0 and $\frac{1}{2}$.

5. (10) When success is certain the probability is 1.

6. (12) When an outcome is impossible the probability is 0.

7. (14) The probability of getting a head or a tail on a single toss of a coin is 0.

8. (16) The probability of tossing a 7 with a single toss of a normal die is 1.

9. (18) You will obtain exactly 4 heads in 8 tosses of an honest coin.

10. (20) The longer you continue to toss a coin the closer you expect to have 50% of the tosses produce a head.

11. (22) If the probability of an outcome is $\frac{1}{3}$ then the probability of the outcome not occurring is $\frac{2}{3}$.

12. (24) A sample space must contain all possible outcomes.
13. (26) A sample space may consist of a set of ordered pairs.

14. (28) If two coins are tossed the sample space has 8 elements.

15. (30) If two dice are thrown the sample space has 36 elements.

II. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

16. (32) The probability of tossing a 4 on one throw of a single die is
   1. $\frac{1}{6}$
   2. $\frac{2}{6}$
   3. $\frac{3}{6}$
   4. $\frac{5}{6}$

17. (34) The probability of not tossing a 4 on one throw of a single die is
   1. $\frac{1}{6}$
   2. $\frac{2}{6}$
   3. $\frac{3}{6}$
   4. $\frac{5}{6}$

18. (36) The probability of tossing a 3 or a 4 on one throw of a single die is
   1. $\frac{1}{6}$
   2. $\frac{2}{6}$
   3. $\frac{3}{6}$
   4. $\frac{4}{6}$

19. (38) The probability of not tossing either a 3 or a 4 on one throw of a single die is
   1. $\frac{1}{6}$
   2. $\frac{2}{6}$
   3. $\frac{3}{6}$
   4. $\frac{4}{6}$
20. (40) The probability of tossing a number less than 4 on one throw of a single die is
   1. 1/6
   2. 2/6
   3. 3/6
   4. 4/6

21. (42) The probability of tossing a number greater than 4 on one throw of a single die is
   1. 1/6
   2. 2/6
   3. 3/6
   4. 4/6

22. (44) The probability of tossing at least a 4 on one throw of a single die is
   1. 1/6
   2. 2/6
   3. 3/6
   4. 4/6

23. (46) The probability of tossing an odd number on one toss of a single die is
   1. 1/6
   2. 2/6
   3. 3/6
   4. 4/6

24. (48) The probability of not tossing an odd number on one toss of a single die is
   1. 1/6
   2. 2/6
   3. 3/6
   4. 4/6

25. (50) The probability of tossing a number that is odd and greater than 4 on one toss of a single die is
   1. 1/6
   2. 2/6
   3. 3/6
   4. 4/6
26. (52) The probability of tossing a number that is odd or even on one toss of a single die is
1. 0
2. 1/6
3. 3/6
4. 1

27. (54) The probability of tossing a number that is odd and also even on one toss of a single die is
1. 0
2. 1/6
3. 3/6
4. 1

28. (56) The probability of tossing a number that is odd or greater than 4 on one toss of a single die is
1. 1/6
2. 2/6
3. 3/6
4. 4/6

29. (58) The probability of tossing a number that is odd or less than 4 on one toss of a single die is
1. 1/6
2. 2/6
3. 3/6
4. 4/6

30. (60) The probability of tossing a number that is odd and is less than 4 on one toss of a single die is
1. 1/6
2. 2/6
3. 3/6
4. 4/6

31. (62) The probability of tossing a 5 on one toss of two dice is
1. 2/36
2. 4/36
3. 6/36
4. 8/36
32. (64) The probability of tossing a 7 on one toss of two dice is
1. 2/36
2. 4/36
3. 6/36
4. 8/36

33. (66) The probability of tossing an 11 on one toss of two dice is
1. 2/36
2. 4/36
3. 6/36
4. 8/36

34. (68) The probability of tossing a 7 or an 11 on one toss of two dice is
1. 2/36
2. 4/36
3. 6/36
4. 8/36

35. (70) The probability of getting exactly 3 heads on three tosses of a coin is
1. 4/8
2. 3/8
3. 2/8
4. 1/8

36. (72) The probability of getting exactly 2 heads on three tosses of a coin is
1. 4/8
2. 3/8
3. 2/8
4. 1/8

37. (74) The probability of getting more heads than tails on three tosses of a coin is
1. 4/8
2. 3/8
3. 2/8
4. 1/8
38. (76) The probability of not getting more heads than tails on three tosses of a coin is

1. $4/8$
2. $3/8$
3. $2/8$
4. $1/8$

39. (78) A box contains one red and two white balls. Two balls are drawn in succession without replacement. The probability of drawing two white balls is

1. $1/6$
2. $2/6$
3. $3/6$
4. $4/6$

40. (80) A box contains one red and two white balls. Two balls are drawn in succession without replacement. The probability of drawing one white ball and one red ball is

1. $1/6$
2. $2/6$
3. $3/6$
4. $4/6$
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Exercise 7-A

1. Indicate whether the following statements are true or false by placing a punch in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

1. (2) A line graph is more appropriate than a histogram for presenting discrete data.

2. (4) Line graphs appear to associate values with all points on a continuous interval rather than just with points with integers as coordinates.

3. (6) The number which divides the data so that half of the scores are above this number and half are below it is called the mean of the distribution.

4. (8) The number computed by finding the sum of the scores and dividing by the number of scores is called the median of the distribution.

5. (10) The number which appears most frequently in a distribution is called the mode of the distribution.

6. (12) The mode, mean and median are all called measures of dispersion.

7. (14) The set of scores 5,6,7,8 has no mode.

8. (16) The set of 2,3,3,4,5,5 is bimodal.

9. (18) The median, is the only one of the three averages that is affected by one low score.

10. (20) The mean should be used as a representative of a set of data when extreme scores should be reflected in the average.

11. (22) The median is the appropriate measure of central tendency to describe the average salary of the workers in a factory that employs 100 people.
12. (24) The median is the appropriate measure to describe the average salary in a shop staffed by the owner and five employees.

13. (26) A measure of central tendency provides information on the variability of a set of data.

14. (28) The range is the difference between the largest and the smallest number in a set.

15. (30) The mean, the median, and the mode all have the same value in any continuous distribution.

16. (32) The standard deviation is a measure of dispersion.

17. (34) If an interval of 2 standard deviations from the mean is considered approximately all of the data is included.

18. (36) An interval of 1 standard deviation about the mean includes approximately 68% of the data in a normal distribution.

19. (38) A disadvantage of the range as a measure of dispersion is that it relies on only two extreme scores.

20. (40) The standard deviation can only be computed for normal distributions.

II. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

21. (42) The probability that a person who is 15 years old will die before the age of 40 is

1. .02
2. .03
3. .04
4. .05

22. (44) The probability that a person who is 30 years old will die before the age of 50 is

1. .07
2. .08
3. .09
4. .10
23. (46) The one of the following that is not a measure of central tendency is

1. mean
2. median
3. range
4. mode

24. (48) The one of the following that is a measure of dispersion is

1. mean
2. standard deviation
3. median
4. mode

25. (50) The set of scores 71, 79, 79, 81, 87, 89, 95 has a mode of

1. 79
2. 81
3. 83
4. 87

26. (52) The set of scores 71, 79, 79, 81, 87, 89, 95 has a median of

1. 79
2. 81
3. 83
4. 87

27. (54) The set of scores 71, 79, 79, 81, 87, 89, 95 has a mean of

1. 79
2. 81
3. 83
4. 87

28. (56) The set of scores 28, 78, 78, 80, 86, 88, 94 has a mode of

1. 76
2. 78
3. 80
4. 86
29. (58) The set of scores 28, 78, 78, 80, 86, 88, 94 has a median of
   1. 76
   2. 78
   3. 80
   4. 86

30. (60) The set of scores 28, 78, 78, 80, 86, 88, 94 has a mean of
   1. 76
   2. 78
   3. 80
   4. 86

31. (62) The set of scores 6, 7, 8, 9, 10, 12, 15, 15, 20, 28 has a mode of
   1. 11
   2. 12
   3. 13
   4. 15

32. (64) The set of scores 6, 7, 8, 9, 10, 12, 15, 15, 20, 28 has a median of
   1. 11
   2. 12
   3. 13
   4. 15

33. (66) The set of scores 6, 7, 8, 9, 10, 12, 15, 15, 20, 28 has a mean of
   1. 11
   2. 12
   3. 13
   4. 15
In exercise 34 through 39, 100 coins are tossed repeatedly, the distribution of the number of heads is a normal one with a mean of 50 and a standard deviation of 5.

34. (68) The percent of the number of heads between 45 and 55 is

1. 50
2. 68
3. 84
4. 95

35. (70) The percent of the number of heads less than 50 is

1. 50
2. 68
3. 84
4. 95

36. (72) The percent of the number of heads above 40 is

1. 84
2. 95
3. 97.5
4. 99.7

37. (74) The percent of the number of heads between 40 and 60 is

1. 84
2. 95
3. 97.5
4. 99.7

38. (76) The percent of the number of heads below 60 is

1. 84
2. 95
3. 97.5
4. 99.7

39. (78) The percent of the number of heads between 35 and 65 is

1. 84
2. 95
3. 97.5
4. 99.7
40. (80) The standard deviation of the set of scores 6, 9, 5, 3, 6, 7 is

1. 2
2. 6
3. $\sqrt{3.33}$
4. $\sqrt{20}$
Indicate whether the following statements are true or false by placing a punch in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

In exercise 1 through 8 we consider \( n \) trials of an experiment in which the probability of a success is \( p \) and the probability of a failure is \( q \).

1. (2) The expected mean number of successes \((\bar{X})\) on such an experiment is found by the formula \( \bar{X} = np \).

2. (4) The standard deviation of the number of successes is given by the formula \( \sigma = \sqrt{npq} \).

3. (6) When \( n \) is large the normal distribution can be used to determine the limits within which the data will fall.

4. (8) For large \( n \) the number of successes will be between \( \bar{X} - 2 \sigma \) and \( \bar{X} + 2 \sigma \) almost 100% of the time.

5. (10) For large \( n \) the number of successes will be between \( \bar{X} - 3 \sigma \) and \( \bar{X} + 3 \sigma \) 95% of the time.

6. (12) We say that we are almost 100% confident that the number of successes will be between \( \bar{X} - 3 \sigma \) and \( \bar{X} + 3 \sigma \).

7. (14) \( \bar{X} - 2 \sigma \) and \( \bar{X} + 2 \sigma \) are called the 95% confidence limits.

8. (16) In industry a control chart is used to tell when a process is in control.

9. (18) As long as most of the samples produce data which falls below the lower limit on the control chart the process is in control.

10. (20) The upper control limit of a control chart represents two standard deviations added to the mean.
11. (22) The correspondence of each positive integer \( n \) with a unique number \( f(n) \) is called a function.

12. (24) A correspondence may be presented as a set of ordered pairs of numbers.

13. (26) The set \( \{(2,1),(2,2),(1,2)\} \) is a relation.

14. (28) The set \( \{(1,1),(1,2),(3,1)\} \) is a function.

15. (30) The domain of \( \{(1,1),(2,2),(1,3)\} \) is \( \{1,2,3\} \).

16. (32) The range of \( \{(1,1),(2,2),(1,3)\} \) is \( \{1,2\} \).

17. (34) A relation is a special kind of function.

18. (36) Any relation such that no two ordered pairs have the same second element and different first elements is called a function.

19. (38) The set consisting of the first elements of the set of ordered pairs in a relation is called the domain of the relation.

20. (40) The set consisting of the second elements of the set of ordered pairs in a relation is called the range of the relation.

II. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

In exercise 20 through 24 we consider the special lunch served in a cafeteria. Experience shows that \( \frac{1}{3} \) of the customers choose the special lunch. On those days when 450 customers are served;

21. (42) The mean number of lunches to be expected is

1. 50
2. 100
3. 150
4. 200
22. (44) The standard deviation of the distribution of the number of lunches ordered is
1. 10  
2. 5  
3. 3  
4. 1

23. (46) We can say with almost 100% confidence that the number of lunches ordered will be between
1. 130 and 170  
2. 125 and 175  
3. 120 and 180  
4. 115 and 185

24. (48) We can say with approximately 95% confidence that the number of lunches will be between
1. 135 and 165  
2. 130 and 170  
3. 125 and 175  
4. 120 and 180

In exercise 25 through 30 we consider the production of a new kind of light bulb. Experience has shown that 1/10 of the light bulbs are defective. We check each batch of 100 light bulbs.

25. (50) The mean number of defective light bulbs to be expected is
1. 50  
2. 3  
3. 5  
4. 10

26. (52) The standard deviation of the distribution of the number of defective bulbs is
1. 3  
2. 5  
3. 10  
4. 50

27. (54) The upper control limit for the control chart is
1. 10  
2. 13  
3. 16  
4. 19
28. (56) The lower control limit for the control chart is

1. 1
2. 3
3. 4
4. 7

29. (58) We say that the process is in control if the number of defectives is between

1. 1 and 19
2. 3 and 16
3. 4 and 13
4. 7 and 10

30. (60) We would suspect that something was wrong if the number of defectives was greater than

1. 1
2. 10
3. 19
4. 29

31. (62) The functional rule representing the mapping 1 \rightarrow 2 is

1 \rightarrow 2
2 \rightarrow 5
3 \rightarrow 8
4 \rightarrow 11

1. \ n \rightarrow \frac{1}{3} \ (n + 1)
2. \ n \rightarrow 3n - 1
3. \ n \rightarrow n + 7
4. \ n \rightarrow 2n

32. (64) The functional rule representing the correspondence 1 \rightarrow 1 is

1 \rightarrow 1
2 \rightarrow 4
3 \rightarrow 9
4 \rightarrow 16

1. \ n \rightarrow n
2. \ n \rightarrow 2n
3. \ n \rightarrow 3n^2
4. \ n \rightarrow n
33. (66) The set of ordered pairs representing the mapping \(1 \rightarrow 2\) is
\[
2 \rightarrow 5
\]
\[
3 \rightarrow 8
\]
\[
4 \rightarrow 11
\]
1. \(\{(2,1),(5,2),(8,3),(11,4)\}\)
2. \(\{(1,1),(2,2),(3,3),(4,4)\}\)
3. \(\{(1,2),(2,5),(3,8),(4,11)\}\)
4. \(\{(1,2),(2,3),(3,4),(4,5)\}\)

34. (68) If \(A = \{1,3\}\) and \(B = \{2,4\}\) then \(A \times B\) is
1. \(\{(1,2),(1,4),(3,2)\}\)
2. \(\{(2,1),(4,1),(4,3),(2,3)\}\)
3. \(\{(1,2),(3,4)\}\)
4. \(\{1,2,3,4\}\)

35. (70) The domain of the relation \(\{(10,1),(9,2),(5,5)\}\) is
1. \(\{10,9,5\}\)
2. \(\{1,2,5\}\)
3. \(\{(1,10),(2,9),(5,5)\}\)
4. \(\{10,1,9,2,5\}\)

36. (72) The range of the relation \(\{(10,1),(9,2),(5,5)\}\) is
1. \(\{10,9,5\}\)
2. \(\{1,2,5\}\)
3. \(\{(1,10),(2,9),(5,5)\}\)
4. \(\{1,2,5,9,10\}\)

37. (74) When \(f(x) = x + 3\) then \(f(2)\) is
1. 2
2. 5
3. 3
4. -1

38. (76) When \(f(x) = 3x + 4\) then \(f(3)\) is
1. 3
2. 4
3. 9
4. 13
39. (78) When \( f(x) = 2x^2 + 4 \) then \( f(3) \) is

1. 40
2. 22
3. 18
4. 4

40. (80) For positive integers \( x \) and the relation expressed by the equation \( y = 2x + 3 \), if \( x \) is increased by 1 then \( y \) is increased by

1. 1
2. 2
3. 3
4. 4
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Exercise 9-A

I. Indicate whether the following statements are true or false by placing a punch in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

1. (2) If a relation is a function then each vertical line intersects the graph of the relation in at most one point.

2. (4) The change in the value of $x$ as $y$ is increased by 1 is designated by $\Delta y$.

3. (6) If $y$ is a linear function of $x$ then $\Delta y$ is a constant.

4. (8) If $y = 2x + 3$ then $\Delta y$ is a constant.

5. (10) If $y = ax + b$ then $b = \Delta y$.

6. (12) A function like $y = 2x^2 + 3$ is called a polynomial function.

7. (14) If $y = x^2 - 2$ then $\Delta y$ is a constant.

8. (16) If $y = 2x^2 - 3$ then $\Delta^2 y$ is a constant.

9. (18) If $\Delta^3 y$ is a constant, then $f(x)$ may be expressed in the form $f(x) = ax^2 + bx + c$.

10. (20) If a relation is a function then its inverse is a function.

11. (22) If a relation is not a function then its inverse is not a function.

12. (24) If we interchange the $x$ and the $y$ in an equation we have the equation for the inverse of the original function.

13. (26) An expression in which the $x$ occurs as an exponent is called a logarithmic function.
14. (28) The inverse of the logarithmic function is the exponential function.

15. (30) The logarithm of a number is the power to which the base is raised to obtain the number.

16. (32) \( \log(a \times b) = \log a + \log b \)

17. (34) \( \frac{\log a}{b} = \frac{\log a}{\log b} \)

18. (36) \( \log a^n = n \log a \)

19. (38) \( \log \sqrt[n]{a} = \frac{\log a}{n} \)

20. (40) \( \log(a - b) = \log a - \log b \)

II. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

21. (42) If \( y = 5x - 3 \) then \( \Delta y \) is
   1. 1
   2. 2
   3. 3
   4. 5

22. (44) If \( y = 3x - 2 \) then \( \Delta y \) is
   1. 1
   2. 2
   3. 3
   4. 5

23. (46) If \( y = ax + b \) and \( \Delta y = 2 \) and \( (2, 5) \) is an ordered pair of the function then the value of \( b \) is
   1. 1
   2. 2
   3. 4
   4. 5

24. (48) If \( f(x) = x^2 - 2 \) then \( f(2) \) is
   1. 0
   2. 2
   3. 4
   4. 5
25. (50) If \( f(x) = 2x^2 - x + 5 \) then \( f(3) \) is

1. 3
2. 15
3. 18
4. 20

26. (52) If \( f(x) = x^2 - 3x + 2 \) then \( f(x + 1) \) is

1. \( x^2 + 1 \)
2. \( x^2 - 3x + 2 \)
3. \( x^2 - x \)
4. \( x^2 - 3x \)

27. (54) If \( f(x) = x^2 - 3x + 2 \) then \( \Delta y \) is

1. 2
2. \( 2x - 2 \)
3. \( 3x \)
4. \( x^2 \)

28. (56) If \( f(x) = x^2 - 3x + 2 \) then \( \Delta^2 y \) is

1. 1
2. 2
3. 3
4. 4

29. (58) If \( f(x) = 2x^2 + 1 \) then \( \Delta^2 y \) is

1. 1
2. 2
3. 3
4. 4

30. (60) The inverse of the relation \( \{(1,2),(2,1),(3,4)\} \) is

1. \( \{(3,4),(2,1),(1,2)\} \)
2. \( \{(1,2),(2,1),(4,3)\} \)
3. \( \{(2,1),(1,2),(3,4)\} \)
4. \( \{(1,2),(2,1),(3,4)\} \)

31. (62) The relation which is also a function is

1. \( \{(1,1),(2,2),(3,3)\} \)
2. \( \{(1,1),(1,2),(1,3)\} \)
3. \( \{(1,2),(2,1),(2,3)\} \)
4. \( \{(1,2),(2,3),(2,1)\} \)
32. (64) The relation whose inverse is a function is
1. \( \{(1,1),(1,2),(1,3)\} \)
2. \( \{(1,1),(2,1),(3,1)\} \)
3. \( \{(1,1),(1,2),(2,2)\} \)
4. \( \{(2,3),(3,2),(3,3)\} \)

33. (66) The inverse of the relation determined by the equation \( y = x + 1 \) is
1. \( y = x + 1 \)
2. \( y = 1 + x \)
3. \( x + 1 = y \)
4. \( y = x - 1 \)

34. (68) The inverse of the relation determined by the equation \( y = 2x + 3 \)
1. \( y = 3 + 2x \)
2. \( y = -3/2 + 1/2x \)
3. \( y = 2x + 3 \)
4. \( 2x + 3 = y \)

35. (70) If \( y = 3^x \) then for \( x = 4 \) the value of \( y \) is
1. 12
2. 64
3. 81
4. 144

36. (72) If \( \log_2128 = x \) then \( 2^x \) is
1. 2
2. 7
3. 128
4. 256

37. (74) The value of \( \log_2256 \) is
1. 8
2. 16
3. 256
4. 512

38. (76) The value of \( \log_21/16 \) is
1. -16
2. -4
3. 1/4
4. 1/2
39. \( \log_2 (16 \times 32) \) is

1. 2
2. 4
3. 5
4. 9

40. \( \log_2 (128^5) \) is

1. 8
2. 16
3. 35
4. 64
APPENDIX C
Mathematics 112

Exercise 1-B

I. Indicate whether the following statements are true or false by placing a punch in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

1. (2) Every counting number is a whole number.
2. (4) Every integer is a whole number.
3. (6) Every integer is a rational number.
4. (8) Every integer is a real number.
5. (10) Every real number is a rational number.
6. (12) Every real number is an irrational number.
7. (14) \(-3 + -5 = 8\)
8. (16) \(-3 + 5 = 2\)
9. (18) \((-3)(5) = -15\)
10. (20) \((-2)(-5) = 10\)
11. (22) The whole numbers may be considered as the union of the counting numbers and zero.
12. (24) The integers may be considered as the union of the whole numbers and the negative integers.
13. (26) Every real number can be represented by a decimal.
14. (28) The set of integers is closed under subtraction.
15. (30) The set of integers is closed under division.
16. (32) The graph of the set of integers between -5 and 4 contains eight points.
17. (34) The set of integers form a field.
18. (36) The set of whole numbers form a group.
19. (38) There is a one-to-one correspondence between the elements of the set of rational numbers and the set of points on the number line.

20. (40) There is a rational number replacement for \( n \) in the statement \( 2n = 9 \).

21. (42) There is an integer replacement for \( n \) in the statement \( n + 5 = 2 \).

22. (44) The set of rational numbers is said to be dense.

23. (46) Real numbers that are rational numbers are represented by terminating or by infinite repeating decimals.

24. (48) The reciprocal of every rational number except \( 0 \) is a rational number.

25. (50) The set of negative integers is the same as the set of opposites of the whole numbers.

II. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3, or 4 to indicate your choice from the four possible replies.

26. (52) The property illustrated by the statement \((xy)z = x(yz)\) is

1. distributive
2. associative
3. commutative
4. inverse

27. (54) The property illustrated by the statement \(xy - xz = x(y - z)\) is

1. distributive
2. associative
3. commutative
4. inverse
28. (56) The property illustrated by the statement $xy = yz$ is
   1. identity
   2. associative
   3. commutative
   4. inverse

29. (58) The property illustrated by the statement $5 + (-5) = -5 + 5 = 0$ is
   1. associative
   2. commutative
   3. identity
   4. inverse

30. (60) The property illustrated by the statement $\frac{2}{3} \times \frac{3}{2} = 1$ is
   1. associative
   2. commutative
   3. identity
   4. inverse

31. (62) The property illustrated by the statement $1 \times 2 = 2 \times 1 = 2$ is
   1. associative
   2. commutative
   3. identity
   4. inverse

32. (64) The number set that does not contain a replacement for $n$ in the statement $n + 3 = 2$ is
   1. whole numbers
   2. integers
   3. rational numbers
   4. real numbers

33. (66) The set of numbers that contains a solution to the equation $n^2 - 1 = 2$ is
   1. whole numbers
   2. integers
   3. rational numbers
   4. irrational numbers
34. (68) The set of numbers that contains a solution to the \( \frac{2}{3}n = 5 \) is

1. whole numbers
2. integers
3. irrational numbers
4. rational numbers

35. (70) The number \( \frac{2}{3} \) can be represented as

1. a terminating decimal
2. a repeating decimal
3. a nonterminating, nonrepeating decimal
4. none of the above

36. (72) The number \( \frac{5}{4} \) can be represented as

1. a terminating decimal
2. a repeating decimal
3. a nonterminating, nonrepeating decimal
4. none of the above

37. (74) The number \( \sqrt{4} \) can be written as

1. a terminating decimal
2. a repeating decimal
3. a nonterminating, nonrepeating decimal
4. none of the above

38. (76) The correct graph of real numbers between -2 and 2 is

1. \[ -2 \quad -1 \quad 0 \quad 1 \quad 2 \]
2. \[ -2 \quad -1 \quad 0 \quad 1 \quad 2 \]
3. \[ -2 \quad -1 \quad 0 \quad 1 \quad 2 \]
4. \[ -2 \quad -1 \quad 0 \quad 1 \quad 2 \]

39. (78) The correct graph for the rational number from -2 through 2 is (see question 38 for the replies).
40. (80) The set of negatives of the integers is

1. the negative integers
2. the integers
3. the whole numbers
4. none of the above
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Exercise 2-B

I. Indicate whether the following statements are true or false by placing a punch in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

1. (2) A statement of equality is called an equation whether it is true or false.

2. (4) An open sentence has a variable as part of the sentence.

3. (6) A placeholder for a member of the replacement set is called an identity.

4. (8) \(a < b\) if and only if there is a positive number \(c\) such that \(a + c = b\).

5. (10) When a variable is replaced by a member of the solution set we may get a false statement.

6. (12) An equation that has the empty set for the truth set is called an identity.

7. (14) The inequality \(x + 2 > x\) is an identity.

8. (16) The equation \(x + 2 = x\) is an identity.

9. (18) The solution set of the inequality \(x + 2 < x\)

10. (20) The solution set for \(2x = 5\) is the empty set when the replacement set is the integers.

11. (22) If \(A = \{2, 4, 6\}\) and \(B = \{4, 5, 6\}\) then \(A \cup B = \{2, 4, 5, 6\}\).

12. (24) If \(A = \{1, 3, 5\}\) and \(B = \{2, 4, 6\}\) then \(A \cap B = \emptyset\).

13. (26) The integer solution set of the statement \(x > 2\) and \(x - 1 < 3\) is \(\{2, 3\}\).

14. (28) The integer solution set of the statement \(x \geq 1\) or \(x + 1 \leq 5\) is all integers.
15. (30) $3 > 1$ and $-2 < -3$.

16. (32) $3 < 7$ and $|-2| = 2$.

17. (34) $-1 > 0$ or $0 > -2$.

18. (36) $2 < 0$ or $|5| = -5$.

19. (38) There are two numbers whose distance from zero is 3, namely -3 and 3.

20. (40) The absolute value of -3 is 3.

21. (42) The symbol $|3|$ represents the distance of 3 from the origin.

22. (44) The solution set of $|x| = 5$ consists of two points.

23. (46) The solution set of $|x| > 3$ consists of the numbers whose distance from the origin is greater than 3.

24. (48) The solution set of $|x - 2| = 3$ consists of the numbers whose distance from the number 2 is 3.

25. (50) The solution set of $|x| = -5$ is a single real number.

II. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

26. (52) $3 - 7 < 4 - 8$ is

1. a true statement of equality.
2. a false statement of equality.
3. a true statement of inequality.
4. a false statement of inequality.

27. (54) $-23/25 > -23/26$ is

1. a true statement of equality.
2. a false statement of equality.
3. a true statement of inequality.
4. a false statement of inequality.
28. (56) $5 - 3 = 8 - 6$ is
   1. a true statement of equality.
   2. a false statement of equality.
   3. a true statement of inequality.
   4. a false statement of inequality.

29. (58) The truth set of the sentence $x + 3 = x$ is
   1. a single number
   2. the empty set
   3. the entire replacement set
   4. none of the above

30. (60) The truth set of the sentence $x < x + 5$ is
   1. a single number
   2. the empty set
   3. the entire replacement set
   4. none of the above

31. (62) The truth set of the sentence $x + 1 > x + 2$ is
   1. a single number
   2. the empty set
   3. the entire replacement set
   4. none of the above

32. (64) The graph of the solution set of the sentence $x + 2 \leq 7$ is
   1. a point
   2. a half-line
   3. a ray
   4. a line

33. (66) The graph of the solution set of the sentence $x \leq x + 3$ is
   1. a point
   2. a half-line
   3. a ray
   4. a line
34. (68) The graph of the solution set of the sentence \( x + 3 = 2 \) is

1. a point
2. a half-line
3. a ray
4. a line segment

35. (70) The graph of the solution set of the sentence \(-5 \leq x \leq 5\) is

1. a point
2. a half-line
3. a ray
4. a line segment

36. (72) The graph of the solution set of the compound statement \( x \geq 5 \text{ or } x \leq 1 \) when the replacement set is the real numbers is

1. two points
2. a line segment
3. union of two rays
4. a line

37. (74) The graph of \( |x| \geq 7 \) is

1. two points
2. a line segment
3. union of two rays
4. a line

38. (76) The graph of \( |x + 3| = 5 \) is

1. two points
2. a line segment
3. unions of two rays
4. a line

39. (78) The graph of \( x \geq 0 \) is

1. two points
2. a line segment
3. union of two rays
4. a line
40. (80) The graph of \( |x - 3| \geq 1 \) is

1. two points
2. a line segment
3. union of two rays
4. a line
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Exercise 3-B

I. Indicate whether the following statements are true or false by placing a punch in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

1. (2) If we multiply both members of an equation by the same number we obtain an equivalent sentence.

2. (4) If \( a = b \) and \( c = d \) then \( a - c = b - d \).

3. (6) If \( a = b \) and \( c \neq 0 \) then \( a/c = b/c \).

4. (8) If \( a < b \) and \( c = d \) then \( a - c < b - d \).

5. (10) If \( a < b \) then \( ac < bc \) if \( c > 0 \).

6. (12) If both members of an inequality are multiplied by a positive number, it is necessary to reverse the sense of the inequality to obtain an equivalent sentence.

7. (14) If a replacement is made for only one of the variables, in an open sentence in two real variables, the sentence remains an open sentence.

8. (16) The set of all ordered pairs whose coordinates belong to the given set \( U \) is \( U \times U \).

9. (18) The cartesian product \( A \times B \) is the set of all ordered pairs with the first element from \( A \) and the second element from \( B \).

10. (20) The graph of the cartesian product \( U \times U \) for \( U = \{1, 2, 3\} \) consists of 9 points.

II. Consider the steps in the solutions of \( 3x + 5 = 9 \). Select the column on the answer card corresponding to the step and punch the number 1, 2, 3 or 4 to indicate your choice from the four possible reasons.
11. (22) \((3x + 5) + -5 = 9 + -5\)
   1. addition, =
   2. addition
   3. associative, +
   4. zero, +

12. (24) \(3x + (5 + -5) = 9 + -5\)
   1. addition, =
   2. addition
   3. associative, +
   4. zero, -

13. (26) \(3x + 0 = 4\)
   1. addition, =
   2. addition
   3. associative, +
   4. zero, -

14. (28) \(3x = 4\)
   1. addition, =
   2. addition
   3. associative, +
   4. zero, -

15. (30) \(\frac{1}{3}(3x) = \frac{1}{3} \cdot 4\)
   1. multiplication, =
   2. multiplication
   3. associative, x
   4. one, x

16. (32) \((1/3 \cdot 3)x = 1/3 \cdot 4\)
   1. multiplication, =
   2. multiplication
   3. associative, x
   4. one, x

17. (34) \(1x = 4/3\)
   1. multiplication, =
   2. multiplication
   3. associative, x
   4. one, x
18. (36) \( x = \frac{4}{3} \)

1. multiplication, =
2. multiplication
3. associative, x
4. one, x

III. Consider the steps in the solution of \( 5x + 4 = 9 \).
Select the column on the answer card corresponding to the step and punch in the number 1, 2, 3 or 4 to indicate your choice from the four possible reasons.

19. (38) \( (5x + 4) + -4 = 9 + -4 \)

1. addition,>
2. addition
3. associative, +
4. zero, +

20. (40) \( 5x + (4 + -4) = 9 + -4 \)

1. addition,>
2. addition
3. associative, +
4. zero, +

21. (42) \( 5x + 0 = 5 \)

1. addition,>
2. addition
3. associative, +
4. zero, +

22. (44) \( 5x = 5 \)

1. addition,>
2. addition
3. associative, +
4. zero, +

23. (46) \( \frac{1}{5} (5x) = \frac{1}{5} \cdot 5 \)

1. multiplication,>
2. multiplication
3. associative, x
4. one, x
24. (48) \( \frac{1}{5} - 5 \times \frac{1}{5} \cdot 5 \)
   1. multiplication, \( > \)
   2. multiplication
   3. associative, \( x \)
   4. one, \( x \)

25. (50) \( 1x > 1 \)
   1. multiplication, \( > \)
   2. multiplication
   3. associative, \( x \)
   4. one, \( x \)

26. (52) \( x > 1 \)
   1. multiplication, \( > \)
   2. multiplication
   3. associative, \( x \)
   4. one, \( x \)

IV. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

27. (54) The solution for the open sentence \( x + 6 = 3 \) is
   1. \(-3\)
   2. \(3\)
   3. \(9\)
   4. \(-9\)

28. (56) The solution for the open sentence \( x - 6 = 3 \) is
   1. \(-3\)
   2. \(3\)
   3. \(9\)
   4. \(-9\)

29. (58) The solution for the open sentence \( 2x + 1 = 7 \) is
   1. \(-3\)
   2. \(3\)
   3. \(\frac{8}{2}\)
   4. \(-\frac{8}{2}\)
30. (60) The solution for the open sentence $2x - 1 = 7$ is
   1. $-3$
   2. $3$
   3. $4$
   4. $-4$

31. (62) The solution for the open sentence $-2x - 1 = 7$ is
   1. $-4$
   2. $4$
   3. $3$
   4. $-3$

32. (64) If $2x + 1 < 3$ then
   1. $x > -1$
   2. $x < 1$
   3. $x < 2$
   4. $x > -2$

33. (66) If $-2x + 1 < 3$ then
   1. $x > -1$
   2. $x < 1$
   3. $x < 2$
   4. $x > -2$

34. (68) The solution for the open sentence $2/5x - 3 = 7$ is
   1. $25$
   2. $4$
   3. $8/5$
   4. $10$

35. (70) The solution for the open sentence $2/5x + 3 = 7$ is
   1. $25$
   2. $4$
   3. $8/5$
   4. $10$

36. (72) If $U$ is $\{2,3\}$ then $U \times U$ is
   1. $\{2,3\}$
   2. $\{(2,3)\}$
   3. $\{(2,3),(3,2)\}$
   4. $\{(2,2),(3,3),(2,3),(3,2)\}$
37. (74) If the replacement set is \( \{2,3,4\} \) then the solution set for \( x + y = 5 \) is

1. \( \{(2,3)\} \)
2. \( \{(2,3),(3,2)\} \)
3. \( \{5\} \)
4. empty set

38. (76) If the replacement set is \( \{2,3,4\} \) then the solution set for \( x = y \) is

1. \( \{2,3,4\} \)
2. \( \{2\} \)
3. \( \{(2,2),(3,3),(4,4)\} \)
4. empty set

39. (78) If the replacement set is the set \( \{2,3,4\} \) then the solution set for \( x < y \) is

1. \( \{(2,2),(3,3),(4,4)\} \)
2. \( \{2,3,4\} \)
3. \( \{(2,3),(3,2),(3,4),(4,3)\} \)
4. \( \{(2,3),(2,4),(3,4)\} \)

40. (80) If \( A = \{3\} \) and \( B = \{2\} \) then \( A \times B \) is

1. \( \{(3,2)\} \)
2. \( \{(2,3),(3,2)\} \)
3. \( \{(2,2),(3,2)\} \)
4. \( \{(2,3)\} \)
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Exercise 4-B

I. Indicate whether the following statements are true or false by placing a punch, in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

1. (2) If U is the set of real numbers then the graph of \( U \times U \) consists of the entire plane.

2. (4) The numbers of each ordered pair of integers are the coordinates of a point of the plane.

3. (6) Each point of a plane can be represented by an ordered pair of real numbers.

4. (8) Each ordered pair of real numbers identify a unique point of the plane.

5. (10) The point \((x,y)\) is the term used to designate the point with the coordinates \((x,y)\).

6. (12) If U is the real numbers then the graph of \( y = 2x + 3 \) is a straight line.

7. (14) Any equation that can be expressed in the form \( ax + by + c = 0 \) where \( a \) and \( b \) are not both zero has a straight line as its graph.

8. (16) Any equation that can be expressed in the form \( ax + by + c = 0 \) where \( a \) and \( b \) are not both zero has a straight line as its graph.

9. (18) In order to graph a linear equation we must determine two members of the solution set.

10. (20) The graph of \( y = 3x + 2 \) has a y intercept of 2.

11. (22) The graph of \( 3x - 2y + 3 = 0 \) has an x intercept of -1.

12. (24) The point \((1,3)\) is on the graph of \( 3x - 2y + 3 = 0 \).
13. (26) The graph of \( y < 3x + 2 \) consists of all of the points in the half plane below the line \( y = 3x + 2 \).

14. (28) The point \((3,5)\) is in the solution set of \( y \leq 2x + 3 \).

15. (30) To solve a set of two simultaneous linear equations we find the set of ordered pairs that are solutions of one of the equations.

16. (32) An ordered pair that satisfies both of the equations in a set of two simultaneous linear equations is a member of the solution set.

17. (34) When the solution set of two simultaneous linear equations is the empty set the graphs of the two equations are parallel.

18. (36) To solve the system of linear equations
\[
\begin{align*}
x + y - 5 &= 0 \\
x - y - 1 &= 0
\end{align*}
\]
is to find the solution set of the compound sentence \( x + y - 5 = 0 \) and \( x - y - 1 = 0 \).

19. (38) When we find the solution set of a system of two linear equations we find the union of the solution sets of the two linear equations.

20. (40) The solution set of the sentence \((x - y - 1)(x - y - 2) = 0\) is equivalent to the solution set of the statement \( x - y - 1 = 0 \) or \( x - y + 2 = 0 \).

21. (42) The union of the solution sets of two linear equations is represented by the total graph of the two equations.

22. (44) The solution set of the sentence \((x - y - 1)(x - y + 2) = 0\) is the intersection of the solution set of \( x - y - 1 = 0 \) with the solution set \( x - y + 2 = 0 \).

23. (46) A sentence that may be expressed in the form \( y = ax^2 + bx + c \) is called a linear equation.

24. (48) The graph of \( x = y^2 \) is a parabola.
25. (50) The x axis is the axis of symmetry for the graph $y = x^2$.

26. (52) The point (2,4) is the vertex of the graph of $y = x^2$.

27. (54) The point (1,8) is on the graph of $y = (x - 3)^2 + 1$.

28. (56) The axis of symmetry for the graph of $y = (x - 3)^2$ is $x = 3$.

29. (58) The graph of $y = (x + 2)^2 - 1$ can be obtained from the graph of $y = (x + 2)^2$ by translating the graph of $y = (x + 2)^2$ one unit down.

30. (60) The graph of $y = (x - 2)^2$ has the same shape as $y = x^2$.

II. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

31. (62) The x intercept of the graph of $2x - 3y = 6$ is

1. -2
2. 0
3. 3
4. -3

32. (64) The y intercept of the graph $2x - 3y = 6$ is

1. -2
2. 0
3. 3
4. -3

33. (66) The x intercept of the graph $2x + y - 5 = 0$ is

1. 0
2. 5/2
3. -5/2
4. 5
34. (68) The y intercept of the graph $2x + y - 5 = 0$ is
1. 0
2. $5/2$
3. $-5/2$
4. 5

35. (70) When the sentence $3x + y = 9$ is solved for $y$ we get
1. $y = 9$
2. $y = 9 - 3x$
3. $y = 9 + 3x$
4. $y = 3$

36. (72) When the sentence $9x - 3y = 9$ is solved for $y$ we get
1. $y = 3 - 3x$
2. $y = 3x + 3$
3. $y = 9x - 9$
4. $y = -3 + 3x$

37. (74) When the sentence $x + 3y \leq 6$ is solved for $y$ we get
1. $y \leq 6 - x$
2. $y \leq 2 - x$
3. $y \geq 2 - 1/3x$
4. $y \leq 2 - 1/3x$

38. (76) When the sentence $3x + 3y = 6$ is solved for $y$ we get
1. $y = 2 - x$
2. $y = 6 + 3x$
3. $y = 2 + 3x$
4. $y = 2 - 3x$

39. (78) The axis of symmetry for $y = (x + 2)^2$ is
1. y axis
2. $x = 2$
3. $x = -2$
4. $x = 3$
40. (80) The vertex of the graph \( y = (x + 2)^2 + 3 \) is

1. \((0,0)\)
2. \((-2,3)\)
3. \((2,-3)\)
4. \((2,3)\)
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Exercise 5-B

I. Indicate whether the following statements are true or false by placing a punch, in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

Questions 1-12 refer to the mathematical system represented by the table

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>a</td>
<td>a</td>
<td>a</td>
</tr>
<tr>
<td>b</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>c</td>
<td>a</td>
<td>c</td>
<td>b</td>
</tr>
</tbody>
</table>

1. (2) The system is closed under multiplication.

2. (4) \(a \times b = a\).

3. (6) Multiplication is commutative.

4. (8) The multiplicative identity is \(c\).

5. (10) The multiplicative identity is \(b\).

6. (12) The multiplicative identity is \(a\).

7. (14) \(a \times (b \times c) = b\).

8. (16) \(a \times (b \times c) = (a \times b) \times c\).

9. (18) The multiplicative inverse of \(b\) is \(a\).

10. (20) The multiplicative inverse of \(c\) is \(b\).

11. (22) The multiplicative inverse of \(a\) is \(a\).

12. (24) The system forms a commutative group.

13. (26) The statement \(\neg p\) is true when \(p\) is false.

14. (28) The statement \(q\) is the conclusion of the statement \(p \rightarrow q\).

15. (30) Any statement that is always true is a tautology.
16. (32) A conditional statement and its inverse always have the same truth values.

17. (34) A statement in logic must be identifiable as true or as false.

18. (36) The statement \( p \land q \) is false when both \( p \) and \( q \) are false and is true otherwise.

19. (38) The compound statement \( p \lor q \) is false when both \( p \) and \( q \) are false and is true otherwise.

20. (40) When \( p \) is true and \( q \) is false the statement \( p \rightarrow q \) is defined to be true.

21. (42) The statement \( p \leftrightarrow q \) is true when \( p \) and \( q \) are alike and is false otherwise.

22. (44) The statement \( p \land (\sim p) \) is a tautology.

23. (46) If a conditional statement is false then its converse must be false.

24. (48) If a conditional statement is false then its inverse must be true.

25. (50) If a conditional statement is false then the contrapositive must be false.

II. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

26. (52) The compound statement \( q \land p \) is true when
   1. \( p \) is true and \( q \) is true
   2. \( p \) is true and \( q \) is false
   3. \( p \) is false and \( q \) is true
   4. \( p \) is false and \( q \) is false

27. (54) The compound statement \( q \lor p \) is false when
   1. \( p \) is true and \( q \) is true
   2. \( p \) is true and \( q \) is false
   3. \( p \) is false and \( q \) is true
   4. \( p \) is false and \( q \) is false
28. (56) The compound statement $q \rightarrow p$ is false when
1. $p$ is true and $q$ is true
2. $p$ is true and $q$ is false
3. $p$ is false and $q$ is true
4. $p$ is false and $q$ is false

29. (58) The compound statement $p \lor (\neg q)$ will be false when
1. $p$ is true and $q$ is true
2. $p$ is true and $q$ is false
3. $p$ is false and $q$ is true
4. $p$ is false and $q$ is false

30. (60) The compound statement $(\neg p) \land q$ will be true when
1. $p$ is true and $q$ is true
2. $p$ is true and $q$ is false
3. $p$ is false and $q$ is true
4. $p$ is false and $q$ is false

31. (62) The compound statement $q \land (q \rightarrow p)$ is true when
1. $p$ is true and $q$ is true
2. $p$ is true and $q$ is false
3. $p$ is false and $q$ is true
4. $p$ is false and $q$ is false

32. (64) The converse of the statement, if $x = 2$ then $x^2 = 4$ is
1. if $x \neq 2$ then $x^2 \neq 4$
2. if $x^2 = 4$ then $x = 2$
3. if $x \neq 2$ then $x^2 = 4$
4. if $x^2 \neq 4$ then $x \neq 2$

33. (66) The inverse of the statement, if $x = 2$ then $x^2 = 4$ is
1. if $x \neq 2$ then $x^2 \neq 4$
2. if $x^2 = 4$ then $x = 2$
3. if $x \neq 2$ then $x^2 = 4$
4. if $x^2 \neq 4$ then $x \neq 2$

34. (68) The contrapositive of the statement, if $x = 2$, then $x^2 = 4$ is
1. if $x \neq 2$ then $x^2 \neq 4$
2. if $x^2 = 4$ then $x = 2$
3. if $x \neq 2$ then $x^2 = 4$
4. if $x^2 \neq 4$ then $x \neq 2$
35. (70) The name given to the compound statement 
\( x = 2 \) or \( x^2 = 4 \) is
1. conjunction
2. conditional
3. disjunction
4. biconditional

36. (72) The name given to the compound statement if 
\( x = 2 \) then \( x^2 = 4 \) is
1. conjunction
2. conditional
3. disjunction
4. biconditional

37. (74) The name given to the statement \( x = 2 \) and 
\( x^2 = 4 \) is
1. conjunction
2. conditional
3. disjunction
4. biconditional

38. (76) The name given to the compound statement 
\( x = 2 \) if and only if \( x^2 = 4 \) is
1. conjunction
2. conditional
3. disjunction
4. biconditional

39. (78) The tautology among the following statement is
1. \((p \leftrightarrow q) \rightarrow (p \rightarrow q)\)
2. \(p \lor (p \rightarrow q) \rightarrow q\)
3. \(q \land (p \rightarrow q)\)
4. \((p \rightarrow q) \land (q \rightarrow r)\)

40. (80) The property that is not a property of all 
groups in
1. closure
2. inverse
3. associativity
4. distributive
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Exercise 6-B

I. Indicate whether the following statements are true or false by placing a punch, in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

1. (2) The results (heads, tails) obtained by rolling a honest coin is usually considered to be a set of equally likely events.

2. (4) By probability of success we mean the ratio of the number of ways a success can occur to the total number of events.

3. (6) By probability of failure we mean the ratio of the number of events to the number of ways a failure can occur.

4. (8) The probability of any outcome is between 0 and 1.

5. (10) When success is inevitable the probability is 1.

6. (12) When an outcome cannot occur the probability is 0.

7. (14) The probability of getting a head or a tail on a single toss of a normal die is 0.

8. (16) The probability of tossing a 7 with a single toss of a normal die is 0.

9. (18) Since probability means "probability in the long run." You would not expect exactly 4 heads in 8 tosses of an honest coin.

10. (20) Despite minor fluctuations in the long run the graphs of the per cent of heads and tails will come closer and closer to the 50% line.

11. (22) If the probability of an outcome is 1/3 then the probability of the outcome not occurring is 2/3.

12. (24) The list of possible outcomes when two dice are rolled is called a sample space.
13. (26) The possible outcome for the roll of two dice is represented as ordered pairs of numbers.

14. (28) If three coins are tossed the sample space has 4 elements.

15. (30) For each way the first die may come up the second die may come up in 6 ways.

II. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

16. (32) The probability of tossing a 5 on one throw of a single die is

   1. 1/6
   2. 2/6
   3. 3/6
   4. 5/6

17. (34) The probability of not tossing a 5 on one throw of a single die is

   1. 1/6
   2. 2/6
   3. 3/6
   4. 5/6

18. (36) The probability of tossing a 4 or a 5 on one throw of a single die is

   1. 1/6
   2. 2/6
   3. 3/6
   4. 4/6

19. (38) The probability of not tossing either a 4 or a 5 on one throw of a single die is

   1. 1/6
   2. 2/6
   3. 3/6
   4. 4/6
20. (40) The probability of tossing a number less than 5 on one throw of a single die is

1. 1/6
2. 2/6
3. 3/6
4. 4/6

21. (42) The probability of tossing a number greater than 5 on one throw of a single die is

1. 1/6
2. 2/6
3. 3/6
4. 4/6

22. (44) The probability of tossing at least a 5 on one throw of a single die is

1. 1/6
2. 2/6
3. 3/6
4. 4/6

23. (46) The probability of tossing an even number on one toss of a single die is

1. 1/6
2. 2/6
3. 3/6
4. 4/6

24. (48) The probability of not tossing an even number on one toss of a single die is

1. 1/6
2. 2/6
3. 3/6
4. 4/6

25. (50) The probability of tossing a number that is even and greater than 5 on one toss of a single die is

1. 1/6
2. 2/6
3. 3/6
4. 4/6
26. (52) The probability of tossing a number less than 4 or greater than 5 on one toss of a single die is
1. 1/6
2. 2/6
3. 3/6
4. 4/6

27. (54) The probability of tossing a number that is less than 4 and greater than 5 on one toss of a single die is
1. 0
2. 1/6
3. 3/6
4. 1

28. (56) The probability of tossing a number that is even or greater than 5 on one toss of a single die is
1. 1/6
2. 2/6
3. 3/6
4. 4/6

29. (58) The probability of tossing a number that is even or less than 3 on one toss of a single die is
1. 1/6
2. 2/6
3. 3/6
4. 4/6

30. (60) The probability of tossing a number that is even and less than 5 on one toss of a single die
1. 1/6
2. 2/6
3. 3/6
4. 4/6
31. (62) The probability of tossing a 4 on one toss of two dice is

1. 1/36
2. 3/36
3. 5/36
4. 7/36

32. (64) The probability of tossing a 6 on one toss of two dice is

1. 1/36
2. 3/36
3. 5/36
4. 7/36

33. (66) The probability of tossing an 8 on one toss of two dice is

1. 1/36
2. 3/36
3. 5/36
4. 7/36

34. (68) The probability of tossing a 4 or a 6 on one toss of two dice is

1. 4/36
2. 6/36
3. 8/36
4. 10/36

35. (70) The probability of getting exactly 3 tails on three tosses of a coin is

1. 4/8
2. 3/8
3. 2/8
4. 1/8

36. (72) The probability of getting exactly 2 tails on three tosses of a coin is

1. 4/8
2. 3/8
3. 2/8
4. 1/8
37. (74) The probability of getting more tails than heads on three tosses of a coin is

1. $\frac{4}{8}$
2. $\frac{3}{8}$
3. $\frac{2}{8}$
4. $\frac{1}{8}$

38. (76) The probability of not getting more tails than heads on three tosses of a coin is

1. $\frac{4}{8}$
2. $\frac{3}{8}$
3. $\frac{2}{8}$
4. $\frac{1}{8}$

39. (78) A box contains two red and one white balls. Two balls are drawn in succession without replacement. The probability of drawing two red balls is

1. $\frac{1}{6}$
2. $\frac{2}{6}$
3. $\frac{3}{6}$
4. $\frac{4}{6}$

40. (80) A box contains two red and one white balls. Two balls are drawn in succession without replacement. The probability of drawing one white ball and one red ball is

1. $\frac{1}{6}$
2. $\frac{2}{6}$
3. $\frac{3}{6}$
4. $\frac{4}{6}$
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Exercise 7-B

I. Indicate whether the following statements are true or false by placing a punch in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

1. (2) A bar graph is more appropriate than a line graph for presenting discrete data.

2. (4) Line graphs are more appropriate for continuous data rather than with discrete data.

3. (6) The number which divides the data so that half of the scores are above this number and half are below it is called the mean of the distribution.

4. (8) The number computed by finding the sum of the scores and dividing by the number of scores is called the mode of the distribution.

5. (10) The number which appears most frequently in a distribution is called the mean of the distribution.

6. (12) The range and standard deviation are called measures of dispersion.

7. (14) The set of scores 4, 5, 6, 7 has no mode.

8. (16) The set 2, 3, 3, 4, 5, 6 is bimodal.

9. (18) The mode is the only one of the three averages that is affected by one low score.

10. (20) When the mean is used as a representative of a set of data extreme scores are reflected in the average.

11. (22) The mean is the appropriate measure of central tendency to describe the average salary of the workers in a factory that employs 100 people.

12. (24) The mean is the appropriate measure to describe the average salary in a shop staffed by the owner and five employees.
13. (26) A measure of central tendency provides information on the variability of a set of data.

14. (28) The range provides information on the variability of a set of data.

15. (30) The mean, the median, and the mode all have the same value in a normal distribution.

16. (32) The standard deviation is a measure of central tendency.

17. (34) If an interval of 3 standard deviations from the mean is considered approximately all of the data is included.

18. (36) About 68% of the data in a normal distribution lies in an interval of 1 standard deviation about the mean.

19. (38) A disadvantage of the standard deviation as a measure of dispersion is that it relies on only two extreme scores.

20. (40) The standard deviation can be computed for any distribution.

II. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

21. (42) The probability that a person who is 5 years old will die before the age of 49 is

   1. .05
   2. .06
   3. .07
   4. .08

22. (44) The probability that a person who is 25 years old will die before the age of 35 is

   1. .02
   2. .03
   3. .04
   4. .05
23. (46) The one of the following that is not a measure of central tendency is

1. standard deviation
2. mean
3. mode
4. median

24. (48) The one of the following that is a measure of dispersion is

1. mean
2. range
3. mode
4. median

25. (50) The set of scores 52, 53, 53, 55, 59, 59, 61 has a mode of

1. 53
2. 54
3. 55
4. 56

26. (52) The set of scores 52, 53, 53, 55, 59, 59, 61 has a median of

1. 53
2. 54
3. 55
4. 56

27. (54) The set of scores 52, 53, 53, 55, 59, 59, 61 has a mean of

1. 53
2. 54
3. 55
4. 56

28. (56) The set of scores 10, 13, 13, 14, 15 has a mode of

1. 11
2. 12
3. 13
4. 14
29. (58) The set of scores 10, 13, 13, 14, 15 has a median of

1. 11
2. 13
3. 14

30. (60) The set of scores 10, 13, 13, 14, 15 has a mean of

1. 11
2. 12
3. 13
4. 14

31. (62) The set of scores 7, 8, 9, 10, 12, 12, 14, 16 has a mode of

1. 10
2. 11
3. 12
4. 13

32. (64) The set of scores 7, 8, 9, 10, 12, 12, 14, 16 has a median of

1. 10
2. 11
3. 12
4. 13

33. (66) The set of scores 7, 8, 9, 10, 12, 12, 14, 16 has a mean of

1. 10
2. 11
3. 12
4. 13

In exercise 34 through 39 we consider a normal distribution of 10,000 test scores with a mean of 500 and standard deviation of 100.

34. (68) The percent of the number of scores between 400 and 600 is

1. 50
2. 68
3. 84
4. 95
35. (70) The percent of the number of scores less than 500 is

1. 50
2. 68
3. 84
4. 95

36. (72) The percent of the number of scores above 300 is

1. 84
2. 95
3. 97.5
4. 99.7

37. (74) The percent of the number of scores between 300 and 700 is

1. 84
2. 95
3. 97.5
4. 99.7

38. (76) The percent of the number of heads below 700 is

1. 84
2. 95
3. 97.5
4. 99.7

39. (78) The percent of the number of heads between 200 and 800 is

1. 84
2. 95
3. 97.5
4. 99.7

40. (80) The standard deviation of the set of scores 8, 10, 12, 16, 19 is

1. 4
2. \( \sqrt{80} \)
3. 13
4. 16
I. Indicate whether the following statements are true or false by placing a punch in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

In exercise 1 through 8 we consider \( n \) trials of an experiment in which the probability of a success is \( p \) and the probability of a failure is \( q \).

1. (2) The expected mean number of successes (\( \mu \)) on such an experiment is found by the formula \( \mu = np \).

2. (4) The standard deviation of the number of successes is given by the formula \( \sigma = \sqrt{pn} \).

3. (6) When \( n \) is large the binomial distribution can be approximated by the normal distribution.

4. (8) For large \( n \) the number of successes will be between \( \mu - \sigma \) and \( \mu + \sigma \) almost 100% of the time.

5. (10) For large \( n \) the number of successes will be between \( \mu - 2 \sigma \) and \( \mu + 2 \sigma \) 95% of the time.

6. (12) We say that we are almost 95% confident that the number of successes will be between \( \mu - 2 \sigma \) and \( \mu + 2 \sigma \).

7. (14) \( \mu - 2 \sigma \) and \( \mu + 2 \sigma \) are called the 100% confidence limits.

8. (16) In industry a control chart is used to tell when a process is not in control.

9. (18) As long as most of the samples produce data which falls above the upper limit on the control chart the process is in control.

10. (20) The upper control limit of a control chart represents three standard deviations added to the mean.
11. (22) The mapping of each positive integer \( n \) to a unique number \( f(n) \) is called a function.

12. (24) A mapping may be presented as a set of ordered pairs of numbers.

13. (26) The set \( \{(1, 2), (2, 1), (1, 3)\} \) is a relation.

14. (28) The set \( \{(1, 2), (2, 1), (1, 3)\} \) is a function.

15. (30) The domain of \( \{(1, 1), (2, 2), (3, 1)\} \) is \( \{1, 2\} \).

16. (32) The range of \( \{(1, 1), (2, 2), (3, 1)\} \) is \( \{1, 2, 3\} \).

17. (34) A function is a special kind of relation.

18. (36) Any relation such that no two ordered pairs have the same first element and different second elements is called a function.

19. (38) The set consisting of the first elements of the set of ordered pairs in a function is called the domain of the function.

20. (40) The set consisting of the second elements of the set of ordered pairs in a function is called the range of the function.

II. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.

In exercise 20 through 24 we consider crossing garden peas having the gene pair (red, white) gene pair. \( \frac{1}{4} \) of such crosses are expected to have white flowers. When 48 plants from such a cross are examined;

21. (42) The mean number of white flowers is

1. 3
2. 6
3. 9
4. 12

22. (44) The standard deviation of the distribution of the number of white flowers is

1. 3
2. 6
3. 9
4. 12
23. (46) We can say with almost 100% confidence that the number of white flowers will be between
1. 3 and 15
2. 6 and 18
3. 3 and 21
4. 9 and 15

24. (48) We can say with almost 95% confidence that the number of white flowers will be between
1. 3 and 15
2. 6 and 18
3. 3 and 21
4. 9 and 15

In exercise 25 through 30 we consider the production of a new kind of golf ball. Experience has shown that 1/5 of the golf balls are defective. We check each batch of 225 golf balls.

25. (50) The mean number of defective golf balls to be expected is
1. 45
2. 100
3. 200
4. 250

26. (52) The standard deviation of the distribution of the number of defective golf balls is
1. 3
2. 6
3. 9
4. 12

27. (54) The upper control limit for the control chart is
1. 45
2. 51
3. 57
4. 63

28. (56) The lower control limit for the control chart is
1. 27
2. 33
3. 39
4. 45

29. (58) We say that the process is in control if the number of defectives is between
1. 33 and 37
2. 27 and 63
3. 39 and 51
4. 39 and 63
30. (60) We would suspect that something was wrong if the number of defectives was greater than
   1. 45
   2. 51
   3. 57
   4. 63

31. (62) The functional rule representing the mapping 1 \rightarrow 2 is
   1. \( n \rightarrow n \)
   2. \( n \rightarrow 3n - 1 \)
   3. \( n \rightarrow 2n \)
   4. \( n \rightarrow 3n - 2 \)

32. (64) The functional rule representing the correspondence 1 \rightarrow 1 is
   1. \( n \rightarrow 3n - 2 \)
   2. \( n \rightarrow n \)
   3. \( n \rightarrow 2n \)
   4. \( n \rightarrow 2n + 1 \)

33. (66) The set of ordered pairs representing the mapping 1 \rightarrow 2 is
   1. \( \{(1,2),(2,3),(3,4),(4,5)\} \)
   2. \( \{(1,1),(2,2),(3,3),(4,4)\} \)
   3. \( \{(2,1),(4,2),(6,3),(8,4)\} \)
   4. \( \{(1,2),(2,4),(3,6),(4,8)\} \)

34. (68) If \( A = \{2, 4\} \) and \( B = \{1, 3\} \) then \( A \times B \) is
   1. \( \{(1,2),(1,4),(3,4),(3,2)\} \)
   2. \( \{(2,1),(4,1),(4,3),(2,3)\} \)
   3. \( \{(1,2),(3,4)\} \)
   4. \( \{1,2,3,4\} \)
35. (70) The domain of the relation \( \{(1,10),(2,9),(5,5)\} \) is
1. \( \{10,9,5\} \)
2. \( \{1,2,5\} \)
3. \( \{(10,1),(9,2),(5,5)\} \)
4. \( \{10,1,9,2,5\} \)

36. (72) The range of the relation \( \{(1,10),(2,9),(5,5)\} \) is
1. \( \{10,9,5\} \)
2. \( \{1,2,5\} \)
3. \( \{(10,1),(9,2),(5,5)\} \)
4. \( \{10,1,9,2,5\} \)

37. (74) When \( f(x) = x + 4 \) then \( f(2) \)
1. 2
2. 4
3. 6
4. 3

38. (76) When \( f(x) = 3x + 5 \) then \( f(3) \) is
1. 14
2. 10
3. 5
4. 4

39. (78) When \( f(x) = 2x^2 + 5 \) then \( f(3) \) is
1. 5
2. 19
3. 23
4. 41

40. (80) For positive integers \( x \) and the relation expressed by the equation \( Y = 3x + 3 \) if \( x \) is increased by 1 then \( Y \) is increased by
1. 2
2. 3
3. 4
4. 5
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Exercise 9-B

I. Indicate whether the following statements are true or false by placing a punch in the column corresponding to the question. Punch a 1 if the answer is true and punch a 2 if the answer is false.

1. (2) The graph of a relation intersects a vertical line in at most one point.

2. (4) The change in the value of y as x is increased by 1 is designated by \( \Delta y \).

3. (6) If \( \Delta y \) is a constant then y is a linear function of x.

4. (8) If \( y = 2x + 3 \) then \( \Delta y = 3 \).

5. (10) If \( y = ax + b \) then \( \Delta y = a \).

6. (12) A function like \( y = 2x^2 + 3 \) is called an exponential function.

7. (14) If \( y = x^2 + 2x \) the \( \Delta y \) is a constant.

8. (16) If \( y = x^2 \) then \( \Delta^2 y \) is a constant.

9. (18) If \( \Delta^2 y \) is a constant, then \( f(x) \) may be expressed in the form \( f(x) = ax^2 + bx + c \).

10. (20) Some functions have inverses that are not functions.

11. (22) Some non functional relations have inverses that are functions.

12. (24) After interchanging the x and the y in an equation we usually solve for y to get the equations for the inverse of the original function.

13. (26) An expression in which the x occurs as an exponent is called a polynomial function.

14. (28) The inverse of the exponential function is the logarithmic function.
15. (30) If $x$ is the exponent for $b$ which gives $y$ then $y = \log_b x$

16. (32) $\log (ax + b) = (\log a)(\log b)$

17. (34) $\log \frac{a}{b} = \log a - \log b$

18. (36) $\log a^n = (\log a)^n$

19. (38) $\log \sqrt[n]{a} = \sqrt[n]{\log a}$

20. (40) $\log (a - b) \neq \log a - \log b$

21. (42) If $y = 4x - 2$ then $\Delta y$ is
   1. 1
   2. 2
   3. 3
   4. 4

22. (44) If $y = 2x - 3$ then $\Delta y$ is
   1. 1
   2. 2
   3. 3
   4. 4

23. (46) If $y = ax + b$ and $\Delta y = 2$ and $(2, b)$ is an ordered pair of the functions then the value of $b$ is
   1. 1
   2. 2
   3. 3
   4. 4

24. (48) If $f(x) = x^2$ then $f(4)$ is
   1. 14
   2. 16
   3. 18
   4. 20

11. Indicate the answer to the following by placing a punch in the column corresponding to the question. Punch the number 1, 2, 3 or 4 to indicate your choice from the four possible replies.
25. (50) If \( f(x) = 3x^2 - 2x + 4 \) then \( f(2) \) is
1. 4
2. 8
3. 12
4. 16

26. (52) If \( f(x) = x^2 + 2x - 3 \) then \( f(x + 1) \) is
1. \( x^2 + 2x \)
2. \( x^2 + 4x \)
3. \( x^2 + 4x + 3 \)
4. \( x - 3 \)

27. (54) If \( f(x) = x^2 + 2x - 3 \) then \( \Delta y \) is
1. \( x^2 + 2x - 3 \)
2. \( x^2 + 4x \)
3. \( 2x \)
4. \( 2x + 3 \)

28. (56) If \( f(x) = x^2 - 3x + 2 \) then \( \Delta^2 y \) is
1. 2
2. 3
3. 7
4. 8

29. (58) If \( f(x) = 2x^2 - 1 \) then \( \Delta^2 y \) is
1. 1
2. 2
3. 3
4. 4

30. (60) The inverse of the relation \( \{(3,4),(3,5),(4,1)\} \) is
1. \( \{(4,1),(3,5),(3,4)\} \)
2. \( \{(4,3),(5,3),(4,1)\} \)
3. \( \{(5,3),(4,3),(1,4)\} \)
4. \( \{(1,4),(3,5),(4,3)\} \)

31. (62) The relation which is also a function is
1. \( \{(1,2),(2,2),(3,2)\} \)
2. \( \{(1,2),(1,3),(2,3)\} \)
3. \( \{(1,2),(2,2),(2,3)\} \)
4. \( \{(1,2),(2,3),(2,4)\} \)
32. (64) The relation whose inverse is a function is

1. \{(1,2),(2,2),(3,2)\}
2. \{(1,2),(1,3),(2,3)\}
3. \{(1,2),(2,2),(2,3)\}
4. \{(1,2),(2,3),(2,4)\}

33. (66) The equation for the inverse of the relation determined by the equation \(y = x - 1\) is

1. \(y = x + 1\)
2. \(y = -x + 1\)
3. \(x - 1 = y\)
4. \(y = x - 1\)

34. (68) The inverse of the relation determined by the equation \(y = \frac{1}{2}x + 3\) is

1. \(y = x - 3\)
2. \(y = 2x - 6\)
3. \(3 + \frac{1}{2}x = y\)
4. \(y = \frac{1}{2}x + 3\)

35. (70) If \(y = 4^x\) then for \(x = 3\) the value of \(y\) is

1. 4
2. 12
3. 64
4. 128

36. (72) If \(\log_2 256 = \) then \(2^x\) is

1. 2
2. 8
3. 256
4. 512

37. (74) The value of \(\log_2 64\) is

1. 2
2. 6
3. 8
4. 64

38. (76) The value of \(\log_2 1/32\) is

1. -5
2. 2
3. 5
4. 32
39. \( \log_2 (8 \times 32) \) is
   1. 3
   2. 5
   3. 256
   4. 8

40. \( \log_2 (256)^6 \) is
   1. 6
   2. 8
   3. 48
   4. 256
1. Student’s name ____________________________

2. Circle your classification
   Freshman, Sophomore, Junior, Senior

3. Program at Winona State (check one)
   a. B. S. teaching ( )
   b. B. S. nursing ( )
   c. B. A. ( )
   d. Other ( ) explain__________________________

4. College major______________________________

5. College minor______________________________

6. High school graduated from____________________

7. Location of high school graduated from________

8. Favorite high school subject____________________

9. Total number of years of mathematics taken in grades 9 through 12____________________________

10. Use a check to indicate the grade in which mathematics was taken 9th ( ) 10th ( ) 11th ( ) 12th ( )
APPENDIX E
Indicate your answer to the following by placing an X in the correct box after the number on the answer sheet corresponding to the question.

1. The property illustrated by the statement $(x + y) + z = x + (y + z)$ is
   a. distributive
   b. associative
   c. commutative
   d. inverse

2. The property illustrated by the statement $n + (-n) = (-n) + n = 0$ is
   a. identity
   b. associative
   c. commutative
   d. inverse

3. The number set that does not contain a replacement for $n$ in the sentence $5 + n = 2$ is
   a. whole numbers
   b. integers
   c. rational numbers
   d. real numbers

4. The set of numbers that contains a solution to the equation $5n = 7$ is
   a. whole numbers
   b. integers
   c. rational numbers
   d. irrational numbers

5. $4 - 8 < 8 - 4$ is
   a. a true statement of equality
   b. a false statement of equality
   c. a true statement of inequality
   d. a false statement of inequality
6. The truth set of the sentence $x + 3 \neq x$ is
   a. single number
   b. the empty set
   c. the entire replacement set
   d. none of the above

7. The graph of the solution set of the sentence $x - 3 > 9$ is
   a. a point
   b. a half-line
   c. a ray
   d. a line

8. The graph of the solution set of the sentence $-5 \leq x \leq 2$ is
   a. a point
   b. a half-line
   c. a ray
   d. a line segment

9. The solution for the open sentence $x + 5 = 3$ is
   a. 5
   b. 3
   c. 2
   d. -2

10. The solution for the open sentence $4x + 1 = 9$ is
    a. 2
    b. $10/4$
    c. 4
    d. 6

11. The solution for the open sentence $-3x - 1 = 5$ is
    a. 2
    b. -2
    c. $4/3$
    d. $-4/3$
12. If $-3x + 1 > 2$ then
   a. $x > \frac{-1}{3}$
   b. $x > \frac{1}{3}$
   c. $x < \frac{-1}{3}$
   d. $x < \frac{1}{3}$

13. The $x$ intercept of the graph of $2x - 3y = 6$ is
   a. -3
   b. 3
   c. 2
   d. -2

14. The $x$ intercept of the graph of $2x + y - 5 = 0$ is
   a. 0
   b. $\frac{5}{2}$
   c. $-\frac{5}{2}$
   d. 5

15. When the sentence $3x + y = 8$ is solved for $y$ we get
   a. $y = 8$
   b. $y = 8 - 3x$
   c. $y = 8 + 3x$
   d. $y = \frac{8}{3}$

16. A point on the graph of $y = (x + 2)^2 + 1$ is
   a. (2,1)
   b. (-2,9)
   c. (4,9)
   d. (0,5)

17. The compound statement $p \lor q$ is false when
   a. $p$ is true and $q$ is true
   b. $p$ is true and $q$ is false
   c. $p$ is false and $q$ is true
   d. $p$ is false and $q$ is false

18. The compound statement $p \rightarrow q$ is false when
   a. $p$ is true and $q$ is true
   b. $p$ is true and $q$ is false
   c. $p$ is false and $q$ is true
   d. $p$ is false and $q$ is false
19. The compound statement \( p \land (p \to q) \) is true when

a. \( p \) is true and \( q \) is true
b. \( p \) is true and \( q \) is false
c. \( p \) is false and \( q \) is true
d. \( p \) is false and \( q \) is false

20. The inverse of the statement, if \( x = 2 \) then \( x^2 = 4 \) is

a. if \( x \neq 2 \) then \( x^2 \neq 4 \)
b. if \( x^2 = 4 \) then \( x = 2 \)
c. if \( x \neq 2 \) then \( x^2 = 4 \)
d. if \( x^2 \neq 4 \) then \( x \neq 2 \)

21. The name given to the compound statement \( x = 2 \) or \( x^2 = 4 \) is

a. conjunction
b. conditional
c. disjunction
d. biconditional

22. The property that is not a property of all groups is

a. closure
b. inverses
c. associativity
d. distributive

23. The probability of not tossing a 3 on one throw of a single die is

a. 1/6
b. 2/6
c. 3/6
d. 5/6

24. The probability of tossing a number less than 3 on one throw of a single die is

a. 1/6
b. 2/6
c. 3/6
d. 5/6
25. The probability of tossing a 3 or a 4 on one toss of two dice is
   a. 1/36
   b. 2/36
   c. 5/36
   d. 7/36

26. The probability of getting exactly 2 tails on three tosses of a coin is
   a. 1/8
   b. 2/8
   c. 3/8
   d. 4/8

27. A box contains two red and two white balls. Two balls are drawn in succession without replacement. The probability of drawing two red balls is
   a. 1/12
   b. 2/12
   c. 3/12
   d. 4/12

28. The one of the following that is not a measure of central tendency is
   a. mean
   b. median
   c. mode
   d. range

29. The set of scores 71, 71, 79, 82, 87, 89, 95 has a mean of
   a. 71
   b. 79
   c. 82
   d. 83

30. The percentage of the number of heads between 190 and 210 is
   a. 50
   b. 68
   c. 84
   d. 95
31. The percentage of the number of heads between 180 and 220 is
   a. 84
   b. 95
   c. 97.5
   d. 99.7

32. The percentage of the number of heads between 170 and 230 is
   a. 84
   b. 95
   c. 97.5
   d. 99.7

33. We consider the production of a new kind of radio. Experience shows that 1/6 of the radios are defective. When we examine 720 radios we can say with 68% confidence that the number of defectives will be between
   a. 600 and 720
   b. 115 and 125
   c. 590 and 610
   d. 110 and 130

34. The functional rule representing the correspondence
   \[ 1 \rightarrow 2 \]
   \[ 2 \rightarrow 5 \]
   \[ 3 \rightarrow 10 \]
   \[ 4 \rightarrow 17 \]
   a. \( n \rightarrow n \)
   b. \( n \rightarrow 2n \)
   c. \( n \rightarrow 2n + 1 \)
   d. \( n \rightarrow n^2 + 1 \)

35. If \( A = \{2,3\} \) and \( B = \{3,4\} \) then \( A \times B \) is
   a. \( \{(2,2),(3,3),(4,4)\} \)
   b. \( \{(3,2),(3,3),(4,2),(4,3)\} \)
   c. \( \{(2,2),(2,3),(3,2)\} \)
   d. \( \{(2,3),(2,4),(3,3),(3,4)\} \)

36. When \( f(x) = x + 4 \) then \( f(3) \) is
   a. 1
   b. 7
   c. 12
   d. 3x
37. For positive integers \( x \) and the relations expressed by equation \( y = 3x + 4 \), if \( x \) is increased by 1 then \( y \) is increased by

- a. 1
- b. 2
- c. 3
- d. 4

38. If \( f(x) = 2x + 3 \) then \( f(x + 1) \) is

- a. \( x + 3 \)
- b. \( 2x + 2 \)
- c. \( 2x + 3 \)
- d. \( 2x + 5 \)

39. The equation for the inverse of the relation determined by the equation \( y = x - 2 \) is

- a. \( y = x + 2 \)
- b. \( y = x - 2 \)
- c. \( x - 2 = y \)
- d. \( y = -x + 2 \)

40. \( \log_2 32 \) is

- a. 2
- b. 5
- c. 16
- d. 32
APPENDIX F
Indicate your answer to the following by placing an X in the correct box after the number on the answer sheet corresponding to the question.

1. The property illustrated by the statement 
   \((x + y) + z = x + (y + z)\) is 
   
   a. distributive  
   b. associative  
   c. commutative  
   d. inverse  

2. The property illustrated by the statement 
   \(x(y + z) = xy + xz\) is 
   
   a. distributive  
   b. associative  
   c. commutative  
   d. inverse  

3. The property illustrated by the statement 
   \(n + (-n) = (-n) + n = 0\) is 
   
   a. identity  
   b. associative  
   c. commutative  
   d. inverse  

4. The property illustrated by the statement 
   \(1 \times n = n \times 1 = n\) is 
   
   a. identity  
   b. associative  
   c. commutative  
   d. inverse  

5. The number set that does not contain a replacement for 
   \(n\) in the sentence \(5 + n = 2\) is 
   
   a. whole numbers  
   b. integers  
   c. rational numbers  
   d. real numbers
6. The set of numbers that contains a solution to the equation \( n^2 + 1 = 6 \) is
   a. whole numbers
   b. integers
   c. rational numbers
   d. irrational numbers

7. The set of numbers that contains a solution to the equation \( 5n = 7 \) is
   a. whole numbers
   b. integers
   c. rational numbers
   d. irrational numbers

8. The number \( \frac{2}{3} \) can be represented as
   a. a terminating decimal
   b. a repeating decimal
   c. a nonterminating, nonrepeating decimal
   d. none of the above

9. The number \( \frac{1}{2} \) can be represented as
   a. a terminating decimal
   b. a repeating decimal
   c. a nonterminating, nonrepeating decimal
   d. none of the above

10. The number \( \sqrt{2} \) can be written as
    a. a terminating decimal
    b. a repeating decimal
    c. a nonterminating, nonrepeating decimal
    d. none of the above

11. \( 4 - 8 < 8 - 4 \) is
    a. a true statement of equality
    b. a false statement of equality
    c. a true statement of inequality
    d. a false statement of inequality
12. \(-16/21 \geq -17/21\) is
   a. a true statement of equality
   b. a false statement of equality
   c. a true statement of inequality
   d. a false statement of inequality

13. The truth set of the sentence \(x + 3 \neq x\) is
   a. a single number
   b. the empty set
   c. the entire replacement set
   d. none of the above

14. The truth set of the sentence \(x \geq x + 4\) is
   a. a single number
   b. the empty set
   c. the entire replacement set
   d. none of the above

15. The graph of the solution set of the sentence \(x - 3 > 9\) is
   a. a point
   b. a half-line
   c. a ray
   d. a line

16. The graph of the solution set of the sentence \(x - 3 = 8\) is
   a. a point
   b. a half-line
   c. a ray
   d. a line segment

17. The graph of the solution set of the sentence \(-5 \leq x \leq 2\) is
   a. a point
   b. a half-line
   c. a ray
   d. a line segment
18. The graph of the solution set of the compound statement $x > 2$ or $x < 6$ when the replacement set is the real numbers is
   a. two points
   b. a line segment
   c. union of two rays
   d. a line

19. The graph of $|x| \leq 5$ is
   a. two points
   b. a line segment
   c. union of two rays
   d. a line

20. The graph of $|x - 3| \leq 5$ is
   a. two points
   b. a line segment
   c. union of two rays
   d. a line

21. The solution for the open sentence $x + 5 = 3$ is
   a. 5
   b. 3
   c. 2
   d. -2

22. The solution for the open sentence $x - 6 = 4$ is
   a. -10
   b. 10
   c. -6
   d. 6

23. The solution for the open sentence $4x + 1 = 9$ is
   a. 2
   b. $10/4$
   c. 4
   d. 6
24. The solution for the open sentence $3x - 2 = 5$ is
   a. 2
   b. $7/3$
   c. 1
   d. $-7/3$

25. The solution for the open sentence $-3x - 1 = 5$ is
   a. 2
   b. -2
   c. $4/3$
   d. $-4/3$

26. If $2x + 3 < 2$ then
   a. $x < -1/2$
   b. $x > -1/2$
   c. $x < 1/3$
   d. $x > -1/3$

27. If $-3x + 1 > 2$ then
   a. $x > -1/3$
   b. $x > 1/3$
   c. $x < -1/3$
   d. $x < 1/3$

28. The solution for the open sentence $2/3x - 3 = 5$ is
   a. 12
   b. 3
   c. $16/3$
   d. 8

29. If $U = \{1,2,3\}$ then the solution set for $x + y = 4$ is
   a. $\{(1,3)\}$
   b. $\{(1,3),(3,1)\}$
   c. $\{(1,2),(1,3)\}$
   d. $\{4\}$

30. If $U = \{1,2,3\}$ then the solution set for $x < y$ is
   a. $\{(1,2),(1,3),(2,3)\}$
   b. $\{(1,2),(2,3)\}$
   c. $\{1,2,3\}$
   d. $\{(1,1),(2,2),(3,3)\}$
31. The x intercept of the graph of $2x - 3y = 6$ is
   a. -3
   b. 3
   c. 2
   d. -2

32. The y intercept of the graph of $2x - 3y = 6$ is
   a. -3
   b. 3
   c. 2
   d. -2

33. The x intercept of the graph of $2x + y - 5 = 0$ is
   a. 0
   b. $5/2$
   c. $-5/2$
   d. 5

34. The y intercept of the graph of $2x + y - 5 = 0$ is
   a. 0
   b. $5/2$
   c. $-5/2$
   d. 5

35. When the sentence $3x + y = 8$ is solved for $y$ we get
   a. $y = 8$
   b. $y = 8 - 3x$
   c. $y = 8 + 3x$
   d. $y = 8/3$

36. When the sentence $x + 3y \leq 6$ is solved for $y$ we get
   a. $y \leq 6 - x$
   b. $y \leq 2 - x$
   c. $y \geq 2 - 1/3x$
   d. $y \leq 2 - 1/3x$

37. A point on the graph of $y = (x + 2)^2 + 1$ is
   a. (2, 1)
   b. (-2, 9)
   c. (4, 9)
   d. (0, 5)
38. The axis of symmetry of the graph of \( y = (x - 3)^2 + 2 \) is

   a. \( y \) axis
   b. \( y = 2 \)
   c. \( x = 3 \)
   d. \( x = 2 \)

39. The vertex of the graph of \( y = (x - 3)^2 + 2 \) is

   a. \( (0,0) \)
   b. \( (-3,2) \)
   c. \( (3,-2) \)
   d. \( (3,2) \)

40. The axis of symmetry of the graph of \( y = (x + 2)^2 + 1 \) is

   a. \( y \) axis
   b. \( x = 2 \)
   c. \( x = -2 \)
   d. \( y = 1 \)

41. The compound statement \( p \lor q \) is false when

   a. \( p \) is true and \( q \) is true
   b. \( p \) is true and \( q \) is false
   c. \( p \) is false and \( q \) is true
   d. \( p \) is false and \( q \) is false

42. The compound statement \( p \land q \) is true when

   a. \( p \) is true and \( q \) is true
   b. \( p \) is true and \( q \) is false
   c. \( p \) is false and \( q \) is true
   d. \( p \) is false and \( q \) is false

43. The compound statement \( p \rightarrow q \) is false when

   a. \( p \) is true and \( q \) is true
   b. \( p \) is true and \( q \) is false
   c. \( p \) is false and \( q \) is true
   d. \( p \) is false and \( q \) is false

44. The compound statement\( \neg p \lor q \) will be false when

   a. \( p \) is true and \( q \) is true
   b. \( p \) is true and \( q \) is false
   c. \( p \) is false and \( q \) is true
   d. \( p \) is false and \( q \) is false
45. The compound statement $p \land (p \rightarrow q)$ is true when
   a. $p$ is true and $q$ is true
   b. $p$ is true and $q$ is false
   c. $p$ is false and $q$ is true
   d. $p$ is false and $q$ is false

46. The converse of the statement, if $x = 2$ then $x^2 = 4$ is
   a. if $x \neq 2$ then $x^2 \neq 4$
   b. if $x^2 = 4$ then $x = 2$
   c. if $x \neq 2$ then $x^2 = 4$
   d. if $x^2 \neq 4$ then $x \neq 2$

47. The inverse of the statement, if $x = 2$ then $x^2 = 4$ is
   a. if $x \neq 2$ then $x^2 \neq 4$
   b. if $x^2 = 4$ then $x = 2$
   c. if $x \neq 2$ then $x^2 = 4$
   d. if $x^2 \neq 4$ then $x \neq 2$

48. The contrapositive of the statement, if $x = 2$ then $x^2 = 4$
   a. if $x \neq 2$ then $x^2 \neq 4$
   b. if $x^2 = 4$ then $x = 2$
   c. if $x \neq 2$ then $x^2 = 4$
   d. if $x^2 \neq 4$ then $x \neq 2$

49. The name given to the compound statement $x = 2$ and $x^2 = 4$
   is
   a. conjunction
   b. conditional
   c. disjunction
   d. biconditional

50. The name given to the compound statement if $x = 2$ then $x^2 = 4$ is
   a. conjunction
   b. conditional
   c. disjunction
   d. biconditional
51. The name given to the compound statement \( x = 2 \) or \( x^2 = 4 \) is
   a. conjunction
   b. conditional
   c. disjunction
   d. biconditional

52. The tautology among the following statements is
   a. \( p \rightarrow q \)
   b. \( p \lor \neg p \)
   c. \( p \land \neg p \)
   d. \( p \leftrightarrow q \)

53. The property that is not a property of all groups is
   a. closure
   b. inverse
   c. associativity
   d. distributive

54. The probability of tossing a 3 on one throw of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 5/6

55. The probability of not tossing a 3 on one throw of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 5/6

56. The probability of tossing a 3 or a 4 on one throw of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 4/6
57. The probability of tossing a number less than 3 on one throw of a single die is

a. 1/6  
b. 2/6  
c. 3/6  
d. 5/6

58. The probability of not tossing a 3 or a 4 on one throw of a single die is

a. 1/6  
b. 2/6  
c. 3/6  
d. 4/6

59. The probability of tossing a 3 on one toss of two dice is

a. 1/36  
b. 2/36  
c. 3/36  
d. 4/36

60. The probability of tossing a 4 on one toss of two dice is

a. 1/36  
b. 2/36  
c. 3/36  
d. 4/36

61. The probability of tossing a 3 or a 4 on one toss of two dice is

a. 1/36  
b. 2/36  
c. 5/36  
d. 7/36

62. The probability of getting exactly 3 tails on three tosses of a coin is

a. 1/8  
b. 2/8  
c. 3/8  
d. 4/8
63. The probability of getting exactly 2 tails on three tosses of a coin is
a. 1/8
b. 2/8
c. 3/8
d. 4/8

In questions 64 through 66 a box contains two red and two white balls. Two balls are drawn in succession without replacement;

64. The probability of drawing one white and one red ball is
a. 8/12
b. 9/12
c. 10/12
d. 11/12

65. The probability of drawing two red balls is
a. 1/12
b. 2/12
c. 3/12
d. 4/12

66. The probability of drawing two balls of the same color is
a. 1/12
b. 2/12
c. 3/12
d. 4/12

67. The one of the following that is not a measure of central tendency is
a. mean
b. median
c. mode
d. range

68. The one of the following that is a measure of dispersion is
a. mean
b. median
c. standard deviation
d. mode
69. The set of scores 71, 71, 79, 82, 82, 94, 99 has a median of
   a. 71
   b. 79
   c. 82
   d. 83

70. The set of scores 71, 71, 79, 82, 87, 89, 95 has a mode of
   a. 71
   b. 79
   c. 82
   d. 83

71. The set of scores 71, 71, 79, 82, 87, 89, 95 has a mean of
   a. 71
   b. 79
   c. 82
   d. 83

72. The set of scores 6, 8, 10, 12, 14 has a median of
   a. 9
   b. 10
   c. 11
   d. 12

In exercise 73 through 78, 400 coins are tossed repeatedly, the distribution of the number of heads is a normal one with a mean of 200 and a standard deviation of 10.

73. The percentage of the number of heads between 190 and 210 is
   a. 50
   b. 68
   c. 84
   d. 95

74. The percentage of the number of heads less than 200 is
   a. 50
   b. 68
   c. 84
   d. 95
75. The percentage of the number of heads between 180 and 220 is
   a. 84
   b. 95
   c. 97.5
   d. 99.7

76. The percentage of the number of heads above 190 is
   a. 84
   b. 95
   c. 97.5
   d. 99.7

77. The percentage of the number of heads between 170 and 230 is
   a. 84
   b. 95
   c. 97.5
   d. 99.7

78. The percentage of the number of heads below 210 is
   a. 84
   b. 95
   c. 97.5
   d. 99.7

In exercise 79 through 82 we consider the production of a new kind of radio. Experience shows that 1/6 of the radios are defective. When we examine 720 radios;

79. The mean number of defectives to be expected is
   a. 20
   b. 120
   c. 600
   d. 720

80. The standard deviation of the distribution of the number of defectives is
   a. 6
   b. 5
   c. 10
   d. 20
81. We can say with 68% confidence that the number of defectives will be between
    a. 600 and 720
    b. 115 and 125
    c. 590 and 610
    d. 110 and 130

82. We can say with 95% confidence that the number of defectives will be between
    a. 100 and 140
    b. 590 and 610
    c. 90 and 150
    d. 700 and 740

83. The functional rule representing the correspondence
    \[ 1 \rightarrow 2 \]
    \[ 2 \rightarrow 5 \]
    \[ 3 \rightarrow 10 \]
    \[ 4 \rightarrow 17 \]
    \[ \ldots \ldots \]
    a. \( n \rightarrow n \)
    b. \( n \rightarrow 2n \)
    c. \( n \rightarrow 2n + 1 \)
    d. \( n \rightarrow n^2 + 1 \)

84. The set of ordered pairs representing the mapping \( 1 \rightarrow 1 \) is
    \[ 2 \rightarrow 4 \]
    \[ 3 \rightarrow 9 \]
    \[ 4 \rightarrow 16 \]
    \[ \ldots \ldots \]
    a. \[ \{(1,1),(2,4),(3,9),(4,16)\ldots\} \]
    b. \[ \{(1,1),(4,2),(9,3),(16,4)\ldots\} \]
    c. \[ \{(1,2),(2,3),(3,4),(5,6)\ldots\} \]
    d. \[ \{(1,4),(4,9),(9,16),(16,25)\ldots\} \]

85. If \( A = \{2,3\} \) and \( B = \{3,4\} \) then \( A \times B \) is
    a. \[ \{(2,2),(3,3),(4,4)\} \]
    b. \[ \{(3,2),(3,3),(4,2),(4,3)\} \]
    c. \[ \{(2,2),(2,3),(3,2)\} \]
    d. \[ \{(2,3),(2,4),(3,3),(3,4)\} \]
86. The domain of the relation \{(1,2),(2,3),(3,4)\} is
   a. \{2,3,4\}
   b. \{1,2,3\}
   c. \{(2,1),(3,2),(3,4)\}
   d. \{1,2,3,4\}

87. When \( f(x) = x + 4 \) then \( f(3) \) is
   a. 1
   b. 7
   c. 12
   d. 3x

88. When \( f(x) = 3x + 5 \) then \( f(2) \) is
   a. 2
   b. 3
   c. 5
   d. 11

89. When \( f(x) = 2x^2 + 3 \) then \( f(3) \) is
   a. 39
   b. 21
   c. 17
   d. 3

90. For positive integers \( x \) and the relations expressed by equation \( y = 3x + 4 \), if \( x \) is increased by 1 they \( y \) is increased by
   a. 2
   b. 1
   c. 3
   d. 4

91. If \( y = 6x - 2 \) then \( \Delta y \) is
   a. 2
   b. 4
   c. 6
   d. 8

92. If \( y = ax + b \) and \( \Delta y = 3 \) and \((2,7)\) is an ordered pair of the function then the value of \( b \) is
   a. 0
   b. 1
   c. 2
   d. 3
93. If $f(x) = 2x + 3$ then $f(x + 1)$ is
   a. $x + 3$
   b. $2x + 2$
   c. $2x + 3$
   d. $2x + 5$

94. If $f(x) = x^2 - 3x - 2$ then $f(4)$
   a. 1
   b. 2
   c. 3
   d. 4

95. If $f(x) = x^2 + 4x - 3$ then $\Delta^2 y$ is
   a. $2x + 1$
   b. $2x + 3$
   c. 2
   d. 4

96. The inverse of the relation $\{(2,4),(3,4),(4,5)\}$ is
   a. $\{(4,5),(3,4),(2,4)\}$
   b. $\{(4,2),(3,4),(4,5)\}$
   c. $\{(4,2),(4,3),(4,5)\}$
   d. $\{(4,2),(4,3),(5,4)\}$

97. The equation for the inverse of the relation determined by the equation
   a. $y = x + 2$
   b. $y = x - 2$
   c. $x - 2 = y$
   d. $y = -x + 2$

98. If $\log_2 64 = x$ then $2^x$ is
   a. 64
   b. 32
   c. 5
   d. 2
99. $\log_2 32$ is
   a. 2
   b. 5
   c. 16
   d. 32

100. $\log_2 \frac{1}{32}$ is
   a. -2
   b. 2
   c. -5
   d. 5
Mathematics 112

Final Exam

Indicate your answer to the following by placing an X in the correct box after the number on the answer sheet corresponding to the question.

1. The property illustrated by the statement 
   \((x + y) + z = x + (y + z)\) is
   
   a. distributive  
   b. associative  
   c. commutative  
   d. inverse

2. The property illustrated by the statement 
   \(x(y + z) = xy + xz\) is
   
   a. distributive  
   b. associative  
   c. commutative  
   d. inverse

3. The property illustrated by the statement 
   \(n + (-n) = (-n) + n = 0\) is
   
   a. identity  
   b. associative  
   c. commutative  
   d. inverse

4. The property illustrated by the statement 
   \(1 \times n = n \times 1 = n\) is
   
   a. identity  
   b. associative  
   c. commutative  
   d. inverse

5. The number set that does not contain a replacement for 
   \(n\) in the sentence \(5 + n = 2\) is
   
   a. whole numbers  
   b. negative integers  
   c. rational numbers  
   d. real numbers
6. The set of numbers that contains a solution to the equation \( n^2 + 1 = 6 \) is

a. whole numbers  
b. integers  
c. rational numbers  
d. irrational numbers

7. The set of numbers that contains a solution to the equation \( 5n = 7 \) is

a. positive integers  
b. negative integers  
c. rational numbers  
d. irrational numbers

8. The number \( \frac{2}{3} \) can be represented as

a. a terminating decimal  
b. a repeating decimal  
c. a nonterminating, nonrepeating decimal  
d. an irrational number

9. The number \( \frac{1}{2} \) can be represented as

a. a terminating decimal  
b. a repeating decimal  
c. a nonterminating, nonrepeating decimal  
d. an irrational number

10. The number \( \sqrt{2} \) can be written as

a. a terminating decimal  
b. a repeating decimal  
c. a nonterminating, nonrepeating decimal  
d. none of the above

11. \( 4 - 8 < 8 - 4 \) is

a. a true statement of equality  
b. a false statement of equality  
c. a true statement of inequality  
d. a false statement of inequality

12. \( -\frac{16}{21} > -\frac{17}{21} \) is

a. a true statement of equality  
b. a false statement of equality  
c. a true statement of inequality  
d. a false statement of inequality
13. The truth set of the sentence \( x + 3 \neq x \) is
   a. a single number
   b. the empty set
   c. the entire replacement set
   d. none of the above

14. The truth set of the sentence \( x > x + 4 \) is
   a. a single number
   b. the empty set
   c. the entire replacement set
   d. none of the above

15. The graph of the solution set of the sentence \( x - 3 > 9 \) is
   a. a point
   b. a half-line
   c. a ray
   d. a line

16. The graph of the solution set of the sentence \( x - 3 = 8 \) is
   a. a point
   b. a half-line
   c. a ray
   d. a line segment

17. The graph of the solution set of the sentence \(-5 \leq x \leq 2\) is
   a. a point
   b. a half-line
   c. a ray
   d. a line segment

18. The graph of the solution set of the compound statement \( x > 2 \) or \( x < 6 \) when the replacement set is the real numbers is
   a. two points
   b. a line segment
   c. union of two rays
   d. a line

19. The graph of \( x \leq 5 \) is
   a. two points
   b. a line segment
   c. union of two rays
   d. a line
20. The graph of $|x - 3| \leq 5$ is
   a. two points
   b. a line segment
   c. union of two rays
   d. a line

21. The solution for the open sentence $x + 5 = 3$ is
   a. 8
   b. -8
   c. 2
   d. -2

22. The solution for the open sentence $x - 6 = 4$ is
   a. -10
   b. 10
   c. -2
   d. 2

23. The solution for the open sentence $4x + 1 = 9$ is
   a. 2
   b. 10/4
   c. 4
   d. 8

24. The solution for the open sentence $3x - 2 = 5$ is
   a. 7
   b. 7/3
   c. 1
   d. -7/3

25. The solution for the open sentence $-3x - 1 = 5$ is
   a. 2
   b. -2
   c. 4/3
   d. -4/3

26. If $2x + 3 \leq 2$ then
   a. $x \leq -\frac{1}{2}$
   b. $x > -\frac{1}{2}$
   c. $x \leq 1/3$
   d. $x > -1/3$
27. If \(-3x + 1 > 2\) then
   a. \(x > -\frac{1}{3}\)
   b. \(x > \frac{1}{3}\)
   c. \(x < -\frac{1}{3}\)
   d. \(x < \frac{1}{3}\)

28. The solution for the open sentence \(\frac{2}{3}x - 3 = 5\) is
   a. 12
   b. 3
   c. \(\frac{16}{3}\)
   d. 8

29. If \(U = \{1, 2, 3\}\) then the solution set for \(x + y = 4\) is
   a. \(\{(1, 3)\}\)
   b. \(\{(1,3), (3,1), (2,2)\}\)
   c. \(\{(3,1)\}\)
   d. \(\{(0,4), (4,0)\}\)

30. If \(U = \{1, 2, 3\}\) then the solution set for \(x < y\) is
   a. \(\{(1,2), (1,3), (2,3)\}\)
   b. \(\{(1,2), (2,3)\}\)
   c. \(\{1,2,3\}\)
   d. \(\{(1,1), (2,2), (3,3)\}\)

31. The \(x\) intercept of the graph of \(2x - 3y = 6\) is
   a. -3
   b. 3
   c. 2
   d. -2

32. The \(y\) intercept of the graph of \(2x - 3y = 6\) is
   a. -3
   b. 3
   c. 2
   d. -2

33. The \(x\) intercept of the graph of \(2x + y - 5 = 0\) is
   a. 0
   b. \(\frac{5}{2}\)
   c. \(-\frac{5}{2}\)
   d. 5
34. The $y$ intercept of the graph of $2x + y - 5 = 0$ is
   a. 0
   b. $5/2$
   c. $-5/2$
   d. 5

35. When the sentence $3x + y = 8$ is solved for $y$ we get
   a. $y = 8$
   b. $y = 8 - 3x$
   c. $y = 8 + 3x$
   d. $y = 8/3$

36. When the sentence $x + 3y \leq 6$ is solved for $y$ we get
   a. $y \leq 6 - x$
   b. $y \leq 2 - x$
   c. $y \geq 2 - 1/3x$
   d. $y \leq 2 - 1/3x$

37. A point on the graph of $y = (x + 2)^2 + 1$ is
   a. (2,1)
   b. (-2,9)
   c. (4,9)
   d. (0,5)

38. The axis of symmetry of the graph of $y = (x - 3)^2 + 2$ is
   a. $y$ axis
   b. $y = 2$
   c. $x = 3$
   d. $x = 2$

39. The vertex of the graph of $y = (x - 3)^2 + 2$ is
   a. (0,0)
   b. (-3,2)
   c. (3,-2)
   d. (3,2)

40. The axis of symmetry of the graph of $y = (x + 2)^2 + 1$ is
   a. $y$ axis
   b. $x = 2$
   c. $y = 1$
   d. $x = -2$
41. The compound statement $p \lor q$ is false when
   a. $p$ is true and $q$ is true
   b. $p$ is true and $q$ is false
   c. $p$ is false and $q$ is true
   d. $p$ is false and $q$ is false

42. The compound statement $p \land q$ is true when
   a. $p$ is true and $q$ is true
   b. $p$ is true and $q$ is false
   c. $p$ is false and $q$ is true
   d. $p$ is false and $q$ is false

43. The compound statement $p \to q$ is false when
   a. $p$ is true and $q$ is true
   b. $p$ is true and $q$ is false
   c. $p$ is false and $q$ is true
   d. $p$ is false and $q$ is false

44. The compound statement $\neg p \lor q$ will be false when
   a. $p$ is true and $q$ is true
   b. $p$ is true and $q$ is false
   c. $p$ is false and $q$ is true
   d. $p$ is false and $q$ is false

45. The compound statement $p \land (p \to q)$ is true when
   a. $p$ is true and $q$ is true
   b. $p$ is true and $q$ is false
   c. $p$ is false and $q$ is true
   d. $p$ is false and $q$ is false

46. The converse of the statement, if $x = 2$ then $x^2 = 4$ is
   a. if $x_2 \neq 2$ then $x^2 \neq 4$
   b. if $x = 4$ then $x = 2$
   c. if $x_2 \neq 2$ then $x = 4$
   d. if $x^2 \neq 4$ then $x \neq 2$

47. The inverse of the statement, if $x = 2$ then $x^2 = 4$ is
   a. if $x_2 \neq 2$ then $x^2 \neq 4$
   b. if $x = 4$ then $x = 2$
   c. if $x_2 \neq 2$ then $x = 4$
   d. if $x^2 \neq 4$ then $x \neq 2$
48. The contrapositive of the statement, if \( x = 2 \) then \( x^2 = 4 \) is
   a. if \( x^2 \neq 2 \) then \( x^2 \neq 4 \)
   b. if \( x^2 = 4 \) then \( x = 2 \)
   c. if \( x^2 \neq 2 \) then \( x^2 = 4 \)
   d. if \( x^2 \neq 4 \) then \( x \neq 2 \)

49. The name given to the compound statement \( x = 2 \) and \( x^2 = 4 \) is
   a. conjunction
   b. conditional
   c. disjunction
   d. biconditional

50. The name given to the compound statement if \( x = 2 \) then \( x^2 = 4 \) is
   a. conjunction
   b. conditional
   c. disjunction
   d. biconditional

51. The name given to the compound statement \( x = 2 \) or \( x^2 = 4 \) is
   a. conjunction
   b. conditional
   c. disjunction
   d. biconditional

52. The tautology among the following statements is
   a. \( p \to q \)
   b. \( p \lor \lnot p \)
   c. \( p \land \lnot p \)
   d. \( p \leftrightarrow q \)

53. The property that is not a property of all groups is
   a. closure
   b. inverses
   c. associativity
   d. distributive
54. The probability of tossing a 3 on one throw of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 5/6

55. The probability of not tossing a 3 on one throw of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 5/6

56. The probability of tossing a 3 or a 4 on one throw of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 4/6

57. The probability of tossing a number less than 3 on one throw of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 5/6

58. The probability of not tossing a 3 or a 4 on one throw of a single die is
   a. 1/6
   b. 2/6
   c. 3/6
   d. 4/6

59. The probability of tossing a 3 on one toss of two dice is
   a. 1/36
   b. 2/36
   c. 3/36
   d. 4/36
60. The probability of tossing a 4 on one toss of two dice is
   a. 1/36
   b. 2/36
   c. 3/36
   d. 4/36

61. The probability of tossing a 3 or a 4 on one toss of two dice is
   a. 1/36
   b. 3/36
   c. 5/36
   d. 7/36

62. The probability of getting exactly 3 tails on three tosses of a coin is
   a. 1/8
   b. 2/8
   c. 3/8
   d. 4/8

63. The probability of getting exactly 2 tails on three tosses of a coin is
   a. 1/8
   b. 2/8
   c. 3/8
   d. 4/8

In questions 64 through 66 a box contains two red and two white balls. Two balls are drawn in succession without replacement.

64. The probability of drawing one white and one red ball is
   a. 8/12
   b. 9/12
   c. 10/12
   d. 11/12

65. The probability of drawing two red balls is
   a. 1/12
   b. 2/12
   c. 3/12
   d. 4/12
66. The probability of drawing two balls of the same color is
   a. 1/12
   b. 2/12
   c. 3/12
   d. 4/12

67. The one of the following that is not a measure of central tendency is
   a. mean
   b. median
   c. mode
   d. range

68. The one of the following that is a measure of dispersion is
   a. mean
   b. median
   c. standard deviation
   d. mode

69. The set of scores 71, 71, 79, 82, 82, 94, 99 has a median of
   a. 71
   b. 79
   c. 82
   d. 83

70. The set of scores 71, 71, 79, 82, 87, 89, 95 has a mode of
   a. 71
   b. 79
   c. 82
   d. 83

71. The set of scores 71, 71, 79, 82, 87, 89, 95 has a mean of
   a. 71
   b. 79
   c. 82
   d. 83
72. The set of scores 6, 8, 10, 12, 14 has a median of
   a. 9
   b. 10
   c. 11
   d. 12

In exercise 73 through 78, 400 coins are tossed repeatedly, the distribution of the number of heads is a normal one with a mean of 200 and a standard deviation of 10.

73. The percentage of the number of heads between 190 and 210 is
   a. 50
   b. 68
   c. 84
   d. 95

74. The percentage of the number of heads less than 200 is
   a. 50
   b. 68
   c. 84
   d. 95

75. The percentage of the number of heads between 180 and 220 is
   a. 84
   b. 95
   c. 97.5
   d. 99.7

76. The percentage of the number of heads above 190 is
   a. 84
   b. 95
   c. 97.5
   d. 99.7

77. The percentage of the number of heads between 170 and 230 is
   a. 84
   b. 95
   c. 97.5
   d. 99.7
78. The percentage of the number of heads below 210 is
   a. 84
   b. 95
   c. 97.5
   d. 99.7

In exercise 79 through 82 we consider the production of a new kind of radio. Experience shows that 1/6 of the radios are defective. When we examine 720 radios;

79. The mean number of defectives to be expected is
   a. 100
   b. 120
   c. 360
   d. 720

80. The standard deviation of the distribution of the number of defectives is
   a. 6
   b. 5
   c. 10
   d. 20

81. We can say with 68% confidence that the number of defectives will be between
   a. 600 and 720
   b. 115 and 125
   c. 590 and 610
   d. 110 and 130

82. We can say with 95% confidence that the number of defectives will be between
   a. 100 and 140
   b. 590 and 610
   c. 90 and 150
   d. 700 and 740

83. The functional rule representing the correspondence 1 \rightarrow 2 \text{ is }
    a \rightarrow 2
    b \rightarrow 5
    3 \rightarrow 10
    4 \rightarrow 17
    \ldots

   a. n \rightarrow 3n - 1
   b. n \rightarrow 2n
   c. n \rightarrow 2n + 1
   d. n \rightarrow n^2 + 1
84. The set of ordered pairs representing the mapping \( 1 \rightarrow 1 \) is

\[ 2 \rightarrow 4 \]
\[ 3 \rightarrow 9 \]
\[ 4 \rightarrow 16 \]

a. \( \{(1,1),(2,4),(3,9),(4,16)\ldots\} \)
b. \( \{(1,1),(4,2),(9,3),(16,4)\ldots\} \)
c. \( \{(1,2),(2,3),(3,4),(5,6)\ldots\} \)
d. \( \{(1,4),(4,9),(9,16),(16,25)\ldots\} \)

85. If \( A = \{2,3\} \) and \( B = \{3,4\} \) then \( A \times B \) is

a. \( \{(2,2),(3,3),(4,4)\} \)
b. \( \{(3,2),(3,3),(4,2),(4,3)\} \)
c. \( \{(2,2),(2,3),(3,2)\} \)
d. \( \{(2,3),(2,4),(3,3),(3,4)\} \)

86. The domain of the relation \( \{(1,2),(2,3),(3,4)\} \) is

a. \( \{2,3,4\} \)
b. \( \{1,2,3\} \)
c. \( \{(2,1),(3,2),(3,4)\} \)
d. \( \{1,2,3,4\} \)

87. When \( f(x) = x + 4 \) then \( f(3) \) is

a. 1
b. 7
c. 12
d. 3x

88. When \( f(x) = 3x + 5 \) then \( f(2) \) is

a. 2
b. 3
c. 5
d. 11

89. When \( f(x) = 2x^2 + 3 \) then \( f(3) \) is

a. 39
b. 21
c. 17
d. 3
90. For positive integers $x$ and the relations expressed by equation $y = 3 + 4$, if $x$ is increased by 1 then $y$ is increased by

a. 1  
b. 2  
c. 3  
d. 4

91. If $y = 6x - 2$ then $\Delta y$ is

a. 2  
b. 4  
c. 6  
d. 8

92. If $y = ax + b$ and $\Delta y = 3$ and $(2, 7)$ is an ordered pair of the function then the value of $b$ is

a. 0  
b. 1  
c. 2  
d. 3

93. If $f(x) = 2x + 3$ then $f(x + 1)$ is

a. $x + 3$  
b. $2x + 2$  
c. $2x + 3$  
d. $2x + 5$

94. If $r(x) = x^2 - 3x - 2$ then $f(4)$

a. 1  
b. 2  
c. 3  
d. 4

95. If $f(x) = x^2 + 4x - 3$ then $\Delta^2 y$ is

a. $2x + 1$  
b. $2x + 3$  
c. 2  
d. 4
96. The inverse of the relation \{(2,4),(3,4),(4,5)\} is
   a. \{(4,5),(3,4),(2,4)\}
   b. \{(4,2),(3,4),(4,5)\}
   c. \{(4,2),(4,3),(4,5)\}
   d. \{(4,2),(4,3),(5,4)\}

97. The equation for the inverse of the relation determined by the equation \(y = x - 2\) is
   a. \(y = x + 2\)
   b. \(y = x - 2\)
   c. \(x - 2 = y\)
   d. \(y = -x + 2\)

98. If \(\log_2 64 = x\) then \(2^x\) is
   a. 64
   b. 32
   c. 5
   d. 2

99. \(\log_2 32\) is
   a. 2
   b. 5
   c. 16
   d. 32

100. \(\log_2 1/32\) is
    a. -2
    b. 2
    c. -5
    d. 5
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## APPENDIX G

### Table 36

Data for Experimental and Control Groups

| Student | Group | Years of Pretest ACT Composite Post-test |
|---------|-------|-----------------------------|-----------------------------|
|         |       | High School Scores          | Scores                      |
| E       | C     | Mathematics                |                             |
| 1.      | X     | 3                           | 17                          | 22                          | 78                          |
| 2.      | X     | 2                           | 15                          | 20                          | 59                          |
| 3.      | X     | 3                           | 17                          | 24                          | 91                          |
| 4.      | X     | 3                           | 26                          | 24                          | 98                          |
| 5.      | X     | 2                           | 14                          | 17                          | 60                          |
| 6.      | X     | 3                           | 18                          | 23                          | 92                          |
| 7.      | X     | 3                           | 14                          | 17                          | 85                          |
| 8.      | X     | 2                           | 14                          | 27                          | 69                          |
| 9.      | X     | 4                           | 21                          | 21                          | 98                          |
| 10.     | X     | 4                           | 24                          | 24                          | 96                          |
| 11.     | X     | 2                           | 15                          | 19                          | 58                          |
| 12.     | X     | 3                           | 16                          | 25                          | 86                          |
| 13.     | X     | 3                           | 20                          | 23                          | 92                          |
| 14.     | X     | 4                           | 17                          | 17                          | 86                          |
| 15.     | X     | 2                           | 19                          | 18                          | 93                          |
| 16.     | X     | 3                           | 17                          | 19                          | 88                          |
| 17.     | X     | 1                           | 16                          | 18                          | 78                          |
| 18.     | X     | 1                           | 15                          | 20                          | 64                          |
| 19.     | X     | 2                           | 8                           | 16                          | 37                          |
| 20.     | X     | 1                           | 14                          | 16                          | 70                          |
| 21.     | X     | 4                           | 19                          | 25                          | 88                          |
| 22.     | X     | 2                           | 10                          | 17                          | 44                          |
| 23.     | X     | 4                           | 21                          | 27                          | 94                          |
| 24.     | X     | 3                           | 20                          | 25                          | 92                          |
| 25.     | X     | 1                           | 18                          | 26                          | 92                          |
| 26.     | X     | 4                           | 19                          | 24                          | 92                          |
| 27.     | X     | 4                           | 17                          | 23                          | 81                          |
| 28.     | X     | 4                           | 16                          | 15                          | 70                          |
| 29.     | X     | 2                           | 18                          | 2 21                         | 86                          |
| 30.     | X     | 2                           | 17                          | 21                          | 84                          |
| 31.     | X     | 4                           | 25                          | 21                          | 86                          |
| 32.     | X     | 2                           | 12                          | 17                          | 65                          |
| 33.     | X     | 2                           | 13                          | 13                          | 55                          |
| 34.     | X     | 4                           | 16                          | 27                          | 91                          |
| 35.     | X     | 1                           | 22                          | 23                          | 85                          |
| 36.     | X     | 2                           | 21                          | 20                          | 89                          |
| 37.     | X     | 3                           | 15                          | 17                          | 54                          |
| 38.     | X     | 2                           | 20                          | 24                          | 87                          |
| 39.     | X     | 1                           | 11                          | 13                          | 37                          |
| 40.     | X     | 2                           | 19                          | 24                          | 81                          |
Table 36 (continued)

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