A photomechanics system for nondestructive three-dimensional stress analysis

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A PHOTOMECHANICS SYSTEM FOR NONDESTRUCTIVE
THREE-DIMENSIONAL STRESS ANALYSIS

by

David LaMoyne Olson

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
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DOCTOR OF PHILOSOPHY

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Dean of Graduate College

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INTRODUCTION

A product engineer in one of today's highly competitive manufacturing industries must be able to economically produce a machine of known life and reliability. This machine may be composed of simple as well as complex parts. Each part comprises a link in the machine's reliability chain. If one link in the chain fails, the machine fails.

The stresses which each machine part must endure depend upon the loads experienced by the machine. The company's product engineers must be able to relate the stresses in each part to the expected and/or actual machine loads which were determined by the field engineers.

The product engineer may need to determine internal stresses in a composite model, thermal stresses in an engine piston, or contact stresses in a hypoid gear set. He may need to make a parametric study of an involute spline coupling with various engagement lengths and internal spline wall thicknesses. A gusseted frame may be subjected to various loading geometries and ratios of loading forces. When pre-stressing a thick-walled pressure vessel so as to allow for higher working pressures, a specified amount of plastic flow may have to be introduced.

Providing that the geometry of the parts does not become too intricate, the engineer may be able to calculate some approximate stresses in terms of the machine loads. Other-
wise, experimental stress analysis techniques must be relied upon.

The experimental stress analysis techniques which may be used are strain gages, brittle coatings, and photo-mechanics. With strain gages, brittle coatings, photoelasticity coatings, and moiré, only surface stresses on accessible surfaces may be analyzed. Two-dimensional photoelasticity offers the advantage of determining full field stresses but only for two-dimensionally stressed components. The stress-freezing technique of photoelasticity permits the analysis of internal stress on a three-dimensional basis but requires destruction of the model.

The objective of the research reported in this dissertation was to develop a nondestructive, three-dimensional stress analysis system which would aid the product engineer.
PHOTOMECHANICS BACKGROUND

Photomechanics is an experimental stress analysis method based on the property of temporary double refraction that certain transparent isotropic materials exhibit when they are subjected to stress or strain.

Theory

The optical phenomenon of double refraction, when viewed in a field of polarized, monochromatic light, manifests itself in the form of interference fringes or alternate dark and light bands. These fringes are ordered according to the darkness-brightness cycles that take place at the given point as the load or deformation is increased from zero to its final value.

As light enters a doubly refracting medium, the light is resolved along two perpendicular optical axes which have different indices of refraction, \( n_p \) and \( n_q \). At a distance \( s \) from where the light entered the model, the light which is propagating along the \( n_p \) optical axis is retarded \( N \) wavelengths with respect to the light which is propagating along the \( n_q \) optical axis. This photomechanics optic law is expressed mathematically as follows:

\[
N = \frac{1}{\lambda} \int_0^s (n_p - n_q) ds
\]
where \( \lambda \) is the light wavelength. In the general case where one of the principal optical axes in the model is not parallel to the light propagation axis, the photomechanics optic law is expressed in terms of the "secondary" indices of refraction. These are the maximum, \( n_p \), and the minimum, \( n_q \), of those indices of refraction in the planes which are perpendicular to the light propagation axis.

Depending on the material, the optical phenomenon of temporary double refraction may be due to stress or strain, or both. But for linear elastic materials, where stress and strain are linearly related, the optical effect can be referred to either stress or strain. This temporary double refraction is also referred to as temporary birefringence or the photomechanics effect.

For linear elastic materials, the principal optical axes coincide with the directions of the principal stresses, and the differences between the indices of refraction are proportional to the applied load. The principal stresses (and strains) are therefore linearly related to the indices of refraction.

Equation 2 expresses the fundamental relation between stress and optical effect and is known as the stress-optic law of photoelasticity,

\[
 n_1 - n_0 = C_1 \sigma_1 - C_2 (\sigma_2 + \sigma_3)
\]
\[ n_2 - n_0 = C_1 \sigma_2 - C_2 (\sigma_3 + \sigma_1) \quad (2) \]

\[ n_3 - n_0 = C_1 \sigma_3 - C_2 (\sigma_1 + \sigma_2) \]

where:

- \( n_1, n_2, n_3 \) = principal indices of refraction
- \( n_0 \) = index of refraction before loading
- \( C_1, C_2 \) = stress-optical coefficients
- \( \sigma_1, \sigma_2, \sigma_3 \) = principal stresses

In practice, however, the differences between the principal indices of refraction are more commonly used. Eliminating \( n_0 \) from Equation 2 yields Equation 3,

\[ n_1 - n_2 = C (\sigma_1 - \sigma_2) \]

\[ n_2 - n_3 = C (\sigma_2 - \sigma_3) \quad (3) \]

\[ n_3 - n_1 = C (\sigma_3 - \sigma_1) \]

where:

\[ C = C_1 + C_2 \] = relative stress-optical coefficient

For the general case of secondary principal planes, Equation 3 is expressed as Equation 4,

\[ n_p - n_q = C (\sigma_p - \sigma_q) \quad (4) \]
where \( \sigma_p \) and \( \sigma_q \) are the maximum and minimum secondary principal stresses respectively.

The photomechanics stress-optic law equation for linear elastic materials is obtained by substituting Equation 4 into Equation 1,

\[
N = \frac{1}{\lambda} \int_0^S C(\sigma_p - \sigma_q) ds
\]  

(5)

For two-dimensional photoelasticity stress analysis, Equation 5 is usually presented in its integrated form,

\[
N = \frac{C}{\lambda} (\sigma_p - \sigma_q)t
\]  

(6)

where:

\( t = \) model thickness

or

\[
N = \frac{\sigma_p - \sigma_q}{f_\sigma} t
\]  

(7)

where:

\( f_\sigma = \lambda/C = \) the material fringe value

When analyzing model materials for which stress and strain are not linearly related, such as materials used in plasticity studies, the stress-optic law and/or the strain-optic law must be determined for the particular model material.
Stress-Freezing Technique

One technique for analyzing three-dimensional stress problems is the so-called "stress-freezing" technique which utilizes certain di-phase plastics which behave as if they were composed of two components with different mechanical and thermal properties. When the model is slowly heated to a "critical" temperature, one component, an elastic infusible skeleton, remains relatively rigid and carries the load, while the other component, a fusible matrix, becomes flexible. When the model is loaded at the critical temperature and then slowly cooled, the fusible component rehardens around the deformed skeleton and "locks-in" the state of deformation corresponding to the elastic state of stress at the elevated temperature. This state of stress is not disturbed by careful slicing of the model.

Thin slices are removed from the model wherever the stress distribution is required. A fringe pattern, which corresponds to the three-dimensional stress distribution in the model at the time of stress freezing, is obtained by using two-dimensional photoelasticity analysis techniques.
Scattered-Light Technique

The scattered-light technique of photomechanics is based on the scattering characteristics of a wave of light as it passes through a doubly refracting medium.

**Polarization by scattering**

Suppose that a light ray which is passing through a scattering medium (a model) is viewed transversely as shown in Figure 1. Since the incident light vibration is in the transverse plane, the vibrating vector OA can be divided into two components, one along the line of vision OC and the other perpendicular to the line of vision OB. The scattered light that the eye perceives arises entirely from the component which is perpendicular to the line of vision. Consequently, this light is completely polarized, and the vibration is perpendicular to the plane containing the line of vision and the incident ray.

**Light ellipse**

When a ray of polarized light traverses a stressed, doubly refractive medium, the light at any point is generally elliptically polarized in the plane of the wave front. The ellipse traced by the light vector is called the light ellipse and has the equation,

\[ \frac{p^2}{a^2} + \frac{Q^2}{b^2} - \frac{2pq}{ab} \cos 2\pi N = \sin^2 2\pi N \]  

(8)
Figure 1. Polarization phenomena connected with scattering of light
where:

\[ P = a \cos \omega t \]

and

\[ Q = b \cos(\omega t + 2\pi N) \]

The parameters \( P \) and \( Q \) are the component vibrations along the directions of the secondary principal axes \( p \) and \( q \) in the plane of the incident wave front.

The axes of the light ellipse are oriented at angles \( \beta \) and \( \beta + (\pi/2) \) to the \( p \) axis (Figure 2) such that

\[ \tan 2\beta = \frac{2ab}{a^2 - b^2} \cos 2\pi N \]  

(9)

When \( N \) is a full fringe order or a half fringe order, the light is plane polarized, and the light vector is inclined at an angle \( \pm \beta \) to the \( p \) axis such that \( \tan \beta = \pm b/a \).

The light scattered perpendicular to the wave normal is plane polarized and has an intensity proportional to the square of the amplitude obtained by projecting the light ellipse onto a line which is perpendicular to the direction of observation. For observation along a line that makes an angle \( \alpha \) (Figure 2) with the \( p \) axis, the intensity is given by

\[ I = K(a^2 \sin^2 \alpha + b^2 \cos^2 \alpha - 2ab \sin \alpha \cos \alpha \cos 2\pi N) \]  

(10)

where \( K \) is a constant which depends upon the light transmission and scattering properties of the model material.
Figure 2. Light ellipse and direction of observation
Optic laws

The photomechanics optic law in formula form for scattered-light is the following differential form of Equation 1:

\[ \frac{dN}{ds} = \frac{1}{\lambda} (n_p - n_q) \]  \hspace{1cm} (11)

A mathematical expression for the scattered-light stress-optic law with nonrotating secondary principal axes may be derived from Equation 5:

\[ \frac{dN}{ds} = \frac{C}{\lambda} (\sigma_p - \sigma_q) \]  \hspace{1cm} (12)

Jessop (1, p. 252) gives the following mathematical alteration for the scattered-light stress-optic law when the secondary principal axes are rotating:

\[ \frac{dN}{ds} = \frac{dN^*}{ds} [1 + (\frac{a}{b} - \frac{b}{a}) \sin 2\pi N \frac{da}{dN^*}] \]  \hspace{1cm} (13)

where \( dN \), the relative retardation acquired over a distance \( ds \), is partly due to the retardation \( dN^* \) which is caused by stresses alone and partly due to the rotation \( da \) of the secondary axes over the distance \( ds \); \( a \) and \( b \) are the amplitudes of the light vectors, and \( N \) is the scalar order existing at \( s \).

Aderholt, McKinney, Ranson, and Swinson (2) suggested a technique which eliminates any error resulting from rotation
of the secondary axes. As the scalifs are ordered along
the light beam, the angle of observation, $\alpha$, must always be
such that $\tan \alpha = b/a$ (Figure 2).
LITERATURE REVIEW

With Weller's (3) development of the scattered-light stress-optic law in 1939, scattered-light photomechanics began its emergence as a useful laboratory tool. Since 1939, this tool for nondestructive three-dimensional experimental stress analysis has been in a continuous stage of development.

Theory

An examination of the theory of the photoelastic effect and of the conditions affecting the appearance of interference fringes in basic scattered-light photoelasticity was conducted by Jessop (1), Swinson (4), Srinath (5), and Srinath and Frocht (6).

Rotation of Secondary Principal Stress Directions

The stress-optic law of scattered-light photoelasticity states that the scalar space gradient for nonrotating secondary principal stress directions is proportional to the difference of secondary principal stresses in the plane normal to the light propagation axis.

Using the simplified vibration theory of light and assuming that the changes in relative retardation due to stress and the changes in orientation of the stress directions took place in alternate small increments, Jessop (1) derived a
differential equation which mathematically expresses the observed rate of change of phase difference with respect to the rates of change of stress directions and of phase difference due to stress only. He then explained why the errors introduced by measurement of the positions of the scalifs would not exceed an order of 1% when the scalifs are sharply defined over a range of two or more scalifs. The effect of stress axes rotation was proven to be minimized by using as large a model as possible.

A wave propagation approach was used by Drucker and Mindlin (7) to show that if the rate of rotation of secondary principal stress directions with respect to the rate of retardation due to secondary principal stress differences only is small, the errors due to neglecting the effects of rotation are small.

The effects of rotation of secondary principal stress directions on the methods of determination of secondary principal stress directions and secondary principal stress differences for basic scattered-light photoelasticity were discussed by Srinath (8). Of the several methods discussed for determining the secondary principal stress directions, the zero intensity method was considered the best. A dual observation modification of the constant-intensity method was shown to be exact for rotating secondary principal stress directions. Among the three methods discussed for deter-
mining the magnitudes of the secondary principal stress differences, the compensator method was considered the best although it is a point-by-point method.

The concept of light waves following the secondary principal axes as the axes rotate was substantiated with experimental evidence and used by Aderholt, McKinney, Ranson, and Swinson (2) in their development of a technique for eliminating any error resulting from rotation of the secondary axes. (See Photomechanics Background, Optic Laws.)

General Methods

The reviewed literature revealed four general methods by which the basic data for scattered-light photoelasticity could be obtained. The most suitable method for a particular application would depend upon the finances available, the types of problems to be solved, and the information desired for the problem solution.

Basic

A simple, basic method for determining the secondary principal stress directions and birefringence was given by Swinson (4), Srinath (5), and Srinath and Frocht (6). This straightforward method does not require any sophisticated electronics, although a low intensity light meter would be helpful for determining minimum intensities.
Polariscopes designed with this approach in mind were discussed by Swinson (4), Srinath (5), and Swinson and Bowman (9).

Swinson's polariscope was designed for manual movement of the model with respect to the light beam in three mutually perpendicular directions and rotation of the model about the light beam axis. The vertical range of movement was 6.5 inches, and the transverse ranges were 4.2 inches and 6.5 inches. The plexiglass immersion tank, which was assembled with screws and sealed with epoxy, developed leaks after two months of service. The tank was then placed in an aluminum frame and heavily sealed with epoxy. This lasted about one month before leaks began to appear again.

Srinath's relatively small, manually operated polariscope used a 1000 watt mercury arc lamp with a lens system as a light source. Model motion in three mutually perpendicular directions and rotation about the light beam were permitted. For compensation and accurate determination of secondary principal stress directions, a compensator and photometer were incorporated in the polariscope design.

Both polariscopes described above were designed for analyzing stress-frozen models.

Specified axes

Jenkins (10) developed a method for finding the rectangular normal-stress differences and shearing stresses.
This method requires observations of scalar positions only. For the general case, a circularly polarized sheet of monochromatic light must be projected along three paths parallel to the X-Z plane, three paths parallel to the Y-Z plane, and one path parallel to the Z axis. Jenkins' particular problem contained two planes of symmetry, so only three paths of observation were required for determining the unknown normal-stress differences and shear.

Rather than moving his model, which was fixed in a tensile machine, Jenkins used fixed mirrors and horizontal and vertical motion of his laser light source to observe points of interest from specified angles. His polariscope was applicable only to problems which were similar to his tensile specimen which contained a cylindrical glass inclusion.

Dual observation

The dual observation method developed by Cheng (11) does not require rotation of the direction of polarization of the light beam or rotation of the model. Point-by-point raw photoelasticity data are obtained by measuring the scattered light intensity while the model is both under load and free of load. These measurements must be made from two directions which are 45 degrees from each other.

A flexible relay optic, sophisticated photon counting equipment, and computer techniques were incorporated in the polariscope for automatic data collecting and interpreting.
Spinning analyzer

All the above methods use the polarizing effect of light scattering as an analyzer. Robert and Guillemet (12), in their development of the spinning analyzer method, used this polarizing effect as a polarizer for their polariscope. The energy of the emergent light beam was mathematically expressed as the sum of two components, one independent of the analyzer orientation and one dependent on the analyzer orientation. A photomultiplier was used to convert these components into a continuous alternating voltage which was measured by parallel DC and AC voltmeters. The photoelastic raw data parameters, obtained on a point-by-point basis, were proportional to these voltages.

An extremely expensive commercial polariscope has just recently been marketed which uses a similar system with a spinning polarizer.

Models

Most models used for scattered-light photomechanics studies have been fabricated from cast epoxies. Swinson (4) and Swinson and Bowman (9) used Ciba Araldite 6020 while Davis and Swinson (13) and Aderholt, Ranson, and Swinson (14) used Bakelite ERL 2774. Aderholt, Ranson, and Swinson's model was a composite model of a rocket motor fabricated by casting Solithane 113 around a wax core inside an ERL 2774 shell.
Davis and Swinson (13) offered some good suggestions for annealing and storing ERL 2774 models and also tabulated some of the material properties of ERL 2774 for various temperatures.

Photography

Most scattered-light photoelasticity patterns which were photographed by Swinson (4), Swinson and Bowman (9), Davis and Swinson (13), Aderholt, Ranson, and Swinson (14), and Bhonsle and Work (15) were recorded with a 35 mm camera and Tri-X film.

Data Reduction

Since the difference in secondary principal stresses is proportional to the scalif space gradient, $dN/ds$, the natural method of data reduction would be to obtain a scalif order versus distance curve or relationship and from this obtain the slope or derivative.

Srinath (5) and Jenkins (10) plotted curves and fitted tangents to them. Jenkins commented that he could not obtain a satisfactory least squares polynomial fit.

Swinson (4), Swinson and Bowman (9), Davis and Swinson (13), and Aderholt, Ranson, and Swinson (14) fit polynomials by least squares methods but ran spot checks by using the curve plotting and tangent fitting method.
Stress Separation

Apparently the only method of stress separation used for general three-dimensional states of stress is the shear-difference method which was developed by Frocht and Guernsey (16). Srinath (5) and Srinath and Frocht (6) used a rectangular coordinate system for the shear-difference method while Bhonsle and Work (15) used a cylindrical coordinate system and Davis and Swinson (13) used a spherical coordinate system.

Spline Coupling

Most published information concerning the design of involute spline couplings has been compiled from experiences with the performance of the couplings in practice or under laboratory fatigue tests.

Dudley (17, 18) categorized the possible ways in which a spline coupling could fail and, after some simplifications, calculated the magnitudes of these critical stresses. To account for the simplifications and loading history, he modified the allowable working stresses. After observing many of these splines perform, he constructed charts, graphs, and "rules of thumb" which the design engineer may refer to while keeping in mind some of the suggested manufacturing considerations.

A design treatment of shafts and splines from the fatigue
point of view was given by Burke and Fisher (19). In correlating test and analytical curves, the concept of nominal stress was used to introduce simplicity and perspective into the interpretation of such complex fatigue life factors as stress concentrations, physical properties, and modes of loading. A qualitative graphical presentation was given of the torsional shear stress and the transverse bending stress in the spline fillet along the shaft-hub interface.

Yoshitake (20) obtained the stress concentrations in a torqued splined shaft by using the wedge method of photoelasticity in conjunction with the compensating deformation method of model fabrication. Using the stress freezing and slicing technique, he acquired the distribution of bending stresses along the spline root for a parallel spline coupling. The magnitude of the bending stress was estimated by extrapolation of data to be more than three times that of the nominal shearing stress in the shaft.
EQUIPMENT

The equipment discussed in this chapter was designed and assembled by the author, since at the time his research program was initiated there were no commercial scattered-light polariscopes or scalif recorders available. In fact, there were no proven designs which met the author's specifications.

Polariscope

The polarscope, which is shown in Figures 3 and 4, was designed to obtain secondary principal direction and birefringence measurements by using the basic scattered-light method. This method permitted a simpler machine and a more fundamental approach to data acquisition than the other methods of scattered-light photomechanics.

The size of the constructed machine was dictated somewhat by the minimum size of the involute spline coupling model which could be readily fabricated. Although the polarscope was definitely designed with this model in mind, any model which could have been manipulated within the immersion tank could have been analyzed if a suitable loading frame had been constructed. If a stress-frozen model would have been analyzed, it would have been necessary to construct only a model holder.
Figure 3. Scattered-light polariscope (front view)
Figure 4. Scattered-light polariscope (back view)
Carriage

Electric motors and slide bars (Figure 5) were used to move the model in one or more of three orthogonal directions, one of which was parallel to the laser light beam. The Y- and Z-axis slide bars had a travel range of 12.50 inches whereas the X-axis slide bars were limited to a travel range of 9.50 inches. Synchronizing the movement of the two X-axis slide bars was done by connecting their lead screws with a cogged belt. After some shimming, these inexpensive dovetail slide bars performed satisfactorily.

Motorized model rotation about the light beam was accomplished by rotating the turntable to which the orthogonal slide bars were mounted. Obstruction of the operator's line of vision by the loading frame limited the range of rotation.

In scattered-light photomechanics, the plane of polarization of the light beam is rotated 90° to change from a dark to a light field or vice versa. If rotating stresses are encountered, the plane of polarization may be oriented such that the alteration term for the stress-optic law in Equation 13 is minimized. Since the laser light was plane polarized, the polarization plane was rotated by rotating the laser turntable.
Figure 5. Polariscope's motor and slide bar system
Motors

The motors which powered the lead screws on the X- and Y-axis slide bars are shown in Figures 6 and 7. The Z-axis motor was mounted in the aluminum block which held the Z axis to the Y axis. The turntable motors were mounted in the turntable support blocks which are shown in Figure 8. These small permanent magnet 27 VDC motors had a 40 in-lb stall torque and operated over a speed range of 0 to 22 rpm when powered by an SCR controller.

The wiring diagram for the SCR controller is shown in Figure 9.

Counters

Counters, which are shown mounted in Figure 10, were used to indicate the position of the light beam in the model. Since these inexpensive counters could not subtract, one counter recorded motion in the positive direction and another recorded motion in the negative direction.

When these 48 VDC military surplus counters were wired as shown in Figure 11, they could be energized with 110 VAC.

The counters were activated by a microswitch which closed when its follower arm dropped into a detent. These detents were located on detent wheels as shown in Figure 12, on a drive coupler as shown in Figure 13, and on the turntable surface as shown in Figure 14.
Figure 6. X-axis slide bar motor

Figure 7. Y-axis slide bar motor

Figure 8. Model turntable motor
Figure 9. SCR controller wiring diagram

28 VDC Motor

115 VAC
3/4 Amp

Control Potentiometer

SCR Controller
Figure 10. Model position counters
115 VAC

Counter
48 VDC

Diodes: 500 ma, Type G100G, PT-5, or F-6
Resistor: 75 Ω, 10 Watt

Figure 11. Counter wiring diagram
Figure 12. Detent wheel

Figure 13. Drive coupler with detents

Figure 14. Turntable with detents
When the polariscope light axis was parallel to the X or Y model axis, the model coordinate axes were, obviously, no longer parallel to their respective polariscope axes. This meant that model movement caused by the polariscope X-axis slide bars would have moved the model in the negative Z direction. The coordinate transformation switching system shown schematically in Figure 15 permitted the operator to continue observing the correct model coordinate position on the counters as they are labeled in Figure 10.

**Electrical circuit**

The major portion of the polariscope's electrical system (Figure 16) was located in the control drawer (Figures 17 and 18) which was covered with a combination safety shield and writing surface. If extensive work was to be performed on or within the control drawer, the drawer was removed from the polariscope after unplugging an umbilical cord. The umbilical plug terminals, which are labeled in Appendix A, provided a good trouble-shooting station as did the control drawer.

All polariscope motion was controlled from the control drawer. A given slide or turntable was moved in a given direction by first moving the corresponding four-pole, double-throw, on-off-on switch in the corresponding direction. To complete the motor power circuit, the motor switch was
Figure 15. Model-polariscope coordinate transformation switch wiring diagram
Figure 16. Control drawer wiring diagram
Figure 17. Control drawer interior (mirror image)

Figure 18. Control drawer switch panel
moved to the continuous operating position or the motor switch was moved to the momentary position and the motor momentary switch was closed.

If the corresponding counter was to advance simultaneously with the motor, the counter switch was moved to the automatic position. If a counter missed a count, the counter was corrected by moving the motor switch to the off position, moving the corresponding motor-counter switch to its corresponding position, moving the counter switch to the advance position, and then pressing the counter advance switch the correct number of times.

If a slide bar or turntable drive motor stopped with the microswitch in detent position and the motor-counter switch was moved to the off position, a count signal was completed. If this drive motor and counter were reactivated in the same direction, another count signal was completed for the same detent. The relay shown in the control drawer wiring diagram, Figure 16, prevented double counting by not allowing a drive motor to stop with a microswitch closed.

If the SCR motor controller had failed, an emergency DC power source could have been used. The installation of the emergency DC power source required that the motor power switch be moved to the 28 VDC position and that the DC power source be plugged into the exterior jacks. Any voltage source less than 28 VDC could have been used, but the motors
would have operated more slowly.

**Light source**

The polariscope light source was a helium-neon laser whose output power was guaranteed greater than 15 milliwatts at 6,328 angstroms wavelength. Linear to better than one part in 1,000 was the guarantee on the polarization of the output. Measured at the $1/e^2$ points, the laser beam diameter was 1.1 millimeters.

Mounting the laser as shown in Figure 19 permitted the laser to be translated in a plane perpendicular to its light axis and to be rotated about two axes which were perpendicular to the light axis. These three degrees of freedom were required for alignment of the polariscope. A polariscope alignment procedure is given in Appendix B.

A lens system was used to converge the light beam to a point at the area of interest in the model.

The light sink, shown schematically in Figure 20, was placed at the bottom of the immersion tank to absorb the light beam after its emergence from the model. India ink, which appeared to absorb light very well, was used to paint the entire light sink with the exception of the steel cone mirror.
Figure 19. Laser mount
Figure 20. Light sink
Immersion tank

The immersion tank walls were constructed by rolling a 1/8 inch thick by 24 inch wide aluminum sheet into a 30 inch diameter cylinder and then welding the longitudinal seam. A 3/8 inch thick aluminum disk was welded inside and at one end of the cylinder to form the bottom of the tank. Sandblasting the tank interior removed all reflecting areas.

A reservoir to hold any tank window seal leakage was formed by inserting the 1/4 inch plate glass window inside the tank and by not extending the viewing hole to the bottom of the tank. Because the glass was not extended to the top of the tank, any tank overflow would have passed over the tank viewing window into the leak reservoir.

The window was sealed and bonded to the tank wall and bottom by using Silastic RTV 732 from Dow Corning. As a precautionary measure, a mechanical clamp (Figure 21) was installed.

Immersion fluid

The immersion fluid was Aroclor 1248 mixed with light mineral oil. A very small proportion of mineral oil was required to match the index of refraction of the fluid to that of the model.

The method used for matching fluid and model indices of refraction is presented in Figure 22. Since all three
Figure 21. Immersion tank window seal and clamp
Figure 22. Matching immersion fluid index of refraction to model index of refraction.
models contained cylindrical portions, the models themselves were used for index matching.

This immersion fluid mixture had a tendency to blister and lift paint as well as swell rubber parts such as O-rings.

A positive pressure general purpose pump was initially used to transfer the immersion fluid between the storage drums and immersion tank. Although the pump impeller was advertised as being oil resistant, severe swelling problems were encountered. To circumvent these problems, the immersion fluid was transferred to and from the immersion tank by pressurizing and vacuuming respectively the interior of the storage drums.

Since dust and other foreign material had a tendency to accumulate in the immersion fluid, a filtering system using an automotive crankcase oil filter was constructed. The filter base was designed to thread into the storage drum as shown in Figure 23.

By heating the immersion fluid, its index of refraction could be lowered slightly. For close index of refraction matching, a stirring motor and a thermostatically controlled 300 watt heating element were installed in the immersion tank. The 300 rpm stirrer was installed to maintain a uniform fluid temperature and to prevent scorching of the immersion fluid.
Figure 23. Immersion fluid filter
Because Aroclor 1248 and mineral oil did not mix easily, the stirring rod was connected to the stirring motor's 1800 rpm output shaft when the two fluids were being mixed.

**Camera**

The polariscope camera was a 5 x 7 monorail view camera with a 21 centimeter f3.5 lens and shutter speeds from 1/2 to 1/50 second and time and bulb settings.

The camera was mounted on the polariscope base plate or suspended from a mounting bridge. To maintain the camera lens axis near the point of interest in the model, the camera was elevated or lowered with mounting spacers.

**Loading frame**

The hydraulic torsional loading frame shown in Figure 24 was designed to apply a specified deflection angle which could be varied by adjusting the screws which stopped the torque lever arm. The ratio of the deflection angle in the loading frame back plate to the deflection angle in the involute spline coupling model was calculated to be approximately 0.004.

The torque lever was forced to the stop screws by four hydraulic pistons which received pressurized oil through passages drilled in the structural members of the loading frame.

For creating hydraulic energy, a manual pump was used to charge an accumulator to a pressure significantly higher
Figure 24. Torsional loading frame
than the pressure required by the loading frame for the desired model deflection. The accumulator and a valve at the polariscope operator's station permitted the model to be loaded without the operator leaving his station. The accumulator also maintained pressure in the hydraulic system even though there was some valve leakage. In the event of a loading crisis, or simply to remove the model load, a solenoid dump valve was actuated from the operator's station.

One of the more intricate machining procedures required for the construction of the loading frame was the machining of the races for the large ball bearing. This bearing permitted relatively friction-free rotation of the torque lever with respect to the top structural member.

Because of space limitations, the hydraulic pistons had only one O-ring for sealing. According to Society of Automotive Engineers standards (21, p. 265), this seal was sufficient for fluid pressures up to 1,500 psi, although when the seals were tested at 2,000 psi, no leakage was detected. With 2,000 psi hydraulic pressure, the loading frame was capable of applying 6,280 in-lb of torque.

The active length of the loading frame was varied in 1/2 inch increments by moving the bottom model support plate up or down on the back plate. The method used to attach the model to the model support plate and to the torque lever or ball bearing inner race also offered a means of varying the
active length of the loading frame.

Obtaining data for a general point in a model required that the light beam be directed along three mutually perpendicular axes. Assume, for explanation purposes, that the Z model axis was the model's longitudinal axis, which was initially vertical. The model's Y axis was rotated to the vertical position by rotating the entire loading frame on its mounting bearing. The model was then rotated about its Z axis to vertically orient its X axis. One means of accomplishing this was to unfasten and rotate the bottom model support plate and the torque lever. Another means was to unfasten and rotate the model fixtures with respect to the bottom model support plate and the torque lever.

Scalif Recorder

The scalif recorder shown in Figure 25 was designed by the author so that scalif positions could be accurately determined.

The foundation of the scalif recorder was an X-Y recorder, which had a time-based X-axis movement. An inexpensive photoconductive cell, which was mounted on the recorder's traversing bar, provided a strong, zero-based signal when it was incorporated in the circuit shown schematically in Figure 26.

The process for accurately determining the scalif centers
Figure 25. Scalif recorder
Figure 26. Photocell wiring diagram
involved placing a positive transparency of the scalif pattern, such as the one shown in Figure 27, on the scalif recorder transparency table which was illuminated with a two-tube fluorescent desk lamp. As the recorder bar, with photocell attached, traversed in the X direction on a time basis, the photocell passed under the scalif pattern on the correctly positioned transparency. Simultaneously, the signal from the photocell was recorded as the Y-axis variable. The pen trace on the graph paper thus provided a means for scaling the transparency density versus distance curve. Figure 28 shows the output of the scalif recorder for the scalif pattern shown in Figure 27.
Figure 27. Scalif pattern
Figure 28. Scalif recorder output
MODELS

The first official release of an American involute spline standard appeared in the 1946 edition of the Society of Automotive Engineers Handbook. Since then, and before, the involute spline coupling has been used for transmitting torque from shaft to shaft, shaft to wheel, shaft to lever, and vice versa. However, the stress distribution in the involute spline coupling is still not well known. (See Literature Review, Spline Coupling.)

The scattered-light technique of photomechanics offered a means whereby the stresses inside the coupling could be measured. But, before the expensive involute spline coupling model was fabricated, a simplified experimental model was analyzed.

A grooved shaft model, which consisted of a circular shaft with a semicircular groove, was used for determining the optimum smoothing parameter for the cubic spline function algorithm. (See Data Reduction, Spline Function Algorithm.)

Experimental Model

The experimental model, which is shown in Figure 29, consisted of a solid cylindrical shaft bonded to the interior of a hollow cylindrical shaft. This model was designed and
Figure 29. Experimental model
fabricated so that:

(1) the distance over which the involute spline coupling influenced the torsional stress distribution in the externally splined shaft could be estimated,

(2) under constant deformation the relaxation of the scalif order with respect to time could be measured for the model material, and

(3) the assembled photomechanics system could be debugged and calibrated.

Design

To accomplish the first and third objectives, the experimental model had to, first of all, resemble the envisioned involute spline coupling model. Consequently, the coupling area and the loading ends of the experimental model were 1/5 scale models of the involute spline coupling model with the exception that the solid shaft was cemented inside the hollow shaft instead of splined. The two shafts were as long as their respective counterparts in the envisioned involute spline coupling model so that the end and coupling effects had sufficient distance to dissipate.
Fabrication

The experimental model was fabricated from hexahydrophthalic, phthalic anhydride cured ERL 2774 which was cast according to Leven's (22, p. 151) recommendations. Some "cab-o-sil" (a silica gel) was cast into the epoxy to improve its scattering properties. Bonding of the shaft ends to the aluminum loading frame fixtures was accomplished with the cement recommended by Leven (22, p. 159). With the loading frame serving as the positioning fixture, this recommended cement was also used to bond the solid shaft to the hollow shaft. (See Discussion, Model Fabrication, Cementing.)

Involute Spline Coupling Model

The involute spline coupling, shown in Figure 30, was to be analyzed so that more design information for agricultural implement spline couplings, particularly those experiencing fatigue loading conditions, would be available. Engagement length, female shaft wall thickness, and proximity of the external spline runout to the coupling were the main design parameters of interest.

Similitude

Because most agricultural splines are fabricated from steel, and an epoxy model was required for the scattered-light
Figure 30. Involute spline coupling model
technique, principles of similitude were used to transfer
the epoxy model data to the steel prototype.

The pertinent variables considered for the involute
spline coupling study were:

\( \sigma_e \) = Hencky-von Mises effective stress

\( \tau_{\text{max}} \) = maximum torsional shear stress in the external
spline shaft

\( E \) = modulus of elasticity

\( \nu \) = Poisson's ratio

\( d \) = external spline major diameter

\( l \) = spline engagement length

\( t \) = internal spline wall thickness

\( \lambda \) = general geometry

Therefore,

\[ \sigma_e = f(\tau_{\text{max}}, E, \nu, d, l, t, \lambda) \quad (14) \]

The Buckingham Pi Theorem (23, p. 36) permitted Equation
14 to be expressed as,
\[ \frac{\sigma_e}{\tau_{\text{max}}} = g\left(\frac{\tau_{\text{max}}}{E}, \nu, \frac{l}{d}, \frac{t}{d}, \lambda\right) \]  

Equation (15)

The load variable \( \tau_{\text{max}} \) was used instead of a loading torque so that the determination of the material fringe value and the calibration of the loading frame could be eliminated. Also, the meaning of the Pi term \( \sigma_e/\tau_{\text{max}} \) is more readily comprehended than that of a Pi term containing \( \sigma_e \), torque, and length terms.

The four variables, \( d, l, t, \) and \( \lambda \), assured geometrical similarity between model and prototype.

Since the torque was transmitted from one coupling member to the other member through deformable contact surfaces (the splines), the coupling material modulus of elasticity and Poisson's ratio were considered pertinent variables. The effect of this consideration is manifested in Equations 14 and 15. Usually the ratio of a stress to the load which causes the stress is a constant for a linearly elastic body, but this was not so for the involute spline coupling. Equation 15 mathematically shows that the ratio, \( \sigma_e/\tau_{\text{max}} \), was a function of the applied load Pi term, \( \tau_{\text{max}}/E \).

Because the model Poisson's ratio (0.36) did not equal the prototype Poisson's ratio (0.30), the model was distorted. Three possible procedures for handling distorted models were listed by Young (24, p. 28):
(a) Neglect certain variables that may be only slightly significant but lead to distortion.

(b) Determine the effect of distortion, analytically.

(c) Determine the effect of the distortion, empirically.

Determining the effect of distortion analytically would have required an analytical relationship between the distortion factor and the prediction factor. For as complex a structure as an involute spline coupling, determining this relationship would have required an analytical stress solution. Consequently, the experimental stress solution would not have been necessary in the first place.

After investigating the influence of Poisson's ratio in a plate with three loaded holes and in the three-dimensional problem of a cylinder with a deep external hyperbolic groove subjected to uniform stress in the direction of the longitudinal axis, Clutterbuck (25, p. 329) reported

> The main conclusion which can be drawn from this work is that differences in experimentally determined stress distributions due to Poisson's ratio effects are less than the errors which would be introduced by other factors.

After examining studies by other authors, Kenny (26, p. 706) concluded,

> The general result from the above studies was a maximum increase in peak stress values of up to 10% for the Poisson's ratio value of 0.5 compared with the stresses obtained with a Poisson's ratio value of 0.3.
After using the frozen-stress photoelastic technique to study the stresses on the inner surfaces of pipe T-junctions subjected to different internal pressures, Fessler and Lewin (27, p. 273) reported,

Within the limits of experimental accuracy, the peak strains were found to be independent of Poisson's ratio.

Several attempts by various authors to determine the magnitude of the error resulting from differences in Poisson's ratio between a photoelastic model and a prototype structure were studied by Sanford (28, p. 100) before he concluded,

The results of these studies indicate that violation of the Poisson model law does not restrict the use of the photoelastic model method except in those cases in which high precision measurements are required. However, it must be observed that in all of the above investigations only the first boundary-value problem was considered. Therefore, the conclusions reached apply to this class of problems only and should not, without supporting evidence, be generalized to include other classes of problems.

Sanford then considered the stress distribution in an infinite plate subjected to a uniaxial stress and containing a bonded elastic circular disk of a different material. By changing Poisson's ratio from 1/3 to 1/2, the change in the Hencky-von Mises effective stress was less than 7%. But Sanford reported instead an increase of 86% for a minor stress; in fact, its magnitude was less than 1/3 the magnitude of the major principal stress.
Hypoid gear sets (29), reactor closure heads (30, p. 316), and multi-component pressure vessels (31, p. 275), all of which contained contact stresses, were satisfactorily analyzed by using the stress-freezing technique of photoelasticity. If these steel contact stress prototypes were satisfactorily modeled with models which had Poisson's ratios of 0.50 instead of 0.30, the steel involute spline coupling should have been satisfactorily modeled with a model which had a Poisson's ratio of 0.36. Consequently, the expensive, time-consuming method of empirically determining the distortion effect was abandoned in favor of neglecting the distortion caused by Poisson's ratio.

Design

The smallest involute spline coupling which could be readily fabricated was 13 tooth, 6/12 pitch. So that there was a plane of symmetry and so that inspection measurements were made on a diameter, an even number of teeth, 14, was chosen. The involute spline coupling model shown in Figure 30 was therefore fabricated according to Society of Automotive Engineers' specifications for the following spline:

- flat root side fit
- 14 teeth
- 6/12 pitch
- 30° pressure angle
2.333 inch pitch diameter
2.463 inch external spline major diameter

The coupling and loading frame designs permitted $l/d$ ratios of 1.42, 1.22, 1.02, 0.81, and 0.61. At $l/d$ equal to 1.42, the internally splined member came within 0.20 inches (0.08d) of the external spline runout.

So that an internally splined member which was relatively rigid with respect to the externally splined member could be modeled, the wall thickness of the internally splined member was as large as the loading frame and available casting molds would permit. This gave a relative rigidity ratio greater than 40. For further studies, the wall thickness could be reduced by decreasing the outside diameter.

Fabrication

Hexahydro phthalic, phthalic anhydride cured ERL 2774, which was cast in the laboratory according to Leven's (22, p. 151) recommendations, was the material used for the model. Some "cab-o-sil" was added to the epoxy to improve its scattering properties.

When hobbing the external spline and shaper cutting the internal spline, an aluminum end plate was affixed to the shaft to prevent end chipping. Even then the shafts had to be cut 1/4 inch longer than was required so that after machining this chipped 1/4 inch length could be removed.

Because of stroke limitations on the shaper cutter used
to cut the internal splines and the chip evasion procedure mentioned above, only 4.75 inches of internal splines were cut. Even so, for maximum coupling engagement, there still remained 1.25 inches (0.507d) of internal splines beyond the end of the externally splined member.

Mounting

Because the rigidity of the externally splined shaft was to remain constant throughout the study, this shaft was mounted to the bottom model support plate with a 2° rotational offset. As a result, the coupling axes coincided with the polariscope axes when the model was loaded.

Bonding the model fixtures to the model was done with the cement recommended by Leven (22, p. 159).

Grooved Shaft Model

The grooved shaft model shown in Figure 31 was used for determining the optimum smoothing parameter for the spline function (See Data Reduction, Spline Function Algorithm.) and to prove that large models could be used in scattered-light photomechanics studies.

Design

The theoretical solution for the grooved shaft (32, p. 126) stipulated that the groove had to be semicircular with its center located on the shaft circumference. Also, the
Figure 31. Grooved shaft model
area of interest had to be loaded in pure torsion.

Because difficulties had been experienced with light transmission through 2.50 inches of the involute spline coupling model (See Discussion, Model Fabrication, Casting.), the grooved shaft diameter was made 3.50 inches to prove that large models could be used in scattered-light photo-mechanics studies.

Fabrication

The material used for the grooved shaft model was Bakelite ERL 2774 cured with hexahydro phthalic and phthalic anhydride according to Leven's (22, p. 151) recommendations. In an attempt to improve light transmission through the model, no "cab-o-sil" was added.

After the epoxy cylinder was turned to size on a lathe with a tungsten carbide bit, the groove was machined with a ball end mill in a vertical mill. While milling, the ball end mill was submersed in a lard base oil.

In an effort to eliminate diffusion of the light beam as it passed through the model and to eliminate bright spots where the light beam passed into and out of the model, the model was polished until one could see through it. (Notice in Figure 31 that the model fixture can be seen through the model.) For initial smoothing, 320 grit wet and dry carbide paper flooded with water followed by 600 grit wet
and dry carbide paper flooded with water were used. The final polishing was done with pumice powder and mineral oil and then Brasso.

Mounting of the grooved shaft model into the loading frame was similar to the mounting of the other two models.
DATA ACQUISITION

Data acquisition, the sensing and handling of the information data, was, for the photomechanics system, logically handled with photographic techniques. But first of all, it was necessary to determine what data was to be collected, how it was to be identified, how it was to be recorded, and how it was to be prepared for reduction.

Secondary Principal Directions

With the general three-dimensional state of stress, there was always the possibility that the secondary principal directions were rotating. The directions of these axes were determined by the uniform intensity method which is based on the fact that for observations along a secondary principal axis, the intensity of the scattered light is independent of the birefringence. Mathematical proof of this concept is given by letting $\alpha$ equal zero or $\pi/2$ in Equation 10.

Birefringence

After the secondary principal directions were determined at the point of interest, the direction of observation with respect to these directions was set at 45°. The light source plane of polarization was then rotated to obtain maximum contrast. This assured that $a$ and $b$ in Equation 13 were
approximately equal and the birefringence effect due to rotating stresses was insignificant.

Equation 10 may be used to prove that for $\alpha$ equal to $45^\circ$, maximum intensity amplitude is obtained for $a$ equal to $b$.

Data Identification

For data identification purposes and computer computations, a ten-digit numerical code was formulated. The significance of each digit or group of digits in the code is explained in Table 1.

Table 1. Data identification code

<table>
<thead>
<tr>
<th>Digit</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1, to maintain a ten-digit code</td>
</tr>
<tr>
<td>BCD</td>
<td>film number</td>
</tr>
<tr>
<td>E</td>
<td>polariscope field: 1 = dark, 2 = light, 3 = mean of light and dark</td>
</tr>
<tr>
<td>F</td>
<td>model axis parallel to the light beam: 1 = X, 2 = Y, 3 = Z</td>
</tr>
<tr>
<td>GH</td>
<td>not used, available for usage</td>
</tr>
<tr>
<td>IJ</td>
<td>model number</td>
</tr>
</tbody>
</table>
For example, the identification code 1012130005 means that the raw data for model 5 was recorded on film 12 with a dark field and with the light beam propagating parallel to the Z model axis.

The first digit of the identification code was no greater than one or the computer would have overflowed. If the first digit had been zero, the computer would have printed a nine-digit identification code. Without knowing whether the number should have consisted of nine or ten digits, it would have been difficult to decipher the code at a glance.

For identification purposes, the film negatives were consecutively numbered with pencil in the film identification notch corner on the emulsion side. The positive transparencies were numbered correspondingly.

On the computer data sheets used for the recording of data during the operation of the polariscope were recorded the identification code, all counter readouts, the laser turntable position, and a brief comment.

All computer cards contained the identification code. If the code wasn't needed as program input, the code was punched in the last ten positions of the card.
Photography

Half-frame photography was used for data recording because the polariscope camera (See Equipment, Polariscope, Camera.) used a large film and was easily adjusted for half-frame photography. The film centerline was offset from the camera lens axis which intersected the polariscope light beam axis.

For consistency, the dark field birefringence pattern was always recorded on the notched half of the film. The model axis parallel to the light beam was increasing towards the notched end of the film. Prior to recording the light field pattern, the camera back with conventional film holder was rotated 180 degrees.

An exposure of 1/5 second at f5.6 was required for Tri-X film when recording patterns along the one inch diameter of the experimental model. The film was tank developed four minutes in DK-50 stock solution at 68° F.

A camera magnification ratio of approximately 1.5 was possible with the immersion tank in the polariscope.

Enlarging

When necessary, the photographic negatives were enlarged as positive transparencies with a 4 x 5 enlarger having a 135 mm f4.5 lens. A glass sandwich film holder was constructed
for holding the 5 x 7 negatives.

The positive transparencies were printed on 5 x 7 Contrast Process Ortho film so that safelights could be used in the darkroom and with the enlarger when positioning the transparency film with respect to the enlarged image. An exposure of four seconds at f8.0 was required when magnifying the negative four times. Although DK-50 stock solution was not recommended for developing the Contrast Process Ortho film, a five minute development at 68° F. gave good results. And, only one developer was required for all development work.

To reduce the chances for errors, the dark and light field patterns were positioned on the positive transparency in the same relative positions and directions as on the negative.

The scalif space gradient had to be calculated using the actual model distances. Therefore, when enlarging to make scalif measuring more accurate, the product of the camera and enlarger magnification ratios had to be known. This product was measured by inserting into the positioned and focused enlarger a scaled negative which had been exposed with the polariscope camera in its operating position and with the scale at the polariscope light axis.
Scalif Recording

Manually determining and measuring accurately the distances between scalif centerlines with a mechanical scale on the photographic negatives or positive transparencies would have been tedious and difficult, so a scalif recorder was designed and constructed. This recorder plotted a transparency density versus distance curve on coordinate paper. (See Equipment, Scalif Recorder.)

Selection of a photocell diaphragm size was one of the first steps in the preparation of the scalif recorder for usage. The effects of model bright spots were reduced and a smoother curve was obtained if the ratio of the minimum scalif spacing on the transparency to the diaphragm size was approximately two.

With the lighted desk lamp in position and the darkest area of the transparency over the diaphragm, the photocell circuit potentiometer was adjusted for zero output signal.

For most scalif patterns, the maximum photocell traversing speed was 0.05 inches per second. Diaphragm size, scalif spacing, photocell inertia, and recorder pen inertia governed the speed.

Figure 28 (page 55) shows the scalif recorder output for the scalif pattern shown in Figure 27 (page 54). On the lower portion of the graph sheet was recorded the dark field pattern.
with the curve valleys representing the dark areas and the peaks representing the light areas. The battery leads to the photocell circuit were then reversed. On the upper portion of the graph sheet was then recorded the light field pattern with the curve valleys representing the light areas and the peaks representing the dark areas. This meshing of the two patterns permitted a larger Y-axis amplitude and an easy comparison of the dark and light field patterns.

The numerical scaling of the graph abscissa was immaterial. However, the zero position had to correspond to the model position which was used as the reference position in the data reduction computer programs. Secondly, the number of abscissa units representing one model inch was required information for the computer programs so that the correct scalar space gradients would be calculated.
DATA REDUCTION

Data reduction, the processing and computation of the test data, was computerized so that a large quantity of data could be rapidly reduced with the best known techniques and with as little human error as possible. The spline function program could be used for all scattered-light photomechanics data, whereas the effective stress ratio program and particularly the torsion function programs were specialized.

Effective Stress Ratio

The final form in which the results of a stress analysis are presented depends on the particular problem investigated. Determination of surface stresses was sufficient for those problems involving fatigue (33, p. 1, 107). For other problems, all six cartesian components of stress were found. Before this route was taken by the author, the time, expense, and skill required for using the three-dimensional shear-difference method of stress separation were considered.

An alternative to the shear-difference method was using an effective stress which was based on the Hencky-von Mises failure criterion which is defined as follows:

\[ \sigma_e^2 = \frac{1}{2}[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 ] \]  \hspace{1cm} (16)

The Hencky-von Mises failure criterion, also known as
the Mises-Hencky, von Mises, shear strain energy, and octahedral shear failure criterion, has gained wide acceptance in the field of fatigue failure prediction. Forrest (33, p. 113) states,

> For ductile metals the closest correlation with the experimental value of the ratio of fatigue strengths in torsion to fatigue strengths in bending ... is given by the shear strain energy or von Mises criterion ... . This criterion is equivalent to the average shear stress on different planes and in different directions for all the crystals in the metal and also to the condition that slip shall occur in all the crystals.

Marin (34, p. 213) states,

> When sufficient material constants are not available, the octahedral shear theory ... rather than the modified octahedral shear theory is recommended. This is justified since in most cases the octahedral shear theory is more conservative than the modified octahedral shear theory.

Shigley (35, p. 185) graphically showed that, given the bending endurance limit, the Hencky-von Mises failure criterion more closely predicted the torsion endurance limit than did the maximum shear stress criterion. Shigley (35, p. 187) also states,

> Soderberg once recommended the maximum-shear-stress theory but now proposes the von Mises-Hencky theory.

Chang, Pimbley, and Conway (36, p. 136) state,

> A number of investigations have been made on fatigue under combined stresses. These indicate that if two specimens have different states of combined stress, but the same octahedral shearing stress, they will have the same life. ...if
two volume elements of a material having equal octahedral shearing stress undergo fatigue, then two elements consume the same plastic hysteresis energy per cycle.

One of the Pi terms used in the similitude analysis of the involute spline coupling was the ratio of the effective stress at a given point to the maximum shear stress in the pure torsion section of the cylindrical section of the externally splined member. Using this ratio as a Pi term eliminated the determination of the material fringe value and the calibration of the loading frame.

Dally and Riley (30, pp. 272-274) showed how Equation 16 was expressed as a function of photoelasticity data collected for three mutually orthogonal directions at the point of interest in the model. To form the equation for calculating the above Pi term, the equation given by Dally and Riley was divided by the aforementioned maximum shearing stress, $\tau_{\text{max}}$. The parameters in the resulting equation were converted to scattered-light photoelasticity data parameters, thereby obtaining the following equation for the Pi term:

$$\frac{\sigma_e}{\tau_{\text{max}}} = \frac{1}{\frac{dN}{dR}} \left[ \left( \frac{dN}{dX} \right)^2 (2 + \sin^2 2\theta_X) + \left( \frac{dN}{dY} \right)^2 (2 + \sin^2 2\theta_Y) + \left( \frac{dN}{dZ} \right)^2 (2 + \sin^2 2\theta_Z) \right]^{1/2}$$  (17)
where:

\[
\frac{dN}{dR} = \text{the maximum scalif space gradient for the light beam propagating along a diameter of the pure torsion section of the externally splined member}
\]

\[
\frac{dN}{dx} = \text{scalif space gradient at the point of interest for the light propagating in the X direction}
\]

\[
\theta_X = \text{orientation with respect to the Y or Z axis of the secondary principal axes in the Y-Z plane}
\]

**Torsion Function**

Since all three models had portions of their lengths loaded in torsion only, an analysis procedure for torsional shear stresses was developed.

Drucker and Frocht (37) showed the equivalence of photoelastic scattering patterns and membrane contours for torsion. The equation for the membrane surface is analogous to the torsion stress function from which the shear stresses are calculated. Therefore, the form of the equation which described the scalif order versus distance relationship was easily determined.

**Circular cross section**

The torsion stress function, \( \phi \), for a circular cross section is as follows in cylindrical coordinates:
\[ \phi = \frac{G \theta}{2L} (R_o^2 - R^2) \quad (18) \]

where:
- \( G \) = material modulus of rigidity
- \( \theta / L \) = angular deflection per unit shaft length
- \( R_o \) = radius of circular shaft
- \( R \) = radial position coordinate

The torsional shear stress, \( \tau \), is related to the torsion stress function as follows:

\[ \tau = \frac{3\phi}{3R} \quad (19) \]

and to the scattered-light photoelasticity parameters as follows:

\[ \tau = \frac{\sigma_1 - \sigma_2}{2} = \frac{f \sigma}{2} \frac{dn}{dR} \quad (20) \]

An equation which mathematically described the scalar order versus radial distance relationship,

\[ N = \frac{G \theta}{f \sigma L} (R_o^2 - R^2) \quad (21) \]

was obtained by substituting Equations 18 and 20 into Equation 19, rearranging, and integrating. Equation 21 is of the general form

\[ y = a + bx^2 \quad (22) \]
where:

\[ a = \frac{G\theta}{f_0 L} R^2 \]

\[ b = -\frac{G\theta}{f_0 L} \]

An equation of even more general form,

\[ y = a + b(x + c)^2 \] \hspace{1cm} (23)

was fitted by least squares to the data obtained from the scalif recorder. By adding the extra parameter, \( c \), the circular torsion function program automatically, by least squares techniques, referenced the scalif recorder data.

By analogy

\[ R = x + c \] \hspace{1cm} (24)

To determine \( \frac{dN}{dR} \) for Equation 17, Equations 21 and 23 were differentiated,

\[ \frac{dN}{dR} = -2 \frac{G\theta}{f_0 L} R \] \hspace{1cm} (25)

\[ \frac{dy}{d(x+c)} = 2b(x + c) \] \hspace{1cm} (26)

Analogous relationships between these two equations provided

\[ \frac{dN}{dR} = -2bR_0 \] \hspace{1cm} (27)
for the maximum scalar space gradient along a diameter of
the pure torsion section of a circular shaft.

Circular cross section with semicircular groove

The modified torsion stress function, $\psi$, for the
grooved shaft shown in Figure 32 is given by Sokolnikoff
(32, p. 126) as follows:

$$\psi = a(x - \frac{b^2 x}{x^2 + y^2}) + \frac{b^2}{2}$$

The torsion stress function is obtained from the modified
torsion stress function in the following manner:

$$\phi = \psi - \frac{(x^2 + y^2)}{2}$$

Thus

$$\phi = a(x - \frac{b^2 x}{x^2 + y^2}) + \frac{b^2}{2} - \frac{(x^2 + y^2)}{2}$$

for the circular cross section with a semicircular groove.

The torsional shear stress, $\tau$, is related to this torsion
stress function as follows:

$$\tau_{zx} = \frac{G\theta}{L} \frac{\partial \phi}{\partial y}, \quad \tau_{zy} = -\frac{G\theta}{L} \frac{\partial \phi}{\partial x}$$

and to the scattered-light photoelasticity parameters as
follows:

$$\tau_{zx} = \frac{f}{2} \frac{dN}{dy}, \quad \tau_{zy} = -\frac{f}{2} \frac{dN}{dx}$$
Figure 32. Grooved shaft sketch
An equation which mathematically described the scalif order versus distance relationship,

\[ N = \frac{2G\theta}{\sigma_0 L} [a(x - \frac{b^2 x}{x^2 + y^2}) + \frac{b^2}{2} - \frac{(x^2 + y^2)}{2}] \quad (33) \]

was obtained by substituting Equations 30 and 32 into Equation 31, rearranging, and integrating.

For a given model subjected to a given load

\[ \frac{2G\theta}{\sigma_0 L} = C_1 = \text{constant} \quad (34) \]

Therefore

\[ N = C_1 [a(x - \frac{b^2 x}{x^2 + y^2}) + \frac{b^2}{2} - \frac{(x^2 + y^2)}{2}] \quad (35) \]

Equation 35 was fitted with respect to \( C_1 \) by least squares to the data obtained from the scalif recorder. Once \( C_1 \) was known, a theoretical scalif space gradient was available for comparison purposes.

Spline Function Algorithm

One of the main difficulties encountered by experimental stress analysts using the scattered-light technique of photomechanics was the determination of the scalif space gradient so that the secondary principal stress differences could be calculated. Fitting tangents to hand-plotted curves was tedious, time-consuming, and subjective.
Differentiating least-squares fit polynomials required the selection of the proper polynomial degree for fitting and an occasional hand checking of the derivative. Numerical differentiation techniques amplified the experimental errors contained in the data.

Fitting a curve to the data would smooth the data, but what type of curve should be used was the question. What was needed was a general set of curves.

Spline functions, which were mainly used for interpolation but which have been developed to replace strict interpolation by smoothing, appeared to be the general curves desired. A spline function algorithm (38) was used by the author to solve the scattered-light technique differentiation problem.

The formulation of the algorithm required only that the abscissas, \( s_i (i = 1, \ldots, n) \), be presented in ascending order. The smoothing function, \( f(s) \), was then constructed such that

\[
\int_{s_1}^{s_n} [f''(s)]^2 ds
\]

was minimized and such that

\[
\sum_{i=1}^{n} \frac{f(s_i) - N_i}{\delta N_i}^2 \leq S
\]

(36)
where:

\[ 0 < \delta N_i \] = individual data point smoothing parameter

\[ 0 \leq S \] = overall smoothing parameter.

For \( S \) equal to zero, the algorithm interpolates.

From the calculus of variations solution of the above integral and Equation 36, the optimal function \( f(s) \) was obtained such that:

\[
\begin{align*}
    f'''(s) &= 0, \quad s_i < s < s_{i+1} \quad i = 1, \ldots, n-1 \\
    f''(s_i^+) - f''(s_i^-) &= 0 \quad i = 2, \ldots, n-1 \\
    f'(s_i^+) - f'(s_i^-) &= 0 \quad i = 2, \ldots, n-1 \\
    f(s_i^+) - f(s_i^-) &= 0 \quad i = 2, \ldots, n-1
\end{align*}
\]  

(37)

where:

\[ f''(s_i^+) = \lim_{h \to 0} f''(s_i^+ h), \text{ etc.} \]

The function \( f(s) \) was thus composed of cubic parabolas

\[
f(s) = a_i + b_i (s-s_i) + c_i (s-s_i)^2 + d_i (s-s_i)^3,
\]

\[ s_i \leq s < s_{i+1} \]

which joined at their common endpoints such that \( f, f', \) and \( f'' \) were continuous. Hence, the solution was a cubic spline.

A parameter evaluation procedure was devised whereby the optimum smoothing parameters for this particular application of the algorithm were estimated. (See Results,
Spline Function Algorithm.

The Fortran subroutine which utilized this cubic spline function algorithm is listed in Appendix C.
RESULTS

The results of this research endeavor to develop a photomechanics system for nondestructive, three-dimensional stress analysis are given in detail in the Equipment, Data Acquisition, and Data Reduction chapters of this dissertation. The results of specific tests and the search for the optimum cubic spline function smoothing parameters are reported in this chapter.

Optical Creep

Leven (22, p. 162) states that the total optical creep for hexahydro phthalic, phthalic anhydride cured ERL 2774 epoxy is approximately 12% under constant stress conditions at room temperature. Most of this creep reportedly occurs during the first hour after loading. If this optical creep did not occur under constant deflection conditions, data acquisition could commence immediately after the deflection of the model.

As a check on this possibility, the experimental model was held under a constant torsional deflection for a two hour period. During this two hour period, the maximum scalif order, as determined from the data by the circular torsion function program, increased from 9.25 to 9.42 or less than 2%. This was considered insignificant, so data acquisition
commenced immediately after the model was deflected.

Torsional Stress Distribution

The torsional stress distribution check was conducted on the experimental model so that the distance over which the involute spline coupling influenced the torsional stress distribution in the externally splined shaft could be more accurately estimated.

When the maximum scalif order and the maximum scalif gradient, as determined by a least squares torsion fit, were plotted versus longitudinal shaft distance from the shoulder as in Figures 33 and 34, the effect of the shaft shoulder on the torsional stress distribution was shown to exist for approximately 0.25 small shaft diameters from the shoulder.

Loading Frame Repeatability

If the loading frame deflection were significantly repeatable, only one calibration data set would have been necessary for a given model and loading frame deflection. Therefore, the loading frame deflection was checked for repeatability with the involute spline coupling model.

The maximum scalif order, as determined from the data by using the circular torsion function program, varied less than 1% for ten model deflections. The particular
Figure 33. Maximum scalif order versus longitudinal distance from shaft shoulder
Figure 34. Maximum scalif gradient versus longitudinal distance from shaft shoulder.
deflection used gave a maximum scalif order of 9.57 in the pure torsion segment of the round section of the externally splined member.

Spline Function Algorithm

Recommended values for the smoothing parameters, $\delta N_i$ and $S$, in Equation 36 were given (38, p. 177) for using the cubic spline function algorithm to obtain the optimum curve fit—not the curve fit for the optimum derivative. The suggested value for $\delta N_i$ was an estimate of the standard deviation of the ordinate $N_i$. In this case, $S$ was recommended to be

$$n - \sqrt{2n} \leq S \leq n + \sqrt{2n}$$  \hspace{1cm} (39)

where $n$ was the number of data points.

A parameter evaluation procedure was devised whereby the optimum smoothing parameters for this particular application of the algorithm were estimated more accurately. Experimental data was collected from all three models at positions for which a theoretical solution existed; the pure torsion section along the small shaft of the experimental model, the circular pure torsion section of the externally splined member of the involute spline coupling model, and the pure torsion section of the grooved shaft model.

When the respective torsion computer programs fit the
respective theoretical curves by least squares to this data, the b's for the circular shafts and the C_1's for the grooved shaft varied from their means by less than 1%. Therefore the "true" scalif space gradient used in calculating the spline function derivative's coefficient of determination, \( r^2 \), was the scalif space gradient as determined by differentiating the respective least squares theoretical solution.

The recommended values for the smoothing parameters, \( \delta N_i \) and \( S \), were tried, but the statistical population available for calculating the \( \delta N_i \)'s was only two, the light and dark field data. Consequently, this method for evaluating the individual smoothing parameters gave erratic results and was therefore abandoned.

Since the \( \delta N_i \)'s and \( S \) were redundant and no conceivable means of individually evaluating the \( \delta N_i \)'s remained, Equation 36 was simplified by setting all \( \delta N_i \)'s equal to one. The overall smoothing parameter, \( S \), was then set equal to the product of the number of data points and the estimated standard deviation of the data set's \( N_i \)'s. Again, the statistical population was evidently too small because \( r^2 \) was more uniform and a higher mean \( r^2 \) was obtained when \( S \) equalled the number of data points multiplied by a constant which was the same for all data sets.

With \( \delta N_i \)'s equal to one and \( S \) optimized at 0.0008 times the number of data points, the mean \( r^2 \) was 0.9987
and the standard deviation of the \( r^2 \)'s was 0.0007 for 24 data groups which were comprised of a total of 67 independent data sets.

Figures 35 through 49 show some representative scalif patterns and plots of \( r^2 \) versus the smoothing parameter \( S \) divided by the number of data points in the respective data set. After studying Figures 50 and 51 in which reduced data sets 88 and 95 (Figure 44) are respectively plotted, one can estimate the relative changes in accuracy as well as the accuracy to be expected when using the cubic spline function algorithm.

**Spline Coupling Shear Stress Concentration**

The shear stress concentration factor along the center of the external spline root in the involute spline coupling is shown graphically in Figure 52. As shown in the plot, the nominal shear stress used for calculating the concentration factor was the maximum shear stress in the pure torsion section of the circular section of the externally splined member.

Determination of the maximum shear stress along the minor diameter was done with the smoothing parameter of the cubic spline function equal to 0.0010 times the number of data points instead of 0.0008. Poor definition of the
Figure 35. Involute spline coupling scalif pattern (No. 71)

Figure 36. Experimental model scalif pattern (No. 88)

Figure 37. Experimental model scalif pattern (No. 95)

Figure 38. Grooved model scalif pattern (No. 315, Y=-0.6)
Figure 39. Grooved shaft model scalif pattern (No. 326, \( Y=0.0 \))

Figure 40. Grooved shaft model scalif pattern (No. 341, \( Y=0.6 \))

Figure 41. Grooved shaft model scalif pattern (No. 361, \( X=0.75 \))

Figure 42. Grooved shaft model scalif pattern (No. 376, \( X=1.50 \))
Figure 43. Derivative accuracy versus algorithm smoothing parameter (Nos. 71-80)
Figure 44. Derivative accuracy versus algorithm smoothing parameter (Nos. 87-95)
Figure 45. Derivative accuracy versus algorithm smoothing parameter (Nos. 311, 313, 315)
Figure 46. Derivative accuracy versus algorithm smoothing parameter (Nos. 326, 328, 330)
Figure 47. Derivative accuracy versus algorithm smoothing parameter (Nos. 341, 343, 345)
Figure 48. Derivative accuracy versus algorithm smoothing parameter (Nos. 361, 363, 365)
Figure 49. Derivative accuracy versus algorithm smoothing parameter (Nos. 376, 378, 380)
Figure 50. Theoretical and experimental normalized stress versus normalized radius (Experimental model, No. 88, S/N=0.0008, $r^2=0.9954$)
Figure 51. Theoretical and experimental normalized stress versus normalized radius (Experimental model, No. 95, S/N=0.0008, \( r^2=0.9998 \))
Figure 52. Involute spline coupling external spline minor diameter shear stress concentration factor versus longitudinal distance from coupling
scalifs in this particular model was the reason for allowing a looser fitting spline function curve. For an explanation of the poor scalif quality, see Discussion, Model Fabrication, Casting.
DISCUSSION

During the development of the photomechanics system, many techniques and methods had to be developed or collected. Although these techniques and methods were relatively minor on an individual basis, they were very significant in terms of the overall system performance. Therefore they are mentioned in this chapter along with discussions of some of the more significant techniques and methods which were used.

Spline Function Algorithm

When deciding upon the overall smoothing parameter for the cubic spline function algorithm, it was considered better to choose a value which was too large rather than too small. The reason for this may be seen by observing the slopes of the curves in Figures 44, 47, and 49. A smoothing parameter which was too large did not decrease the derivative accuracy near as much as a smoothing parameter which was, by the same magnitude, too small.

With the improvements in model casting and preparation techniques which were learned throughout this research program, the model boundaries were much more easily defined (Compare Figures 37 and 38.). Therefore the stress errors due to inaccurate boundary determinations would be significantly reduced.
Loading of the Grooved Shaft Model

The grooved shaft model was not subjected to pure torsional loading. This was first suspected while taking data in the low scalif space gradient areas of the model. Previous to the light beam's passing through the zero secondary shear stress region of the model, the scalifs were well defined. Beyond the zero secondary shear stress region, the scalifs were very difficult to detect. Instead of a spatially instantaneous 90 degree rotation of the secondary principal stress axes, the rotation was gradual enough that the resolving of the light vectors during their rotation produced unequal amplitudes along the individual secondary axes.

Upon inspection of the scalif patterns similar to those in Figure 38, it was obvious that the scalif orders at the two model boundaries were not equal. The fractional scalif order error proved the insignificance of the axial component of the load, while the undetectable change in secondary principal directions showed the insignificance of the bending component of the load.

Model Fabrication

If large complex models are to be analyzed with the scattered-light technique of photomechanics, the models
must possess sufficient light transmittance—perhaps even through cemented joints.

Casting

The conclusion was drawn during the literature search that a scattering agent should be added to the epoxy to improve its poor light scattering property. Therefore, silica gel (0.007 micron size) was added to the epoxy used for the experimental and the involute spline coupling models. If the polariscope beam did not have to penetrate the model more than two inches, no adverse effects were noticed. Over longer distances the light beam became noticeably diffused and cone shaped.

After experiencing the above mentioned difficulty, particularly with the involute spline coupling model, the grooved shaft model was fabricated from a cylinder of epoxy which contained no "cab-o-sil". A comparison of the light transmission qualities of the two models may be made by comparing Figures 35 and 38.

Cementing

The experimental model was fabricated by cementing the solid shaft to the interior of the hollow shaft. When the polariscope light beam tried penetrating this cement joint, extreme reflection and diffusion problems were encountered.
The major cause of the reflection and diffusion problems was the difference in the indices of refraction of the model material and the cement. Air bubbles in the cement would cause similar effects, but they were eliminated in later cement joints and no significant improvements were seen.

A recommendation for fabrication of complex models would therefore be to eliminate cement joints from areas of interest. Normally areas of interest are high stressed areas from which cement joints should be eliminated anyway because of their structural weakness.

Polariscope

Generally speaking the polarscope performed very satisfactorily. Although the machine would not have had to have been as elaborate as it was, those conveniences were later appreciated during its operation.

As usual, there were some conveniences or features which could have been added to make the operation of the polarscope even more enjoyable. A photomultiplier would have made determination of secondary principal directions easier, more objective, and less subjective. Indicator lights by the four-pole, motor-counter switches would have prevented unintentional activation of two slide bars and two counters simultaneously.

A thin sheet of light instead of a pencil beam would
have been more convenient and informative at times, but the incorporation of a cylindrical lens into the polariscope's optical system was abandoned because of the foreseen mounting problems. If only a cylindrical lens had been incorporated, a uniform thickness, diverging (fan-shaped) sheet of light would have been produced. If the divergence of the sheet of light had been corrected with a collimating lens, the sheet of light would have no longer been of uniform thickness. Positioning the thinnest section of the sheet of light at the area of interest in the model would have required a long optical tube for the correct positioning of the collimating lens relative to the cylindrical lens.
CONCLUSIONS

When used by competent personnel, the developed photomechanics system for nondestructive three-dimensional stress analysis is capable of solving industrial problems.

From the ERL 2774 epoxy mixture an excellent scattered-light photomechanics model was fabricated.

After initial adjustments and familiarization, the polariscope was indeed a pleasure to operate.

Similitude offered an efficient means of transferring data from the model to the prototype while simultaneously eliminating model material and loading frame calibrations.

The use of the computer permitted not only rapid and error-free data reduction, but also machine plotting of the reduced data. Although the cubic spline function algorithm was a great technological advance in the area of scalif space gradient determination, more experience and study are necessary to reveal the algorithm's full potential.

The major bottleneck of the system is the transferring of data from the scalif recorder graphs to the computer.
The nondestructive, three-dimensional stress analysis system developed was based on the scattered-light technique of photomechanics. With this technique, stresses can be analyzed on the surface or inside the model during live loading under any number or combination of loading conditions because stress freezing and physical slicing of the model is not necessary.

To demonstrate the feasibility of the developed system, a motorized scattered-light polariscope with digital position readout and a laser light source was designed and assembled by the author. No proven designs or commercial machines were available when this project was initiated.

A scalif (scattered-light fringe) recorder was constructed for more accurately measuring the positions of the scalifs. An X-Y recorder, a photocell, and an enlarged positive transparency, which was printed from the photographic negative of the model scalif pattern, were the main components of the scalif recorder.

The data from the scalif recorder was used as input for a computer program which, before fitting a cubic spline function to the scalif-versus-distance data, referenced and scaled the data distances. Because the optimum smoothing parameters for this cubic spline algorithm were unknown for
this particular application of the algorithm, an effort was made to determine the smoothing parameters which would most efficiently utilize the algorithm for scattered-light photo-mechanics.

A scattered-light photoelasticity effective stress ratio concept, which was based on the Hencky-von Mises failure criteria, was developed for a point of generalized stress. The data needed for calculating this ratio were the scalar-versus-distance relationships and the directions of the secondary principal stresses for three mutually perpendicular planes at the point of interest.

The particular models studied were an involute spline coupling and a circular shaft with a semicircular groove. Live torsional loading was used for both models. Similitude theory provided the means for data transition from the model to the prototype.
BIBLIOGRAPHY


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The National Science Foundation provided a Traineeship which made my years of graduate study financially possible.

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The Engineering Mechanics Department very graciously provided laboratory space and equipment along with an invigorating educational environment.

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The Mechanical Engineering Shop was very cooperative with fabricating machine parts and allowing me to use their equipment. Special acknowledgment is given to Leon Girard, the shop superintendent, and Bill Reed, who fabricated the polariscope and loading frame.

John Deere Waterloo Tractor Works, under the supervision of Ross Brown, fabricated the experimental model and the involute spline coupling model.
Special acknowledgment is due my wife, Bev, for her patience, sacrifices, and assistance shown over the years. My son and daughter, Jon and Renae, have had to forego some much deserved fatherly companionship.

The author is, indeed, very grateful and wishes to express sincere thanks to the individuals and organizations mentioned as well as some who are not named.
APPENDIX A: UMBILICAL PLUG TERMINALS
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<th>Terminal No.</th>
<th>Electrical connection</th>
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<tr>
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<tr>
<td>2</td>
<td>X motor, negative terminal</td>
</tr>
<tr>
<td>3</td>
<td>Y motor, positive terminal</td>
</tr>
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Table 2 (Continued)

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<td>motor indicator lamp</td>
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<tr>
<td>25</td>
<td>counter indicator lamp</td>
</tr>
<tr>
<td>26</td>
<td>common lamp (live)</td>
</tr>
</tbody>
</table>
APPENDIX B: POLARISCOPE ALIGNMENT PROCEDURE
1. Obtain an accurate level, a vernier caliper, and a pair of acetylene welding goggles.
2. Level the laser turntable.
3. Using the vernier caliper, parallel the model turntable with respect to the laser turntable.
4. Don the acetylene welding goggles.
5. Turn on the laser.
6. Adjust the pupil scram lens as to obtain the desired light beam configuration.
7. Install the 0.02 inch diaphragm in the stabilizer plate center hole.
8. Using the laser mount adjustments, horizontally move the laser so that the center of its light beam passes through the center of the diaphragm.
9. At a position as far from the diaphragm as conveniently possible, mark the center of the light beam.
10. Rotate the laser turntable 180°.
11. Repeat Steps 8 and 9.
12. Using the laser mount adjustments, tilt the laser so as to move the center of the light beam to a point midway between the two marked spots.
13. Remove the two spot markings and mark the present center of the light beam.
14. Repeat Steps 10, 11, 12 and 13 but observe and record the magnitude and direction of the horizontal laser movement.

15. Horizontally move the laser turntable 1/2 the distance in the opposite direction of that recorded in Step 14.

16. Repeat Steps 11 through 15 until the light beam does not move or change intensity as the laser turntable is rotated.

17. Intercept the light beam with the model or loading frame.

18. Mark the center of the light beam on the model or loading frame.

19. Rotate the model turntable 180°.

20. Mark the center of the light beam.

21. Horizontally move the model turntable so that the light beam is midway between the two spots.

22. Remove the two spot markings and mark the present center of the light beam.

23. Repeat Steps 19 through 22 until a spot on the model or loading frame remains stationary with respect to the light beam as the model turntable is rotated.

24. Using the level and the light beam, align the loading frame.
APPENDIX C: CUBIC SPLINE FUNCTION ALGORITHM LISTING
SUBROUTINE REIN

C
C REFERENCE -
C SMOOTHING BY SPLINE FUNCTIONS
C CHRISTIAN H. REINSCH
C NUMERISCHE MATHEMATIK
C V10, PP177-183, 1967
C
C S = ABSISSA DATA
C Y = ORDNATE DATA
C N1 = INDEX OF FIRST DATA POINT, N1>1
C N2 = INDEX OF LAST DATA POINT
C Q = SMOOTHING LIMIT
C A = POLYNOMIAL CONSTANT COEFFICIENTS
C B = POLYNOMIAL FIRST ORDER COEFFICIENTS
C C = POLYNOMIAL SECOND ORDER COEFFICIENTS
C D = POLYNOMIAL THIRD ORDER COEFFICIENTS

COMMON S(60), N1, N2, Q, A(60), B(60), C(60), D(60), Y(60)
DIMENSION R(60), R1(60), R2(60), T(60), T1(60), U(60),
#ON(60), V(60)
DATA DN/60*1.0/
M1=N1-1
M2=N2+1
R(M1)=0.0
R(N1)=0.0
R(N2)=0.0
R(M2)=0.0
R1(M1)=0.0
R1(N1)=0.0
R1(N2)=0.0
R1(M2)=0.0
R2(M1)=0.0
R2(N1)=0.0
R2(N2)=0.0
R2(M2)=0.0
U(M1)=0.0
DO 7 I=N1,M2
    G=H
    H=(U(I+1)−U(I))/(S(I+1)−S(I))
    V(I)=(H−G)*DN(I)**2
7 E=E+V(I)*(H−G)
    V(N2)=−H*DN(N2)**2
    E=E−V(N2)*H
    G=F2
    F2=E*P**2
    IF (F2−Q) 8,12,12
8 IF (F2−G) 12,12,9
9 F=0.0
    H=(V(M1)−V(N1))/(S(M1)−S(N1))
    DO 10 I=M1,M2
    G=H
    H=(V(I+1)−V(I))/(S(I+1)−S(I))
    G=G−R1(I−1)*R(I−1)−R2(I−2)*R(I−2)
    F=F+R(I)*G**2
10 R(I)=G
    H=E−P*F
    IF (H) 12,12,11
11 P=P+(Q−F2)/((SQRT(Q/E)+P)*H)
    GO TO 4
12 DO 13 I=N1,N2
    A(I)=Y(I)−P*V(I)
13 C(I)=U(I)
    H=S(M1)−S(N1)
    D(N1)=0.0
    B(N1)=(A(M1)−A(N1))/H−C(N1)*H
    DO 14 I=M1,M2
    H=S(I+1)−S(I)
    D(I)=(C(I+1)−C(I))/(3.0*H)
14 B(I)=(A(I+1)−A(I))/H−(H*D(I)+C(I))*H
    H=S(N2)−S(M2)
    D(N2)=0.0
    B(N2)=B(M2)+(2.0*C(M2)+3.0*D(M2)*H)*H
    RETURN