The nonlinear properties of a viscoelastic material under impact loading

Chin-Tza Tang

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The nonlinear properties of a viscoelastic material under impact loading

by

Chin-Tza Tang

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of DOCTOR OF PHILOSOPHY

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I. INTRODUCTION

A. Background

In low-modulus viscoelastic material the propagation velocity of a stress wave is less than that in conventional photoelastic materials which are more rigid by an order of magnitude\(^1\), \(^2\). Thus, in recent years, many investigators have used these materials to analyze dynamic phenomena photoelastically since they have more time to photograph the dynamic event within a given field of view.

For some years considerable effort has been devoted to the mechanical and optical characterization for low-modulus viscoelastic materials. Nolle\(^3\) developed methods for measuring dynamic mechanical properties; Hopkins\(^4\) investigated dynamic shear properties; Volterra and Barton\(^5\) introduced the double pendulum method; Dally, Riley, and Durelli\(^6\) modified the double pendulum method; Theocaris and Mylonas\(^7\) established the mechanical and optical characteristics as a function of strain rate and temperature; Arenz, Ferguson, Kunio, and Williams\(^8\) and Williams and Arenz\(^9\) described the interrelation between the stress and strain optic coefficients; Brown and Selway\(^10\) determined frequency response; Williams, Beebe, Arenz, and Ferguson\(^11\) and Arenz, Ferguson, and Williams\(^12\) reported the mechanical and optical characterization for a typical Solithane 113 composition; Ferguson\(^13\) analyzed stress wave propagation in
Solithane 113 by photoviscoelastic techniques; San Miguel and Duran(14) discussed the mechanical properties of some low-modulus birefringent resins; Williams(15) studied the structural analysis of viscoelastic materials; Arenz(16) discussed uniaxial wave propagation in realistic viscoelastic materials; McGuirt and Lianis(17) proposed a new form of constitutive equation for incompressible viscoelastic materials under isothermal conditions; McGuirt and Lianis(18) also discussed the viscoelastic behavior of a Styrene-Butadiene Rubber under finite uniaxial and equal biaxial deformations for nonisothermal case; Theocaris(19) investigated the viscoelastic properties of epoxy resins derived from creep and relaxation tests at different temperatures.

B. Mechanical Behavior of Viscoelastic Materials

Some materials display a pronounced sensitivity to the rate of loading, the strain being larger if the stress has grown more slowly to its final value. They also display creep; i.e., an increasing deformation under sustained load, the rate of strain depending on the stress. Such materials are called viscoelastic.

The constitutive equations of viscoelastic materials may be either linear or nonlinear of which linear behavior is the simplest case. The behavior of linear materials in uniax-
ial stress can be understood in terms of mechanical models built from discrete elastic and viscous elements, represented by springs and dashpots, respectively.

Sometimes viscoelastic materials are said to possess "memory" which means that the stress and strain distributions depend on the whole loading-time history prior to the moment of observation. This fact suggests that the materials may behave quite differently under transient and repeated sinusoidal loading since in the earlier case the duration of loading is extremely short and the "memory" property is not predominant. The "memory" property would be most predominant in the sinusoidal loading situation.

C. Hysol 8705

Hysol 8705 is a soft low-modulus polyurethane rubber with high photoelastic stress sensitivity. It can be purchased from Hysol, Inc. or it can be manufactured by casting a mixture of 100 parts of Hysol 2085 as base material and 24 parts of Hysol 3562 as hardner. The mixture is cured at 280°F for two hours and post-cured at 210°F for four hours. It does not exhibit viscous flow at room temperature, and has a high coefficient of thermal expansion. It can be easily machined and cemented to other materials. It is free from any measurable time-edge effect when properly machined.

Durelli and Riley(20) and Daniel and Durelli(21) investigated the coefficient of thermal expansion, modulus of
elasticity, Poisson's ratio, and the material fringe values of Hysol 8705 as functions of temperatures. In recent years, Hysol 8705 has been used by numerous investigators(22, 23) for dynamic photoelasticity studies. Since it is quite popular for this use, it is chosen as the test material in this investigation.

D. Scope of This Study

As explained above the test material Hysol 8705 might behave quite differently under impact and sinusoidal loadings. In order to clarify this point, these two kinds of loading tests were performed on the material in this study.

Since the behavior of the material under impact loading is more akin to stress wave propagation, the impact loading will be investigated in greatest detail. Effort will be taken to search for a mathematical model consisting of certain kind of spring and dashpot combination which can successfully reproduce the experimental results of impact loading. Then the same model will be tried to see whether it can also successfully reproduce the experimental results of sinusoidal loading. If so, the mechanical properties of the material under these two kinds of loadings can be related by a single mathematical model. It is possible, however, that two distinctly different mathematical models may be required; one for each type of loading.
II. THEORY OF VISCOELASTICITY

A. Spring and Dashpot

A spring (Fig. 2.1a) is an ideal linear elastic element. When a tensile force is applied to it, the increase in its length is proportional to the force. We have the relation

\[ \sigma = k \varepsilon \]  \hspace{1cm} (2.1)

in which \( k \) is the spring constant. In this case stress \( \sigma \) and strain \( \varepsilon \) are preferred since this removes length and cross-sectional area from our consideration, and thus gives them more generality.

![Diagram of a spring and dashpot](image)

Fig. 2.1. Spring and dashpot

A dashpot (Fig. 2.1b) is an ideal linear viscous element. When a tensile force is applied to it, the sides moves apart at a rate which is proportional to the force. We have

\[ \sigma = c \dot{\varepsilon} \]  \hspace{1cm} (2.2)

in which \( c \) is a constant, known as the viscosity of the dashpot. The quantity \( \dot{\varepsilon} \) is called strain rate.

The behavior of viscoelastic materials is a mixture of these two simple cases described by Eqs. 2.1 and 2.2.
B. Maxwell Fluid

When a spring and a dashpot are connected in series (Fig. 2.2), Eq. 2.3 is the required relationship between \( \sigma \) and \( \varepsilon \):

\[
\sigma + p_1 \dot{\varepsilon} = q_1 \dot{\varepsilon}
\]

(2.3)
in which \( p_1 = c/k \), and \( q_1 = c \).

![Fig. 2.2. Maxwell Fluid](image)

Equation 2.3 is interpreted by performing a two-stage test. In the first stage, known as the creep phase of the test, a constant stress \( \sigma_0 \) is suddenly applied at \( t = 0 \). The equation has the solution

\[
\varepsilon = \frac{\sigma_0 (p_1 + t)}{q_1}
\]

which is represented in Fig. 2.3 by the curve for \( 0 < t < t_1 \). The dotted lines indicate what would happen if this stage were to be extended beyond \( t = t_1 \) and clearly indicates that the material described by Eq. 2.3 shows a typical property of fluid; i.e., its capability of unlimited deformation under finite stress. Thus it is called Maxwell fluid in honor of J. C. Maxwell who first proposed such a model.

In the second stage (the relaxation phase of the test which begins at \( t = t_1 \)), the strain is fixed at whatever value it has, say \( \varepsilon_1 \). The solution of Eq. 2.3 in this case is
\[ \sigma = \sigma_0 \exp[-(t - t_1)/\rho_1], \quad t > t_1 \]

which shows that stress decreases under constant strain, that is, the material relaxes.

![Graph showing stress relaxation over time](image)

**Fig. 2.3.** Creep phase and relaxation phase of Maxwell fluid

**C. Kelvin Solid**

When a spring and a dashpot are connected in parallel (Fig. 2.4), we have

\[ \sigma = \sigma_0 \dot{\varepsilon} + \eta_1 \ddot{\varepsilon} \]  \hspace{1cm} (2.4)

in which \( \sigma_0 = k \), and \( \eta_1 = c \).

In the creep phase, Equation 2.4 has the solution

\[ \varepsilon = \frac{\sigma_0 [1 - \exp(-\sigma_0 t/\rho_1)]}{\sigma_0} \]
If this phase were to be extended to \( t \to \infty \), strain would not grow indefinitely, but would gradually approach the limit \( \sigma_0/q_0 \) (Fig. 2.5). This is almost the behavior of an elastic solid, and the material represented by Eq. 2.4 is known as Kelvin solid (sometimes called a Voigt solid) in honor of Lord Kelvin who first proposed such a model.

In the relaxation phase, the solution of Eq. 2.4 is

\[
\sigma = \sigma_0 [1 - \exp(-q_0 t_1/q_1)]
\]

which shows the relaxation is incomplete since stress is immediately relaxed by a certain amount and remains at this value so long as the strain remains \( \varepsilon_1 \).

D. Three-Parameter Solid

Figure 2.6 shows a spring and a Kelvin element connected in series. Considering the strains in both parts, we have

\[
\sigma = k_1 \varepsilon'
\]

and

\[
\sigma = k_2 \varepsilon'' + c \dot{\varepsilon}''
\]

Applying the Laplace transformation to both sides of both equations and multiplying each of these equations by a suitable constant and adding, we have
Fig. 2.5. Creep phase and relaxation phase of Kelvin solid

\[ \sigma = \sigma_0 \]

\[ 0 \leq t \leq t_1 \]

Fig. 2.6. Three-parameter solid

\[ \ddot{\sigma}(k_2 + sc) + k_1 \ddot{\sigma} = k_1(k_2 + sc)(\ddot{\varepsilon}' + \ddot{\varepsilon}'') \]

\[ = k_1(k_2 + sc)\ddot{\varepsilon} \]

where \( \ddot{\varepsilon} \) is the Laplace transformation of total strain.

Transforming back into the physical plane gives
\[(k_1 + k_2)\sigma + \dot{\sigma} = k_1k_2\varepsilon + k_1\dot{\varepsilon}\]

whose normalized form is
\[
\sigma + p_1\dot{\sigma} = q_0\varepsilon + q_1\dot{\varepsilon}
\]

where \(p_1 = c/(k_1 + k_2)\), \(q_0 = k_1k_2/(k_1 + k_2)\), and \(q_1 = k_1c/(k_1 + k_2)\).

In the creep phase, Eq. 2.5 has the solution
\[
\varepsilon = \sigma_0[1 - (1 - p_1q_0/q_1)\exp(-q_0t/q_1)]/q_0
\]
which shows asymptotic elastic behavior (Fig. 2.7). Thus, the material represented by Eq. 2.5 qualifies as a solid and is called three-parameter solid.

![Diagram](image1)

**Fig. 2.7.** Creep phase and relaxation phase of three-parameter solid
In the relaxation phase, the solution of Eq. 2.5 is

\[ \sigma = q_0 e_1 \left[ 1 - \exp\left\{ -(t - t_1)/p_1 \right\} \right] \]
\[ + \sigma_0 \exp\left\{ -(t - t_1)/p_1 \right\}, \quad t > t_1 \]

which shows the material relaxes asymptotically to \( q_0 e_1 \).

E. The Generalized Maxwell and Kelvin Models

With springs and dashpots more complicated models can be built up. Some typical models with their differential equations and other information are given in (24, 25, 26, 27).

There are two ways of systematically building up more complicated models from the generalized Maxwell model (Fig. 2.8) and Kelvin chain (Fig. 2.9). It has been shown (24, 25) that the differential equation of any model made from the Maxwell and Kelvin types must assume the form

\[ \sigma + p_1 \dot{\sigma} + p_2 \ddot{\sigma} + \cdots = q_0 \dot{e} + q_1 \ddot{e} + q_2 \dddot{e} + \cdots \]

Fig. 2.8. Generalized Maxwell model

Fig. 2.9. Kelvin chain
F. Hereditary Integral

Consider the case shown in Fig. 2.10, stress $\sigma_0$ is suddenly applied at $t = 0$, which produces a strain $\varepsilon = \sigma_0 J(t)$. The function $J(t)$ is called creep compliance, the strain per unit of applied stress. For $t < 0$, $J(t) = 0$. For $t > 0$, $J(t)$ is a monotonically increasing function since a tension bar under sustained load will never get shorter. At $t = t'$, some more stress $\Delta \sigma'$ is added. Then, for $t > t'$, additional strain will be produced which is proportional to $\Delta \sigma'$ and depends on the same creep compliance. If the material is linear, the total strain for $t > t'$ is

$$\varepsilon(t) = \sigma_0 J(t) + \Delta \sigma' J(t - t'), \quad t > t'$$

The stress diagram in Fig. 2.11 can be divided into a basic part $\sigma_0 \Delta(t)$, where $\Delta(t)$ is a unit step function, and a sequence of infinitesimal step functions $\Delta \sigma'[\Delta(t - t')]$ where $d\sigma' = (d\sigma'/dt)_{t=t'} dt' = (d\sigma'/dt') dt'$. The corresponding strain at any time $t$ is then the sum of the strains caused by all the steps that have taken place at $t' < t$; that is

$$\varepsilon(t) = \sigma_0 J(t) + \int_{t'}^{t} J(t - t') d\sigma' dt'$$  \hspace{1cm} (2.6)

Equation 2.6 is called hereditary integral which shows how the strain at any given time depends on all that has happened before. This is quite different from what happens in an elastic material whose strain at any time depends solely on the stress at that time. Following the same procedure, the stress can be expressed in a similar equation involving the
the strain history.

Hereditary integrals are discussed further in (24, 25, 26, 27, 28).

Fig. 2.10. Superposition of step input

Fig. 2.11. Derivation of hereditary integral
III. DYNAMIC TESTS OF HYSOL 8705

A. Impact Loading

To determine the mechanical properties of low-modulus viscoelastic materials under different rates of loading, the double pendulum method employed by Volterra and Barton(5) can be conveniently used. The method employed in this investigation differed from the double pendulum method in several respects. Instead of using two pendulums, only one pendulum was used. Figure 3.1 is a schematic diagram of the setup. The pendulum, 11\(\frac{1}{2}\) in. long and \(\frac{3}{4}\) in. in diameter, was made of Aluminum Alloy 2024-T6 with 1\(\frac{1}{16}\) in. long steel end part to work with electro-magnetic pendulum release. It was released from several different heights above its equilibrium position to produce axial impulse-loading to the specimen. Each pendulum height was selected to correspond to a certain initial strain rate. The pendulum with everything on it weighted .595 lb. In order to increase its weight, weights were added to obtain weights of 1.28 lbs. and 2.98 lbs., respectively.

The specimen was made of Hysol 8705, its size was .985 in. x .418 in. x .499 in. The specimen was glued to a small 1\(\frac{1}{4}\) in. thick aluminum cap which was attached to a Kistler Model 912 quartz load cell. The load cell was used to measure the output impulsive force transmitted through the specimen from the pendulum, and it was screwed to a 24 in. x 25 in. x 1.5
Fig. 3.1. Schematic diagram of the setup for impact loading
in. steel plate whose weight was approximately 250 lbs. The plate was cemented to the brick wall of the building. At the front end of the pendulum another Kistler Model 912 quartz load cell which was adjusted to remove its acceleration sensitivity was used to measure the input impulsive force acting on the specimen and another small \( \frac{1}{4} \) in. thick aluminum cap was attached to the front end of the load cell. Each load cell was connected to a Kistler (Model 568 or 504A) charge amplifier. The signals from the charge amplifiers were recorded on Polaroid film with a Tektronix Model 502 dual-beam oscilloscope and Tektonix Model C-12 camera.

For the purpose of having a check on the velocities of the pendulum at the beginning and the end of the impact, the signal from the load cell attached at the front end of the pendulum was integrated by employing an analog computer. The integrating circuit is shown in Fig. 3.2. After careful calibration the sensitivity of the integrating circuit was found to be 1806 mv/(lb.-sec.). In order to measure the deformation of the specimen during impact, a PhysiTech Model 39 electro-optical tracker was used which tracked the motion of the pendulum during impact. The signal from the tracker was recorded on the oscilloscope. Photographs of the experimental setup are shown in Figs. 3.3 and 3.4.

Since the specimen was in compression during impact, all measured forces and deformations were for compression and were defined to be positive.
Fig. 3.2. Integrating circuit for integrating the input force-time history

B. Sinusoidal Loading

To subject the specimen to a forced sinusoidal oscillation an ADI electro-magnetic shaker (Model AV50) was used. The schematic diagram of the setup for the test is shown in Fig. 3.5. One of the Kistler load cells used in impact loading was mounted on the moving head of the shaker while the other Kistler load cell was fixed to the heavy foundation of the shaker. Each end of the specimen was glued to one of the two aluminum caps used in impact and each of the caps was connected to one of the load cells. The output of the load cell mounted on the moving head of the shaker was compensated so that the base acceleration sensitivity of the load cell could be eliminated. The signals from the load cells were sent to the oscilloscope and a B & K (Brüel and Kjaer) Model 2416 electronic voltmeter.

The frequency of the oscillation and vibration amplitude of the shaker was controlled by a B & K Model 1042 sine-
Fig. 3.3. Experimental setup used for impact loading
Fig. 3.4. A close look of the pendulum and the specimen
Fig. 3.5. Schematic diagram of the setup for sinusoidal loading
random generator via a power amplifier, and the frequency was checked with a Hewlett-Packard Model 5233L electronic counter. In order to have a good comparison with the results from impact loading, the velocity amplitude of the oscillation was kept constant over the frequency range by employing a B & K Model 2502 vibration meter in the control feedback loop. The phase angle between the displacement of the oscillation and the compensated force can be determined by the phase angle between the acceleration and the force, since displacement is 180 degrees out of phase with acceleration. The phase angle was measured by using a Deltron Model 100A phasemeter. Sometimes the signal from the accelerometer built into the shaker was not strong enough to operate the phasemeter. Therefore, the signal was sent to an analog computer where it was amplified 10 times before being sent to the phasemeter and recorded with the oscilloscope.

A complete list of equipment used in the experiments is given in Table 1.
<table>
<thead>
<tr>
<th>Item</th>
<th>Mfr.</th>
<th>Model</th>
<th>Characteristics</th>
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</thead>
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<tr>
<td>Quartz load cell</td>
<td>Kistler</td>
<td>912</td>
<td>Limited to 500 lbs. tension</td>
</tr>
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<td>Kistler</td>
<td>912</td>
<td>Limited to 500 lbs. tension</td>
</tr>
<tr>
<td>Charge amplifier</td>
<td>Kistler</td>
<td>568</td>
<td>Frequency response: DC to 150 KHz Ranges: .1 to 100 mv/psi, g, lb.</td>
</tr>
<tr>
<td>Charge amplifier</td>
<td>Kistler</td>
<td>504A</td>
<td>Frequency response: DC to 100 KHz Ranges: 1 to 5000 psi, g's, lbs. per volt</td>
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<tr>
<td>Electro-optical tracker</td>
<td>PhysiTech Inc.</td>
<td>39</td>
<td>Working distance: .100&quot; to infinity depends on lens system L135 + 45</td>
</tr>
<tr>
<td>Electro-magnetic shaker</td>
<td>AGAC-Derritron Inc.</td>
<td>AV50</td>
<td>Total excursion: .5&quot;(±.25&quot;) Frequency range: 5 Hz to 20 KHz</td>
</tr>
<tr>
<td>Power amplifier</td>
<td>AGAC-Derritron Inc.</td>
<td>N-300</td>
<td>Frequency range: 1.5 Hz to 20 KHz Power output: 300 watts rms continuous</td>
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</table>

Dual-beam

General purpose designed for desk-top use

Output: 0-5.0 V.D.C. proportional to target position
<table>
<thead>
<tr>
<th>Item</th>
<th>Mfr.</th>
<th>Model</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine-random generator</td>
<td>Brüel &amp; Kjaer</td>
<td>1042</td>
<td>Frequency range: 5 Hz to 10 KHz</td>
</tr>
<tr>
<td>Vibration meter</td>
<td>Brüel &amp; Kjaer</td>
<td>2502</td>
<td>Tolerance: ±1% from 20 dB below to 10 dB above full scale deflection</td>
</tr>
<tr>
<td>Accelerometer preamplifier</td>
<td>AGAC-Derritron Inc.</td>
<td>VM-12R</td>
<td>Frequency response: ±1% 5 Hz to 5 KHz, 2% 2 Hz to 20 KHz</td>
</tr>
<tr>
<td>Frequency counter</td>
<td>Hewlett-Packard</td>
<td>5233L</td>
<td>Frequency range: 0 to 2 MHz</td>
</tr>
<tr>
<td>Electronic voltmeter</td>
<td>Brüel &amp; Kjaer</td>
<td>2416</td>
<td>Frequency response: linear within 2% rms from 2 Hz to 200 KHz</td>
</tr>
<tr>
<td>Phasemeter</td>
<td>Deltron Inc.</td>
<td>100A</td>
<td></td>
</tr>
</tbody>
</table>
IV. ANALYSIS OF EXPERIMENTAL RESULTS

A. Results from Impact Loading

1. Calculation of important parameters

During impact loading many sets of data were taken at different initial strain rates for three different pendulum weights.

The signals from the load cells, recorded with a dual-beam oscilloscope, are shown in Fig. 4.1. The upper beam is the output force-time record coming from the load cell with its front end connected to the specimen and another end fixed to the wall while the lower beam is the input force-time record from the load cell connected to the front end of the pendulum.

In Fig. 4.2, the upper beam is the displacement-time record at initial strain rate 10.61 in./in./sec. and pendulum weight 2.98 lbs., the lower beam is the input force-time record. The initial and final slopes of the displacement-time record give us the velocities of the pendulum \( v_i \) and \( v_f \) at the beginning and the end of impact, respectively. The initial velocity \( v_i \) is defined to be positive, and accordingly \( v_f \) should be negative, since \( v_i \) and \( v_f \) are opposite in sense. The initial and final strain rates \( \hat{\varepsilon}_i \) and \( \hat{\varepsilon}_f \) can be calculated from the following equations:

\[
\hat{\varepsilon}_i = \frac{v_i}{l_0}
\]
\[ \dot{\varepsilon}_i = 10.61 \text{ in./in./sec.}, \quad w = 2.98 \text{ lbs.} \]

Fig. 4.1. Force-time record of impact loading (sweep rate: 5 msec./division)

\[ \dot{\varepsilon}_i = 10.61 \text{ in./in./sec.}, \quad w = 2.98 \text{ lbs.} \]

Fig. 4.2. Displacement-time and input force-time records of impact loading (sweep rate: 5 msec./division)
where \( l_0 \) is the initial length of the specimen, and \( l_f \) is its final length. Based on the signs of \( v_i \) and \( v_f \), the initial strain rate \( \dot{\varepsilon}_i \) is positive and the final strain rate \( \dot{\varepsilon}_f \) is negative.

Under loading, the deformation of the specimen was not small as compared to its size, therefore true stress and true strain should be considered. Since Poisson's ratio is nearly \( \frac{1}{3} \) for the material under consideration (10, 11), it is assumed that the volume of the specimen remains constant during deformation so that the cross-sectional area of the specimen \( A(t) \) can be computed from the following relationship:

\[
A(t) = \frac{V_0}{l(t)}
\]

where \( V_0 \) is the initial volume of the specimen, and \( l(t) \) is its actual length at the instant of consideration, and can be calculated as follows:

\[
l(t) = l_0 - \delta(t)
\]

where \( \delta(t) \) is the compressive deformation of the specimen at the instant and is defined to be positive.

The true stress \( \sigma(t) \) is given by

\[
\sigma(t) = \frac{F(t)}{A(t)} \frac{l(t)}{V_0}
\]

where the compressive force \( F(t) \) and the compressive stress \( \sigma(t) \) are all defined to be positive. The true strain \( \varepsilon(t) \)
is given by

\[ \varepsilon(t) = \ln \left( \frac{L(0)}{L(t)} \right) \]  \hspace{1cm} (4.2)

where \( \varepsilon(t) \) is a compressive strain, and is defined to be positive.

Hence, by using Eqs. 4.1 and 4.2, the stress and strain can be computed as a function of time from Fig. 4.2, as shown in Figs. 4.3 and 4.4 for three different pendulum weights.

The input force-time record \( F(t) \) was integrated with an analog computer (see Fig. 3.2). The integrated curve which is essentially a momentum-time curve is the upper beam in Fig. 4.5 while the lower beam is the input force-time record. The change in velocity \( \Delta v = v_f - v_i \) can be computed from the following equation:

\[ \Delta v = -\frac{1}{m} \int_0^\tau F(t) \, dt \]  \hspace{1cm} (4.3)

where \( m \) is the mass of pendulum in slug, and \( \tau \) is the duration of impact which can be easily determined from Fig. 4.1 or Fig. 4.2. The integral in the above equation is given by

\[ \int_0^\tau F(t) \, dt = \frac{G}{s_v} \]

where \( G \) can be determined from the integrated curve whose unit is \( mv \), and the sensitivity of the integrating circuit \( s_v \) was found to be 1806 \( mv/(lb.-sec.) \) as mentioned in Chapter III. The velocity difference, \( \Delta v \), given by Eq. 4.3, can be used to check velocities \( v_i \) and \( v_f \) as measured from the
\[ \dot{\varepsilon}_1 = 33 \, \text{in./in./sec} \quad (v_1 = 32.5 \, \text{in./sec}) \]

- Circle: pendulum weight 0.595 lb.
- Square: pendulum weight 1.28 lbs.
- Triangle: pendulum weight 2.98 lbs.

Fig. 4.3. Stress-time history of impact loading
\[ \dot{e}_1 = 33 \text{ in./in./sec.} (v_1 = 32.5 \text{ in./sec.}) \]

- Circle: pendulum weight 0.595 lb.
- Square: pendulum weight 1.28 lbs.
- Triangle: pendulum weight 2.98 lbs.

**Fig. 4.4.** Strain-time history of impact loading
slopes of the displacement-time history in Fig. 4.2.

The loop in Fig. 4.6 is the input force-deformation curve at the same initial strain rate of 10.61 in./in./sec. \(w = 2.98\) lbs.). The area enclosed in this hysteresis loop is the energy loss in the specimen due to internal friction of the material. The initial and final moduli of elasticity of the material can be determined from the initial and final slopes of the loop, respectively. The true stress-strain curves can be obtained either by eliminating time in Figs. 4.3 and 4.4 or from the hysteresis loop in Fig. 4.6. In either approach, Eqs. 4.1 and 4.2 must be used.

Figures 4.2 and 4.6 clearly indicate that at the end of impact the specimen did not come back to its original length; i.e., there was some permanent set left in the specimen. The material needs more time to relax to its original size than is allowed during the few milliseconds of relaxation allowed during the last half of the impact.

As explained above, from the pictures taken, initial and final velocities \(v_i\) and \(v_f\), initial and final strain rates \(\dot{\epsilon}_i\) and \(\dot{\epsilon}_f\), maximum input force \(F_{\text{max}}\), maximum deformation \(\delta_{\text{max}}\), permanent set \(\epsilon\), duration of impact \(\tau\), input energy \(w\), the maximum area under the increasing load-deformation curve in Fig. 4.6), the hysteresis energy loss \(\Delta W\), the initial and final moduli of elasticity \(E_i\) and \(E_f\), and the initial kinetic energy of the pendulum based on \(v_i\)
$\dot{\varepsilon}_i = 10.61 \text{ in.}/\text{in.}/\text{sec.}, \ w = 2.98 \text{ lbs.}$

Fig. 4.5. Integrated force-time and force-time records of impact loading (sweep rate: 5 msec./division)

$\dot{\varepsilon}_i = 10.61 \text{ in.}/\text{in.}/\text{sec.}, \ w = 2.98 \text{ lbs.}$

Fig. 4.6. Input force-deformation record of impact loading
((E_k)_i) can all be calculated, and they are tabulated in Table 2.

In order to determine the static modulus of elasticity of Hysol 8705, the specimen must be loaded statically. A typical static force-deformation curve is shown in Fig. 4.7 where approximately 90 seconds were used to load and unload the specimen to a maximum true stress and strain of 55.6 psi and .1134 in./in., respectively. The static modulus of elasticity was found to be 490 psi from this straight line curve.

2. Determination of initial modulus of elasticity as a function of initial strain rate

A plot of the initial modulus of elasticity $E_i$ as a function of initial strain rate $\dot{\varepsilon}_i$ for the material is shown in Fig. 4.8. The curve starts at 490 psi and becomes a straight line when the initial strain rate exceeds 35 in./in./sec. The dotted line is parallel to the straight part of the curve, and its slope is 3.51 psi/in./in./sec. Therefore $E_i$ can be treated as the sum of two parts: a linear part $E_l$ under the dotted line, and a nonlinear part $E_n$ above the dotted line. It is quite clear, the relation between $E_l$ and $\dot{\varepsilon}_i$ is

$$E_l = 490 + 3.51\dot{\varepsilon}_i$$

The nonlinear part $E_n$ can be written in an exponential form of $\dot{\varepsilon}_i$ with good accuracy (see Fig. 4.9):

$$E_n = 108[1 - \exp(-\dot{\varepsilon}_i/12.5)]$$
Table 2. Mechanical properties of Hysol 8705 under impact

Pendulum weight 0.595 lb.

<table>
<thead>
<tr>
<th>$v_i$ (in./sec.)</th>
<th>$v_f$ (in./sec.)</th>
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<th>$\dot{v}_f$ (in./in./sec.)</th>
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### Table 2. (Continued) Mechanical properties of Hysol 8705 under impact

Pendulum weight 1.28 lbs.

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Table 2. (Continued) Mechanical properties of Hysol 8705 under impact
Pendulum weight 2.98 lbs.

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Fig. 4.7. Force-deformation curve of static loading test
Since \( E_i = E_l + E_n \), the relationship between \( E_i \) and \( \dot{\varepsilon}_i \) is

\[
E_i = 490 + 3.51\dot{\varepsilon}_i + 108[1 - \exp(-\dot{\varepsilon}_i/12.5)] \quad (4.4)
\]

where the units of \( E_i \) and \( \dot{\varepsilon}_i \) are psi and in./in./sec., respectively.

Figure 4.10 is a plot of the final modulus of elasticity \( E_f \) as a function of final strain rate \( \dot{\varepsilon}_f \). Like \( E_i \), the relationship between \( E_f \) and \( \dot{\varepsilon}_f \) is

\[
E_f = 490 + 1.36\dot{\varepsilon}_f + 258[1 - \exp(-\dot{\varepsilon}_f/8.82)] \quad (4.5)
\]

where the units of \( E_f \) and \( \dot{\varepsilon}_f \) are psi and in./in./sec., respectively. Only magnitude of \( \dot{\varepsilon}_f \) is considered in Eq. 4.5, although it was defined to be negative in Chapter III.

3. Determination of empirical relations

Log-log plots of maximum force \( F_{max} \) and maximum deformation \( \delta_{max} \) vs initial kinetic energy of the pendulum \((E_k)_i\) are shown in Figs. 4.11 and 4.12. The empirical relations between \( F_{max} \) and \((E_k)_i\), \( \delta_{max} \) and \((E_k)_i\) are

\[
F_{max} = 16.3(E_k)_i^{0.574} \quad (4.6)
\]

\[
\delta_{max} = 0.107(E_k)_i^{0.43} \quad (4.7)
\]

where the units of \( F_{max} \), \( \delta_{max} \), and \((E_k)_i\) are lb., in., and in.-lb., respectively. Eliminating \((E_k)_i\) from Eqs. 4.6 and 4.7 gives the relationship between \( \delta_{max} \) and \( F_{max} \):

\[
\delta_{max} = 0.0132F_{max}^{0.749} \quad (4.8)
\]

Log-log plots of input energy \( W \)(the maximum area under the increasing load-deformation curve in Fig. 4.6) and energy
Eq. 4.4

108 psi

Initial strain rate (in./in./sec.)

Initial modulus of elasticity (psi)

○ pendulum weight 0.595 lb.
□ pendulum weight 1.28 lbs.
△ pendulum weight 2.98 lbs.

Fig. 4.8. Initial modulus of elasticity as a function of initial strain rate for impact loading
Fig. 4.9. Comparison of experimental and theoretical data of the nonlinear part of initial modulus of elasticity.

- Experimental data
- Theoretical data: $108(1 - \exp(-\dot{e}_1/12.5))$
Fig. 4.10. Final modulus of elasticity as a function of final strain rate for impact loading.
Initial kinetic energy of the pendulum (in.-lb.)

Eq. 4.6

Fig. 4.11. Log-log plot of maximum force vs initial kinetic energy of the impact pendulum
Initial kinetic energy of the pendulum (in.*lb.)

Fig. 4.12. Log-log plot of maximum deformation vs initial kinetic energy of the impact pendulum

- ○ pendulum weight 0.595 lb.
- □ pendulum weight 1.28 lbs.
- △ pendulum weight 2.98 lbs.

Eq. 4.7
loss $\Delta W$ (the area enclosed in the hysteresis loop) vs the initial kinetic energy of the pendulum $(E_k)_i$ are shown in Fig. 4.13, and the following empirical relations can be established:

$$W = 0.925(E_k)_i^{0.963} \quad (4.9)$$
$$\Delta W = 0.296(E_k)_i^{0.963} \quad (4.10)$$
$$\Delta W = 0.32W \quad (4.11)$$

where the units of $W$, $\Delta W$, and $(E_k)_i$ are all in.-lb. Combining Eqs. 4.7, 4.9, and 4.10, we have

$$W = 138\delta_{\text{max}}^{2.24} \quad (4.12)$$
$$\Delta W = 44.2\delta_{\text{max}}^{2.24} \quad (4.13)$$

Figure 4.14 is a log-log plot of maximum stress $\sigma_{\text{max}}$ vs initial kinetic energy of the pendulum $(E_k)_i$, and we have

$$\sigma_{\text{max}} = 75.2(E_k)_i^{0.567} \quad (4.14)$$

where the unit of $\sigma_{\text{max}}$ is psi. Eliminating $(E_k)_i$ from Eqs. 4.10 and 4.14 yields the relationship between $\Delta W$ and $\sigma_{\text{max}}$:

$$\Delta W = 1.933x10^{-4}\sigma_{\text{max}}^{1.7} \quad (4.15)$$

Robertson and Yorgiadis(29) found that $\Delta W$ was nearly independent of the frequency and was proportional to the third power of the stress amplitude. Kimball and Lovell(30) found that $\Delta W$ varied as the second power of the stress amplitude. Dally, Riley, and Durelli(6) found that $\Delta W$ varied as the
Fig. 4.13. Log-log plots of input energy and energy loss vs initial kinetic energy of the impact pendulum.
Fig. 4.14. Log-log plot of maximum stress vs initial kinetic energy of impact pendulum

- ○ pendulum weight 0.595 lb.
- □ pendulum weight 1.28 lbs.
- ▲ pendulum weight 2.98 lbs.

Eq. 4.14

Initial kinetic energy of the pendulum (in.-lb.) vs Maximum stress (psi)
two and half power of the stress amplitude. Equation 4.15 shows that \( \Delta W \) varies as the one point seven power of stress amplitude, but the stress amplitudes observed in (29), (30), and (6) were smaller than those in this study so that no direct comparison can be made.

Figure 4.15 is a log-log plot of final strain rate \( \dot{\varepsilon}_f \) vs initial strain rate \( \dot{\varepsilon}_i \), the relationship between them is

\[ \dot{\varepsilon}_f = \dot{\varepsilon}_i^{0.93} \quad (4.16) \]

where the units of \( \dot{\varepsilon}_f \) and \( \dot{\varepsilon}_i \) are all in./in./sec.

From the log-log plot of permanent set \( e \) and final strain rate \( \dot{\varepsilon}_f \), shown in Fig. 4.16, the relationship between them is

\[ e = 3.39(10)^{-3}\dot{\varepsilon}_f^{0.59} \quad (4.17) \]

where the units of \( e \) and \( \dot{\varepsilon}_f \) are in. and in./in./sec., respectively. Combining Eqs. 4.16 and 4.17 gives the relationship between \( e \) and \( \dot{\varepsilon}_i \):

\[ e = 3.39(10)^{-3}\dot{\varepsilon}_i^{0.549} \quad (4.18) \]

B. Results from Sinusoidal Loading

1. Calculation of important parameters

The motion of the moving head of a shaker can be described by the following equations:

\[ \delta = \delta_0 \sin \omega t \quad (4.19) \]
\[ v = \delta_0 \omega \cos \omega t = v_0 \cos \omega t \quad (4.20) \]
Fig. 4.15. Log-log plot of final strain rate vs initial strain rate of impact loading
Pig. 4.16. Log-log plot of permanent set
vs final strain rate of impact loading

- ○ pendulum weight 0.595 lb.
- □ pendulum weight 1.28 lbs.
- △ pendulum weight 2.98 lbs.

Fig. 4.16. Log-log plot of permanent set vs final strain rate of impact loading.
\[ a = -\delta \omega^2 \sin \omega t = -a_0 \sin \omega t \] (4.21)

where \( \delta \) is displacement, \( v \) is velocity, \( a \) is acceleration, \( \omega \) is frequency, and \( t \) is time.

As mentioned previously, \( v_0 \) was kept constant while the frequency of motion was changed to various discrete values. Knowing \( v_0 \), the displacement amplitude \( \delta_0 \), and the acceleration amplitude \( a_0 \) for a certain frequency \( \omega \) can be calculated from the following standard vibration equations:

\[ \delta_0 = \frac{v_0}{\omega} \] (4.22)

\[ a_0 = v_0 \omega \] (4.23)

where \( a_0 \) was also measured using the accelerometer built into the shaker head.

Pictures were taken from the oscilloscope traces. Two of these photos are shown in Figs. 4.17 and 4.18 for conditions of \( v_0 = 7.07 \) in./sec., \( f = 200 \) Hz. The upper beam in Fig. 4.17 is the compensated force acting on the specimen while the lower beam is the acceleration of the motion (amplified 10 times). Figure 4.18 is the hysteresis loop of the force and acceleration in Fig. 4.17. Since \( \delta = -a_0 \omega^2 \), the hysteresis loop in Fig. 4.18 can be converted into a force-displacement diagram, and the energy loss \( \Delta W \) due to the internal friction of the specimen can be calculated from the area enclosed in the loop. For a simple linear material the slope at point A in Fig. 4.18 will give us a
Fig. 4.17. Force and acceleration records of sinusoidal loading (sweep rate: 1 msec./division)

\[ v_0 = 7.07 \text{ in./sec.}, f = 200 \text{ Hz} \]

Fig. 4.18. The hysteresis loop of the force and acceleration in Fig. 4.17

\[ v_0 = 7.07 \text{ in./sec.}, f = 200 \text{ Hz} \]
modulus of elasticity of the material $E_g$ for sinusoidal loading which should be comparable to the initial modulus of elasticity of the material for impact loading.

The mechanical properties of the material under sinusoidal loading are listed in Table 3.

2. Empirical relations and plots

The results of the phase difference measurements between the compensated force and the displacement are shown in Figs. 4.19 and 4.20. The data points reveal that the phase difference is less than 20 degrees over the frequency and velocity ranges investigated. Fig. 4.19 shows that the phase difference depends on velocity, and this indicates that the material is nonlinear since for a simple linear viscoelastic material phase difference should be velocity independent.

In Fig. 4.21 modulus of elasticity $E_g$ is plotted against velocity amplitude $v_o$ which shows that $E_g$ is independent of frequency. Figure 4.21 is extremely different in character from Fig. 4.8 which is a plot of $E_i$ vs $\dot{u}_i$. Therefore the material behaves very differently when it is under impact compared to sinusoidal oscillation. The empirical relationship between $E_g$ and $v_o$ is:

$$E_g = 1300 - 23.5v_o \quad (4.24)$$

the units of $E_g$ and $v_o$ are psi and in./sec., respectively.

Equation 4.24 looks extremely different from Eq. 4.4.
Table 3. Mechanical properties of Hysol 8705 under sinusoidal loading

<table>
<thead>
<tr>
<th>$V_0$ Peak value in./sec.</th>
<th>$f$ Hz</th>
<th>Force amplitude (compensated) lb.</th>
<th>$a_0$ g's</th>
<th>$\delta_0$ in.</th>
<th>Stress amplitude psi</th>
<th>Strain amplitude in./in.</th>
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Table. 3. (Continued) Mechanical properties of Hysol 8705 under sinusoidal loading

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<th>$V_0$ Peak value in./sec.</th>
<th>$f$ Hz</th>
<th>$\Delta W$ in.-lb.</th>
<th>$E_s$ psi</th>
<th>Phase difference (compensated force vs displacement) degrees</th>
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Fig. 4.19. Phase difference as a function of frequency for sinusoidal loading.
Fig. 4.20. Phase difference as a function of velocity amplitude for sinusoidal loading
Fig. 4.21. Modulus of elasticity $E_g$ as a function of velocity amplitude for sinusoidal loading
\[ E_i = 490 + 3.51\dot{\varepsilon}_i + 108\left[1 - \exp\left(-\dot{\varepsilon}_i/12.5\right)\right] \quad (4.4) \]

Figure 4.22 is a log-log plot of energy loss \( \Delta W \) vs velocity amplitude \( v_o \) at three different frequencies, and the following empirical equations can be obtained:

\[
\begin{align*}
\Delta W &= 0.00137v_o^{1.495} \quad (f: 50 \text{ Hz}) \\
\Delta W &= 0.000528v_o^{1.495} \quad (f: 100 \text{ Hz}) \\
\Delta W &= 0.000191v_o^{1.495} \quad (f: 200 \text{ Hz})
\end{align*}
\]

the units of \( \Delta W \) and \( v_o \) are in.-lb. and in./sec., respectively. By employing Eq. 4.22 and \( \omega = 2\pi f \), the above equations can be rewritten as

\[
\begin{align*}
\Delta W &= 7.45\delta_o^{1.495} \quad (f: 50 \text{ Hz}) \quad (4.25) \\
\Delta W &= 7.88\delta_o^{1.495} \quad (f: 100 \text{ Hz}) \quad (4.26) \\
\Delta W &= 7.81\delta_o^{1.495} \quad (f: 200 \text{ Hz}) \quad (4.27)
\end{align*}
\]

the units of \( \Delta W \) and \( \delta_o \) are in.-lb. and in., respectively.

Figure 4.23 is a log-log plot of energy loss \( \Delta W \) vs frequency \( f \) at three different velocity amplitudes \( v_o \), and the empirical relations between \( \Delta W \) and \( f \) are

\[
\begin{align*}
\Delta W &= 3.3f^{-1.48} \quad (v_o: 3.54 \text{ in./sec.}) \\
\Delta W &= 6.6f^{-1.48} \quad (v_o: 5.66 \text{ in./sec.}) \\
\Delta W &= 8.7f^{-1.48} \quad (v_o: 7.07 \text{ in./sec.})
\end{align*}
\]

where the units of \( \Delta W \) and \( f \) are in.-lb. and Hz, respectively. By employing Eq. 4.22 and \( \omega = 2\pi f \), the above equations can be
Fig. 4.22. Log-log plot of energy loss vs velocity amplitude for sinusoidal loading.
Fig. 4.23. Log-log plot of energy loss vs frequency for sinusoidal loading
written in terms of displacement amplitude $\delta_0$:

$$\Delta W = 7.62\delta_0^{1.48} \quad (v = 3.54 \text{ in./sec.}) \quad (4.28)$$

$$\Delta W = 7.71\delta_0^{1.48} \quad (v = 5.66 \text{ in./sec.}) \quad (4.29)$$

$$\Delta W = 7.42\delta_0^{1.48} \quad (v = 7.07 \text{ in./sec.}) \quad (4.30)$$

where the units of $\Delta W$ and $\delta_0$ are in.-lb. and in., respectively. Allowing inevitable experimental errors, the six equations (Eqs. 4.25-4.30) are essentially one equation, which is quite different from Eq. 4.13:

$$\Delta W = 44.2\delta_{\max}^{2.24} \quad (4.13)$$

and this reiterates the fact that the material has dual-personality under dynamic loading of impact and sinusoidal oscillation.
V. MODEL REPRESENTATION OF HYSOL 8705 UNDER IMPACT

A. Three-Parameter Solid

In searching for a mathematical model of this material (i.e., a certain kind of spring and dashpot combination) to reproduce the mechanical properties measured under impact, the key point is that the model must predict the correct initial and final moduli of elasticity, maximum stress and strain, and permanent set for a given initial strain rate. The duration of impact should also be predicted when the corresponding pendulum mass is used.

Several models were investigated, they all failed to show any amount of permanent set. Finally the three-parameter solid discussed in Chapter II (p. 8) was investigated. Judging the solid physically from its $E_i$ and $E_f$ curves (see Figs. 4.8 and 4.10), we suspect that at a given loading rate during impact the dashpot will be deformed. Accordingly there will be some deformation stored in the dashpot, and afterwards, there is insufficient time to release all the deformation stored in the dashpot so that there will be some permanent set left in the material at the end of impact. From this point of view, the permanent set is governed by the viscosity $c$ of the dashpot and spring constant $k_2$.

The differential equation determining the properties of the solid was derived in Chapter II to be given by
\[
\sigma + p_1 \dot{\sigma} = q_0 \varepsilon + q_1 \dot{\varepsilon}
\]

(2.5)

where \( p_1 = \frac{c}{(k_1 + k_2)} \), \( q_0 = \frac{k_1 k_2}{(k_1 + k_2)} \), and
\( q_1 = \frac{k_1 c}{(k_1 + k_2)} \).

Now the problem is how to determine the spring constants \( k_1 \) and \( k_2 \), and the viscosity \( c \).

B. Determination of \( k_1 \), \( k_2 \), and \( c \)

The coefficient \( q_0 \) in Eq. 2.5 is in the order of magnitude of the static modulus of elasticity. When the model is loaded statically, the dashpot will be out of action, and the model is reduced to two springs in series. In this case \( q_0 \) is exactly the equivalent spring constant of \( k_1 \) and \( k_2 \) which are in series, and should be equal to the static modulus of elasticity of the material; i.e., 490 psi as reported in Chapter IV. At the very beginning of impact \( \sigma \) and \( \varepsilon \) are both extremely small, therefore the first term on either side of Eq. 2.5 is very small as compared with other terms, and can be neglected. Thus, initially we find

\[
\frac{\dot{\sigma}}{\dot{\varepsilon}} = E_i = \frac{q_1}{p_1} = k_1
\]

so that the initial modulus of elasticity is controlled solely by the spring constant \( k_1 \). From Eq. 4.5 we have experimentally found that the initial modulus of elasticity is related to the initial strain rate. Thus, we have

\[
k_1 = 490 + 3.51 \dot{\varepsilon}_1 + 108[1 - \exp(-\dot{\varepsilon}_1/12.5)]
\]

(5.1)
Statically the equivalent spring constant of \( k_1 \) and \( k_2 \) in series is equal to 490 psi, therefore statically \( k_2 \) can be determined from the following relationship:

\[
\frac{1}{490} = \frac{1}{k_1} + \frac{1}{k_2}
\]

or

\[
k_2 = \frac{490k_1}{k_1 - 490}
\]  \( (5.2) \)

When the model is under dynamic loading, \( k_2 \) should be in the order of magnitude determined by Eq. 5.2. The exact value of \( k_2 \) is determined empirically when Eq. 2.5 is solved numerically.

Since \( c \) and \( k_2 \) control the permanent set \( e \), the viscosity \( c \) can be evaluated from Eq. 2.5. At the end of impact \( \sigma = 0 \) and \( e = e_{p.s.} = \ln(l_0/(l_0 - e)) \), where \( e \) can be determined from Eq. 4.18 for a given initial strain rate \( \dot{e}_1 \). Therefore Eq. 2.5 can be rewritten as

\[
c = \frac{k_1k_2e_{p.s.}}{\dot{\sigma}_f - k_1\dot{e}_f}
\]  \( (5.3) \)

where \( \dot{\sigma}_f \) and \( \dot{e}_f \) are final stress rate and final strain rate, respectively. The final stress rate \( \dot{\sigma}_f \) can be determined from the final slope of the force-time record in Fig. 4.1 or 4.2. Since the final slope is negative, the final stress rate \( \dot{\sigma}_f \) is a negative quantity. The final strain rate \( \dot{e}_f \) is negative since \( v_f \) is defined to be negative. However, there are times when it is rather difficult to obtain an accurate final slope from the record since the curvature of the
force-time record changes continuously during the last period of loading. Hence, a scheme was worked out where the values of \( k_1, k_2 \), and \( c \) were selected on the basis of Eqs. 5.1, 5.2, and 5.3. The values of \( c \) and \( k_2 \) were then adjusted so that the numerical results gave the same permanent set and final modulus of elasticity. The following numerical scheme was used.

C. Numerical Computation Scheme to Evaluate \( k_1 \), \( k_2 \), and \( c \)

Solving Eq. 2.5 for stress rate \( \dot{\sigma} \), we have

\[
\dot{\sigma} = \frac{1}{c}[k_1 k_2 \varepsilon + k_1 c \dot{\varepsilon} - (k_1 + k_2)\sigma]
\]

As explained above spring constant \( k_1 \) must be equal to initial modulus of elasticity \( E_1 \) of the material in order for the mathematical model to reproduce \( E_1 \) accurately. Therefore \( k_1 \) was set equal to \( E_1 \) (Eq. 5.1) when Eq. 5.4 was being solved numerically.

For the first computer run, the values of \( k_2 \) and \( c \) were calculated from Eqs. 5.2 and 5.3, respectively. Thus, in the numerical computation scheme, the inputs were: initial strain rate \( \dot{\varepsilon}_i \), final stress rate \( \dot{\sigma}_f \), and pendulum weight \( w \). Given \( \dot{\varepsilon}_i \), the values of \( k_1 \) and \( k_2 \) can be determined from Eqs. 5.1 and 5.2, respectively. Knowing \( \dot{\sigma}_f \), the final slope of the force-time record in Fig. 4.1 or 4.2, the value of \( c \) can be calculated from Eqs. 4.18 and 5.3. The reason to have
w among the inputs will be given next.

Figure 5.1 is a free-body diagram of the pendulum during impact loading. Applying Newton's second law of motion, we have

\[ a_n = -386\frac{F_n}{w} \]  \hspace{1cm} (5.5)

where the units of acceleration \( a_n \), force \( F_n \), and pendulum weight \( w \) are in./sec^2, lb., and lb., respectively. The force acting on the specimen during impact is the reaction of force \( F_n \) since the magnitude of \( F_n \) is equal to stress \( \sigma_n \) times the cross-sectional area \( A_n \) of the specimen. From Eq. 5.4, stress rate \( \sigma_n \) can be calculated with an iteration scheme for a small time increment \( \Delta t \) by using the following relationship

\[ \sigma_n = \sigma_{n-1} + \dot{\sigma}_{n-1}\Delta t, \quad n = 1, 2, 3, \ldots \]  \hspace{1cm} (5.6)

since the stress \( \sigma_n \) and the force \( F_n \) are related by \( F_n = \sigma_n A_n \).

\[ \text{Fig. 5.1. Free-body diagram of the pendulum} \]

To calculate strain \( \varepsilon_n \), the following relationships must be employed:

\[ \delta_n = \delta_{n-1} + \dot{\delta}_{n-1}\Delta t + \frac{1}{2}a_{n-1}(\Delta t)^2 \]  \hspace{1cm} (5.7)
\[ \dot{\delta}_n = \dot{\delta}_{n-1} + a_{n-1} \Delta t \]  
\[ l_n = l_0 - \delta_n \]  
\[ \varepsilon_n = \ln \left( \frac{l_0}{l_n} \right) \]

where \( \delta_n \) is the deformation of the specimen, \( \dot{\delta}_n \) is deformation rate, and \( l_0 \) is the initial length of the specimen with units of in., in./sec., and in., respectively. In Eqs. 5.7 and 5.8 the higher order terms are neglected.

The time increment \( \Delta t \) used was 0.0001 sec. Experimental results indicated that the duration of impact depends mainly on pendulum weight \( w \); its value being within the range of 0.008 to 0.022 sec. Figure 5.2 is the flow diagram of the numerical iteration scheme.

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**Fig. 5.2. Flow diagram of the numerical iteration scheme**
As mentioned in Section B there are times when it is difficult to obtain viscosity $c$ accurately, and Eq. 5.2 gives the value of $k_2$ corresponding to static loading case. Therefore, during the first computer run, only the initial modulus of elasticity can be reproduced accurately by the mathematical model. The numerical results of maximum stress and strain, and duration of impact will not be far off from the experimental data since these values are largely controlled by spring constant $k_1$ and pendulum weight $w$. The values of permanent set and final modulus of elasticity will be in error compared to the experimental data since the permanent set is controlled by $k_2$ and $c$ whose values are not accurate for the first computer run.

The scheme to obtain the correct value of $k_2$ is to introduce a correction factor $\lambda$ in Eq. 5.2:

$$k_2 = \lambda \frac{490k_1}{k_1 - 490} \quad (5.11)$$

where $\lambda$ is a dimensionless number and is determined empirically by comparing experimental and numerical results.

The numerical data obtained from different computer runs indicated that $k_2$ had more effect on permanent set than $c$ did. Therefore $\lambda$ was first determined to make the numerical result of permanent set close to its experimental value. Then, the value of $c_f$ in Eq. 5.3 was modified to obtain the correct value of $c$ which would make the model
reproduce permanent set accurately. Sometimes \( \dot{\sigma}_f \) could be determined accurately from the final slope of the force-time record. In this situation, it was only necessary to determine \( \lambda \).

Once permanent set was accurately reproduced by the model, the values of duration of impact and final modulus of elasticity would be accurately reproduced as well. However, the values of maximum stress and strain reproduced by the model were always greater than the experimental results. The reason will be given in Section D.

D. Results Reproduced by the Model

Six different initial strain rates were chosen for a pendulum weight 0.595 lb., four for a pendulum weight 1.28 lbs., two for a pendulum weight 2.98 lbs. in order to evaluate the numerical computation scheme and mathematical model as outlined in Section C. The three-parameter mathematical model reproduced the impact experimental data satisfactorily. The results for three different initial strain rates are shown in Figs. 5.3, 5.4, and 5.5, which show that the model reproduced the initial and final moduli of elasticity, the permanent set, and the duration of impact with good accuracy. However, the maximum stress and strain predicted by the model were larger than the experimental data. This can be explained as follows: During the impact loading there was energy loss due to (a) air
Fig. 5.3. Comparison of reproduced and experimental data for initial strain rate 19.5 in./in./sec.
Fig. 5.4. Comparison of reproduced and experimental data for initial strain rate 33.1 in./in./sec.
Fig. 5.5. Comparison of reproduced and experimental data for initial strain rate 83.4 in./in./sec.
friction and sound and (b) energy transmitted to the wall on which the specimen was attached. Therefore, the total energy input to the specimen was less than the initial kinetic energy of the pendulum. This explains why the following empirical relation in Chapter IV has coefficients other than unity; i.e.,

$$W = 0.925(E_k)_i^{0.963}$$

(4.9)

where $W$ is the input energy defined as the maximum area under the increasing load-deformation curve and $(E_k)_i$ is the initial kinetic energy of the pendulum. This kind of energy loss takes place outside of the specimen; and, hence the mathematical model cannot sense it. If there were no such energy loss, the experimental data of maximum stress and strain should be accurately reproduced by the model. In other words, the input energy reproduced by the model, $W_p$ (which is the maximum area under the increasing stress-strain curve reproduced by the model times the volume of the specimen), should be equal to the initial kinetic energy of the pendulum $(E_k)_i$. Figure 5.6 is a plot of $W_p$ vs $(E_k)_i$ which clearly clarifies the point since $W_p = (E_k)_i$.

The effective initial velocity of the pendulum $v_{eff}$ can be calculated from the measured stored energy $W$. Then with the initial strain rate based on $v_{eff}$ and using the same mathematical model, the numerical data (marked with crosses in Figs. 5.3, 5.4, and 5.5) followed the experimental
**Fig. 5.6.** The input energy reproduced by the model vs the initial kinetic energy of the pendulum.

Input energy reproduced by the model (in.-lb.)

Initial kinetic energy of the pendulum (in.-lb.)

- ○ pendulum weight 0.595 lb.
- □ pendulum weight 1.28 lbs.
- △ pendulum weight 2.98 lbs.
data (marked with circles) accurately in all respects.

Figure 5.7 is a plot of the duration of impact reproduced by the model vs the experimental data where the data points can be connected with a straight line with slope equal to one.

As mentioned above, the exact value of spring constant \( k_2 \) was determined empirically by matching the experimental and computed permanent set. In doing that a correction factor \( \lambda \) was introduced in Eq. 5.2. A log-log plot of correction factor \( \lambda \) vs initial strain rate \( \dot{\varepsilon}_i \) is shown in Fig. 5.8 which gives the following empirical equation:

\[
\lambda = 0.0916 \dot{\varepsilon}_i^{0.7}
\]

where \( \lambda \) is dimensionless and the unit of \( \dot{\varepsilon}_i \) is in./in./sec.
Fig. 5.7. Plot of the duration of impact reproduced by the model vs the experimental data.
Fig. 5.8. Log-log plot of correction factor vs initial strain rate
VI. CONCLUSIONS AND PROSPECT

A. Conclusions

In summary we have the following conclusions:

1. The test material Hysol 8705 is nonlinear and highly strain rate dependent.

2. The material has extremely different mechanical properties under impact and sinusoidal loadings.

3. There is no simple way to relate these vastly different behaviors under impact and sinusoidal loadings at this time.

4. A nonlinear three-parameter model with its parameters properly chosen was found to reproduce the initial and final moduli of elasticity, the permanent set, the duration of impact, and the stress-strain behavior of the material under impact quite successfully.

B. Prospect

As mentioned at the beginning of this dissertation, it was anticipated that a mathematical model could be found to relate the mechanical properties of the material under impact and sinusoidal loadings. After several computer trials the nonlinear three-parameter solid failed to reproduce the sinusoidal experimental data. The problem is how to determine a model that is adequate for sinusoidal loading; and what modifications, if any, can be made to the three-para-
In the impact loading case parameters $k_2$ and $c$ are controlled by the deformation left in the specimen at the end of impact loading. For sinusoidal loading, it is a totally different story. Whenever viscosity $c$ exists, energy loss must take place. Therefore quite possibly $k_2$ and $c$ are tied up with energy loss, and this needs further investigation.

Many investigators have investigated the material photoelastically. The question is: Could we use the three-parameter mathematical model to study the stress-optic behavior of the material? This is another recommendation which can be made concerning research in this field.
VII. REFERENCES


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