Effective medium modeling and experimental characterization of multilayer dielectric with periodic inclusion

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Effective medium modeling and experimental characterization of multilayer dielectric with periodic inclusion

by

Teng Zhao

A dissertation submitted to the graduate faculty in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

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Ames, Iowa
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Multi-layered dielectric structures are widely seen in modern semiconductor industry. Their applications cover multi-scaled areas such as in chips, packagings and printed circuit boards. Effective medium modeling for these structures enables fast yet accurate analysis so that the computational cost are reduced dramatically. In this dissertation, the numerical modeling techniques and experimental validations are explored for a general multilayer dielectric with periodic inclusions structure.

The effective medium model based on the same reflection and transmission coefficients is introduced. The extraction procedure for effective constitutive parameters is described in detail. The branch cut issue in extraction procedure is examined and the solutions are provided so that the results are physically reasonable. Expressions of effective permittivity and permeability are given for both normal and oblique incidence, parallel and perpendicular polarizations. Analytical forms for low frequency limits of effective parameters are derived and numerical simulations are provided to verify the results.

Based on the summary of Maxwell-Garnett mixing rule (MG formula), the asymptotic forms for the frequency dependent term in MG formula is derived. Then the transition of periodic dielectric to periodic metal objects is studied. Numerical results are presented to verify the transition. A multi-effective layer model is also evaluated. And the numerical results show that under certain conditions this model achieves better accuracy than simple MG formula.

To validated our numerical modeling approach, experimental tests are performed. Using standard 4 layer printed circuit boards (PCB) technology, we are able to build periodic metal patches embedded in multi-layered dielectric. Different testing structures such as parallel plate capacitors and micro-strip with de-embedding structures are fabricated and measured. We’ve
proposed an analysis procedure based on micro-strip with de-embedding structure and full wave modeling of micro-strip line. The approach has the advantage of easy material preparation, broadband results for both complex permittivity and permeability. The experiment results are compared with our modeling results and good agreement has been achieved.
CHAPTER 1. INTRODUCTION

1.1 Review on effective medium models

Multi-layered substrate embedded with periodic inclusion structures are widely involved in the applications of modern information industry. Such applications include but not limited to multi-band absorber [1], thermal emission control [2], radio frequency (RF) component design [3], and antenna design [4]. The structures are usually multi-scaled so that it demands intensive computational resources when simulated by traditional approaches such as finite element method or finite difference method. Effective medium theory is one solution to this problem. By replacing the actual complicated metal embedded multi-layered structure with an effective medium, CPU time and memory are saved for further simulation.

Different effective medium models are studied intensively over the decades. Here we list a few recent published work on this topic. Campione et al. apply dual dipole approximation approach to homogenize 3D periodic metal cubes [5]. Felbacq et al. use asymptotic approach for homogenization of 2D layers [6]. Wang et al. adopt the effective medium model based on the scattering data to design 3D metamaterial in THz [7]. In [8], the tensor form of effective permittivity is considered in effective medium modeling. In our research, the effective medium theory based on the same S parameter is adopted [9, 10]. The original structure is abstracted as one layer homogeneous medium with effective permittivity and permeability. The new structure is effective on the sense that it has the same S parameters as the original one when imposed by a plane wave. To accurately find the S parameter of original structure, different methods such as T-matrix methods [11, 12], integral equation methods [13, 14, 15], and finite element
methods [16, 17, 18] are usually considered. In our effective medium modeling approach, an integral equation method combining equivalence principle and connection scheme (EPACS) is applied [19, 20, 21] to solve the scattering problem. The EPACS avoids evaluation of the multi-layered periodic Green’s Function so it’s very efficient. Another feature of EPACS is its ability to model 3D structures, which can be perfect electric conductor (PEC), real metal with finite conductivity or general dielectric inclusions. EPACS applies to finite multilayer structure or semi-infinite layered structures. Moreover, EPACS can be further accelerated by a logarithm algorithm if the layers are identical.

After the scattering problem of the original structure is solved, the S parameters are obtained to extract the effective permittivity and permeability of the medium. The concept of effective medium theory is usually used to describe composite materials [22, 23]. The applications of effective medium theory usually fall in the following cases: the original structure is complicate or the structure has non-conventional electromagnetic properties. In either case, the usage of effective medium theory simplifies the structure and brings in some physical insights. There are different ways to define the effective medium [22]. Different models will give different results for the effective constitutive parameters. In our modeling approach, the effective medium is defined to have the same S parameters as original structure. And the effective constitutive parameter is extracted on this basis.

1.2 Review on experimental method to determine the effective parameter

Characterizing the EM properties of materials has been intensively studied over the years due to the wide applications in the field of new material, efficient circuit design. Different methods have been proposed to determine the permittivity and/or permeability of material at RF or microwave frequency range. Such methods include free space measurement, waveguide measurement, coaxial probes, transmission line techniques etc. [24, 25, 26, 27].
To use micro-strip line pair to characterize the constitutive parameter, one can apply the basic structure described in [28]. The formula from the phase delay to extract the permittivity was provided in the same reference. This method only extracts the real part of the permittivity. Mondal et al. developed an algorithm which was applied to similar testing structure and output the complex propagation constant [29]. The complex effective permittivity is solved from the complex propagation constant assuming that the relative permeability equals to 1. The algorithm treats any discontinuity due to impedance mismatch to the effect of test fixture and de-embeds it. Thus the actual characteristic impedance is not utilized. Researchers further developed the approach by specifying the length of the micro-strip line and applied the idea to coaxial line holder [30]. They share the same assumption that the relative permeability equals to one and explicit expressions for relative permittivity were provided. This assumption is fair when the material is homogeneous and known as non-magnetic. However, in a more general case we cannot simply apply this assumption because periodic structures formed by non-magnetic material still can have effective permeability different from 1 [23]. For example, Maxwell-Garnett formula gives prediction that for periodic sphere structure may have effective permeability less than 1 [31]. Some periodic metal structures behave negative effective permeability [9]. Under these cases, it is also important to evaluate the effective permeability.

To simultaneously acquire both complex permittivity and permeability requests to make full use of measured S parameters. Electro-magnetic models for the test structures are usually needed. Popular test structures are coaxial line, coupled transmission line, waveguide and other structures [32, 33, 34, 35, 36]. The coaxial line and waveguide structure are usually favored by researchers because the feeding system generates less discontinuity as long as the test sample can be fit in a cylindrical ring shape or thick brick shape cavity. On the other hand, the microstrip line structure is easy for sample preparation but special care is needed to handle the test fixture.

We propose a characterization method using the measurement of microstrip line pair. Two microstrip lines of different lengths are fabricated on the specimen material and measured. A
cascaded two-port network model is used to de-embed the test fixtures. From the de-embedded S parameter, we extract two complex parameters: characteristic impedance and propagation constant of the microstrip line. Then these two parameters are used in the integral equation approach based model of the microstrip line and complex permittivity and permeability of the substrate, i.e. the specimen material, are solved.
CHAPTER 2. EFFECTIVE MEDIUM MODELING APPROACHES

In this chapter, we firstly introduce two effective medium modeling approaches, which are the model based on the same propagation constant and the one based on the same scattering parameter. We present the integral equation based approach to solve the scattering problems of our interested structure: multi-layer dielectric with periodic inclusions. Using tensor form of permittivity to characterize the anisotropy of effective medium is briefly discussed. Numerical examples are provided to validate the efficiency and accuracy of the effective medium modeling approach.

2.1 Effective medium model based on the same propagation constant

Figure 2.1 shows a two-layer dielectric between two parallel PEC (perfect electric conductor) plates. The effective permittivity is defined as both cases (a single layer with effective parameters and two layers as shown in Fig. 2.1) have the same propagation constant for the dominant quasi-TEM (transverse electromagnetic) mode: \( \beta = \omega \sqrt{\varepsilon_{\text{eff}} \mu_{\text{eff}}} \).

It is found that the low frequency limit of this model likes a parallel plate capacitor (with same capacitance):

\[
\frac{1}{\varepsilon_{\text{reff}}} = \frac{1}{\varepsilon_1 \, h_1 + h_2} + \frac{1}{\varepsilon_2 \, h_1 + h_2} \quad (2.1)
\]

2.2 Effective medium model based on the same scattering parameter

In our approach, the effective medium theory is based on the same S parameters, as illustrated in Fig. 2.2. The S parameters can be defined upon the reflection and transmission. When
Figure 2.1 Effective medium model based on the same propagation constant. The electric field of incident wave is perpendicular to the interface of dielectric slabs.

The multi-layered structure is abstracted as one layer homogeneous medium with the permittivity and permeability. The two structures have the same S parameter. Once we know the S parameters, the effective permittivity and permeability can be calculated from the following equations [9, 10, 22]:

\[ \varepsilon_r = \frac{n}{\bar{\eta}}, \quad \mu_r = n\bar{\eta} \]  \hfill (2.2)

\[ \bar{\eta}_{1,2} = \pm \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}} \]  \hfill (2.3)

\[ n = j \frac{\ln(\varphi)}{k_0h} \]  \hfill (2.4)

\[ \varphi_{1,2} = \frac{1 + S_{21}^2 - S_{11}^2 \pm \sqrt{(1 + S_{21}^2 - S_{11}^2)^2 - 4S_{21}^2}}{2S_{21}} \]  \hfill (2.5)
where $h$ is the thickness of the medium and $k_0$ is the propagation constant in free space. We notice that both $\bar{\eta}$ and $\varphi$ have two possible results. The passive medium criteria help us to choose result which makes physical sense.

![Figure 2.2](image)

**Figure 2.2** Effective medium theory based on the same S parameters. The electric field of incident wave is parallel to the interface of dielectric slabs. The total thickness for original structure and the effective layer are both $d$.

A novel integral equation method, EPACS, is used to solve the scattering problem of the original multi-layered substrate with periodic metal inclusion to get the S parameters. The structure of interest is as illustrated in Fig. 2.3. The structure is assumed to be infinitely large in the XY plane while the metal inclusions are doubly periodical in this plane as well. The parameters of each dielectric layer, such as the permittivity, permeability, and thickness can be different. One period is divided into $N$ cells based on number of layers of substrate. With the help of equivalence principle, our computation domain is constrained in just one cell. To make a more general case, the inclusions are assumed to be dielectric objects which have their own permittivity and permeability but not limited to PEC. This enables us to model real metal with finite conductivity or even dielectric inclusion. Thus we have equivalent source on both the surface of the cell $S_o$ and the surface of the inclusion $S_i$ as well.
Figure 2.3 The structure of multilayer substrate with periodic inclusion and unknown distribution in one cell [41].

The electric field integral equation (EFIE) and magnetic field integral equation (MFIE) on the outer surface of cell1 $S_{o1}$, can be derived as follows [19, 20, 21]:

\[
E_{o1} = L_1 (\bar{\eta}_1 \bar{J}_{o1}) - K_1 (M_{o1}) - L_1 (\bar{\eta}_1 \bar{J}_{i1}) + K_1 (M_{i1}) + \frac{1}{2} \hat{n} \times M_{o1} \tag{2.6}
\]

\[
H_{o1} = \frac{1}{\bar{\eta}_1} L_1 (M_{o1}) + K_1 (\bar{J}_{o1}) - \frac{1}{\bar{\eta}_1} L_1 (M_{i1}) - K_1 (\bar{J}_{i1}) - \frac{1}{2} \hat{n} \times \bar{J}_{o1} \tag{2.7}
\]

where $\bar{J} = J \eta_0$ and $\bar{\eta}_1 = \eta_1 / \eta_0$. The PMCHWT formulation is applied on the surface of inclusion [20]. The equations are expressed as [19, 20, 21]:

\[
\begin{bmatrix}
L_1 (\bar{\eta}_1 \bar{J}_{i1}) + L_{1,m} (\bar{\eta}_{1,m} \bar{J}_{o1}) - K_1 (M_{i1}) \\
-K_{1,m} (M_{i1})
\end{bmatrix} = \begin{bmatrix}
L_1 (\bar{\eta}_1 \bar{J}_{o1}) - K_1 (M_{o1})
\end{bmatrix} \tag{2.8}
\]

\[
\begin{bmatrix}
L_1 (M_{i1}/\bar{\eta}_1) + L_{1,m} (M_{i1}/\bar{\eta}_{1,m}) + K_1 (\bar{J}_{i1}) \\
+K_{1,m} (\bar{J}_{i1})
\end{bmatrix} = \begin{bmatrix}
L_1 (M_{o1}/\bar{\eta}_1) - K_1 (\bar{J}_{o1})
\end{bmatrix} \tag{2.9}
\]
The definitions of operators \( L_{i,(m)} \) and \( K_{i,(m)} \) for Cell \( i \) are defined as follows [20]:

\[
L_{i,(m)} (X) = jk_{i,(m)} \int_S \left[ X (r') + \frac{\nabla \nabla' \cdot X (r')}{k_{i,(m)}^2} \right] G_{i,(m)} dS'
\]

\[
K_{i,(m)} (X) = \int_S X (r') \times \nabla G_{i,(m)} dS'
\]

\[
G_{i,(m)} (R) = e^{-jk_{i,(m)}R} / (4\pi R), \quad R = |r - r'|
\]

\[
\bar{\eta}_{i,(m)} = \sqrt{\mu_{ri,(m)} / \varepsilon_{ri,(m)}}, \quad k_{i,(m)} = k_0 \sqrt{\mu_{ri,(m)} \varepsilon_{ri,(m)}}
\]

(2.10)

Then the integral equations are discretized. The unknowns on the surface of inclusions and the four sides of outer surface (the surface of dielectric box) are eliminated by applying the periodic boundary condition. After that, the connection scheme is adopted to enforce the tangential continuity of neighboring cells so that the relation between the top layer and bottom layer is determined [19, 20, 21]. Finally, the tangential fields on the topmost and bottommost surface are matched to get the matrix equation as [20]:

\[
\left( A^{(N)} - W \right) \begin{bmatrix} J_t^{(1)} \\ M_t^{(1)} \\ J_b^{(N)} \\ M_b^{(N)} \end{bmatrix} = \begin{bmatrix} E^{inc} \\ H^{inc} \\ 0 \\ 0 \end{bmatrix}
\]

(2.11)

The coefficient matrix \( A^{(N)} \) is obtained by a recursive way when one performs the connection scheme to the structure, as discussed above. The matrix \( W \) is the coefficient generated when matching the tangential fields of the topmost and bottommost layer. Solving the matrix equation for the equivalent source and the reflection/transmission coefficients are calculated in terms of the equivalent sources.

### 2.3 Effective parameter in tensor form

We’ve introduced two different effective medium modeling approach. One is based on the same propagation constant and the other is based on the same reflection coefficient. These two models usually give different results for effective parameters. In general cases, it’s more
accurate to use tensor form of effective permittivity and permeability in effective medium model.

As for the structure illustrated in Fig. 2.3, multiple dielectric layers stack together. For each layer, the material is homogeneous, isotropic. The permittivity, permeability, and thickness for \( i \)’th layer are \( \varepsilon_i, \mu_i, h_i \), respectively. If we use one homogeneous effective layer to replace the multilayer structure, the tensor form effective parameters give better results because they can model different wave behavior.

As shown in Fig. 2.5, the electric field in the layers can be parallel to the interface or perpendicular to the interface. Effective parameters for these two cases are usually different.

\[
\mathbf{\bar{\varepsilon}}_{\text{eff}} = \begin{bmatrix}
\varepsilon_{xx} & 0 & 0 \\
0 & \varepsilon_{yy} & 0 \\
0 & 0 & \varepsilon_{zz}
\end{bmatrix}
\] (2.12)

When the E field is perpendicular to the interface, the results are the same as effective medium model based on the same propagation constant and at the low frequency, the results
Electric fields directions in two different effective medium modeling. $E$ is the electric field direction. $k$ is the wave propagation direction. In one configuration, the E field is perpendicular to the interface of the dielectric layers (left) while in the other configuration, the E field is parallel to the interface of the layers (right).

are reduced to the parallel plate capacitor model. So we have:

$$\frac{1}{\varepsilon_{zz}} = \sum \frac{1}{\varepsilon_i h_i}$$  \hspace{1cm} (2.13)

When the E field is parallel to the interface, the results are the same as effective medium model based on the same scattering coefficients and similar to [22], we can write the expression for effective permittivity at the low frequency as:

$$\varepsilon_{xx} = \varepsilon_{yy} = \sum \varepsilon_i \frac{h_i}{h}$$  \hspace{1cm} (2.14)

2.4 Numerical results

Effective medium model can greatly simplify the original structure (multilayer or periodic), which provides computational savings in the simulation. To show the advantage of the effective medium model, we start from a 4 dielectric layer with metal inclusion case. The physical structure and its effective medium model are illustrated in Fig. 2.6. Both structures are built in HFSS and the simulated S parameters are as shown in Fig. 2.7 (for $S_{11}$) and Fig. 2.8 (for $S_{21}$), respectively. Meanwhile the computational costs are recorded in Table 2.1. Both simulations are performed in HFSS with the same accuracy configuration. We can see from the results that the S parameters are almost no change while the effective medium model saves lots of computational cost.
Figure 2.6  Metal embedded in 4-layer substrate and embedded in effective substrate. For 4-layer structure, the dielectric constant for each layer (from top to bottom) is 3.8, 6.5, 3.8, 6.8, respectively. The dielectric thickness (µm) for each layer (from top to bottom) is 0.45, 0.06, 0.6, 0.065, respectively.

Table 2.1  Computational cost comparison between multilayer structure and its effective medium model

<table>
<thead>
<tr>
<th></th>
<th>Multilayer structure</th>
<th>Effective model</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU time (s)</td>
<td>50</td>
<td>17</td>
</tr>
<tr>
<td>Memory</td>
<td>446</td>
<td>205</td>
</tr>
<tr>
<td>Number of tetrahedra</td>
<td>26933</td>
<td>1083</td>
</tr>
</tbody>
</table>

In previous cases, the electro-magnetic (EM) field distribution follows a simple pattern where the field is either perpendicular or parallel to the surface of the substrate but not both. In the cases where more complex structures are involved, the direction of the EM field should be considered. In other words, we need consider not only the basic reflection and forward transmission of the circuit but also the cross coupling. To target this, a micro-strip line pair is simulated.

The structure illustration is as shown in Fig. 2.9. Each microstrip (MS) is 1 mm wide and 10.5 mm long. The separation between them is 4.2 mm. The substrate is a two layers of homogeneous dielectric. Top layer has a relative permittivity of 4 and bottom layer’s dielectric constant is 9. Each dielectric layer is 14 mils thick. When replaced with effective medium, a 28 mil-thick dielectric with effective permittivity is used.

Two different effective media are simulated. They are based on different effective models. The first effective medium is based on the same propagation constant. The low frequency limit
Figure 2.7 Simulated $S_{11}$ of multilayer structure and its effective medium model. The structure is as shown in Fig. 2.6. Simulation frequency goes from 600 GHz to 1 THz.

of the permittivity for this media is calculated using Eq. 2.1. The second effective medium is based on the same scattering. The low frequency limit of permittivity is calculated using Eq. 2.15. In our case, result from Eq. 2.1 is 6.22 and result from Eq. 2.15 is 7.10. The formulations for relative permittivity are listed below [22].

$$\varepsilon_{reff} = \varepsilon_{r1} \frac{h_1}{h_1 + h_2} + \varepsilon_{r2} \frac{h_2}{h_1 + h_2}$$

(2.15)

The multilayer structure and two effective media are simulated in ADS. The results for basic reflection and forward transmission are as shown in Fig. 2.10. From Fig. 2.10, it is clear that the effective medium based on the same propagation constant gives better performance for our case. It is because that in this ‘effective’ medium model the wave direction and field direction are most close to the original multilayer substrate scenario. In Fig. 2.11, we show the wave direction and field direction for these two ‘effective’ medium models, for better illustration.
As mentioned above, we care the basic reflection and forward transmission but also the 
cross coupling between two microstrips. The results for cross coupling are as shown in Fig. 2.12. Although for cross coupling the EM fields we concern are oblique to the surface of the 
substrate, the results still show that effective medium model based on the same propagation 
constant works better than the other one.

One last issue to mention is that in our cases the permittivities are calculated using formula 
for low frequency limit. It works in relative low frequency. When frequency goes high, it 
should be modified accordingly.

The idea of effective medium can also be applied to periodic structures. A patch antenna 
is used to simulate the effective medium for periodic structures. The physical structure for 
this patch antenna is as shown in Fig. 2.13. By applying the effective medium theory, the 
physical structure can be simplified as shown in Fig. 2.14. Both physical structure and effective 
structure are simulated and the $S_{11}$ of the patch antenna are reported in Fig. 2.15 and Fig. 2.16,
Figure 2.9  Microstrip line built on two dielectric layer substrate in ADS. Each microstrip (MS) is 1 mm wide and 10.5 mm long. The separation between them is 4.2 mm. The substrate is a two layers of dielectric. Top layer has a relative permittivity of 4 and bottom layer’s dielectric constant is 9. Each dielectric layer is 14 mils thick.

respectively. Meanwhile the computational costs are recorded and summarized in Table 2.2. From the simulation results we can see that after applying the effective medium theory, the computational cost has dramatically drops while the difference in resonance frequency is less than 1%. Both simulations are performed using Agilent ADS momentum.

Table 2.2  Computational cost comparison between periodic structure and its effective medium model

<table>
<thead>
<tr>
<th></th>
<th>Periodic structure</th>
<th>Effective model</th>
</tr>
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<tbody>
<tr>
<td>CPU time</td>
<td>50 min 17 sec</td>
<td>36 sec</td>
</tr>
<tr>
<td>Matrix size</td>
<td>26933</td>
<td>1083</td>
</tr>
</tbody>
</table>
Figure 2.10  Reflection and transmission of simulated microstrip line pair. The structure is as shown in Fig. 2.9. Simulation frequency goes from DC to 5 GHz.

Figure 2.11  Wave direction (k) and E field direction for effective medium based on the same propagation constant (left) and the same reflection and transmission coefficients (right).
Figure 2.12  Cross coupling of simulated microstrip line pair. The structure is as shown in Fig. 2.9. Simulation frequency goes from DC to 5 GHz.
Figure 2.13  Physical structures for patch antenna built on the periodic structures. Periodic metal patches embedded in dielectric slabs work as substrate. The patch antenna is designed in X band. Two layers of periodic metal patches embedding in dielectric slab work as the substrate. The period is 10 mil by 10 mil by 28 mil. And the metal patch is 5 mil by 5 mil by 1.4 mil.
Figure 2.14  Patch antenna built on effective medium substrate. The physical size of the antenna is the same as the one in Fig. 2.13. The substrate is homogeneous and the effective permittivity is solved using our effective model approach.
Figure 2.15  $S_{11}$ of the patch antenna on periodic structure. The structure is as shown in Fig. 2.13. Simulation frequency goes from 8 GHz to 14 GHz.
Figure 2.16 $S_{11}$ of the patch antenna on effective medium substrate. The structure is as shown in Fig. 2.14. Simulation frequency goes from 8 GHz to 14 GHz.
CHAPTER 3. EFFECTIVE PARAMETERS EXTRACTION

In this chapter, multiple value issue (branch cut issue) in effective parameter extractions is studied. Passive medium criteria and phase continuity are used to choose solutions which are physically reasonable. The expressions of characteristic impedance for oblique incidence are provided. Based on these, the low frequency limit of effective parameters are derived. In the end, numerical results are provided for low frequency limit validation.

3.1 Branch cut discussion in extraction procedure

For the model based on the same reflection coefficient and transmission coefficient, or S parameter, extracting effective permittivity and permeability from S parameters is an important step. In this section, we generally review the procedure to extract permittivity and permeability from the reflection and transmission coefficients (or S matrix). Special efforts are paid to deal with the branch cut issue in the extraction procedure.

Assume that the S parameters are obtained from measurement or simulation. The following equations are used to get the effective permittivity and permeability [10, 22].

\[
S_{11} = \Gamma \frac{1 - \varphi^2}{1 - \Gamma^2 \varphi^2}, \quad S_{21} = \varphi \frac{1 - \Gamma^2}{1 - \Gamma^2 \varphi^2}
\]  

(3.1)

where

\[
\Gamma = \frac{\bar{\eta}_1 - 1}{\bar{\eta}_1 + 1}, \quad \varphi = e^{-j\beta d} = e^{-j\omega d}
\]

\[
\bar{\eta}_1 = \frac{\eta_1}{\eta_0}, \quad \bar{\eta}_1 = \sqrt{\frac{\mu_r}{\epsilon_r}}, \quad n = \sqrt{\epsilon_r \mu_r}
\]
Directly solving equations (3.1) we can get

$$\Gamma_{1,2} = \frac{1 + S_{21}^2 - S_{11}^2}{S_{11}^2} \pm \frac{\sqrt{(1 + S_{11}^2 - S_{21}^2)^2 - 4S_{11}^2}}{2S_{11}}$$ \hspace{1cm} (3.2)

$$\varphi_{1,2} = \frac{1 + S_{21}^2 - S_{11}^2}{2S_{21}} \pm \sqrt{(1 + S_{21}^2 - S_{11}^2)^2 - 4S_{21}^2}$$ \hspace{1cm} (3.3)

We can see that both $\Gamma$ and $\varphi$ have two possible values. The passive media requirement can be used to determine the right result.

The passive media requirement can be expressed as: for passive media, the intrinsic impedance $\bar{\eta}_1$ should have a positive real part, and the refraction index $n$ should have a negative imaginary part.

The intrinsic impedance $\bar{\eta}_1$ and $\Gamma$ are related by:

$$\bar{\eta}_1 = \frac{1 + \Gamma}{1 - \Gamma}$$ \hspace{1cm} (3.4)

plugging (3.2) into (3.4) we have:

$$\bar{\eta}_1 = \pm \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}}$$ \hspace{1cm} (3.5)

From (3.5) we can see that only one of the possible result for $\bar{\eta}_1$ has a real positive part.

As for ‘negative imaginary part of refraction index $n$’, it is equivalent to ‘the magnitude of $\varphi$ is less than 1’. From (3.3) we have:

$$\varphi_1 \varphi_2 = 1$$ \hspace{1cm} (3.6)

Thus for lossy media, there is one and only one possible result for $\varphi$ has a magnitude less than 1. For lossless case, in which $|\varphi_{1,2}| = 1$, we will use the following formula to determine the value of $\varphi$:

$$\varphi = \frac{S_{21}}{1 - S_{11}\Gamma}$$ \hspace{1cm} (3.7)

Equation (3.7) can be directly derived from equation set (3.1), so it’s always valid. And we can always use (3.7) to calculate $\varphi$ once we determined the value of $\Gamma$. It turns out that
for lossy media case, the value calculated from (3.7) is the same as the one calculated from passive media requirement.

It’s also worth mentioning that there is another multi-branch issue in our solution path:

\[ n = j \frac{\ln \varphi}{k_0 d} = -\frac{\angle \varphi + 2m\pi}{k_0 d} + j \frac{\ln |\varphi|}{k_0 d} \quad (3.8) \]

where \( \angle \varphi \) is the principle value of the angle of \( \varphi \), and \( m \) is an integer. We can see from (3.8) that the imaginary part of \( n \) is well defined while its real part may have many possible values. To determine the right branch of \( n \), we can use the approach called ‘mathematical continuity’[10]. Supposing that we have \( n \) at previous frequency \( f_0 \) correct, we do a Taylor expansion for \( \varphi \) at current frequency point \( f_1 \):

\[ e^{-jn(f_1)k_0(f_1)d} \approx e^{-jn(f_0)k_1(f_0)d}(1 + \Delta + \Delta^2) \quad (3.9) \]

where

\[ \Delta = -jn(f_1)k_1(f_1)d + jn(f_0)k_0(f_0)d \quad (3.10) \]

By solving (3.9), we have:

\[ \Delta_{1,2} = -1 \pm \sqrt{\frac{2}{\varphi(f_1)} - 1} \quad (3.11) \]

One of the value in (3.11) gives \( n \) whose imaginary part is more closed to \( \frac{\ln |\varphi|}{k_0 d} \), so it can be used as a prediction of \( n(f_1) \). Then we can choose the suitable \( m \) in (3.8) so that the difference between the predicted \( n \) and \( n(f_1) \) is minimized.

### 3.2 Oblique incidence and low frequency limit of parameters

Without loss of generality, we assume that the characteristic (wave) impedance and wave number along the \( z \) direction for each layer are \( \tilde{\eta}_n \) and \( k_{zn} \), respectively. The background medium is \( \tilde{\eta}_b = \tilde{\eta}_0 \). For oblique incidence the wave number along \( z \)-direction is given as [42]:

\[ k_{zn} = k_n \cos \theta_{tn}, \quad k_{z0} = k_0 \cos \theta_i, \quad k_n = k_0 \sqrt{\varepsilon_r \mu_r} \quad (3.12) \]
Figure 3.1 Two-layer medium illuminated by a plane wave. The thickness of two layers are $h_1$ and $h_2$. The permittivity of two layers are $\epsilon_1$ and $\epsilon_2$.

The wave impedance is

$$\tilde{\eta}_n = \frac{\eta_n}{\cos \theta_{tn}}$$

(3.13)

for perpendicular polarization, and

$$\tilde{\eta}_n = \eta_n \cos \theta_{tn}$$

(3.14)

for parallel polarization, where $\eta_n$ and the refraction angle $\theta_{tn}$ are given by

$$\eta_n = \eta_0 \sqrt{\mu_{rn}/\epsilon_{rn}}, \quad \sqrt{\epsilon_{rn}\mu_{rn}} \sin \theta_{tn} = \sin \theta_i$$

(3.15)

In this section, we will present analytical derivation of the input impedance and transmission coefficient of multilayer structure. The approximations of these two parameters at low frequency are given. Based on that, we will further provide the low frequency limit of effective permittivity and permeability for both perpendicular polarization and parallel polarization.
3.2.1 Low frequency limit of input impedance

Using the transmission line model, the input impedance shown in Fig. 3.1 is given by

\[ Z_1 = \tilde{\eta}_0 \]  

(3.16)

The input impedance \( Z_2 \) is given by

\[ Z_2 = \tilde{\eta}_1 Z_1 + j \tilde{\eta}_1 \tan(k_{z1} h_1) \] \[ \frac{\tilde{\eta}_1}{\eta_1} + j Z_1 \tan(k_{z1} h_1) \]  

(3.17)

When \( k_{z1} h_1 \ll 1 \), applying Taylor series expansion to (3.17) and taking the first-order approximation yield

\[ Z_2 \approx Z_1 \left( 1 + j \frac{\tilde{\eta}_1}{\eta_0} k_{z1} h_1 \right) \left( 1 - j \frac{\tilde{\eta}_0}{\eta_1} k_{z1} h_1 \right) \approx Z_1 M_1 \]  

(3.18)

where

\[ M_1 = 1 + j \left( \frac{\tilde{\eta}_1}{\eta_0} - \frac{\tilde{\eta}_0}{\eta_1} \right) k_{z1} h_1 \]  

(3.19)

For the input impedance \( Z_3 \), one can have

\[ Z_3 \approx Z_1 \left[ 1 + j \left( \frac{\tilde{\eta}_2}{\eta_0} - \frac{\tilde{\eta}_0}{\eta_2} \right) k_{z2} h_2 + j \left( \frac{\tilde{\eta}_1}{\eta_0} - \frac{\tilde{\eta}_0}{\eta_1} \right) k_{z1} h_1 \right] = Z_1 M_2 \]  

(3.20)

where

\[ M_2 = 1 + j \left( \frac{\tilde{\eta}_2}{\eta_0} - \frac{\tilde{\eta}_0}{\eta_2} \right) k_{z2} h_2 + j \left( \frac{\tilde{\eta}_1}{\eta_0} - \frac{\tilde{\eta}_0}{\eta_1} \right) k_{z1} h_1 \]  

(3.21)

Similarly, for the \( N \)-layer dielectric stack-up, one can obtain

\[ Z_{N+1} \approx Z_1 M_N \]  

(3.22)

\[ M_n = 1 + \sum_{i=1}^{n} j \left( \frac{\tilde{\eta}_i}{\eta_0} - \frac{\tilde{\eta}_0}{\eta_i} \right) k_{zi} h_i \]  

(3.23)

with

\[ M_0 = 1 \]  

(3.24)
3.2.2 Low frequency limit of transmission coefficient

For the $N$-layer medium, as shown in Fig. 3.1, one can represent the transverse component of the electric field as [45]

$$E_n = A_n \left(e^{jkzz_n} + R_n e^{j2kzz_n} e^{-jkzz_n}\right) e^{-jk_0zx}$$

(3.25)

where

$$R_n = \frac{Z_n - \tilde{\eta}_n}{Z_n + \tilde{\eta}_n}, \quad R_0 = 0 \quad (3.26)$$

$$k_0x = k_0 \sin \theta_i$$

The transmission coefficient is defined as

$$T = \frac{A_0}{A_{N+1}} \frac{e^{jkzn_1}}{e^{jk_{z,N+1}z_{N+1}}} = \frac{A_0}{A_{N+1}} e^{jk_0(z_1 - z_{N+1})} \quad (3.27)$$

From the boundary condition at $z = z_n$, we have

$$A_{n-1} \left(e^{jk_{z,n-1}z_n} + R_{n-1} e^{jk_{z,n-1}(2z_{n-1} - z_n)}\right) = A_n B_n \quad (3.28)$$

where

$$B_n = e^{j(kz_n - kz_{n-1})z_n} \frac{1 + R_n}{1 + R_{n-1} e^{-j2kz_{n-1}z_{n-1}}}. \quad (3.29)$$

$A_{N+1} = 1$, we can rewrite the transmission coefficient in (3.27) as

$$T = A_1 (1 + R_1) e^{jk_{z,N+1}z_{N+1}} \quad (3.30)$$

Substituting (3.22) and (3.26) into (3.29) and taking the first-order approximation yield

$$B_n \approx \frac{C_{n-1}}{C_n} \left[1 + jk_{0z} \mu_{r,n-1} h_{n-1} + D_n - D_{n-1} + E_n - E_{n-1}\right] \quad (3.31)$$

where

$$C_n = 1 + \tilde{\eta}_n, \quad \tilde{\eta}_n = \frac{\tilde{\eta}_n}{\tilde{\eta}_0}, \quad \tilde{\eta}_{N+1} = 1$$

$$D_n = \frac{\tilde{\eta}_n}{1 + \tilde{\eta}_n} \sum_{i=1}^{n-1} \left(\frac{\tilde{\eta}_i}{\tilde{\eta}_0} - \frac{\tilde{\eta}_0}{\tilde{\eta}_i}\right) k_{2i} h_i, \quad D_1 = 0$$

$$E_n = jk_{z,n} z_n$$
Applying (3.28) and (3.30) and taking the first-order approximation give

\[ A_n = A_{N+1} \prod_{i=N+1}^{n+1} B_i \approx \frac{C_n}{C_{N+1}} \]  

(3.33)

\[ C = 1 + jk_0 \sum_{i=N}^{n} \mu_{ri} h_i + D_{N+1} - D_n + E_{N+1} - E_n \]

Then the transmission coefficient can be found

\[ T \approx 1 + \frac{1}{2} j \sum_{n=1}^{N} \left( \frac{\eta_n}{\eta_0} - \frac{\eta_0}{\eta_n} \right) k_{zn} h_n - jk_0 \sum_{n=1}^{N} \mu_{rn} h_n \]  

(3.34)

3.2.3 Effective constitutive parameters for perpendicular polarization

Submitting (3.13) and (3.12) into (3.22) yields

\[ \frac{Z_{N+1}}{Z_1} \approx 1 + jk_0 \sum_{n=1}^{N} \left( \mu_{rn} - \epsilon_{rn} \frac{\cos^2 \theta_{tn}}{\cos^2 \theta_i} \right) h_n \cos \theta_i \]  

(3.35)

For one-layer slab with thickness \( h_t \) and the effective permittivity \( \epsilon_{reff} \) and permeability \( \mu_{reff} \) the effective input impedance can be approximated as

\[ \frac{Z_{eff}}{Z_1} \approx 1 + jk_0 \left( \mu_{reff} - \epsilon_{reff} \frac{\cos^2 \theta_{teff}}{\cos^2 \theta_i} \right) h_t \cos \theta_i \]  

(3.36)

where

\[ \sqrt{\epsilon_{reff} \mu_{reff}} \sin \theta_{teff} = \sin \theta_i \]  

(3.37)

If the reflection from an effective single slab is the same as from \( N \)-layer slab, the input impedances given in (3.35) and (3.36) should be same. Then we have

\[ \mu_{reff} - \epsilon_{reff} \frac{\cos^2 \theta_{teff}}{\cos^2 \theta_i} = \sum_{n=1}^{N} \left( \mu_{rn} - \epsilon_{rn} \frac{\cos^2 \theta_{tn}}{\cos^2 \theta_i} \right) h_n / h_t \]  

(3.38)

Submitting (3.13), (3.12) and (3.35) into (3.34) yields

\[ T = 1 - \frac{1}{2} jk_0 \sum_{n=1}^{N} \left( \mu_{rn} + \epsilon_{rn} \frac{\cos^2 \theta_{tn}}{\cos^2 \theta_i} \right) h_n \cos \theta_i \]  

(3.39)
On the other hand, the transmission coefficient for single layer slab with effective permeability and permittivity is given as

\[ T_{\text{eff}} = 1 - \frac{1}{2} j k_0 \left( \mu_{\text{eff}} + \epsilon_{\text{eff}} \frac{\cos^2 \theta_{\text{teff}}}{\cos^2 \theta_i} \right) \cos \theta_i h_t \]  

(3.40)

Equating (3.39) and (3.40) yields

\[ \mu_{\text{eff}} + \epsilon_{\text{eff}} \frac{\cos^2 \theta_{\text{teff}}}{\cos^2 \theta_i} = \sum_{n=1}^{N} \left( \mu_{rn} + \epsilon_{rn} \frac{\cos^2 \theta_{tn}}{\cos^2 \theta_i} \right) \frac{h_n}{h_t} \]  

(3.41)

For the model based on the same reflection and transmission coefficients, solving (3.38) and (3.41) together and using (3.15) and (3.37) yield

\[ \mu_{\text{eff}} = \sum_{n=1}^{N} \mu_{rn} \frac{h_n}{h_t} \]  

(3.42)

\[ \epsilon_{\text{eff}} = \sum_{n=1}^{N} \epsilon_{rn} \frac{h_n}{h_t} - \sin^2 \theta_i \left( \sum_{n=1}^{N} \frac{h_n}{h_t \mu_{rn}} - \frac{1}{\mu_{\text{eff}}} \right) \]  

The effective permeability is the weighted average of each layer, but the effective permittivity has more complex form. It not only depends on the permittivity of each layer, but also depends on the permeability. For \( N \)-layer non-magnetic slab, \( \mu_{rn} = 1 \), the formulas are similar to the case of normal incidence [22].

\[ \mu_{\text{eff}} = 1, \quad \epsilon_{\text{eff}} = \sum_{n=1}^{N} \epsilon_{rn} \frac{h_n}{h_t} \]  

(3.43)

The effective permeability for non-magnetic multilayer slab is 1 and the effective permittivity is the weighted average of each layer. Both of them do not depend on the incident angle.

### 3.2.4 Effective constitutive parameters for parallel polarization

Applying similar approach, we have

\[ \epsilon_{\text{eff}} = \sum_{n=1}^{N} \epsilon_{rn} \frac{h_n}{h_t} \]  

(3.44)

\[ \mu_{\text{eff}} = \sum_{n=1}^{N} \mu_{rn} \frac{h_n}{h_t} - \sin^2 \theta_i \left( \sum_{n=1}^{N} \frac{h_n}{h_t \epsilon_{rn}} - \frac{1}{\epsilon_{\text{eff}}} \right) \]
The effective permittivity is the weighted average of each layer, but the effective permeability has a more complex form. One can find that the effective permeability is not only relative to the average of each layer’s permeability, but also depends on the permittivity of each layer. For \( N \)-layer non-magnetic slab, \( \mu_{rn} = 1 \), we have:

\[
\epsilon_{reff} = \sum_{n=1}^{N} \epsilon_{rn} h_n / h_t, \quad \mu_{reff} = 1 - \sin^2 \theta_i \left( \sum_{n=1}^{N} \frac{h_n}{h_t \epsilon_{rn}} - \frac{1}{\epsilon_{reff}} \right)
\]  

(3.45)

For the parallel polarization, the effective permeability for non-magnetic multi-layer slab is slightly less than 1 and depends on the incident angle and the permittivity of each layer.

### 3.3 Numerical results

As the structure shown in Fig. 3.1, two-layer slabs with permittivity \( \epsilon_{r1} = 2, \epsilon_{r2} = 4 \) respectively and the same permeability \( \mu_{r1} = \mu_{r2} = 1 \) is tested. The thickness of each layer is 4 \( \mu \text{m} \). The background is free space and the frequency is 10 GHz. Because the thickness of the slab is in the order of micron, 10 GHz could be still seemed as the low frequency situation.

The results for the parallel polarization are plotted in Fig. 3.2. The effective permittivity is simply equal to the average of the permittivity. When each layer has the same permeability of 1, the effective permittivity is slightly less than 1 and depends on the incident angle and the permittivity of each layer, although each layer has the same value equaling to 1. The effective permeability is about 0.96 when the incident angle is close to 90 degrees. From the figure, we also can see a good agreement with the exact results and low frequency limit.

For more discussion, the case of 1 THz is tested and the results are shown in Fig. 3.3. The results show that the effective permittivity has a negative imaginary part and the effective permeability has a positive imaginary part. To investigate the sign of the imaginary parts of the effective constitutive parameters, for the perpendicular case, two cases are considered. First one is that the parameters are \( \epsilon_{r1} = 4, \epsilon_{r2} = 2; \mu_{r1} = \mu_{r2} = 1; h_1 = h_2 = 4 \mu \text{m} \). The second
Figure 3.2 Extracted effective permittivity and permeability as functions of incident angle for parallel polarizations from a 2-layer slab using low frequency limit and analytical results. $\epsilon_r^1 = 2$, $\epsilon_r^2 = 4$, $\mu_r^1 = \mu_r^2 = 1$, $h_1 = h_2 = 4 \mu m$, and $f = 10$ GHz.

Table 3.1 S parameters and effective parameters of the two-layered medium (Normal incidence; Frequency = 1 THz)

<table>
<thead>
<tr>
<th></th>
<th>$\epsilon_r^1 = 2, \epsilon_r^2 = 4$</th>
<th>$\epsilon_r^1 = 4, \epsilon_r^2 = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ Real</td>
<td>-0.033467</td>
<td>-0.046796</td>
</tr>
<tr>
<td>$R$ Imaginary</td>
<td>-0.159863</td>
<td>-0.156480</td>
</tr>
<tr>
<td>$T$ Real</td>
<td>0.973221</td>
<td>0.9732208</td>
</tr>
<tr>
<td>$T$ Imaginary</td>
<td>-0.161757</td>
<td>-0.161757</td>
</tr>
<tr>
<td>$\epsilon_{ref}^f$ Real</td>
<td>2.9954</td>
<td>3.0092</td>
</tr>
<tr>
<td>$\epsilon_{ref}^f$ Imaginary</td>
<td>0.0430</td>
<td>-0.0436</td>
</tr>
<tr>
<td>$\mu_{ref}^f$ Real</td>
<td>0.9933</td>
<td>1.0065</td>
</tr>
<tr>
<td>$\mu_{ref}^f$ Imaginary</td>
<td>-0.0406</td>
<td>0.0400</td>
</tr>
</tbody>
</table>

one is to switch the location of two layers, whose the parameters are $\epsilon_r^1 = 2$, $\epsilon_r^2 = 4$; $\mu_r^1 = \mu_r^2 = 1$; $h_1 = h_2 = 4 \mu m$.

Table 3.1 gives the specific results for the two cases, when the incident angle is equal to 0 degree (normal incidence) and the frequency is equal to 1 THz. It is clear that when the locations of the two layers are exchanged, the real parts of the permittivity and permeability are almost the same; the imaginary parts of the permittivity and permeability have different signs.
Figure 3.3  Extracted effective permittivity and permeability as functions of incident angle for parallel polarizations from a 2-layer slab using low frequency limit and analytical results. \( \varepsilon_r^1 = 2, \varepsilon_r^2 = 4, \mu_r^1 = \mu_r^2 = 1, h_1 = h_2 = 4\mu m, \) and \( f = 1 \) THz.
CHAPTER 4. EFFECTIVE MEDIUM MODELS FOR PERIODIC STRUCTURE

In this chapter, we start from the Maxwell-Garnett mixing rule (MG formula) in literature. We summarize MG formula for PEC inclusion, quasi-static dielectric and frequency-dependent dielectric. Followed by the study of the effect of inclusion’s filling rate, shape distribution and polarization. By deriving the asymptotic form of the frequency dependent term in the MG formula, we then look into the transitions from periodic dielectric to periodic metal objects. In the end, we briefly study the multi-effective layer model based on MG formula and investigate its accuracy.

4.1 Maxwell-Garnett formula based model

The Maxwell-Garnett (MG) formula is used to calculate the effective permittivity and permeability of spherical composed material [31, 50]. It can be derived basing on the dipole approximation. And it neglects the interactions of the inclusions. Results show that the MG formula works well in the region of low volume fraction.

The MG formulas are shown as [50]:

\[
\varepsilon_{\text{eff}} = \varepsilon_{r,m} \frac{\varepsilon_{r,i}(1 + 2p)F(\theta) - 2(p - 1)\varepsilon_{r,m}}{\varepsilon_{r,i}(1 - p)F(\theta) + \varepsilon_{r,m}(2 + p)} \tag{4.1}
\]

\[
\mu_{\text{eff}} = \mu_{r,m} \frac{\mu_{r,i}(1 + 2p)F(\theta) - 2(p - 1)\mu_{r,m}}{\mu_{r,i}(1 - p)F(\theta) + \mu_{r,m}(2 + p)} \tag{4.2}
\]
where the subscripts _m and _i represent the permittivity and permeability of the matrix material and inclusion material, respectively. \( p \) is the volume fraction of the inclusion. \( F(\theta) \) is the frequency dependent term which is defined as:

\[
F(\theta) = \frac{2(\sin \theta - \theta \cos \theta)}{(\theta^2 - 1) \sin \theta + \theta \cos \theta}
\]

\[
\theta = k_0 a \sqrt{\varepsilon_{r,i} \mu_{r,i}}
\]

where \( a \) is the radius of the sphere inclusion.

When the frequency is low, the value of \( \theta \) is small. The limit of \( F(\theta) \) for small argument is obtained by applying the L' Hospital’s rule:

\[
\lim_{\theta \to 0} F(\theta) = 1
\]

Substituting 4.5 into 4.1 and 4.2 yields the asymptotic forms for the effective permittivity and permeability for quasi-static:

\[
\varepsilon_{reff} = \varepsilon_{r,m} \frac{1 + 2p}{1 - p} + \varepsilon_{r,i}(2 + p)
\]

\[
\mu_{reff} = \mu_{r,m} \frac{1 + 2p}{1 - p} + \mu_{r,i}(2 + p)
\]

The PEC limit of the MG formula is presented in [23]:

\[
\varepsilon_{reff} = \varepsilon_{r,m} \frac{1 + 2p}{1 - p}, \quad \mu_{reff} = \mu_{r,m} \frac{1 - p}{1 + p/2}
\]

Equation 4.6 can be derived from 4.1 and 4.2 by making the large conductivity approximation. Under the condition that the conductivity of inclusion is large, \( \theta \) is approximated by

\[
\theta \approx k_0 a \sqrt{\mu_{r,i} \omega_0 \varepsilon_{r,m} \frac{-j \sigma}{\omega_0 \varepsilon_0}} = \frac{a}{\delta} (1 - j)
\]

where \( \delta = \sqrt{2/(\sigma \omega_0 \mu_i)} \) is the skin depth of the object, \( \sigma \) is the conductivity.

Substituting 4.7 into 4.3 and taking the exponential form of sine and cosine functions, one gets the approximation of \( F \) at high conductivity:

\[
F(\theta) \approx -2j/\theta
\]
So at high conductivity, the value of $F$ goes to 0 at the rate of $2/|\theta|$, as illustrated in Fig. 4.1. Under the condition of $F$ going to zero, Eq. 4.2 is reduced to the PEC limit in Eq. 4.6. Meanwhile, $\varepsilon_{r,i}F(\theta)$ may still be very large at high conductivity condition because:

$$|\varepsilon_{r,i}F(\theta)| \approx 2 |\varepsilon_{r,i}/\theta| \approx 2 \sqrt{|\varepsilon_{r,i}| / (k_0 a)}$$

(4.9)

![Graph showing $F(\theta)$ as a function of the inclusion's conductivity. Frequency = 200 GHz, $a = 2.399 \, \mu m$, Re($\varepsilon_{r,i}$) = 2. The conductivity of the inclusion is swept from 1 to $10^8 \, S/m$.](image)

**4.2 Discussion of the filling rate and shape polarization**

In this section, we simulate several cases to explore the role of inclusion filling rate and shape polarization in MG formula. The first case is periodic dielectric sphere. The relative permittivity of the sphere is 40. The period cell is a cube with side length set to be 0.1 mm and the solution frequency is set to be 2 GHz. The radius of the sphere is adjusted to fulfill different filling rate (FR), which is defined as the ratio of inclusion volume over cell volume.
When the inclusion is spherical shape and the FR is small, the MG formula works as good approximation. We change the size of the sphere while keeping the same period so that we get permittivity at different FRs. The results are compared with both MG formula [23] and results from [46]. The curve labeled 'EPACS' is full wave solution result, which is obtained using the approach described in Section 2.3. These three results are illustrated in Fig. 4.2. As we can see, when the filling rate is below 35%, all these three results have little difference. When the filling rate goes up, the MG formula results show larger difference while the other two stay close. The reason is that when the filling rate is high, the interactions between inclusions become strong but the MG formula cannot catch the interaction. Wu and Whites [46] extracted the permittivity based on electric flux and averaged electric field while in our proposed method, the effective permittivity is extracted from the scattering parameter. Thus two approaches give slightly different results.

Figure 4.2 Effective permittivity of periodic dielectric sphere as a function of filling rate. Dielectric spheres are placed in a period of 0.1 mm by 0.1 mm by 0.1 mm cube. The frequency in our modeling is 2 GHz. The results are compared with MG formula and those from [46].
In the EPACS approach, the inclusions are described using the surface mesh. This enables us to simulate various geometries but not limited to ellipsoid. Moreover, EPACS accepts complex permittivity input so that we can model different types of inclusions: lossless dielectric, lossy dielectric, metal with finite conductivity or perfect electric conductor (PEC). The following simulation case is periodic PEC cubes in lossless dielectric slab with a relative permittivity of 2. We also change the volume of the cube inclusion so that we get results at different FRs. The extracted permittivity are shown in Fig. 4.3 and compared with MG formula. Though the two results still agree with each other when filling rate is low, but the difference become obvious at even lower FR comparing to spherical inclusion case. This is due to the MG formula is derived based on sphere inclusion rather than other shapes.

![Effective permittivity of periodic metal cubes as a function of filling rate. The period and solution frequency are configured the same way as the case shown in Fig. 4.2.](image)

We have noticed that the effective permittivity changes for binary mixing structures even with the same filling rate. The reason is the shape of the inclusion affects the field distribution
so that the effective permittivity changes. This phenomenon is not caught by the Maxwell-Garnett formula because the formula is derived for spherical inclusion. For spherical inclusions, once the filling rate is fixed, the effective permittivity can be calculated. For cuboid inclusions, it is no longer the case.

To explore how the inclusion’s shape affect the effective permittivity, we propose the following simulation. Several cases of periodical metal inclusions embedded in dielectric slab are simulated. In all the cases the filling rate are set to be the same while the shapes of the metal inclusion are different. In Fig. 4.4 we illustrated the detailed geometry of the cuboids. There are actually only 3 different cuboids, in the simulation we are just illuminating the different faces of the cuboids with plane wave, and the electric field (E field) is parallel to different edges. We are treating them as five different types of inclusions because it’s easier for us to analyze them in the same Cartesian coordinate. The structure is placed in Cartesian coordinate and the incident wave comes from z direction with electric field polarized to x direction or y direction, as illustrated in Fig. 4.5. Five of them have the same volume. The host dielectric is set to have a relative permittivity of 4.5. The period is set to be 10 mm by 10 mm by 10 mm, which gives us a filling rate of 6.4%. The EPACS is run to get the S parameter and then effective medium theory presented is applied to get the effective permittivity. The results are plotted in Fig. 4.6 and Fig. 4.7 for different polarizations.

![Figure 4.4](image)

**Figure 4.4** Different cuboid metal inclusions, lengths of the edges are provided, unit: mm.

The results for incident E field parallel to y direction is as shown in Fig. 4.6. From this figure, we have one observation that the effective permittivity gets larger if the inclusion has a longer edge parallel to the incident E field. The green curve, representing a cuboid with
Figure 4.5 One period of simulated structures in Cartesian coordinate. The wave comes from the $z$ direction. The structure is doubly periodic in $x$ and $y$ directions.

an 8 mm edge parallel to the incident E field, shows the largest permittivity among all these inclusion types. Then it comes the red and black curves, both representing cuboids with a 4 mm edge parallel to the incident E field. Following them, we have the cyan and blue curves, both for cuboids with a 2 mm edge parallel to the incident E field. The smallest permittivity value, comes from the cuboid with a 1 mm edge parallel to the incident E field, which is the pink curve. The MG formula results are also provided as reference. And as we expected, among all the cuboids, the cube results (4 mm by 4 mm by 4 mm) are most closed to the MG formula results. Since from the symmetry point of view, the cube is most closed to the sphere.

We also run the case when the incident E field is parallel to $x$ direction. The results are as shown in Fig. 4.7. From Fig. 4.7, firstly we can confirm the observation from Fig. 4.6: longer edge parallel to the incident E field, large permittivity we get. The pink, green, red and blue curves represent cuboids with a 8 mm edge parallel to the incident E field, thus their results are generally larger than the cyan and black curves, which both representing a 4 mm edge
Figure 4.6  Effective permittivity for different cuboid shapes. The incident E field is parallel to $y$ direction. The effective permittivity for six different shapes are color coded as shown in the legend. The dimension showed in legend is in $mm$. The period is 10 $mm$ by 10 $mm$ by 10 $mm$ cube. The relative permittivity for the dielectric layer is 4.5. Filling rate is 6.4%. The MG formula results are plotted in yellow dashed line. Simulation frequency is from 1 GHz to 10 GHz.

parallel to the incident E field. However, it is also quite clear that there exist two groups in the largest 4 sets of results. The pink and green curves form a group that there permittivity results are clearly larger than the red and blue curves’ results, even though all of them have the same length of edge parallel to the incident E field. To examine why those groups form, we need to look into the mathematical expression which can catch the shape effect of the inclusion.

There are lots of publications over decades addressing the shape issues of the inclusion. Reynolds and Hough claimed that ‘Unfortunately, it is only possible to calculate it exactly for the case of parallel slabs or the case of infinitely diluted dispersions of particles of ellipsoidal shape.’ in their early work related to this topic [47]. Here by ‘calculating exactly’, the authors were looking for analytical results. Although researchers may not come up with analytical expression for inclusions with complex shape, the problem can be solved using full wave solutions such as FEM [48] or MoM [20]. For the scope of qualitative comparison effective permittivity of different inclusion shapes, we are using analytical formula for ellipsoid
Figure 4.7 Effective permittivity for different cuboid shapes. The incident E field is parallel to x direction. The effective permittivity for six different shapes are color coded as shown in the legend. The dimension showed in legend is in mm. The period is 10 mm by 10 mm by 10 mm cube. The relative permittivity for the dielectric layer is 4.5. Filling rate is 6.4%. The MG formula results are plotted in yellow dashed line. Simulation frequency is from 1 GHz to 10 GHz.

For the case of ellipsoid inclusions, if the incident E field is parallel to one of the inclusion’s principle axis, the formula for effective permittivity is [49]:

\[
\varepsilon_{\text{eff}} = \frac{p\beta \varepsilon_i + (1 - p) \varepsilon_m}{p\beta + (1 - p)}
\]

(4.10)

where \(p\) is the filling rate, \(\varepsilon_i\) and \(\varepsilon_m\) are permittivity for the inclusion and host medium, respectively. The parameter \(\beta\) is defined by:

\[
\beta = \sum_{j=1}^{3} \frac{\cos^2 \alpha_j}{1 + L_j (\varepsilon_i/\varepsilon_m - 1)}
\]

(4.11)

where \(\alpha_j\) is the angle between the incident E field and ellipsoid’s principle axis and they satisfy \(\sum_{j=1}^{3} \cos^2 \alpha_j = 1\). And parameter \(L_j\) is defined by:

\[
L_j = \frac{x_1 x_2 x_3}{2} \int_0^\infty \frac{d\zeta}{(\zeta + x_j^2) \sqrt{(\zeta + x_1^2)(\zeta + x_2^2)(\zeta + x_3^2)}}
\]

(4.12)

where \(x_j\) are the semi-axes of the ellipsoid.
Under the condition that the incident E field is parallel to one of the principle axis, denoting as $x_p$, the expression for $\beta$ is:

$$\beta = \frac{1}{1 + L \left( \frac{\varepsilon_i}{\varepsilon_m} - 1 \right)}$$  \hspace{1cm} (4.13)

and $L$ is

$$L = \frac{x_1x_2x_3}{2} \int_0^\infty \frac{d\zeta}{\left( \zeta + x_p^2 \right) \sqrt{\left( \zeta + x_1^2 \right) \left( \zeta + x_2^2 \right) \left( \zeta + x_3^2 \right)}}$$  \hspace{1cm} (4.14)

where $x_p$ is whichever that is parallel to the incident E field.

When we consider the metal inclusions, by taking the high conductivity approximation $\varepsilon_m/\varepsilon_i \ll 1$, we can get:

$$\beta = \frac{1}{1 + L \left( \frac{\varepsilon_i}{\varepsilon_m} - 1 \right)} \approx \frac{\varepsilon_m}{L \varepsilon_i}$$  \hspace{1cm} (4.15)

Based on analysis above, the expression for $\varepsilon_{eff}$ for metal ellipsoid inclusion case under the condition that incident E field is parallel to one of the principle axis is:

$$\varepsilon_{eff} = \varepsilon_m \frac{1 + \left( 1/L - 1 \right) p}{1 - p}$$  \hspace{1cm} (4.16)

where $L$ is defined in Eq. 4.14.

For spherical inclusion ($L = 1/3$), Eq. 4.16 can be reduced to MG formula for PEC limit as in Eq. 4.6 [23].

If we approximate the cuboid inclusion with ellipsoid one, under the condition that they have the same volume and aspect ratio, Eq. 4.14 is rewritten as:

$$L = \frac{x_1x_2x_3}{2} \int_0^\infty \frac{d\zeta}{\left( \zeta + x_1^2 \right)^{3/2} \sqrt{\zeta^2 + \left( x_2^2 + x_3^2 \right) \zeta + x_2^2 x_3^2}}$$  \hspace{1cm} (4.17)

the incidence E field direction is assumed to be parallel to the $x_1$.

Eq. 4.16 suggests that the smaller $L$ gives larger $\varepsilon_{reff}$. Eq. 4.17 indicates that if we have longer edge parallel to the incident E field, the denominator in the integral in $L$ gets larger, which results in a smaller $L$, thus larger $\varepsilon_{reff}$. This explains the observation we got from Fig. 4.6. And it also explains why we have groups in Fig. 4.7. For instance, for the cuboid with dimension of 8 mm by 8 mm by 1 mm, the effective permittivities are similar regardless the
wave is illuminating the 8 mm by 8 mm face or the 8 mm by 1 mm face, as long as the incident E field is parallel to one of the 8 mm edge. Eq. 4.16 and 4.17 also explain why in Fig. 4.7, the 8 mm by 8 mm by 1 mm cuboid gives larger permittivity than the 8 mm by 4 mm by 2 mm cuboid, even though the incident E fields are all parallel to an edge of 8 mm long. Because the 8 mm by 4 mm by 2 mm cuboid gives larger integrand, which results in a smaller $\epsilon_{\text{reff}}$.

Relative permittivity for these six different cases calculated using Eq. 4.16 and Eq. 4.17 are as listed in Table 4.1 and compared with results of our modeling approach using EPACS. From this table we can see that the analytical formula Eq. 4.16 can catch the character for different shapes and polarizations. We do see the difference between the approximate results and our modeling results, which is due to the analytical formula uses ellipsoid to approximate the cuboid shape.

Table 4.1 Comparison of effective permittivity calculated by approximate analytical formula 4.16, 4.17 and EPACS

<table>
<thead>
<tr>
<th>Shape</th>
<th>$x_1 \times x_2 \times x_3$</th>
<th>$L$</th>
<th>$\epsilon_{\text{reff}}$</th>
<th>$\epsilon_{\text{reff}}$ (EPACS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 x 4 x 4</td>
<td>0.333</td>
<td>5.423</td>
<td>5.622</td>
<td></td>
</tr>
<tr>
<td>8 x 2 x 4</td>
<td>0.112</td>
<td>7.239</td>
<td>8.533</td>
<td></td>
</tr>
<tr>
<td>8 x 4 x 2</td>
<td>0.112</td>
<td>7.239</td>
<td>8.529</td>
<td></td>
</tr>
<tr>
<td>8 x 1 x 8</td>
<td>0.085</td>
<td>8.139</td>
<td>9.435</td>
<td></td>
</tr>
<tr>
<td>8 x 8 x 1</td>
<td>0.085</td>
<td>8.139</td>
<td>9.366</td>
<td></td>
</tr>
<tr>
<td>4 x 2 x 8</td>
<td>0.285</td>
<td>5.581</td>
<td>5.724</td>
<td></td>
</tr>
</tbody>
</table>

4.3 From periodic metal objects to periodic dielectric objects

In the first section of this chapter, we discuss different MG formula and their expressions under quasi-static and PEC conditions. We explore in detail how the frequency dependent term play a role in these expressions.

A number of numerical tests are done to validate our conclusions. The structure adopted in the test is one layer of doubly periodic dielectric spheres embedded in a dielectric slab. The relative permittivity of the slab is 3.9. Real part of the relative permittivity of the sphere is 2.
The radius of the sphere is 2.399 µm and the distance between centers of adjacent spheres is 10 µm. The volume fraction is 5.8%. All materials have unit relative permeability. The simulations are run at 200 GHz. Figures 4.8 and 4.9 show the effective permittivity and permeability of the whole structure as the function of conductivity of the embedded spheres, respectively. At large conductivity, both parameters merge to the PEC limit, which is represented in long dashed black lines in the figures. To make a solid conclusion, the results for permittivity and permeability are not only calculated by MG formula but also by EPACS.

![Figure 4.8 Effective relative permittivity of the periodic dielectric spheres embedded in a slab as a function of the conductivity of the inclusion. \( \sigma = 2.3993 \, \text{µm}; \, p = 5.8\%. \) Re(\( \epsilon_{r,\text{eff}} \)) = 2; \( \epsilon_{r,\text{m}} = 3.9 \). Frequency: 200 GHz. Long dashed line represents results of PEC limit.]

As shown in Fig.4.8, when the conductivity is low, the results are around the low frequency approximation. With the increase of the conductivity, a transition region appears and then the results approach to the PEC limit. The results for relative permeability as shown in Fig.4.9
behave the same trend as predicted. When the conductivity is low, the relative permeability is around 1. With the increase of conductivity, the value of relative permeability decreases and merges to the PEC limit. The location of transition region can be predicted using the criteria drew in the last section combining the approximation forms 4.8 and 4.9. Comparing Fig. 4.1 and transition region in Fig. 4.8 and Fig. 4.9, results show that our theory and approximations work well on predicting the transition region.

![Graph showing effective relative permeability as a function of conductivity](image)

**Figure 4.9** Same as Fig. 4.8, effective relative permeability as a function of the conductivity of the inclusion.

We then took a step further to explore the transition region of the effective properties. By examining Eq. 4.8 and 4.9, one can draw the conclusion that: When the frequency increases, the transition region for permittivity goes to higher conductivity while transition region for permeability goes to lower conductivity. Another case similar to Fig. 4.8 and Fig. 4.9 are run except the frequency is now down to 20 GHz. The results are shown in Fig. 4.10 and Fig. 4.11, for permittivity and permeability, respectively. And our conclusion is clearly verified in Fig. 4.10 and Fig. 4.11.
4.4 Multi-effective layer model

The MG formula is designed to evaluate the effective medium property of two materials mixing together. However, researchers proposed multi-effective layer model [51] and claim that the approach would achieve better accuracy than single effective layer model.

To illustrate the multi-effective layer model, a periodic sphere embedded in dielectric slab is considered, as shown in Fig. 4.12. One way to analyze the structure is to directly apply the mixing rules for two materials such as MG formula to solve for the effective parameter of the whole structure as in Fig. 4.12 (left). On the other hand, we can divide the whole structure in 3 layers and apply the mixing rules only to the central layer which have two materials. As shown in Fig. 4.12 (right), Layer 1 and Layer 3 are thus homogeneous. Once we apply the mixing rule for layer 2 and get the effective homogeneous layer, the total reflection and transmission for the 3 layer stack-up structure can be obtained analytically. Therefore, the modeling accuracy is
improved.

To study the multi-effective medium layer model, a 3D periodic sphere embedded in dielectric slab is considered. The structure is the same as Fig. 4.12. In our case, the radius of sphere is 3.75 mm. Period is 15 mm by 15 mm by 15 mm. Dielectric constant of the slab is 2. And dielectric constant of sphere inclusion is 4. The reflection of the structure is calculated using one layer model (MG formula directly) and three layer model. Also we use full wave solution to the structure and the results work as reference. The results are as shown in Fig. 4.13. It’s shown that the 3 layer model has advantage over the 1 layer model for this case.

However, we notice that the 3 layer model loses its edge under high dielectric contrast cases. We change the dielectric constant (DK) of the inclusion to 20 and 70, repeat the whole procedure and the results are as shown in Fig. 4.14 and Fig. 4.15 respectively. We can see that in DK=20 case, the 3 layer model results are still close enough to integral equation (IE) based full wave results while for DK=70 case, the 3 layer model no longer show advantage. To better
illustrate, we sweep the DK from 20 to 70 in this case and report the reflection coefficient for frequency of 3 GHz. And the results are shown in Fig. 4.16. We can see that the 3 layer model only work well under relatively low dielectric contrast situations.

In this end, we simulated the PEC inclusions case. The physical dimension are the same as the case in Fig. 4.13. Results for PEC inclusions are shown in Fig. 4.17. Once again we see that 3 layer model only work as good approximation at relative low frequency.
Figure 4.12  Simple MG based 1 layer model (Top) vs multi-effective layer model (Bottom).
Figure 4.13  Multi-effective layer model for 3D dielectric sphere. The radius of sphere is 3.75 mm. Period is 15 mm by 15 mm by 15 mm. Dielectric constant of the slab and the sphere inclusions are 2 and 4, respectively. One layer model and 3 effective layer model results are plotted using blue and green dashed lines, respectively. Our full wave modeling approach results are plotted in red circle. Simulation frequency is from 1 GHz to 5 GHz.
Figure 4.14 Multi-effective layer model for 3D dielectric sphere. Dielectric constant of sphere is 20. And other configurations are the same as in Fig. 4.13. One layer model and 3 effective layer model results are plotted using blue and green dashed lines, respectively. Our integral equation based full wave modeling approach results are plotted in red circle (denoted as ‘IE’ in legend). Simulation frequency is from 1 GHz to 5 GHz.
Figure 4.15 Multi-effective layer model for 3D dielectric sphere. Dielectric constant of sphere is 70. And other configurations are the same as in Fig. 4.13. One layer model and 3 effective layer model results are plotted using blue and green dashed lines, respectively. Our integral equation based full wave modeling approach results are plotted in red circle (denoted as ‘IE’ in legend). Simulation frequency is from 1 GHz to 5 GHz.
Figure 4.16  Multi-effective layer model for 3D dielectric sphere. Reflection coefficients as a function of spheres’ dielectric constant. The physical dimensions are the same as in Fig. 4.13. Simulation frequency is 3 GHz. The one layer model results are in blue dot. The 3 effective layer model results are in green circles. Our integral equation based full wave modeling approach results are plotted in red circle (denoted as ‘IE’ in legend).
Figure 4.17  Multi-effective layer model for 3D PEC sphere. The physical dimensions are the same as in Fig. 4.13. One layer model and 3 effective layer model results are plotted using blue and green dashed lines, respectively. Our integral equation based full wave modeling approach results are plotted in red circle (denoted as ‘IE’ in legend). Simulation frequency is from 1 GHz to 5 GHz.
CHAPTER 5. EXPERIMENT CHARACTERIZATION

Starting with a brief review of testing approaches to characterize material, we study the effective parameter through experiments in this chapter. The topic include the design of testing structure, the approaches to analyze experiment data, theoretical analysis of measurement uncertainty. Two fabricated printed circuit boards are presented. The measurement data are analyzed using published approaches and our integral equation based analysis approach.

5.1 Test scheme consideration and testing board design

A summary of testing schemes can be referred in Fig. 5.1 [24]. We are going to determine the effective parameter using both parallel plate capacitor and transmission line. Our first PCB board design is as shown in Fig. 5.2. It's a screen-shot of Gerber file, and it's the top view of multilayer PCB board.

The whole board is 6 inches by 2 inches. The design contains capacitors and transmission lines, so we can work on both test schemes in this design. The bottom layer is full metal working as ground, as the red color part in Fig. 5.2. The inner layers contain periodic structure, and all inner layers are identical, as the pink color dots in Fig. 5.2. We can have two inner layers for a 4-layer board or four inner layers for a 6-layer board.

Because it is the top view, we can see that the periodic structures exist only in the upper half part of the design. The periodic structure is made up with periodic cells. Each periodic cell is 10 mils by 10 mils dielectric (the thickness is determined by fabrication technology) and with a 5 mil-by-5 mil metal patch inside (thickness of the metal patch is 1.4 mil). The
Figure 5.1 Categories of different test schemes to characterize EM parameter of material [24].

The top layer, in the color of yellow, contains the designed transmission lines and capacitors. In the uppermost region of top layer, there are T-Line (Transmission line)-like structures forming a THRU and an OPEN structure. These structures are good for de-embedding. Right below this region, there are two transmission lines of different lengths. By measuring the $S_{21}$ phase difference of these two transmission lines and the physical length difference, we can find the effective permittivity.

Below the T-Line region, there is a parallel plate capacitor. By measuring the capacitance or $S_{11}$, we can also determine the effective parameter. The lower half part of this design is almost symmetrically the same as the upper half. But in the lower half region, there are no periodic structure in the inner layers. This part serves to determine the original permittivity of this PCB board. Another difference is that in this region, there are THRU and OPEN structures and a capacitor but only one transmission line instead of two with different lengths. This is
Figure 5.2  Screenshot PCB design file. The whole board is 6 inches by 2 inches. The top layer metal is plotted in yellow, bottom metal is in red and inner metal is in pink. For substrate with periodic metal, 3 microstrip lines are designed. The lengths are 0.5 inch, 1.7 inch and 2 inches, respectively. For substrate without periodic metal, 2 microstrip lines with the lengths of 0.5 inch and 2 inches are designed. The ‘Open’ structure on board is 0.25 inch long. The width of all the microstrip lines are 120 mils. The dimension of parallel plate capacitors is 1 inch by 2 inch.

because we can use the THRU part as the other T-Line. Measuring the original permittivity is very important because in our effective media model the effective permittivity is related to the original permittivity and periodic structure. The transmission line has a width of 130 mils which will give nearly 50 ohm characteristic impedance in our interested frequency band.

The photo of the board is as shown in Fig. 5.3.

5.2  Analysis based on parallel plate capacitor structure

We have included parallel plate capacitors in our PCB design. Thus we can use the capacitor model to calculate the permittivity of the dielectric. The formula for parallel plate capacitor
Figure 5.3 Photo of fabricated PCB board. The PCB structure is the same as in Fig. 5.2.

is:

\[ \varepsilon_r = \frac{Cd}{\varepsilon_0 A} \]  

(5.1)

where \( C \) is the capacitance, \( d \) is the thickness of the dielectric material, and \( A \) is the area of the parallel plate. In the experiment we could only measure the total capacitance which includes the fringing capacitance. Equation 5.1 doesn’t take fringing effect into account thus its results are not accurate. Lots of researches have been done to address the fringing effect issue. In [54], there is analytical formula provided including the fringing effect. The formula is based on conformal mapping and the reported error is within several percent. The formula is listed here [54]:

\[ C_{tot} = \frac{2\varepsilon}{\pi} \ln \left( \frac{2R_b}{R_a} \right) \]

\[ \ln (R_a) = -1 - \frac{\pi w}{2h} - \frac{p+1}{\sqrt{p}} \tanh^{-1} \left( \frac{1}{\sqrt{p}} \right) - \ln \left( \frac{p-1}{4p} \right) \]

\[ R_b = \eta + \frac{p+1}{2\sqrt{p}} \ln \Delta \]

\[ \eta = \sqrt{p} \left( \frac{\pi w}{2h} + \frac{p+1}{2\sqrt{p}} \left( 1 + \ln \frac{4}{p-1} \right) - 2 \tanh^{-1} \frac{1}{\sqrt{p}} \right) \]

\[ \Delta = \max (\eta, p) \]

\[ p = 2B^2 - 1 + \sqrt{(2B^2 - 1)^2 - 1} \]

\[ B = 1 + \frac{t}{h} \]

We measured the capacitance using Agilent 4263B LCR meter. The measurement range of this equipment is up to 100 kHz. Two capacitors were measured. One is with the original FR4 as dielectric insulator. And for the other, the insulator is FR4 embedded with periodic metal structure. To acquire better accuracy, we did all the measurement in 10 kHz. The measured results and calculated relative permittivity from Changs model [54] are provided in Table 5.1. The difference is less than 5%. And the results show clearly that the embedded periodic metal structure results in an increase in the effective relative permittivity.

**Table 5.1  Measured capacitance and extracted relative permittivity**

<table>
<thead>
<tr>
<th>Capacitance</th>
<th>Relative Permittivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>dielectric without metal embedded</td>
<td>41.1 pF</td>
</tr>
<tr>
<td>dielectric with metal embedded</td>
<td>42.3 pF</td>
</tr>
</tbody>
</table>

5.3 Analysis based on microstrip line pair structure

5.3.1 Data validation

It’s crucial that our measured data are valid, physically reasonable. There are some qualitative properties in the measured data should be expected before we go to further data processing. For example, we expect a phase difference in the forward transmitting coefficient, \( S_{21} \), between
the TML of the same length but supported by a) original substrate and b) periodic metal embedded substrate. And the phase difference is approximately proportional to the frequency. Furthermore, we have quantitative criteria to evaluate the measured data. We designed our PCB board with both the ‘open’ structure and the ‘through’ structure. The ‘open’ structure is physically half of the ‘through’ structure. Literature [37, 44] reports a correlation between the S parameters of these two structures as:

\[ S_{11\text{open}} = S_{11\text{through}} + S_{21\text{through}} \]  

Equation 5.3 is used to evaluate the measured data for self-validation. The results are shown in Fig. 5.4. And we can see that the measurement results follow Eq. 5.3 very well.

Figure 5.4 Validating the measured data using the S parameter of open and through structures. The lefthand side and righthand side of Eq. 5.3 are plotted in blue solid line and red circles, respectively.
5.3.2 Extract effective parameter based on micro-stripe line pair measurement

Matched transmission line (T-Line) has simple form of scattering matrix (S matrix) and scattering transfer matrix (T matrix). We can relate the matrix elements to the effective permittivity with two different approaches. One approach makes use of the S matrix and the other uses the information of T matrix directly. These two approaches share similar assumptions so we can see the results agree reasonably well. The S matrix and T matrix of matched T-Line are given as:

\[
S = \begin{bmatrix}
0 & e^{-\gamma l} \\
e^{-\gamma l} & 0 
\end{bmatrix} \quad (5.4)
\]

\[
T = \begin{bmatrix}
e^{-\gamma l} & 0 \\
0 & e^{\gamma l} 
\end{bmatrix} \quad (5.5)
\]

where \( \gamma^2 = -k_0^2 \varepsilon_{\text{req}} = -\omega^2 \varepsilon_0 \mu_0 \varepsilon_{\text{req}} \).

We can see that for lossless case, the phase of \( S_{21} \) is nothing but the phase delay of the matched T-Line which related to effective permittivity. So we can measure \( \Delta \varphi \), the phase difference of \( S_{21} \) between two different transmission lines with physical length difference \( \Delta l \), and apply the following equation [28]:

\[
\varepsilon_{\text{req}} = \left( \frac{\Delta \varphi c}{2\pi f \Delta l} \right)^2 \quad (5.6)
\]

In previous approach, what we get is only the approximated real part of complex \( \gamma \) under the assumption that the T-lines are matched. With two sets of the S matrix, we have enough information to determine the complex \( \gamma \) so that we can get the complex \( \varepsilon_{\text{req}} \). The value of complex \( \gamma \) can be calculated as [29]

\[
\gamma = \ln \left[ \left( A \pm \sqrt{A^2 - 4} \right) / 2 \right] / (l_1 - l_2) \quad (5.7)
\]

where \( A = (T_{11(1)}T_{22(2)} + T_{11(2)}T_{22(1)}) - (T_{21(1)}T_{12(2)} + T_{21(2)}T_{12(1)}) \), and \( T_{ij(k)} \) is the T matrix element \((i,j)\) of the \( 'k' \)th T-Line while \( l_k \) is the T-Line’s physical length.
The T matrix is calculated from S matrix:

\[
\begin{align*}
T_{11}(i) &= -\det(S(i))S_{21}(i), \\
T_{12}(i) &= S_{11}(i)/S_{21}(i), \\
T_{21}(i) &= -S_{22}(i)/S_{21}(i), \\
T_{22}(i) &= 1/S_{21}(i).
\end{align*}
\] (5.8)

This approach uses the T matrix of the shorter T-Line for de-embedding.

Both approaches discussed above utilize two micro-strip lines with different lengths and they both get the equivalent permittivity (as in strip line model) first then go to the substrate permittivity (as in micro-strip line model). The difference between strip line model and micro-strip model are demonstrated in Fig. 5.5.

And equations for the correlation between equivalent permittivity and substrate permittivity are given as follows [56]:

\[
\begin{align*}
\varepsilon_{req} &= \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2} \left[1 + 12 \frac{h}{w_{eq}} \right]^{-\frac{1}{2}} \\
\frac{w_{eq}}{h} &= \frac{w}{h} + \frac{1.25}{\pi} \frac{t}{h} \left[1 + \ln \left( \frac{2h}{t} \right) \right].
\end{align*}
\] (5.9)

where \(w\) is the width of the signal strip, \(h\) is the thickness of substrate, \(t\) is the thickness of the strip, \(\varepsilon_r\) is the relative permittivity (or effective relative permittivity) of the substrate.

5.3.3 Measurement data analysis

Two approaches described before will give similar results for equivalent permittivity, as shown in Fig. 5.6. The results for equivalent permittivity for substrate with and without pe-
rioric metal embedded are shown in Fig. 5.7. As we can see from the results, the embedded metal structure has increased the equivalent permittivity. This means that it's very likely that the effective permittivity of the substrate increases due to the embedded metal structure. Applying Eq. 5.9, we can get the effective substrate permittivity from the equivalent permittivity. The results are shown in Fig. 5.8. Several observations can be made based on the results. First, the embedded metal structure has increased the effective permittivity. Second, in this frequency range, we see the dispersion property of the substrate. The permittivity decreases when the frequency goes up. This trend agrees with what reported in [55]. Third, the extracted substrate permittivity is in the range of 4 to 4.5, which is also reasonable as reported in [55].

![Figure 5.6](image.png)

**Figure 5.6** Equivalent permittivity of raw substrate from two approaches. The results for phase difference approach (Eq. 5.6) is plotted in blue solid line. And the results for de-embedding approach (Eq. 5.7) is plotted in red dashed line. Measured results are from 200 MHz to 2 GHz.

The measured results are also compared with EPACS results. In the EPACS we built the model so that the incident direction and polarization are the same as we did the experiment. The EPACS results are listed in Table 5.2 and compared with measurement results. As we
Figure 5.7 Equivalent permittivity for substrates with and without periodic metal embedded. The results for phase difference approach (Eq. 5.6) is plotted in blue solid line. And the results for de-embedding approach (Eq. 5.7) is plotted in red dashed line. Measured results are from 200 MHz to 2 GHz.

can see from Table 5.2, when we set the substrate permittivity close to what we measured, the EPACS model for the effective relative permittivity of the metal embedded substrate is also pretty close to our experiment results.

<table>
<thead>
<tr>
<th>(Effective) Relative permittivity</th>
<th>Measured at 200 MHz</th>
<th>EPACS</th>
</tr>
</thead>
<tbody>
<tr>
<td>substrate without metal embedded</td>
<td>4.48</td>
<td>set to 4.5</td>
</tr>
<tr>
<td>substrate with metal embedded</td>
<td>4.61</td>
<td>4.64</td>
</tr>
</tbody>
</table>

The EPACS algorithm is designed to model the doubly periodic structure. Based on the EPACS results, effective model based on the same reflection and transmission coefficients are built to retrieve the effective permittivity and permeability of the periodic structure. The measurement of TML also aims to measure the reflection and transmission coefficients. Though we may not build a real infinitely large doubly periodic structure, it’s still possible to approxi-
Figure 5.8 Effective substrate permittivity of for substrates with and without periodic metal embedded. Results for raw substrate are plotted in red dashed line and the results for substrate with periodic inclusions are plotted in blue solid line. Results are from 200 MHz to 2 GHz.

mate the “infinitely large” by sufficient number of periods. Another important issue is that we need to make sure the wave incidence direction and polarization are the same between the measurement and EPACS model. This is because the reflection and transmission coefficients are usually depend on these two directions unless the structure has spherical symmetry. Figure 5.9 illustrates structure differences between the fabricated and EPACS model. We know that the fabricated structure has limited periods over all the three dimensions while the EPACS model will have infinite periods in two dimensions and finite in the third dimension. In our configuration the EPACS model will have infinite period in the x and y directions, as the coordinate shown in Fig. 5.9, while finite periods in the z direction (or k direction). This configuration makes the polarization and the incidence identical between the measurement and EPACS model so that they have the least difference. In the y direction, the fabricated structure has sufficient number of periods to approximate ‘infinite periods’. In the x direction the fabricate
structure has less periods. However, the EPACS models show that two layers of periodic structures are already a good approximation to more layers of periodic structures [57]. And the effective media model also assumes that the effective parameter should not change with the number of layers [9].

Figure 5.9 Structure difference between fabricated structure and EPACS model. ‘k’ indicates the wave propagation direction and ‘E’ represents the direction of electric field.

The model as illustrated in Fig. 5.9 is built in EPACS. The EPACS results are listed in Table 5.3. For the comparison purpose, the results of MG formula are also listed. In Table 5.3, $\epsilon_{r,sub}$ is the relative permittivity of the substrate without metal inclusion while $\epsilon_{reff}$ is the effective relative permittivity of periodic metal embedded substrate. Several observations can be made through the measurement results. First, the relative permittivity of the substrate material (FR4) decreases when the frequency increases. The same dispersive trend is reported in [28]. Second, with the periodic metal embedded, the effective relative permittivity rises, but the dispersive trend is still the same. Comparing with the measurement, our EPACS model gives very accurate prediction. The largest difference is around 2% while in the other points the differences are within 1%. Table 5.3 also shows that the MG formula results have larger difference to the measurement compared with the EPACS. The largest difference is around 10%. This is because the MG formula is an approximated formulation for sphere inclusion.

From the modeling we know that when the filling rate of the inclusion changes, the effective permittivity changes accordingly. And MG mixing rule suggests that the higher filling rate, the larger of the effective permittivity is expected. To experimentally explore the effect of
filling rate and verify our modeling approach, periodic metal patch structures corresponding to different filling rates are designed, fabricated and the relative permittivity of those structures are evaluated. We used standard 4 metal layer printed circuit board (PCB). The inner two metal layers are designed to have periodic metal patches. Meanwhile the top metal and the bottom metal form microstrip (MS) pairs which are used to determine the substrate permittivity. With this configuration, the substrate of the MS pairs becomes layered media embedded with periodic objects. We obtained the effective relative permittivity of the homogenized substrate by analyzing the propagation constant of MS pairs.

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>$\varepsilon_{r,\text{sub}}$</th>
<th>$\varepsilon_{r,\text{eff}}$</th>
<th>EPACS</th>
<th>MG formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.200</td>
<td>4.486</td>
<td>4.646</td>
<td>4.750</td>
<td>5.007</td>
</tr>
<tr>
<td>0.506</td>
<td>4.390</td>
<td>4.615</td>
<td>4.648</td>
<td>4.893</td>
</tr>
<tr>
<td>0.803</td>
<td>4.287</td>
<td>4.550</td>
<td>4.540</td>
<td>4.779</td>
</tr>
<tr>
<td>1.001</td>
<td>4.186</td>
<td>4.469</td>
<td>4.433</td>
<td>4.666</td>
</tr>
<tr>
<td>2.000</td>
<td>4.040</td>
<td>4.216</td>
<td>4.278</td>
<td>4.521</td>
</tr>
</tbody>
</table>

Figure 5.10 Photo of fabricated PCB and illustration for periodic metal patches. Top and bottom metal layers form MS pairs while two inner metal layers are configured as periodic metal patches with different filling rate. All microstrip lines are 109 mils wide. The length for short microstrip is 0.5 inch while the long microstrip is 1 inch. Periodic metal patches are designed to fulfill the filling rate of 0 (no inclusions), 1.25% and 3.47%.
The photo of the fabricated board and structure illustration are as shown in Fig. 5.10. By changing the size of the metal patches in inner layers, we have implemented three different filling rates. One configuration is to place the metal patch of 0.127 mm by 0.127 mm by 0.035 mm in a period of 0.254 mm by 0.254 mm by 0.711 mm, which gives us a filling rate of 1.25%. Another configuration is to make the metal patch size to be 0.635 mm by 0.635 mm by 0.035 mm while the period to be 0.762 mm by 0.762 mm by 0.711 mm, which gives a filling rate of 3.47%. There are three sets of microstrip line pairs, corresponding the filling rate in the substrate to be 0 (no metal inclusion), 1.25% and 3.47%, respectively.

The effective permittivity of substrate is the permittivity for the homogenized substrate. The experiment results are then compared with our proposed modeling results.

![Effective permittivity of periodic metal patches](image)

Figure 5.11 Effective permittivity of periodic metal patches embedded in layered media, measurement results and modeling results for two different filling rate. Relative permittivity for raw substrate is plotted in blue dot. Experiment results for filling rates of 1.25% and 3.47% are plotted in black circles and red star, respectively. The modeling results for filling rates of 1.25% and 3.47% are plotted in black solid line and red dashed line, respectively. Results are from 200 MHz to 2 GHz.

The measurement results are shown in Fig. 5.11. The measurement is performed in frequency band 200 MHz to 2 GHz. Measured results are plotted in discrete circles and crosses, for filling rate of 1.25% and 3.47%, respectively. The results for raw substrate are plotted in
discrete dots, for reference. Modeling results are also provided in Fig. 5.11 using solid and dashed line for filling rate of 1.25% and 3.47%, respectively. From Fig. 5.11, both experiment results and modeling results suggest that the effective permittivity rises when periodic metals are embedded. The results also suggest that higher filling rate gives higher effective permittivity. The difference between experiment and modeling results is less than 3%. This good correlation between the experiment and our numerical modeling approach validates our proposed homogenized approach.

5.4 Analysis of microstrip line using integral equation approach

Previously described approaches have assumption that relative permeability of the substrate equals to 1 and the empirical formula are involved to solve the effective permittivity from the equivalent propagation constant. Those characteristics may limit the application of approaches. Here we propose a characterization method using the measurement of microstrip line and de-embedding structure. The de-embedding structure is generally a ’THRU’ circuit of the testing fixtures. The cascaded two-port network theory is used to de-embed the test fixtures. From the de-embedded S parameter, we extract two complex parameters: characteristic impedance and propagation constant of the microstrip line. Then these two parameters are used in the integral equation approach based model of the microstrip line and complex permittivity and permeability of the substrate, i.e. the specimen material, are solved.

5.4.1 De-embedding microstrip line pair

In our method, an integral equation (IE) based approach is used to analyze the physical structure of microstrip line. The constitutive parameters are solved by equalizing the modeled results with measured results. It is clear that there are attenuation and phase delay due to the test fixture in the measurement. And it will give error in the results of the permittivity and permeability if the raw measurement data is used to do the extraction. Considering this, to
de-embed the effect of test fixtures is important.

We can measure a second microstrip line of a different length to remove the test fixtures’ effect. As shown in Fig. 5.12, two microstrip lines with length of $l_1$ and $l_2$ (assuming that $l_1 > l_2$) are measured. The microstrip $l_2$ is treated as cascaded test fixtures while the microstrip $l_1$ is treated as cascaded of test fixtures and an fixture-free microstrip with length of $l_1 - l_2$.

So the scattering transfer Matrix (T matrix) of two microstrip lines can be expressed as:

$$ T_{l_2} = [A] [B] $$
$$ T_{l_1} = [A] [C] [B] $$

The T matrix can be calculated from the S matrix [29]. Taking the assumption that the test fixtures are reciprocal and symmetrical, we have $[A] = [B]$ , and $[A]$ can be calculated from the S parameter of microstrip line $l_2$. Use the similar approach as in [37], we have the T matrix
of the fixture:

\[
A = \begin{bmatrix}
\frac{S_{11}^2 - S_{12} - S_{21}^2}{U} & \frac{S_{11}}{U} \\
-\frac{S_{11}}{U} & \frac{1 + S_{12}}{U}
\end{bmatrix}
\]

where \( S_{11} \) and \( S_{21} \) are scattering parameter of microstrip line \( l_2 \), \( U = \pm \sqrt{S_{12} \left[ (1 + S_{12})^2 - S_{11}^2 \right]} \), the sign is determined so that the value of ‘\( U \’ shows a phase delay. The T matrix for fixture-free microstrip can be calculated by:

\[
[C] = [A]^{-1} [T_{l1}] [A]^{-1}
\]

From matrix \( C \), the S matrix of fixture free microstrip is obtained.

### 5.4.2 Extraction of characteristic impedance and propagation constant

We assume that the cross section profile of the microstrip line doesn’t change along the propagation direction, as we can see in Fig. 5.13. Thus the complex characteristic impedance (normalized) \( \bar{\eta} \) and complex propagation constant \( \beta \) are fixed. With the S matrix of fixture-free microstrip line, the complex \( \bar{\eta} \) and \( \beta \) can be determined by the following equations [10, 22]:

![Test fixture-free microstrip line.](image)

Figure 5.13 Test fixture-free microstrip line. The (effective) permittivity and permeability of the substrate are \( \epsilon_1 \) and \( \mu_1 \), respectively; the substrate thickness is \( h \).
\[
\tilde{\eta}_1 = \frac{\eta_1}{\eta_0} = \pm \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}}
\]

\[
\beta = j \ln(\varphi)/l
\]

\[
\varphi_{1,2} = \frac{(1 + S_{21}^2 - S_{11}^2) \pm \sqrt{(1 + S_{21}^2 - S_{11}^2)^2 - 4S_{21}^2}}{2S_{21}}
\]

where \( l \) is the length of the microstrip line. We notice that both \( \tilde{\eta} \) and \( \beta \) (which is solved through the value of \( \varphi \)) have two possible results. The passive medium criteria help us to choose result which makes physical sense. It’s straightforward for \( \tilde{\eta} \) because equation for impedance shows that only one of the possible \( \tilde{\eta} \) has a positive real part which means passive material. For \( \varphi \), we find that \( \varphi_1 \varphi_2 = |\varphi_1| |\varphi_2| e^{j(\angle \varphi_1 + \angle \varphi_2)} = 1 \), which means that in lossy case only one of the possible \( \varphi \) has a magnitude less than 1, that’s the one to choose. For lossless case in simulation, we can examine the phase to choose \( \varphi \) value which has a phase delay.

Now we’ve extracted the measured complex propagation constant and characteristic impedance, which are used in next step to solve the complex permittivity and permeability of the substrate material.

### 5.4.3 Determining the permittivity and permeability

For a microstrip line having fixed cross-section profile along propagation direction, the characteristic impedance and propagation constant are fixed and can be solved by analyzing the cross-section profile. In spectral domain we have [38]:

\[
G_{xx}(\alpha_n, \beta) \tilde{J}_x(\alpha_n) + G_{xz}(\alpha_n, \beta) \tilde{J}_z(\alpha_n) = \tilde{E}_{x1}(\alpha_n, h)
\]

\[
G_{zx}(\alpha_n, \beta) \tilde{J}_x(\alpha_n) + G_{zz}(\alpha_n, \beta) \tilde{J}_z(\alpha_n) = \tilde{E}_{z1}(\alpha_n, h)
\]

The Green’s functions can be referred from [39].

At \( y = h \), we have:

\[
E_x(x) J_x(x) = E_z(x) J_z(x) = 0
\]
And applying the Parseval’s theorem, we have:

\[
\begin{bmatrix}
K^{xx} & K^{xz} \\
K^{zx} & K^{zz}
\end{bmatrix}
\begin{bmatrix}
a \\
b
\end{bmatrix} = 0
\]

The \( K \) matrix is calculated by solving:

\[
K_{pq}^{ij} = \sum_{n=1}^{\infty} \tilde{J}_{pi}(\alpha_n) G_{pq}(\alpha_n, \beta) \tilde{J}_{qj}(\alpha_n)
\]

where: \( p, q \in \{x, z\} \).

\( a = [a_i] \) and \( b = [b_i] \) are column vectors for the current expansion:

\[
\tilde{J}_x(\alpha_n) = \sum_{i=1}^{M_x} a_i \tilde{J}_{xi}(\alpha_n) \quad \tilde{J}_z(\alpha_n) = \sum_{i=1}^{M_z} b_i \tilde{J}_{zi}(\alpha_n)
\]

Finally the propagation constant can be determined by:

\[
\det \begin{bmatrix}
K^{xx} & K^{xz} \\
K^{zx} & K^{zz}
\end{bmatrix} = 0
\]

The characteristic impedance is obtained by a voltage-current definition. The modeled propagation constant and characteristic impedance are both the function of relative permittivity and permeability of the substrate. By equalizing the modeled results with measurement results, we have:

\[
\bar{\eta}_{\text{model}}(\varepsilon_r, \mu_r) = \bar{\eta}_{\text{meas}}, \quad \beta_{\text{model}}(\varepsilon_r, \mu_r) = \beta_{\text{meas}}
\]

### 5.4.4 Numerical verification

Our proposed method utilizes the S parameter of de-embedded microstrip line to extract the permittivity and permeability of the substrate. Firstly we use simulated S parameter to verify this approach. We use Agilent ADS momentum to simulate the S parameter of structure as show in Fig. 5.14. The top part is the 'THRU' structure which is used for de-embedding the bottom microstrip line. The de-embedded microstrip line has a width of 2.96 mm and length of 5 mm. The substrate is 32 mils thick. Simulation frequency is set to be 3 GHz. Two different
substrates are simulated. The first case is conventional material which has permittivity to be 40 while permeability to be 1. And the second case is magnetic material which has permittivity equals to 1 while permeability to be 40. After the ADS momentum finished simulation. We adopt our proposed method to analyze the S parameters. The extracted results are as shown in Table 5.4. We compare the configuration in momentum and our extracted results. As we can see that the difference is within 3%. Nonetheless, our proposed method presents good results for substrate material characterization in both cases.

![Figure 5.14](image)

**Figure 5.14** The structure used in ADS momentum simulation. The top part is the ’THRU’ structure and bottom is embedded microstrip.

<table>
<thead>
<tr>
<th></th>
<th>Configuration in ADS momentum</th>
<th>Extracted results of proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>$\varepsilon_r = 40, \mu_r = 1$</td>
<td>$\varepsilon_r = 38.979, \mu_r = 0.994$</td>
</tr>
<tr>
<td>Case II</td>
<td>$\varepsilon_r = 1, \mu_r = 40$</td>
<td>$\varepsilon_r = 0.997, \mu_r = 40.946$</td>
</tr>
</tbody>
</table>
5.5 Full wave approach to analyze measurement data

To validate our proposed characterization approach, a printed circuit board (PCB) is designed, fabricated and measured. The photo of the fabricated board is as shown in Fig. 5.15. The PCB has 4 metal layers. The top metal and the bottom metal form microstrip lines (MS). The inner metal layers can be used to form periodic structures as a different substrate other than FR4.

The proposed method is used to analyze the measured data of microstrip pair and determine the complex relative permittivity and permeability of the substrate material. The material can be homogeneous or composite material such as periodic structures which can be characterized by effective permittivity and effective permeability. In our experiment design, two different substrates are used and analyzed by proposed method. Firstly, the FR4 material is characterized. Secondly, we utilize the inner metal layers to form a periodical metal patches structure and use it as the substrate. The period dimension is 10 mil by 10 mil and thickness is 28 mil. In each period, a metal patch is embedded. The metal patch size is 5 mil by 5 mil, and thickness is 1.4 mil. For this case we want to demonstrate that periodic structures formed by non-magnetic material may have an effective relative permeability not equal to one.

Figure 5.15 The photo of fabricated microstrip lines. The shorter microstrip has the length of 0.5 inch and is used for de-embedding the longer one, which is 2 inches long.
5.5.1 Measurement results

Firstly, the FR4 substrate is measured. In this case, we have prior knowledge that the material is non-magnetic so that we can configure the program to solve permittivity only and permeability is set to unit. In this case we optimize the value of $\varepsilon_r$ only to minimize the difference between modeled results and measured results of $\bar{\eta}$ and $\beta$.

Fig. 5.16 shows the extracted characteristic impedance and complex propagation constant, respectively. We can see that the normalized characteristic impedance is close to 1 which means that the device under test is close to but not perfect at the impedance matching point. The imaginary part of the complex propagation constant is negative and close to zero, which means the substrate material is low loss.

From the extracted characteristic impedance and propagation constant, we can solve the relative permittivity. The results are as shown in Fig. 5.17. We can see that the relative permittivity has negative imaginary parts which count for dielectric loss. The measurement data are also processed by the approach presented in [29] as a comparison. The approach in [29] took the assumption of $\mu_r = 1$ and extracted the complex permittivity based on the complex propagation constant. We can see that the extracted complex relative permittivities from those two approaches agree well.

For the periodic metal patches structure, the characteristic impedance and propagation constant are as shown in Fig. 5.18. Comparing with raw FR4, we can see that both characteristic impedance and propagation constant change. The absolute value of imaginary part of the propagation constant increases. It shows that the MS is more lossy, which is due to the inclusion of periodic metal patches. The real part of the propagation constant increases and the conclusion is the same as in reference [40]. The extracted complex permittivity and permeability are as shown in Fig. 5.19. The effective permittivity results are compared with results using approach described in [29]. Our results agree well with reference results. The approaches in [28],[29] take the assumption that permeability is unit. But for periodic structures the effective permeability can be different. The MG (Maxwell-Garnett) formula listed in [23] predicts that
the effective permeability is less than unit when periodic metal are examined. The MG formula is derived for spherical inclusion yet it works as reference for low filling rate case. Our results show that at relative low frequency the effective permeability is close to 1 and at higher frequency the results approach to MG formula.

5.5.2 Discussion

The measurement uncertainty in experiments affects the results of extracted parameter. The uncertainty comes from different reasons such as background noise, fabrication tolerance etc.
Figure 5.17 Relative permittivity of FR4. Our proposed integral equation (IE) based approach results are compared with results extracted using approach in [29]. The real part and imaginary part of results from our full wave analyzing approach are plotted in blue solid line and green circles, respectively. The real part and imaginary part of results from reference approach are plotted in red dot dashed line and cyan dashed line, respectively.

The difference between the model and test bench could also be counted into the measurement uncertainties. With all these measurement uncertainties, the S parameter got from the experiment is different from the S parameter in the model. It’s important for us to understand how these measurement uncertainties affect the results.
Figure 5.18 Normalized complex characteristic impedance (top) and propagation constant (bottom) of periodic structure. The real part and imaginary part of normalized η and β are plotted in blue solid line and green dashed line, respectively. The results are from 200 MHz to 2 GHz.

For a homogeneous dielectric slab with a normalized characteristic impedance \( \bar{\eta} \) and propagation constant \( \beta \), the S parameters could be calculated by:

\[
S_{11} = \Gamma \frac{1 - \varphi^2}{1 - \Gamma^2 \varphi^2}, \quad S_{21} = \varphi \frac{1 - \Gamma^2}{1 - \Gamma^2 \varphi^2}
\]

where \( \Gamma = (\bar{\eta} - 1)/(\bar{\eta} + 1) \) and \( \varphi = e^{-j\beta l} \), ‘l’ is the material thickness in wave propagation direction. Our goal is to find the coefficients matrix in the following equation:

\[
\begin{bmatrix}
\frac{d\bar{\eta}}{dS_{11}} & \frac{d\eta}{dS_{21}} \\
\frac{d\beta}{dS_{11}} & \frac{d\beta}{dS_{21}}
\end{bmatrix}
\begin{bmatrix}
\frac{dS_{11}}{ds} \\
\frac{dS_{21}}{ds}
\end{bmatrix}
\]
Figure 5.19  Effective relative permittivity (top) and permeability (bottom) of periodic structure. The permittivity results are compared with results extracted using approach in [29] and the permeability results are compared with MG formula results, which is calculated using Eq. 4.6.

We find that the coefficients look sophisticated if expressed by S parameters. So we express the coefficients using intermediate parameters $\Gamma$ and $\varphi$. We start by looking for the coefficients matrix of the following:

$$
\begin{bmatrix}
  dS_{11} \\
  dS_{21}
\end{bmatrix} =
\begin{bmatrix}
  A & B \\
  C & D
\end{bmatrix}
\begin{bmatrix}
  d\Gamma \\
  d\varphi
\end{bmatrix}
$$
All the coefficients A to D are defined and expressed as the following:

\[
A = \frac{\partial S_{11}}{\partial \Gamma} = \frac{(1-\varphi^2)(1+\Gamma^2\varphi^2)}{(1-\Gamma^2\varphi^2)^2}
\]

\[
B = \frac{\partial S_{11}}{\partial \varphi} = \frac{-2\Gamma \varphi(1-\Gamma^2)}{(1-\Gamma^2\varphi^2)^2}
\]

\[
C = \frac{\partial S_{21}}{\partial \Gamma} = \frac{-2\Gamma \varphi(1-\varphi^2)}{(1-\Gamma^2\varphi^2)^2}
\]

\[
D = \frac{\partial S_{21}}{\partial \varphi} = \frac{(1-\Gamma^2)(1+\Gamma^2\varphi^2)}{(1-\Gamma^2\varphi^2)^2}
\]

Inversing the matrix yields:

\[
\begin{bmatrix}
\frac{d\Gamma}{d\varphi} \\
\end{bmatrix} = \frac{1}{\det} \begin{bmatrix}
D & -B \\
-C & A
\end{bmatrix}
\begin{bmatrix}
\frac{dS_{11}}{dS_{21}} \\
\end{bmatrix}
\]

The determinant of the matrix can be calculated:

\[
\det = AD - BC = \frac{(1-\Gamma^2)(1-\varphi^2)}{(1-\Gamma^2\varphi^2)^2}
\]

with the help of the following:

\[
\begin{bmatrix}
\frac{d\bar{\eta}}{d\beta} \\
\end{bmatrix} = \begin{bmatrix}
\frac{2}{(1-\Gamma)^2} & 0 \\
0 & j \frac{1}{\varphi l}
\end{bmatrix}
\begin{bmatrix}
\frac{d\Gamma}{d\varphi} \\
\end{bmatrix}
\]

finally we get:

\[
\begin{bmatrix}
\frac{d\bar{\eta}}{d\beta} \\
\end{bmatrix} = \begin{bmatrix}
\frac{2(1+\Gamma^2\varphi^2)}{(1-\Gamma^2)(1-\varphi^2)} & \frac{4\Gamma \varphi}{(1-\Gamma^2)(1-\varphi^2)} \\
\frac{j 2\Gamma}{l(1-\Gamma^2)} & \frac{j 1+\Gamma^2\varphi^2}{l(1-\Gamma^2)}
\end{bmatrix}
\begin{bmatrix}
\frac{dS_{11}}{dS_{21}} \\
\end{bmatrix}
\]

By examining the coefficient matrix we find that we can minimize the measurement uncertainties’ effect by reducing the value of \( \Gamma \). This explains when we design our microstrip line; we want it in a region near impedance matching point. Ideally at perfect matching point, coefficients in the last equation go to either 0 (off diagonal) or minimum (diagonal), which means that measurement error will not affect the final results significantly. Though we may never get to the perfect matching point, in our design we should do our best to match the microstrip line to our measurement system.
CHAPTER 6. CONCLUSION

The problem of effective medium model for periodic inclusions in multi-layered structure is introduced. Several models are presented based on different definitions of 'effectiveness'. Analytical forms of effective parameters for multi-layered material only are derived. Several cases are simulated to show the efficiency improvement by adopting the effective medium model.

We present the work in effective parameter extraction for effective medium model based on the same reflection and transmission coefficients (which equivalent to the same scattering parameters model). Firstly, we discuss the branch cut issue in extraction procedure and present the solutions. Then the normal and oblique incidence cases are explored. The low frequency limit for input impedance and transmission coefficient are derived. The analytical forms for effective parameters in both parallel polarization and perpendicular polarization are given. Based on the directions of electric field and propagation, we present the effective parameter in tensor form. Numerical results are provided to verify the analytical low frequency limits.

Several effective medium models for periodic structure in are explored. We start from Maxwell-Garnett mixing rule (MG formula) which gives effective permittivity and permeability, in the sense of field average. We analyze different forms of MG formula. We also explore the filling rate and shape polarization of inclusions. Based on the frequency dependent model, we’ve derived both the quasi-static and high-frequency asymptotic forms of the frequency dependent term in MG formula and thus predict the transition region where the effective parameter goes from near-dielectric results to near-metal results. The case of periodic lossy dielectric spheres are used as example to verify our findings. In the end, we combine MG
formula with multi-layered effective medium model approach and show that the multi-layered effective medium model could have achieved better results.

Experimentation characterization of unknown planar material is practiced. This part of research works as verification for modeling of periodic material in multi-layered substrate so that our modeling approach is validated. Different testing structures such as parallel plate capacitor, micro-strip line pair and micro-strip line with de-embedding structure are carefully designed and fabricated using 4-layer printed circuit board technology. The periodic metal patches are fulfilled using inner metal layers of this technology and complex permittivity and permeability of the periodic structure are extracted by analysis of measurement data. Different metal patches are designed and implemented so that we are able to analyze how the filling rate of the metal inclusion will affect the effective permittivity and permeability. Several different testing schemes are performed and analyzed. We start from following approaches in literatures which output complex permittivity at low frequency, real or complex permittivity in RF range. In the end, we developed our approach based on micro-strip line with de-embedding structure and full-wave analysis of micro-strip line. Our approach is able to extract both complex permittivity and permeability from raw measurement data of testing structure. Our approach is compared with approaches from literature and results are validated. The measurement uncertainty is also considered. Using effective medium theory, the analytical results show that the micro-strip line should be designed near the matching point to reduce measurement uncertainty. The experimental extracted results are compared with our modeling results and good agreement is observed.
APPENDIX A. GRAPHIC USER INTERFACE FOR EFFECTIVE MEDIUM EXTRACTOR

This Effective Parameter Extractor (EPE) with Graphic User Interface (GUI) is developed under Matlab and is designed to work with our in-house Fortran program (the EPACS solver) and extract effective permittivity $\epsilon_{\text{reff}}$ and effective permeability $\mu_{\text{reff}}$ from the reflection and transmission coefficients of the structure. The following 3 cases can be handled: 1) multi-layer slab structure; 2) periodic 3D PEC structure; 3) periodic 3D dielectric structure. Besides the effective constitutive parameters, the geometry of embedded object (if any) and the information of the whole structure can also be displayed. A top level GUI with step by step guide to use the EPE is designed.

In 3D periodic structures problems, mesh files are required to describe the embedded objects. The Matlab GUI can display the geometry of the objects as well. For the GUI to display geometry of embedded objects, the mesh file is needed. It’s the same mesh file used in Fortran program.

Configuration files for EPE describe the problem, define physical parameters. For 3D periodic structures, the Configuration file for EPE is the same as the input file for Fortran program. For multi-layer slab case, an input file with similar format is defined and served for the EPE.

The examples for this EPE handling all the three cases are provided. For each case, detailed problem description is given. The needed files are listed. The screen-shot for the cases are presented.

For each case, different analytical results can be chosen for comparison. For multi-layer
slab case, the capacitor model at low frequency (the ‘capacitor model’) and low frequency limit of model based on the same reflection and transmission coefficient (the ‘low frequency limit’) are the choices. For periodic 3D structures, the MG formula is the choice. MG formula for PEC is different from the one for dielectric. A summary of all formulations of these analytical results can be found in the end of this appendix.

A.1 Structure of EPE

The entire EPE consists of 3 programs: Configuration File Generator, Extractor and EPE-main. Configuration File Generator is the graphic user interface to configure the problem, define the physical structure and numerical solver parameters. Extractor is the program to extract the effective parameters from reflection coefficient and transmission coefficient. EPE-main is the top level program which guides the user through the steps to solve the whole problem.

A.1.1 EPE-main

The EPE-main is the top level program. Its screen-shot is as shown in Fig. A.1. It clearly indicates the total three steps to solve for the effective parameter.

The first step is to configure the program. Clicking the ‘Config’ button will lead the user to Configuration File Generator. The detail of Generator will be discussed in next subsection.

Step 2 involves with the calculation of reflection and transmission coefficient (R/T file). Our effective media model is based on the same reflection and transmission coefficient. So R/T file is one of the inputs for extractor. For 3D periodic structure, the R/T file is generated by the EPACS Fortran program, and for multilayer slab problem, a Matlab subroutine is developed to generate R/T file. But this subroutine is not open to user. It’s automatically called to run extracting the effective parameter.
Step 3 is extracting the effective media parameter. Clicking the ‘Extract’ button will lead the user to the Extractor.

A.1.2 Configuration file generator

The screen-shot of the configuration file generator GUI is shown in Fig. A.2. It’s shown that the GUI generally consists of two parts: basic options and advanced options. The items in basic option are all left blank because these items are what we need to change according to problems. On the other hand, the items in advanced option all have defaulted value because in most of the cases they can be the same. The defaulted values are optimized for the Fortran program.
The first step is to choose ‘cases’ in basic option part. After the problem case is chosen, irrelevant input items will be disabled automatically. For instance, if the problems case is ‘3D PEC’, then the ‘constitutive parameters of inclusion’ will be disabled. Similarly, for frequency and angle input, once the ‘single’ option is chosen, input items for ‘sweep’ are disabled. All the input items correspond to some item in the input file. Help information can be found in earlier parts in this section.
A.1.3 Extractor

Figure A.3 is the screen-shot for the Extractor. It has two panels. User can click the ‘>>’ button to switch between the two panels. After the configuration, the extracted effective $\epsilon_r$ is shown in the bottom left part in panel 1 and the effective $\mu_r$ is shown in the bottom right in panel. In panel 2, the geometry of the embedded objects (if any) will be shown in bottom left part and the bottom right of the panel two will show the summary of the whole structure.

This Extractor can handle the following 3 cases: 1) Multi-layer slab case 2) periodic 3D PEC case and 3) periodic 3D dielectric case. For case 1), choose inclusion type to be ‘None’ and choose inclusion type to be ‘PEC’ or ‘Dielectric’ for Case 2) and 3), respectively.

Meanwhile, user can pick up some analytical reference for the case you have chosen. For case 1), capacitor model and low frequency limit can be picked as reference. For case 2) and 3), the available choice is MG formula. However, the MG formula for case 2) and 3) are different. The GUI will decide which MG formula to use based on the inclusion type. Clicking ‘choose problem configuration file’ button, a new window will pop up and the current directory is open. Choose the configuration file for you case, it should be the same one used in Fortran program.
A.1.4 Files Needed for the Extractor

Before working on the Extractor (last step of EPE), the following files should be prepared:

a) Input file for EPE. It contains the physical parameters of the problem we want to solve. For 3D periodic structures, it is the input file for EPACS Fortran program.

b) Output file of the Fortran program. The EPE is to extract effective constitutive parameters from the reflection coefficient and transmission coefficient of the structure. For 3D periodic structures, these coefficients are saved in the output file of the Fortran program. For multi-layered slab structure, the coefficients are calculated by a subroutine in the Matlab GUI, so no output file needed for multi-layered slab case.

c) Mesh file. For 3D periodic structures problem, the mesh file is needed for the Fortran program. The same mesh file is also needed for this Matlab GUI. It is for display the geometry and estimating the filling rate.

All the three files mentioned above are needed to make the Extractor work. With these files presented, Extractor can work on any operating system as long as a Matlab is installed properly.

A.2 Examples for EPE

We start with a multilayer structure case for EPE. Simulation case: 2 layer dielectric, each has the thickness of 4 \( \mu \)m, dielectric constants are 4 and 2, respectively. Simulation frequency 1 GHz to 10 THz.

The 1st step it to open the epe-main window. Click ‘Config’ button (Step 1 in EPE main window), the configuration file generator window will pop up. Input the parameters according to the problem. When the configuration file is correctly generated, a message window will pop up. When this step is finished, user can close the configuration file generator window. Then lick ‘extract’ button (Step 3 in EPE main window). The Extractor window will pop up, choose the ‘No inclusion’ in inclusion type option. Then choose ‘Capacitor model’ or ‘Low
frequency limit’ as reference (optional). Click ‘choose problem configuration file’, and choose the configuration file generated by configuration file generator. The final result will show in the Extractor window.

Then the GUI will calculate the reflection and transmission coefficients of the problem and extract the effective constitutive parameters of this structure. The results will show in the figures. After this step, you will get a window as shown in Fig. A.4. You can click the panel switch button to see the information of this problem, as shown in Fig. A.5. In this case, we don’t have inclusions so that the geometry window is empty.

Then we look into a 4 layer of periodic PEC sphere embedded in a dielectric layer case. The total thickness of dielectric is 24 µm and dielectric constant is 2. Simulation frequency is 100 GHz to 1 THz.

Open and run the epe-main script, the main window of this GUI will appear. Click ‘Config’ button (Step 1 in EPE main window), the configuration file generator window will pop up. Input the parameters according to the problem. When the configuration file is correctly
generated, a message window will pop up. When this step is finished, user can close the configuration file generator window. Call the Fortran program to calculate the reflection and transmission coefficients. This step is not done in Matlab. When the R/T data is calculated by Fortran, copy the file to the EPE program’s directory. Click ‘extract’ button (Step 3 in EPE main window). The Extractor window will pop up, choose the ‘PEC’ in inclusion type option. Then choose ‘MG formula’ as reference (optional). Click ‘choose problem configuration file’, and choose the configuration file generated by configuration file generator. The final result will show in the Extractor window.

Then the EPE will read the reflection and transmission coefficients calculated by Fortran program and extract the effective constitutive parameters of this structure. The results will show in the figures. After this step, you will get a window as shown in Fig. A.6. You can click the panel switch button to see the information of this problem, as shown in Fig. A.7. Then we can see the structure summary and also the geometry of the inclusion.

Figure A.5  EPE results window for mulilayer dielectric case, panel 2. Simulation information are the same as in Fig. A.4.
Figure A.6  EPE results window for periodic PEC case, panel 1. Simulation structure: 4 layer of periodic PEC sphere embedded in a dielectric layer. The total thickness of dielectric is 24 $\mu$m and dielectric constant is 2. Simulation frequency is 100 GHz to 1 THz.

In the end, we look into a 4 layer of periodic dielectric sphere in the air case. The total thickness of structure is 40 $\mu$m. Dielectric constant of sphere is 40. Simulation frequency is 10 GHz to 100 GHz.

Follow similar steps as PEC case. Instead of choosing ‘PEC’, choose ‘Dielectric’ in configuration and extractor window. The screen-shots for both panels are shown in Fig. A.8 and Fig. A.9.

You can click the panel switch button to see the information of this problem, as shown in Fig. A.9. Then we can see the structure summary and also the geometry of the inclusion.

### A.3 Formulations for references in EPE

For different structures, there are different analytical references results available. We’ve integrated several of them in the EPE. And the formulations for these analytical references can be referred here:
Capacitor model:

\[
\frac{1}{\varepsilon_{\text{reff}}} = \frac{\sum_{n=1}^{N} h_n}{h_t \varepsilon_{r_n}}
\]

where \( h_n \) is the thickness of \( n \)th layer and \( h_t \) is the total thickness of all dielectric layers. \( \varepsilon_{r_n} \) is the relative permittivity of \( n \)th layer.

Low frequency limit (based on the same reflection and transmission coefficient):

a) For perpendicular polarization:

\[
\mu_{\text{reff}} = \sum_{n=1}^{N} \frac{\mu_{r_n} h_n}{h_t}
\]
\[
\varepsilon_{\text{reff}} = \sum_{n=1}^{N} \frac{\varepsilon_{r_n} h_n}{h_t} - \sin^2 \theta_i \left( \sum_{n=1}^{N} \frac{h_n}{h_t \mu_{r_n}} - \frac{1}{\mu_{\text{reff}}} \right)
\]

b) For parallel polarization:

\[
\varepsilon_{\text{reff}} = \sum_{n=1}^{N} \frac{\varepsilon_{r_n} h_n}{h_t}
\]
\[
\mu_{\text{reff}} = \sum_{n=1}^{N} \frac{\mu_{r_n} h_n}{h_t} - \sin^2 \theta_i \left( \sum_{n=1}^{N} \frac{h_n}{h_t \varepsilon_{r_n}} - \frac{1}{\varepsilon_{\text{reff}}} \right)
\]

where \( \theta_i \) is the incident angle, \( h_n \) is the thickness of \( n \)th layer and \( h_t \) is the total thickness of all dielectric layers. \( \varepsilon_{r_n} \) is the relative permittivity of \( n \)th layer.
Figure A.8  EPE results window for periodic dielectric case, panel 1. Simulation structure: 4 layer of periodic dielectric sphere in the air. The total thickness of structure is 40 µm. Dielectric constant of sphere is 40. Simulation frequency is 10 GHz to 100 GHz.

MG formula for PEC:

\[
\varepsilon_{r,eff} = \varepsilon_{r,m} \frac{1 + 2p}{1 - p}, \\
\mu_{r,eff} = \mu_{r,m} \frac{1 - p}{1 + p/2},
\]

where \( p \) is the filling rate, \( \varepsilon_{r,m} \) and \( \mu_{r,m} \) are the relative permittivity and permeability of dielectric material.

MG formula for dielectric objects:

\[
\varepsilon_{r,eff} = \varepsilon_{r,m} \frac{\varepsilon_{r,i}(1 + 2p) - 2(p - 1)\varepsilon_{r,m}}{\varepsilon_{r,i}(1 - p) + \varepsilon_{r,m}(2 + p)}, \\
\mu_{r,eff} = \mu_{r,m} \frac{\mu_{r,i}(1 + 2p) - 2(p - 1)\mu_{r,m}}{\mu_{r,i}(1 - p) + \mu_{r,m}(2 + p)},
\]

where \( p \) is the filling rate, \( \varepsilon_{r,m} \) and \( \mu_{r,m} \) are the relative permittivity and permeability of dielectric material while \( \varepsilon_{r,i} \) and \( \mu_{r,i} \) are the relative permittivity and permeability of inclusion material.
Modified MG formula for dielectric objects:

\[
\varepsilon_{\text{reff}} = \frac{\varepsilon_{r,i}(1 + 2p)F - 2(p - 1)\varepsilon_{r,m}}{\varepsilon_{r,i}(1 - p)F + \varepsilon_{r,m}(2 + p)}
\]

\[
\mu_{\text{reff}} = \frac{\mu_{r,i}(1 + 2p)F - 2(p - 1)\mu_{r,m}}{\mu_{r,i}(1 - p)F + \mu_{r,m}(2 + p)}
\]

where:

\[
F(\theta) = \frac{2(\sin \theta - \theta \cos \theta)}{(\theta^2 - 1) \sin \theta + \theta \cos \theta}
\]

\[
\theta = k_0 a \sqrt{\varepsilon_{r,i} \mu_{r,i}}
\]

\(p\) is the filling rate, \(\varepsilon_{r,m}\) and \(\mu_{r,m}\) are the relative permittivity and permeability of dielectric material while \(\varepsilon_{r,i}\) and \(\mu_{r,i}\) are the relative permittivity and permeability of inclusion material.
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