Vibration modeling for vibrothermography

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Vibration modeling for vibrothermography

by

Jyani Somayajulu Vaddi

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

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I dedicate this dissertation to my mother, for her constant support and innumerable sacrifices to make this all possible.
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ABSTRACT

Vibrothermography is a nondestructive evaluation method for identifying defects such as fatigue cracks and delaminations, primarily in aerospace components. When a specimen with crack is subjected to mechanical vibrations, friction and/or adhesion hysteresis between vibrating crack faces generates heat. An infrared camera can capture this heat and identify the defect. Vibrothermography has at times, proved to be an effective method for detecting tight and short cracks that other methods may fail to detect. However, long standing issues such as lack of repeatability and incomplete understanding of physics behind crack heat generation have so far failed to instill confidence in this method for use in industry.

In this research, we address the questions of how to measure and predict specimen vibration. We propose the use of viscoelastic coatings to identify specimen resonant mode shapes and map vibration distribution. We develop a numerical model for specimen vibration in vibrothermography. This is part of a larger physics based hybrid numerical/empirical model we developed at Iowa State University to predict crack heating in vibrothermography. Specimen vibration in vibrothermography is often affected by external factors like mounting and transducer coupling. We show that using compliant couplant and isolators at the contact points on specimen eliminates the effect of mounting and transducer characteristics on specimen resonances and makes the specimen vibration more repeatable. In addition, we show that isolators act as absorptive springs in parallel to the specimen and increase the effective specimen stiffness and in turn, the resonance frequency. We characterize the couplant and isolators with the use of simplified electrical circuits and explain their effect on specimen vibration based on analogous electrical circuit principles. Based on these observations, we develop a linear vibration model for vibrothermography. We also develop a linear inversion process to quantify isolator and couplant damping. Finally, we validate the vibration model against physical and simulation experiments.
Our empirical model for vibrothermography describes crack heat intensity as a function of specimen vibration and other crack related parameters. Crack heat intensity is therefore one of the required input parameters to the model. We propose an inversion process to estimate heat intensity from the measured crack surface heating. Normally, direct inversion of measured surface heating is an ill-posed problem because of the diffusion. However, we make certain assumptions within the scope of which, the inversion process is tractable and is capable of accurately reconstructing the measured heating.
CHAPTER 1. GENERAL INTRODUCTION

1.1 Introduction

Vibrothermography is a nondestructive evaluation (NDE) method for detecting cracks and delaminations, primarily in aerospace components. When a specimen with crack is subjected to mechanical vibrations, friction and/or adhesion hysteresis between contacting crack faces generates heat, which can be detected with an infrared camera. Vibrothermography, also called as Sonic IR and Thermosonics was first introduced by Henekke et al. [Henneke et al. (1986); Reifsnider et al. (1980)] in 1980s for damage detection in composite materials. With the advent of affordable infrared cameras, the interest in vibrothermography was rekindled during the 2000s [Shepard et al. (2004); Favro et al. (2000); Rantala et al. (1996); Holland (2007)]. Currently, several research groups are working to develop this method to be a reliable NDE method in industry.

Vibrothermography has shown promise especially in its ability to detect relatively closed cracks compared to other methods such as fluorescent penetrant inspection (FPI). Applications of vibrothermography in the aerospace industry include detecting fatigue cracks in turbine disks and blades of aircraft engines, delaminations in composite panels of aircraft structures and in the nuclear industry for weld inspection.

Vibrothermography inspection is a three stage process: vibration, crack heat generation and heat flow. The vibration energy applied to the specimen is locally converted to heat at the crack due to friction and hysteresis, which flows to the surface. Therefore, for heat to occur, there should be relative motion between two contacting (crack) surfaces. All the three stages of the process affect crack detectability and therefore are equally important. For example, if the vibration amplitude is too low to generate adequate relative motion at the crack faces,
no heating occurs. Similarly, if the crack faces are locked tight, there is no relative motion between the faces and no heat is generated. If the heat is generated at regions too deep below the surface, there may not be any detectable heat diffused to the surface. The general rule of thumb in any thermographic method is that a flaw must be at least as large as it is deep in order for it to be detected [Maldague (2001)]. All these aspects make vibrothermography a complicated process. Furthermore, the physics behind crack generation is in itself complicated and thus far, there hasn’t been a complete understanding of what exactly causes crack heating or the ability to accurately predict how much heat is generated. One reason for this lack of understanding is because of poor repeatability and consistent experiment procedures. Much of the research has been focused on qualitatively identifying defects than to actually understand the underlying physics.

The objective of our research at Iowa State University (ISU) is to perform quantitative research in vibrothermography. Over the past decade, significant progress has been made in understanding the mechanisms involved in generating crack heating. Crack heating in vibrothermography generally depends on several factors including, but not limited to: dynamic vibrational stress at the crack, vibration frequency, crack length, crack closure state and crack roughness [Holland et al. (2011); Zhang (2010); Renshaw et al. (2009a, 2011)]. Holland et al. [Holland et al. (2011)] have shown that crack heating increases with increasing dynamic vibrational stress at the crack. Zhang et al. [Zhang (2010)] have shown that crack heating increases with increasing vibration frequency: Higher crack heating was observed in Ti 6-4 and Inconel 718 alloy specimens when vibrated at 5th flexural mode than at 3rd flexural mode. The aforementioned factors are extrinsic to the crack and therefore are experimentally more controllable to some extent. Intrinsic factors that influence crack heating are crack closure, morphology and roughness. Renshaw et al. [Renshaw et al. (2009b)] discovered that heating does not occur uniformly along the crack and that maximum heat generation is concentrated in areas where the crack is neither fully closed or fully open. In the region of high closure stress, the crack faces are locked against each other and no relative motion occurs and hence no heat is generated. In the region where the crack faces are fully open, they never come into contact and no heat is generated either. Therefore, crack heating in vibrothermography occurs
in regions of tenuous contact where the crack faces are neither fully closed nor fully open. It was also discovered that in general, long cracks generate higher net heating than short cracks.

Despite the progress, much work still needs to be done to make vibrothermography a reliable inspection method for industrial applications. As a step towards achieving this, we developed a physics based hybrid computational/empirical model for vibrothermography and implemented a simulation tool called ‘VibroSim’ based on this model. VibroSim is analogous to a real experiment in that it simulates all the three aspects of vibrothermography: vibration, crack heat generation and heat flow.

The goal of this research is to develop a quantitative numerical model for vibrothermography. Since vibration is the first part of our forward model, it is critical to get this step right. To develop a precision vibration model, we investigate all the factors that influence specimen vibration. We then measure all the relevant material properties and develop the numerical model. Finally, the accuracy of the model is validated against experiments.

### 1.2 Vibrothermography Experiment

The essential components to perform a vibrothermography inspection are an excitation system and an imaging system. Specimen vibration is typically generated using a piezoelectric transducer attached to a power amplifier and the crack heating is observed using an infrared camera. Commercially available ultrasonic welder and horn assembly connected to a power amplifier is commonly used to excite the specimen. The welder is configured to operate at a single narrowband frequency range, typically centered at 20, 40 or 60 kHz.

Although quick to assemble and readily available commercially, the welder type excitation system has some limitations. A major disadvantage is that since it relies on contact non-linearity for generating resonant motion in the specimen, the vibration is notrepeatable and is often difficult to perform any quantitative analysis. Moreover, acoustic chaos has been reported by Han et al. [Han et al. (2004)] as a method to improve defect detectability in vibrothermography. Also, an ultrasonic welder applies large amount of mechanical energy to the specimen during the vibration process, which, sometimes has known to cause additional
To overcome the limitations of an ultrasonic welder, we developed a broadband excitation system at ISU for generating specimen vibrations [Holland (2007)]. The advantage of using a broadband excitation system is that the transducer can be electrically tuned to vibrate explicitly at any frequency unlike a single narrowband frequency range of a welder. Therefore, instead of relying on contact non-linearity, specimen resonances can be explicitly excited. Because of this frequency selectivity, this process is more controllable and hence more repeatable. Also, since sufficiently high amplitude vibration can be generated with relatively low power excitation at resonances, this significantly improves the transducer life. The broadband transducers used in vibrothermography are known to degrade with prolonged usage over a period of time [Vaddi et al. (2012)].

Specimen is typically mounted in metallic grips with rubber pins or layers of cardstock material called ‘isolators’ sandwiched between the specimen and grips. Similarly, a layer of ‘couplant’ material is placed at the contact between transducer tip and specimen. Researchers
Figure 1.2  Specimen velocity spectrum as a function of frequency. Peaks in the spectrum correspond to specimen resonance frequencies, as indicated by the arrows.

have observed that using a couplant layer improves the consistency of vibration and also prevents surface damage to the specimen arising from the contact against transducer.

A laser vibrometer measures the specimen velocity at a single surface point. Calculating the time integral of measured velocity gives the displacement. We use a FLIR SC6000 infrared camera for measuring crack heating in our vibrothermography experiments at ISU. Figure 1.1 shows a schematic of our experiment set up.

The specimen is first vibrated with a broadband frequency sweep excitation and the velocity at a single point is measured as a time waveform. The Fourier Transform of this velocity waveform gives the velocity spectrum. Figure 1.2 shows a typical measured velocity spectrum of a specimen in vibrothermography. The peaks in the spectrum correspond to resonance frequencies. The desired specimen resonance is then selected and the specimen is again vibrated, this time with a tone burst excitation at resonance. The IR camera is calibrated so that it measures specimen surface temperature in degrees Kelvin. A total of 150 frames are recorded at a frame rate of 90 fps. The first 20 frames record the background before excitation begins, and the last 41 frames record the cool down after the excitation ends.
Vibrational strain is calculated from the measured displacement according to a procedure developed by Holland et al. [Holland et al. (2011)]. If the mode shape of the flexural resonant mode is known, the dynamic vibrational strain across the entire specimen length can be measured using the displacement measured at a single point. Often, multiple resonances are tested to ensure sufficient vibration coverage in the specimen.

The average of the background frames is calculated and subtracted from all the thermal images to obtain the effective temperature rise in the field of view. In case a crack is present in the specimen, the temperature rise at the crack is much higher than the rest of the image. The background subtracted images are further processed using a physics based signal reconstruction algorithm developed by Holland [Holland (2011)]. This algorithm improves the signal to noise ratio and enables semi-automatic crack detection in vibrothermography.

### 1.3 Vibration Measurement and Consistency

An experimental procedure has to be repeatable and consistent to perform any meaningful analysis. This is especially true for an inherently complicated process like vibrothermography. As we saw earlier, several extrinsic and intrinsic factors affect crack heating. Intrinsic factors such as crack closure and morphology are driven by the fatigue process that generates the crack and the user has little control over them. However, extrinsic factors such as specimen set up and vibration generation must be controlled such that consistent and repeatable results ensue.

One of the key objectives of this research is to develop methods to improve the consistency and repeatability of specimen vibration in vibrothermography. Consistent and repeatable experimental procedures enable quantitative validation of numerical model.

As explained in section 1.2, vibrothermographic testing at ISU is predicated on testing at resonance frequencies. The knowledge of specimen resonance frequencies and the corresponding mode shapes is a pre-requisite for quantitative vibration analysis. Even for qualitative inspection of real world components, the position of a flaw relative to the resonant mode shape significantly affects its detectability. For example, a crack present at an antinode (region of maximum vibration) of a resonant mode is more likely to be detected than one present at a
node (region of minimum vibration) when vibrated at that resonance frequency. Since vibrational strains are not uniform across the specimen surface, measuring vibration coverage is a key aspect in determining frequency selectivity of vibrothermography.

The ability to accurately predict the resonance frequencies is critical for numerical vibration modeling. Vibration amplitude in the specimen at resonance is usually orders of magnitude higher than that off resonance, especially at high Q vibrations with small damping. Hence exciting the specimen at a wrongly predicted resonance frequency may affect the amount of heat generated at the crack and in turn the crack detectability. Therefore, the ability to predict resonance frequencies is a key to precision vibration model. Further, as we show in chapter 4, isolators act as springs in parallel with the specimen to increase the effective specimen stiffness and in turn the resonance frequency.

Also equally important is the amount of damping present in the system, which determines the magnitude of vibration at resonance. In an ideal no damping condition, the magnitude of vibration becomes unbounded and the specimen once impacted, continues to ring forever. However, real systems always have some form of damping and the vibration eventually attenuates to zero. Vibration damping in vibrothermography primarily occurs due to the presence of isolators and couplant. System damping can be calculated as either the half power peaks in the spectrum in frequency domain or in the time domain as the logarithm of the ratio of successive peaks of the decaying velocity/displacement signal once the excitation ends [Vierck (1979); Findeisen (2013)]. In chapter 4, we develop a linear inversion method to estimate the loss factors of couplant and isolators from the measured system damping.

1.3.1 Methods to Improve Consistency and Repeatability of Vibration Measurement

As discussed in section 1.1, one of the key aspects that need to be addressed to make vibrothermography a reliable NDE method is to identify consistent and repeatable experiment procedures. Specimen vibration is sensitive to external factors such as specimen mounting and excitation [Vaddi et al. (2015)].

During the course of this research, we developed several procedures to improve the efficiency
and repeatability of our vibrothermography experiments at ISU. Following these procedures ensures that specimen vibration is predictable and hence modelable.

1.3.1.1 Use of Viscoelastic Coatings for Identifying Resonant Mode Shapes

The most common way to measure vibrational strains in a vibrating object is based on single point measurements using strain gages, accelerometers or laser vibrometers. Full field dynamic strain measurement is often tedious and time consuming and often involves scanning the specimen surface. Optics based methods are capable of measuring vibration in a region but require high quality cameras and may involve complicated data processing.

In chapter 2, we show that viscoelastic polymer adhesive coatings can be used to measure the vibration distribution and identify resonance mode shapes in the specimen in vibrothermography. Viscoelastic materials exhibit properties of both viscous and elastic nature. When subjected to vibration, these materials absorb energy and dissipate heat. We measure vibration coverage by applying a single layer of vibration damping adhesive to the specimen surface and vibrate the coated specimen. We show that hysteretic heating generated in the coating maps to the dynamic strain in the specimen. This is a simple method to measure the vibration coverage in a specimen in vibrothermography. This method can be used to optimize the excitation frequency and transducer position to maximize defect detectability in vibrothermography. We envisage this as a first step in any vibrothermography experiment to select an appropriate resonance to test at.

A common way to characterize these materials is to represent their Young’s Modulus as a complex number. The real part of the modulus, called storage modulus and the imaginary part, called loss modulus represent the elastic and viscous nature respectively. The complex modulus of viscoelastic materials is typically measured with a Dynamic Mechanical Analyzer (DMA). Time temperature superposition is applied to generate master curves of storage and loss modulii as a function of frequency [Ferry (2008)]. We developed a novel method to measure the loss modulus of viscoelastic coatings based on the hysteretic heating generated by the coatings, In chapter 3, we explain how the vibrational strain and thermal power dissipated by the coating can be used to calculate the loss modulus.
Thermal power is the total heat dissipation rate by the coating. We calculate the thermal power from the measured surface temperature using Green’s function solution of heat equation [Beck et al. (1992)]. The thermal power dissipated by the coating is proportional to the loss modulus, dynamic strain amplitude square and the vibration frequency. The loss modulus is then calculated using the measured vibrational strain and the calculated thermal power. We compare the calculated loss modulus against that measured using DMA.

1.3.1.2 Compliant Couplant and Isolators Eliminate Sympathetic Resonances

We observed that using a couplant material in vibrothermography inspection not only prevents surface damage to specimen, but also improves the repeatability of specimen vibration and makes the resonance frequencies independent of the transducer used.

In order to understand the role played by the couplant, we model the excitation system as a simple equivalent electrical circuit and calculate the specimen velocity using the principles of electric circuit analysis. We show that when the mobility of the couplant is much higher than that of the transducer and specimen, specimen velocity is no longer influenced by the transducer resonance structure.

Similar to couplant, compliant isolators eliminate sympathetic resonances between mount and specimen. In addition, we observed that pre-deforming the isolators with a static load higher than any used in the testing results in a more consistent specimen vibration. We show that isolators behave like non-linear springs acting in parallel with the specimen and increase the effective specimen stiffness. Therefore, adding isolators results in an increase in resonance frequency. We observed that resonance frequency increases with the amount of static pre-load on the isolators.

Chapter 4 gives more details about the characterization of couplant and isolators and justifies why compliant couplant eliminates sympathetic resonances.
1.4 Numerical Modeling of Vibrothermography

Numerical models are necessary to simulate a process which in itself is not repeatable experimentally. Such models are useful for developing physics based probability of detection (POD) analysis. Numerical models are also useful in assisting practitioners in optimizing inspection parameters in a time and cost efficient manner.

Several researchers have performed full simulation of vibrothermography process [Mian et al. (2004); Mabrouki et al. (2009); Morbidini et al. (2006); Pieczonka et al. (2010)]. Mian et al. [Mian et al. (2004)] simulated the response of a delamination in a graphite/epoxy composite specimen to a sonic IR pulse. They modeled the delamination as two contacting faces and calculated the frictional energy dissipated at the interface and the resulting surface temperature. Pieczonka et al. (2010) performed numerical simulation to detect impact damage in composite specimen, also based on frictional heating. However, they modeled the damage based on measured X-radiography testing. Mabrouki et al. [Mabrouki et al. (2009)] developed a numerical model to calculate crack heating in steel specimens based on Coulomb friction between contacting crack faces. Morbidini et al. [Morbidini et al. (2006)] predicted crack heating numerically based on vibration damping measurements. They calculated the vibrational loss factor due to the presence of a crack and calculated the resulting energy dissipation which results in crack heating.

The common feature in all the works cited above is that their analyses were nonlinear and were performed in time domain. At high excitation frequencies, the computational cost can get prohibitively expensive, often restricting the usability of the model to academic purposes. Furthermore, flaws (delamination, crack or impact damage) were characterized by a single universal parameter in the form of either a friction coefficient or a loss factor. However, the physics behind heat generation in vibrothermography is too complicated to be accurately characterized by a single parameter. Therefore, there is a need for a more realistic physics based crack heating model for vibrothermography that is computationally feasible yet reasonably accurate.
1.4.1 VibroSim

VibroSim is a physics based simulation tool developed at ISU to simulate vibrothermography process. Given a specimen and a crack configuration, VibroSim predicts crack heating at the specimen surface. VibroSim implements a hybrid numerical and empirical model we developed and simulates all the three steps involved: vibration, crack heat generation and heat flow. Vibration and heat flow are modeled numerically and the crack heat generation is represented by an empirical model determined experimentally. VibroSim is implemented using COMSOL Multiphysics and MATLAB software tools [COMSOL Inc (2015); MATLAB (2014)].

Vibration model of VibroSim predicts the resonance frequency and dynamic strain on the specimen surface. These parameters, along with experimentally calculated intrinsic crack parameters like closure gradient and crack mobility are the inputs for the empirical model. The empirical model represents the crack heating as a product of vibration, crack mobility and closure gradient, \( p = v \times m \times c \) where \( p \) is the total heat dissipation rate (thermal power) at the crack, \( v \), \( m \) and \( c \) denote the contribution from vibration, crack mobility and crack closure respectively. Thermal power is calculated from the numerically calculated vibration parameters and experimentally determined mobility and closure gradient. Surface temperature is then evaluated numerically from the thermal power. Further details about the empirical model are out of scope of the current dissertation and are explained elsewhere in a greater detail by Lesthaeghe et al. [Lesthaeghe et al. (2015)].

Since this is a forward numerical model, all the three components: vibration, crack mobility, and crack closure contribute towards crack heating and hence it is critical to model all the aspects accurately. Error in any of the three aspects results in an error in the final outcome of the model.

1.4.1.1 Quantitative Vibration Model for Vibrothermography

The core topic of this dissertation is quantitative vibration modeling of specimen vibration in vibrothermography (the vibration aspect of VibroSim). In order to develop a quantitative model, all factors that affect specimen vibration must be understood. Once implemented, the
model must be validated against experiment to ensure the accuracy of underlying assumptions and hypotheses.

In this research, we implement a linear harmonic model of specimen vibration using COMSOL Multiphysics and MATLAB software.

The metrics to evaluate the vibration model are resonance frequency and vibration amplitude. As discussed in section 1.2, specimen in vibrothermography is vibrated at resonances. Extrinsic factors such as mounting and excitation affect resonance frequency. Vibration amplitude is determined by the amount of damping present in the system. Primary sources of damping are: isolators, couplant, viscoelasticity in specimen, presence of crack, friction at contact points, radiation losses etc. Therefore, to develop a precision vibration model, it is critical to understand the effect of mounting, exciter coupling and the damping introduced thereby.

In chapter 4, we explain how couplant and isolators are modeled. We show that when sufficiently compliant couplant and isolators are in place, mount can be modeled as a rigid foundation supporting the specimen via isolators. Isolators can be therefore modeled as non-linear spring foundation boundary condition and transducer excitation can be modeled as a prescribed displacement boundary condition. Damping is included in the model as complex stiffness of isolators and couplant. We experimentally measure the stiffness of isolators and couplant and develop an inversion algorithm to calculate the corresponding loss factors. The numerical model is then validated against independent vibration experiments.

1.4.1.2 Estimation of Thermal Power From Measured Surface Temperature

The last stage in our hybrid model is modeling the heat flow. The output of empirical model is the heat dissipation rate at the crack, also called as thermal power. When the specimen vibrates, the contacting crack faces generate heat which flows towards the specimen surface. To estimate various terms of our empirical heating model, we must be able to calculate the thermal power from the measured temperature maps of the crack. This is an inverse problem in which thermal power must be estimated from the measured temperature rise. However, inverting the measured temperature to calculate thermal power is an ill-posed problem because
of the diffusive nature of heat flow. Instead, we invert the heat map immediately after the heat flow starts and calculate the thermal power. At this time scale, since there hasn’t been sufficient time for heat to diffuse, the crack heating can be resolved better and regions of heat generation can be identified with relative ease. In chapter 5, we describe an algorithm to calculate the thermal power from the measured heat maps using linear inversion. We show that quantitative thermal power can be calculated from the measured surface heating at the crack.
CHAPTER 2. VISCOELASTIC ABSORPTIVE COATINGS FOR FULL FIELD VIBRATION COVERAGE MEASUREMENT IN VIBROTHERMOGRAPHY

A paper to be submitted to *NDT&E International*

Jyani S. Vaddi, Stephen D. Holland and Michael R. Kessler

2.1 Abstract

Vibrothermography is a rapidly emerging nondestructive evaluation technique for detecting fatigue cracks in aircraft components. Externally applied mechanical vibrations in a specimen generate crack heating due to the friction between vibrating crack faces. Crack heating in vibrothermography depends on the vibrational stresses, and therefore the amount of vibration generated in the specimen. A method to measure the vibration distribution from the hysteretic heating of viscoelastic polymer adhesive applied to a specimen surface is described. The heat generated by the adhesive coating maps directly to the vibrational strain distribution in the substrate. This is a fast method of vibration coverage measurement for vibrothermographic testing.

2.2 Introduction

Vibrothermography is a nondestructive evaluation method for finding cracks in aircraft components by vibration induced frictional heating. This method is effective in finding surface cracks in aircraft components like turbine blades and turbine disks and subsurface cracks such
as delaminations in composite panels [Reifsnider et al. (1980); Bolu et al. (2010); Han et al. (2010)]. When a specimen with a crack is subjected to mechanical vibrations, the friction between rubbing crack faces generates heat which is detected by an infrared camera. Vibrations are usually generated using a broadband piezoelectric transducer or an ultrasonic welder/horn assembly [Favro et al. (2000); Holland (2007)]. The amount of heat generated by the crack depends on the dynamic vibrational strain and excitation frequency [Li et al. (2011)]. So a quick and easy way to measure dynamic strain is important. Since the specimen vibrates primarily in resonant modes, the vibrational strains are not uniform across the specimen and the probability of detecting a crack located near the node of a resonant mode is much smaller than that for a crack located at an antinode. Therefore, to be able to predict whether a crack at a location is detectable when vibrated at a given resonance, we must know the vibration distribution in the specimen. The existing non-contact methods of full field dynamic vibration measurement are either too time consuming or are not practical for application in vibrothermography. Strain gages are good for point measurement but covering the entire specimen with strain gages in order to get full surface strains is highly impractical. Laser velocimetry methods [Renshaw and Holland (2008)], although dynamic and non-contact, require scanning the entire specimen surface three times to get the full vibration field which is time consuming and error prone. Other optical and interference based methods are not practical for measuring strains at acoustic and near ultrasonic frequencies used in vibrothermography since that would require high frame rate monitoring cameras and cumbersome data processing [Hung (1982); Preston Jr and Kreuzer (1967)]. Finite element based methods are also time consuming and typically unable to accurately model the complicated transducer-specimen coupling and, up until now, have not been very accurate for modeling the vibration.

In this paper, we propose a method of vibration distribution measurement for vibrothermography using viscoelastic absorptive coatings. We define the term ‘vibration coverage’ to describe how completely the vibration is distributed in the specimen. Ideally, vibration coverage is a quantitative mapping between the dynamic strain on the specimen surface and the resonant mode shape of the specimen for a given vibrothermographic excitation. Having the knowledge of vibration coverage for a given specimen configuration enables one to determine
if cracks present in the regions of interest are detectable at that test configuration. Practi-
tioners have used tapes and adhesives as qualitative indicators of vibration for a long time, but no procedure for measuring vibration coverage has been reported so far. Quantitatively determining the vibrational strain across the entire specimen surface will be a critical step for performing crack detectability and probability of detection studies for vibrothermography.

We measured vibration coverage by applying an absorptive adhesive coating on the specimen surface and observing the heating of this coating when the specimen was vibrated. We quantitatively correlated the vibrational strains in the specimen with the temperature rise in the adhesive coating for a simple rectangular bar geometry. Extrapolation of these results for more complicated geometries can be used to achieve qualitative mapping of vibration coverage.

### 2.3 Absorptive Viscoelastic Coatings

Viscoelastic materials are a class of materials that exhibit both viscous and elastic nature. When subjected to a load, the relaxation effects inside the material cause the deformation to lag the applied load resulting in hysteresis. These materials are modeled as having complex Young’s modulus $E^* = E' + jE''$ that is a function of temperature and frequency [Ferry (2008)]. The real part of the modulus, called storage modulus ($E'$) is equivalent to the Young’s modulus of purely elastic materials and it determines the in-phase elastic strain in the material. The imaginary part, called loss modulus ($E''$) determines the out-of-phase(viscous) component of strain. The quantity $\delta \equiv \text{ArcTan} \left( \frac{E''}{E'} \right)$ is the phase angle between stress and strain, $\delta$ being $0^\circ$ for elastic materials, $90^\circ$ for viscous materials and $0^\circ < \delta < 90^\circ$ for viscoelastic materials. The frequency dependent modulus, $E$ can be measured experimentally using dynamic mechanical analysis [Corsaro and Sperling (1990)]. When a viscoelastic material is subjected to oscillatory loading, stress in the material leads the strain and the viscous relaxation, represented by the imaginary part of the Young’s modulus is determined by the strain rate. In this loading, only a fraction of the energy applied to the material is recovered at the end of the cycle and the rest of it is dissipated as heat [Brinson and Brinson (2008)]. This behavior is analogous to an AC circuit in electrical engineering [Huelsman (1972)]: stress and strain in the material are
complex exponentials analogous to voltage and current. The recovered energy after one cycle is analogous to reactive power and the dissipated heat is analogous to real power. The ratio of energy dissipated to energy recovered is determined by the value of \( \tan \delta \), which is analogous to the power factor of an AC circuit.

When an elastic specimen undergoes a cyclic excitation, the specimen responds in phase with the loading and all the energy stored in the specimen during the positive half cycle of loading is released during the negative half cycle. Therefore, the net energy dissipated during the vibration cycle is zero. However, if the same specimen is coated with a layer of viscoelastic material and is again subjected to the same cyclic loading, the specimen does not respond in phase with the applied load and all the energy is not restored in the material at the end of the cycle; a part of it is dissipated as heat. This delayed viscous relaxation creates hysteresis in the specimen. The thermal power dissipated per unit volume, \( g \) due to hysteresis is calculated by solving the integral:

\[
g = \sum_i \frac{1}{\tau} \int_0^\tau \sigma_i \epsilon_i dt \tag{2.1}
\]

where \( \tau \) is the time period of oscillation and the summation index \( i \) is over all the components \( \sigma_i \) and \( \epsilon_i \) of the stress and strain tensors respectively. Dot indicates time derivative. If the loading or deformation is uniaxial, all but one term in the above summation vanish and the integrand contains the only nonzero stress or strain corresponding to the loading axis. Based on the above relation, when the axis of stress or deformation is known, the thermal power dissipated by the coating can be used as a metric to quantify the dynamic vibrational strain in the substrate. For a generalized multi-axial loading, evaluation of the above summation over all stress and strain components can be simplified by the use of the first and second strain invariants \( J_1 = tr(\epsilon) \) and \( J_2 = \frac{1}{2} \left( tr(\epsilon^2) - tr(\epsilon)^2 \right) \) \cite{Reismann1980}. In this case, thermal power only is not adequate to quantify both the strain invariants as it would be an ill-conditioned problem. We suggest that using at least two independent measurements i.e., absorptive coatings and another independent strain measurement method (like photoelastic coatings that calculate the difference of principal strains \cite{Gerber2011}) would solve this problem and quantify the strain invariants for more general multiaxial loading. In this
paper, we measure dynamic strains for uniaxial loading of a rectangular bar only and give a qualitative vibration coverage analysis for generalized multiaxial loading.

To quantify vibration coverage, we tested rectangular bar shaped metal specimens excited at flexural resonant frequencies. In a flexural resonance, the axial strain at the surface of the substrate is much higher than all other strain components and hence the loading becomes effectively uniaxial (neglecting all other strain components). The integral of equation 2.1 for the case of axial loading becomes:

\[
g = \frac{1}{\tau} \int_{\tau} \text{Re} \left( \frac{1}{2} \sigma \dot{\epsilon}^* \right) dt \quad (2.2)
\]

where \( \epsilon = \epsilon_0 e^{j(2\pi f)t} \) is the dynamic cyclic strain in longitudinal direction, \( \epsilon_0 \) is the strain amplitude, \( t \) is the time, \( \tau \) is the time period of excitation and \( f = \frac{1}{\tau} \) is the excitation frequency in Hz. The asterisk symbol indicates complex conjugate. Substituting Hooke’s law for stress \( \sigma = \epsilon (E' + jE'') \), the integral becomes

\[
g = \frac{1}{\tau} \int_{\tau} \text{Re} \left( \frac{1}{2} (E' + jE'') \epsilon \dot{\epsilon}^* \right) dt \quad (2.3)
\]

Replacing the strain derivative \( \dot{\epsilon} \) by \( j\omega \epsilon \) for cyclic loading and evaluating the above integral gives the thermal power \( g \). If the substrate has no adhesive layer on it, the Young’s modulus is purely real (\( E'' \) is zero) and the thermal power in the integral evaluates to zero. With the coating applied, however, the thermal power is

\[
g = \pi E'' \epsilon_0^2 f \quad (2.4)
\]

We model strain in the coating as determined by strain on the surface of substrate. Therefore, from equation 2.4, by measuring the thermal power dissipated by a coating with a known loss modulus at a known vibration frequency, we can quantify the vibrational strain in the coating. Following our model, this strain is same as that on the surface of the metal bar. The thermal power dissipated by the coating is a measure of vibrational strain coverage in the substrate underneath.

Temperature rise in the coating increases with loss modulus and coating thickness. However, substantially thicker coatings shift the resonance frequencies of the specimen due to the added
mass and therefore do not represent the specimen’s natural resonance behavior. Also, our assumption that strain is uniform across the thickness of the coating ceases to be valid if the thickness becomes comparable to that of the substrate or is not much smaller than the lateral feature size. On the other hand, coatings that are too thin do not respond to low strain amplitudes and do not dissipate detectable amount of heating. Adding multiple layers of thin coatings increases the heat generated.

Since temperature rise is a more directly measurable quantity than the thermal power, we can rewrite equation 2.4 in terms of peak temperature rise instead of thermal power. The coatings follow heat conduction equation [Carslaw and Jaeger (1959)]:

\[ \alpha \frac{\partial^2 T(x,t)}{\partial x^2} + \frac{1}{\rho c} g = \frac{\partial T(x,t)}{\partial t} \]  

(2.5)

where, \( T \) is the temperature rise in the coating in °C, \( g \) is the thermal power in \( \text{W} \text{m}^{-3} \), \( x \) is along the direction of heat propagation (coating thickness), \( \alpha \) is the thermal diffusivity \( (\text{m}^2 \text{s}^{-1}) \), \( \rho c \) is the volumetric heat capacity of the coating \( (\frac{1}{\text{m}^3 \text{K}}) \) and \( t \) is the time (s). Rearranging equation 2.5, we get

\[ g = \rho c \left( \frac{\partial}{\partial t} - \alpha \frac{\partial^2}{\partial x^2} \right) T(x,t) \]  

(2.6)

Since the derivative and Laplacian are linear operators, the thermal power dissipated varies linearly with the temperature rise observed at a given time and position. Therefore, if we replace the thermal power \( g \) in equation 2.4 with temperature rise and introduce a linear coefficient \( A \), equation 2.4 can be simplified as:

\[ \Delta T = A \epsilon_0^2 f \]  

(2.7)

where \( \Delta T \) is the peak temperature rise of the coating. The coefficient \( A \) depends on the vibration frequency, coating thickness and mechanical and thermal properties of the coating. To further simplify equation 2.7, we introduce an unknown parameter \( \beta \) that encompasses the frequency dependence of the heating within the vibration frequency range of interest, and replace the coefficient \( A \) by \( A_0 \), which is now just a material and thickness dependent constant independent of vibration frequency. Equation 2.7 now is rewritten as:

\[ \Delta T = A_0 \epsilon_0^2 f^\beta \]  

(2.8)
The unknown parameters in this equation are the coefficient $A_0$ and the exponent $\beta$ and the measurable quantities are the temperature rise and dynamic strain. These parameters can be estimated by performing vibration measurements at different excitation frequencies and dynamic strains and fitting equation 2.8 to the measured quantities $\epsilon_0$ and $\Delta T$. We can then use this equation to correlate the heat generated by the coating to the vibrational strain in the substrate. The experiment procedure and the results of fitting the model are explained in the subsequent sections. Once these parameters have been determined for a particular viscoelastic coating, equation 2.8 can be used to measure vibrational strains across the entire specimen surface using only the measured coating heating, the excitation frequency and the estimated model parameters $A_0$ and $\beta$.

### 2.4 Experiment

We performed experiments to calculate the parameters $A$ and $\beta$ for two viscoelastic coatings in the frequency range of interest for vibrothermography. Removable polymer adhesives were used as coatings because of their ease of application and removal on metal surfaces. We tested various types of hot melt adhesives, acrylic transfer tape, silicone adhesive tapes and acrylic vibration damping tapes. Among these, the acrylic vibration damping tapes VHB4914 and VHB4905 (manufactured by 3M inc) performed most optimally in terms of both heat dissipation and usable frequency range. All the subsequent results we show in this paper are for these two coatings. Table 2.1 shows the representative properties of these coatings.

We performed two sets of experiments to characterize the coatings and to measure vibration coverage. Figure 2.1 shows the set up for both the experiments. In the first set of experiments (figure 2.1a), we verified that heat dissipated by the viscoelastic coatings indeed describes the vibration coverage of the substrate it is applied to. Using this data, we quantitatively characterized the relation between the coating heating, dynamic vibrational strain and frequency and calculated the parameters $A$ and $\beta$ in equation 2.8 for both the coatings. We used two rectangular bar specimens made of a titanium alloy as substrates: Specimen A with dimensions $228.2 \times 27.3 \times 7.2$ mm and specimen B with dimensions $153.7 \times 25.3 \times 6.6$ mm.
Table 2.1  Manufacturer specifications of the two vibration damping tapes used for this research.

<table>
<thead>
<tr>
<th>coating name</th>
<th>material</th>
<th>color</th>
<th>thickness (mm)</th>
<th>density ($Kg/m^3$)</th>
<th>frequency range (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VHB4914</td>
<td>Acrylic foam</td>
<td>White</td>
<td>0.25</td>
<td>800</td>
<td>12-45</td>
</tr>
<tr>
<td>VHB4905</td>
<td>Acrylic foam</td>
<td>Clear</td>
<td>0.5</td>
<td>960</td>
<td>15-45</td>
</tr>
</tbody>
</table>
Figure 2.1  Experimental setup used for measuring vibration coverage. (a) sample A with a VHB4914 vibration damping tape coating mounted at nodal points. (b) gas turbine blade with a VHB4914 vibration damping tape coating on the airfoil. (c) schematic of rectangular bar specimens with coordinate axes shown. Curved arrows indicate the direction of applied vibration. The flexural strain in the specimen for this excitation is along the X axis.

We first predicted the natural frequencies and mode shapes using an open source finite element analysis software, CalculiX [Dhondt and Wittig (1998)]. Then we performed experiments both to validate the finite element calculations and to measure the heat generated by the coating.

Both the specimens were tested at their natural resonant frequencies as in a real vibrothermographic test. A broadband excitation system was used to generate vibrations at the desired frequencies in the specimen [Vaddi (2011)]. A layer of cardstock material was used as a couplant between transducer tip and the specimen to eliminate transducer-specimen combined system resonances [Vaddi et al. (2011)]. A laser vibrometer measured the specimen vibrations while a calibrated infrared camera was used to measure the temperature rise in the coating.

The specimen was mounted at nodes of the resonant mode shape using compliant rubber pins to ensure that the natural resonances are not severely affected by mounting. We first
measured the motion across the entire specimen surface using a scanning laser vibrometer and verified that the measured vibrational mode shapes and the resonance frequencies are in agreement with those predicted from finite element analysis. We then applied single frequency tone burst excitations with varying vibration amplitudes at 7th(16.59 kHz), 8th(20.1 kHz), 10th(30.49 kHz) and 11th(35.68 kHz) flexural resonances for sample A and 4th(12.38 kHz) and 5th(18.34 kHz) flexural resonances for sample B. All the excited resonances induced primary displacements in the Z direction (see figure 2.1c). Temperature rise in the coating was measured from the thermal images by subtracting the ambient temperature at every pixel on the coating surface. Vibrational strain in the coating was measured by scaling the measured displacement at a point by the ratio of strain and displacement calculated according to finite element software at a vibration point [Holland et al. (2008)].

In our second set of experiments, we measured the vibration coverage of the airfoil of a gas turbine blade using VHB4914 coating. We applied a uniform layer of the coating to the airfoil and securely mounted the blade in a mounting fixture using rubber pins as shown in figure 2.1b. We first applied a frequency sweep excitation and identified the peaks corresponding to different resonances of the blade. We then applied a single frequency excitation and measured the coating heating at each of the resonances in the frequency range between 19 kHz and 38 kHz, just like the rectangular bar specimens.

2.5 Results

Figure 2.2 compares the observed heating of the coating and the measured vibrational mode shape at various resonance frequencies for specimen A. For each mode, the top image in the red and black colormap shows the temperature rise in the coating and the bottom image in the rainbow colormap shows the vibration profile measured using laser vibrometer\(^1\). The bright red regions in the coating heating indicate high temperature rise and the red and blue regions in vibration image indicate high vibrational strain in the substrate (antinodes). The black regions

\(^1\)The heating and vibration profiles shown are not across the entire specimen length. A part of the specimen surface along the outer ends could not be imaged as it was obscured by the mounting points (see figure 2.1a)
in the coating heating indicate zero temperature rise and the green regions in the vibration image indicate zero vibrational strain. From these figures, it can be seen that the temperature rise is maximum at vibrational antinodes and is almost zero at nodes and the heating pattern in the coating matches exactly with the mode shape of the specimen. Figure 2.3 shows the observed heating pattern in the airfoil at various excited resonances. Heating was not observed in some of the modes where vibration was not sufficiently high. The high heating in a few modes along some portions at the corners (bright yellow spots) was because the coating did not bond well with the specimen in those regions and the strain in coating alone was much higher than the rest of the surface (attached to the substrate) resulting in higher heat dissipation. Our general observation is that if the coating does not adhere well to the specimen, the strain is not continuous and the coating exhibits very high strain and temperature rise at the peel points.

We used the measured temperature rise ($\Delta T$) in the coating, measured vibrational strain ($\epsilon_0$) in the substrate and known excitation frequency ($f$) to calculate the unknown coefficients $A_0$ and $\beta$ in equation 2.8. From a linear least squares fit of the measured data to equation 2.8, the estimated values of $A_0$ and $\beta$ for VHB4914 and VHB4905 were determined and are shown in table 2.2 [Kay (2009)].

Figure 2.4 shows the plots of temperature rise in the coating against measured strain at
Figure 2.3  vibration coverage in a turbine blade measured using VHB4914 coating at different resonant modes from 19 to 38 kHz.

various frequencies. Plots in figure 2.4a are for VHB4914 coating and the plots in figure 2.4b are for VHB4905 coating. The scatter plots represent measured value and the solid curves represent the value from the fit. The value of $\beta$ is higher than 1.0 for both the coatings to account for the frequency dependence of the viscoelastic heating, as discussed earlier. This additional frequency dependence comes from loss modulus of the coating, as shown in equation 2.4. In order to confirm this frequency dependence, we independently measured the loss modulus of both coatings using dynamic mechanical analysis and time-temperature superposition [Ferry (2008)]. The results of this measurement are shown in figure 2.5.

The experiments show that hysteretic heating in viscoelastic polymers indeed follows the model described in equation 2.8 and strain calculations using this model for uniaxial loading are valid. With the parameters $A_0$ and $\beta$ estimated for any viscoelastic coating, this model can be used to estimate uniaxial vibrational strain in the substrate given the temperature rise in the coating under similar test conditions.
Table 2.2 parameters of power law model for the coatings VHB4914 and VHB4905 calculated from experimental measurements of strain and coating heating at various flexural resonant frequencies.

<table>
<thead>
<tr>
<th>Coating Type</th>
<th>$\log_{10}(A_0)$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VHB4914</td>
<td>1.13 ± 0.17</td>
<td>1.22 ± 0.03</td>
</tr>
<tr>
<td>VHB4905</td>
<td>1.22 ± 0.23</td>
<td>1.16 ± 0.05</td>
</tr>
</tbody>
</table>

Figure 2.4 Temperature rise in the rectangular bar samples plotted against measured strains at various frequencies using (a) VHB4914 coating and (b) VHB4905 coating. The scatter plots are experimental data and the solid curves are the corresponding analytical expressions calculated from the least squares fit. The frequencies 12.3 kHz and 18.1 kHz correspond to 4th and 5th resonant modes of specimen B and the frequencies 16.5 kHz, 20.8 kHz and 30.5 kHz correspond to 7th, 8th and 10th modes of specimen A.

2.6 Conclusions

Viscoelastic materials dissipate heat proportional to the excitation frequency and square of the vibrational strain when they are subjected to sinusoidal loading. Vibration coverage in specimens for vibrothermography can be measured using this property of viscoelastic coatings. This method can serve as a qualitative test to optimize transducer position and excitation frequency in order to maximize vibration generation and defect detection in vibrothermography. Where the strain or stress is known to be uniaxial, or when combined with an independent dynamic strain measurement, this method can quantify the dynamic vibrational strain in vi-
Figure 2.5  Loss modulus measurement of VHB3905 and VHB3914 coatings. The useful frequency range for vibrothermographic testing is between 10 kHz and 40 kHz.

2.7 Acknowledgments

This material is based upon work supported by the Air Force Research Laboratory under Contract #FA8650-04-C-5228 at Iowa State University’s Center for Nondestructive Evaluation.
CHAPTER 3. THERMAL MEASUREMENT OF LOSS MODULUS OF A POLYMER ADHESIVE ON A METAL SUBSTRATE AT ACOUSTIC AND ULTRASONIC FREQUENCIES

A paper to be submitted to Polymer Testing

Jyani S. Vaddi, Stephen D. Holland and Michael R. Kessler

3.1 Abstract

In this paper, a novel method is proposed to measure the loss modulus of viscoelastic polymer adhesives at kilohertz frequencies. Resonant mechanical vibrations were generated in a metal substrate coated with a layer of polymer adhesive. The vibrating coating, because of its absorptive nature, generates hysteretic heating. This heating is proportional to the loss modulus, the dynamic strain in the coating and the vibration frequency. The measured temperature rise in the coating was inverted to calculate the thermal power dissipated. The loss modulus was then calculated from the thermal power and the vibrational strain in the coating as a function of vibration frequency. This method is demonstrated with two different polymer adhesives at several frequencies in the range of 12 - 30 kHz and these values were compared with results from a dynamic mechanical analysis experiment.

3.2 Introduction

Viscoelastic materials exhibit characteristics of both elastic and viscous materials and are absorptive in nature. When subjected to load, relaxation effects in the material cause the
deformation to lag the load. A common model to characterize these materials is to represent their Young’s modulus as a complex number and a function of frequency and temperature. The real and imaginary components of the modulus, called *storage modulus* and *loss modulus* represent the elastic and viscous nature of the material respectively. Young’s modulus is often measured using dynamic mechanical analysis (DMA) [Menard (2008)]. Oscillatory loads are applied to the specimen at a pre-defined set of frequencies at various temperatures inside a temperature controlled chamber and the response is measured. Young’s modulus is calculated from this data using the principle of ‘Time-temperature superposition’ (TTSP), which explains the equivalence of time and temperature domains in viscoelastic materials [Ferry (2008)]. This is often a lengthy experimental procedure and takes multiple tests to measure the frequency dependent modulus. In this paper, we introduce an alternate method to measure the loss modulus of a polymer adhesive at acoustic and near ultrasonic frequencies. We use hysteretic heating generated by the coating due to viscoelastic absorption to determine its loss modulus.

When a solid is subjected to a load, the energy required to deform a unit volume of that solid is called ‘strain energy density’. If the solid is perfectly elastic and the loading is cyclic, the total energy dissipated by the solid is zero. However, when cyclic loading is applied to a viscoelastic material, relaxation effects within the material cause the deformation to lag behind the applied force resulting in hysteresis in the material. Energy is lost to the deformation process which results in net heat energy dissipation at the end of each loading cycle. The total power dissipated by a solid subjected to uni-axial cyclic loading is

\[
g = \frac{1}{\tau} \int_0^\tau \text{Re} \left( \frac{1}{2} \sigma \epsilon^* \right) dt
\]

where, \(\sigma\) and \(\epsilon\) are the elastic stress and strain components along the loading direction and \(\tau\) is the time period of cyclic loading. Here, asterisk indicates complex conjugate and dot indicates time derivative. For cyclic loading, the time derivative of strain can be written as \(\dot{\epsilon} = j2\pi f \epsilon\) where \(f\) is the frequency of loading in Hz. Writing the stress in terms of strain, we can rewrite this equation as

\[
g = \frac{1}{\tau} \int_0^\tau \text{Re} \left( \frac{1}{2} \left( E' + jE'' \right) \epsilon \left( j\omega \epsilon \right)^* \right) dt
\]

where \(E = E' + jE''\) is the complex Young’s modulus. If the material is perfectly elastic,
the imaginary part of the Young’s modulus is zero and this integral evaluates to zero. For viscoelastic materials, however, this integral evaluates to

$$g = \pi f E'' \epsilon_0^2$$  

(3.3)

where $\epsilon_0$ is the strain amplitude [Brinson and Brinson (2008)].

Based on equation 3.3, the loss modulus at any frequency can be calculated if the vibrational strain and the average power dissipated in a cyclic loading are known. In order to measure the loss modulus, we apply a layer of the polymer adhesive to a metal substrate and apply mechanical vibrations on the substrate. The heat generated by the coating is measured using an infrared camera. The dynamic strain is measured using a laser vibrometer. Based on these measured quantities, our method of calculating the loss modulus is as follows:

1. measure the heat intensity of the coating in response to vibration,
2. measure the vibrational strain in the substrate,
3. calculate loss modulus from the measured thermal power and vibrational strain.

The following section gives a more detailed description of this process.

3.2.1 Loss Modulus Measurement from Thermal Power and Vibrational Strain

The temperature rise in the polymer adhesive coating follows a well defined boundary value problem. Mathematically, we can approximate this as a case of one dimensional heat conduction with plane heat sources uniformly distributed across the coating thickness, whose intensity is the average power dissipated by the coating. Our goal is to solve this boundary value problem to calculate the average power required to generate the measured temperature rise in the coating. We begin by calculating the analytical solution for unit source heat intensities. We then fit this analytical solution to the measured temperature rise in the coating and determine the exact value of heat intensity. The interface between the adhesive and the metal substrate is modeled as a constant temperature heat sink (Dirchilet boundary condition, $T = 0$) and the other
surface of the polymer adhesive is modeled such that the net heat flux into the surroundings through this surface was zero (Neumann boundary condition, \( \frac{\partial T}{\partial x} = 0 \)). We also assume zero initial conditions since the coating starts to heat only after the vibration begins. The boundary value problem to solve is therefore:

\[
\alpha \frac{\partial^2 T(x,t)}{\partial x^2} - \frac{1}{\rho c g} \frac{\partial T(x,t)}{\partial t} = 0 \quad (3.4)
\]

\[
\frac{\partial T}{\partial x} = 0 \text{ at } x=0 \quad (3.5)
\]

\[
T = 0 \text{ at } x=0 \quad (3.6)
\]

\[
T = 0 \text{ at } t=0 \quad (3.7)
\]

where, \( T \) is the temperature rise in the coating (°C), \( g \) is the heat source intensity per unit volume (W/m³), \( L \) is the coating thickness (m), \( x \) is the coordinate along the direction of heat propagation (coating thickness), \( \alpha \) is the thermal diffusivity (m²/s), the product term \( \rho c \) is the volumetric heat capacity of the coating, (J/m³K) and \( t \) is time, (s).

We use Green’s function approach to solve this boundary value problem. The appropriate Green’s function for the specified boundary conditions, representing the temperature field caused by a thin planar volumetric heat source at \( x = x' \) which delivers an energy pulse at \( t = \tau \) is [Beck et al. (1992)]:

\[
G_{X21}(x,t|x',\tau) \approx \begin{cases} 
(4\pi\alpha(t-\tau))^{-1} \left( \exp \left[ \frac{-(x-x')^2}{4\alpha(t-\tau)} \right] + \exp \left[ \frac{-(x+x')^2}{4\alpha(t-\tau)} \right] - \exp \left[ \frac{-(2L-x-x')^2}{4\alpha(t-\tau)} \right] \right), \\
\text{if } \frac{\alpha(t-\tau)}{L^2} < 0.022 \\
\frac{2}{L} \sum_{m=1}^{3} \exp \left( \frac{\beta_m^2 \alpha(t-\tau)}{L^2} \right) \cos \left( \beta_m \frac{x}{L} \right) \cos \left( \beta_m \frac{x'}{L} \right), \\
\text{if } \frac{\alpha(t-\tau)}{L^2} > 0.022
\end{cases} 
\]

(3.8)

where \( \beta_m = \pi(m - 0.5) \). Using Green’s function approach, the solution to the boundary value problem in equations 3.4 through 3.7 is given by the convolution of equation 3.8 with the temporal and spatial distribution of the heat source. Calculating the convolution between the Green’s function and the heat source distribution gives the solution to the set of PDEs in equations 3.4 through 3.7. The convolution is calculated for unit intensity heat sources uniformly distributed across the entire coating thickness. The heat dissipation begins at the
onset of vibration \((t = t_0)\) and ends at the end of the vibration \((t = t_1)\). The heat source distribution is a rectangular function in both time and space whose limits are \(\tau = [t_0, t_1]\) and \(x' = [0, L]\) respectively:

\[
g(x', \tau) = \begin{cases} 
1.0 & t_0 \leq \tau \leq t_1, 0 \leq x' \leq L \\
0 & \text{otherwise}
\end{cases} \quad (3.9)
\]

The convolution between Green’s function and the unit intensity heat source distribution is:

\[
T(x, t) = \int_{t_0}^{t} \int_{0}^{L} (G_{X21}(x, t|x', \tau)) dx' d\tau \quad t_0 < t \leq t_1
\]

\[
= \int_{t_0}^{t_1} \int_{0}^{L} G_{X21}(x, t|x', \tau)) dx' d\tau \quad t > t_1
\]

Evaluating the spatial integral of the above equation using computer algebra, we get:

\[
T(x, t) = \int_{t_0}^{t} G_0(x, t, \tau) d\tau \quad t_0 < t \leq t_1
\]

\[
= \int_{t_0}^{t_1} G_0(x, t, \tau) d\tau \quad t > t_1
\]

where \(G_0\) is the result of evaluating the spatial integral:

\[
G_0(x, t, \tau) = \frac{2}{15\pi\rho c} \left[ 5 \exp \left( \frac{4(t-\tau)\pi^2\alpha}{L^2} \right) \sin \left( \frac{3(L-x)\pi}{2L} \right) + 5 \exp \left( \frac{4(t-\tau)\pi^2\alpha}{L^2} \right) \sin \left( \frac{3(L+x)\pi}{2L} \right) \\
+ 15 \exp \left( \frac{6(t-\tau)\pi^2\alpha}{L^2} \right) \sin \left( \frac{(L-x)\pi}{2L} \right) + 15 \exp \left( \frac{6(t-\tau)\pi^2\alpha}{L^2} \right) \sin \left( \frac{(L+x)\pi}{2L} \right) \\
+ 3 \sin \left( \frac{5(L-x)\pi}{2L} \right) + 3 \sin \left( \frac{5(L+x)\pi}{2L} \right) \right] \exp \left( -\frac{25(t-\tau)\pi^2\alpha}{4L^2} \right)
\]

Since no closed form solution exists for the temporal integral in equation 3.11, it has to be evaluated numerically to obtain the coating temperature as a function of \(x\) and \(t\).

The temperature rise due to unit intensity heat source distribution can be calculated by subtracting the ambient temperature from the analytical solution. To calculate the actual heat intensity dissipated by the coating, the temperature rise is scaled to match the experimentally measured heating and the resulting scale factor gives the actual dissipated heat intensity \(g\) of the coating.

The vibration along the primary axis of deformation is measured at a single point on the surface of the coating using a laser vibrometer. The ratio of any local strain component
anywhere on the substrate to the corresponding velocity at the vibrometer location can be extracted from the finite element calculation. This ratio, multiplied by the measured velocity from the vibrometer, gives the measured local dynamic strain component [Holland et al. (2008)]. The resonant mode shapes are calculated using CalculiX, an open source finite element software [Dhondt and Wittig (1998)]. We assume that the coating is perfectly bonded to the substrate surface and the dynamic strain in the coating is determined by the substrate. Using the measured thermal power $g$, measured vibrational strain $\epsilon_0$ and known excitation frequency $f$, loss modulus of the coating can be calculated from equation 3.3 as:

$$E'' = \frac{g}{\pi \epsilon_0^2 f}$$

(3.13)

### 3.3 Experiment

We measured the loss modulus of two different polymer adhesives VHB3914 and VHB3095 at several frequencies between the range of 12 kHz and 30 kHz. We used a 228 mm $\times$ 25 mm $\times$ 6 mm rectangular titanium alloy bar as the substrate. Table 3.1 lists the nominal properties of these adhesives as specified by the manufacturer.

<table>
<thead>
<tr>
<th>Adhesive</th>
<th>thickness (mm)</th>
<th>density (Kg/m$^3$)</th>
<th>color</th>
</tr>
</thead>
<tbody>
<tr>
<td>VHB4914</td>
<td>0.25</td>
<td>800</td>
<td>white</td>
</tr>
<tr>
<td>VHB4905</td>
<td>0.5</td>
<td>960</td>
<td>clear</td>
</tr>
</tbody>
</table>

The first set of our experiments was to measure the mode shape of the coated substrate and validate the resonance frequency against the numerical calculation. To measure the mode shapes experimentally, the substrate was subjected to swept frequency excitations as a laser vibrometer scanned across the vibrating substrate surface measuring the displacement at each point. Resonance frequencies were determined from the peaks in the vibration spectrum. Only transverse flexural resonance modes were used for measuring the loss modulus.

The substrate was subjected to single frequency vibrations at each of the selected resonance modes and the coating heating was measured. Figure 3.1 shows the experimental setup we used
Figure 3.1  Experimental setup used for measuring the loss modulus of a viscoelastic adhesive. A piezoelectric transducer generates mechanical vibrations in the specimen with adhesive coating on it. The infrared camera monitors the temperature of the coating surface and the laser vibrometer measures the vibration amplitude of the coating surface reflected off the mirror.

for this measurement. A broadband piezoelectric transducer was used to generate vibrations in the substrate. The excitation lasted for 1 second. A calibrated infrared camera was used to record the surface temperature of the coating as the substrate was vibrating. The camera started recording the temperature at t=0 s and captured 150 frames at a frame rate of 90 fps. The thermal images were later post processed to calculate the net temperature rise on the coating surface.

We also measured the Young’s modulus of both coatings independently using dynamic mechanical analysis on a TA Q800 DMA apparatus. We tested the adhesives with frequency sweeps of 1 Hz to 100 Hz at temperatures of -30°C to 30°C with 10°C increments. We applied the principle of ‘time-temperature superposition’ (TTSP) at reference temperature of 20°C and generated master curves for loss modulus as a function of frequency at this temperature.

3.4 Results and Discussion

Figure 3.2a shows the observed heating in the coating. The bright red bands indicate regions of high coating heating and the dark black regions indicate regions where the coating did not heat. Figure 3.2b shows the vibration amplitude across the coating at the measured
Figure 3.2 (a) Temperature rise observed in VHB4914 acrylic adhesive film applied on a Ti-6-4 substrate subjected to resonant excitation at 20880 Hz 8th order flexural resonance. (b) vibration amplitude of the substrate surface measured with a scanning laser vibrometer; regions in blue indicate vibrational nodes and regions in red indicate vibrational anti-nodes. Horizontal axis is along the substrate length and vertical axis is along the substrate width.

From these figures, it can be observed that the coating generated high heat in the regions where vibration amplitude was high and almost no heat in the regions where the vibration amplitude was low. This is in accordance with theory that the heat generation in the coating depends on the vibrational strain in the substrate. Figure 3.3 shows the observed temperature rise as a function of time at a single point on the coating surface and the fitted Green’s function solution scaled to match measured heating. The temperature in the coating began to rise at the onset of excitation(0.2 s), continued to rise and started cooling down at the end of excitation(1.2 s). From the figure, and many others like it, the analytical solution fits well with the measured data and hence validates our thermal power calculation.
Table 3.2 lists the values of the loss modulii of VHB4914 and VHB4905 coatings in the frequency range 12-30 kHz measured from equation 3.13 and those measured using DMA measurement. Figure 3.4a shows a graphical comparison of loss modulus values between our method and DMA. Figure 3.4b illustrates the procedure for generating the master curve for loss modulus of VHB4914 coating using Time Temperature Superposition (TTSP). It is to be noted that since the relation between loss modulus and the vibrational strain is quadratic, any error in strain measurement affects the loss modulus data significantly. Also, when the coating is not an ideal black body, the calculated thermal power may not match its true value. The DMA measurement depends on several external factors such as pre load, oscillation amplitude etc. and is predicated on the assumptions behind TTSP. Despite these possible sources of measurement and calculation errors, the estimated values of loss modulii correlate well.

### 3.5 Conclusions

A new technique for measuring the loss modulus of a polymer adhesive was introduced. Resonant vibrations of the underlying substrate have been measured at various frequencies in
Figure 3.4  (a) Comparison of loss modulus measured from thermal power dissipation and DMA measurement for VHB4914 and VHB4905 adhesives at room temperature. Solid line shows the modulus of VHB4905 and dashed line shows the modulus of VHB4914 from DMA data. The circles and squares show modulus of 4905 and 4914 measured from thermal analysis respectively. Inset shows master curves for DMA measurement of 4914 and 4905 on a broader frequency scale. (b) Master curve for loss modulus of VHB4914 adhesive generated using time temperature superposition. The arrows indicate horizontal shifting of the loss modulii at all measurement temperatures to generate the master curve at a reference temperature of 20°C.

the range of 12-30 kHz. These values compare well with those measured using DMA. This method can be quicker than conventional DMA and still gives the loss modulus values with similar accuracy. This may be useful for materials where the accuracy of TTSP is questionable. One disadvantage of this method is that for the coating to generate detectable amount of heat, sufficiently high vibrational amplitude has to be applied to the substrate. This restricts the measurement frequencies only to the resonant frequencies of the substrate where it is possible to achieve high amplitude vibrations. Also, polymers with very low loss modulus may not generate detectable heat even at substantially high vibration amplitudes.
<table>
<thead>
<tr>
<th>Adhesive</th>
<th>frequency (Hz)</th>
<th>Loss modulus, thermal (MPa)</th>
<th>Loss modulus, DMA (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VHB4914</td>
<td>12380</td>
<td>81.5±0.82</td>
<td>92.2</td>
</tr>
<tr>
<td></td>
<td>16590</td>
<td>103.8±3.26</td>
<td>98.6</td>
</tr>
<tr>
<td></td>
<td>18350</td>
<td>94.2±1.81</td>
<td>100.9</td>
</tr>
<tr>
<td></td>
<td>20880</td>
<td>100.6±1.67</td>
<td>104.1</td>
</tr>
<tr>
<td></td>
<td>30490</td>
<td>101.4±1.13</td>
<td>113.4</td>
</tr>
<tr>
<td>VHB4905</td>
<td>12340</td>
<td>60.7±1.14</td>
<td>52.3</td>
</tr>
<tr>
<td></td>
<td>18140</td>
<td>57.1±0.74</td>
<td>57.5</td>
</tr>
<tr>
<td></td>
<td>20710</td>
<td>62.3±0.48</td>
<td>59.2</td>
</tr>
</tbody>
</table>

Table 3.2 Thermal measurement of loss modulii of VHB4914 and VHB4905 at all the measured frequencies.

### 3.6 Acknowledgments

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CHAPTER 4. QUANTITATIVE NUMERICAL MODELING OF SPECIMEN VIBRATION IN VIBROTERMOMOGRAPHY

A paper to be submitted to *Journal of Sound & Vibration*

Jyani S. Vaddi, Stephen D. Holland, Tyler J. Lesthaeghe, Gabriel Murray, John C. Aldrin

4.1 Abstract

Vibrothermography is a nondestructive evaluation method to detect cracks and delaminations, primarily in aerospace components. When the component to inspect is subjected to mechanical vibration, friction and/or adhesion hysteresis between the contacting flaw faces generates heat. Crack heating in vibrothermography depends on the amount of dynamic vibrational strain generated at the crack. High vibrational strains are generated in the specimen when vibrated at its natural resonances. However, extrinsic factors such as specimen mounting and excitation coupling affect specimen vibration and often makes the process non-repeatable and hence unpredictable. We propose a quantitative numerical model to predict specimen vibration in vibrothermography. We develop experimental procedures to improve the consistency and repeatability of specimen vibration and build the numerical model following these procedures. We represent the behavior of specimen and mounting materials as analogous electrical circuits and characterize specimen vibration using mobility theory. The numerical model predicts the vibration amplitudes and resonance frequencies of the specimen. Finally, we validate the numerical model against independent experiments.
4.2 Introduction

Vibrothermography is a nondestructive evaluation method for detecting cracks and delaminations, primarily in aerospace components [Reifsnider et al. (1980)]. The specimen is subjected to mechanical vibrations, during which friction and/or adhesion hysteresis at the contacting asperities generates heat. An infrared camera is used to detect the heat and hence the presence of flaw. Crack heating in vibrothermography is controlled by local vibrational strain at the crack, along with other parameters like vibration frequency, crack closure and crack roughness [Zhang (2010); Renshaw et al. (2009a, 2011)]. The vibrational strain is dependent on specimen geometry, resonances and other extrinsic factors such as specimen mounting and excitation transducer.

In this paper, we develop a quantitative numerical model for specimen vibration in vibrothermography. Accurate modeling of specimen vibration is a challenging task because vibration process is sensitive to extrinsic factors like specimen mounting and excitation and is highly variable. Therefore, to make the vibrothermography process amenable to modeling, the experiment procedures need to be adjusted to be more consistent and repeatable. We develop experimental procedures that make the vibration process more repeatable so that a numerical model can be developed on the basis of measured experimental data. In addition, we perform independent experiments to characterize the mounting materials and incorporate these into our numerical model. Based on these procedures, a numerical vibration model is developed using COMSOL Multiphysics and MATLAB software. The model is validated against independent experiments and the results are quantitatively compared.

4.3 Vibration Process in Vibrothermography

Specimen vibration in vibrothermography is generated using a piezoelectric transducer. Commercially available ultrasonic welder and horn assembly, in conjunction with a power amplifier is often used to excite the specimen. Contact non-linearity between the welder tip and specimen often results in chaotic and non-repeatable vibrations and therefore, it is hard to perform any quantitative analysis with this type of excitation [Han et al. (2004)].
Figure 4.1  Specimen velocity spectrum as a function of frequency. Peaks in the spectrum correspond to specimen resonance frequencies, as indicated by the arrows.

In order to overcome the limitation of generating non-repeatable vibrations when using welder excitation system, we developed a broadband excitation system for specimen vibration generation for vibrothermography [Holland (2007)]. The advantage of using broadband transducers is that they can be electrically controlled to vibrate the specimen explicitly at resonances instead of relying on contact non-linearity to excite various resonances. Therefore, this process gives more control over specimen vibration generation. Also, since high amplitude specimen vibrations can be generated at relatively low excitation voltages, this significantly improves the transducer life [Vaddi et al. (2012)].

The specimen is typically mounted in metallic grips and a static pre-load is applied to the mounts for support. Sheets of soft material called ‘isolators’ are placed between specimen and grips to provide vibration isolation. Mechanical vibration in the specimen is typically generated using a piezoelectric transducer with a layer of ‘couplant material’ sandwiched between the transducer tip and specimen. Practitioners have observed that using a layer of couplant improves the repeatability of their vibrothermography experiments and prevents surface damage to the specimen due to contact from the transducer.
Figure 4.2 Solid model of the experiment set up showing the specimen, transducer, isolators and the couplant. 25.4 mm diameter stainless steel rollers are used for specimen mounting. Five layers of 110 lb cardstock material are used as isolators and one layer of 110 lb cardstock material is used as couplant. The arrow on the top indicates the direction of laser vibrometer.

A laser vibrometer measures the specimen vibration at a single surface point. Calculating the time integral of measured velocity gives the displacement. The specimen is first vibrated with a broadband frequency sweep excitation and the surface velocity is measured as a time waveform. Fourier Transform of this velocity waveform gives the velocity spectrum. Peaks in the velocity spectrum correspond to resonance frequencies of the specimen. Figure 4.1 shows a typical measured specimen velocity spectrum, with the arrows pointing at resonance peaks. From the velocity spectrum, the desired specimen resonance is then selected and the specimen is again vibrated, this time with a tone burst excitation at resonance.

Figure 4.2 shows the set up we use for the vibration characterization experiments in this research. We use a single layer of 110 lb cardstock as couplant and five layers of 110 lb cardstock as isolator materials for this research. Stainless steel rollers are used as mounts and a broadband piezoelectric transducer is used to generate specimen vibration. Couplant is placed at the contact between transducer and specimen and four sets of isolators are placed at the contact between rollers and specimen. A laser vibrometer is mounted on the top to measure specimen vibration. The direction of the laser is indicated by the red arrow in the figure.
4.4 Procedures for Improving Vibration Consistency

Vibration variability comes from a number of factors:

1. System resonances which include sympathetic resonances of the specimen with the transducer and/or mounts.

2. Mounting variability and the complicated mechanical impedance characteristic of the mounts, including the stiffening effect of static load on nonlinear isolator materials.

3. Extreme non-linearity and chaos in the contact between transducer tip and specimen.

4. Variations in damping resulting from mounting, transducer coupling, and/or isolators.

These factors can be at least partially addressed by using carefully characterized compliant isolators and couplant as justified in detail below. In our testing we found that the use of 110 lb cardstock, pre-deformed with a static load larger than any used in the testing as isolators and couplant gave us relatively repeatable and predictable results. Our hypothesis is that since cardstock material is fibrous in nature, pre-deforming it compresses the fibers and eliminates any micro-voids present with in the material.

4.4.1 Theory of Compliant Mounts

The reduction in variability when using compliant couplant and isolators can be explained through the theory of mechanical mobilities [Beranek (1996)]. This theory represents linearized mechanical system components by their frequency-dependent ratio of relative velocity to force (mechanical mobility is the reciprocal of mechanical impedance). Mechanical assemblies can then be analyzed using mobilities (velocity/force) equivalently to how AC electrical circuits are analyzed using the theory of impedances (voltage/current). This is because Kirchoff’s current law (sum of currents into a node = 0) maps to Newton’s law (sum of forces on a mass less node = 0) and Kirchoff’s voltage law (sum of voltages around a loop=0) maps to the mechanical consistency requirement that the sum of velocity differences around a loop must sum to zero.

To illustrate why compliant mounts are important, consider the mechanical contacts between specimen and mount illustrated in Fig. 4.3a. The equivalent mechanical circuits are
shown in Fig. 4.3b. The vibrating specimen is represented by a velocity source $v_s$ (which represents the motion the specimen would have with no mount present) in series with the mechanical mobility of the specimen at the mount point $M_s$. With no compliant isolator material present, the velocity of the contact is $v_s M_m / (M_m + M_s)$ (velocity-divider formula, analogous to voltage-divider formula in electrical circuit theory) and this motion will be very large when $M_m + M_s$ is zero, which corresponds to system (“sympathetic”) resonances between the mount and specimen. Because the mount mobility is complicated, hard to predict, and dependent on the exact details of the mounting and apparatus, in this case the specimen response will also be hard to predict and dependent on the exact details of the mounting and apparatus.

However, when a compliant isolator material is present, the velocity of the specimen side of the contact is $v_s (M_m + M_i) / (M_m + M_i + M_s)$. So long as the magnitude of the isolator mobility, $M_i$ is larger than the magnitudes of $M_m + M_s$, it is no longer possible for the denominator to go to zero and thus system resonances between mount and specimen are broken. So long as the mobility of the mount $M_m$ is small by comparison to the isolator (i.e., it is much stiffer), the mount can be neglected and thus the mount can be modeled or simulated as a fixed boundary condition, shown in Fig. 4.4. The complicated frequency- and orientation-dependent dynamic response of the mounting apparatus can be replaced by a simple fixed-displacement boundary condition, so long as the isolator is sufficiently compliant.

We now explain the effect of isolator and mount on specimen vibration. We assume the isolator can be represented as an (absorptive) spring. When the specimen is vibrating at res-
Recall the equation for resonant frequency of mass on a spring system: \( f = \frac{1}{2\pi} \sqrt{k/m} \). As the effective stiffness \( k \) increases, the resonant frequency increases with \( \sqrt{k} \). While the specimen resonances are much more complicated than a simple mass on a spring, the same principles apply. The effect of the isolators on the specimen vibration is to increase the resonant frequencies of those resonant modes that have large displacements at the isolator positions.

4.4.2 Theory of Compliant Mounts: Experimental and Vibrational Simulation Results

To test the hypothesis that isolators act as springs, we performed both experiments and vibrational simulations and measured the resonance frequency as a function of the pre-load on
isolators. We tested a 246 mm × 26 mm × 13 mm Ti 6Al-4V alloy rectangular bar sample at the 9th order flexural resonance frequency. We placed five layers of 110 lb cardstock isolators in one case at resonant nodes and in the other case at resonant anti-nodes of the 9th flexural resonance of the sample and tested a range of normal pre-load values. Since the cardstock deformation is somewhat nonlinear under very large loads, the effective stiffness increases with load. As illustrated in Fig. 4.5, when the isolators were placed at nodal points, the resonant frequency only changed by 200 Hz as load was increased, indicating that with the isolator located at a point where deformation is zero, its spring-like behavior has little effect on the resonant frequency. In contrast, when the isolators were placed at antinodes – the positions of maximum motion – the resonant frequency increased by 1200 Hz with increasing load (increasing isolator stiffness).

We performed simulations of specimen vibration under the same conditions as experiments above based on the linearized stiffnesses of the cardstock at different static loads. In the simulation, the isolator stiffness has no effect on resonant frequency when placed at the resonant node whereas when placed at the the resonant antinode the effect on resonant frequency is substantial as illustrated in Fig. 4.6). This suggests that the effect of isolator is to increase

Figure 4.5 Experimentally Measured Shift of 9th Order Flexural Resonance Frequency With Static Load on Isolators.
Figure 4.6  Simulated Shift of 9th Order Flexural Resonance Frequency With Static Load on Isolators.

resonant frequencies slightly and to increase overall damping. Therefore in the numerical vibration model, isolators can be modeled explicitly as layers of compliant but absorptive material, or implicitly as absorptive spring boundary conditions on the specimen boundary.

4.5 Transducer Consistency and Modeling

The use of cardstock or a similar compliant material between transducer and specimen breaks specimen-transducer system resonances [Vaddi et al. (2011)]. It also reduces non-linearity and the potential for unpredictable “chaos” at the tip-specimen contact. Figure 4.7a shows the schematic of specimen, transducer and the couplant. Figure 4.7b shows the equivalent circuit representation of the same. The couplant is in series between the transducer and specimen, thereby resulting in a velocity drop. Using velocity-divider formula, the velocity of the specimen at the transducer location is

\[ v_s = \frac{M_s}{M_s + M_t + \frac{j\omega}{k_c}} v_{dc} \]  (4.1)
Figure 4.7  Diagram of Couplant Model: (a) Physical Illustration, (b) Mechanical Circuit.

where $v_{oc}$ is the transducer “open-circuit” velocity and $M_s$ is the specimen mobility as seen from the transducer contact point. Open circuit velocity is defined as the transducer tip velocity when no specimen is attached to it. This quantity can be measured by exciting the transducer with a harmonic frequency waveform and measuring the tip motion with a laser vibrometer. $M_t$ is the “mobility” of the transducer and can be calculated from the open circuit velocity and the measured velocity of a point mass (mobility = $\frac{1}{j\omega m}$) attached to the transducer tip.

When the couplant stiffness is sufficiently small (mobility is sufficiently high) that the term $\frac{k_c}{k_c}$ in the denominator of equation 4.1 is much higher than the sum of the mobilities of transducer and specimen, the two terms $M_t$ and $M_s$ can be neglected and the specimen velocity becomes $v_s \approx k_c M_s \frac{1}{j\omega} v_{oc}$. As with compliant isolators, using a compliant couplant eliminates the transducer-specimen sympathetic resonances.

To validate the effect of compliant couplant, we experimentally measured the velocity spectrum of a rectangular bar specimen using four different transducers, once without a couplant and once with a single layer of 110 lb cardstock material as couplant. Specimen velocity spectrum was calculated as the magnitude of the Fourier Transform of measured surface velocity. Figure 4.8a shows the velocity spectrum when no couplant was in place. Peaks in the spectrum indicate resonances. Despite all other excitation parameters being identical, the velocity spectrum was not repeatable and varied substantially across transducers. The measured resonances in this case were sympathetic transducer-specimen resonances that arose due to the
Figure 4.8 Demonstration of compliant couplant criterion: (a) Specimen velocity spectrum when excited using four different transducers, no couplant present (b) Specimen velocity spectrum when excited with four different transducers with a single layer of cardstock as couplant (c) Comparison of stiffness of transducer and specimen against that of cardstock. For the sympathetic resonances to vanish, the couplant stiffness should be much smaller than the transducer and specimen combined.

absence of a couplant. Figure 4.8b shows the velocity spectrum of the same specimen when couplant was in place. The specimen resonance frequencies in this case were very repeatable and independent of the transducer. Figure 4.8c shows the comparison of couplant stiffness against that of transducer and specimen combined. For the couplant we used, the couplant stiffness was indeed much smaller than the stiffness of transducer and specimen and therefore, the sympathetic resonances vanished, as predicted.

With compliant couplant in place, the transducer excitation can be modeled according to a similar process as the isolators. The primary difference is the explicit inclusion of the vibration source in the mechanical circuit and in simulation, the use of the integral of the measured transducer open-circuit velocity $v_{oc}$ as a displacement boundary condition on the couplant in
place of the fixed boundary condition on the isolators.

4.6 Characterization of Couplant and Isolator Materials

In order to accurately model specimen vibration, the mounting materials have to be characterized and the relevant mechanical properties must be measured. Isolators act like springs in parallel with the specimen to increase the effective specimen stiffness and in turn the resonance frequency. Hence, following the discussion in section 4.4.1, we model the isolators and couplant in our numerical model as lossy springs with complex stiffness. The elastic stiffness acts to increase the resonance frequency and the loss factor introduces damping in specimen vibration. In this section, we develop procedures for measuring the stiffness and loss factors of the isolators and couplant in the context of vibrothermography modeling. We measure the stiffness from the compressive force versus displacement data of the cardstock and we develop an inversion process to experimentally evaluate the loss factors.

4.6.1 Evaluation of Isolator and Couplant Stiffness

Since cardstock is a fibrous material, it is inherently anisotropic and its properties are different along the in-plane and out of plane directions [Mann et al. (1979); Fällström et al. (2002)]. In vibrothermography, the couplant and isolator materials undergo compressive loading and hence compressive stiffness in the out of plane direction is the primary mechanical property that needs measured. We measure this quantity by applying a compressive static force on the isolators and calculating the linearized slope of the force – deformation curve obtained. We use five layers of cardstock, sandwiched between a stainless steel half-roller and a flat plate (to replicate the conditions during specimen loading) as the test geometry. Since we pre-deform the isolators prior to vibrothermography testing, we must characterize the stiffness of pre-deformed cardstock. Therefore, we perform a two step measurement to calculate stiffness: First, the isolators are pre-loaded to 3336 N (the same pre-load applied in the vibration experiments) and are held at this load for ten minutes. After unloading, the pre-deformed isolators are subjected to a maximum compressive load of 2224 N. The force and deformation data is recorded during
both the measurement steps. Compressive stiffness of the isolators is calculated as the linearized slope of force–deformation curve of the pre-deformed isolators. A detailed description isolator stiffness measurement procedure is explained in the appendix.

Couplant in vibrothermography usually subjected to a much smaller static pre-load than the isolators. A compressive load of approximately 100 N is usually applied to the transducer and couplant using a pneumatic cylinder to improve the efficiency of vibration coupling into the specimen. In our experiments, the couplant is the same material as of the isolators. Since we use five layers of cardstock as isolators and only one layer as couplant, we calculate the couplant stiffness as five times the isolator stiffness, after correcting for the couplant geometry.

4.6.2 Evaluation of Isolator and Couplant Damping

Vibration amplitude at resonance is determined by the amount of damping present in the system. Large vibrational amplitudes are observed when the damping is small and vice-versa. Some sources of specimen damping in vibrothermography are: absorptive isolators and couplant, absorption in the specimen, damping introduced due to the presence of crack, damping due to the pneumatic cylinder, friction at contact points, acoustic losses [Rivin (2003)]. Since all the above factors act in combination, it is impractical to attempt to individually quantify all the damping sources. Instead, in the context of our numerical model, we assume that the entire system damping is contributed by just the isolators and couplant and that other sources such as specimen damping are negligible. We model the isolators and couplant as absorptive springs with complex stiffness, \( k = k_r(1 + j\eta) \) where \( k_r \) is the elastic stiffness and \( \eta \) is the loss factor of the spring.

We evaluate the isolator and couplant loss factors from the measured Q-factor of specimen vibration. In terms of energies, Q-factor is defined as [Findeisen (2013)]:

\[
Q\text{-factor} = \frac{\text{Maximum energy stored per cycle}}{\text{Energy dissipated per cycle}} \tag{4.2}
\]

Q-factor can be measured from the vibration spectrum as the ratio of the resonance frequency and the half power bandwidth of that resonance, \( Q = \frac{\omega_0}{\Delta\omega} \), where \( \omega_0 \) is the peak angular frequency in the vibration spectrum and \( \Delta\omega \) is the band width at the half-power point in the
vibration spectrum. \[\text{Findeisen (2013)}\]. In time domain, Q-factor is approximately \(Q \approx \frac{\omega_0^2}{2\alpha}\) where \(\omega_0\) is the angular excitation frequency and \(\alpha\) is the exponential decay rate of the specimen vibration after the excitation ends \[\text{Vierck (1979)}\]. We measure the Q-factor in time domain at resonance and use equation 4.2 to evaluate the individual loss factors of couplant and isolators from the measured Q-factor.

For harmonic loading, the total elastic potential energy stored in the specimen, \(E_s^s\) is defined as

\[
E_s^s = \int_V \text{Re} \left[ \frac{1}{2} \sigma_{ij} \epsilon_{ij}^s \right] dV
\]  
(4.3)

where \(\sigma_{ij}\) and \(\epsilon_{ij}\) are the stress and strain components respectively and the integration is over the volume of the specimen \[\text{Reismann and Pawlik (1980)}\]. In the above equation, the quantity \(\sigma_{ij} \epsilon_{ij}^s\) indicates summation of product of all the individual stress and conjugate of strain components.

The evaluation of specimen stored energy requires that all the stress and strain components be known. Therefore, we use numerically evaluated mode shapes from an eigenfrequency analysis of the specimen to calculate all the stress and strain components over the entire specimen volume. The dynamic stresses, strains and the surface displacements calculated from the eigenfrequency analysis are quantified by scaling them with the ratio of numerical and measured displacement amplitude at a single surface point \[\text{Holland et al. (2011)}\]. The specimen strain energy \(E_s^s\) can then be evaluated using these calculated stress and strain components.

Since we model the couplant and isolators as springs, the elastic potential energy stored in these components is \(E_s^c = \frac{1}{2} k^c |z_s^c|^2\) and \(E_s^i = \frac{1}{2} k^i |z_s^i|^2\), where \(k^c\) and \(k^i\) represent stiffnesses of couplant and isolator respectively, and \(|z_s^c|\) and \(|z_s^i|\) represent the magnitude of out of plane specimen displacement at couplant and isolator. These quantities can be directly calculated from the known stiffnesses and displacement at isolator and couplant points evaluated by the same procedure as the strains and stresses above.

According to our assumption that only isolators and couplant contribute to damping, energy loss occurs only in these components. The dissipated energy per cycle of couplant and isolator is \(E_d^c = \frac{1}{2} k^c \eta^c |z_s^c|^2\) and \(E_d^i = \frac{1}{2} k^i \eta^i |z_s^i|^2\) respectively; \(\eta^c\) and \(\eta^i\) are the unknown couplant and isolator loss factors \[\text{Rivin (2003); Vierck (1979)}\].
In our experiments, we mount the specimen using four sets of isolators and a single layer of couplant placed between transducer and specimen. Therefore, from the above definitions, the total stored and dissipated energies in the system are respectively:

\[ E_{stot} = E_s^s + E_c^c + 4E_i^i \]  \hspace{1cm} (4.4)  \\
\[ E_{dtot} = E_d^c + 4E_d^i \]  \hspace{1cm} (4.5)  \\
\[ E_{stot} \]  \hspace{1cm} (4.6)

and the Q-factor is:

\[ \text{Q-factor} = \frac{E_{stot}}{E_{dtot}} = \frac{E_s^s + \frac{1}{2}k_c^c|w_s^c|^2 + 2k_i^i|w_s^i|^2}{\frac{1}{2}k_c^c\eta_c^c|w_s^c|^2 + 2k_i^i\eta_i^i|w_s^i|^2} \]  \hspace{1cm} (4.7)

The only unknowns in this equation are the couplant and isolator loss factors, \( \eta_c^c \) and \( \eta_i^i \).

We follow a two step procedure to estimate these quantities. We first measure the couplant loss factor independently and use this quantity to evaluate isolator loss factor as a function of pre-load applied to isolators from equation 4.7.

To evaluate the loss factors, we performed two sets of experiments. In the first set, we mounted a Ti 6-4 rectangular bar specimen was sandwiched between the tips of two identical transducers at the center, one each on the top and bottom faces. We placed a single layer of couplant at each contact point. The top transducer was held fixed and a static pre-load of approximately 100 N was applied to the bottom transducer using a pneumatic cylinder. The specimen was vibrated with the bottom transducer at the 9th flexural resonance frequency and the Q-factor of vibration was calculated. Since there are no isolators in this experiment, energy dissipation occurred because of the two couplants only and the Q-factor in this case was \( \frac{E_s^s + \frac{1}{2}k_c^c|w_s^c|^2}{\frac{1}{2}k_c^c\eta_c^c|w_s^c|^2} \). Couplant loss factor, \( \eta_c^c \) was directly calculated from the measured Q-factor and the calculated stored energies in specimen and couplants.

In the second set of experiments, we mounted the same specimen using four isolators placed symmetrically from the center with the top and bottom sets of isolators placed equidistant from the edges (see figure 4.2). The mounting points were at antinode positions of the 9th flexural resonant mode of the specimen. A single transducer and couplant were placed at the specimen center. Specimen vibration and the corresponding Q-factors were measured at 9th flexural
mode at ten different isolator pre-loads in the range of 224 N and 2224 N, at steps of 224 N. The measured Q-factors, along with the couplant loss factor $\eta^c$ from above was used to calculate isolator loss factors $\eta^i$ at each load.

Since we measured the loss factors at the 9th flexural resonance of the specimen (at approximately 33 kHz), in order to eliminate any frequency dependence on the measured value, the equivalent dashpot coefficient was calculated. The equivalent dashpot coefficient $c$ is defined as $c = \frac{\eta k}{\omega}$, where $\eta$ is the couplant or isolator loss factor, $k$ is the isolator or couplant stiffness, and $\omega$ is the angular frequency at which the loss factors were measured (9th flexural resonance frequency of the specimen). The loss factors of couplant and isolator at any desired angular frequency, $\omega_{desired}$ can be calculated from the calibrated value of $c$ as $\eta_{desired} = \frac{C_{desired}}{k}$.

At a static pre-load of 100 N, the measured couplant stiffness was 54 kN/mm and the equivalent dashpot coefficient of the couplant was approximately 158 Ns/m. Figures 4.9a and 4.9b show the measured stiffness and the equivalent dashpot coefficient of the isolators as a function of the static pre-load applied to the isolators. The fact that the stiffness and dashpot coefficient increased with increasing pre-load indicates that isolators were acting stiffer and were inducing more damping at high pre-loads.
4.7 Numerical Vibration Model

We build a linear harmonic numerical model for specimen vibration based on the measured characteristics of transducer, isolators, and couplant using COMSOL Multiphysics and MATLAB software. Specimen geometry in the model can be built using the Solid Mechanics interface, or can be imported as a CAD file for complex specimen geometries. Isolators and couplant are modeled as equivalent spring foundation boundary conditions applied to the specimen surface. Stiffness of isolators is evaluated at the desired pre-load.

To model vibration excitation, we first measure the broad band open circuit displacement of the transducer tip, calculate the Fourier transform of the response and normalize the Fourier Transform with that of the excitation waveform. This gives a transfer function response of the transducer in frequency domain. Figure 4.10 shows the transducer transfer function as a function of frequency measured at an excitation voltage of 1V. Within linear excitation regime, this transfer function, multiplied by the desired excitation voltage is applied as the prescribed
displacement to the couplant. This is implemented in the numerical model as equivalent spring force on the specimen surface generated due to the transducer. The force on the specimen surface is therefore, $F = k_c(u_t - u_s)$, where $k_c$ is the couplant stiffness, $u_t$ is the transducer open circuit displacement (transfer function value multiplied by excitation voltage amplitude) and $u_s$ is the specimen displacement at couplant. A mesh refinement study is performed and specimen mesh size is chosen that does not affect the computed results.

After building the model geometry and applying appropriate boundary conditions, an eigen-frequency analysis is performed to calculate the resonance frequencies and the corresponding mode shapes. Then, a narrow band frequency sweep vibration is performed centered around the desired resonance frequency to get a more accurate estimate the resonance frequency. Finally, harmonic analysis is performed at the resonance frequency and the vibration amplitude and vibrational strains are recorded at this frequency. These analysis steps simulate the experiment procedure we follow for vibrothermography testing. All the steps are automated and can be executed from a MATLAB script.

4.8 Evaluation of Predictive Capacity of the Numerical Model

To evaluate the numerical model, we performed vibration experiments and simulations on a Ti 6-4 rectangular bar specimen of dimensions 246.2 mm $\times$ 26.2 mm $\times$ 12.6 mm. The specimen was tested at three different mounting positions and five isolator static pre-loads at 7th, 9th and 11th flexural resonant modes. Specimen was mounted at 40 mm, 45 mm and 50 mm from the ends and was tested at isolator static pre-loads of 224 N to 1112 N at steps of 224 N. The same set up from figure 4.2 was used for these set of experiments.

In each test, the resonance frequency and vibration amplitude were measured. To ensure that desired resonances were being excited, specimen motion was measured at two different points along the surface. Point A, near the long edge of the specimen surface (coordinates: 79 mm, 26 mm) is neither a node nor an antinode of torsional or flexural resonant modes. Point B, at the center of the specimen surface (coordinates: 125 mm, 15 mm) is a node of a torsional mode but an antinode of an odd order flexural mode. Therefore, when specimen vibrates
in flexural mode, both points have similar vibration amplitudes and at torsional mode, the vibration amplitude at the specimen edge (point A) is much higher than that at the specimen center (point B). Figure 4.11a shows a drawing of the specimen surface with the measurement points A and B marked.

The same set of experiments were simulated using the numerical model built in section 4.7. Figure 4.11b shows a snapshot of the geometry in COMSOL Multiphysics software interface with the isolators and couplant annotated. Isolators and couplant were modeled as equivalent spring foundation boundary conditions on the specimen surface. Specimen excitation was applied as a prescribed displacement boundary condition on the couplant spring foundation. This prescribed displacement was determined by the frequency dependent transducer open-circuit displacement measured independently. Elastic properties of the specimen were measured using ultrasound time of flight measurements [Krautkrämer and Krautkrämer (2013)] and are
listed in table 4.1. Stiffness and loss factors of isolator and couplant measured in section 4.6 were used in the numerical model. The frequency normalized isolator and couplant loss factors were scaled to simulation frequency.

![Table 4.1 Material properties of Ti 6-4 rectangular bar specimen.](image)

<table>
<thead>
<tr>
<th>Component</th>
<th>Young’s Modulus(GPa)</th>
<th>Density( kg/m$^3$)</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti 6-4 bar</td>
<td>114.8</td>
<td>4386</td>
<td>0.342</td>
</tr>
</tbody>
</table>

4.8.1 Results and Discussion

Figures (4.12 to 4.14) show the measured and predicted displacement amplitudes at point A as a function of isolator static pre-load when the specimen was mounted at 40 mm, 45 mm and 50 mm respectively. Figures (4.15 to 4.17) show the same set of plots at point B.

Figure (4.18 to 4.20) show the measured and predicted resonance frequencies of the specimen as a function of isolator pre-load.

In each case, there was a good correlation between measured and predicted displacement amplitudes. To perform a more rigorous error analysis, we calculated the relative error between the simulated and measured displacement amplitudes. This quantity is defined as

$$
e = \left(100 \ast \left(\frac{w_{sim} - w_{meas}}{w_{meas}}\right)\right)\%$$

(4.8)

where $w_{meas}$ and $w_{sim}$ represent the measured and simulated absolute out of plane displacement at a single specimen point (point A or point B). We calculated the relative error as a function of all variables in the experiment: isolator offset, static pre-load, resonance frequency and measurement position. For each variable, we calculated the average and standard error of the relative error over all the other variables. For example, the average error at 220 N static pre-load was calculated as the average of the relative error in displacement amplitude over all the measurements at 220 N including all offsets, all resonant modes and at both measurement points A and B. Figure 4.21 shows the relative error as a function of static load, isolator offset, resonant mode shape and measurement position. The error bars indicate one standard deviation from the mean relative error.
Figure 4.12  Comparison of measured and predicted surface displacement amplitudes at point A as a function of isolator static pre-load. The isolators were placed at an offset of 40 mm from the edges.

From these plots, it can be concluded that overall, the model under-estimated the displacement amplitudes by approximately 20% than the measured values. Despite the simplified nature of our assumptions, the numerical model was able to predict specimen vibration with a reasonable accuracy. We hypothesize that a more involved damping measurement procedure including specimen damping and damping introduced due to transducer might give a more accurate prediction of displacement amplitudes. Further, our displacement amplitude measurement assumes that there is no non-linearity in the specimen vibration and therefore no harmonics and sub-harmonics are generated. This is not necessarily true and some non-linearity often exists in the specimen vibration due to the inherently non-linear nature of the piezoelectric transducer and the contact points at isolators and couplant do add a certain degree of contact
non-linearity, no matter how compliant the mounting materials are.

The resonance frequencies increased with increasing static pre-load as expected. The model, however, generally under-estimated the resonance frequencies at all the mounting positions tested. Even though the relative error between measured and predicted resonances is small (less than 1% in most cases), one would expect a better estimate than was predicted by our model. One reason for the prediction error can be explained by the empirically observed fact that isolator stiffness is sensitive to a number of extrinsic factors such as the exact amount of pre-load applied, the duration of which the pre-deforming load was applied and the relative humidity. Considering these constraints, it is not practical to characterize the exact stiffness, and therefore, the resonance frequencies of the isolators used in the experiment. A more rigorous
Figure 4.14  Comparison of measured and predicted surface displacement amplitudes at point A as a function of isolator static pre-load. The isolators were placed at an offset of 50 mm from the edges.

Isolator stiffness measurement study would give better confidence bounds on isolator stiffness measurement. Alternatively, isolator materials with a more predictable behavior can be used for vibration damping.

4.9 Conclusions

Specimen vibration in vibrothermography is affected by mounting and excitation coupling. Contact non-linearity between transducer and specimen and mounting grips and specimen often introduces non-repeatable system resonances. Using sufficiently compliant couplant and isolator materials eliminates these system resonances and significantly improves the repeatability
Figure 4.15  Comparison of measured and predicted surface displacement amplitudes at point B as a function of isolator static pre-load. The isolators were placed at an offset of 40 mm from the edges.

of specimen vibration. Isolators were shown to act as compliant springs acting in parallel with the specimen and increase the effective stiffness and hence the resonance frequency. Methods to characterize the complex stiffness of isolators and couplant were shown. Based on the measured characteristics of specimen and mounting materials, a linear harmonic specimen vibration model was built for vibrothermography using COMSOL Multiphysics and MATLAB software. The model was validated against independent experiments. It is shown that, despite the simplified assumptions, the model was able to predict the vibrational amplitudes and resonance frequencies with a reasonable accuracy.
Figure 4.16  Comparison of measured and predicted surface displacement amplitudes at point B as a function of isolator static pre-load. The isolators were placed at an offset of 45 mm from the edges.

4.10 Acknowledgments

This material is based on work supported by the Air Force Research Laboratory under Contract #FA8650-10-D-5210, Task Order #023, and performed at Iowa State University.
Figure 4.17 Comparison of measured and predicted surface displacement amplitudes at point B as a function of isolator static pre-load. The isolators were placed at an offset of 50 mm from the edges.
Figure 4.18  Measured and predicted resonance frequencies of the specimen of the 7th, 9th and 11th flexural modes. Specimen was mounted at an offset of 40 mm from the edges.

Figure 4.19  Measured and predicted resonance frequencies of the specimen of the 7th, 9th and 11th flexural modes. Specimen was mounted at an offset of 45 mm from the edges.
Figure 4.20  Measured and predicted resonance frequencies of the specimen of the 7th, 9th and 11th flexural modes. Specimen was mounted at an offset of 50 mm from the edges.
Figure 4.21 Relative error in displacement amplitude as a function of (a) isolator pre-load, (b) isolator offset, (c) mode number and (d) measurement position.
CHAPTER 5. DETERMINING CRACK HEAT SOURCE INTENSITY FROM MEASURED SURFACE TEMPERATURE FOR VIBROTHERMOMETRY

A paper to be submitted to Infrared Physics & Technology

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5.1 Abstract

Vibrothermography is a nondestructive evaluation method for detecting cracks and delaminations, primarily in aircraft components. When mechanical vibrations are applied to a specimen, friction and/or adhesion hysteresis at the contacting crack faces generates heat. This heat flows towards the surface of the specimen and can be detected with an infrared camera. A physics based empirical model that characterizes vibrothermography defines the heat intensity at the crack as a function of vibration and crack parameters. Heat intensity, however, is not directly measurable and has to be estimated from the measured crack surface heating and the knowledge of heat transfer. Inverting the measured surface heating to estimate heat intensity of buried sources is an ill-posed problem because of the inherently diffusive nature of heat flow. An algorithm is proposed that, assuming semi-elliptical subsurface heating profile, makes the inversion process tractable and calculates the heat intensity from the measured surface heating. The algorithm inverts the crack heating at the onset of heat flow when the heat would not yet diffuse to the surface. Heat intensity is estimated by minimizing the squared error between measured and numerically simulated crack heating.
5.2 Introduction

Vibrothermography is an active thermography method for identifying cracks and delaminations based on heat generated from a vibrating crack [Reifsnider et al. (1980)]. Unlike flash thermography, in which an external heat flux is applied to the specimen and the thermal contrast is observed to identify flaws, in vibrothermography, heat is instead generated at the flaw by applying mechanical vibrations to the specimen using a piezoelectric transducer. When the specimen vibrates, friction and/or adhesion hysteresis between contacting asperities generate heat, which can be detected with an infrared camera. Defect detection in vibrothermography is therefore a three step process:

1. specimen is vibrated using a piezoelectric transducer,
2. heat energy is dissipated at contacting asperities,
3. heat diffuses towards the surface and is detected by an infrared camera.

Crack heating in vibrothermography is influenced by several factors, some intrinsic to the crack and some extrinsic. Heating is known to increase with increasing dynamic vibrational strains and increasing vibration frequency. Crack parameters that affect heating are the static closure stress (the contact stresses that keep the crack faces locked against each other) and crack morphology. Static closure stresses are maximum near the crack tips and minimum at the center. In the regions where closure stress is high, the crack faces do not separate and there is no relative motion between them to generate heat. Similarly, in the regions where the crack is fully open, the crack faces do not come into contact during vibration and there is no heat generated in this case either. Instead, heating occurs at regions where the contact is just sufficient to allow relative motion between crack faces. A model for vibrothermography predicts the crack heating for a given geometry and excitation conditions and takes all of the above parameters into consideration.

We develop an empirical crack heating model that describes the heat intensity at a crack as a function of vibration and crack closure state. The empirical model is represented as 

\[ g = v \times c \times m \]

where \( g \) is the total heat intensity at the crack, \( v \) represents the contribution
from vibrational strain and excitation frequency, \( c \) is the closure stress and \( m \) is the crack mobility (the relative motion between crack faces) [Lesthaeghe et al. (2015)].

The parameters of the empirical model are determined based on vibrothermography experiments on fatigue cracks. Vibrational strain can be calculated using the measured displacement at a single point and the corresponding resonant mode shape [Holland et al. (2011)]. Closure stress can be calculated by measuring vibrothermographic heating at several static loads and identifying the regions of heat generation [Renshaw et al. (2009a)]. The heat intensity, however, is not directly measurable. In a vibrothermography experiment, only the surface crack heating can be measured. Therefore, to characterize the empirical model, one must first estimate the heat intensity from crack surface heating. In this paper, we describe an inversion algorithm to estimate heat intensity from the measured surface heating. We numerically pre-calculate crack heating from each of an array of assumed basis source distributions. We then estimate source intensity for each basis entry by linear inversion of the measured surface heating.

### 5.3 Assumptions and Hypotheses

The inverse problem of calculating heat intensity from measured surface heating is ill-posed because of the inherently diffusive nature of heat conduction. Heat generated beneath the crack surface flows away from the crack and blurs the surface temperature [Carslaw and Jaeger (1959)]. Therefore, resolving the crack heating is challenging.

However, at the beginning of the heat flow, crack heating has higher resolution because it has not diffused as much and it is possible to resolve the regions of heat generation at this time scale. Therefore, we calculate the heat intensity at the crack based on surface heating measured 20 ms after the onset of heat flow. We use numerical simulation along with measured crack heating at this time to calculate heat intensity.

Crack heat generation in a fatigue crack is illustrated in figure 5.1. Figure 5.1a shows a typical surface breaking fatigue crack. Shaded region represents the crack face. Figure 5.1b shows the cross-sectional view of this crack. The red and yellow bands indicate regions of heat generation observed on the surface. Figure 5.1c shows the hypothesized crack heating
pattern on the crack face. It was reported that crack heating in vibrothermography occurs along discrete semi-annular fretting bands formed on the crack face [Renshaw et al. (2011)]. These bands are created when asperities come in contact to generate heat and line up with the growth rings of the crack. They can be observed from microscopic studies of fracture surfaces. Based on this, we hypothesize that heating on the crack face follows a uniform concentric semi-annular ring pattern as seen in figure 5.1c.

To accommodate asymmetric heating, instead of assuming uniform energy deposition across the entire semi-annular ring, we assume that heat intensity is uniform across each quarter annular ring. This assumption is depicted by different colors of the hot spots (rings) in figures 5.1b and 5.1c.

We use the geometry in figure 5.1 as our reference crack model for calculating heat intensities. The approach is to use simulation to numerically pre-calculate the surface temperature due to uniform quarter ring heat sources (basis entries). The actual heat source is assumed to be a linear combination of these basis entries. Then we perform linear inversion to calculate the heat intensities of all the quarter rings that generate measured surface heating.
5.4 Numerical Modeling

To numerically pre-calculate the crack surface heating for each basis entry, we assume that heat is generated at individual quarter rings on the crack face and that the heat intensity is uniform across each quarter-ring as highlighted in figure 5.1c. Since heat conduction is linear, the total heat intensity of the entire crack is the sum of heat intensities of all the individual quarter rings.

In this case, to support cracks up to 5 mm in length, we simulate the heat flow in a 5 mm crack discretized into 49 semi-annular rings (98 quarter annular rings), with minimum and maximum ring radii of 0.5 mm and 2.5 mm respectively. The thickness of each ring is therefore 0.05 mm. A total of 98 simulations are performed using COMSOL Multiphysics software [COMSOL Inc (2015)], individually activating each quarter ring with unit heat intensity (for every active ring, the rest of the rings are assigned zero heat intensity). From the simulation, the crack surface heating is calculated at 20 ms after the heat flow begins. Therefore, a total of 98 sets of surface crack heating data are collected, one for each active unit heat intensity quarter ring.

We perform two sets of simulations, one for Titanium 6-4 alloy and one for Inconel 718 alloy. Table 5.1 lists the material properties used in the simulation. These values are referred from the material properties handbook [Welsch et al. (1993)].

With the temperature profiles pre-calculated, our goal now is to estimate the heat intensities of each quarter ring so that the simulated surface heating matches the measured crack heating. For a crack with unknown heat intensities, we represent the measured crack heating as

\[ T^m_j = \sum_{i=1}^{98} g_i T_{ij} \]

where \( T^m_j \) is the measured crack heating at \( j^{th} \) discrete point along the crack, \( g_i \) is the (unknown) heat intensity of the \( i^{th} \) quarter ring, \( T_{ij} \) is the simulated crack heating arising from the \( i^{th} \) active quarter ring at the \( j^{th} \) point along the simulated crack.

We sample a set of \( P \) points along the crack and set up a set of \( P \) linear equations similar
to above summation. Equation 5.1 shows the system of linear equations in matrix form.

\[
\begin{bmatrix}
T_1^m \\
\vdots \\
T_P^m
\end{bmatrix} =
\begin{bmatrix}
T_{11} & T_{21} & \ldots & T_{N1} \\
\vdots & \vdots & \ddots & \vdots \\
T_{1P} & T_{2P} & \ldots & T_{NP}
\end{bmatrix}
\begin{bmatrix}
g_1 \\
\vdots \\
g_N
\end{bmatrix}
\]  
(5.1)

where \((T)^+\) indicates the pseudo-inverse of matrix \(T\) [Kay (2009)]. The columns of matrix \(T\) are pre-calculated once numerically and the vector \(T^m\) is generated from the measured crack heating. The total heat intensity of the crack is the sum of heat intensities of all the quarter rings, vector \(g\).

\[
g = (T)^+ T^m
\]  
(5.2)

5.5 Experiment and Results

To implement the inversion process and calculate heat intensity, we tested fatigue cracked Titanium 6-4 and Inconel 718 alloy specimens using vibrothermography. Specimen vibration was generated using a broadband piezoelectric transducer [Holland (2007)]. A FLIR SC6000 infrared camera was used to record the thermal images. A total of 150 frames were captured at a frame rate of 90 fps. Specimen was vibrated at one of its resonance frequencies for a one second duration between 0.2 s and 1.2 s. We used the thermal image captured at 220 ms (the second camera frame after the excitation began) to estimate the heat intensity. Figure 5.3a shows the measured temperature rise in the Titanium specimen with 3.96 mm long crack.

As discussed in section 5.4, crack heating must be sampled at discrete points along the crack and the heat intensity is evaluated at the sampled points. In order to do this, we reduced the measured two dimensional heat map image into a one dimensional heat map curve through discrete points along the crack. This process is graphically illustrated in figure 5.2. The heat map image was divided into 23 rectangular bins of width 176 \(\mu m\) each along the crack (X
Figure 5.2 Illustration of the procedure for reducing a two dimensional heat map image into a one dimensional heat map curve. The thermal image is divided into uniform rectangular bins. Average is calculated along bin length and an exponential integral function is fitted to this average. Peak value of the fit gives the representative crack heating for that bin. The curve underneath the bins shows the resulting heat map curve.

direction in the illustration above) and height 1 mm perpendicular to the crack (Y direction in the illustration). The bin width is approximately 1.5 times the thermal diffusion length in Ti 6-4. First, the average of the crack heating was computed along the bin width. This gave a single crack heating curve along Y for each bin (perpendicular to the crack). Next, an exponential integral function was fitted to this heat curve, and the peak value of the fit was considered as the single representative crack heating value for the entire bin. In order to overcome the singularity of the exponential integral function when the argument is zero, we perturbed the argument by a small amount so that the function value is bounded. The averaging and curve fitting is repeated for all the bins along the crack, effectively reducing a two dimensional image into a one dimensional curve whose points are sampled from the bins. The same process was used for discretizing the crack heating in simulated heat maps to generate the matrix $T$ in

\footnote{Exponential integral function was chosen to perform the curve fit of heat map curve because this is the functional form of the integration of Green’s function solution to heat equation [Holland (2011)]}
Figures 5.3 and 5.4 show the heat map image and heat map curve at 220 ms for the Titanium and Inconel specimens. The images on the left are the measured heat maps and the plots on the right are the heat map curves obtained from the discretization above. The crack length in these samples was 3.96 and 4.1 mm respectively. The maximum heating of 0.045 K in the Titanium specimen was much smaller than the 0.6 K in the Inconel specimen.

These heat map curves were used to populate the vector $T^m$ in equation 5.1. The columns of matrix $T$ were filled from the numerically pre-calculated heating of annular rings and the heat intensity of the rings was calculated as the product of the pseudo-inverse of the matrix $T$ and the vector $T^m$, equation 5.2.

Figures 5.5a and 5.5b show the estimated crack heat intensity in Titanium and Inconel alloy specimens respectively. Not surprisingly, the thermal power dissipated in the Inconel specimen was substantially higher than that in the Titanium specimen owing to higher crack heating in the former.

The estimated heat intensities of the annular rings were substituted back in the numerical
model and the resulting temperature rise was simulated at 220 ms to validate the heat intensities. Figures 5.6a and 5.6b show the reconstructed heat map images for the Titanium and Inconel specimens. The reconstructed images are much less noisy compared to the experiment. Even for the titanium specimen with a low signal to noise ratio, the algorithm was able to reconstruct the crack heating to a reasonable accuracy. To quantify the difference between measured and reconstructed images, the average crack heating in the entire heat map was calculated. This value was 8 mK for the measured heat map and 9 mK for reconstructed heat map in the Titanium specimen. For the Inconel specimen, the average measured and simulated crack heating was 52 mK and 61 mK respectively. This gives a reconstruction error of 11% for the Titanium specimen and 14% for Inconel specimen. A more appropriate measure of error is to quantify the difference of measured and simulated heat map images. However, since the cracks are not aligned, this would require a registration algorithm to first align the heat maps before the difference image can be calculated. Also, the algorithm models the crack as straight line on the surface and does not take into account, the tortuosity of real cracks, making the registration process non trivial.

The linear inversion algorithm does have its limitations. The reconstruction is dependent on
the assumption that crack heating is uniform in a quarter ring. As a result, the algorithm may wrongly estimate the temperature rise at much later times beyond 220 ms. This is particularly true for cracks where heating occurs for some reason only at a particular depth. The estimation algorithm could be improved in such cases by selecting a more appropriate crack heating profile based on crack fracture studies, or by readjusting the estimate based on thermal images from different times. However, achieving a perfect reconstruction by inverting a diffusion process is not possible in general without unbounded noise gain.

5.6 Conclusion

Heat intensity is a critical parameter in an empirical crack heating model for vibrothermography. We developed an algorithm to estimate heat intensity at a vibrating crack from the measured surface heating in vibrothermography. The algorithm calculates the heat intensities that minimize the least square error between numerically pre-calculated and measured crack heating. Within the scope of the assumptions made, the algorithm reconstructs the measured crack heating to a reasonable accuracy.
Figure 5.6  Reconstructed thermal image in: (a) Titanium alloy (b) Inconel alloy sample.

5.7  Acknowledgments

This material is based on work supported by the Air Force Research Laboratory under Contract #FA8650-10-D-5210, Task Order #023, and performed at Iowa State University.
Table 5.1  Material properties of Ti 6-4 and Inconel 718 used for the numerical simulation of crack heating.

<table>
<thead>
<tr>
<th>Material</th>
<th>density ($kg/m^3$)</th>
<th>thermal conductivity ($W/m/K$)</th>
<th>specific heat capacity ($J/kg/K$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti 6-4</td>
<td>4430</td>
<td>6.7</td>
<td>526.3</td>
</tr>
<tr>
<td>Inconel 718</td>
<td>8190</td>
<td>11.4</td>
<td>435</td>
</tr>
</tbody>
</table>
CHAPTER 6. CONCLUSIONS

6.1 Summary

Vibrothermography is an effective NDE method for identifying cracks and delaminations in aerospace components. Its use as an industry standard is hindered by the non-repeatable and often qualitative nature of defect detection. Crack heating in vibrothermography is affected by several parameters, both intrinsic and extrinsic to the crack. The focus of our research at ISU is to perform quantitative research in vibrothermography and make this a viable inspection method in aerospace industry.

We developed a physics based forward numerical/empirical model for specimen vibration in vibrothermography. All the three components i.e., vibration, heat generation and heat flow were modeled. We implemented this empirical model as a finite element tool called ‘VibroSim’ using COMSOL Multiphysics and MATLAB software. Since vibration is the first step in our model, accurately modeling the specimen vibration was a critical step to ensure the accuracy of rest of the model.

The goal of this research was to develop a numerical model for specimen vibration in vibrothermography as part of the forward model. This dissertation described methods to improve the repeatability and sensitivity of vibrothermography experiments via consistent and repeatable experimental procedures. Based on these methods, a linear harmonic model for specimen vibration was developed using COMSOL Multiphysics and MATLAB software. This chapter discusses the general conclusions of the current research and the scope for future research directions in vibrothermography.
6.2 Viscoelastic Coatings for Mapping Specimen Vibration

Chapter 2 discussed the use of viscoelastic coatings to measure vibration distribution on specimen surface in vibrothermography. Viscoelastic materials are a class of materials that have the properties of both elastic and viscous nature. The absorptive nature of these materials causes heat dissipation when subjected to mechanical vibrations. We used this phenomenon of ‘hysteretic heating’ to map specimen vibration distribution in vibrothermography. When a specimen is applied with a coating of viscoelastic polymer adhesive and subjected to vibrothermographic excitation, the coating dissipates heat according to the resonant mode shape of the underlying specimen. The hysteretic heating, captured by an infrared camera corresponds to the mode shape of the resonance in which the specimen is vibrating.

Using viscoelastic coatings to map vibration is an efficient and quick method to measure the resonant mode shapes and vibration distribution in the specimen for vibrothermography. While using a laser vibrometer to scan specimen surface motion takes hours to perform the same measurement, using viscoelastic coatings only takes a few minutes to achieve the same. This method can be potentially used by inspectors to optimize the excitation resonance frequency and specimen set up so that vibration can be maximized at potential defective regions and thus improve detectability.

Chapter 3 discussed a novel method to measure the loss modulus of viscoelastic coatings from the hysteretic heating generated when subjected to vibration. Loss modulus is the imaginary component of the complex modulus of viscoelastic materials. This quantity characterizes the viscous nature of the material and determines the amount of hysteretic heating generated. Perfectly elastic materials have zero loss modulus and hence they do not generate hysteretic heating. Loss modulus is typically measured using Dynamic Mechanical Analysis (DMA) and using the principle of Time Temperature Superposition (TTSP), master curves of loss modulus can be generated at any temperature as a function of frequency. We calculated the thermal power dissipated by the coating by fitting the measured heating to the analytic solution of corresponding boundary value heat equation problem. The loss modulus was calculated as a function of frequency from the measured dynamic vibrational strain and the calculated ther-
mal power. The loss modulus calculated using this method correlated well with that measured using DMA. This is a quick and easy way to calculate the loss modulus of viscoelastic polymer coatings at high frequencies without having to perform DMA and TTSP.

6.3 Numerical Vibration Modeling for Vibrothermography

Precision vibration modeling is a critical aspect in the development of a forward model for vibrothermography. Since vibration is the first component of the VibroSim model, if the vibration model is not accurate, the rest of the model would not be accurate either. Specimen vibration in vibrothermography is often sensitive to mounting and excitation coupling. Specimen–transducer and specimen–mount interaction often generates unwanted system resonances involving both specimen, transducer and mounts. These system resonances are not repeatable and are therefore not feasible to model.

We observed that using compliant couplant and isolators at the specimen–transducer and specimen–mount contact eliminate these system resonances and makes the specimen motion more repeatable and hence modelable. In chapter 4, we developed simplified equivalent electrical circuits to characterize the specimen motion at the couplant and mount interfaces.

Based on this analysis, we showed that when the couplant and isolators are sufficiently compliant, specimen mounts can be modeled as a rigid foundations and therefore the complexity of explicitly modeling the mounts can be eliminated. We modeled the isolators as nonlinear spring foundations acting in parallel with the specimens. Isolator stiffness depends on the amount of pre-load applied to the mount. Specimen excitation was modeled as a prescribed displacement boundary condition on the free face of the couplant, calculated from the transfer function of the transducer.

Specimen resonance frequency is affected by the amount of pre-load applied to the mounts. At high pre-loads, the isolators are deformed more and have higher stiffness and in turn result in higher resonance frequency than at a low pre-load. Isolator and couplant stiffness is experimentally measured as a function of pre-load. Time integration of independent measurement of transducer open tip velocity gives the specimen excitation waveform.
Specimen vibration amplitude is determined by the amount of damping present in the system. In our numerical model, we assume that the only components contributing towards specimen damping are the couplant and isolators. Using measured isolator and couplant stiffness and measured Q-factors from specimen vibration, we calculated the loss factors of couplant and isolators.

Finally, we evaluated the predictive capacity of the numerical model by performing vibrothermography experiments and compared the simulation predictions against experiments.

This research presents a general way to characterize the components specific to each vibrothermography experiment. This approach is simple enough to be adopted by any researcher developing numerical vibration models for vibrothermography.

### 6.4 Estimation of Crack Heat Intensity

In our empirical model for vibrothermography, crack heat intensity is modeled as the product of vibration, crack closure stress and crack morphology. Therefore, to characterize the empirical model, crack heat intensity must be evaluated experimentally. Since heat intensity is not a directly measurable quantity, it has to be estimated from the measured surface heating at the crack. However, this is an ill-posed problem and directly inverting a diffusion process is not practical without unbounded noise gain. In chapter 5, we develop an inversion algorithm that, under certain assumptions can estimate the heat intensity with reasonable accuracy. Specifically, we invert the measured crack heating immediately after the onset of heat flow as at this time scale, the heat would not have diffused to the surface and has a higher resolution than at a later time step.

We build a hypothetical crack geometry based on experimental evidence and numerically pre-calculate surface crack heating for this geometry. We then estimate the heat intensities that minimize the squared error between the numerical and measured crack heating. We show that with in the scope of our assumptions, the inversion process can reconstruct the measured heating with acceptable accuracy.
6.5 Future Research Directions

In this research, we developed a simple linearized numerical model for specimen vibration in vibrothermography. Even when compliant couplant and isolators are used, specimen motion is often not fully linear and harmonic and sub-harmonic generation does occur. Piezoelectric transducer is inherently non-linear and we observed that the resonance frequency sometimes shifts with increasing transducer excitation voltage. Currently, our model overcomes this by using the transducer tip velocity at specific amplitudes at which the test is performed. Our linear model can be extended to a more general non-linear model that can emulate a welder type excitation system. Also, extensive physical and simulation experiments have to be performed to quantify the uncertainty and determine confidence bounds of our numerical vibration model.

The current research is a first step towards developing precision quantitative models for vibrothermography. More work needs to be done to develop practical simulation tools that can be deployed by researchers and a wider NDE community.
APPENDIX. PROCEDURE TO MEASURE STIFFNESS OF ISOLATOR AND COUPLANT

In order to develop a precision numerical model, it is necessary to accurately characterize the relevant mechanical properties of isolators and couplant. Since the isolators and couplant in vibrothermography are subjected to compressive loading, the quantity of interest here is the out of plane compressive stiffness. We measured this quantity by performing compression tests on five layers of cardstock material in an Instron test frame. We placed the isolator layers sandwiched between a semi-cylindrical roller and a Ti 6-4 flat plate which were in turn sandwiched between the two compression platens of the test frame. Figure A.1 shows the test set up we used for to measure isolator stiffness.

Five layers of 199 gsm cardstock, each 0.25 mm thick were cut in shapes of 25 mm × 25 mm. The cardstock layers were stacked and pre-loaded to 3336 N compressive force and were held under the maximum deformation for ten minutes. The load was then removed and the pre-deformed isolators were loaded back to a maximum compressive force of 2225 N. This process emulates the pre-deformation the isolators undergo prior to and during our vibrothermographic testing. The force and displacement data were recorded for all the measurements.

These measurements give the combined force response of isolators along with the test frame, the roller and the flat plate. The force – deformation curves were fitted to polynomial functions and the combined stiffness was calculated as the slope of the tangent to the polynomial as a function of static force.

The desired quantity, however, is the isolator stiffness without the effect of test frame. Therefore, to compensate for the influence of test frame on the stiffness measurement, we performed an independent compression test with just the roller and flat plate sandwiched between the compression test platens without the isolators and calculated its stiffness.

We use the analogy of series springs to extract isolator stiffness from the combined stiffness.
Figure A.1  Test set up used to measure the stiffness of isolators. Five layers of 110 lb cardstock were placed between a half-roller and a flat plate. Compressive loading was applied to this assembly. Stiffness was calculated as the linearized slope of force-deformation curve.

The isolators and test frame (including the roller and flat plate) can be represented by springs in series. The combined stiffness, $k_c$ is therefore,

$$k_c = \frac{k_i k_f}{k_i + k_f} \quad (A.1)$$

where $k_i$ is the isolator stiffness and $k_f$ is the testframe stiffness. Using equation A.1, the isolator compressive stiffness was calculated as:

$$k_i = \frac{k_f k_c}{k_f - k_c} \quad (A.2)$$

Figure A.2 shows the various stiffnesses measured experimentally. The plot on the top left shows the combined stiffness of isolators, the roller and the test frame ($k_c$). The plot on top right shows the stiffness of the roller and test frame ($k_f$). Finally, the plot in the bottom row shows the stiffness of isolators, $k_i$ calculated using equation A.2.
We model the isolators as spring foundation boundary conditions acting on the specimen surface. In the numerical model, the spring foundation is specified as stiffness per unit area over which the isolators act on. Since the isolators are sandwiched between a cylindrical roller and a flat plate the vibration testing, the contact area is approximately a rectangle, the length of which is same as that of the isolator and whose width is determined by the indentation made on the isolator by the roller. Since this contact width increases with increasing static load, it is difficult to experimentally measure the contact width as a function of load. When the thickness of isolators is thick enough so as to approximate a semi-infinite spring foundation, a closed form expression for the contact width can be calculated using contact mechanics theory [Johnson and Johnson (1987)]. However, when this is not the case, no closed form expression exists for the contact width and it has to be determined numerically.

Therefore, instead of setting up a complicated numerical analysis to estimate the contact width, we use the size of the indentation on isolator layers to approximate the contact width. A contact width of 4 mm and the length of the isolator is used to calculate the area of isolators on the specimen surface in the numerical model.
Figure A.2  (a) Combined stiffness of isolators, roller and the test frame. (b) Stiffness of roller and test frame. (c) Stiffness of isolators calculated using the combined stiffness and roller stiffness.
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