A portfolio explanation of the behavior of stock prices

Michael Kenneth Madden
Iowa State University

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A portfolio explanation of the behavior
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by

Michael Kenneth Madden

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INTRODUCTION

The New York Stock Exchange has both a long and transitional history relative to the other components of the United States complex of financial institutions. Its history is long from the standpoint that it is the one institution that has been in existence in one form or another since before the revolution. It has been transitional in view of the changes in its organization, number of participants and variety of issues included in its listings. The result of this transition of the Stock Market is that its present characteristics include those which enable it to be subject to the economic analysis of a market. This is possible because of the vast number of people now engaged in present day stock market purchases and sales rather than the imperfect market resulting from trading undertaken by an exclusive handful of brokers of earlier periods. Also, as will be discussed later, the importance of the New York Stock Exchange relative to the total American financial spectrum suggests that this market may well be analyzed in a macroeconomic setting. Basically, the purpose of this study is to examine the economic factors that determine the temporal behavior of the New York Stock Market. The following brief history of the New York Stock Exchange serves to demonstrate how the stock market has evolved into a market in the economic sense of the term and consequently is suited for economic analysis.
Period of Formation: 1780-1820

Before stock trading could be a business of economic importance there had to come into existence several issues of stocks in amounts sufficient to attract large investors. The first issues to become available were shares arising from bank capitalization. The Bank of North America was capitalized at $400,000 and increased to $2,000,000 three years later. In 1784 the Bank of New York was established and in 1791 the First Bank of the United States was chartered with $10,000,000 of capital raised by the sale of shares. After 1792 the development of banking progressed rapidly and by the turn of the century total bank capitalization exceeded $33,000,000. By the end of the formative period about one-third of the issues traded in New York were those of banks. Insurance companies constituted the other major private commercial enterprises during this time. By the latter part of the formative period there were 13 chartered insurance companies along with 10 banks listed on the New York Stock Exchange.

The other major types of security issued during this period were obligations of the United States government. Acts passed by Congress in 1790 called for refunding the national debt and provisions for the assumption by the federal government of all state debts. The total amount of bonds issued by the federal government, including both the
national and the assumed state debts, amounted to $77,500,000. Despite the large dollar amount of U.S. bonds listed on the exchange they were seldom traded on a day-to-day basis.

Although capital formation for industrial purposes was progressing vigorously throughout the nation, the financing was not facilitated through the stock market. Each merchant and farmer considered himself a capitalist and would rather put funds into his own business than to invest in companies he could not control. In fact the only industrial stock appearing in the period was that of the New York Manufacturing Company in 1715. Shortly after it left the list and was no longer traded.

The first step toward organization of the activities of stock market participants came during this period in 1792. Twenty-four brokers, who made it a practice to give their services on a commission basis, pledged themselves "not to buy or sell from this day for any person whatsoever, any kind of public stock at a rate less than one quarter percent commission on the specie value and that we will give preference for each other in our negotiation." (53, p. 43)

This brief summary of the formative years of the exchange suggests that it was not a market of sufficient scope and national influence to be studied with reference to macroeconomic factors. The reasons for this can be listed as follows:
1) Capital formation and therefore output capacity was unrelated to events occurring in the stock market except to the extent that bank capitalization would create loanable funds for capital purchases.

2) The three categories of securities (banks, insurance companies and government bonds) did not constitute a representative picture of the United States financial structure in general.

3) In addition to the limited number of issues of stock, the purchasers were also small in number which caused the market to be imperfect from both the supply and demand sides.

4) Competition from exchanges in Boston and Philadelphia limited the significance of the New York Exchange. Thus, in this period New York was not considered the country's financial center as it was in later years.

Accordingly, the New York Exchange was quite unimportant during this span of United States economic history.

Middle Period—1720-1934

This rather long span of time can be identified by the prevalence of the following two properties: first, the behavior of the market as a whole was dictated by individual technological developments occurring through time; second, the market was often vulnerable to the speculative activities of a relatively few investors. Consequently, the behavior of
the market was not related to economic development in general, but often through the unpredictable behavior of stock market participants.

The canal building industry flourished during the beginning of this period due to technology brought over from England. As a result, canal companies dominated trade on the exchange floor and they continued to dominate until the early eighteen thirties. At the height of popularity of canal shares a challenge to that form of transportation presented itself. This was the beginning of the railroads. From the mid-eighteen thirties and for the following two decades, railroad issues took the spotlight. The stock market was now dominated by the rail stocks in much the same manner that canals had previously. As rail stock rose steadily other stock prices rose along with them in response making it apparent that the movement of the market was due to the latest technological developments. So dominant were the railroads by 1857 that the dollar value of rail shares made up well over 60% of all outstanding issues on the exchange. As interest in railroad securities leveled off a bull market developed among the mining issues. By 1855, mines had begun paying large dividends at increasing amounts. A rush to enter the mining boom soon had bullish effects in rails and other indirectly related stocks. Railroads and mine shares continued to dominate interest for the next half-century on the Exchange
as the nation settled westward. Other securities during this period appeared as soon as technology allowed. The industries that made great technological strides as a result of the demand for war-related goods were rapidly shifting into consumer goods. The securities of these industries, along with those of the westward expanding railroads offered the public a medium for investment and formed the basis of a more stable period for the stock market. A further result of this expansion was that as new types of industries listed issues on the exchange, it diminished the impact on the market by the short run changes in an individual industry. The effect of individual industries on the stock market was not eliminated however. Oil companies, automobile, tobacco, food processing and other industries, reflecting either technological developments or increases in the demand for luxuries, each exerted an effect on the market. Notably different was the lesser degree and length of the effect that they exerted in the early twentieth century compared to the nineteenth. By the end of this period (1930) over 1,000 issues were listed on the Exchange representing every industry. The result was that the market as a whole could not be controlled by the market of one company or even a single industry. Also, during this period, individual financiers were able to produce significant effects on the market much like the events within individual industries could. During this period, the market
rose and fell with the whims of leading speculators. First was the rise and subsequent fall of gold prices engineered by Jay Gould and James Fisk. Three years later, in 1872, speculators like Daniel Drew, Commodore Vanderbilt, Gould and Fisk, struggled through manipulation to control the leading railroad companies. Courts were bribed, legislators were bought, stock was watered, and trickery of the most subtle kind was employed. Consequently, market movements were largely the result of these activities and not due to economic conditions. These men and others acting in the latter nineteenth century, lived for profits and had no interest in running firms or knowing how they were run. As a result, most of them knew how the market operated through manipulation and speculation but few if any had any idea of its economic underpinning. Other market leaders that followed, such as J. P. Morgan, took an active interest in the operation of firms and consequently based the value of stock on the internal operations of a firm. Rather than manipulate individual stocks, he would, through his control of directorships, make companies more solvent by refinancing high interest debts, improving organization and insuring investor confidence. The result was an increase in the prices of stocks, to his advantage.

Men, like companies, declined in individual importance during this period and by 1930 the market was relatively
safe from manipulative activities of single individuals. There were of course powerful figures associated with the Exchange, but the mass of small investors gradually gained an important veto power in the market. The result was that the dominant stock market figure by the early nineteen hundreds was the investing public.

The primary developments bringing this about all centered around communication advances. The postwar period saw the first significant beginnings of financial journalism. Fostered by the growing interest in securities, it led more investors and speculators to Wall Street. Such publications as Barron's, Forbes, The American Railroad Journal, The Wall Street Journal, and others became widely available to the general public. Before these financial publications could reach a significant level of importance, companies had to make available sufficient data to make meaningful analysis possible. The Interstate Commerce Act of 1887 brought this about by requiring corporations to submit regular reports concerning their financial condition. (50, p. 29)

Also, during this time the telegraph was gaining wide use, which enabled people from all parts of the nation to obtain and transmit market information rapidly. Later, ticker tape and telephones further improved the process of distributing information. By the early eighteen nineties, information from the floor of the Stock Exchange was almost
instantaneous. Corners were nearly impossible to engineer since communication facilities made secrecy difficult.

To summarize, the development of the New York Stock Exchange during this period was characterized first by the increase through time of the number of companies listing equities through time and second, by the increased number of investors with access to stock trading brought about through the growth of information facilities. Both of these characteristics were necessary for the Stock Exchange to evolve into an economic market.

Modern Period

The passage of the Securities and Exchange Act in 1934 brought about a reversal of the dominant philosophy regarding the purpose of its existence. Prior to the passage of this act, its basic purpose was to furnish an open free market for its members. Providing a readily available market place for equities to be bought and sold by society as a whole was only secondary in purpose. The Securities and Exchange Act reflects a different view of the matter. Because of the shock effect of the great depression, the large growth of security listings, the high number of people investing funds in the market and its close interrelationship to the national financial structure, it was evident that stock exchange transactions are effected with a national public interest. In view of these points, the
law declares that "the prices determined on the exchanges are susceptible to manipulation and control and that their dissemination gives rise to excessive speculation and unreasonable fluctuations in prices. This causes unreasonable contraction and expansion of credit, obstructs the effective operation of the banking (monetary) system, intensifies and prolongs national emergencies, and produces widespread unemployment and dislocation of industry." (53, p. 243)

The new supervision of the New York Stock Exchange made stock registration and financial reports mandatory. The result was that investors could now evaluate stocks more accurately, and consequently remove much of the risk that was so commonly associated with stocks previously. Thus, this juncture in time provides a convenient place to begin a temporal analysis of stock market behavior in its present day institutional setting.

A review of the theoretical and empirical research concerning New York Stock Market pricing patterns will be undertaken in the following two chapters.
THE THEORY OF RANDOM WALKS

The Theory of Random Walks was first formulated by Bachlier in 1900 (5, pp. 17-78). The theory remained almost dormant until 1959 when Osborne gave it a more precise form. Since then it has been restated by many authors and numerous empirical tests as to its relevance have been undertaken in many fields of study.

Two assumptions are basic to the Random Walk theory of movements in stock prices. They are:

1) New pieces of information upon which purchasers base their estimates of actual value arise independently (randomly) through time.

2) The evaluation of this new information would be done independently. That is, the evaluation of one analyst would not influence the evaluation of another.

The plausibility of these assumptions will now be examined. In regard to the first it may be said that information new to the speculators can arrive only in a random, unpredictable way since insofar as information is predictable it is really not new. The second assumption is more obscure. Independence does not imply that each participant acts differently. Rather, new information may bring about an action which is common to many participants. The assumption requires only that the action taken by one participant was not due solely to the influence of others. If we further assume that
purchasers are rational and maximize profits they will use all of the market information available to them the best way they know how. For example, if trends are known to exist in the fundamental market forces that determine price, individual stock market participants will use the knowledge of trends to increase the accuracy of the predictions of future prices and consequently their behavior in the market. Thus, as information is received, price expectations will change and the purchasers will act to bid prices up or down accordingly. Since genuinely new information arrives randomly and independently of past information (and past prices), price changes are random and independent through time. In the language of economic analysis, the price effects of any event on the New York Stock Market are immediately "discounted" when new information vetoing the likelihood of that future event becomes available. Thus, prices and therefore price indices could be expected to follow a random walk.

Fama, in a recent article, points out that the Bachelier and Osborne assumptions (numbers 1 and 2 above) are quite restrictive (5, pp. 37, 38). With regard to the first assumption, it is not likely that new information will arise independently through time. That is to say, good news may tend to be followed by additional good news rather than by bad news in the sequential sense, if not the causal sense. Referring to the second assumption, it is not likely that the evaluations
of all participants are independent of one another. For example, certain individuals or institutions, as leaders, may exert some influence over the opinions of other investors.

Nevertheless, Fama contends, the random walk model could be valid even if the Bachelier-Osborne assumptions are not accepted. In describing why this is so the following three definitions are helpful: (i) Intrinsic value refers to the value of a security or group of securities by analysis based on economic and political factors affecting companies. (ii) Superior intrinsic value analysts are those analysts who are markedly better at predicting the appearance of new information and estimating its effects on intrinsic values than others. (iii) Superior chart readers are individuals who are much better at doing statistical analysis of price behavior than others. Theoretically, both types of participants are serving the same function as will be evident below.

With this foundation it can be seen why the absence of these Bachelier-Osborne (B-O) assumptions do not preclude the existence of random walk patterns in stock market prices.

With regard to dependencies in the information generation process, the sophisticated analysts would learn to predict future information from the pattern of present and past information and would apply their knowledge of all known or predictable information in their intrinsic value determinations. The effect of the intrinsic value analysts acting in
this manner would be statistical independence of successive stock price changes.

With regard to possible dependencies in the information evaluation process (the suggestion that some analysts will influence the opinion and therefore actions of others), Fama maintains that, provided there are sufficiently numerous sophisticated analysts involved in accurately predicting intrinsic values of stocks, there would be a tendency of an instantaneous adjustment of intrinsic values to market price. This instantaneous process would neutralize any dependence in prices through time which one may expect to exist if information is dependent. In other words, if there are many sophisticated traders in the market, they would eventually learn that it is profitable to interpret and utilize both the price effects of current new information and of the future information implied by the dependence in the information generation process. Thus, it is concluded that the actions of traders tend to make price movements independent (5, p. 39).

However, this theoretical analysis of random walks seems not to stop here, since the elimination of the two B-0 assumptions was done at the cost of introducing two more. The argument above required 1) there must be a large number of sophisticated intrinsic value analysts, and 2) the action of these analysts must be assumed to result in instantaneous adjustments of actual market prices to intrinsic values. The
first condition seems plausible; however, it is much more doubtful that these analysts possess the simultaneity and rapidity to bring about instantaneous adjustment. More reasonable is the suggestion that as a larger and larger number of people become more and more certain of the new information, their actions might cause a gradual movement of market prices toward intrinsic values. The gradualness of price movements would be functionally related to the gradualness of the arrival and validity of the new information. Admission of this phenomenon would admit successive price changes that are dependent and not of a random walk. This outcome again can be changed by adding to the foregoing analysis the assumption that the superior analysis anticipate how the lags in information mentioned above will effect the market as a whole. This assumption suggests that the series of price changes may still follow a random walk (6, p. 97). This then, is the crux of the random walk question of stock market behavior.

At this point the random walk theory is subject only to the truth or lack thereof of the assumption of instantaneous response. In the parlance of the economist, one who believes the market is virtually instantaneous believes the stock market is "perfect" or "efficient." One who believes it is not, contends that imperfections (monopolistic elements) exist in the market because of unequal financial resources or uncertain information. As to which is correct, the answer can
only be sought through empirical investigations. The primary work to date will be summarized somewhat later.

**Distributional Properties**

Another hypothesis, in addition to independence, concerning the theory of random walks of stock prices is that these price changes conform to some probability distribution. In the most general sense of the theory the particular distribution need not be specified. The correct distribution should be that one which correctly characterizes the successive price changes. Being able to do this is extremely helpful to analysts and investors. The general shape of the distribution can reveal the degree of risk. For example, measures of variance, skewness and kurtosis will allow a comparison of risk for two price series with identical means. From the academic standpoint, the properties of distributions provides insight into the stability of the process generating price changes through time.

The theoretical treatment of the distributional properties was first developed by Bachelier (5, pp. 17-78) and by Osborne (46, pp. 145-173) who derived the properties independently fifty years later.

The B-O assumptions are: 1) price changes from transaction to transaction for individual securities are independently distributed random variables; 2) transactions
are spread uniformly through time and 3) the number of transactions per unit time are large so that price changes are sums of many independent random variables. If these assumptions are met the central limit theorem is applicable and it would therefore be expected that stock prices through time would have a normal distribution. Moreover, if all stocks had normal distributions then linear combinations (price indices) of stock prices would have normal distributions. This can be shown as follows: Suppose we have $n$ stocks with mean and variance of price changes equal to $\bar{\rho}_i$ and $\sigma_i^2$ ($i = 1,n$). Suppose we take a weighted sum of $m$($m \leq n$) stocks to form a price index. Then,

$$\bar{\rho}_s = \sum_{j=1}^{m} \lambda_j \bar{\rho}_j \quad \text{and} \quad \sigma_s^2 = \sum_{j=1}^{m} \lambda_j^2 \sigma_j^2$$

are the mean and variance of the index respectively. Furthermore, the index of price change will be normally distributed. Although the assumptions do not seem to be extreme, the results of empirical tests rarely have been towards verification of normality. Specifically, the empirical distributions of price changes of individual stocks have too high a frequency of small changes and too high a frequency of extremely large changes. In statistical parlance this deviation from normality is called leptokurtosis.

The discrepancy between the theoretical and empirical distributions can be shown graphically as follows:
The solid curve represents the shape of the unit normal distribution and the dashed line is a typical empirical distribution. According to Fama, whose research is most complete concerning this problem, the points of intersection of the two curves occur at ± .5 standard deviations from the mean and again at somewhere between ± 1.5 to ± 2.0 standard deviations. Using thirty selected stocks listed on the New York Stock Exchange, Fama found that on the average there is 8.4% too much relative frequency in the interval ± .5 standard deviations. In the two intervals beyond ± 2.0 standard deviations there is an almost constant relative frequency up to 5 standard deviations implying there are far too many observations in the tails of the distribution (14, pp. 48, 51).

This discrepancy was noticed long before by researchers, but the general feeling was that the evidence was strong enough to be able to assume approximate normality (30, pp.
This general feeling soon was disputed as more and more tests were made, each consistently reporting leptocurtic empirical distributions. Alexander in 1961 noted that Osborne, who claimed results that were normally distributed, did not subject his frequency distributions to rigorous test. The evidence was overwhelming that the distribution of price changes was not normal. Interestingly, most of the discrepancy between observed and expected frequencies was due to price changes greater than ± 10% (1, pp. 205-213).

Cootner (10), recognizing the problem, developed a theory attempting to provide an explanation for its existence. Cootner begins by dividing investors into either a professional group or a non-professional group. The latter group consists of those investors who do not devote any of their time to stock market research. Thus, these investors take the current price of a stock as the best estimate of its present and future value. Next, he defines the professional investors as those who specialize in the market and are actively engaged in estimating future prices. Consequently, they do have an idea of what the future price will be, but they cannot profit from this knowledge unless the present price deviates enough from the expected price to cover transaction costs. Their profits are the result of observing the random walk of prices until it wanders sufficiently far from
the expected price. Professional investors, by acting in this way, establish barriers, outside of which they will enter the market in such a manner to drive the price back to its expected value. This tendency would cause a larger amount of price change close to the expected change than with the normal distribution, thus bringing about elpto-kurtosis. The interaction between these two disparate groups of investors (professions and non-professionals) modifies the strict randomness that the B-O model suggests (10, pp. 233-239). Using an algorithm to test the Cootner hypothesis, Steiger found that the model is a satisfactory description of stock price behavior (55, pp. 253-261).

The classic approach to this problem was to assume that some other disturbing influence was responsible for generating the extreme values. Each of the extreme values of a time series was treated individually and thrown out of the data if a plausible explanation could be found to rationalize these exclusions. In his work dealing with this problem, Mandelbrot showed that if the extreme values are numerous, the exclusion of them weakens the significance of tests on the remaining data (15, pp. 420-429). Furthermore, since probability distributions are available which can represent all of the data they should be used rather than concentrating on approximations to normality. The distributions that Mandelbrot employed are of the stable Paretoian variety. All of the
mathematical properties of this distribution will not be discussed in this summary. However, one of the four parameters describing its shape is of great importance for stock price data and will be discussed. A parameter, labeled $\alpha$, measures the height of the tails of the distribution and can fall in the interval $0 < \alpha < 2$. If $\alpha = 2$, the Pareto distribution is identical to the normal distribution. The closer $\alpha$ is to 0, the higher are the tails of the distribution. In other words the higher the level of total probability in the extreme tails, the smaller the value of $\alpha$. Another important consequence is that the variance is defined only if $\alpha = 2$, meaning that if a stable Pareto distribution is empirically applicable, the variance is meaningless and statistical tests employing confidence intervals are impossible (14, pp. 43-45).

Rather extensive testing by Fama has shown the stable-Pareto distribution to be supported by the data. In a sample of thirty randomly chosen stocks, every stock contained more relative frequency than expected under conditions of normality, and more important, they all contained higher relative frequency in the tails. As a result, $\alpha$ was shown to be less than two with the average value estimated to be about 1.90 (14, pp. 45-51). If stock prices are more closely characterized by a stable Pareto rather than by a normal distribution, a number of economic implications arise. First, a stock following a normal distribution will have a smooth
trend through time whereas a stock following a Paretoian market will be marked by large disjointed moves through time. This factor suggests that there is reason to expect that there exists economic or other explanatory variables explaining the large moves (wide variance) in stock prices and stock indexes. In different terms, with the combination of independence and a stable Paretoian distribution with \( \alpha < 2 \), the implication would be that actual or intrinsic values often change by large amounts during short periods of time. This strongly suggests that other factors are relevant in evaluating a stock in addition to expected return, dividend rate, and such nebulous concepts as general business conditions. The main body of this study will address itself to the search for more basic economic variables to explain this phenomenon.

To quote Fama,

"In essence, there is as yet no general model of price formation in the stock market which explains price levels and distributions of price changes in terms of the behavior of more basic economic variables. Developing and testing such a model would contribute greatly toward establishing sound theoretical foundations in this area." (14, pp. 98-99)

The Question of Independence

Three major approaches have been used in determining the degree of dependence in successive stock price changes. Most commonly employed have been statistical tests of serial correlation. This, of course, is the statistical measurement
of closeness of the relationship between successive stock market price changes. If it were found that a series of price changes were highly correlated, it would be concluded that the random walk model of stock price behavior is not applicable and technicians would be wise to base their present behavior on past price movements. If, on the other hand, it were found that serial correlation did not exist, or at best was negligible, the random walk hypothesis would apply and market participants would find it no more profitable to look at past prices than to base their behavior on coin tosses. In other words, the fruits of technical analysis would be highly accidental.

Another statistical approach is known as runs testing. A "run" is defined as a sequence of price changes, all consisting of the same sign. Once a sequence of prices changes in sign, the run is said to be terminated. The unit length of a run is the number of consecutive price movements observed of a common sign. The average length of a run can then be compared to a mathematically-determined expected length. If the observed lengths of runs were greater (statistically) than the expected length, the random walk model would be refuted.

A third technique is the simulation of some predetermined technical investment strategy. The profits resulting from this behavior pattern are compared with the profits of a
buy and hold strategy or with a random selection of purchase and sale data. If the profits from simulation are in excess of those necessary to pay for transaction costs, the validity of the random walk hypothesis would be rejected. If simulated profits less transaction costs are no greater than profits from a buy and hold strategy, the random walk hypothesis would apply. A brief review of the research concerning independence will follow. Tests of serial correlation in stock prices have been undertaken since at least 1953. In a study by Kendall, twenty-two lagged serial correlations of first differences were correlated. Most of these were British industrial stock prices on a weekly basis (30, pp. 12-23). What Kendall found was that, contrary to opinion at that time, knowledge of past prices yields virtually no information about future price changes. More specifically, he found that each period's price change was not significantly correlated with earlier price changes strongly suggesting the validity of the random walk hypothesis. There was one exception noted to this conclusion, however. The series on cotton did have a significant amount of autocorrelation. The reason for this was not because the cotton market was fundamentally different from the others, but rather, that data was of a different form. Unlike the other series, this data (monthly) was derived by computing the mean of weekly prices instead of simply taking the monthly closing
prices as was done for the other series. It was subsequently pointed out in the literature that data arrived at in this manner is expected to have a serial correlation of approximately the magnitude observed. Had this series been tested with data of the same type as the other series it also would have failed to produce price dependencies (11, pp. 910-911).

The most complete statistical analysis of stock index dependence was undertaken by Alexander in 1961 (1, pp. 7, 21). Because his data conforms most closely to the basic data of this thesis, his contribution will be dealt with in some detail. A runs test was used on Standard and Poors monthly composite index for the years 1929 to 1959, which is approximately the time period covered in this thesis. It was not appropriate to compare the number of positive or negative runs with the runs derived from a computer generated random walk model of equal probability of rise or fall, due to the fact that stock prices have increased regularly during the time period covered. Consequently, positive changes would be expected more frequently than negative changes. Specifically Alexander noted:

a) The relative frequencies of rising and declining months were .58 and .42 respectively.

b) The relative frequency of rising months among all months for which the preceding month was rising, \( p(+) | + \), was .67, and the relative frequency of declining months for which the preceding month was declining, \( p(-) | - \), was .50.
Computing the number of expected runs with a random walk model, but contingent on the conditional probabilities, \( p(+) = 0.67 \) and \( p(-) = 0.50 \) gives results very strongly in support of serial independence. These results are based on end of the month data and not other means of weekly data. Utilization of the latter type data did give an indication of dependence, but it must be discounted for the reasons discussed above. Alexander concluded therefore, that month to month data is independent and therefore consistent with a random walk. His statistical results are summarized in the table below.

Table 1. Distribution of lengths of runs of monthly Standard and Poor's industrial stock prices. Observed (Obs) versus Expected (Ex)

<table>
<thead>
<tr>
<th>Length of run (months)</th>
<th>February 1927 to December 1959</th>
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<tbody>
<tr>
<td></td>
<td>Up (positive)</td>
</tr>
<tr>
<td></td>
<td>Obs</td>
</tr>
<tr>
<td>1</td>
<td>38</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6 (or longer)</td>
<td>8</td>
</tr>
<tr>
<td>Total</td>
<td>87</td>
</tr>
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</table>

\(^{a}\)Expected totals differ from sum due to rounding errors.
Another method of detecting independence is to compare the active value of the ratio of sequence to reversal of signs. If the series was a random walk with no secular trend in either direction the ratio would be expected to equal 1. However, with the trend that we notice in stock prices, the ratio of sequences to reversals is expected to be 1.04 due to the conditional probability $p(+|+) = 0.67$. The observed value of this ratio is 1.045 which is very close to what is expected under serial independence. Moore, in a later study, ran serial correlation tests on weekly closing averages also using the Standard and Poor's stock index. Reasoning that relative changes are more applicable than absolute changes, natural logs were computed for the 873 Friday closing averages. The changes in logs were computed and a serial correlation test was made yielding a coefficient of .034. The magnitude of the coefficient suggests some positive dependence, but "it appears to be slight" (43, pp. 157-159). A runs test further supported independence of weekly averages. It should be made clear, as Moore does, that a runs test is non-parametric and therefore not as sensitive as serial correlation tests.

Fama, in a later study, tests for serial correlation in daily price movements of thirty stocks listed on the New York Stock Exchange (14, pp. 48-86). Coefficients for daily changes in price were computed with a lag of 1 to 30
days. None of these randomly selected stocks showed evidence of dependence that would be important for investment purposes. Using longer differential intervals of 4, 9 and 16 days again produced very small serial correlation coefficients. Fama admits that serial correlation tests are not completely applicable since in stable Paretian distributions with $\alpha < 2$ the denominator of the correlation coefficient,

$$\tau = \frac{\text{covariance} \left( P_t, P_{t-\tau} \right)}{\text{variance} \left( P_t \right)}$$

is not theoretically defined.

For this reason he points out that a runs test may be more applicable since it does not require a finite variance. A runs test was conducted on the thirty stocks for 1, 4, 9 and 16 day intervals and the actual number of runs, relative to the expected number of runs, decreased with increases in the differencing interval. In no case did the two differ from each other enough to suggest non-randomness.

The foregoing statistical studies suggest that there might be some difficulty in deciding whether or not a price series is dependent or not. Empirically, we could not imagine a series that is completely independent. Thus, in the strict sense the random walk model cannot be an accurate description of the real world. Recognition of this leads one to attempt by some rational means to specify some minimum level of dependence. Fortunately, the stock market trader
has a practical criterion for judging where to "draw the line" for statistical dependence. The random walk model is appropriate so long as the discrepancy from the model does not present an opportunity for profits to be made. More specifically, the independence assumption is an adequate description of reality as long as the actual degree of dependence in the series of price changes is not sufficient to allow the past history of the series to be used to predict the future in a way which makes expected profits greater than would occur under a naive buy and hold policy. It follows that the minimum acceptable level of dependence would be directly related to the commission costs of buying and selling stocks, statistical research expense and the like.

Some theoretical reasoning shows that there is a definite relationship that must exist between the degree of dependence and the costs of making transactions. Suppose that the degree of dependence was high enough so that market analysts could make extra profits by acting on the knowledge of past prices. Market participants would then behave in the market in such a way to assume those profits away and in so doing would eliminate price dependencies by an amount just sufficient to render speculative activity unprofitable. This reasoning provides a basis for determining whether a price series is essentially dependent or independent.

By utilizing simulation techniques to test mechanical
trading rules that are based on systematic price patterns, we can implicitly measure the degree of dependence. If stock price changes are independent there is no stock trading rule that would show a profit. Tests for the possibility of independence in this manner were first made by Alexander in 1961. The mechanical rule which he tested has come to be called the "filter technique" (1, pp. 214-218). This technique is based on the assumption that there are trends in the market, but that they unfortunately are concealed by unexplained fluctuations in the market. To avoid encountering an undue number of these gyrations in the market, a filter could be used to rationally disregard market movements smaller than a specified size and only address oneself to those remaining. Thus, a 5% filter may be one with a rule: If the market moves up 5%, buy and hold on to the stock until it fails at least 5% where you would sell and again not buy until the price moves up by 5%. The idea behind a rule such as this is that an upward move presumably signals more upward moves and likewise for downward moves. Of course, the more stringent the filter the fewer mistakes that are made, but at a cost of not utilizing some signals that just narrowly missed the filter. If a price series was a random walk a filter would yield zero profits or more precisely, vary from zero profits positively or negatively in a random manner. Alexander's results show that a filter rule such
as the one described above will yield profits consistently above a buy and hold policy. Furthermore, profits increase inversely with the size of the filter. The unfortunate problem associated with this is that smaller filter rule signals more transactions, and therefore larger commission expenses to reckon with. Alexander did not take into consideration these expenses in his initial study and concluded tentatively that some serial dependence did exist in the price index he used (1, pp. 217-218). His later study did involve transaction costs and drastically reduced the amount of gains with filters of all sizes (2, pp. 338-372).

Alexander’s results were further questioned by Fama and Blume in 1966 (16, pp. 236-241). Rather than using a stock index as Alexander did, they used the thirty stocks of the Dow-Jones Industrial Average. Applying twenty-four different filters to the data ranging from 1957 to 1962, they found the possibility of nominal profits. Including the costs of making transactions, however, turned most of the profits into losses. The primary reason why their results indicated even less dependence than Alexander’s revised results (2, pp. 338-372) was that they took into consideration the effects of dividends. They should be allowed for because when making a short sale, the borrower of a security typically reimburses the lender for any dividends paid while the debt is outstanding (16, pp. 229-233).

A basic shortcoming of the filter technique is that it
places too much importance on the most recent price in decision making. Cootner suggested a moving average to focus attention on market conditions of a number of the most recent prices (10, pp. 239-251). This technique is especially interesting since it is actually used by a number of investment companies. The rule used in conjunction with moving averages is as follows: If the current price is greater than the moving average, buy the stock. If it is less, sell the stock short. Often the moving average rule is coupled with a tactic to reduce the number of transactions. Thus, the rule could read: when the current price is 5% above the moving average, buy. When it is 5% below the moving average, sell. Conceivably, any percentage figure could be used although 2, 3 and 5% are most common. Another variation is to use a geometric average, the reason being that more recent prices should be stressed more strongly. Another variation is to advance the moving average one to three time periods. In other words, a buy or sell decision for the present period should be based on the moving average computed one to three periods earlier.

It should be pointed out that this technique is centered around the existence of trends. If a stock price moves above the average level it has achieved in the past, this is taken as an indication that an upward trend has started and the rule signals an investor to take advantage of this trend.
at its outset. Likewise, when the current price moves below its moving average, this is an indication of a downside trend and appropriate action is called for (27, pp. 319-320).

The first exploration of this technique was made by Cootner in 1964 (10, pp. 231-252). Using a 200 day moving average and comparing it with a buy and hold strategy for 455 stocks, he found the former far superior if commission costs are not included. When these costs are included, the moving average strategy is inferior. He then added the 5% bands to reduce the number of transactions and found that the gain after commissions is smaller still. Cootner consequently concluded that there were insufficient price trends to produce profits by using moving averages. Two recent studies by Seelenfreund, Parker and VanHorne (49, p. 282) and James (27, pp. 224-226) also failed to turn up any evidence of non-randomness with this technique.

Summary

The purpose of this chapter was to trace the development of the way analysts believe the stock market is structured. The two fundamental questions discussed were 1) What, if any, serial dependencies exist in stock price time series; 2) What distribution properties exist theoretically and empirically. In answering the first question, it can be concluded that there is no indication that a significant
amount of dependence exists and certainly no dependence exists of such a degree to make profits possible. In answer to the second question, Bachelier and Osborne developed the theoretical reasoning by which successive stock price movements should be normally distributed. Empirical work, however, consistently has shown that actual price movements are leptokurtic to the normal distribution. This property suggests that a stable Paretian distribution with $\alpha<2$ is the best description of reality. The combination of these two conclusions, independence and stable-Paretian distributions, provides the foundation for this study, which is to examine the economic disturbances that cause independent random variables to have distributions that are not normal. This will provide interesting insights into rational investment behavior for stock market participants.
ECONOMIC EXPLANATION OF STOCK PRICE DETERMINATION

A number of approaches to explaining changes in stock prices with economic models have been employed. The classical approach to the problem of evaluating a stock has been to use the standard present value approach. Presumably if each stock could be evaluated in this manner a linear combination of stock prices (an index) would reflect the present value of stocks taken as an aggregate. The basic consideration in determining the value of a stock with this approach is the size and date of occurrence of future net receipts, consisting of either dividends or the price of a stock upon liquidation. Since these are future cash flows they must be appropriately re-evaluated or discounted according to present day worth. The standard textbook formulation of this procedure is as follows:

\[ P_t = \sum_{i=1}^{L} \frac{D_i}{(1+r)^i} + \frac{P_L}{(1+r)^L} \]

\( P_t \) is the price in period \( t \), \( P_L \) is the price at liquidation time, \( r \) is the rate of discount achieved through the market process, \( D \) is the annual return or dividends in the case of stocks. This formula for evaluating an asset is almost exclusively reserved for riskless consols in economics textbooks and research journals. However, it has been used as an
approximation in some work concerning stocks (8, p. 212).

There are a large number of properties related to stocks and stock pricing which make the present value approach inappropriate. A discussion of these properties will follow, including references to the major studies connected with this approach.

In the above equation, $D$ refers to the annual return from owning a security. In the case of stocks, the annual return comes in the form of dividends and they are neither constant nor predictable for future periods. It is therefore not correct, even as an approximation, to treat dividends as they are in the above formulation. Furthermore, since companies that issue stock rarely return all of the stockholders returns in dividends and instead retain a portion as undistributed, it is an underestimate of stockholder earnings to discount only dividends. This problem could be resolved by replacing $D$ by the sum $D + E$ or dividends plus retained earnings. This sum would be simple to substitute except that the rate at which stockholders wish to discount retained earnings is not necessarily equal to their discount rate for dividends. This, of course, is true since people commonly value income they actually receive higher than accrued income. Thus, it could be hypothesized that equilibrium in a market for a stock could only be arrived at by discounting dividends at a rate, $r_D$, and retained
earnings by a rate, \( r_E \). If, in fact, dividends are preferred to retained earnings then \( r_D < r_E \). Therefore, on the average, the greater \( D \) is relative to \( D+E \), the higher the price of the stock. Carrying the reasoning one step further it could be inferred that the higher dividends are in relation to total earnings for all stocks in the aggregate, the more appealing stocks would be relative to other types of investments and therefore the higher stock price indexes would be. Although the latter proposition has not been pursued, there have been a large number of research findings concerning the preference of large dividend earnings ratios for individual firms.

Graham and Dodd were first to make the observation that stocks should be at least in part evaluated on the ratio of their dividends relative to earnings. In fact they assert that stock prices should bear a specified relation to earnings and dividends, but they neither present nor cite empirical work to support this generalization (24, p. 99).

The first academic study of this hypothesis was undertaken by Gordon in 1959 (24, pp. 99-105). The basic question he sought to answer was: Is there evidence that prices are related and therefore predictable by dividends and earnings? Specifically, are stock purchases effected by dividends, earnings, or a combination of the two? Cross section data for 1951 and 1954 were used to test these hypotheses. For
four industries, chemicals, steels, foods, and machine tools. He used a multiple regression model of the form:

\[ P = a_0 + a_1 D + a_2 Y \]

where \( P \) is stock price, \( D \) is dividends and \( Y \) is total earnings. His total findings were that \( a_1 > a_2 \), for all but one of the regressions which tells us that \( \frac{\partial P}{\partial D} > \frac{\partial P}{\partial Y} \). Thus, prices are influenced more by dividends than total earnings.

The statistical shortcoming of this analysis however, is that \( D \) is a large portion of \( Y \) so that multicollinearity is present. Further evidence of this is the large and diverse variances noted for \( a_1 \) and \( a_2 \). The multiple correlation coefficients, however, were quite high but this also is possibly a characteristic of multicollinearity (29, pp. 205-206).

To reduce the undesirable effect of interdependence between variables, the regression equation was modified to:

\[ P = a_0 + a_1 D + a_2 (Y-D) \]

The result was that multicollinearity was reduced while the relative size of coefficients \( a_1 \) and \( a_2 \) still indicated that dividends effected price more than retained earnings. The theoretical reinforcements of this conclusion are: 1) Since retained earnings ultimately result in capital gains by reducing debt-equity ratios, and because of the variability
of retained earnings, stockholders, due to a preference for predictable income, will respond by paying high prices for stocks with high dividends. 2) New investments purchased with retained earnings are felt to yield a low marginal profitability. 3) New investments are felt to be much more risky than projects presently employed. 4) As mentioned earlier, current income is preferred to future income (17, pp. 658-659). In a later study, Gordon again finds a buyer preference for dividends (25, pp. 37-51). The approach was slightly more complex in that natural logs were used in both price and the explanatory variables. Also, a method was incorporated to determine if dividend preference was effected by high as opposed to low price stocks. The hypothesis was affirmed, but more interesting was the finding that high priced shares sell at lower prices (relative to their dividends) than low priced shares. This suggests the possibility that purchasers feel that there is some "normal" level for dividends to take. Also it may be that along with this feeling, investors feel that low dividends are more likely to rise in the future than high dividends.

The other school of thought on this issue is that dividends per se have no effect on the capitalization rate. The theory was first put forth by Modigliani and Miller (39). They show mathematically that the dividends-earnings ratio is immaterial in determining the market value of stocks.
Furthermore, they argue that if a share owner is not satisfied with the amount of dividend payment and would rather receive a greater portion of his earnings at present because of the uncertainty of the stocks' future prospects, he is free to sell an amount of stock equal to the difference between the actual and his desired dividends. A sale such as this could be undertaken advantageously since the "high" retained earnings would augment the market price of the stock. This option, in effect, increases the present cash flow or dividend stream relative to the total amount of capital gains brought about by the price increase. Stated more succinctly, investors should be indifferent if the value of the additional future returns resulting from earnings retention equals the value of dividends foregone. Of course this is true only if the added rate of return is the same as the rate before the retained earnings are invested. It may be contended that brokerage fees would prevent, or at best discourage, a relatively small sale for this purpose. Miller and Modigliani answer this by suggesting that there may be just as many investors, if not more, who are instead accumulators, and requiring them to pay brokerage fee in order to reinvest their dividends causes them to dislike a high dividend payout.

Both schools of thought on this issue agree that present tax laws provide a differential between income earned through dividends and income earned through capital gains. To the
extent that stock ownership is to a large extent in the hands of the wealthy, this may be a very significant factor.

In reviewing the literature on this issue, one finds that the crux of the matter is that the "dividends count" supporters feel the four effects mentioned above together far outweigh the tax differential effect. By contrast, Modigliani and Miller feel that if in fact dividends do determine price, it can occur only in view of the tax differential effect, and this effect in theory could reverse the direction of the relationship between stock prices and dividends that is asserted by Gordon (24).

The proposition that dividends are not the crucial variants in determining stock price levels and movements does seem theoretically sound, but as is often the case, the empirical evidence does not confirm the theory. Miller and Modigliani do not have any published empirical work to complement their theoretical analysis.

Recent statistical studies, although coming up with results consistently in favor of the dividends hypothesis, also consistently suffer from a statistical bias (21, p. 660).

Virtually all studies involving this controversy have utilized cross-section data. Specifically, an industry is selected and the stock prices of various companies within the industry are correlated with their respective dividend payout ratios. Results are consistently in favor of the
"dividends do count" school. A possible alternative explanation to the assertion that dividends are preferred is the following: It is conceivable that the price effect of retained earnings is dominated by the effect of high rates of return in a company. This problem arises because we are likely to expect high rates of return to be closely associated with high retained earnings, and in turn, price is correlated with the rate of return.

If the relative effects on price due to dividends (D) and retained earnings (R) are estimated by the equation,

\[ P_t = a_1 + a_2 D_t + a_3 R_t + e_t \]

another statistical bias is quite possibly reflected in the weights \( a_1 \) and \( a_2 \). Implied is the assumption that risk is held constant or completely uncorrelated with the explanatory variables within the sample. With regard to this, a sample of companies within an industry (such as Foods, Steels or Chemicals) contains variation in size, financial structure and product mix. It, therefore, seems unlikely that risk variations are negligible. It is widely known that there is a managerial desire to avoid dividend cuts, so it seems logical that companies facing greater uncertainty about profit performance would adopt lower dividend payout, as a means of hedging the risk of being forced to cut their dividends. Thus, high risk may result in both low payment of
dividends and low price-earnings ratios, whereas low risk may result in high payout and high price-earnings ratios. Consequently, omitting a risk variable in the above regression equation could result in an upward bias in the dividend coefficient. The magnitude of this bias depends upon the extent to which risk varies between companies and on the strength of risk in determining payout (21, pp. 661-663).

A related problem is that of the regression weighting of the coefficients. It is well known in statistical analysis that observed values which differ widely from the mean play the largest part in determining the regression weights. Thus, extreme values are much more important in determining regression results than are those values centered more closely about the sample average.

Now, it is a generally accepted fact that high quality (low risk) stocks are characterized by low per-share values. Further, as discussed above, high quality stocks may tend to pay out a higher proportion of earnings than do low quality (high risk) issues. Thus, it well may be that issues which differ from the average in this sense may introduce a further bias in regression coefficients.

By taking account of these factors and others of lesser importance, Friend and Puckett found little or no explanatory or predictive ability in the variation in dividends and/or retained earnings (21, pp. 679-680).
The foregoing discussion, in effect, presents the problems surrounding stock valuation using dividend and earnings data. In essence there are statistical studies that indicate a relationship between dividends and stock prices that is more significant than the relation between retained earnings and prices but the most recent evidence indicates that there are other variables not included in the regression that produce results which are biased toward the dividend effect. Also, it should be pointed out that the high correlation in the above regression by no means makes a model capable of explaining aggregate stock index movements.

The Rate of Discount

An allied problem to the above consideration is the question of which rate of discount to use, if any, in evaluating future income. As noted above, it would probably be improper to discount dividends at the same rate as dividends in present value formula. Mathematically it may be more correct to replace the \( i \)th term by

\[
\frac{D}{(1+r_D)^i} + \frac{R}{(1+r_R)^i}
\]

Furthermore, following the Friend-Puckett analysis (21, pp. 656-682). Different levels of uncertainty are associated with
dividends than those uncertainty levels which are associated with retained earnings. Therefore, to evaluate a stock under conditions of risk, it may be proper to break the two rates of discount, \( r_D \) and \( r_R \), into \( r'_D = r'_D + r_{DU} \) and \( r_R = r'_R + r_{RU} \) where \( r_{DU} < r_{RU} \). The interpretations of the four new variables are as follows: \( r'_D \) is the rate of discount of future dividends assuming they arrive in a known manner. \( r_{DU} \) is the additional rate of discount due to uncertainty. \( r'_R \) and \( r_{RU} \) are the corresponding variables for retained earnings.

One more problem dealing with the pricing patterns of stocks must be treated before moving on. Adding to the Miller-Modigliani argument, Malkiel considers a situation where annual dividends, \( D \), are not constant, but grow at a constant rate, \( g \), through time (36, pp. 1004-1031). Thus, the appropriate dividend value to be discussed in the \((t+i)\)th time period is:

\[
D_t (1+g)^i
\]

and the numerator of terms unequal to \( i \) are correspondingly modified. Without going into the empirical details, it is easily noted that for given changes in \( r_D \), the changes in price based on present value will be greater the larger \( g \) is. Furthermore, in addition to discounting dividends back to their present value, we can also discount the price at any future date with the use of the following term:
where \( L \) is the length of the time horizon. The growth rate, \( g \), will be the appropriate rate, assuming investors are willing to capitalize earnings at the same rate as price. Thus, Malkiel proposes as an explanation of the observed fact that when stock prices as a whole fluctuate, the issues with highest growth rates fluctuate by the largest amount.

One conceptual problem that should be noted is that if the rate of growth is larger than the rate of discount, \( g > r \), the series does not have a finite sum and, theoretically speaking, there is no price a stock could assume that would not be an underestimate of the present discounted value. A number of proposals for avoiding this paradox are:

1) Since empirically all stock prices are finite we could assume \( r \) must be greater than \( g \).

2) We can accept the fact that \( g \) is greater than \( r \) (commonly interpreted as the marginal efficiency of capital) for a relatively short time horizon, but that \( r \) is a function of time where \( \frac{dr}{dt} > 0 \). This could be reasonably done by assuming that uncertainty increases as one looks further into the future (i.e., \( r_{DU} \) is increasing). If this is acceptable it only requires that \( r \) increases sufficiently within the time horizon of decisions to render the sum of the series finite.
3) It is also reasonable to assume that investors are aware of the fact that the growth of a company cannot continue at a constant rate indefinitely. In other words it is likely that \( g \) is not a parameter but a function of some explanatory variables in such a way that

\[
\frac{d^2g}{dt^2} < 0
\]

It would appear that both 2) and 3) above would apply under normal circumstances thereby preventing the embarrassment of the present value theory of failing to produce a determinate price.

Like the work of Gordon, Miller and Modigliani, Friend and Puckett and others, the Malkiel model is not intended to explain stock price movements as much as it serves to explain certain types of price discrepancies between stocks that have been classified with respect to risk, growth, dividends, earnings or some combination of these features and others. In particular, Malkiel concludes that growth stocks are intrinsically different from "standard" issues. When the level of share prices changes, the prices of growth stocks must fluctuate more than proportionately if the present value structure of shares is to hold. Thus, although this shows that growth stocks are inherently more volatile than standard issues, it does not provide a basic explanation for the behavior of stock price patterns in general.
The last feature of the present value formula to be examined in the context of share pricing is the length of the time horizon $L$. Now, it is clear that the present value of a stock would vary positively with $L$. Therefore, the arguments determining $L$ are valid subjects of consideration in stock price movements. The time horizon concept has taken on a number of meanings in the literature. Malkiel (36, pp. 1022-1024), believes that the proper interpretation of time horizon is the length of time that one can "reasonably expect growth and earnings predictions to be estimated."

Quoting Malkiel,

"I believe that ordinarily five years should be considered the absolute maximum for the investment horizon,---, I believe that growth rates of 20% or more should be viewed very suspiciously if projected more than three years."

Although these comments are quite a priori, he also employs a crude test of its validity. Using historical growth rates, current prices, and current price earnings ratios, he computed the number of years of growth at the historical rates that would be necessary to justify the present price of a stock. Examination of a number of firms with high growth records in this manner produced widely differing results as to what the investment horizons are for different stock issues. An implication of these findings is that the investment horizon has been a price determining factor from issue to issue. Also, when comparing these imputed horizons of successive
years for the same industry, it is found that they change erratically. Thus, while this consideration suggests the importance of time horizons in determining equity values, the difficulty is that there is no one reasonable horizon. Rather, it is a volatile characteristic produced by market sentiment by any time. Again, this observation serves not to explain stock prices per se, but to give added theoretical support to the tendency for price adjustments in growth issues to exceed adjustments in standard issues.

Taking a more general approach to the problem of time horizons, Johnson and Lambert envision the time horizon to be based primarily on when investors wish to receive the income from selling their stock (28, p. 16). Among the twenty per cent or so of the population who own shares of stock are a large number who use this equity for dis-saving during latter periods of their life cycle. Therefore, the time horizon of an investor would be the intended length of time he expected to hold the stock. This, of course, would vary tremendously from one investor to another and also from one type of stock to another. The time horizon chosen by an investor depends upon both the anticipation of gains through dividends as time passes and the expectation of capital gains through price appreciation, as well as by changes in his financial needs. There are perhaps other considerations determining the length according to Johnson and Lambert, but
at any rate there does seem to be some theoretical aggregative time horizon existing at a point in time for the market.

Next, they argue that due to changes in institutional habits of savers there has been a lengthening of this period of time. The argument runs as follows: Investors have responded to the increasing social and economic complexities of life by following more conservative savings habits. The result of this has been two-fold; first, individuals have been net sellers of common stock in recent years (28, p. 16), and second, the consequential increased use of financial intermediaries by the public has created a rapidly rising amount of investment funds from these sources. This has been followed by very heavy purchases of common stocks by these intermediaries.

In explaining the time horizon effect, assume that the amount of disinvestment of the private investors is exactly equal to the increase in investment of intermediaries even though in reality the latter probably exceeds the former. If the result was simply a shift of ownership and the decision criteria in purchases was the same for both groups we would expect no change to take place in the stock market. The one apparent effect, however, would be that returns of private investors would be reduced by the commission charges of the intermediaries. The crucial difference with respect to the two groups is the criteria upon which their time horizons
are based. As noted above the length of horizon for an individual is dictated by the need for income. The financial intermediary is in a position to look much further into the future due to its more or less perpetual existence. Furthermore, since contributions being paid in are more often than not in excess of outgoing payments to pensioners, and holders of life insurance and savings accounts, for many years in the future the effective time horizon for an intermediary may be indefinitely long. Hence, it is possible for them to discount anticipated increases in earnings, dividends and capital gains much further into the future (28, p. 19).

As indicated above, the relationship between the time horizon, L, and the present value of a stock is positive. Mathematically, the effect of L on $P_t$ using the present value formula above is:

$$\frac{\Delta P_t}{\Delta L} = \frac{D+P_s}{(1+r)^{L+1}}$$

This is the change in $P_t$ we would expect if we assume that $D$ and $P_s$ are constants, however $P_s$ and $D$ as discussed above are expected to increase with time. If this is recognized in the above relationship we would have to modify $D$ and $P_s$ by $D(1 + \frac{\Delta D}{\Delta L})$ and $P_s(1 + \frac{\Delta P_s}{\Delta L})$ respectively.

Although not stated in this manner, Johnson and Lambert use this observation to explain why the long run or secular
increase in stock prices mirrors the shift towards institutional ownership of stock shares.

The process of building more and more complexities into the present value theory of stock prices could be continued virtually without end. For example, dividends, earnings, the various discount rates and the length of the time horizon are all variables that could be functionally related to the business cycle in addition to or in place of the relations suggested above in past literature. However, the cost of adding complexities into economic relationships is high in terms of the benefits received in predicting or explaining dependencies. No better example of this fact is needed than the preceding exploration of the present value approach to stock pricing. It is clear that the intrinsic nature of stocks renders this classical approach non-operational even though it has great theoretical appeal. The uncertainty element is too overwhelming to seriously assert that investors use their knowledge of the future as the basis of stock finance behavior.

Alternative Approaches

For an economic model to be effective in producing empirically meaningful results the following characteristics are desirable: First, the model must be simple both in terms of the number of arguments included and their general con-
ceptual familiarity. Second, the model must be operational. That is, the variables must be quantifiable and their magnitudes must be available or derivable from statistical compilations. If these two characteristics are met statistical hypotheses can be tested and in some instances their applicability in the market can be evaluated. It is the lack of these characteristics that renders the present value model inapplicable in share price behavior.

Not until recently has the movement of share prices been studied with this approach in mind. The first work of particular interest in regard to this thesis is an article by George A. Christy published in 1964 (8, pp. 209-232).

Basically the study consists of the theoretical and empirical relevance of the following four propositions:

1) Investors' long-term expectations and motivations are based upon retrospective, rather than prospective economic data.

2) These expectations and the investment behavior they generate adjust to fundamental economic changes only with a pronounced lag.

3) The adjustments take the form of distinct "steps" rather than of a continuous process.

4) Between successive steps, investors demand renewed verifications of their assumptions concerning economic trends.

In this study it was found, as has been found before, that
current stock price changes are not explained by the current economic data relating to stocks. The economic data referred to are earnings per share, dividends, and movements of the price index. This conclusion was reached using the appropriate data from the years 1946 to 1962. The period was broken down into five cycles and these cycles are rank-correlated with respect to stock price movements versus changes in other variables. This in effect rules out a substantial portion of auto-correlation due to time. Regressing stock price changes or changes in each of the above variables produced insufficient correlation for explanatory or predictive purposes except for the first variable, the price earnings ratio. Although Christy fails to point it out, this correlation is hardly surprising. This can be shown by examining the model as he apparently conceptualized it. In equation form it would be:

\[ P_i = a_0 + a_1 \left( \frac{P_i}{E_i} \right) + E_i \]

where \( P \) refers to price and \( E \) to earnings. The \( i \) subscript would be the ranking of prices in descending order. A priori reasoning alone would make it obvious that the correlation between \( P_i \) and a quotient containing \( P_i \) in the numerator would indeed be high. This is further reinforced when we recognize that earnings, \( (E_i) \), show relative stability through time. (To the extent that \( E_i \) is nearly constant), the above model is
55
effortlessly correlating $P_i$ with $P_i$. The other correlation
which was unusually high was that between $P_i$ and yield which
is simply the same regression equation with the quotient re-
versed. Essentially the same results as above were obtained
by using one year leads and lags in the variables.

Rather than recognize the automatic high correlation
found between stock prices and price-earnings ratios, Christy
makes the following economic interpretation. In the light
of the present value theory, present earnings is only the
first term of an infinite series and is therefore not a
really important variable by itself in determining the value
of a share. The crucial factor to note, says Christy, is
that as progressively larger future earnings are foreseen in
such a series, the present worth (price) of the series divided
by its "first" term (i.e., price-earnings ratio) will rise.
The converse is also true. Thus, Christy concludes that
rising price earnings ratios signal rising future stock prices
(8, pp. 221-222). In this sense, fluctuations in the current
ratio of price to earnings can be said to represent changes in
investors expectations and therefore to changes in their
market behavior. No quantifiable model is put forth by Christy,
so this portion of the research was not tested statistically.
His approach was instead couched around specific market
changes in the last two decades and the concomitant economic
data (principally past price-earnings ratios) that could
account for them. The specific instances he discussed were so varied that a rigorous formulation or operational test of the hypothesis would be exceedingly complex.

A more workable and realistic study was put forth by Beryl W. Sprinkel in his book *Money and Stock Prices* (54). This work, which is within the framework of the quantity theory of money, is a historically based study of how changes in the money market influence stock market behavior.

He argues, as does Friedman in his work in monetary theory, that the nominal amount of money circulating in the economy is largely beyond the control of private spending units; the demand for money will vary with such factors as money income and interest rates. The higher the level of income, the larger the amount of money spending units desire to hold. Also, the lower the level of interest rates the greater the amount of money spending units would be willing to hold, since the opportunity cost of maintaining liquidity is reduced.

If at a given time the actual amount of liquidity is less than that which is desired, spending units in the economy will attempt to convert their non-liquid assets into liquid funds. Furthermore, by reducing consumption expenditures relative to incomes, individuals will attempt to build up their liquidity. But these actions will produce downward pressure on less liquid assets such as common stocks.

Conversely, if actual liquidity is more than desired
liquidity, spending units would be induced to exchange money for less liquid forms of assets. This of course, would tend to place upward pressure on the price of less liquid assets such as common stocks. In addition to this effect on stock prices, spending in other parts of the economy would be stimulated.

Thus, it would seem that there is both a direct and an indirect effect on stock prices that is produced by changes in the money supply. The direct effect is the process of altering portfolios to achieve an equilibrium which is disturbed by money supply changes. The indirect effect is the change in stock prices due to changes in general economic activity. Since these two effects tend to complement each other in the event of some change in monetary policy, it appears on the surface that a fundamental relation between money and stock prices exists.

Sprinkel's empirical work notes that both monetary change and changes in stock prices lead the general business cycle turning points. But since monetary changes have a longer lead over business cycle turning points than do stock price changes, it follows that monetary change leads changes in stock prices.

Using National Bureau of Economic Research results, Sprinkel noted that the average lead of changes in monetary growth prior to the business cycle peak is about 19 months
compared to a four month average lead of stock price changes. On the other hand, changes in monetary growth lead cyclical upturns by an average period of about seven months, whereas stock price upturns occur about five months prior to business upturns on the average. It follows that changes in monetary growth lead changes in stock prices by an average of about 15 months prior to a bear market and by about two months prior to bull markets. There are two severe weaknesses of this finding that lessen its value in application. First, these lags are average values and the lags from one time period to another are extremely variable. That is, even though the lags in stock price changes average 15 and 2 months behind monetary peaks and troughs respectively, very few actual lags of this length are found. Second, there have been numerous times in the past when the stock market has completely failed to turn in response to monetary change.

Sprinkel then constructs a simulation model of stock purchases and sales in view of the observed average lags noted above. Specifically, his model signaled a stock sale 15 months after a peak in the rate of change in the money supply to avoid the predicted bear. Correspondingly, the model signaled a stock purchase 2 months after a low turning point of monetary change to get in on a bull market. The results in this retrospective application are quite impressive. However, the use of lag models of this type is highly
inoperable if it is not known for certain when the peaks and troughs really occur. This shortcoming isn't quite so severe in the case of sale signals because an investor always has 15 months of money supply data before him before the rule calls for a sale or non-sale. However, since monetary troughs succeed stock price troughs on the average of only 2 months, it is not reasonable to presume that investors can recognize a monetary trough with this short notice. This suggests that bull markets may often be missed, or entered only with a high level of uncertainty. The results using hindsight are therefore, probably better than results which will result from foresight.

Although the author believes that this model has merit in terms of general approach, he believes it should be substantially modified and improved in view of the following two considerations: First, the existence of long and highly variable lags presents an operational problem that is not easy to ignore or overcome. Second, since Sprinkel basically uses a portfolio approach, other explanatory variables in addition to money may very well be important and therefore worthy of analysis. In particular, since there is a wide range of assets that lie between money and common stocks with respect to degree of liquidity it may be that conditions in other markets more closely related to the stock market in terms of marketability, liquidity, etc., than the money
market will produce more reliable and meaningful results.
It is upon this assertion that the following study is based.
THEORETICAL AND EMPIRICAL ANALYSIS

The general approach, then, suggested by the preceding analysis is that of a portfolio of financial assets where stock equities play a significant part of the financial aggregate. The selection of a particular portfolio is based on expected earnings subject to risk. This follows the approach devised by Markowitz (38, p. 102-115). His application of the central limit theorem demonstrates that an individual will reduce the risk of his asset holdings by diversification. His approach to portfolio selection is basically the following: From the standpoint of a rational investor a particular portfolio differs from another according to two factors: 1) Its expected average yield which is computed by taking a weighted sum of the expected earnings of individual financial assets within the portfolio. 2) Expected risk which is computed by taking a weighted sum of the variances of past asset yields within the portfolio.

For any given level of expected average yield there can be associated with it many different levels of expected risk depending on the mix of financial assets held. A portfolio, however, is efficient at a given expected yield only when the expected risk is minimized. In other words, at any level of expected yield there is a unique mix that will afford this yield at minimum risk. Conceptually, the proper mix can be solved by complex statistical techniques;
however, all that is theoretically required is that rational investors sort out efficient portfolios with this minimiza-
tion in mind. If an investor considers a higher expected yield he must realize that this must be done at the expense of incurring a higher level of minimum expected risk. Therefore, there is a positive locus between minimum risk and expected portfolio yield facing an investor at any given time. A rational investor will choose a particular point on this locus depending on his personal attitudes concerning risk and yield.

If the above analysis describes the behavior of individuals it should also describe the behavior in the aggregate. This suggests that at any point in time the portfolio of an economy has associated with it a unique combination of earnings and risk. It would appear for example that if there were an in-
crease in the yield of a relatively low risk financial asset, such as bonds; the aggregate portfolio, through the choices of individual investors, would experience a shift in funds to these assets and away from relatively high risk assets such as common stocks. That is, the shift would allow the individual to maintain its original yield on financial assets at a new lower level of risk or alternatively, it could main-
tain is original level of risk at a new higher expected yield. Quite likely it would be a move resulting in a combination of the two alternatives. The preceding considerations are
fundamental to the following analysis.

For brevity of presentation the model can be stated mathematically as follows: Let $\bar{A}$, the total stock of marketable financial assets at a given time equal

$$\bar{A} = \sum_{i=1}^{n} S_i \quad i = 1, n \quad (1)$$

The $S_i$ represents the $n$ different classes of financial assets. Generally, the $S_i$ are arranged in the model such that if any two assets are compared such as $S_i$ and $S_j$ and $i < j$, then $S_i$ is an asset more liquid than $S_j$. In other words the assets are arranged according to decreasing liquidity. Following this we will call $S_1$ money, $M$, since it is the most liquid of all assets. $S_n$ is the dollar value of common stocks. It is taken as the market value of all shares listed on the New York Stock Exchange. Admittedly there are other sources of stock purchases, but the shares listed on the New York Stock Exchange constitute by far the largest magnitude of stock values listed in a single market, and it also has the most complete and accurate data available with which to work. It is assumed that at a given point of time $\bar{A}$ is essentially constant, but that the $S_i$, the dollar amount invested in each class of asset does change significantly from time to time. For example, it is of course true that $S_1$ (the supply of money) is changing from time to time, but in a large part these changes are offset by opposite changes in the aggregate value
of United States Bills and Bonds held. To recapitulate, the
dollar value of financial assets held in an economy at a
given time is the sum of the dollar values of all the $S_i$
different classes or types of financial assets. The spectrum
of financial assets, ranked by liquidity would include money
stock (considered as demand deposits plus currency here)
time and savings deposits, savings and loan shares, short and
long term bonds, corporate bonds, stock shares, federally
underwritten mortgages, conventional mortgages, plus many more.

By taking the total differential of (1) we can examine
in some detail the properties of portfolio behavior that have
been treated and examined in past studies and at the same time
cite some properties that have not been treated adequately.
The total differential of (1) is:

$$\frac{dA}{dt} = \sum_{i=1}^{n} \frac{\partial A}{\partial S_i} \frac{dS_i}{dt} \quad i = 1, n$$

Some properties of this can be examined at this point.
In general, the $\partial A/\partial S_i$ term can be economically interpreted
as the marginal rate of return of investment in the $i^{th}$
asset. For simplicity we will consider the time period in­
volved in the analysis to be one year. Thus, for most of the
$n$ assets $\partial A/\partial S_i$ is the yield of one dollar invested in the $i^{th}$
asset or more specifically the rate of interest. These $n$
different rates of return will be classified in view of their
underlying economic interpretation.
First, we assert that:

\[ \frac{\partial A}{\partial S_1} = \frac{\partial A}{\partial M} = 0 \]  

(3)

In other words, the rate of return per period of the sum of demand deposits and currency is zero.

Second, we find that among the family of financial assets, there are some whose rates of interest have shown a strong tendency for being constant through time. That is, either there is no market mechanism producing an equilibrium rate of interest, or the rate is fixed by institutions by statute, or by other considerations. To avoid sounding too restrictive by this assertion it should be noted that these rates of return are admittedly not strictly constant and some rate changes do occur, but quite infrequently. Interest rates on time deposits, savings deposits and savings and loan shares for example, have changed in the past, but they have been stepwise jumps either upward or downward and in both cases remaining constant over long periods of time. These rates will be denoted as

\[ \frac{\partial A}{\partial S_i} = k_i \quad i = 2 \ldots n_1 \]  

(4)

to indicate that they are for the most part constant but not necessarily equal for each of these \( n_1 - 1 \) assets.

Third, we can segment another group of assets that have rates of return which change frequently according to occurrences
in their respective markets. Bonds of course are marketed daily and their price and therefore their yields fluctuate accordingly. This is a property of all bonds whether they be bills, short or long term government bonds, industrial bonds or other debt instruments issued either by the government or privately. For this reason, there is a clear justification for treating these securities differently than the group discussed immediately above. These rates will be symbolized as

\[
\frac{\partial A}{\partial S_i} = r_i \quad i = n_1 + 1 \ldots n-1
\] (5)

There are \( n-(n_1+1) \) of these each having a yield pertaining to it at a given time. For the purposes of this study it is assumed that there is one representative rate of interest for this classification of assets. This is done in view of the following past observations: First, a comparison of yield returns of these assets through time indicates that they move together very closely. Statistical evidence is available in support of this (26, pp. 100-119). Second, if there are a large number of market participants within and between each of these markets and information concerning each of these security markets is widely available we would expect arbitrage to produce similar movements in each market. These characteristics of security markets are widely accepted. Third, for the purpose of this model a treatment of more
than one of these rates would render it necessary to deal with the problem of multicollinearity. Proper handling of this would necessitate eliminating the highly correlated independent variables (29, pp. 102-107). This would ultimately reduce this model to one interest rate. Thus, the question is which interest rate is the proper one to use for the analysis below. In view of these considerations we will modify (5) to

\[ \frac{\partial A}{\partial S_i} = r \quad i = n_1 \ldots n-1 \]

where \( r \) is interpreted theoretically as "the" rate which is representative for these assets at a given point in time.

The last asset, common stocks, which is the \( n \)th term in Equation (2) represents the fourth classification of financial assets and its return will be equal to

\[ \frac{\partial A}{\partial S_n} = x \]  

(6)

to suggest that the return on a dollar's worth of stock in the aggregate is primarily unknown and autonomous. This is true because it is not operationally feasible to attach an accurate magnitude to the return on stock equities until a significant time period has elapsed. (Dividend and earnings statistics are available only after long periods of time have elapsed) and the difficulty is further augmented in indexing these earnings so that they may be compared in the
aggregate with the returns of other assets. From another viewpoint, stock earnings seem to be autonomous since they do not appear to be statistically related to any economic data. There is no significant amount of correlation of stock earnings with the rates of return on the assets discussed immediately above. Neither are stock return indexes related to stock price indexes. It will be assumed then that in the context of this model 'x' will be treated as autonomous and statistically

\[ \rho_{xS_n} = \rho_{xr} = \rho_{xk_1} = 0 \]

This assumption is quite commonly noted in the literature.

Now in view of the above discussion and the initial assumption that \( \bar{A} \) is essentially constant we can rewrite Equation (2) as follows:

\[
\frac{d\bar{A}}{dt} = 0 = \sum_{i=2}^{n_1} k_i \frac{dS_i}{dt} + \sum_{i=n_1+1}^{n-1} r_i \frac{dS_i}{dt} + x \frac{dx}{dt} \quad (7)
\]

The following behavioral assumptions are commonly accepted and verified in economic analysis. For a given interval of time it is expected that:

\[
\frac{\partial S_i}{\partial r} < 0 \quad i = 2...n_1 \quad (8a)
\]

\[
\frac{\partial S_i}{\partial r} > 0 \quad i = n_1+1,...,n-1 \quad (8b)
\]
The sign of the derivative in (8a) says that changes in the yields of marketable bonds bring about changes in the opposite direction of dollars held in assets which have relatively fixed yields (i.e., $S_2 \ldots S_{n_1}$). The possibility of the derivative equaling zero is allowed because it is likely that some of these $n_1 - 1$ assets such as pension r and s are fixed in amount according to institutional criteria. The derivative in (8b) is positive since there is an observed tendency for investors to increase their desire for holdings of assets whose yields increase. On the basis of (8a) and (8b) we can say that:

$$\frac{\partial S_i}{\partial s_j} < 0 \quad i = 2, \ldots, n_1 \quad j = n_1 + 1, \ldots, n-1 \quad (9)$$

Without examining all of the behavioral characteristics of (7) the two properties that provide the backbone of this study will now be examined. They are:

$$\frac{\partial S_n}{\partial x} = 0 \quad (10a)$$

$$\frac{\partial S_n}{\partial r} < 0 \quad (10b)$$

The first relationship has been examined in great detail in the past literature. The most recent and most applicable of this body of literature is the study by Christy (8, pp. 209-232). His results and the results of others have invariably failed to find any quantifiable relationship between
the level of stock price and yield indexes. Part of this is due to the nature of the x data discussed above, but it still remains that investors apparently do not respond in the market to current stock yields.

This leads one to explore the possibility of a relationship expressed in (10b). The question is, do investors individually view the yield of bonds as a major factor influencing their behavior in the stock market. In other words, to the extent that debt capital (bonds) is substitutable for equity capital (stocks) and since earnings accruing from shares viewed in isolation appear operationally meaningless for an aggregate portfolio it may very well be that yields which are known and meaningful in one set of markets (bonds) may be the major decision variable in another market (common stocks).

This possibility has not been considered in the literature. The study most closely resembling this approach was by Sprinkel who related changes in the money stock with changes in equity prices (54). His widely variable results made stock price explanation difficult and price prediction virtually impossible. By referring to the equation:

$$\frac{dA}{dt} = 0 = \sum_{i=2}^{n} k_i \frac{dS_i}{dt} + \sum_{i=n+1}^{n-l} r \frac{dS}{dt} + x \frac{dS_i}{dt}$$

(7a)

allowing for money supply changes we can rewrite (7) as follows:
Rather than assume $dM$ to be equal to zero as above, Sprinkel assumes changes in the money supply and in essence this produces the following chain of events: First, the disequilibrium in the money market brings about readjustments in the other types of assets. He, as well as others, believes that the adjustments are felt more quickly and more pronounced in near-money or liquid assets than in less liquid assets. As time progresses, further adjustments than those deemed most necessary are undertaken. Only then are changes felt in the stock market. Referring to (7a) the process of readjusting this macroportfolio proceeds from the equal sign and to the right through time. That is, the adjustment moves from liquid to less liquid assets. This intuitively explains why the lag discussed in chapter three is so large in his analysis. Again looking at (7a) it seems reasonable that stock adjustments are more proximate to interest rate changes than money supply changes. Further support for this view comes from a correlation study by Hamburger, where it was found that marketable securities, savings deposits and shares are all poor substitutes for money and substitutability among securities is far higher than between money and securities (27, pp. 101-113).

On the basis of Equation (7) a test of a number of
propositions was made which will be cited below. If we have:

$$0 = \sum_{i=2}^{n_1} k_i \frac{dS_i}{dt} + \sum_{i=n_1+1}^{n-1} S_i \frac{dr}{dt} + x \frac{dn}{dt}$$  \hspace{1cm} (7a)$$

and as discussed above the $k_i$ are virtually constant or at least not market determined, the $S_i$ ($i=2,...,n-1$) are determined by, $r$, the interest rate level, and $x$ is an autonomous variable apparently not quantitatively related to any of the variables in the model, we are then left with the proposition: Is the number of dollars invested in common stocks related to the rate of interest? The general proposition is:

$$S_n = S_n(r), \quad \frac{dS_n}{dr} < 0$$  \hspace{1cm} (11)$$

Regression analysis, in a variety of forms, was used to test this proposition statistically. The simple regression formula used was:

$$S_{nt} = a_1 + a_2 r_t + \varepsilon_t \quad \varepsilon = \text{NID}(0, \sigma_s^2)$$  \hspace{1cm} (12)$$

In order to determine the relationship between share prices and the rate of interest, the following modification is necessary. We define $S_n$ as follows:

$$S_n = P_n Q_n$$  \hspace{1cm} (13)$$

In this equation, $P_n$ is an index of stock prices and $Q_n$ is the quantity of shares listed on the New York Stock Exchange. From (13):
Now, if we can assume in the short run (months and quarters) the quantity of shares listed is relatively constant or at least increasing at a constant rate we can assume that \( p_n \), a stock price index, is a linear function of \( s_n \) and therefore we can in a manner similar to (12) test:

\[
p_{nt} = a_1 + a_2 r_t + \varepsilon_t \quad \varepsilon_t = \text{NID}(0, \sigma^2).
\]  

\( (15) \)

Empirical Analysis

Several types of regressions were employed including those which use monthly and quarterly data, lags of the independent variables, alternative interest rates, and the use of first differences in the variables.

The Data

Data for the following statistical analysis were acquired from the following sources: Stock indexes used were Standard and Poor's Index of 425 Industrial Stocks. They were taken from the Federal Reserve Bulletin. Also another index computed by the Center for Research in Security Prices at the University of Chicago was used. This index differs from other indexes in that it includes all of the common stocks listed on the New York Stock Exchange from 1926 through
1960. In addition to the conventional price index formulation, the Center also computed an investment performance index which measures the change in the value of stocks as a function of dividend and earnings changes as well as price changes. Further collaboration on how the indexes are computed and their interpretation can be found in an article published in 1966 by Fisher (17, pp. 191-195). Unless otherwise stated, interest rates were used from Moody's Industrial Manual. Short term and long term government bond yields were used as well as the yields on corporate and industrial bonds. Monetary data was derived from the Federal Reserve Bulletin. In all cases the data concerning money supply was seasonally adjusted to demand deposits plus currency. Data dealing with volume of shares and their dollar value was generously supplied by the statistical division in The Research Department of the New York Stock Exchange. Time spans of the various statistical time series which follow will be given in Appendix A.

**Time Periods Used**

The time periods used were selected on the basis of two considerations. The time period from observation to observation should be long enough so that spurious or non-economic events will not exert a dominant influence over the magnitudes of the variables under consideration. For this
reason day to day observations would be far too short. Much
too frequently the daily closing prices are determined by a
single news headline or a statement by a government official.
Similarly, weekly data will be affected by the same type
of phenomena, although the effects of these spurious events
would probably be diluted by using this longer period.
Monthly data seems to be of sufficient duration for the
effects of these occurrences to be eliminated. That is,
these incidental effects are still present, but either they
are "canceled out" by other incidental events which produce
opposite effects, or the day to day effects are adjusted for
the market in succeeding days during the month. A graphing
of price indexes with time partially verifies the above
reasoning. Since monthly data show few single observations
that are distinctively divergent from preceding and following
months it suggests that incidental effects are essentially
eliminated.

Quarterly data were also used in the regressions that
follow and, of course if monthly data is long enough to
eliminate effects of unusual day to day events, quarterly data
would also satisfy this requirement.

On the other hand, the time interval should not be so long
as to lose relevance with respect to purchasing horizons of
individual investors. Time periods as long as a year are
probably too long for observing occurrences in aggregate
portfolio changes. The empirical findings show that the portfolio analyzed in this study undergoes significant changes in composition in periods shorter than a year. Quarterly data helps to eliminate this possibility of overlooking intra-year relationships between interest rates and stock market activity. Monthly data also was used to determine whether or not investor behavior between the two classes of assets is as the model suggests in this shorter time period.

In summary, the time interval chosen had to be based on both an upward and downward constraint. In view of the empirical behavior of the data it appears that monthly or quarterly data is most appropriate. The following tests will provide some insight into which of the two are best.

Which Rate of Interest to Use

The theoretical problem of determining which interest return to use as the predetermined variable is not easily solved. In fact, valid reasons exist for using each of a large number of different interest rates are worthy of mention.

United States bond yields seem to be the most popular variable in past statistical studies, partially because accurate and up to date sources of data are available. In addition, they are recognized as being relatively riskless in comparison to other securities. This latter property of U.S.
bonds would not, however, render it highly applicable to this study since common stocks are not noted for their risklessness. To the extent that risk differences affect substitutability, it would seem that yields on U.S. securities would not be a good explanatory variable. However, other countervailing factors, such as their being an extremely large proportion of the total make-up of financial assets, could outweigh this. A related question is: If data on United States securities are used, should the data be the yields on long or short term bonds? It could be reasoned that the time horizon of stock market investors should indicate which maturity of bonds to use. If the heaviest buyers of stocks purchase them with the idea in mind of owning them for long periods of time then perhaps long term bonds are closer substitutes and therefore their yields most applicable. To reinforce this argument, we know that stock issues are primarily for long term investment purposes (i.e., plant and equipment, etc.). This is also largely true for long term government bonds so their substitutability could be evaluated with this in mind.

Conversely, short term bond yields may be economically most relevant to the extent that short term bonds and bills have less risk of capital loss and therefore are viewed as being more safely liquidated in the event of favorable economic conditions which bring about a substitution toward
common stocks. In other words, it is useful to test the possibility that short term government obligations are used as a temporary depository for wealth depending on their current rate of return. As an example, if there was an increase in the stock of bonds held by the private sector, interest rates on these bonds will fall, and owners of short term bonds would shift to common stocks, driving up their prices. One may be skeptical of this line of reasoning but nevertheless, it will be explored empirically below.

The second set of interest yields that may be most applicable empirically is that related to corporate and industrial bonds. This would be true if the following considerations are met: First, if there is a class of investors which is primarily concerned with committing investment funds into corporations and are only secondarily concerned as to whether it is held as debt (corporate bonds) or equity (common stocks) we would find shifts from debt to equity in response to the yield on corporate bonds. Second, since corporate debt and equity are simply two different types of corporate liabilities it would appear that a large part of the risk involved would be common to both debt holders and equity holders. These two conditions would lend support to the view that this model should employ corporate yields as the independent variable.

All of the rates discussed have theoretical merit for use in the model developed above. In reality, no single
rate is appropriate, and perhaps a weighted average of these rates along with others would be best. The weighting would have to take into account factors such as volume of each type of debt held, changes in volume, a measure of market activity and many others.

Emphasis in this study will be placed on the fact that high multicolinearity exists in interest rates and since this suggests difficulty in using more than one rate, simple regression analysis will be employed. The alternative of using an index of a number of rates was also rejected; first, because no such index exists and second, if one could be constructed it would not necessarily be the proper index for stock market analysis.

The Choice of Lag Periods

The lags chosen were based on theoretical as well as empirical criteria. Theoretically, it would seem that the lag should be just the length of time necessary for investors to take the appropriate action called for by a change in the rate of interest in the bond market. Since security markets are operated daily, one might at first posit that the effect occurring in the equity market due to a change in one of the bond markets would occur virtually without lag. A number of considerations, however, forces one to modify this view. One important consideration is that a response occurring this
rapidly carries with it the underlying assumption that investors know with a high degree of certainty that changes in bond yields are simply not temporary.

If an investor is quite uncertain about the permanence of a yield change in one of the bond markets, it could not be expected that he would alter his portfolio in the manner postulated in the model of the previous chapter. It is likely that day to day evaluation of the various markets would be dominated by uncertainty. It would seem that this uncertainty could be eliminated only if the behavior of the markets could be examined over a longer span of time. This leads one to conclude that the financial asset portfolio for the economy would not adjust as a result of a yield change until individuals governing the portfolio have had a reasonable length of time to fully assess the relevant market changes. This could be termed the behavioral lag.

Another lag associated with a portfolio change involves the time necessary to list securities and have them sold and the subsequent (or perhaps simultaneous) act of placing orders for purchases. It is hard to determine what the average length of time would be to accomplish these two transactions. The time lag would undoubtedly vary greatly from one pair of transactions to another, but it is reasonable to assume that an investor who is eager to alter his portfolio could easily make the change within a week. This lag could be
termed the mechanical lag and could overlap the behavioral lag in the aggregate.

Although these rather theoretical considerations provide some insight into what the proper lag time is, they are incapable of providing the answer as to just when the full adjustment in the equities market is realized, due to a yield change in one or more of the bond markets.

In determining this, the author relied on empirical analysis which in this case refers to an examination of the raw data in tabular and graphical form.

The monthly time series were examined in the following way: Both the University of Chicago price index and the Standard and Poor's index were plotted with the corresponding monthly yields of various classes of bonds. The five classes of bonds were, short-term government bonds (3 to 5 year maturity), long-term government bonds (at least 10 years to maturity), class AAA corporate bonds, Moody's composite corporate bonds and Moody's composite industrial bonds.

A number of rather unsurprising findings can be separated from some of those that are more interesting. Movements of both price indexes were virtually identical with respect to direction and degree. This is to be expected since both of the indexes include a large number of stocks. Standard and Poor's Index includes 425 individual stocks and the Chicago index includes all of the New York Exchange's listed stocks.
The high similarity of the two series serves quite well to show that the Standard and Poor's Index is an excellent representation of prices in the whole market. Because of this, one of the price indexes was eliminated from further analysis.

The next step was to choose which index was to be retained. The author elected to use the Standard and Poor's Index for the following reasons. First, the vast majority of related research such as was discussed in Chapter three employs this index. Thus, for comparing these findings with previous research, the use of this index prevents the creation of an additional qualitative difference. Second, there is considerable familiarity with this index among the investing public as well as those who have a primarily academic interest in this type of analysis. It would seem then, that for evaluating this analysis independent of other previous work, the use of this index should add an element of meaningfulness. Third, an even more practical consideration is that data dealing with the Chicago index are decidedly more difficult to obtain. This is particularly true of the more recent years which are of great interest in this analysis. Of course the recent data are not impossible to obtain, but even if it were obtained it would not be of substantive benefit due to its nearly complete congruence with the Standard and Poor's Index.

Another unsurprising observation was that the three
time series of non-government bond yields moved together so closely that differences in their movements with the stock price index could not be determined visually. It did not matter therefore, at this stage of the analysis, which of the three yield series was utilized. It will be pointed out below that quantitative differences in their correlation with price index data do appear in the statistical analysis.

At this point the only beneficial comparisons remaining are those of the price index with both long and short term government bond yields and with one of the corporate bond yields.

The most apparent pattern following from an examination of these three comparisons was that all of the series possessed a positive trend. That is, for the time span covered, interest rates and the price index have been rising due to inflation and perhaps to other factors. Because of this factor, there is no substantial knowledge gained by looking at long term trends, but it is better to compare the month by month changes occurring along the trend that is common to both series. In other words, the question which is raised is: do month to month changes in one variable(s) (bond yields) bear any relationship to the corresponding month by month changes in another (the stock price index)? The core of this analysis involves the answer to this question. A clue to the answer lies in a simple visual examination of the
monthly changes occurring in each of the two time series. The changes in the two series appear to move in opposite directions; downward movements in bond yields tend to be accompanied by upward movements in stock prices and vice-versa when one abstracts from the positive trend common to both series. Of course this property of the series does not appear without fail, but usually, when interest rates tend to move in one direction, stock prices in some nearby month tend to begin a move in the opposite direction. The more crucial question concerning the approximate lag, which exists if any, between changes in the two series, it is more difficult to answer. One factor does appear certain, however, and that is that most of the time bond yield changes are followed by opposite stock price changes rather than the reverse. This empirical finding confirms with the line of causation required in the portfolio model developed earlier. This model, it will be recalled, states that prices in the equities market are variables which are dependent on such variables as bond yields, an assertion causally consistent with the empirical results discussed thus far. It is not possible to specify at this point the precise lag time which occurs between the variables, but the data suggest that it seldom is as much as two months and more often it is only one month or no lag at all. The statistical results below provide a much better insight into this issue.
It is interesting to note that not only does one detect an opposite movement of the variables month by month, but that yield movements that occur in one general direction for a large number of months produce movements in stock prices in the opposite direction for roughly the same number of months. Accordingly the effects of price changes in the bond markets are sensitive enough to produce equity price changes in short periods of time as well as over longer periods.

Brief mention should be made concerning the differences that each of the three yield series bear in their relation to the stock price index. It is obvious from looking at each of the comparisons that corporate bond yields follow the pattern outlined above much more closely than either long or short term government bond yields. This, also, is as the theory implies since government bonds are not as close in substitutability to equities as are corporate bonds. Furthermore, since the largest single participant in the government bond markets is the government itself (we include the Federal Reserve System as a functional part of the government) and since the government does not manage its portfolio in the same manner or with the same objectives as private investors, it logically should be expected that its effect on related markets is quite indirect. Another contributing factor is that commercial banks and other financial intermediaries are restricted, legally or voluntarily, as to
the types of assets they can hold. Since financial intermediaries are large holders of government debt, one would not expect government bond yield decreases to induce them to go directly into other higher risk assets such as equities.

The quarterly data were examined graphically in the same fashion as the monthly data and so most of the comments above apply here as well. The data for the quarterly analysis were developed by the averaging of the appropriate monthly data. The corresponding quarterly comparisons show the general inversion pattern in very much the same manner as the monthly inspection did. Of course this result is somewhat forced since one must expect quarterly grouped data to have the same temporal movements as the monthly data from which it was developed.

Visual inspection does not reveal much evidence for lags as was found in the monthly data. In some cases price changes appear to follow changes in bond yields by as much as one quarter, but very seldom by two or more quarters. Once again the statistical analysis will be relied on in determining what the best lag period is.

One problem associated with aggregating monthly data is the possibility of losing or distorting peaks or troughs in time series due to the averaging of data from a number of consecutive months. The problem does not appear to be
a major one in this study, however, since low and high points in the quarterly series mirror quite regularly similar points in the respective monthly series. This can reasonably be expected since the quarterly aggregation only requires three consecutive months for each observation.

The Statistical Model

The preceding discussion provides a foundation for a statistical analysis of the relation of stock prices to interest rates. Although interest rates are of primary concern in this study, they alone are not sufficient to explain stock prices. What is needed next is a theoretical formulation which incorporates other important variables.

Malkiel (36, p. 1011) and later Keran (31, pp. 17-18) have both developed theoretical equations which employ a procedure of discounting expected future dividends and expected future stock prices. By using a derivation of this type of formulation one can integrate the portfolio analysis developed above with other variables that are relevant in explaining stock price behavior. If stock prices are determined by purchasers according to the value of discounted expected future dividends one can express stock prices as
Where $P$ is the stock price, $D_e$ is the expected current dividend, $g$ is the expected rate of dividend growth and $r$ is the rate of interest. By summing this series stock prices would be equal to

$$P = \frac{D_e (1+g)}{r-g}$$

In the equity market, however, investors are not concerned with dividends in the indefinite future but only during the time they expect to hold the stock. They also will discount the expected future price according to when they expect to sell the stock. Therefore assuming that investors continue to be willing to capitalize the future dividends of shares at the same rate $g$ and the price of stock grows at the same rate $g$, the valuation formula becomes

$$P = \sum_{i=1}^{N} D_e \left( \frac{1+g}{1+r} \right)^i + P \left( \frac{1+g}{1+r} \right)^N$$

where $N$ is the finite time horizon. Summing this progression we again have

$$P = \frac{D_e (1+g)}{r-g}$$

Thus we note that the time horizon, $N$, does not influence stock prices given the above assumptions concerning the rate
of growth. Assuming that the supply of shares does not change significantly over short time periods the formula above suggests that purchasers of stock will bid up the price of stock if the expected growth rate increases, if expected dividends increase or if the rate of interest falls. They will bid down stock prices if these variables move in the opposite direction.

Both expected dividends and the rate of interest can be derived directly using an adaptive expectations hypothesis. The expected rate of growth however is not as direct so one must seek a theoretical explanation for a measure of the expected rate of growth.

It is not unreasonable to postulate that the expected rate of growth of dividends and stock prices is influenced by the rate at which earnings are plowed back into corporations. In the aggregate this rate would in turn largely be determined by the level of corporate earnings. A major factor determining expected corporate earnings is the level of and changes in nominal GNP (31, pp. 22-24).

To state the above in general equations:

\[ g = f_1(E) \]  
(17)

Where \( E \) is expected earnings. And

\[ E = f_2(\Delta y, y) \]  
(18)

where \( \Delta y \) and \( y \) are the change in income and the level of
income respectively. And from Equation 16:

$$P = f_3(r, D^e, g)$$  \hspace{1cm} (19)$$

By reducing Equation 17, 18 and 19,

$$P = f_3(r, D^e, \Delta Y, Y)$$  \hspace{1cm} (20)$$

A regression model which employs the Almon distributed lag technique to estimate expectations will be used to test the general relationship expressed in Equation 20. This technique will also answer the questions concerning the length of lags.

The basic equation to be tested is

Model I

$$P_A = a + \left[ \sum_{i=0}^{n_1} b_1 r_{t+i} \right] + \left[ \sum_{i=0}^{n_2} b_2 D_{t-i} \right] + \sum_{i=0}^{n_3} b_3 \Delta Y_{t-i} + b_4 Y_t + \varepsilon_t$$

To determine the effect of changes in rather than levels of the above explanatory variables as well as reducing the possibility of error correlation a test of the following was conducted.

Model II

$$\Delta P_t = a + \sum_{i=0}^{n_1} b_1 \Delta r_{t-i} + \sum_{i=0}^{n_2} b_2 \Delta D_{t-i} + \sum_{i=0}^{n_3} b_3 \Delta Y_{t-i} + \varepsilon_t$$
Model III

\[ P_t = a + \sum_{i=0}^{n_1} b_1 \Delta r_{t-i} + \sum_{i=0}^{n_2} b_2 \Delta D_{t-i} \]

\[ + \sum_{i=0}^{n_3} b_3 \Delta y_{t-i} + \epsilon_t \]

The Statistical Results

The estimates of the above equations appear in the Appendix. The earlier discussion suggested that the coefficients associated with income and dividend variables should be negative, and coefficients associated with interest rates should be positive. All of the polynomial distributed lags have the theoretically expected signs. The income variable in Model I is negative, saying that high levels of current income are associated with low current stock prices. This is perhaps due to the fact that normally a stock price series leads an income series. A more accurate description of the income stock price relationship is found by looking at changes in income.

In Model I the coefficients associated with bond yields and changes in income were not significantly different from zero at the 95% level. They do, however, have the expected sign. A detailed look at the distributed lag coefficients suggests that their positive influence on stock prices occurs with decreasing weight with each coefficient being significant as far back as seven periods. The interest rate influence
shows the same type of descending coefficients although they are not statistically significant. Both of these observations lend support to the view that stock price changes are related to past dividend levels and bond yields. The Durbin-Watson statistic is quite low in this model which tends to weaken the results.

Model II produces a small coefficient of determination which is common when all the variables are in first differences. Although all of the regression coefficients in this model are of the right sign none are statistically significant. This, along with the low R-squared, limits the value of Model II.

Model III, where the stock price index is regressed on lagged changes in the explanatory variables, produces more convincing results than either of the two prior models. The Durbin-Watson statistic for this model falls short of the indeterminant range, indicating the presence of some serial correlation of the residuals. At 1.134, however, the D-W statistic is not so low as to indicate a serious mis-specification of the model. All coefficients are significant and all display the expected sign. In terms of the theoretical analysis outlined above the results of this regression indicate that past changes in yields of corporate bonds, dividends and growth all play important roles in explaining stock prices. The individual coefficients of the distributed lag show that past interest rate changes exert an influence on
stock prices a number of quarters forward. This is because
the effect as indicated by the regression coefficients are
initially small but become larger for the three to six quarter
lags and then again become smaller. This suggests that
interest rate changes do not exert their maximum impact until
a few quarters have elapsed. The lag pattern of dividends
is somewhat different. The regression coefficients remain
statistically significant and relatively constant for seven
of the ten past periods taken into consideration.

This analysis is similar to recent work by Keran (31),
there are, however, some important conceptual differences.
Keran takes the view that monetary factors are important in
forming stock prices. Interest rates are in part determined
by monetary conditions. Therefore, his use of the interest
rate was justified because it is a monetarily determined
variable. In contrast, this study views the importance of
interest rates as arising from the fact that it is a measure
of return on an asset which is a close substitute for stocks.
Because the return on stocks is also theoretically important
in the pricing of stocks, a measure of dividends was also
included. The statistical results of this study show that
both dividends and the return on debt are important in the
formation of stock prices.
SUMMARY AND CONCLUSION

There are a number of implications of this study which are relevant to the body of thought concerning stock price behavior.

The first to be discussed will be its relation to the leptokurtic distributions that stock price movements have been found to follow. Chapter two indicated that a leptokurtic distribution differs from a normal distribution in that an excessive number of observations centered around the mean and in the extreme tails. Distributions like this follow stable Paretoian distributions with the proper choice of parameters.

To put the matter more simply, a movement of a price series through time which follows a stable Paretoian distribution would contain a large number of observations that do not vary a great deal from each other, but also marked by observations resulting from an extraordinarily high number of large, disjointed changes. As we have seen, Fama believes that there must exist basic economic data responsible for this property. The study of variables directly associated with equities have proven less than satisfactory to this end, and as in many cases, a more complex and indirect approach has demonstrated an ability to provide new insight. The approach taken has been indirect, in the sense that explanatory variables have been employed which do not relate to stocks,
but rather to assets that are good substitutes for stocks. The merits of these variables have been discussed above. An explanation of how these variables relate to the property of leptokurtosis is as follows: The yield changes occurring in bond markets, which are substitutes for equities, produce abnormal changes in investors valuation of stocks; these are changes that do not occur due to events in the stock market. These occasional outside impacts produce the occasional large variations which appear on the tails of stock price distributions. When intertemporal changes in bond yields are very slight stock prices tend to be stable, accounting for the high frequency of distributions occurring about the mean.

One final comment seems appropriate regarding the applicability of this study to present economic conditions. The relationship between bond yields and stock prices during the past four years is an excellent example of the real world relevance of the results in this study. Almost without fail the stock market has taken a bearish turn each time the price of debt increases. A skeptic of my hypothesis may argue that the market has fallen because liquidity demands have forced sales in stocks. The fact is, however, that stock prices started falling before a restrictive monetary policy was initiated. This study suggests that the high bond yields (which may or may not have been due to tight money)
have attracted investment capital toward debt and away from equity. The conditions that have occurred in the bond markets have caused the investing public to drastically change their valuations of stocks.

In closing, it is interesting to recall the answer that Financier J. P. Morgan gave to a man who asked him how he thought (stock) prices would move. Morgan replied, "They will fluctuate, son; they will fluctuate." It is hoped that this thesis has provided some understanding as to why they fluctuate as they do.


APPENDIX

Statistical Results of Regressions Relating Stock Prices to AA Corporate, Dividends and Money Income

Period for which data covers is 1950 to 1969 inclusive. All data is in quarterly intervals.
Model I

\[ P_t = -13.310 - 0.178 Y_{t-1} - \sum_{i=0}^{2} 2.877 r_{t-i} + \]
\[ (5.23) (-2.753) \]

\[ \sum_{i=0}^{8} 10.346 D_{t-i} + \sum_{i=0}^{3} 1.061 \Delta Y_{t-i} \]
\[ (5.614) (0.351) \]

\[ R^2 = 0.9683 \]
\[ S.E. = 4.58 \]
\[ D-W = 0.709 \]

**Detailed Distributed Lag Results**

<table>
<thead>
<tr>
<th>Estimated Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 ( r_0 )</td>
<td>1.871</td>
</tr>
<tr>
<td>1 ( r_1 )</td>
<td>0.682</td>
</tr>
<tr>
<td>2 ( r_2 )</td>
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<td>( \sum r_{t-i} )</td>
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<tr>
<td>0 ( D_0 )</td>
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</tr>
<tr>
<td>1 ( D_1 )</td>
<td>1.774</td>
</tr>
<tr>
<td>2 ( D_2 )</td>
<td>1.558</td>
</tr>
<tr>
<td>3 ( D_3 )</td>
<td>1.345</td>
</tr>
<tr>
<td>4 ( D_4 )</td>
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<tr>
<td>5 ( D_5 )</td>
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<tr>
<td>6 ( D_6 )</td>
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<tr>
<td>7 ( D_7 )</td>
<td>0.533</td>
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<tr>
<td>8 ( D_8 )</td>
<td>0.339</td>
</tr>
<tr>
<td>( \sum D_{t-i} )</td>
<td>10.346</td>
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<td>Estimated Coefficient</td>
<td>Standard Error</td>
</tr>
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<td>-----------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>$\Delta y_0$</td>
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<tr>
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<tr>
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<td>.136</td>
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<tr>
<td>$\sum \Delta y_i$</td>
<td>1.061</td>
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</table>

Note: "t" statistics appear with each Regression Coefficient enclosed by parentheses. The absolute value of a "t" value must be 1.664 or larger to be statistically significant.
Model II

\[ \Delta P_t = \sum_{i=0}^{4} \Delta r_{t-i} + \sum_{i=0}^{4} .994 \Delta D_{t-i} + \sum_{i=0}^{9} 1.440 \Delta y_{t-i} \]

\[ r^2 = .066 \]

\[ S.E. = 2.054 \]

\[ D-W = 3.780 \]

**Detailed Distributed Lag Results**

<table>
<thead>
<tr>
<th>Estimated Coefficient</th>
<th>Standard Error</th>
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</thead>
<tbody>
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<td>( \Delta r_0 )</td>
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<tr>
<td>( \Delta r_1 )</td>
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<td>( \Delta r_2 )</td>
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<tr>
<td>( \Delta r_4 )</td>
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<tr>
<td>( \Sigma r_{t-i} )</td>
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<tr>
<td>( \Delta D_0 )</td>
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<tr>
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<tr>
<td>( \Delta D_3 )</td>
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<tr>
<td>( \Delta D_4 )</td>
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<tr>
<td>( \Sigma D_{t-i} )</td>
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<td>Estimated Coefficient</td>
<td>Standard Error</td>
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<tr>
<td>-----------------------</td>
<td>----------------</td>
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<td>$\Delta y_7$</td>
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<tr>
<td>$\Delta y_8$</td>
<td>-.270</td>
</tr>
<tr>
<td>$\Delta y_9$</td>
<td>-.247</td>
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</tbody>
</table>

$\sum \Delta y_i = -1.440$

Note: "t" statistics appear with each Regression Coefficient enclosed by parentheses. The absolute value of a "t" value must be 1.664 or larger to be statistically significant.
Model III

\[ P_t = \sum_{i=0}^{9} -99.478 \Delta r_{t-i} + \sum_{i=0}^{7} 38.910 \Delta D_{t-i} \]

\[ + \sum_{i=0}^{7} 3.877 \Delta y_{t-i} \]

\[ R_2 = .332 \]

\[ S.E. = .806 \]

\[ D-W = 1.134 \]

**Detailed Distributed Lag Results**

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<tr>
<td>(\bar{\Delta r_i})</td>
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<tr>
<td>Estimated Coefficients</td>
<td>Standard Error</td>
</tr>
<tr>
<td>------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>( \Delta D_0 )</td>
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<td>( \Delta D_7 )</td>
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<tr>
<td>( \sum \Delta D_i )</td>
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<tr>
<td>( \Delta Y_0 )</td>
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<tr>
<td>( \sum \Delta Y_i )</td>
<td>3.877</td>
</tr>
</tbody>
</table>

Note: "t" statistics appear with each Regression Coefficient enclosed by parentheses. The absolute value of a "t" value must be larger to be statistically significant.