One pion production in pp interactions at 2.85 GeV/c

Elizabeth Scott Hafen
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One pion production in $\bar{p}p$ interactions
at 2.85 GeV/c

by

Elizabeth Scott Hafen

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DOCTOR OF PHILOSOPHY

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I. INTRODUCTION

This thesis is an Investigation of the reactions

\[ \bar{p}p \rightarrow \bar{p}p\pi^0 \]  
\[ + \bar{p}n\pi^+ \]  
\[ + \bar{n}p\pi^- \]  

at an incident antiproton momentum of 2.85 GeV/c. Two experiments are involved, both performed at Brookhaven National Laboratory in 1967 using the 31" hydrogen bubble chamber at the AGS, one in June and the other in September. Hereafter, the first experiment is referred to as Experiment A and the second as Experiment B. Details of the beam and the film have been previously reported (1-3). The elastic scattering has been reported elsewhere (4). A preliminary sample of reactions (1)-(3) of Experiment A was used to test some Veneziano models (5).

Early experiments investigating these reactions demonstrated the production of the \(\Delta(1236)\) resonance (6,7). The resonance was originally thought to be produced via (for example) \(\bar{p}p \rightarrow (\bar{n}\pi^-)p + \bar{n}\Delta^0 + \bar{n}(\pi^-p)\); that is, via exchanging one pion between the nucleon and the antinucleon. The values of spin density matrix elements of the observed \(\Delta\) and the \(t\)-dependence predicted from this assumption about the production mechanism did not match the data even for the initial experiments (7), which had typically about 200 events in each reaction. Two recent experiments (8,9) raised the possibility that the ratio of charged \(\Delta(1236)\) to neutral \(\Delta(1236)\) does not match that predicted from the invariance of the strong interaction under isospin transformations. We have tried to measure
accurately the cross sections for $\Delta^+$ and $\Delta^0$ production.

One pion production in $\bar{p}p$ interactions has been studied over a wide range of incident antiproton momenta: 1.23 GeV/c (10), 1.30 GeV/c (10), 1.36 GeV/c (10), 1.43 GeV/c (10), 1.51 GeV/c (11), 1.61 GeV/c (12-13), 1.65 GeV/c (11), 1.80 GeV/c (11), 1.95 GeV/c (11), 2.15 GeV/c (11), 2.2 GeV/c (9), 2.45 GeV/c (11), 2.60 GeV/c (11), 2.70 GeV/c (8), 2.75 GeV/c (11), 2.90 GeV/c (11), 3.0 GeV/c (6), 3.28 GeV/c (14), 3.6 GeV/c (15-16), 3.66 GeV/c (14), 4.0 GeV/c (7), 5.7 GeV/c (17), and 6.94 GeV/c (18-19).

Reactions (1)-(3) provide rather sensitive tests of C and CP invariance of the strong interaction because under various combinations of C, P, and R the momentum and angular distributions of the particles transform from that of one final state into that of another (20). C is the charge conjugation operator, which transforms a particle into its antiparticle; P is the parity operator which inverts the x, y, and z coordinate axes; R is a rotation of 180° about an axis perpendicular to the direction of the incident $\bar{p}$. Whenever statistical accuracy has been adequate, reactions (1)-(3) have been studied for CP invariance, with no statistically significant deviations being found. This is also true in the present case, subject to the limitations discussed in Appendix A.

If CP invariance is accepted as valid it can be used as a check for biases, as is done in Appendix A. It can also be used to increase the statistical accuracy of the experiment by adding together equivalent histograms, as is done here and has also been done in every other experiment involving these reactions.

In general, at all energies, some form of a one pion exchange model
has been used to represent the data. In all cases having significant statistical accuracy, it was concluded that the form of one pion exchange in vogue was inadequate to describe the production and decay angular distributions of the $\Delta$ resonance. Several recent experiments have indicated that a one pion exchange model was also inadequate to explain the different structure of the effective mass plots for the $\Delta^0$ compared to the $\Delta^+$ (9, 10, 12). Some of the difference, in this experiment and in the experiment discussed in the Sears paper (8), appears to be due to contamination of the final states involved with spurious fits to other final states, as is discussed at length in Appendix A. The remaining difference is apparently due to the mass difference of the two charge states $\Delta^+$ and $\Delta^0$. We have fitted the masses of the $\Delta^+$ and the $\Delta^0$ as $1233 \pm 4$ MeV/c$^2$ and $1220 \pm 7$ MeV/c$^2$ respectively, with a width of $135 \pm 15$ MeV/c$^2$. The cross section for $\Delta^+$ production was found to be $2.08 \pm 0.04$ mb. Spin density matrix elements have also been calculated.

Chapter II of this thesis discusses the experimental details. Chapter III gives experimental results such as cross sections and CP plots. Chapter IV contains the study of the $\Delta$ resonance. Appendix A discusses the ambiguity problem, and Appendix B discusses fitting philosophy.
II. EXPERIMENTAL DETAILS

A. Beam and Film

The film used in this experiment was taken in two sections, both at Brookhaven National Laboratory in 1967 using the 31" hydrogen bubble chamber at the AGS. Between the first 40,000 pictures, taken in June, and the last 110,000 pictures, taken in September, the bubble chamber was warmed up, dismantled, cleaned, and reassembled. Because of this the two sections of film had different beam momenta and different optics constants. In addition to these differences, the scanning, measuring, and processing procedures were changed between the processing of the 2 prongs in the first 40,000 pictures and the processing of the 2 prongs in the next 40,000 pictures. The data obtained from the two sections of film thus must be considered to be two separate experiments, at least as far as normalization and possible experimental biases are concerned. This was in fact the purpose in changing the processing procedures; the first experiment, Experiment A, had raised the possibility of interesting structure both in the elastics (4) and in the one pion production final states, discussed in this paper. The second experiment, Experiment B, was intended as an independent check, in so far as possible, of Experiment A. After bias studies and corrections, the two experiments did support each other, and have been added together for analysis purposes.

The beam and target properties have been reported previously (1-3). The \( \pi, \mu, \text{ and } K \) contamination was < 1% in Experiment A and < 2.5% in Experiment B, as reported previously (1).
B. Scanning and Measuring Procedures

In both experiments, the beam flux was determined from counting the beam tracks entering the chamber in every tenth frame. This flux was then corrected for the number of interactions occurring in the chamber but before the beginning of the fiducial volume. The average number of beam tracks in the chamber was $8.76 \pm 0.05$ per frame for 37,189 frames in Experiment A and $9.31 \pm 0.05$ beam tracks per frame for 33,351 frames in Experiment B. The errors given are statistical only. A beam count correlation was attempted, and the efficiency of the beam count was found to be $98 \pm 2\%$, consistent with 100\%. All measured events in 5,897 frames of Experiment A were examined for "non-events", such as interactions induced by early or late or off-momentum incident antiprotons. The number of events measured was then corrected for these "non-events." In Experiment B, all measured events were examined, and "non-events" were eliminated from the final data sample.

In Experiment A, 37,189 frames were scanned for all topologies, and 9,886 frames were scanned twice, also for all topologies. The efficiency of scan 1 for 2 prongs was $90.6\% \pm 1.5\%$ and of scan 2 was $92.6\% \pm 1.5\%$. The approximate vertex location of each event was recorded at the scanning table by noting the location of the vertex within a grid having squares approximately 2" on a side in real space. Only events occurring in the first scan and between approximately 3" and 18" in the chamber as indicated by the grid location were entered on a measuring list, although all events in the chamber had been included in the scans.

In Experiment B, 33,351 frames were scanned twice for 2 prongs, once without grid locations and once with. All events found within the chamber...
were measured and the fiducial volume was determined from a study of where losses due to high \( \chi^2 \) for poorly measured events began to become important. (See Appendix A.) A third scan within approximately the fiducial volume so determined was done with grid locations, and a fourth scan was done by physicists for stopping positive tracks only, to study the \( t \)-dependence of scan correlations in the elastics. As might be expected, scans were found to be correlated (for elastics only) in the region requiring \( \phi \)-dependent corrections to the \( t \)-distribution; that is, for proton tracks less than approximately 1 cm. \( (-t < 0.04 \text{ (GeV/c)}^2) \). One assumes from this that the inelastic final states involving production of one pion are also uncorrelated since the conditions leading to scan bias for low \( t \) elastics don't hold for the inelastics. The scan efficiency for the uncorrelated events was \( 96.0 \pm 1.0\% \).

After Experiment A had been measured and processed, those events which might have been poorly-measured elastics were classified as "lost protons" and remeasured. That classification consisted of (a) those events about which either the scanner or the measurer had commented that the positive track was dark; (b) those events found upon ion-checking to be inconsistent with the fits obtained; (c) events having elastic fits with \( \chi^2 \) greater than 15. Comments on the negative tracks were ignored. The "no fit" category was not ion-checked until much later, and lost proton, antiproton, and kaon events from this classification were not remeasured in Experiment A. This lack is thought to have no effect on the final data sample; see the comments below on Experiment B. Experiment A had heavy losses in transferring the data tapes from the EMR 6050, which
controlled the measuring, to the IBM 360/65, which did the processing, because the tape drives were not tuned compatibly at that time.

Experiment B was measured in 4 passes. The first pass included all those events found in the first scan. The second pass consisted of those events about which the scanner or measurer had commented that the positive track was dark. The third pass consisted of those events about which the scanner or measurer had commented that the negative track was dark, the events having no ionization-consistent fits, and elastic fits having $\chi^2$ greater than 12. The fourth pass consisted of the lost proton, antiproton, and kaon events found by ion checking the "no fit" category and of those events not previously measured but which were found in any one of the 4 scans and had a proton comment by any one of the scanners. This last pass was different from Experiment A and is thus of interest. It added 101, 101, and 168 events, or about 7% to reactions (1)-(3) respectively, and added about 800 events, or about 7% to the elastics. The qualitative structure in the effective mass and in the angular distributions did not change in any of these states, so presumably the results of Experiment A would not have been changed by this addition either. The new events found in the scan correlations were not included in the final samples of reactions (1)-(3) because the contribution to the inelastics from that source is not known; including them would in essence make the scanning efficiency for the inelastics unknown.

C. Geometric Reconstruction

The events were processed using a locally written version of HGEOM. There were 3 differences between the 2 experiments:
(1) The hydrogen density determined from the length of \( \mu \) tracks in \( \pi \mu e \) decays was \( 0.0660 \pm 0.0007 \text{ gm/cm}^3 \) for Experiment A and \( 0.0608 \pm 0.0002 \text{ gm/cm}^3 \) for Experiment B. See Figure 1. The larger error on the hydrogen density in A is due partly to the fact that only 225 \( \mu \)'s were measured in Experiment A and 394 in Experiment B, and partly due to the better control of measuring quality for the \( \mu \) tracks in Experiment B over those in Experiment A; the \( \mu \) tracks in Experiment A were measured off-line, before the computer-controlled measuring system was implemented while the \( \mu \) tracks in Experiment B were measured under computer control.

(2) The momentum for Experiment A was \( 2885 \pm 80 \text{ MeV/c} \) and for Experiment B was \( 2820 \pm 80 \text{ MeV/c} \). The momentum for the summed experiments was \( 2850 \pm 90 \text{ MeV/c} \). See Figure 2.

(3) A study of the stretches for the elastics for Experiment A and Experiment B revealed that optics constants had to be made roll-dependent in Experiment B but not in A.

For both experiments, wherever possible, the momentum of the positive track was determined from its range, since vertices can be measured more accurately than can the curvature of short tracks.

D. Kinematic Reconstruction

The kinematic reconstruction was done by GUTS (21). An elastic fit was tried if the missing mass obtained from the measured quantities was within 5 standard deviations from 0. Any event which had a fit to an elastic hypothesis with \( \chi^2 \) less than 40 was assumed to be an elastic and no other fits were tried. If the event did not fit an elastic hypothesis, all other fits were tried and accepted under the following conditions.
Figure 1. Muon length from $\pi$ decay in Experiment A (Figure 1a) and in Experiment B (Figure 1b). The vertical errors shown are statistical. The horizontal errors shown are the bin widths. The curve is a fit to a gaussian plus a constant. There are 225 events in Figure 1a and 394 events in Figure 1b.
Figure 2. Beam momentum of elastic fits for Experiment A (Figure 2a), for Experiment B (Figure 2b), and for both (Figure 2c). The curve is a fit to a gaussian plus a constant.
The $\bar{p}p\pi^0$, $\bar{p}n\pi^+$, and/or $p\pi^-$ final states were tried if the missing mass was within 5 standard deviations from the mass of the appropriate neutral and was accepted if the $\chi^2$ for the fit was less than 9. The $K^+K^-$ hypothesis was tried if the missing mass was within 5 standard deviations from 0 and accepted if the $\chi^2$ for the fit was less than 40. The $\pi^+\pi^-$ hypothesis was tried if the missing mass was within 3 standard deviations from 0 and accepted if $\chi^2$ was less than 40. The $\pi^+\pi^-\pi^0$ hypothesis was tried if the missing mass was within 3 standard deviations from the $\pi^0$ mass and accepted if the $\chi^2$ was less than 9. The numbers of events obtained and the cross sections for these final states are given in Chapter III.

E. Ionization Consistence

All events having a track with momentum less than 1300 MeV/c or with dip greater than 50° were examined for ionization consistence of the fits obtained to it. All $K^+K^-$ fits were examined. Whenever an event was ionization consistent with more than one fit, and that choice was between a one constraint fit and the "no fit" possibility, the one constraint hypothesis was chosen preferentially.

F. Bookkeeping

To eliminate the duplicates arising from the many measuring passes, the GUTS output tapes were sorted on roll and frame numbers, then the reconstructed vertices of events in nearby frames were compared. If 2 vertices were less than 0.2" apart, the fits were assumed to be from the same event. The fits obtained for each event were then examined. If an elastic fit was obtained on any measurement, only the elastic fit with
the lowest $\chi^2$ was retained and all other fits to any other final state were discarded. If no elastic fit was obtained, the measurement having the lowest $\chi^2$ for a given hypothesis was retained, then these hypotheses were examined for ionization consistence whenever possible. Any ambiguities remaining among the final states after such examination were decided by keeping a fit only if its $\chi^2$ probability was greater than 5 times the $\chi^2$ probability of all other acceptable fits to that event. If this criteria was not satisfied for one of the remaining fits, the event was discarded. See Appendix A for any possible effect this would have on the data. For those events for which all tracks were minimum-ionizing, the "no fit" category was chosen only when no other fits were obtained. Ambiguities were resolved as for those events which could be examined for ionization consistence.
III. EXPERIMENTAL RESULTS

A. Cross Sections

The cross section is given by

\[ \sigma = \frac{AN}{\rho A_o \beta fh} \] (4)

where

- \( A = 1.008 \text{ gm/mole} \)
- \( A_o = 6.0226 \times 10^{23} \text{ atoms/mole} \)
- \( \rho = \text{density of hydrogen (gm/cm}^3) \)
- \( \beta = \text{average beam tracks per picture} \)
- \( f = \text{total number of frames} \)
- \( \bar{h} = \text{average length of a beam track, corrected for interactions and for track curvature.} \)

The correction for losses due to interactions is given by

\[ \bar{h} = (1-\alpha)S_{FV} + \alpha S_i = S_{FV} - \alpha(S_{FV} - S_i) \]

\( \alpha = \text{probability a beam track will interact within the fiducial volume} \)

\( S_{FV} = \text{track length of non-interacting beam tracks in the fiducial volume} \)

\( S_i = \text{average track length of interacting beam tracks.} \)

The correction for track curvature is given by

\[ s = L + 1.5 \times 10^{-8}(B/p)^2L^3 \]

\( L = \text{length of the fiducial volume (in cm) along the direction of the beam when it first enters the chamber} \)

\( B = \text{average magnetic field, in gauss} \)

\( P = \text{beam momentum, in MeV/c} \)
\[ N = \text{number of real events in the fiducial volume} = \frac{N'}{\varepsilon_s \varepsilon_p \varepsilon_c} \]

\[ N' = \text{number of events in the selected final states} \]

\[ \varepsilon_s = \text{scanning efficiency} \]

\[ \varepsilon_p = \text{processing efficiency} \]

\[ \varepsilon_c = \text{probability a real event will pass the cuts described in Appendix A and a fake one will not} \]

Only the last term, \( \varepsilon_c \), depends on the final state. All other terms for the two experiments are given in Table 1. The factor \( \varepsilon_c \) is discussed in Appendix A. If we define

\[ F = \frac{A}{\rho A_\beta \sigma \varepsilon_s \varepsilon_p} \tag{5} \]

then \( F \) is a conversion factor from corrected number of events to total or partial cross sections, applicable to all final states. The factors \( F \) for the two experiments separately and for the summed experiments are also given in Table 1. The conversion factors \( F^c \) to apply to individual final states are given in Table 2, which also contains the cross sections.

**B. CP Invariance**

One can define distribution functions, \( W \), of any 5 independent kinematical variables in the 3 body final states of \( \bar{p}p \rightarrow 1+2+3 \) as the probability that an event will be seen with those values. If these 5 variables are chosen as \( E_1 \), the energy of particle 1 in the center of mass; \( \theta_1 \), the angle particle 1 makes in the center of mass with the incident \( \bar{p} \) momentum; \( E_2 \), the energy of particle 2 in the center of mass; \( \theta_2 \),
<table>
<thead>
<tr>
<th>Quantity</th>
<th>Experiment A</th>
<th>Experiment B</th>
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<tr>
<td>$\rho$</td>
<td>$0.06605 \pm 0.00070 \text{ gm/cm}^3$</td>
<td>$0.06075 \pm 0.0002$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$8.455 \pm 0.047^a$</td>
<td>$9.094 \pm 0.051^a$</td>
</tr>
<tr>
<td>$f$</td>
<td>37,189</td>
<td>33,351</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$0.1107 \pm 0.0016$</td>
<td>$0.1516 \pm 0.0032$</td>
</tr>
<tr>
<td>$B$</td>
<td>$22.46 \times 10^3 \text{ gauss}$</td>
<td>$22.46 \times 10^3 \text{ gauss}$</td>
</tr>
<tr>
<td>$P$</td>
<td>$2885 \pm 80 \text{ MeV/c}$</td>
<td>$2820 \pm 80 \text{ MeV/c}$</td>
</tr>
<tr>
<td>$L_{FV}$</td>
<td>$36.576 \text{ cm}$</td>
<td>$54.610 \text{ cm}$</td>
</tr>
<tr>
<td>$S_{FV}$</td>
<td>$36.620 \pm 0.022 \text{ cm}$</td>
<td>$54.765 \pm 0.006$</td>
</tr>
<tr>
<td>$L_1$</td>
<td>$17.927 \text{ cm}$</td>
<td>$26.648 \text{ cm}$</td>
</tr>
<tr>
<td>$S_1$</td>
<td>$17.932 \pm 0.0001 \text{ cm}$</td>
<td>$26.666 \pm 0.001 \text{ cm}$</td>
</tr>
<tr>
<td>$\bar{N}$</td>
<td>$34.551 \pm 0.466 \text{ cm}$</td>
<td>$50.505 \pm 0.090 \text{ cm}$</td>
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<tr>
<td>$\epsilon_5$</td>
<td>$0.906 \pm 0.015$</td>
<td>$0.960 \pm 0.010$</td>
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<tr>
<td>$\epsilon_p$</td>
<td>$0.935 \pm 0.007$</td>
<td>$1.026 \pm 0.006^c$</td>
</tr>
<tr>
<td>$F$</td>
<td>$2.752 \pm 0.060 \text{ mb/ev}$</td>
<td>$1.826 \pm 0.023$</td>
</tr>
<tr>
<td>$F_{A+B}$</td>
<td>$1.095 \pm 0.014$</td>
<td></td>
</tr>
</tbody>
</table>

$^a$Corrected for losses due to interactions occurring in the chamber but before the beginning of the fiducial volume.

$^b$ $\alpha$ is the ratio of total number of interactions to total number of beam tracks. The errors on $\beta$ and $\alpha$ are not independent.

$^c$The processing efficiency is basically the ratio of number of events in all final states to the number of events found in the scan. In Experiment B few events missed all four measuring passes. Since the measurers were allowed to add events found while measuring, the net efficiency is greater than 1.
Table 2. Total cross sections and conversion factors from events to microbarns

<table>
<thead>
<tr>
<th>Final State</th>
<th>$\bar{p}p\pi^0$</th>
<th>np$\pi^+$</th>
<th>pn$\pi^-$</th>
<th>$\pi^+\pi^-\pi^0$</th>
<th>$\pi^+\pi^-$</th>
<th>$K^+K^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conversion factor $F_c$, in $\mu$b/event</td>
<td>1.225$^{+0.020}_{-0.020}$</td>
<td>1.341$^{+0.026}_{-0.026}$</td>
<td>1.264$^{+0.023}_{-0.023}$</td>
<td>1.095$^{+0.014}_{-0.014}$</td>
<td>1.095$^{+0.014}_{-0.014}$</td>
<td>1.095$^{+0.014}_{-0.014}$</td>
</tr>
<tr>
<td>Total cross section, in mb</td>
<td>2.90$^{+0.08}_{-0.08}$</td>
<td>2.83$^{+0.08}_{-0.08}$</td>
<td>2.96$^{+0.08}_{-0.08}$</td>
<td>0.704$^{+0.029}_{-0.029}^a$</td>
<td>0.016$^{+0.004}_{-0.004}$</td>
<td>0.0066$^{+0.0027}_{-0.0027}$</td>
</tr>
</tbody>
</table>

$^a$Upper limit. This state may have some contamination from $\pi^+\pi^-+2\pi^0$ events.
the angle particle 2 makes in the center of mass with the incident $\overline{p}$ momentum; and $\phi_{12}$, the angle in the plane perpendicular to the incident $\overline{p}$ between the projections of the momenta of particles 1 and 2 on that plane, then Pais has shown (20) that applying the charge conjugation operator $C$ and the parity operator $P$ and the rotation operator $R$ to the final state $1+2+3$ relates some of the distribution functions to each other in a simple manner if the strong interaction is invariant under the appropriate operation. Some of these relations are given below.

CP invariance predicts

$$W(E_1, \theta_1, E_2, \theta_2, \phi_{12}) = W(E_{\pi^-}, -\theta_{\pi^-}, E_{\pi^+}, -\theta_{\pi^+}, \phi_{12})$$

and CR predicts

$$W(E_1, \theta_1, E_2, \theta_2, \phi_{12}) = W(E_{\pi^-}, -\theta_{\pi^-}, E_{\pi^+}, -\theta_{\pi^+}, -\phi_{12})$$

or alternatively, either one predicts

$$W(E_1, \cos\theta_1, E_2, \cos\theta_2, \cos\phi_{12}) = W(E_{\pi^-}, -\cos\theta_{\pi^-}, E_{\pi^+}, -\cos\theta_{\pi^+}, \cos\phi_{12})$$

Integrating over energies and angles this gives

$$W(\cos\theta_p, \cos\theta_{\overline{p}}) = W(-\cos\theta_p, \cos\theta_{\overline{p}})$$

and

$$W(\cos\theta_{\pi^0}) = W(-\cos\theta_{\pi^0})$$

for the $p\overline{p}\pi^0$ state. One can transform variables to obtain the relations

$$W(p_{CM}(p)) = W(p_{CM}(\overline{p}))$$
\[ W(p_{\pi^0}) = W(-p_{\pi^0}) \] for the \( p\pi^0 \) state, \hspace{1cm} (12)

where \( p_{CM}^i \) is the momentum of particle \( i \) in the center of mass and \( M_{12} \) is the effective mass of the particle combination \( 1+2 \).

These relations mean that Figure 3a should match Figure 3b if Figure 3b is reflected about \( \cos \theta = 0 \), Figure 3c should be symmetric about \( \cos \theta = 0 \), Figure 4a should match Figure 4b, and Figure 5a should match Figure 5b. After the cuts described in Appendix A, these and analogous expressions for the \( \bar{p}n\pi^+ \) and \( p\pi^- \) states hold within the statistical resolution of the experiment. Figures 3, 4 and 5 are the final data samples after the cuts.
Figure 3. Histograms of the center of mass scattering angle for each particle in the three final states ppπ₀, npπ⁺, and pnπ⁻.
Figure 4. Histograms of the momentum in the center of mass for each particle in the three final states $pp\pi^0$, $np\pi^+$, and $pn\pi^-$. 
Figure 5. Histograms of the squares of the effective masses of all the two-particle combinations in the final states $p\bar{p}^0$, $np^+$, and $p\bar{n}^-$.
IV. PRODUCTION AND DECAY OF THE $\Delta(1236)$

A. Isospin and Charge Conjugation Arguments

An examination of the effective mass plots (Figures 5, 6 and 7) suggests that what occurs in the reactions

$$\text{pp} \rightarrow \bar{\text{pp}}\pi^0$$
$$\rightarrow \bar{\text{pn}}\pi^+$$
$$\rightarrow \bar{\text{pn}}\pi^-$$

(1) (2) (3)

can be described as

$$\bar{\text{pp}} \rightarrow \bar{\text{p}}\Delta^+$$
$$\rightarrow \bar{\text{n}}\Delta^0$$
$$\rightarrow \bar{\text{p}}\Delta^+$$
$$\rightarrow \bar{\text{n}}\Delta^0$$
$$\rightarrow \bar{\text{pp}}\pi^0$$
$$\rightarrow \bar{\text{pn}}\pi^+$$
$$\rightarrow \bar{\text{pn}}\pi^-$$

(13) (14) (15) (16) (17) (18) (19)

with the $\Delta(\bar{\Delta})$ subsequently decaying to a nucleon (antinucleon) and pion. Reactions (17)-(18) are background terms in which no resonance is produced. Charge conjugation invariance requires that (13) and (15) have the same production cross sections. Similarly, reaction (14) must have the same cross section as (16), and reaction (18) must have the same cross section as (19).

The total transition amplitude from initial $\bar{\text{pp}}$ state to a final nucleon-antinucleon-pion state via a $\Delta$ or $\bar{\Delta}$ resonance can presumably be written as a product of amplitudes describing separately the resonance
production and the mass and angular dependence of its decay. The isospin invariance of the strong interaction, as well as the charge conjugation invariance mentioned above, can be used to relate the production amplitudes of the four resonances produced and their decay amplitudes into the three observed final states.

Since the strong interaction can be represented by operators which are isoscalars, we can write the production part of the amplitude for the reaction \( 1+2 \rightarrow 3+4 \rightarrow 5+6 \) in terms of reduced matrix elements \( T(l) \) and Clebsch-Gordan coefficients \( \langle m_1 m_2 \mid \text{Im} \rangle \),

\[
\langle f \mid T \mid i \rangle = \sum_i \langle m_1 m_2 \mid \text{Im} \rangle \langle m_1 \mid T \mid m_2 \rangle,
\]

where \( m_1 \) is the third component of the isospin \( I_1 \) of particle \( i \), \( I \) is the total isospin of the two-particle state, and \( m \) is the third component of \( I \). The cross section for the reaction is proportional to \( |\langle f \mid T \mid i \rangle|^2 \).

To examine the effect of the isospin of the \( \Delta \), we need only consider reactions (13) and (14). For reaction (13), the initial state is

\[
|l> = |pp> = |\frac{3}{2}, -\frac{3}{2}> \times |\frac{3}{2}, \frac{3}{2}> = \sqrt{\frac{3}{2}} |1, 0> - \sqrt{\frac{3}{2}} |0, 0>,
\]

and the final state is

\[
|f> = |p\Delta^+> = |\frac{1}{2}, -\frac{1}{2}> \times |\frac{1}{2}, \frac{1}{2}> = \sqrt{\frac{1}{2}} |2, 0> + \sqrt{\frac{1}{2}} |1, 0>. \]

The transition amplitude is

\[
|\langle \overline{p\Delta^+} \mid T \mid pp \rangle| = \frac{1}{2} T(1).
\]
the final state is

\[ |f> = |\bar{n}\Delta^0> = |\frac{3}{2},\frac{3}{2}> \times |\frac{1}{2},-\frac{1}{2}> = \sqrt{\frac{3}{2}}|2,0> - \sqrt{\frac{1}{2}}|1,0>, \tag{24} \]

giving for the transition amplitude

\[ <\bar{n}\Delta^0|T|pp> = -\frac{1}{2}T(1). \tag{25} \]

From this we see that

\[ |<\bar{n}\Delta^0|T|pp>|^2 = \frac{1}{4}|T(1)|^2 = |<\bar{p}\Delta^+|T|pp>|^2, \tag{26} \]

so the probability of producing a \( \Delta^+ \) in reaction (13) equals the probability of producing a \( \Delta^0 \) in reaction (14), or an equal number of \( \Delta^+ \) and \( \Delta^0 \) should be produced by the two reactions. Including the charge conjugation requirements described previously leads us to the conclusion that reactions (13)-(16) each have the same probability.

Now consider the decay of the \( \Delta^+ \) and the \( \Delta^0 \). Let \( T' \) be the transition operator for the decay \( \pi \rightarrow \gamma+6 \). The isospin decomposition of the decay amplitude is given by

\[ <f|T'|i> = <\frac{3}{2},\frac{3}{2}|16|m_6|T'|14,m_4> = \sum_i <m_6|m_4|1m>|T'(i). \tag{27} \]

For the \( \Delta^+ \), \( |i> = |\frac{3}{2},\frac{3}{2}> \) and \( |f> is |p\pi^0> or |n\pi^+>. \)

\[ |p\pi^0> = |\frac{3}{2},\frac{3}{2}> \times |1,0> = \sqrt{\frac{3}{2}}|\frac{3}{2},\frac{3}{2}> - \sqrt{\frac{1}{2}}|\frac{3}{2},\frac{3}{2}> \tag{28} \]

\[ |n\pi^+> = |\frac{1}{2},-\frac{1}{2}> \times |1,1> = \sqrt{\frac{1}{2}}|\frac{3}{2},\frac{3}{2}> + \sqrt{\frac{1}{2}}|\frac{3}{2},\frac{3}{2}>. \tag{29} \]

So \( <p\pi^0|T'|\Delta^+> = \sqrt{\frac{3}{2}}T'(\frac{3}{2}) \) and \( <n\pi^+|T'|\Delta^+> = \sqrt{\frac{1}{2}}T'(\frac{3}{2}) \)

For the \( \Delta^0 \), \( |i> = |\frac{3}{2},-\frac{3}{2}> \) and \( |f> is |p\pi^-> or |n\pi^0>. \)
\begin{align}
|p\pi^-\rangle &= |{1\over 2}, -{1\over 2}\rangle \times |1, -1\rangle = \sqrt{2\over 3} |{3\over 2}, -{1\over 2}\rangle - \sqrt{2\over 3} |{1\over 2}, -{1\over 2}\rangle \\
|n\pi^0\rangle &= |{1\over 2}, -{1\over 2}\rangle \times |1, 0\rangle = \sqrt{2\over 3} |{3\over 2}, -{1\over 2}\rangle + \sqrt{2\over 3} |{1\over 2}, -{1\over 2}\rangle
\end{align}

So \(<p\pi^-|^T|^\Delta^+> = \sqrt{4\over 3} T'(\frac{3}{2}) \) and \(<n\pi^0|^T|^\Delta^+ > = \sqrt{2}\ T'(\frac{3}{2})
\]

The \(\pi^0\) and \(\bar{\pi}^0\) decay modes are not observable in this experiment, so only \(\Delta^0\) and \(\bar{\Delta}^0\) are observed. Thus:

\[
\frac{\text{total number of observed } \Delta^+}{\text{total number of observed } \Delta^0} = \frac{|<p\pi^0|^T|^\Delta^+>|^2 + |<p\pi^-|^T|^\Delta^+>|^2}{|<p\pi^-|^T|^\Delta^0>|^2}
\]

\[
= \frac{|T'(\frac{3}{2})|^2 \left[\frac{2}{3} + \frac{1}{3}\right]}{\left|T'(\frac{3}{2})\right|^2 \left[-\frac{1}{3}\right]} = \frac{3}{T}
\]

From charge conjugation invariance,

\[
\frac{\text{number of observed } \bar{\Delta}^+}{\text{number of observed } \bar{\Delta}^0} = \frac{\text{number of observed } \Delta^+}{\text{number of observed } \Delta^0} = \frac{3}{T}
\]

B. Mass Dependence of the Decay

As commented previously, the transition amplitude for the decay of the \(\Delta\), written above as \(T'(\frac{3}{2})\), is presumably separable into a mass-dependent part and an angular-dependent part. In somewhat more explicit form, this means we can write the differential cross sections for the \(p\pi^0\) state and the summed \(p\pi^+\) and \(p\pi^-\) states as
\[
\frac{d^5\sigma}{\sigma_{\pi\pi}} = \frac{2}{3} A H(t_{\pi\pi}) G(\alpha_{\pi\pi}) E(M_{\pi\pi}) \, dM_{\pi\pi}^2 \, \frac{d^2\sigma}{\sigma_{\pi\pi}} \, dM_{\pi\pi} \, dt_{\pi\pi}
\]

\[+ \frac{2}{3} A H(t_{\pi\pi}) G(\alpha_{\pi\pi}) E(M_{\pi\pi}) \, dM_{\pi\pi}^2 \, \frac{d^2\sigma}{\sigma_{\pi\pi}} \, dM_{\pi\pi} \, dt_{\pi\pi} \, dM_{\pi\pi}^2 \] (37)

\[
+ \left( \frac{\pi}{2} \right)^2 \frac{C_1}{E_{CM}} \, \frac{d^2\sigma}{\sigma_{\pi\pi}} \, d\phi_{\pi\pi} \, dt_{\pi\pi} \, dM_{\pi\pi} \, dt_{\pi\pi} \, dM_{\pi\pi}^2 \, dt_{\pi\pi} \right)
\]

where \( A \) is the probability of producing a \( \pi\pi \), \( \alpha \) is an as yet unspecified choice of 2 angular variables appropriate to the system of particles \( i \) (where \( i \) is one or two final state particles, as indicated above), \( t_{\pi} \) is the 4-momentum transfer to the system of particles \( i \), \( M_{\pi} \) is the effective mass of the system of particles \( i \), \( C_1 \) is the probability of
producing nonresonant $pp\pi^0$ background, $C_2$ is the probability of producing nonresonant $p\pi^+$ (or $p\pi^-$) background, $E(M_i)$ is the mass dependence of the decay of the resonance (the explicit form is given in Equation (41)) for the mass combination $i$, $G(\alpha_i)$ is the angular dependence appropriate to the decay of the $\Delta$ formed by mass combination $i$ (explicit form given in Equation (44)), and $H(t_i)$ is the $t$-dependence of the $\Delta$ production amplitude (explicit form given in Equation (48)). $H$, $G$, and $E$ must be normalized such that the integral over the range of their variables is 1. The factors multiplying $C_1$ and $C_2$ are the explicit normalization factors which normalize the integral over phase space to 1. $E_{CM}$ is the total center of mass energy available--2.72 GeV/c$^2$.

This form for the differential cross section can be integrated over some or all of the variables to obtain expressions for various distributions in the remaining variables. For example, integrating over all variables gives the total cross section. Extracting the $\Delta^+$ from this gives:

$$\sigma_{\Delta^+} = \frac{1}{3} A_{\sigma p\pi^-} + \frac{2}{3} A_{\sigma pp\pi^0} = \frac{1}{6} A(\sigma p\pi^- + \sigma p\pi^+) + \frac{2}{3} A_{\sigma pp\pi^0}.$$  

Integrating over $t$ and $\alpha$ gives an expression for the 2-dimensional Dalitz plot density.

$$\frac{1}{\sigma_{pp\pi^0}} \frac{d^2 \sigma_{pp\pi^0}}{dM_{p\pi^-}^2 dM_{p\pi^0}^2} = \frac{2}{3} A x E(M_{p\pi^-} o) + \frac{2}{3} A x E(M_{p\pi^0} o)$$

$$+ \left(\frac{\pi}{2E_{CM}}\right)^2 C_1.$$  

(39)
The expression for the mass dependence of the decay amplitudes, $E$, is from Jackson (22). If $M$ and $m$ are the masses of the nucleon and pion respectively of the particle combination $(N\pi)$ being considered, and $M_o$ and $\Gamma_o$ are the mass and width respectively of the $\Delta$ formed, then $E(M_{N\pi})$ is given by

$$E(M_{N\pi}) = \left(\frac{\pi}{2E_{CM}}\right)^2 \frac{M_{N\pi}}{P(M_{N\pi};M^2,m^2)} \frac{\Gamma(M_{N\pi})}{(M_{N\pi}^2 - M_o^2)^2 + M_o^2 \Gamma_o^2(M_{N\pi})}$$  \hspace{1cm} (41)$$

$$P(M_{N\pi};M^2,m^2) = \frac{1}{2}[M_{N\pi}^2 - 2(M^2 + m^2) + (M^2 - m^2)^2/M_{N\pi}^2]^{1/2}$$  \hspace{1cm} (42)$$

$\Gamma(M_{N\pi})$ is the empirical form of the mass dependence of the width, and is given by

$$\Gamma(M_{N\pi}) = \Gamma_o \left[ \frac{P(M_{N\pi};M^2,m^2)}{P(M_o;M^2,m^2)} \right]^3 \times \left[ \frac{A_1 m^2 + [P(M_o;M^2,m^2)]^2}{A_1 m^2 + [P(M_{N\pi};M^2,m^2)]^2} \right].$$  \hspace{1cm} (43)$$
The \((A_1 m^2)^{-1/2}\) is the radius of interaction of the \(N\pi\) system; \(A_1\) has an estimated value \(A_1 = 2.2\) (22).

The forms obtained for the Dalitz plot densities by substituting these expressions for \(P\), \(\Gamma\) and \(E\) into Equations (39) and (40) were used to fit the experimental Dalitz plot densities for all three final states as displayed in Figures 6 and 7 and as listed in Tables 3 and 4. The results obtained are given in Table 5.

The first series of fits (fits 1-4) were to the \(pp\pi^0\) state. It was found that the \(\Delta\) width was highly correlated with the value of \(A_1\), and that \(A_1\) was itself poorly determined but consistent with 0. The relatively high value of \(\chi^2\) for all fits is possibly an indication that the data has sufficient resolution to distinguish between different forms for \(E(M_{N\pi})\) and that the chosen form is not a valid one. The fit to the \(pp\pi^0\) Dalitz plot with \(A_1\) fixed at 0 (fit 4) is displayed in Figure 8 superposed on the effective mass histograms of all particle combinations of all three final states. The fit to the \(pp\pi^0\) state reproduces the general features of all three final states, with the exception of the mass region of the \(\Delta^0\) in the \(\pi\pi^+\) and the \(\pi\pi^-\) final states. The lower curve in each case is the contribution from the flat background, the middle curve is the background plus the reflected \(\Delta(A)\) and the top curve is the full differential cross section.

The second series of fits (fits 5-6) is to the summed \(\pi\pi^+\) and \(\pi\pi^-\) Dalitz plot. Permitting a different mass and amplitude for the \(\Delta^+\) and the \(\Delta^0\) gave a much better fit but the amplitude for the \(\Delta^+\) was less well determined than when the \(pp\pi^0\) state is fitted. Note that in fit 6 the ratio
Figure 6. The $\bar{p}p\pi^0$ Dalitz plot. The units are $(\text{GeV}/\text{c}^2)^2$. The curve shown is the average boundary.
Figure 7. The summed $\Sigma p\pi^+$ and $\Sigma\bar{p}\pi^-$ Dalitz plots. The units shown are \((\text{GeV/c}^2)^2\). The curve shown is the average boundary. The points outside the curve are mostly from the $p\pi\pi$ state.
Table 3. Density distribution, normalized to 1, of the $pp\pi^0$ final state

<table>
<thead>
<tr>
<th>$M_{pp\pi}^2$ in $(\text{GeV}/c^2)^2$</th>
<th>1.1 - 1.45</th>
<th>1.45 - 1.60</th>
<th>1.60 - 1.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 - 1.45</td>
<td>.0346 ± .0038</td>
<td>.0266 ± .0034</td>
<td>.0367 ± .0039</td>
</tr>
<tr>
<td>1.45 - 1.60</td>
<td>.0346 ± .0038</td>
<td>.0236 ± .0032</td>
<td>.0325 ± .0037</td>
</tr>
<tr>
<td>1.60 - 1.80</td>
<td>.0356 ± .0039</td>
<td>.0236 ± .0032</td>
<td>.0160 ± .0026</td>
</tr>
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<td>1.8 - 2.05</td>
<td>.0241 ± .0032</td>
<td>.0220 ± .0030</td>
<td>.0249 ± .0032</td>
</tr>
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<td>2.05 - 2.30</td>
<td>.0279 ± .0034</td>
<td>.0228 ± .0031</td>
<td>.0198 ± .0029</td>
</tr>
<tr>
<td>2.30 - 2.70</td>
<td>.0190 ± .0028</td>
<td>.0460 ± .0044</td>
<td>.0295 ± .0035</td>
</tr>
<tr>
<td>2.70 - 3.20</td>
<td>0 ± .0004</td>
<td>.0165 ± .0026</td>
<td>.0198 ± .0029</td>
</tr>
<tr>
<td>$M^2_{\rho\pi^0}$ in (GeV/c$^2$)$^2$</td>
<td>1.80 - 2.05</td>
<td>2.05 - 2.30</td>
<td>2.30 - 2.70</td>
</tr>
<tr>
<td>----------------------------------</td>
<td>------------</td>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>.0266 + .0034</td>
<td>.0198 + .0029</td>
<td>.0224 + .0031</td>
<td>0 + .0004</td>
</tr>
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<td>.0232 + .0031</td>
<td>.0236 + .0034</td>
<td>.0473 + .0045</td>
<td>.0101 + .0021</td>
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<td>.0194 + .0029</td>
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<td>.0139 + .0024</td>
<td>.0122 + .0023</td>
<td>.0144 + .0025</td>
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<td>.0190 + .0028</td>
<td>.0144 + .0025</td>
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<td>.0245 + .0032</td>
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Table 4. Density distribution, normalized to 1, of the summed $\pi^+$ and $\pi^-$ final states

<table>
<thead>
<tr>
<th>$M^2_{[\pi^+ + \pi^-]}$ (GeV/c^2)^2</th>
<th>1.1 - 1.45</th>
<th>1.45 - 1.60</th>
<th>1.60 - 1.80</th>
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</thead>
<tbody>
<tr>
<td>1.1 - 1.45</td>
<td>0.0263 ± 0.0024</td>
<td>0.0245 ± 0.0023</td>
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<tr>
<td>1.45 - 1.60</td>
<td>0.0213 ± 0.0022</td>
<td>0.0177 ± 0.0020</td>
<td>0.0150 ± 0.0018</td>
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<tr>
<td>1.60 - 1.80</td>
<td>0.0245 ± 0.0023</td>
<td>0.0168 ± 0.0019</td>
<td>0.0128 ± 0.0017</td>
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<tr>
<td>1.80 - 2.05</td>
<td>0.0267 ± 0.0024</td>
<td>0.0180 ± 0.0020</td>
<td>0.0184 ± 0.0020</td>
</tr>
<tr>
<td>2.05 - 2.30</td>
<td>0.0233 ± 0.0023</td>
<td>0.0227 ± 0.0023</td>
<td>0.0218 ± 0.0022</td>
</tr>
<tr>
<td>2.30 - 2.70</td>
<td>0.0220 ± 0.0022</td>
<td>0.0366 ± 0.0029</td>
<td>0.0343 ± 0.0028</td>
</tr>
<tr>
<td>2.70 - 3.20</td>
<td>0 ± 0.0002</td>
<td>0.0081 ± 0.0013</td>
<td>0.0211 ± 0.0022</td>
</tr>
</tbody>
</table>
\[ M_{\pi^+ + n\pi^-}^2 \text{ in (GeV/c}^2\text{)}^2 \]

<table>
<thead>
<tr>
<th></th>
<th>1.80 - 2.05</th>
<th>2.05 - 2.30</th>
<th>2.30 - 2.70</th>
<th>2.70 - 3.20</th>
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<td>+0.0153</td>
<td>+0.0019</td>
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<td>+0.0155</td>
<td>+0.0135</td>
<td>+0.0267</td>
<td>+0.0024</td>
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<tr>
<td></td>
<td>+0.0175</td>
<td>+0.0135</td>
<td>+0.0218</td>
<td>+0.0022</td>
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<td>+0.1710</td>
<td>+0.0218</td>
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<td>+0.0330</td>
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<td>+0.0319</td>
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<td></td>
<td></td>
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<td>+0.0094</td>
<td>+0.0015</td>
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Table 5. Fits to the mass dependence of the $\overline{p}p\pi^0$, $p\pi^+$ and $p\pi^-$ Dalitz plots$^a$

<table>
<thead>
<tr>
<th>Fit numbers</th>
<th>States Fitted</th>
<th>$M_{\Delta}^+ \text{ in MeV/c}^2$</th>
<th>$M_{\Delta}^0 \text{ in MeV/c}^2$</th>
<th>$\Gamma_0 \text{ in MeV/c}^2$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>$\overline{p}p\pi^0$</td>
<td>$1238 \pm 4$</td>
<td>149 $^{+16}_{-16}$</td>
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<td>2</td>
<td>$\overline{p}p\pi^0$</td>
<td>$1246 \pm 31$</td>
<td>200 $^{+112}_{-112}$</td>
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<tr>
<td>3</td>
<td>$\overline{p}p\pi^0$</td>
<td>$1236 \pm 4$</td>
<td>120 (fixed)</td>
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<tr>
<td>4$^b$</td>
<td>$\overline{p}p\pi^0$</td>
<td>$1239 \pm 3$</td>
<td>126 $^{+10}_{-10}$</td>
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</tr>
<tr>
<td>5</td>
<td>$p\pi^+ + p\pi^-$</td>
<td>$1227 \pm 5$</td>
<td>114 $^{+15}_{-15}$</td>
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<tr>
<td>6</td>
<td>$p\pi^+ + p\pi^-$</td>
<td>$1243 \pm 5$</td>
<td>1219 $^{+10}_{-10}$</td>
<td>143 $^{+17}_{-17}$</td>
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<td>7</td>
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<td>$1234 \pm 3$</td>
<td>145 $^{+15}_{-15}$</td>
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<tr>
<td></td>
<td>$+ p\pi^-$</td>
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</tr>
<tr>
<td>8</td>
<td>$\overline{p}p\pi^0 + p\pi^+$</td>
<td>$1236 \pm 4$</td>
<td>1224 $^{+5}_{-5}$</td>
<td>142 $^{+14}_{-14}$</td>
</tr>
<tr>
<td></td>
<td>$+ p\pi^-$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9$^c$</td>
<td>$\overline{p}p\pi^0 + p\pi^+$</td>
<td>$1233 \pm 4$</td>
<td>1220 $^{+7}_{-7}$</td>
<td>135 $^{+15}_{-15}$</td>
</tr>
<tr>
<td></td>
<td>$+ p\pi^-$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$The errors displayed here are from the square root of the diagonal elements of the covariance matrix obtained when Bevington's $\lambda$ is set equal to 0. They are typically within 10% of the value obtained from a full error analysis, where the error is defined as that increment in the concerned parameter such that the reminalized $\chi^2$ is 1 unit greater than the value at the minimum.

$^b$This fit is the one displayed in Figure 8.

$^c$This fit is the one displayed in Figure 9.

$^d$These angles are for the $p\pi^+$ state. For the summed $p\pi^+$ and $p\pi^-$ states, $\alpha$ is $146^\circ \pm 3^\circ$ and $\beta$ is $51^\circ \pm 4^\circ$. 


<table>
<thead>
<tr>
<th>$A_1$</th>
<th>$A_\Delta^+$</th>
<th>$\alpha$ (degrees)</th>
<th>$\beta$ (degrees)</th>
<th>$\chi^2/\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2 (fixed)</td>
<td>.66 + .05</td>
<td>--</td>
<td>--</td>
<td>2.10</td>
</tr>
<tr>
<td>15. ± 59.</td>
<td>.58 + .04</td>
<td>--</td>
<td>--</td>
<td>2.14</td>
</tr>
<tr>
<td>.5 ± .8</td>
<td>.59 + .02</td>
<td>--</td>
<td>--</td>
<td>2.17</td>
</tr>
<tr>
<td>0.0 (fixed)</td>
<td>.60 + .03</td>
<td>--</td>
<td>--</td>
<td>2.19</td>
</tr>
<tr>
<td>0.0 (fixed)</td>
<td>.52 + .05</td>
<td>--</td>
<td>--</td>
<td>3.52</td>
</tr>
<tr>
<td>0.0 (fixed)</td>
<td>.84 + .08</td>
<td>--</td>
<td>--</td>
<td>2.29</td>
</tr>
<tr>
<td>[${A_\Delta o = .30 + .05}$] &amp;</td>
<td>&amp;</td>
<td>&amp;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.2 (fixed)</td>
<td>.65 + .01</td>
<td>--</td>
<td>--</td>
<td>2.75</td>
</tr>
<tr>
<td>2.2 (fixed)</td>
<td>.64 + .01</td>
<td>--</td>
<td>--</td>
<td>2.72</td>
</tr>
<tr>
<td>2.2 (fixed)</td>
<td>.717 + .006</td>
<td>107 ± 6$^d$</td>
<td>29 ± 11$^d$</td>
<td>2.53</td>
</tr>
</tbody>
</table>
Figure 8. The fit to the $p\bar{p}n^0$ Dalitz plot with $A_1$ fixed at 0. Note the shift of the $\Delta^0$ (Figure 8e) and the $\Delta^0$ (Figure 8g) toward the edge of phase space.
between the observed $\Delta^+$ production and the observed $\Delta^0$ production is $8.4 \pm 1.6$, consistent with Sears' result of 7 and inconsistent with isospin invariance. The isospin invariance of the strong interaction is such a basic idea that one hesitates to question it until all other possibilities have been discarded. One such possibility is that the differential cross section should be written as a coherent sum of the various processes occurring ($\Delta^+$ production, $\Delta^0$ production, and background production) rather than an incoherent sum. This has been suggested by other authors (12) for different reasons.

When the incoherent sum of the $\Delta$ and $\bar{\Delta}$ and background mass dependences failed to give satisfactory fits, a coherent sum was tried, with a relative phase $\alpha$ between the $\Delta$ and $\bar{\Delta}$ and a relative phase $\beta$ between the $\Delta$ and the background. The amplitudes $\varepsilon$ for the $\Delta$ and the $\bar{\Delta}$ decays were written as

$$E(M_{\pi\pi}) = |\varepsilon_{\pi\pi}|^2$$

$$\varepsilon_{\pi\pi} = [E(M_{\pi\pi})]^{\frac{1}{2}} e^{i\delta_{\pi\pi}} ,$$

where

$$\tan \delta_{\pi\pi} = \frac{M \Gamma(M_{\pi\pi})}{M^2 - M_{\pi\pi}^2}$$

Fit 9 is to the form (for the $p\bar{p}\pi^0$ state)

$$d^2\sigma \propto \frac{2}{3} A E(M_{p\pi^0}) + \frac{2}{3} A E(M_{\bar{p}\pi^0}) + C_1 + 2[\frac{2}{3} A C_1 E(M_{p\pi^0})]^{\frac{1}{2}} \cos(\beta - \delta_{p\pi^0})$$
\[ + 2 \left( \frac{2}{3} A C_1 E(M_{p\pi^{-}}) \right) \frac{1}{2} \cos(\beta-\alpha-\delta-p_{\pi^{-}}) + \frac{4}{3} A [E(M_{p\pi^{-}})E(M_{p\pi^{+}})]^{1/2} \]
\[ \times \cos(\alpha+\delta-p_{\pi^{-}}) \]
where
\[ C_1 = \left( \frac{\pi}{2E_{CM}} \right)^2 C_1. \]

The phase shift \( \delta \) must be set to 0 to obtain a fit at all. Fit 9 is displayed in Figure 9 superposed on the effective mass histograms of all particle combinations of all three final states. The agreement with the \( \Delta^0 \) peak is much better. The lower curve in each case is the contribution from the flat background and the top curve is the full differential cross section, with interference effects.

C. Decay Angular Distributions

If we define an angle \( \theta \) in the rest frame of a \( \Delta(\bar{\Delta}) \) resonance which decays to a nucleon (antinucleon) and a pion as the angle between the nucleon (antinucleon) decay product and the direction of the initial proton (antiproton) momentum as viewed in that frame, and an angle \( \phi \) as the azimuthal angle measured from the \( x \)-axis in the \( x-y \) plane of the right-handed coordinate system having the \( z \)-axis along the direction of the initial proton (antiproton) momentum and the \( y \)-axis along the normal to the production plane, then the decay angular distribution of the \( \Delta(\bar{\Delta}) \) is given by (23)

\[ |G(\alpha)|^2 = W(\theta,\phi) = \frac{3}{4\pi} N [\rho_{33} \sin 2\theta + \left( \frac{1}{2} - \rho_{33} \right) \left( \frac{1}{3} + \cos^2\theta \right) \]
\[ - \frac{2}{\sqrt{3}} \text{Re} \rho_{31} \sin 2\theta \cos 2\phi - \frac{2}{\sqrt{3}} \text{Re} \rho_{31} \sin 2\theta \cos \phi \]

(49)
Figure 9. Results of the fit to the $p\pi^0$ and $\bar{p}n^+$ and $\bar{p}n^-$ Dalitz plots with a coherent sum of amplitudes. The lower curve is the background contribution and the upper curve is the full amplitude with all interference effects included.
where \( N \) is the number of events in the distribution. The \( \rho_{ij} \) are the spin-density matrix elements. This definition of \( (\Theta, \Phi) \) is referred to as \( \alpha \) in Equations (36) and (37) of the last section. Equation (49) can be integrated on \( \Phi \) to give

\[
W_1(\cos \Theta) = N[(\frac{1}{4} + \rho_{33}) + 3(\frac{1}{4} - \rho_{33})\cos^2 \Theta]
\]  

(50)

and integrated on \( \cos \Theta \) to give

\[
W_2(\Phi) = \frac{N}{2\pi} [1 - \frac{4}{\sqrt{3}} \Re \rho_{3,-1} \cos 2\Phi] .
\]  

(51)

The quantity \( \Re \rho_{31} \) can be obtained by integrating Equation (49) over only half the range of \( \cos \Theta \), from 0 to 1, and noting that the distribution so obtained is identical to that obtained by integrating \( W(\Theta, \Phi + \pi) \) over the other half of the range of \( \cos \Theta \), from -1 to 0. Thus we have

\[
W_3(\phi') = \frac{N}{2\pi} (1 - \frac{4}{\sqrt{3}} \Re \rho_{3,-1} \cos 2\phi' - \frac{4}{\sqrt{3}} \Re \rho_{31} \cos \phi')
\]  

(52)

where \( \phi' = \phi \) for \( \Theta < \pi/2 \) and \( \phi' = \phi + \pi \) for \( \Theta > \pi/2 \). The normal to the production plane is defined as

\[
\hat{n} = \frac{\vec{p}_1 \times \vec{p}_3}{|\vec{p}_1 \times \vec{p}_3|}
\]  

(53)

where \( \vec{p}_1 \) is the momentum of the incident proton (antiproton) and \( \vec{p}_3 \) is the momentum of the antinucleon (nucleon) as viewed in the \( \Delta (\bar{\Delta}) \) rest frame. The distribution \( W_3(\phi') \) corresponds to defining \( \phi \) with respect to the coordinate system
\[
(i, j, k) = \left( \hat{n} \times \hat{p}_1 / |\hat{p}_1|, \hat{n}, \hat{p}_1 / |\hat{p}_1| \right)
\]  
(54)

for \(0 < \pi/2\), and defining \(\phi\) with respect to the coordinate system

\[
(i, j, k) = \left( -\hat{n} \times \hat{p}_1 / |\hat{p}_1|, \hat{n}, \hat{p}_1 / |\hat{p}_1| \right).
\]  
(55)

Unfortunately, it turns out that \(\phi\) is distributed according to \(W_3(\phi')\) while \(\phi'\) is distributed according to \(W_2(\phi)\).

The \(\Delta\) and \(\bar{\Delta}\) states were separated for this study by including those \(\Delta(\bar{\Delta})\) events which were within the square of one fitted width, \(\Gamma_0^2\), of the fitted mass value squared, \(M_0^2\), of the \(\Delta(\bar{\Delta})\) and not within the equivalent region of the \(\bar{\Delta}(\Delta)\). The regions used are sketched in Figures 10 and 11. Those distributions which are required to be identical by charge conjugation invariance were added before doing any fits. Only the distributions obtained from the \(pp\pi^0\) state were used to fit the histograms shown in Figure 12 to Equations (50), (51), and (52), as these distributions were thought to have little contribution from background.

Simultaneously fitting \(W_3(\phi')\) to the \(\phi'\) distribution and \(W_2(\phi)\) to the \(\phi\) distribution gave a \(\chi^2\) of 269 for 22 degrees of freedom; the \(\phi\) distribution contributed 254 to that value. The \(\phi'\) and \(\phi\) distributions were then "parameterized" according to \(W_2(\phi')\) and \(W_3(\phi)\), with the results listed in Table 7. The result of fitting the \(\cos \theta\) distribution to \(W_1(\cos \theta)\) is also given in Table 7. The histograms fitted are listed in Table 6. These last fits are displayed in Figure 12, superposed on the distributions from all of the mass regions.
Figure 10. The mass regions used in the fits to the $\Delta^+$ decay angular distributions.
Figure 11. The mass regions used in the fits to the $\Delta^0$ decay angular distributions.
Figure 12. The angular distributions for \( \Delta \) production and decay. The curves are the fits discussed in the text.
Table 6. Production and decay angular distributions for the $\Delta^+$ and $\Delta^+$ which decay into the $\Lambda^0\pi^-$ state

<table>
<thead>
<tr>
<th>$\cos \theta$</th>
<th>$N$</th>
<th>$\phi$ in degrees</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.0 to -0.8</td>
<td>151. $\pm$ 12.</td>
<td>0. to 30.</td>
<td>32. $\pm$ 5.7</td>
</tr>
<tr>
<td>-0.8 to -0.6</td>
<td>117. $\pm$ 11.</td>
<td>30. to 60.</td>
<td>37. $\pm$ 6.1</td>
</tr>
<tr>
<td>-0.6 to -0.4</td>
<td>100. $\pm$ 10.</td>
<td>60. to 90.</td>
<td>90. $\pm$ 9.5</td>
</tr>
<tr>
<td>-0.4 to -0.2</td>
<td>99. $\pm$ 10.</td>
<td>90. to 120.</td>
<td>112. $\pm$ 11.</td>
</tr>
<tr>
<td>-0.2 to 0.0</td>
<td>82. $\pm$ 9.1</td>
<td>120. to 150.</td>
<td>127. $\pm$ 11.</td>
</tr>
<tr>
<td>0.0 to 0.2</td>
<td>81. $\pm$ 9.0</td>
<td>150. to 180.</td>
<td>121. $\pm$ 11.</td>
</tr>
<tr>
<td>0.2 to 0.4</td>
<td>105. $\pm$ 10.</td>
<td>180. to 210.</td>
<td>107. $\pm$ 10.</td>
</tr>
<tr>
<td>0.4 to 0.6</td>
<td>85. $\pm$ 9.2</td>
<td>210. to 240.</td>
<td>116. $\pm$ 11.</td>
</tr>
<tr>
<td>0.6 to 0.8</td>
<td>83. $\pm$ 9.1</td>
<td>240. to 270.</td>
<td>116. $\pm$ 11.</td>
</tr>
<tr>
<td>0.8 to 1.0</td>
<td>97. $\pm$ 9.8</td>
<td>270. to 300.</td>
<td>67. $\pm$ 8.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>300. to 330.</td>
<td>46. $\pm$ 6.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>330. to 360.</td>
<td>29. $\pm$ 5.4</td>
</tr>
<tr>
<td>$\phi'$ in degrees</td>
<td>N</td>
<td>$-t'$ in (GeV/c)$^2$</td>
<td>$N/(GeV/c)^2$</td>
</tr>
<tr>
<td>---------------------</td>
<td>-------</td>
<td>---------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>0. to 30.</td>
<td>66. ± 8.1</td>
<td>0. to 0.15</td>
<td>3927. ± 162.</td>
</tr>
<tr>
<td>30. to 60.</td>
<td>62. ± 7.9</td>
<td>0.15 to 0.30</td>
<td>1293. ± 93.</td>
</tr>
<tr>
<td>60. to 90.</td>
<td>93. ± 9.6</td>
<td>0.30 to 0.45</td>
<td>467. ± 56.</td>
</tr>
<tr>
<td>90. to 120.</td>
<td>95. ± 9.7</td>
<td>0.45 to 0.60</td>
<td>240. ± 40.</td>
</tr>
<tr>
<td>120. to 150.</td>
<td>102. ± 10</td>
<td>0.60 to 0.75</td>
<td>133. ± 30.</td>
</tr>
<tr>
<td>150. to 180.</td>
<td>92. ± 9.6</td>
<td>0.75 to 0.90</td>
<td>107. ± 27.</td>
</tr>
<tr>
<td>180. to 210.</td>
<td>73. ± 8.5</td>
<td>0.90 to 1.20</td>
<td>100. ± 18.</td>
</tr>
<tr>
<td>210. to 240.</td>
<td>91. ± 9.5</td>
<td>1.20 to 1.50</td>
<td>36.7 ± 11.</td>
</tr>
<tr>
<td>240. to 270.</td>
<td>113. ± 11</td>
<td>1.50 to 1.80</td>
<td>43.3 ± 12.</td>
</tr>
<tr>
<td>270. to 300.</td>
<td>84. ± 9.2</td>
<td>1.80 to 2.10</td>
<td>20.0 ± 8.2</td>
</tr>
<tr>
<td>300. to 330.</td>
<td>71. ± 8.4</td>
<td>2.10 to 2.40</td>
<td>16.7 ± 7.5</td>
</tr>
<tr>
<td>330. to 360.</td>
<td>58. ± 7.6</td>
<td>2.40 to 3.00</td>
<td>20.0 ± 5.8</td>
</tr>
</tbody>
</table>
Table 7. Fits to angular distributions of events in the Δ and Δ̅ region of the p\(\bar{p}\)π\(^0\) state

<table>
<thead>
<tr>
<th>Distributions Fitted</th>
<th>Functions Used</th>
<th>Parameters</th>
<th>(\chi^2/\nu)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi) and (\phi')</td>
<td>(W_2(\phi), W_3(\phi'))</td>
<td>(\text{Re } \rho_{3,1} = 0.12 \pm 0.01; \text{ Re } \rho_{3,-1} = -0.07 \pm 0.01)</td>
<td>12.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(\phi) and (\phi')</td>
<td>(W_2(\phi'), W_3(\phi))</td>
<td>(\text{Re } \rho_{3,-1} = 0.068 \pm 0.013; \text{ Re } \rho_{3,1} = -0.250 \pm 0.008)</td>
<td>1.8</td>
<td>1.0%</td>
</tr>
<tr>
<td>(\cos \theta)</td>
<td>(W_1(\cos \theta))</td>
<td>(\rho_{33} = 0.196 \pm 0.007)</td>
<td>2.8</td>
<td>0.4%</td>
</tr>
<tr>
<td>(t' = t - t_{\text{min}})</td>
<td>(H(t'))</td>
<td>(c = 3.92 \pm 0.20 \times 10^5 \text{ events/(GeV/c)}^2; \text{ Re } \rho_{3,1} = 6.46 \pm 0.26 \text{ (GeV/c)}^{-2})</td>
<td>4.4</td>
<td>35.0%</td>
</tr>
</tbody>
</table>

\(^a\)Only the first 6 points were used in this fit.
D. Production Angular Distributions

The form chosen for $H(t)$ was the simple exponential form which is usually applicable to peripheral interactions at low values of $t$.

$$H(t) = Ce^{bt} \quad (56)$$

The $\Delta$ and $\bar{\Delta}$ states were separated as described in the last section. The fit is listed in Table 7 and displayed in Figure 12.

E. Summary

One pion production in $\bar{p}p$ interactions has been studied over the momentum range 1.23 GeV/c to 6.94 GeV/c. At the lower energies the three final states $\bar{p}p\pi^0$, $\bar{p}n\pi^+$ and $\bar{p}n\pi^-$ are dominated by $\Delta(1236)$ and $\bar{\Delta}(1236)$ production. The $\bar{p}n\pi^+$ and $\bar{p}n\pi^-$ states are plagued at all energies with contamination problems, which can be seen indirectly as a problem with ambiguous events in Sears et al. at 2.7 GeV/c (8) and Ferbel et al. at 3 - 4 GeV/c (14) and can be seen more directly in the study of the Monte Carlo generation of fake events done by Lynch et al. at 1.61 GeV/c (12) and by Böckmann et al. at 5.7 GeV/c (17). The contaminating events have enough of the properties of bad fits--i.e., slow momentum of the missing neutral particle (24), high values of $\chi^2$, equally good fits to other final states, beam momentum far from the central value--that they can be substantially eliminated by judiciously chosen cuts. The contaminating events appear in specific regions of mass and angular distributions which are different for the different final states contaminated. This leads to apparent violations of the isospin and charge conjugation invariance of the strong interactions; conversely, any real violations would be masked.
by the contaminating events.

Enough questions about the applicability of one pion exchange to these reactions have been raised that such an assumption about the production mechanism has been avoided in so far as possible in this thesis. Unfortunately, no other models exist for these states; even the generally accepted parametrization of the mass dependence of the decay of the $\Delta(1236)$ depends on this assumption (22).

An incoherent sum of the mass dependence of the decay of the $\Delta(1236)$, of the decay of the $\Delta(1236)$, and of a constant background with only one mass for both the $\Delta^+$ and the $\Delta^0$ does not reproduce the mass dependence of the $\Delta^0$ very well. A coherent sum improves the agreement between the fitted function and the mass histograms, but is still not as good as it might be. The decay angular distributions, coupled with the fact that when the radius parameter $A_1^+$ is freed it chooses a value consistent with zero, suggest that a form for the mass dependence of the $\Delta(1236)$ decay which assumes vector meson exchange might be in order (12).

In conclusion, little is known with certainty about the one pion production states $pp\pi^0$, $p\pi^+$, and $p\pi^-$, although the states are rich in consequences of the invariance properties of the strong interaction. Contrary to the suggestions of some recent authors, it is not necessary to discard basic ideas like charge conjugation and isospin invariance of the strong interaction to explain the mass and angular dependence of the one pion production states, but it is not clear what the best alternate possibility is.
V. BIBLIOGRAPHY


VI. ACKNOWLEDGMENTS

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VII. APPENDIX A: AMBIGUITIES

An examination of the scattering angle in the center of mass of each particle with respect to the incident $\overline{p}$ direction reveals an apparent violation of charge conjugation invariance; there are more $\pi^-$ going forward in the center of mass (Figure 13h) than there are $\pi^+$ going backward (Figure 13d). This could be due to backward $\pi^+$ events not getting through the processing system, or it could be due to too many events of other final states receiving spurious fits to the $p\pi^-$ state, or perhaps charge conjugation is really violated. This last possibility is so repugnant that a lengthy study of the other possibilities is necessary. The two experiments must be studied separately because they had different scanning and measuring procedures, and those procedures must now be questioned. See Chapter II for the details of the differences. The original study was done on Experiment B, and the figures which will be shown in this discussion are for Experiment B only, but the statements made below are valid for both experiments. The cuts determined from the study were applied to each experiment separately to check their validity before applying them to the summed experiments to obtain the final data sample.

The difference between the forward $\pi^-$ fits to the $p\pi^-$ state and the backward $\pi^+$ fits to the $\overline{p}\pi^+$ state is about 200 events out of 1800 observed in the $\overline{p}\pi^-$ state. If this is a loss of slow $\pi^+$ events, it is a substantial one and should be easy to trace down. When this discrepancy was first observed, there existed a classification of lost events (see Chapter II, Scanning and Measuring Procedures) which had not been
Figure 13. Angular distributions in the center of mass for Experiment B. θ is the angle the momentum of the given particle makes with the direction of the incident antiproton. Note that the reaction is highly peripheral (the nucleon and antinucleon retain their initial directions) and that there is an excess of forward π⁻ events in the pπ⁺π⁻ final state. The upper histograms are the uncut data and the smaller histograms are the events excluded by the cuts discussed in the text.
remeasured and which might contain contributions to the $\bar{n}_p\pi^+$ final state if the $\pi^+$ was slow and steeply dipping, so that it was called a "lost proton" or "dark track" even after examination of the tracks for ionization consistence. Note that if Figure 13h is to match Figure 13d by adding events to the $n_p\pi^+$ state, the $\pi^+$ must go backward in the center of mass, and must do so with high momentum for Figure 14d to match Figure 14h as required by charge conjugation invariance. Thus the events needed could very well have been falsely labeled "lost protons" if they were steeply dipping as well as slow in the laboratory. The "lost proton" category was remeasured and again checked for ionization consistence of the fits, this time with special attention paid to steeply dipping positive tracks. No appreciable contribution to the $n_p\pi^+$ state developed from this; in fact, Figures 13-16 include the events from this remeasure.

Those events which did have a $n_p\pi^+$ fit but had been classified after ion checking as "lost protons" were further examined by treating each of the charged tracks in turn as unmeasured and examining the missing mass against the other track. All such events were found to be elastics; i.e., the missing mass was approximately 938 MeV/c$^2$ for both tracks. Thus few, if any, $n_p\pi^+$ events were lost or misclassified, and the possibility that the difference between the $p_n\pi^+$ and the $n_p\pi^-$ states is due to a lack of events in the $p_n\pi^+$ state can be discarded.

The $p_n\pi^-$ state was examined in greater detail. It turns out that there is a high correlation among the following:

(a) Events late in the chamber, where the outgoing tracks cannot be well measured;
Figure 14. The distributions of the momenta in the center of mass for Experiment B. Note the excess of high momentum $\pi^-$ events in the $p\pi^-\pi^+$ final state. The upper histograms are the uncut data and the smaller histograms are the events excluded by the cuts discussed in the text.
Figure 15. The effective mass (squared) distributions for Experiment B. Note the excess of events at $M_{\pi\pi}^2 = 2.8 \text{ (GeV/c}^2)^2$ and at $M_{\pi\pi}^2 = 4.0 \text{ (GeV/c}^2)^2$. The upper histograms are the uncut data and the smaller histograms are the events excluded by the cuts discussed in the text.
Figure 16. The momentum of the neutral particle as viewed from the target rest frame and as viewed from the beam rest frame in Experiment B. Note the excess of slow antineutrons in the laboratory frame. The upper histograms are the uncut data and the smaller histograms are the events excluded by the cuts discussed in the text.
(b) Events having fits to more than one final state, which could not be resolved by ion checking;
(c) Events having a high $\chi^2$ for the fit obtained;
(d) Events with a slow $\bar{n}$ in the laboratory frame $^1$ (24);
(e) Events having a missing mass before the fit far from the appropriate neutral particle mass;
(f) Events having a beam momentum far from the central value;
(g) Events with a $\rho\pi^-$ fit with the $\pi^-$ going forward in the center of mass;
(h) Events with a $\rho\pi^-$ fit with a $\rho\pi^-$ mass combination in the region $M_{\rho\pi^-}^2 = 2.8$ (GeV/c$^2$)$^2$.

We would like to remove as many of the questionable fits (a)-(f) as possible but in a manner such that we know how to correct for the loss of real events. A $\chi^2$ cut and a fiducial volume cut immediately suggest themselves. The fiducial volume cut is suggested in Figure 17; the ambiguous events are concentrated above a vertex position of 20 inches. The $\chi^2$ cut is suggested by Figure 18. Below a $\chi^2$ of 1, the distribution of the $\bar{n}$ momentum in the laboratory pretty well matches the distribution at $\chi^2 = 0$. In contrast, the relative population of slow neutrals above $\chi^2 = 3$ exceeds the relative population of slow neutrals near $\chi^2 = 0$. The cut chosen is as follows. If there was an ambiguity remaining after ion

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$^1$The $\rho\pi^-$ final state has a preponderance of fast neutrals, so an excess of slow ones is much more noticeable in this state than it would be in say the $\rho\pi^+$ state, where the neutrals are mostly slow. Slow neutrals are subject to question; i.e., likely to be spurious fits. See Figures 15 and 16.
Figure 17. The vertex distributions for unique fits and for ambiguous fits for the three states $\bar{p}p\pi^0$, $\bar{p}n\pi^+$, and $p\pi^-$ in Experiment B.
Figure 18. The distribution of $\chi^2$ versus laboratory momentum of the $\bar{n}$ in the $p\pi^-$ final state.
checking, a fit was chosen only if its $\chi^2$ probability was greater than 5 times that for any other remaining acceptable fit. Otherwise all fits to the event were discarded. For the remaining sample, if the vertex was beyond 20" and the $\chi^2$ for the fit was greater than 2, the fit was discarded. These cuts were made on all 3 of the final states $\bar{p}p\pi^0$, $\bar{p}n\pi^+$, and $p\pi^-$. The effect of the cuts on Experiment B are indicated by the histograms in Figures 13-16. In each case, the upper histogram is the data from Experiment B before any cuts and the lower histogram is the events removed by the cuts. Qualitatively similar histograms resulted when these same cuts were applied to Experiment A.

It is of interest to note in the Sears paper (8) that the ambiguous events had a similar effect on the effective mass distributions at 2.7 GeV/c as they do in the present experiment; that is, the ambiguities (and thus also other questionable fits) are concentrated in the region of 1.55 - 1.71 GeV/c$^2$ in the plot of the effective mass of the $p\pi^-$ in the $\bar{p}p\pi^-$ state, as they are in Figure 15g, giving rise to an apparent excess of events in that mass region. When a function having the form of Equation (40) is fitted to such a contaminated final state, the resulting fit would have a relative deficiency of events in the low mass region of the $\Delta(1236)$.

Fiducial volume cuts of 3.8" to 18.2" for Experiment A and of 3.5" to 25.0" for Experiment B were also imposed. These fiducial volume cuts are necessary in order to obtain a small error on $F$ (see Chapter III) as part of the normalization of the experiment. The effects of these fiducial volume cuts are not included in the study in Table 8. Beam momentum cuts
of 2 standard deviations from the central value were also imposed; these
cuts are included in the study in Table 8.

The cuts applied to Experiments A and B had not yet been determined
when the analysis in reference 5 was done on Experiment A.

To find the normalization factors to turn numbers of events into
cross sections, we need to know how many real and how many fake fits are
in the final data samples, and how many real events were discarded by the
cuts. There are two ways of doing this with the cuts chosen. The first
one is to use the theoretical shapes of the $\chi^2$ distribution and of the
gaussian beam momentum distribution to estimate the area under the curves
outside of the cuts as an estimate of the lost real events. The excess
of events removed by the cuts over this estimated number of real events
lost would be the number of spurious fits removed. Unfortunately, even
for a perfect experiment, the tails of the $\chi^2$ and beam momentum distribu­
tions would not match the theoretical ones. Also this technique gives no
way of estimating the remaining contamination, and no way of estimating
the number of real events lost by the ambiguity cut. The second way is
to use the losses in regions of variables where few questionable events
appear to estimate the lost real events, and use CP Invariance to examine
the regions where many questionable events appear in order to estimate the
remaining contamination of spurious fits. We have already seen that the
apparent CP violations between the $p\bar{n}\pi^-$ state and the $\bar{p}n\pi^+$ state diminish
when any plausible cut to eliminate spurious fits is applied.

Since slow neutrals can be easily faked by other final states (24),
one might expect to see contamination in the $p\bar{p}\pi^0$ final state as an
Table 8. Study of the effectiveness of the cuts described in Appendix A

<table>
<thead>
<tr>
<th></th>
<th>$\bar{p}p\pi^0$</th>
<th>$np\pi^+$</th>
<th>$p\bar{n}\pi^-$</th>
<th>$\pi^+\pi^-\pi^0$</th>
<th>$\pi^+\pi^-$</th>
<th>$K^+K^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of fits</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>before any cuts</td>
<td>2782</td>
<td>2668</td>
<td>3056</td>
<td>760</td>
<td>17</td>
<td>8</td>
</tr>
<tr>
<td>Estimated fake</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fits present</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>originally (U = unknown)</td>
<td>73 ± 38</td>
<td>U</td>
<td>404 ± 45</td>
<td>U</td>
<td>U</td>
<td>U</td>
</tr>
<tr>
<td>Number of events</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>after all cuts</td>
<td>2369</td>
<td>2111</td>
<td>2344</td>
<td>643</td>
<td>15</td>
<td>6</td>
</tr>
<tr>
<td>Estimated real</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>events discarded</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>by cuts</td>
<td>282 ± 29</td>
<td>471 ± 37</td>
<td>363 ± 35</td>
<td>--</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Apparent contamina-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tion removed</td>
<td>131 ± 29</td>
<td>86 ± 37</td>
<td>349 ± 35</td>
<td>U</td>
<td>U</td>
<td>U</td>
</tr>
<tr>
<td>Estimated fake</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fits remaining</td>
<td>54 ± 36</td>
<td>U</td>
<td>36 ± 37</td>
<td>U</td>
<td>U</td>
<td>U</td>
</tr>
<tr>
<td>Corrected number of</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>real events present</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>in initial sample$^a$</td>
<td>2651 ± 29</td>
<td>2582 ± 37</td>
<td>2707 ± 35</td>
<td>&lt; 643</td>
<td>15</td>
<td>6</td>
</tr>
<tr>
<td>$\epsilon_c$</td>
<td>.894 ± .010</td>
<td>.817 ± .012</td>
<td>.866 ± .011</td>
<td>1.</td>
<td>1.</td>
<td>1.</td>
</tr>
</tbody>
</table>

$^a$The uncertainty here is only the uncertainty in the determination of the number of lost real events; to avoid counting the statistical uncertainty twice, it is to be folded in every time the factors $F_c$ in Table 2 of Chapter III are applied to experimental numbers or distributions, such as the numbers in Tables 3 and 4, or in the calculation of the total cross sections listed in Table 2.
asymmetry in the scattering angle of the \( \pi^0 \) in the center of mass \( \cos\theta_{CM} \) (Figure 3c). The first and last ten bins of the histogram of the \( \cos\theta_{CM} \) of the \( \pi^0 \) were therefore compared to each other to get an indication of the remaining contamination of the \( \overline{p}p\pi^0 \) state. CP invariance predicts such a difference should be zero for an uncontaminated sample of \( \overline{p}p\pi^0 \) fits. Since CP invariance also says that the number of forward \( \pi^- \) events should equal the number of backward \( \pi^+ \) events, the ten forward bins of the \( \cos\theta_{CM} \) histogram of the \( \pi^- \) were compared to the ten backward bins of the \( \pi^+ \) to estimate the remaining contamination of the \( \overline{p}n\pi^- \) state. What such a comparison actually tests is the relative contamination between the two states; the \( \overline{n}\pi^+ \) state must be assumed to be relatively clean in order to be compared to the \( \overline{p}n\pi^- \) state. This seems reasonable since the principle contamination of the \( \overline{p}n\pi^- \) state is from spurious fits to events which are really elastics, but these spurious fits were carefully searched for and either remeasured or classified as "lost protons." The numbers of real events eliminated by each cut in each of the three states were estimated by comparing the number of events before and after each cut in the center twenty bins about \( \cos\theta_{CM} = 0 \) for the \( \pi^- \). These regions were chosen because the questionable events (the lower histograms in Figure 13) had little contribution in these regions. This study of the cuts described revealed that only the ambiguity cut discarded fake events for the \( \overline{p}p\pi^0 \) state and the \( \overline{p}n\pi^+ \) state; the beam momentum cut and the \( \chi^2 \) cut discarded primarily real events. However, for the \( \overline{p}n\pi^- \) state all three cuts discarded more fake events than real. For the \( \chi^2 \) and beam momentum cuts, the estimates of the number of lost real events so obtained were
consistent with the numbers obtained from the areas under the $\chi^2$ and gaussian tails.

The estimated overall effects of these cuts are listed in Table 8 along with the efficiency factor $\epsilon_{\text{cuts}}$ needed in the normalization of Chapter III. The quantity "estimated fakes" is the estimate of the contamination which was obtained by comparing the forward and backward $\pi$ distributions in the center of mass. The quantity "estimated real events discarded" comes from the difference between the central parts of the $\pi$ histograms before and after the cuts. The quantity "apparent contamination removed" is the difference between the number of fits removed by the cuts and the estimated number of real events discarded by the cuts. This last number is to be used as an estimate of the combined accuracy of the techniques for estimating remaining contamination and for estimating the number of real events discarded.

In summary, several apparent independent violations of CP invariance were related to each other, and seemed to be due to contamination of the $\pi\pi^-\pi^+$ state with spurious fits to events of some other state. Any cut designed to eliminate such spurious fits decreased the amount of apparent CP violation. A combination of cuts was found which decreased the relative contamination without seriously reducing the number of real events remaining. A way of estimating the contamination and the efficiency of the cuts in removing spurious fits versus real events was devised and used to estimate the number of real events present before the cuts. This number is necessary for calculation of the total cross section.
The basic philosophy of calculating theoretical density functions is to consider every bin of the histograms to represent an area in n-dimensional space. The number of events in the bin is thus an n-dimensional integral of the density function from the lower to the upper bounds of the bin, rather than the value of the density function at the center of the n-dimensional bin. For example, if $y_c(x)$ is the theoretical density function describing data presented as a l-dimensional histogram having bins of width $\Delta x$ and centers $x_i$, then the theoretical histogram height for the ith bin should be:

$$y_c(x_i) = \frac{1}{\Delta x} \int_{x_i - \Delta x/2}^{x_i + \Delta x/2} y_c(x) \, dx .$$

(57)

The data can then be binned in whatever width is appropriate to the statistical accuracy desired without losing the correspondence between the theoretical function and the finite numbers of events. For example, in the region of the $\Delta(1236)$ mass where the number of nucleon-antinucleon-pion events is large, a small bin size can be used, whereas a larger bin size is needed far from the $\Delta(1236)$ mass to avoid losing statistical accuracy. Thus one can use $\chi^2$ to do density fits in n-dimensional space in a relatively small amount of computation time, as opposed to the large amount of computation time required for a maximum likelihood fit. The bins chosen for the nucleon-antinucleon-pion final states are illustrated by the heavy lines in Figure 19 and the bin limits are listed in Tables 3 and 4, where the fitted data is listed. The bins have been chosen such
Figure 19. Illustration of the 2-dimensional histogram bins (heavy lines) and the subdivisions (faint lines) used in fitting the $\Delta^0$ and the $\Delta^+$ production amplitudes.
that the numbers of events in each bin are approximately equal.

This philosophy brings up the problem of doing fast accurate inte­
grals. One wishes to sample the theoretical function at as few points
in an n-dimensional bin as possible and still get a result deviating sig­
nificantly less from the real value than the typical data point does. The
method chosen is to chop each n-dimensional histogram bin into an arbitrary
number of subdivisions and then use a 2-point n-dimensional Gauss quadra­
ture formula with variable limits in each subdivision. The sum of the
results for the subdivisions corresponds to the number of events in the
bin. The number of subdivisions is then adjusted so that the result is
just barely a smooth curve. The subdivisions used in the amplitude fits
to the 2-dimensional nucleon-antinucleon-pion mass (squared) histograms
are illustrated by the faint lines in Figure 19.

The essence of an integration technique using a Gauss quadrature
formula is that the points at which the integrand is evaluated can be used
to obtain additional information about the function; an n-point Gauss
quadrature formula gives an exact integral for a polynomial of degree 2n-1,
whereas an n-point polynomial approximation to the integrand is only exact
for a polynomial of degree n-1.

A 2-point Gauss quadrature formula is given by (25):

\[ \int_{-1}^{1} f(x) \, dx = f\left(\frac{1}{\sqrt{3}}\right) + f\left(-\frac{1}{\sqrt{3}}\right) \]  \hspace{1cm} (58)

or after some simple transformations, by:

\[ \int_{a}^{b} f(x) \, dx = \frac{(b-a)}{2} \left[ f(x_+) + f(x_-) \right], \]  \hspace{1cm} (59)
where

\[ x_{\pm} = \frac{1}{2} \left( a + b \pm \frac{b-a}{\sqrt{3}} \right). \]  

(60)

When the subdivisions described in the preceding paragraph have been adjusted by the method described, they have been adjusted to a size such that the density function being integrated can be approximated by a parabola.

The Gauss quadrature formula used gives trouble near the boundaries because a step function cannot be approximated by a parabola. This problem can be eliminated by adjusting the sides of each subdivision so that at least one side lies entirely within the region over which the function is defined. The side which is entirely within the region is then taken as the outermost integral. Again note Figure 19 to see that the subdivisions on the boundaries have been so adjusted.

The fitting program used was a locally written generalized version of Bevington's CURFIT (26).