A hybrid-mode characterization of microstrip transmission lines

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A hybrid-mode characterization of microstrip transmission lines

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I. INTRODUCTION

A. Purpose

The purpose of this work is to investigate the dynamic electrical properties of microstrip transmission line. A full-wave formulation that accounts for the dispersion characteristics of the propagation constant and characteristic impedance will be developed. This model will account for both propagating and evanescent modes. Therefore, it is amenable to mode-matching techniques used in the solution of microstrip discontinuity problems.

The model to be developed uses an assumed current density on the strip conductor. When appropriate approximations are made in the form of this current density, the computation time required to determine the propagation constants and characteristic impedance will be considerably less than that of similar models. Also, the storage requirements of this simplified model can be reduced sufficiently to allow a solution on a programmable desk calculator.

B. Statement of the Problem

There are several related parallel conducting strip transmission line configurations of current interest in microwave engineering. These are illustrated in Figure 1. Usually region 1 is filled with a dielectric material and region 2 is free space, but there has been considerable interest in the case where region 1 is a biased ferrite.
Figure 1. Parallel strip transmission line.
The case where region 1 is a semi-conducting material has also been investigated. In addition to this basic configuration, lines with several dielectric layers and with two or more coupled parallel conducting strips have been considered.

When both regions are filled with the same material, the system is called strip line, with the special case $s = a/2$ called balanced strip line. Microstrip denotes the configuration where the two regions are filled with different materials. Usually $s = d$ for microstrip, but the material of region 1 is occasionally coated over the narrow strip for protection. When $a \rightarrow \infty$ and region 2 is free space, the configuration is called open microstrip. The term shielded microstrip originally denoted the case where $a$ is finite, but in recent years it has also been used to describe the system with finite width upper and lower conductors, connected at their edges by perpendicular conducting surfaces. This configuration has also been called closed microstrip and "microstrip in a box".

In this work, we will be concerned with the development of a theoretical model to account for the high frequency characteristics of this type of structure. The effects of varying the microstrip parameters and the enclosing waveguide size will be investigated, and a model for open microstrip which is easily solved on a programmable desk calculator will be developed.
C. Method of Solution

In order to find the fields on the microstrip, a source-free normal mode expansion of TE and TM modes in inhomogeneous rectangular waveguide will be made. The current on the strip will be assumed known, and a Fourier series solution for the fields will be obtained. This method of solution is a variation of Denlinger's [16] - [18] techniques for open microstrip.

Throughout this work, a time dependence of $e^{j\omega t}$ will be assumed and will be suppressed. Thus, the field quantities which will be used in this analysis will be in phasor form. They can be converted to the real-time field expressions by standard transform equations [27, pp. 13-16].
II. MICROSTRIP FIELDS

A. Historical Background

Although microstrip was first described in the early 1950's [2], [25], [39], it was not widely used until the middle 1960's because of high radiation and dielectric losses and the inability to maintain the necessary tolerances on physical dimensions. These problems have been largely surmounted, and the use of microstrip has become increasingly popular for a wide range of applications because of its simplicity and its adaptability to microwave integrated circuit techniques.

Most early analyses of microstrip involved a quasi-static, or TEM mode, assumption although the presence of the dielectric-air interface precludes the existence of this mode except at zero frequency. This assumption reduces the problem to two-dimensions and its solution has been approached by a variety of methods. Assadourian and Rimai [2] and Wheeler [54], [55] used a conformal mapping technique. Caulton, Hughes, and Sobol [7] used Wheeler's theory to present detailed design data for a wide range of geometries and substrate materials. They also included corrections for finite width conductors.

Silvester [49] used image theory coupled with a Green's function approach to determine the charge distribution on the strip and thus the propagation characteristics of the line. Gunston and Weale [26]
present a similar solution. Hill, Reckord, and Winner [31] also
used image theory to investigate coupled lines of finite width.

The relaxation method was used by Green [24], Stinehelfer [50],
and Baier [3]. These solutions assumed the microstrip was enclosed
in a conducting box distant from the line. Yamashita [58], Yamashita
and Atsuki [59], [60], and Yamashita and Mittra [61] considered
a variational method to investigate a variety of microstrip and
microstrip-related problems.

Byrant and Weiss [5], [6] derive a "dielectric Green's function"
to express the fields at the dielectric interface. From this, they
obtained the charge distribution on the strip, and in a manner similar
to Silvester's [49], they arrived at the propagation properties of
microstrip. Other workers have used variations of these and similar
methods to investigate the quasi-static microstrip problem [4], [8],
[20], [21], [36] - [38], [40], [41], [44], [47].

Quasi-static solutions have been relatively accurate at low
frequencies (up to a few GHz) but they fail to predict the dispersion
characteristics of microstrip at higher frequencies. While the
development of high-dielectric-constant substrates has reduced the
radiation loss of open microstrip and made it more attractive for
microwave circuits, it has also increased the amount of dispersion
observed at lower frequencies. Therefore, much of the recent work
on microstrip has been directed toward a more accurate solution for
higher frequencies.
In 1954 Deschamps [19] qualitatively considered propagation on microstrip. He noted that pure TE or TM modes were not possible and that at least one surface-wave mode would be necessary. Hartwig, Masse, and Pucel [28] made a series of measurements in 1968 that confirmed the existence of a surface wave mode. The lowest order mode was found to propagate at zero frequency, with higher order modes having a non-zero cutoff frequency.

The existence of surface wave modes was theoretically confirmed by Heller [29] in 1969. He expressed the fields in microstrip as an expansion of the normal modes of the dielectric slab waveguide. The fields were shown to be a three-dimensional generalization of two-dimensional surface wave modes. A mode with zero frequency cutoff was shown to exist and he was able to predict the cutoff frequency and propagation properties of the higher order modes.

The first published attempt to qualitatively describe microstrip without resorting to a quasi-static approximation was by Wu [57] in 1957. He arrived at two coupled integral equations to be solved for the longitudinal and transverse current densities on the strip. He was apparently unable to solve these equations, possibly because of an error as pointed out by Delogne [15].

In 1968 Zysman and Varon [62] presented a solution for microstrip enclosed in a rectangular waveguide. They used a superposition of the normal modes in an inhomogeneously filled waveguide to obtain a set of hybrid modes. Application of the boundary conditions on the strip resulted in a pair of coupled homogeneous Fredholm integral equations
of the first kind which were numerically solved for the propagation constant. Mittra and Itoh [45] suggested a method initially similar to that of Zysman and Varon in 1971. After the boundary conditions on the dielectric interface were met, the resulting infinite set of homogeneous simultaneous equations was transformed, using singular integral equation techniques, into an auxiliary set of equations. The convergence properties of the resulting infinite set of equations were such that reasonably accurate parameters for the microstrip mode resulted when as few as two equations were considered.

Denlinger [16] - [18] considered the dynamic behavior of single and coupled open microstrip on both dielectric and ferrite substrates. The fields were expressed in terms of hybrid modes and the current density was assumed. Fourier transforms of the fields and currents were obtained, and the boundary conditions on the dielectric interface and conducting strip were met. The resulting pair of coupled integral equations was solved for the propagation constant. Itoh and Mittra [34] suggested a similar approach except the boundary conditions were matched in the spectral domain and a more easily solved coupled pair of algebraic equations was obtained.

Hornsby and Gopinath [32] suggested a solution similar to that of both Denlinger and Zysman and Varon. They assumed the line to be in a waveguide and used Fourier series techniques. When the boundary conditions on the interface and strip conductor were met, an infinite
set of linear equations in terms of the propagation constant and a set of unknown Fourier coefficients were obtained. The infinite series were truncated, resulting in a finite set of linear equations. A similar solution was presented by Krage and Haddad [42]. In their method, the Fourier coefficients were those of the unknown current density, expanded in terms of a set of assumed basis functions.

Hornsby and Gopinath [33], Gelder [22], Daly [13], and Wharton and Rodrigue [53] used a finite-difference technique to analyze the dispersion characteristics of microstrip. In addition to the quasi-TEM mode, Daly also considered the lowest order waveguide mode. Corr and Davies [12], [14] used a finite difference approach in conjunction with a variational method in order to allow a more efficient solution of the resulting matrix equation. In addition to the dominant mode, several higher order modes were considered.

Several attempts have been made to arrive at closed form solutions for microstrip. Arnold [1] presented an empirically derived model for alumina substrates, and Chudobiak, Jain, and Makios [9] gave a somewhat more general linear model, applicable only above about 5 GHz. Schneider [48] used known details of the variation of phase velocity with frequency to arrive at an algebraic model. Getsinger [23] assumed the dominant microstrip mode was longitudinal-section electric (LSE) to obtain a closed form expression.
B. Normal Mode Expansion

The structure to be considered is shown in Figure 2. It can be described as a waveguide of width $b$ and height $a$, inhomogeneously filled with two slab dielectric regions. One region has dielectric constant $\varepsilon_{r1}$ and thickness $d$ while the other has dielectric constant $\varepsilon_{r2}$ and thickness $a - d$. The dielectrics are assumed to be isotropic, homogeneous, and lossless, so $\varepsilon_{r1}$ and $\varepsilon_{r2}$ are real scalar quantities. The strip conductor is sandwiched between the dielectrics and centered in the waveguide. It has width $w$ and is assumed to have negligible thickness. Both the waveguide walls and the strip are assumed to be perfect conductors.

If $\varepsilon_{r1}$ and $\varepsilon_{r2}$ were equal a TEM mode could exist in this structure, but because of the inhomogeneity in the dielectric and the presence of the conducting strip, it is not possible for pure TEM, TE, or TM modes to exist [19], [28], [29]. Collin [11, pp. 204-208, 229-232] shows that the TE and TM modes in rectangular waveguides form a complete orthogonal set. Therefore, an arbitrary field can be expressed as a linear combination of these modes.

We will construct solutions in each dielectric region in terms of the complete set of source-free modes in a homogeneously filled waveguide. Each of these modes will be chosen to independently satisfy the wave equation and the boundary conditions on the perfectly conducting waveguide walls. The remaining boundary conditions on the strip
Figure 2. Shielded microstrip line configuration.
and along the dielectric interface will be satisfied by properly summing an infinite set of these normal modes.

Harrington [27, pp. 129-132] shows that in a source-free homogeneous region, the electromagnetic fields can be expressed in terms of a magnetic vector potential \( \vec{A} \) and an electric vector potential \( \vec{F} \) as

\[
\vec{E} = -\nabla \times \vec{F} + \frac{1}{j\omega} \nabla \times \nabla \times \vec{A}
\]

(1)

and

\[
\vec{H} = \nabla \times \vec{A} + \frac{1}{j\omega} \nabla \times \nabla \times \vec{F}
\]

(2)

where \( \mu \) and \( \epsilon \) are the permeability and permittivity respectively of the region, \( \omega \) is the angular frequency, and \( j = \sqrt{-1} \). The rectangular components of the wave potentials individually satisfy the scalar wave equation, or Helmholtz equation,

\[
\nabla^2 \psi + k^2 \psi = 0
\]

(3)

where \( k^2 = \omega^2 \mu \epsilon \).

The TE\(_z\) or transverse electric to \( z \), fields are found by choosing \( \vec{A} = 0 \) and \( \vec{F} = \hat{z} \psi_h \). The field components are related to the wave function \( \psi_h \) by [27, p. 130]

\[
\begin{align*}
E_{xh} &= -\frac{\partial \psi_h}{\partial y} \\
E_{yh} &= \frac{\partial \psi_h}{\partial x} \\
E_{zh} &= 0 \\
H_{xh} &= \frac{1}{j\omega \mu} \frac{\partial^2 \psi_h}{\partial y \partial z} \\
H_{yh} &= \frac{1}{j\omega \mu} \frac{\partial^2 \psi_h}{\partial x \partial z} \\
H_{zh} &= \frac{1}{j\omega \mu} \left( \frac{\partial^2 \psi_h}{\partial z^2} + k^2 \right) \psi_h
\end{align*}
\]

(4)
where the $h$ subscript indicates $TE_z$ fields. Similarly, the $TM_z$, or transverse magnetic to $z$, fields are found by choosing $\mathbf{F} = 0$ and $\mathbf{A} = \mathbf{u}_z \psi_e$. The field components are then related to the wave function $\psi_e$ by

\[ E_{xe} = \frac{1}{jw} \frac{\partial^2 \psi_e}{\partial x \partial z} \quad H_{xe} = \frac{\partial \psi_e}{\partial y} \]
\[ E_{ye} = \frac{1}{jw} \frac{\partial^2 \psi_e}{\partial y \partial z} \quad H_{ye} = -\frac{\partial \psi_e}{\partial x} \]
\[ E_{ze} = \frac{1}{jw} \left( \frac{\partial^2}{\partial z^2} + k^2 \right) \psi_e \quad H_{ze} = 0 \]

where the $e$ subscript indicates $TM_z$ fields.

Now, we must construct solutions of (3) for each dielectric region in such a way that the fields given by (4) and (5) satisfy the correct boundary conditions on the waveguide walls. These conditions force the tangential $\mathbf{E}$ and the normal $\mathbf{H}$ to be zero on the conductor surface, or

\[ E_x(y=0,b) = E_y(x=0,a) = E_z(x=0,a; y=0,b) = 0 \quad (6) \]
\[ H_x(x=0,a) = H_y(y=0,b) = 0 \quad (7) \]

A suitable pair of wave functions for the $TE_z$ modes are

\[ \psi_{1h} = A_n \cos \alpha_1 x \cos \frac{ny}{b} e^{-jkz} \quad (8) \]
\[ \psi_{2h} = B_n \cos[\alpha_2 (a-x)] \cos \frac{ny}{b} e^{-jkz} \quad (9) \]
where \( n = 0, 1, 2, \ldots \) and the subscripts 1 and 2 indicate dielectric region 1 and 2 respectively. Similarly, the TM\(_z\) modes can be found from

\[
\psi_{1e} = C_n \sin \beta_1 x \sin \frac{n \pi y}{b} e^{-jk z} \tag{10}
\]

\[
\psi_{2e} = D_n \sin[\beta_2 (a-x)] \sin \frac{n \pi y}{b} e^{-jk z} \tag{11}
\]

where \( n = 1, 2, 3, \ldots \). The coefficients \( A_n, B_n, C_n, \) and \( D_n \) are independent of position but will depend on \( n \). The \( y \) and \( z \) wave numbers have been assumed equal in each dielectric to ensure continuity of tangential \( E \) at the dielectric interface.

When (8)-(11) are substituted into the wave equation (3) the following separation equations result.

\[
\alpha_1^2 + \left( \frac{n \pi}{b} \right)^2 + k_z^2 = k_{z1}^2 = \omega^2 \mu_0 \varepsilon_1 \tag{12}
\]

\[
\alpha_2^2 + \left( \frac{n \pi}{b} \right)^2 + k_z^2 = k_{z2}^2 = \omega^2 \mu_0 \varepsilon_2 \tag{13}
\]

\[
\beta_1^2 + \left( \frac{n \pi}{b} \right)^2 + k_z^2 = k_{z1}^2 = \omega^2 \mu_0 \varepsilon_1 \tag{14}
\]

\[
\beta_2^2 + \left( \frac{n \pi}{b} \right)^2 + k_z^2 = k_{z2}^2 = \omega^2 \mu_0 \varepsilon_2 \tag{15}
\]

In (12)-(15), each value of \( n \) corresponds to one normal mode, and each mode in the microstrip line is made up of an infinite summation of these normal modes. Each microstrip mode will be characterized by a single, frequency dependent, value of \( k_z \). For
propagating modes, \( k_z \) will be real, while \( k_z \) imaginary results in evanescent modes. Then, for each value of \( n, k_x, \) and \( \omega \) there will be a corresponding value of \( \alpha_1, \alpha_2, \beta_1, \) and \( \beta_2. \) Equations (12) and (14) require \( \alpha_1 \) and \( \beta_1 \) to be equal, and (13) and (15) result in \( \alpha_2 = \beta_2. \) Let

\[
\alpha_1 = \beta_1 = \alpha \tag{16}
\]

\[
\alpha_2 = \beta_2 = \beta \tag{17}
\]

Note that for a given microstrip mode at a given frequency, \( k_1, k_2, \) and \( k_z \) are constant, and as \( n \) becomes large, \( \alpha^2 \) and \( \beta^2 \) become negative. Then, \( \alpha \) and \( \beta \) are imaginary quantities, and the trigonometric function in (8)-(11) must be replaced by the appropriate hyperbolic functions.

The total field will be made up of a superposition of \( \text{TE}_z \) and \( \text{TM}_z \) fields. For example, \( E_{x1} = E_{x1h} + E_{x1e}. \) The other field components are found in a similar manner. When (4) and (5) are applied to the wave functions given by (8)-(11) the total fields are given by the following set of equations, where the common \( z \)-dependent term \( e^{-jk_zz} \) has been dropped.

\[
E_{x1n} = \left[ \frac{n\pi}{b} A_n + \frac{k_z \alpha}{w_2} C_n \right] \cos \alpha z \cos \frac{n\pi y}{b} \tag{18}
\]

\[
E_{x2n} = \left[ \frac{n\pi}{b} B_n + \frac{k_z \beta}{w_2} D_n \right] \cos \left[ \beta(a-z) \right] \sin \frac{n\pi y}{b} \tag{19}
\]

\[
E_{y1n} = - \left[ \alpha A_n + \frac{n\pi}{b} \frac{k_z}{w_2} C_n \right] \sin \alpha z \cos \frac{n\pi y}{b} \tag{20}
\]

\[
E_{y2n} = - \left[ \beta B_n + \frac{k_z C_n}{w_2} \frac{n\pi}{b} \right] \cos \alpha z \sin \frac{n\pi y}{b} \tag{21}
\]
\[ E_{y2n} = \left[ \beta B_n - \frac{n \mu_0 k_z^2}{b} D_n \right] \sin \left[ \beta (a-x) \right] \cos \frac{n \pi y}{b} \quad (21) \]

\[ E_{z1n} = \frac{1}{j \omega} \left[ k_1^2 - k_z^2 \right] \left[ k_1^2 - k_2^2 \right] C_n \sin \alpha \times \sin \frac{n \mu_0 y}{b} \quad (22) \]

\[ E_{z2n} = \frac{1}{j \omega} \left[ k_2^2 - k_z^2 \right] \left[ k_1^2 - k_2^2 \right] D_n \sin \left[ \beta (a-x) \right] \sin \frac{n \pi y}{b} \quad (23) \]

\[ H_{x1n} = \frac{\alpha k}{j \omega \mu_0} A_n + \frac{n \pi}{b} C_n \sin \alpha \times \cos \frac{n \pi y}{b} \quad (24) \]

\[ H_{x2n} = \left[ - \frac{\beta k_z^2}{\omega \mu_0} B_n + \frac{n \pi}{b} D_n \right] \sin \left[ \beta (a-x) \right] \cos \frac{n \pi y}{b} \quad (25) \]

\[ H_{y1n} = \frac{n \pi \mu_0 k_z^2}{b} A_n + \frac{n \pi}{b} C_n \cos \alpha \times \sin \frac{n \pi y}{b} \quad (26) \]

\[ H_{y2n} = \left[ \frac{n \pi k_z^2}{b \omega \mu_0} B_n + \beta D_n \right] \cos \left[ \beta (a-x) \right] \sin \frac{n \pi y}{b} \quad (27) \]

\[ H_{z1n} = \frac{1}{j \omega \mu_0} \left[ k_1^2 - k_z^2 \right] A_n \cos \alpha \times \cos \frac{n \pi y}{b} \quad (28) \]

\[ H_{z2n} = \frac{1}{j \omega \mu_0} \left[ k_2^2 - k_z^2 \right] B_n \cos \left[ \beta (a-x) \right] \cos \frac{n \pi y}{b} \quad (29) \]

The necessary boundary conditions at the dielectric interface, \( x = d \), are

\[ E_{z1}(d) = E_{z2}(d) \quad (30) \]

\[ E_{y1}(d) = E_{y2}(d) \quad (31) \]

\[ H_{z1}(d) - H_{z2}(d) = J_y(y) \quad (32) \]
\[ H_{y1}(d) - H_{y2}(d) = - J_z(y) \] \hspace{1cm} (33)

where \( J \), the current density on the interface, has been assumed to have the same \( z \)-dependance as the fields. This is necessary in order to match the boundary conditions for arbitrary \( z \). Note that \( J(y) \) is non-zero only for \( |2y - b| < w \).

When the boundary conditions (30)-(33) are applied to the fields given by (18)-(29), the following equations result, where \( L = a - d \). Note that it is necessary to consider a superposition of the fields in order to match these conditions.

\[ \frac{1}{\varepsilon_1} (k_1^2 - k_2^2) C_n \sin \alpha d - \frac{1}{\varepsilon_2} (k_2^2 - k_2^2) D_n \sin \beta L \] \hspace{1cm} (34)

\[ 0 = (\alpha A_n + \frac{n \pi k_z}{b} C_n) \sin \alpha d + (\beta B_n + \frac{n \pi k_z}{b} D_n) \sin \beta L \] \hspace{1cm} (35)

\[ J_y(y) = \frac{1}{j \omega \mu_0} \sum_{n=1}^{\infty} \left[ (k_1^2 - k_2^2) A_n \cos \alpha d \right. \\
- (k_2^2 - k_2^2) B_n \cos \beta L] \cos \frac{n \pi y}{b} \] \hspace{1cm} (36)

\[ -J_z(y) = \sum_{n=1}^{\infty} \left[ \frac{n \pi k_z}{b} A_n - \alpha C_n \right] \cos \alpha d \\
- \left( \frac{n \pi k_z}{b} B_n + \beta D_n \right) \sin \frac{n \pi y}{b} \] \hspace{1cm} (37)

In (34) and (35) it was recognized that each term in the series must be individually zero in order to make tangential \( E \) continuous across the interface.
Consider equations (36) and (37). They are in the form of Fourier cosine and sine series respectively, and the terms in brackets can be expressed in terms of the current densities using standard Fourier series techniques \[10, \text{Ch. 4}\]. For example, consider equation (37). If we let \( K_n \) equal the term in brackets, (37) becomes

\[
-J_z(y) = \sum_{n=1}^{\infty} K_n \sin \frac{n\pi y}{b}.
\]  

(38)

\( J_z(y) \) is the longitudinal component of conduction current density on the \( x = d \) plane. Since we have assumed perfect dielectrics, it will be nonzero only on the conducting strip. There will be current flow in the waveguide walls, but since it is finite, the component in the \( x = d \) plane is zero.

Since the waveguide walls act to shield out the effects of any electromagnetic fields outside the waveguide, we are free to define \( J_z(y) \) in such a way that it is a periodic function. Let it be an odd function of \( y \) with period \( 2b \). Note that this is a natural choice because of the Fourier sine series form of (38) and also because the image of a current element parallel to a perfectly conducting plane is an oppositely directed current element of equal magnitude at an equal distance on the opposite side of the plane \[27, \text{pp. 103-106}\]. Then, we have

\[
K_n = -\frac{1}{b} \int_{-b}^{b} J_z(y) \sin \frac{n\pi y}{b} \, dy.
\]

(39)
Equation (36) can be treated similarly except the result will be a Fourier cosine series. Then, if we let the term in brackets be $j \omega \mu_0 \frac{H_n}{n}$, we obtain

$$H_n = \frac{1}{b} \int_{-b}^{b} J_y(y) \cos \frac{n \pi y}{b} \, dy. \quad (40)$$

Here $J_y(y)$ is assumed to be an even periodic function of $y$ with period $2b$ and zero average value. This again is a natural choice when image theory is considered.

We have assumed perfect conductors and lossless dielectrics, so $k_z^2$ will be real. Let

$$k_z = M k_0 = M \omega \sqrt{\mu_0 \varepsilon_0} \quad (41)$$

where $\mu_0$ and $\varepsilon_0$ are the permeability and permittivity respectively of free space. $M$ will be real for propagating modes and imaginary for evanescent modes.

When $k_z$ is replaced by $M k_0$, and (38)-(40) are considered, (34)-(37) become

$$0 = \varepsilon_2 (\varepsilon_1 - M^2) C_n \sin \alpha d - \varepsilon_1 (\varepsilon_2 - M^2) D_n \sin \beta L \quad (42)$$

$$0 = (\alpha A_n + \frac{n \pi M}{b} \eta_0 C_n) \sin \alpha d + (\beta B_n - \frac{n \pi M}{b} \eta_0 D_n) \sin \beta L \quad (43)$$

$$H_n = (\varepsilon_1 - M^2) A_n \cos \alpha d - (\varepsilon_2 - M^2) B_n \cos \beta L \quad (44)$$

$$K_n = \frac{n \pi M}{b} A_n - \alpha C_n \cos \alpha d - \frac{n \pi M}{b} \eta_0 B_n + \beta D_n \cos \beta L \quad (45)$$
where $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ is the intrinsic impedance of free space, and

$$\bar{H}_n = \frac{H_n}{\omega \epsilon_0}.$$  (46)

The transverse wave numbers $\alpha$ and $\beta$ are expressed in terms of $k_z$ in the separation equations (12)-(15). Equations (42)-(45) can now be solved for the field potential coefficients $A_n$, $B_n$, $C_n$, and $D_n$ in terms of the normalized propagation constant, $M$, and the Fourier coefficients $K_n$ and $\bar{H}_n$. This solution is carried out in Appendix A. The result is

$$A_n = \frac{\bar{H}_n [V_n \cot \alpha d - \left(\frac{n\pi}{b}\right)^2 \Delta \epsilon_r] - K_n \frac{n\pi}{b} M \eta_0 \Delta \epsilon_r}{U_n \epsilon_r (\epsilon_{r1} - M^2) \cos \alpha d}$$  (47)

$$B_n = \frac{\bar{H}_n [V_n \beta \cot \beta L + \left(\frac{n\pi}{b}\right)^2 \Delta \epsilon_r] + K_n \frac{n\pi}{b} M \eta_0 \Delta \epsilon_r}{U_n \epsilon_{r2} (\epsilon_{r2} - M^2) \cos \beta L}$$  (48)

$$C_n = \frac{\bar{H}_n \frac{n\pi}{b} \eta_0 U_n - K_n (V_n - M^2 U_n)}{U_n \epsilon_{r1} (\epsilon_{r1} - M^2) \sin \alpha d}$$  (49)

$$D_n = \frac{\bar{H}_n \frac{n\pi}{b} \eta_0 U_n - K_n (V_n - M^2 U_n)}{U_n \epsilon_{r2} (\epsilon_{r2} - M^2) \sin \beta L}$$  (50)

where $\Delta \epsilon_r = \epsilon_{r1} - \epsilon_{r2}$ and

$$U_n = \alpha \tan \alpha d + \beta \tan \beta L$$  (51)

$$V_n = \epsilon_{r2} \alpha \tan \alpha d + \epsilon_{r1} \beta \tan \beta L$$  (52)
The fields are given by the following equations, where the upper term in the brackets refers to region 1, the lower term to region 2, and the common z-dependent term $\epsilon$ has been omitted.

\begin{equation}
W_n = \alpha \cot \alpha d + \beta \cot \beta L.
\end{equation}

\begin{align*}
E_{xn} &= \frac{n\pi}{b} \frac{H_n + Mn\eta_0 K_n}{V_n} \left\{ \beta \tan \beta L \frac{\cos \alpha x}{\cos \alpha d} \right. \\
&\quad \left. - \alpha \tan \alpha d \frac{\cos \beta(a-x)}{\cos \beta L} \right\} \cos \frac{n\pi y}{b} \\
E_{yn} &= \frac{n\pi}{b} \frac{H_n \left[ \frac{(n\pi)^2}{b} U_n - k_0^2 V_n \right] + K_n Mn\eta_0 \left( \frac{n\pi}{b} U_n \right)}{V_n W_n} \left\{ \frac{\sin \alpha x}{\sin \alpha d} \right. \\
&\quad \left. \frac{\sin \beta(a-x)}{\sin \beta L} \right\} \sin \frac{n\pi y}{b} \\
E_{zn} &= \frac{\left[ \frac{M}{\eta_0} k_0^2 H_n - \frac{n\pi}{b} K_n \right]}{V_n W_n} \left\{ \frac{\sin \alpha x}{\sin \alpha d} \right. \\
&\quad \left. \frac{\sin \beta(a-x)}{\sin \beta L} \right\} \sin \frac{n\pi y}{b} \\
H_{xn} &= \frac{\left[ \frac{M}{\eta_0} k_0^2 H_n - \frac{n\pi}{b} K_n \right]}{W_n} \left\{ \frac{\cos \beta(a-x)}{\cos \beta L} \right. \\
&\quad \left. \frac{\cos \beta(a-x)}{\cos \beta L} \right\} \cos \frac{n\pi y}{b}
\end{align*}
\[ H_{yn} = \frac{1}{V \omega n} \left[ -\frac{1}{H_n} \left( \frac{n \pi}{b} \right) k_0^2 \frac{M}{\eta_0} \Delta \varepsilon_r \right. \]
\[ + K_n \left\{ \varepsilon_{r1} W_n \frac{\beta \tan \beta L + (\frac{n \pi}{b})^2 \Delta \varepsilon_r}{\cos \alpha d} \right\} \left\{ \cos \frac{\alpha x}{\cos \alpha d} \right\} \left\{ \sin \frac{n \pi y}{b} \right\} \]
\[ + K_n \frac{n \pi}{b} M \eta_0 \Delta \varepsilon_r \left\{ \varepsilon_{r2} W_n \frac{\alpha \tan \alpha d + (\frac{n \pi}{b})^2 \Delta \varepsilon_r}{\cos \beta (a-x)} \right\} \left\{ \cos \frac{\beta (a-x)}{\cos \beta L} \right\} \]
\[ (58) \]

\[ H_{zn} = \frac{j \omega e_0}{V \omega n} \frac{1}{H_n} \left[ -\frac{1}{V_n} \frac{\varepsilon_{z1} W_n \frac{\alpha \cot \alpha d + (\frac{n \pi}{b})^2 \Delta \varepsilon_r}{\cos \alpha d}} {\varepsilon_{z1} W_n \frac{\beta \cot \beta L + (\frac{n \pi}{b})^2 \Delta \varepsilon_r}{\cos \beta (a-x)}} \right] \left\{ \cos \frac{\alpha x}{\cos \alpha d} \right\} \left\{ \cos \frac{n \pi y}{b} \right\} \]
\[ (59) \]

Once the propagation constant is known, the total fields are found by taking an infinite sum of these terms.

The final boundary conditions to be satisfied require the tangential electric field be zero on the perfectly conducting strip. That is,
\[ E_y (x = d) = 0 \quad |2y - b| < w \]
\[ E_z (x = d) = 0 \quad \text{if } |2y - b| < w \]
\[ (60) \]

Using (54) and (55), these conditions become, for \(|2y - b| < w|\),
\[ 0 = \sum_{n=1}^{\infty} \frac{H_n \left( \left( \frac{n\pi}{b} \right)^2 U_n - k_o^2 V_n \right) + K_n M \eta_o \left( \frac{n\pi}{b} \right) U_n}{V_n W_n} \cos \frac{n\pi y}{b} \quad (61) \]

\[ 0 = \sum_{n=1}^{\infty} \frac{H_n \left( \frac{n\pi}{b} \right) M \eta_o U_n - K_n \left( V_n - M^2 U_n \right)}{V_n W_n} \sin \frac{n\pi y}{b} \quad (62) \]

Equations (61) and (62) are similar to those obtained by Krage and Haddad [42]. In their approach, the longitudinal current density was expanded in terms of a set of Legendre polynomial basis functions and the transverse current density in terms of a sine expansion. The conditions expressed in (61) and (62) were matched at a finite set of points on the strip, where the number of points determined the number of unknown coefficients of the assumed basis functions. The result was a set of homogeneous linear equations which was solved for the propagation constant by setting the determinant equal to zero. Itoh and Mittra's [34] solution is similar, except their equations were derived for open microstrip and are formulated in the spectral domain.

Essentially the same approach will be taken here. Since one of the objectives of this work is to obtain a faster and less complex solution, a single term, rather than a series, will be assumed for the unknown current density.
C. Current on Microstrip

Several approximations have been suggested for the form of the current density on microstrip lines. Denlinger [16], [18] and Kammler [37] considered an expression given by Maxwell [43] for the static charge distribution on an isolated conducting strip. When modified for the geometry under consideration here, their expression is

\[
\sigma(y) = \begin{cases} 
\sigma_0 \frac{1}{\pi \sqrt{1 - \left(\frac{2y-b}{w}\right)^2}}, & |2y-b|<w, \\
0, & |2y-b|>w, \ 0 < y < b.
\end{cases}
\] (63)

Assume that the dynamic longitudinal current density can be approximated by \( J_z(y) \approx \frac{v}{p} \sigma(y) \), where \( v \) is the phase velocity. Then,

\[
J_z(y) = \begin{cases} 
\frac{J_{zo}}{\sqrt{1 - \left(\frac{2y-b}{w}\right)^2}}, & |2y-b|<w, \\
0, & |2y-b|>w, \ 0 < y < b
\end{cases}
\] (64)

where \( J_{zo} \) is related to the frequency and the total charge on the strip. Denlinger [16], [18] demonstrated that the form of the longitudinal current density given by (64) agrees well with that obtained by Bryant and Weiss [5] using a Green's function solution.

The following expression was used by Yamashita [58] as a trial function for the static charge distribution in his variational treatment of microstrip.
\[ \sigma(y) = 1 + \left| \frac{2y-b}{w} \right|^3 \]  

(65)

When this distribution is used, the longitudinal current density becomes

\[
J_z(y) = \begin{cases} 
J_{20} \left( 1 + \left| \frac{2y-b}{w} \right|^3 \right), & |2y-b|<w \\
0, & |2y-b|>w, \ 0 < y < b 
\end{cases}
\]  

(66)

The two forms of longitudinal current density are compared in Figure 3. The form of \( J_z(y) \) in (66) is seen to give a lower value of current near the edges of the strip. For strips of large w/d ratio (66) is probably a more accurate expression since (64) was derived assuming an isolated strip, and a nearby conductor would have the effect of making the current distribution more nearly uniform \[16\], \[30\]. This can be seen by considering a statically charged parallel plate capacitor. As the plate separation becomes smaller, the fringing fields decrease and the electromagnetic field becomes more nearly uniform. Thus, the charge distribution approaches uniformity. For the shielded microstrip case under consideration here, the current density given by (66) should be better even for smaller values of w/d because of the presence of the shielding conductors.

Continuity of \( H_z \) at the edges of the strip requires the transverse current density to be zero on the strip edges. Also, since the fundamental microstrip mode is even, the transverse current density must be zero at the center of the strip and have odd symmetry. A form
Figure 3. Comparison of assumed longitudinal current densities on microstrip.
of the transverse current density suggested by Itoh and Mittra \[34\] which meets these conditions is shown in Figure 4. It can be expressed by

\[
J_y(y) = \begin{cases} 
-j \frac{2 J y_0}{w} (2y-b+w) , & -w < 2y-b < -\frac{w}{2} \\
\frac{j}{w} \frac{2 J y_0}{w} (2y-b) , & |2y-b| < \frac{w}{2} \\
-j \frac{2 J y_0}{w} (2y-b-w) , & \frac{w}{2} < 2y-b < w \\
0 , & |2y-b| > w , 0 < y < b
\end{cases}
\]  \tag{67}

The factor \(j = \sqrt{-1}\) has been included in (67) in order to make \(J_y\) real. Consideration of the continuity equation indicates that \(J_y\) and \(J_z\) are 90° out-of-phase for propagating modes. The propagation constant, \(k_z\), becomes imaginary for evanescent modes. Then, since \(v_p = \omega/k_z\), the phase velocity and thus \(J_{zo}\) become imaginary.

Denlinger \[18\] presents a qualitatively similar transverse current density; however, because of the extremely slow convergence of the resulting boundary condition series, it won't be considered here.

The Fourier coefficients \(H_n\) and \(K_n\) are evaluated in Appendix B.

The resulting \(K_n\) for the Maxwell current given in (64) is

\[
K_n = -J_{zo} \frac{n\pi w}{b} \sin \frac{n\pi}{2} J_{o} \left(\frac{n\pi w}{2b}\right), \tag{68}
\]

where \(J_{o} \left(\frac{n\pi w}{2b}\right)\) is the zero order Bessel function. For the Yamashita current distribution in (66),
Figure 4. Assumed transverse current.
\[ K_n = - \frac{32b^3 J_0}{\pi^4w^3} \sin \frac{n\pi}{2} T_n \] 

(69)

where

\[ T_n = \frac{1}{4} \left\{ 6 + 2\left(\frac{n\pi w}{2b}\right)^2 \left[ \left(\frac{n\pi w}{2b}\right)^2 - 3\right] \sin \frac{n\pi w}{2b} 
+ 3 \left[ \left(\frac{n\pi w}{2b}\right)^2 - 2\right] \cos \frac{n\pi w}{2b} \right\} \] 

(70)

For the transverse current distribution (67), \( H_n \) can be determined as

\[ H_n = j \frac{32b^2 J_0}{\pi^2w} \sin \frac{n\pi}{2} S_n \] 

(71)

where

\[ S_n = \frac{1}{2} \frac{\sin \frac{n\pi w}{4b} \left(\cos \frac{n\pi w}{4b} - 1\right)}{n^2} \] 

(72)

Since the Fourier coefficients for the assumed current densities all contain the term \( \sin \frac{n\pi w}{2} \), the field components have only odd numbered terms. This results in \( H_x \) and \( H_y \) zero over the plane of symmetry, \( y = \frac{b}{2} \), as would be expected for an even mode of propagation. Also, note that the longitudinal field components, \( E_z \) and \( H_z \), will be zero at zero frequency, and, for this case, the fields reduce to a TEM mode as was anticipated.

Because of the Bessel function in (68), the Maxwell longitudinal current distribution requires considerably longer computation time than the Yamashita assumption. It will be seen that the results obtained using the two currents are in good agreement, and the
Yamashita current will be used in the programmable desk calculator solution. We will also observe that good results can be obtained by neglecting the transverse current. Thus, \( H_0 = 0 \) will be assumed for the simplified model.

D. Microstrip Parameters

When the assumed currents are considered, the final boundary conditions given in (61) and (62) become

\[
0 = \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{2} \cos \frac{n\pi}{b} V_n}{\cos \frac{n\pi}{b} W_n} \left\{ \frac{R S b^2}{2 M^2} \left[ \frac{n^2}{b^2} U_n - k_0^2 V_n \right] - n T_n U_n \right\} \quad (73)
\]

\[
0 = \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{2} \sin \frac{n\pi}{b} V_n}{\sin \frac{n\pi}{b} W_n} \left\{ n S_n U_n + T_n (V_n - M^2 U_n) \right\} \quad (74)
\]

where \( T_n \) is as given in (70) for the Yamashita current assumption and \( T_n \) is \( J_{\frac{n\pi}{2b}} \) for the Maxwell form. \( R \) is related to the ratio of the longitudinal and transverse current density magnitudes by

\[
R = \frac{J_{\sqrt{2}}}{J_{\sqrt{2}}} \frac{32 M}{\pi W k_0 C_K} \quad (75)
\]

where

\[
C_K = \begin{cases} 
-\frac{\pi W}{b} & \text{(Maxwell current)} \\
-\frac{32 b^3}{4 \pi W} & \text{(Yamashita current)}
\end{cases} \quad (76)
\]
The boundary conditions can no longer be matched over the entire strip because of the assumed current. In fact, (73) and (74) can be matched at only one point. If the current assumptions are reasonable, a match at one point will result in a near match over the entire strip. Note that (73) is automatically satisfied at the center of the strip, \( y = \frac{b}{2} \).

Equations (73) and (74) can be expressed as

\[
0 = R S'_1 + S'_2 \quad (77)
\]

\[
0 = R S'_3 + S'_4 \quad (78)
\]

where

\[
S'_1 = \frac{b^2}{\pi^2 M^2} \sum_{n=1}^{\infty} \sin \frac{n\pi}{2} \cos \frac{n\pi y}{b} \frac{1}{\nu^n \omega^n} S_n \left[ \left( \frac{n\pi}{b} \right)^2 \omega_n - k_0^2 \omega_n \right] \quad (79)
\]

\[
S'_2 = - \sum_{n=1}^{\infty} \sin \frac{n\pi}{2} \cos \frac{n\pi y}{b} \frac{1}{\nu^n \omega^n} n \nu_n \omega_n \quad (80)
\]

\[
S'_3 = \sum_{n=1}^{\infty} \sin \frac{n\pi}{2} \sin \frac{n\pi y}{b} \frac{1}{\nu^n \omega^n} n \nu_n \omega_n \quad (81)
\]

\[
S'_4 = \sum_{n=1}^{\infty} \sin \frac{n\pi}{2} \sin \frac{n\pi y}{b} \frac{1}{\nu^n \omega^n} \nu_n \omega_n \quad (82)
\]

For a given value of \( y \), (77) and (78) are functions of the normalized propagation constant, \( M \), only. In order for a solution to exist, the
determinant of the coefficients must be zero. That is,

\[ S_1^' S_4^' - S_2^' S_3^' = 0 \]  

(83)

A digital computer will be used to solve (83) at a given point on the strip by iterating on \( M \). Once \( M \) is determined, \( R \) is found by solving

\[ R = - \frac{S_2^'}{S_1^'} = - \frac{S_4^'}{S_3^'} \]  

(84)

Note that, because of symmetry, if the boundary conditions are satisfied at a point \( y = \frac{b}{2} + y' \), they are also satisfied at \( y = \frac{b}{2} - y' \). Then the boundary conditions are actually matched at two points on the strip.

Since microstrip is a slow-wave structure, we expect \( k_z \leq k_{z'} \leq k_l \) for the propagating mode, where \( k_z \) has been assumed to be less than \( k_l \). This will normally be the case, since region 2 is usually air with \( \varepsilon_{r2} = 1 \). Then

\[ \sqrt{\varepsilon_{r2}} \leq M \leq \sqrt{\varepsilon_{r1}} \]  

(85)

The phase velocity and guide wavelength are given by

\[ v_p = \frac{\omega}{k_z} = \frac{c}{M} \]  

(86)

\[ \lambda_g = \frac{v_p}{f} = \frac{\lambda_0}{M} \]  

(87)

where \( c \) is the speed of light in free space and \( \lambda_0 \) is the free-space wavelength. An examination of (86) and (87) shows that \( M^2 \) can be
thought of as an effective dielectric constant for the propagating mode.

A quasi-static expression for the characteristic impedance of the microstrip can be obtained by considering the following equation for TEM mode transmission lines.

\[ Z_0 = \frac{1}{vC} \]  

(88)

where \( Z_0 \) is the characteristic impedance, \( v \) is the velocity of propagation along the line, and \( C \) is the line capacitance per unit length. For a transmission line containing a single uniform dielectric, the line capacitance is related to the free-space line capacitance, \( C_o \), by

\[ C = \epsilon_r C_o \]  

(89)

where \( \epsilon_r \) is the relative dielectric constant of the dielectric. \( C_o \) is related to the characteristic impedance of an air-filled line by (88) with \( v \) replaced by \( c \), the speed of light in free space, and \( C \) replaced by \( C_o \). If we use \( M^2 \) as the effective dielectric constant of a uniform dielectric line, (88) and (89) give

\[ Z_0 = \frac{Z_o'}{M} \]  

(90)

where \( Z_o' \) is the characteristic impedance of the same line when it is filled with air.

The expression for characteristic impedance given in (90) is valid only for TEM transmission lines and assumes that no axial displacement current flows in the line. It will be shown in Section IV
that this is a good assumption at low frequencies, and good results can be obtained using this equation.

Another definition for characteristic impedance, suggested by Denlinger [16], [18], for non-TEM transmission lines is

\[ Z_0 = \frac{P}{I^2} \]  

(91)

where \( P \) is the power flowing down the line, and \( I \) is the total longitudinal current. The power flowing through a surface can be found from [27, pp. 19-23]

\[ P = \iint \hat{S} \cdot d\hat{S} \]  

(92)

where \( \hat{S} = \hat{E} \times \hat{H}^* \) is the poynting vector. Here the asterisk indicates the complex conjugate. Note that this equation assumes rms field intensities.

The total current flowing down the line is composed of both conduction current and displacement current. The conduction current can be obtained by integrating the assumed longitudinal current density across the width of the strip. The longitudinal displacement current density is equal to \( j \omega \varepsilon_0 E_z \). The axial displacement current is found by integrating this term over the guide cross section where \( E_z \) is given by (56). Then,

\[
P = \int_0^b \int_0^d \left( E_{x1} H_{y1}^* - E_{y1} H_{x1}^* \right) dx + \int_0^a \left( E_{x2} H_{y2}^* - E_{y2} H_{x2}^* \right) dx \int_0^d \left( E_{x2} H_{y2}^* - E_{y2} H_{x2}^* \right) dy
\]  

(93)
\[
I = \frac{b + w}{2} \int_{b-w}^{b+w} J_z(y) dy + jw \epsilon_0 \int_{0}^{b} [\epsilon_r1 \int_{0}^{r1} E_{z1} dx + \epsilon_r2 \int_{0}^{r2} E_{z2} dx] dy. 
\] (94)

These integrations are carried out in Appendix D. The power is given by

\[
P = \frac{b M n_o J_z o^2 C_k^2}{4} \sum_{n=1}^{\infty} \frac{1}{\n} \left[ \frac{\beta}{\alpha \cos^2 \beta L} \left( K_{1n} - K_{2n} \cot^2 \alpha d \right) \right.
\]
\[
+ \frac{L}{\cos^2 \beta L} \left( K_{3n} - K_{2n} \cot^2 \beta L \right)
\]
\[
+ \left( \frac{K_{1n}}{\alpha \cot \alpha d} + \frac{K_{2n}}{\beta \cot \beta L} + \frac{K_{3n}}{\beta \tan \beta L} + \frac{K_{2n}}{\beta \tan \beta L} \right) \] (95)

where

\[
K_{1n} = \frac{\beta \tan \beta L}{\nu_n} \left[ - \frac{R_n}{M^2} S_n + T_n \right] \left[ R k_o^2 \Delta \epsilon_r \nu_n \right.
\]
\[
+ T_n \left( \epsilon_r1 \beta \tan \beta L \nu_n + \left( \frac{n \pi}{b} \right)^2 \Delta \epsilon_r \nu_n \right) \] (96)

\[
K_{2n} = \frac{1}{\nu_n^2} \left\{ \frac{R_n S_n}{M^2} \left[ \left( \frac{n \pi}{b} \right)^2 u_n - k_o^2 v_n \right] \right.
\]
\[
- \left( \frac{n \pi}{b} \right)^2 T_n U_n \left[ k_o^2 R_n \nu_n + \left( \frac{n \pi}{b} \right)^2 T_n U_n \right] \] (97)

\[
K_{3n} = \frac{\alpha \tan \alpha d}{\nu_n} \left[ \frac{R_n S_n}{M^2} - T_n \right]\left[ R k_o^2 \Delta \epsilon_r \nu_n \nu_n \right.
\]
\[
+ T_n \left( -\epsilon_r2 \alpha \tan \alpha d \nu_n + \left( \frac{n \pi}{b} \right)^2 \Delta \epsilon_r \nu_n \right) \] (98)
When $\alpha$ and $\beta$ are imaginary, the trigonometric functions must be replaced by the appropriate hyperbolic function. As pointed out in Appendix D, this expression for the power is real for propagating modes ($M$ real) and imaginary for evanescent modes ($M$ imaginary) as expected for a lossless waveguide.

The expression for total current is

$$I = J_0 \omega C_I - \frac{2 k^2 b c K J_0}{c} \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{2}}{n \sqrt{n \nu_n}} \left[ \frac{\varepsilon_r_1 (\sec \alpha d - 1)}{\alpha \tan \alpha d} + \frac{\varepsilon_r_2 (\sec \beta L - 1)}{\beta \tan \beta L} \right]$$

where

$$C_I = \begin{cases} 
1.25 & \text{(Yamashita current)} \\
\frac{\pi}{2} & \text{(Maxwell current)}
\end{cases}$$

(100)

The integrations leading to equation (99) are carried out in Appendix D.

The expression for the current is also a function of the propagation constant only. When $M$ is determined using (77) and (78), the characteristic impedance can be evaluated using (91), (95), and (99). This will be done in Section IV, and the characteristic impedance given by (91) will be compared with the quasi-static result given by (90).
As a check on the applicability of the shielded microstrip results to open microstrip problems, we will investigate the current flow in the waveguide walls. The total wall current should be equal in magnitude of the current calculated using (99), and should flow in the negative z-direction. If the shielded microstrip is to effectively approximate the open case, essentially all of the current should flow back on the ground plane.

The current density in the walls of the guide can be found from [11, pp. 13-15], [27, pp. 32-35]

\[ \hat{J}_g = \hat{n} \times \vec{H} \]  \hspace{1cm} (101)

where \( \hat{n} \) is a unit vector normal to the guide walls and \( \vec{J}_g \) is the surface current density on the guide walls. The wall current is given by

\[ I_w = \int_{\Gamma} |\vec{J}_g| dL \]  \hspace{1cm} (102)

where the line integral is taken around the perimeter of the guide. This integral is evaluated in Appendix D, and the current is given by

\[ I_w = -\frac{2b}{\pi} J_{zo} C_K \sum_{n=1}^{\infty} \frac{\sin \frac{\pi n}{2}}{w_n} \left\{ I_{ws} \left( \frac{\cos \alpha d-1}{\alpha \sin \alpha d} + \frac{\cos \beta L-1}{\beta \sin \beta L} \right) \right. \\
+ \left. \frac{I_{wt}}{n v_n \cos \beta L} - \frac{I_{wb}}{n v_n \cos \alpha d} \right\} \]  \hspace{1cm} (103)
where

\[ I_{ws} = R k_o^2 \frac{S_n}{n^2} + T_n \frac{n^2}{b^2} \]  \hspace{1cm} (104)

\[ I_{wt} = k_o^2 R \Delta \epsilon_r n S_n - T_n [\epsilon_{r2} \tan \alpha d W_n - \left(\frac{n \pi}{b}\right)^2 \Delta \epsilon_r] \]  \hspace{1cm} (105)

\[ I_{wb} = k_o^2 R \Delta \epsilon_r n S_n + T_n [\epsilon_{r1} \tan \beta L W_n + \left(\frac{n \pi}{b}\right)^2 \Delta \epsilon_r] \]  \hspace{1cm} (106)

As noted in Appendix D, this current is real. The ground plane current is given by the term in (103) containing \( I_{wb} \). This expression will be investigated in Section IV.

Since the solution presented by Mittra and Itoh [45] makes no assumptions about the current on the strip, it will be used to check the values found for the propagation constant using this theory. Their results are summarized in Appendix E.

E. Series Summation

In order to solve for the propagation constant, the infinite series given in (79)-(82) must be evaluated. The characteristic impedance requires the evaluation of the series given in (95) and (99). The wall current involves the series in (103). Although all of these series converge in the region of interest, the convergence is often relatively slow and requires considerable computer time. This is a particularly acute problem in the summation of the boundary condition series where similar expressions must be evaluated many times for different assumed propagation constants.
It is shown in Appendix C that as $n$ increases, the terms in the various series approach expressions that are either independent of or simply related to the propagation constant. The following relationship was used to accelerate the convergence of the series under consideration.

\[
\sum_{n=1}^{\infty} u_n = \sum_{n=1}^{\infty} (u_n - f(M)u_{nL}) + g(f,M) \sum_{n=1}^{\infty} u_{nL}. \tag{107}
\]

Here, $g(f,M)u_{nL}$ is the large $n$ limit of $u_n$, and $u_{nL}$ is independent of the propagation constant. $g(f,M)$ is a function of the microstrip parameters, the frequency, and the normalized propagation constant $M$.

The first series in (107) was found to converge to three significant figures when a few as five or ten terms were considered. The second series must be evaluated only once for a given microstrip since it is independent of the propagation constant.
III. CALCULATOR SOLUTION

A. Propagation Constant

In order to derive a model suitable for programming on a desk calculator, several simplifying approximations must be made in the results of section II. Since a model for open microstrip is desired, assume that the waveguide dimensions are large compared to the strip dimensions. It is shown in Section IV that \( a = 10d \) and \( b = 10w \) are sufficiently large for many cases of interest.

The Yamashita current density will be assumed in order to avoid the necessity of calculating Bessel functions. In addition, the transverse current is assumed negligible. These assumptions are justified in Section IV.

The necessary boundary conditions are obtained by setting \( R \) equal to zero in (73) and (74). It is apparent that both conditions can no longer be matched, even at a single point, except at the center of the strip where (73) is automatically satisfied. When (74) is satisfied at the strip's center, \( y = b/2 \), the boundary condition becomes

\[
0 = \sum_{n=1}^{\infty} \frac{T_n}{V_n W_n} (V_n - M^2 U_n) \quad (108)
\]

where
\[ T_n' = \frac{1}{4} \left\{ 6 + 2 \left( \frac{n \pi}{20} \right) \left[ \left( \frac{n \pi}{20} \right)^2 - 3 \right] \sin \frac{n \pi}{20} \right\} \]

\[ U_n = \alpha \tan \alpha d - \beta \tanh 9 \beta d \]

\[ V_n = \alpha \tan \alpha d - \varepsilon_r \beta \tanh 9 \beta d \]

\[ W_n = \alpha \cot \alpha d + \beta \coth 9 \beta d \]

\[ \alpha = \pi \sqrt{(\frac{2f_c}{c})^2 (\varepsilon_r - M^2) - (\frac{n}{10w})^2} \]

and

\[ \beta = \pi \sqrt{(\frac{n}{10w})^2 + (\frac{2f_c}{c})^2 (M^2 - 1)} \]

It has been noted in (109)-(112) and (114) that, for the propagating microstrip mode, \( 1 \leq M^2 \leq \varepsilon_r \), and \( \beta \) as defined earlier will always be imaginary. The substitutions \( \varepsilon_{r2} = 1 \) and \( \varepsilon_{r1} = \varepsilon_r \) have been made in (110)-(114).

When the limit series discussed in Section II-E and calculated in Appendix C are used to speed the convergence of the series, (108), after some rearrangement, becomes
Since the terms $U_n$, $V_n$, and $W_n$ are functions of $\alpha$ and $\beta$, and therefore of $M^2$, (115) has the form $M^2 = f(M^2)$. An iterative scheme was used to solve this equation. An initial value was assumed for $M^2$, and $f(M^2)$ was evaluated. The new $M^2$ was taken to be the average of the previous estimate and its resulting value of $f(M^2)$. The process was repeated until the difference between the assumed and calculated quantities indicated that the desired accuracy had been obtained.

Note that the right hand side of (115) is independent of $M$ for $f = 0$. Therefore, the static value for the propagation constant can be found in one step, independent of the initial value assumed.

B. Characteristic Impedance

Because of storage limitations in the desk calculator used, the characteristic impedance given by (91), (95) and (99) could not be programmed, even with the simplifying approximations given in the last section. Instead, a quasi-static expression similar to (90) was used. Equations (88) and (89) can be used to derive the following expression, where $M_0$ and $Z_{oo}$ are the normalized propagation constant and the characteristic impedance for the zero-frequency case respectively.
where

\[ U'_n = \tanh \frac{n \pi d}{10w} + \tanh \frac{9n \pi d}{10w} \]  

\[ V'_n = \tanh \frac{n \pi d}{10w} + \varepsilon_r \tanh \frac{9n \pi d}{10w} \]  

\[ W'_n = \coth \frac{n \pi d}{10w} + \coth \frac{9n \pi d}{10w} \]  

When the limit series is used to increase the rate of convergence, (117) becomes

\[
Z_{oo} = \frac{3.2768 \times 10^{10} \eta_o M_o}{\pi 9} \left\{ \sum_{n=1}^{\infty} \frac{T_n^2}{n} \frac{U'_n}{W'_n V'_n} \frac{1}{1 + \varepsilon_r} \right\}
\]

\[
+ \frac{1}{1 + \varepsilon_r} \sum_{n=1}^{\infty} \frac{T_n^2}{n} \frac{1}{1 + \varepsilon_r} \right\}. \tag{121}
\]
$M_0$ can be calculated using (115) with zero frequency or it can be incorporated in the same program with $Z_{oo}$. If $f = 0$ is used in (115), the result is

$$M^2_0 = \sum_{n=1}^{\infty} \frac{T_n'}{2} \left[ \frac{1}{W_n'} - \frac{1}{2} \right] + \sum_{n=1}^{\infty} \frac{T_n'}{n}$$

The limit series in (115), (121), and (122) have been evaluated on a digital computer using double precision arithmetic. The results, accurate to 6 significant figures, are

$$\sum_{n=1}^{\infty} \frac{T_n'}{n} = \frac{1.29118 \times 10^{-3}}{n \text{ odd}}$$

$$\sum_{n=1}^{\infty} \frac{T_n'}{n^2} = \frac{9.51065 \times 10^{-7}}{n \text{ odd}}$$
IV. NUMERICAL RESULTS AND DISCUSSION

A. Introductory Comments

The various equations derived in Section II were solved using the IBM 360/65 computer at the Iowa State University Computation Center. The programs were compiled under the "H" option Fortran compiler, and were written using double precision arithmetic in order to ensure maximum computational accuracy. The computer programs are listed and discussed in Appendix F.

When the limit series technique discussed in Section II.E. was used, convergence to three significant figures could be obtained with as few as five or ten summation terms. Normally 75 to 100 terms were summed, with convergence to six or eight significant figures resulting. This was considerably better accuracy than could be obtained when up to 300 terms were summed in the original series.

When the Yamashita current density was used, the infinite limit series was usually truncated at 3,000 terms. This resulted in convergence to about six or seven significant figures. A comparison between the results obtained using single and double precision arithmetic indicated that computer round off errors limited the maximum accuracy in the series summation to about eight significant figures.

The accuracy of the evaluation of $M^2$ was the limiting factor in the solution using the Yamashita current distribution. The subroutine used to evaluate $M^2$ allowed a variable limit on the upper bound of the error. This was normally set at $5 \times 10^{-6}$, giving five significant
figures for $M^2$.

Less accuracy was obtained with the Maxwell current distribution. Because of the large amount of time necessary to evaluate the Bessel function, $J_{\frac{n\pi w}{2b}}$ was normally found to only three significant figures. The infinite limit series were truncated at 1,250 terms for this case.

B. Comparison of Models

The boundary condition series given in (83) was investigated for several values of strip dimensions and frequency. A typical plot of the sum of this series versus $M^2$ is given in Figure 5. Only one mode propagates for the case shown, since $\Sigma f(M^2) = 0$ represents allowable solutions, and there is only one value of $M^2$ satisfying this condition for $M^2 > 0$. As the frequency increases, the pattern shifts to the right. At sufficiently high frequencies, more than one zero crossing for $M^2 > 0$ results, indicating the appearance of higher order propagating modes. The allowable solutions, $\Sigma f(M^2) = 0$, for $M^2 < 0$ represent evanescent modes. The plot in Figure 5 is similar to those given by Mittra and Itoh [45].

When the value of $M^2$ representing the fundamental microstrip mode was approached from the right, the series were well behaved. For this case, $M^2$ was relatively easy to evaluate. Starting at $M^2 = \varepsilon^r$, the sum of the series was calculated at the endpoints of successive small intervals. An interval containing a solution was located when these sums had opposite signs. Subroutine DRTMI, a subroutine from the
Figure 5. Typical form of the boundary condition series' sum versus $M^2$. 
IBM Scientific Subroutine Package that uses Mueller's iteration method to solve for the roots of nonlinear equations, was used to solve for $M^2$. Once $M^2$ was evaluated, the phase velocity and characteristic impedance were easily calculated using the results derived in Section II.D.

The solution for higher order propagating modes and evanescent modes was somewhat more difficult because of the presence of points where the boundary condition series diverges. For this case, it was necessary to determine whether intervals where the sum changed sign contained a zero or an infinity. The zeros and infinities were often quite close together, so it was also necessary to use much smaller intervals.

The values of $M^2$ found using both the Maxwell and the Yamashita current distributions are compared in Tables I, II, and III for three strip width to dielectric thickness ratios. For these cases, the boundary condition series (83) was forced to equal zero at a point midway between the edge and the center of the strip. The results found using Mittra and Itoh's [45] technique are also shown, along with the values calculated using the Yamashita current distribution with zero transverse current. For this case, the boundary condition used was $E_z = 0$ at the center of the strip. This condition is expressed by $S_4' = 0$ with $x = b/2$; where $S_4'$ is given by equation (82).

The blank spaces in Tables I, II and III represent values of frequency where it was not possible to match the boundary conditions.
with reasonable values of $M^2$. In these regions, $M^2$ dropped rapidly toward zero, then returned to the original curve. The fact that these regions of inconsistent results appear at different values of frequency for the two assumed current distributions indicates that

Table I. Effective dielectric constant for a wide strip (w/d = 3):

a = 3.17 cm, b = 9.51 cm, d = 3.17 mm, w = 9.51 mm, and $\varepsilon_x = 11.7$.

<table>
<thead>
<tr>
<th>f(GHz)</th>
<th>Yamashita Current</th>
<th>Maxwell Current</th>
<th>Mittra and Itoh</th>
<th>Yamashita, $J_x = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>9.835</td>
<td>10.075</td>
<td>-</td>
<td>10.134</td>
</tr>
<tr>
<td>5</td>
<td>10.481</td>
<td>10.742</td>
<td>10.387</td>
<td>10.789</td>
</tr>
<tr>
<td>7</td>
<td>10.811</td>
<td>11.050</td>
<td>10.737</td>
<td>11.175</td>
</tr>
<tr>
<td>9</td>
<td>11.048</td>
<td>11.260</td>
<td>10.966</td>
<td>11.394</td>
</tr>
<tr>
<td>11</td>
<td>11.208</td>
<td>11.397</td>
<td>11.123</td>
<td>11.520</td>
</tr>
<tr>
<td>15</td>
<td>11.402</td>
<td>11.554</td>
<td>11.318</td>
<td>11.637</td>
</tr>
</tbody>
</table>

the problem is at least partially due to a non-exact current assumption. It was also found that these regions were somewhat dependent on the size of the enclosing waveguide and on the point on the strip at which the boundary condition was matched.
Table II. Effective dielectric constant for an intermediate strip 
\( (w/d = 1) \): \( a = b = 3.17 \text{ cm}, w = d = 3.17 \text{ mm} \) and \( \varepsilon_r = 11.7 \).

<table>
<thead>
<tr>
<th>( f(\text{GHz}) )</th>
<th>Yamashita Current</th>
<th>Maxwell Current</th>
<th>Mittra and Itoh</th>
<th>Yamashita, ( J_x = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.785</td>
<td>7.858</td>
<td>7.835</td>
<td>7.764</td>
</tr>
<tr>
<td>3</td>
<td>8.462</td>
<td>8.571</td>
<td>8.532</td>
<td>8.446</td>
</tr>
<tr>
<td>9</td>
<td>9.968</td>
<td>10.093</td>
<td>10.013</td>
<td>10.012</td>
</tr>
<tr>
<td>11</td>
<td>10.281</td>
<td>10.387</td>
<td>10.313</td>
<td>10.343</td>
</tr>
<tr>
<td>13</td>
<td>10.516</td>
<td>-</td>
<td>10.546</td>
<td>10.600</td>
</tr>
<tr>
<td>15</td>
<td>10.733</td>
<td>10.853</td>
<td>10.729</td>
<td>10.798</td>
</tr>
</tbody>
</table>

The only point where Mittra and Itoh's [45] model gave inconsistent results was near 3 GHz for the wide strip illustrated in Table I. The effective dielectric constant remained near the 1 GHz value through \( f = 3.46 \text{ Hz} \), and then increased rapidly toward the expected range. It is felt that this discrepancy is a result of the finite approximations made for the infinite series.

The results obtained using the three models derived in this work agree well with the results found from Mittra and Itoh's theory. The maximum relative difference between this theory and Mittra and Itoh's theory is less than 3% for all three cases considered.
Table III. Effective dielectric constant for a narrow strip \((w/d = 0.1)\):

\(a = b = 3.17 \text{ cm}, \ w = 0.317 \text{ mm}, \ d = 3.17 \text{ mm} \) and \(\varepsilon_r = 11.7\).

<table>
<thead>
<tr>
<th>(f(\text{GHz}))</th>
<th>Yamashita Current</th>
<th>Maxwell Current</th>
<th>Mittra and Itoh</th>
<th>Yamashita, (J_y = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.006</td>
<td>7.023</td>
<td>7.015</td>
<td>6.990</td>
</tr>
<tr>
<td>3</td>
<td>7.329</td>
<td>7.355</td>
<td>7.345</td>
<td>7.310</td>
</tr>
<tr>
<td>5</td>
<td>7.682</td>
<td>7.717</td>
<td>7.703</td>
<td>7.660</td>
</tr>
<tr>
<td>7</td>
<td>8.068</td>
<td>8.108</td>
<td>8.090</td>
<td>8.044</td>
</tr>
<tr>
<td>9</td>
<td>8.489</td>
<td>8.530</td>
<td>8.508</td>
<td>8.466</td>
</tr>
<tr>
<td>11</td>
<td>8.931</td>
<td>8.968</td>
<td>8.945</td>
<td>8.911</td>
</tr>
<tr>
<td>13</td>
<td>-</td>
<td>9.393</td>
<td>9.370</td>
<td>-</td>
</tr>
</tbody>
</table>

Note that for the wide strip considered in Table I, the results obtained using the Yamashita current distribution are in somewhat better agreement with Mittra and Itoh's model than those obtained when the Maxwell current was used. This agrees with our earlier observation that for wide strips, the presence of the ground plane will tend to make the current distribution more nearly uniform.

The results obtained when the transverse component of the current is neglected are in good agreement with the results when it is included. This agrees with Denlinger's \([16],[18]\) conclusions. The best agreement occurs for narrow strips. This is as expected, since a narrow
strip would tend to have a smaller component of transverse current than a wide strip.

The characteristic impedance values for the three strip width to dielectric thickness ratios considered in Tables I - III are summarized in Tables IV, V and VI. It was found that the quasi-static characteristic impedance calculated using equation (90) with \( Z_0' \) obtained from Wheeler's [55] theory gave essentially the same results as that calculated using equation (116). It was relatively simple to include

Table IV. Characteristic impedance for a wide strip (\( w/d = 3 \)):

\[ a = 3.17 \text{ cm}, \ b = 9.51 \text{ cm}, \ d = 3.17 \text{ mm}, \ w = 9.51 \text{ mm} \text{ and} \]
\[ \varepsilon_r = 11.7 \text{ (Values in ohms)}. \]

<table>
<thead>
<tr>
<th>( f(\text{GHz}) )</th>
<th>Maxwell Current</th>
<th>Yamashita Current</th>
<th>Yamashita, ( J_x = 0 )</th>
<th>Quasi-Static Yamashita, ( J_x = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.46</td>
<td>23.58</td>
<td>23.85</td>
<td>23.57</td>
</tr>
<tr>
<td>3</td>
<td>25.53</td>
<td>-</td>
<td>23.29</td>
<td>22.45</td>
</tr>
<tr>
<td>5</td>
<td>26.63</td>
<td>22.59</td>
<td>23.39</td>
<td>21.76</td>
</tr>
<tr>
<td>7</td>
<td>26.87</td>
<td>22.33</td>
<td>23.67</td>
<td>21.38</td>
</tr>
<tr>
<td>9</td>
<td>23.09</td>
<td>22.21</td>
<td>24.00</td>
<td>21.17</td>
</tr>
<tr>
<td>11</td>
<td>23.17</td>
<td>22.12</td>
<td>24.32</td>
<td>21.06</td>
</tr>
<tr>
<td>13</td>
<td>23.30</td>
<td>22.07</td>
<td>24.60</td>
<td>20.99</td>
</tr>
<tr>
<td>15</td>
<td>-</td>
<td>22.06</td>
<td>24.85</td>
<td>20.95</td>
</tr>
</tbody>
</table>
Table V. Characteristic impedance for an intermediate strip (w/d = 1):
\(a = b = 3.17\ \text{cm}, w = d = 3.17\ \text{mm}, \text{and } \varepsilon_r = 11.7\) (Values in ohms).

<table>
<thead>
<tr>
<th>f(GHz)</th>
<th>Maxwell Current</th>
<th>Yamashita Current</th>
<th>Yamashita, (J_x = 0)</th>
<th>Quasi-Static Yamashita, (J_x = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>44.17</td>
<td>44.97</td>
<td>44.94</td>
<td>45.32</td>
</tr>
<tr>
<td>3</td>
<td>40.70</td>
<td>41.17</td>
<td>41.12</td>
<td>43.46</td>
</tr>
<tr>
<td>5</td>
<td>36.39</td>
<td>36.41</td>
<td>36.43</td>
<td>41.94</td>
</tr>
<tr>
<td>7</td>
<td>32.82</td>
<td>32.43</td>
<td>32.59</td>
<td>40.78</td>
</tr>
<tr>
<td>9</td>
<td>30.12</td>
<td>29.40</td>
<td>29.80</td>
<td>39.91</td>
</tr>
<tr>
<td>11</td>
<td>27.94</td>
<td>27.07</td>
<td>27.80</td>
<td>39.27</td>
</tr>
<tr>
<td>13</td>
<td>-</td>
<td>25.13</td>
<td>26.37</td>
<td>38.79</td>
</tr>
</tbody>
</table>

equation (116) in the computer program that calculated the characteristic impedance, and these values are also included in Tables IV - VI.

Although Mittra and Itoh's method could be used to calculate the characteristic impedance, a large number of terms would have to be included in the series for the fields in order to ensure reasonable accuracy. This would require the solution of large matrix equations, and would require a considerable amount of computer time. For this reason, no attempt was made to solve for the fields obtained with their model. It is expected that their model would give results close to the values given in Tables IV - VI because of the good agreement.
noted for the effective dielectric constant.

The characteristic impedance decreases with increasing frequency in every case except when the Maxwell current is assumed on the wide strip in Table IV. This is in qualitative agreement with the quasi-static model expressed in equations (90) and (116), where the characteristic impedance is inversely proportional to the effective dielectric constant. The quasi-static model and the Yamashita

Table VI. Characteristic impedance for a narrow strip (w/d = 0.1):

\[ a = b - 3.17 \text{ cm}, w = 3.17 \text{ mm}, d = 0.317 \text{ mm}, \text{ and } \varepsilon_r = 11.7 \]

(Values in ohms).

<table>
<thead>
<tr>
<th>f(GHz)</th>
<th>Maxwell Current</th>
<th>Yamashita Current</th>
<th>Yamashita, ( J_x = 0 )</th>
<th>Quasi-Static Yamashita, ( J_x = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>96.79</td>
<td>97.98</td>
<td>97.94</td>
<td>99.50</td>
</tr>
<tr>
<td>3</td>
<td>82.90</td>
<td>83.38</td>
<td>83.65</td>
<td>97.30</td>
</tr>
<tr>
<td>5</td>
<td>62.74</td>
<td>63.13</td>
<td>62.91</td>
<td>95.05</td>
</tr>
<tr>
<td>7</td>
<td>44.06</td>
<td>44.00</td>
<td>43.70</td>
<td>92.75</td>
</tr>
<tr>
<td>9</td>
<td>29.99</td>
<td>29.65</td>
<td>29.31</td>
<td>90.41</td>
</tr>
<tr>
<td>11</td>
<td>20.80</td>
<td>20.36</td>
<td>20.06</td>
<td>88.12</td>
</tr>
<tr>
<td>13</td>
<td>15.38</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>15</td>
<td>12.31</td>
<td>11.97</td>
<td>11.80</td>
<td>84.29</td>
</tr>
</tbody>
</table>

dynamic model with non-zero transverse current give results that are within 5% for the wide strip. This agrees with our earlier
observations, and indicates that the wide strip is acting much like a parallel-plate waveguide operating in the TEM mode.

The dynamic model indicates that for narrower strips, the characteristic impedance will decrease more rapidly with increasing frequency than predicted by the quasi-static model. It will be seen that this conclusion is verified by experimental results.

It was found that the solution for $M^2$ was dependent on the point at which the boundary conditions were matched. This is illustrated in Tables VII and VIII.

Table VII. Effect of matching the boundary conditions at different points on the strip: $a = 7.295$ mm, $b = 3.17$ cm, $w = 0.7295$ mm, $d = 1.585$ mm, and $\epsilon_r = 10$ (Yamashita current).

<table>
<thead>
<tr>
<th>f(GHz)</th>
<th>Fraction of the distance from the center to the edge of the strip</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td>0</td>
<td>6.370</td>
</tr>
<tr>
<td>4</td>
<td>6.764</td>
</tr>
</tbody>
</table>

When the match point was within five-eights of the distance from the center to the edge of the strip, the values obtained for $M^2$ disagreed by less than 4% for all of the cases considered. As the
Table VIII. Effect of matching the boundary conditions at different points on the strip: \( a = 1 \) mm, \( b = 4 \) mm, \( w = 0.1 \) mm, \( d = 0.2 \) mm, and \( \varepsilon_r = 10 \) (Yamashita current).

<table>
<thead>
<tr>
<th>( f )(GHz)</th>
<th>Midpoint between center and edge of strip</th>
<th>Strip edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.267</td>
<td>8.411</td>
</tr>
<tr>
<td>10</td>
<td>6.356</td>
<td>8.583</td>
</tr>
<tr>
<td>20</td>
<td>6.497</td>
<td>8.823</td>
</tr>
<tr>
<td>30</td>
<td>6.626</td>
<td>9.036</td>
</tr>
<tr>
<td>40</td>
<td>6.750</td>
<td>9.241</td>
</tr>
<tr>
<td>50</td>
<td>6.874</td>
<td>9.443</td>
</tr>
</tbody>
</table>

match point approached the edge of the strip, the disagreement increased rapidly. It is felt that this problem is a consequence of the non-exact assumed current density. The form of the current assumed is least exact near the edge of the strip. A comparison with the results obtained using Mittra and Itoh's technique indicated that the best results were obtained when a point about half-way between the center and the edge of the strip was used.
C. Comparison of Model with Other Theories and with Experimental Results

The results obtained using the model with zero transverse current and the Yamashita longitudinal current distribution were compared with published results in order to further verify its validity. Effective dielectric constant is plotted versus frequency in Figure 6 and phase velocity is shown in Figure 7 for values of strip width, dielectric thickness, and dielectric constant considered by Denlinger [16], [18]. Note that this is the intermediate strip considered in Tables II and V.

The excellent agreement between the two theories is not surprising since the model developed here is similar to Denlinger's. Denlinger compared his theoretical phase velocity with experimental results and found good agreement. The characteristic impedance curve for these parameters in shown in Figure 8. Note that there is good agreement between the quasi-static characteristic impedance and the value obtained from the modal expansion model at low frequencies. As the frequency increases, the value of the characteristic impedance predicted by the dynamic model becomes smaller. The waveguide height and width for these calculations were taken to be ten times the dielectric thickness and strip width respectively.

A second case considered by Denlinger is shown in Figures 9 and 10. The phase velocity, plotted in Figure 9, again shows excellent agreement. Denlinger [16] reported negligible difference between the quasi-static characteristic impedance given by equation (91) and the dynamic prediction expressed in equation (95). This conclusion is
Figure 6. Effective dielectric constant versus frequency. (a = b = 3.17 cm, w = d = 3.17 mm, $\epsilon_r = 11.7$.) Theoretical values by Denlinger for open microstrip with d = 3.17 mm, w/d = 0.96, $\epsilon_r = 11.7$. 

ER = 11.7, w/d = 1
DENLINGER
THEORETICAL VALUES -x
Figure 7. Phase velocity versus frequency. \((a = b = 3.17 \text{ cm}, w = d = 3.17 \text{ mm}, \varepsilon_r = 11.7.)\)
Figure 8. Characteristic impedance versus frequency. \( a = b = 3.17 \text{ cm}, \)
\( w = d = 3.17 \text{ mm}, \varepsilon_r = 11.7. \)
Figure 9. Phase velocity versus frequency. \(a = 1.5 \text{ cm}, b = 2.5 \text{ cm}, d = 1.016 \text{ mm}, w = 0.552 \text{ mm}, \epsilon_r = 15.87\). Theoretical values by Denlinger for open microstrip with \(d = 0.1016 \text{ mm}, w/d = 0.543, \epsilon_r = 15.87\).
Figure 10. Characteristic impedance versus frequency. \( (a = 1.5 \text{ cm}, \, b = 2.5 \text{ cm}, \, d = 1.016 \text{ mm}, \, w = 0.522 \text{ mm}, \, \varepsilon_r = 15.87. ) \) ** Theoretical values by Denlinger for open microstrip with \( d = 0.1016 \text{ mm}, \, w/d = 0.543, \, \varepsilon_r = 15.87. \)
not verified here. The characteristic impedance values in Figure 10 show good agreement between Denlinger's results and our quasi-static curve, but the dynamic model predicts lower values.

The effective dielectric constant obtained for microstrip parameters considered by Zysman and Varon [62] is plotted in Figure 11. Their analysis was for microstrip enclosed in a shielded box, and the box dimensions used here are the same as theirs. Although the agreement is not as good as with Denlinger's results, it is within 4 percent. It was necessary to read their values off a small graph, and some error could have resulted. The phase velocity and characteristic impedance for these parameters are shown in Figures 12 and 13. Note that the difference between the quasi-static and dynamic theories is not quite as pronounced for this case, where the dielectric constant of the substrate is lower.

Figure 14 gives a comparison with the experimental results on alumina substrates ($\varepsilon_r = 9.9$) presented by Troughton [52]. The values of the strip widths used were not given, but the lower two curves were specified as 50 Ω lines and the upper curve as 25 Ω, where the impedance values were from Wheeler's [55] static theory. The 50 Ω lines were assumed to have a strip width to dielectric thickness ratio of 1 to 1. This ratio was taken to be 3 to 1 for the 25 Ω line. The calculated DC impedances for these lines were 49.83 Ω and 26.26 Ω respectively.

The agreement with the 25 Ω line is seen to be excellent. Good agreement (within 2.5%) is obtained for the 50 Ω wide substrate line,
Figure 11. Effective dielectric constant versus frequency. ($a = 1.27 \ \text{cm}$, $b = 1.22 \ \text{cm}$, $d = 1.27 \ \text{mm}$, $w = 1.22 \ \text{mm}$, $\varepsilon_r = 9.7$). *Theoretical values from Zysman and Varon for enclosed microstrip with the same dimensions.*
Figure 12. Phase velocity versus frequency. \( a = 1.27 \) cm, \( b = 1.22 \) cm, \( d = 1.27 \) mm, \( w = 1.22 \) mm, \( \epsilon_r = 9.7. \)
Figure 13. Characteristic impedance versus frequency. (a = 1.27 cm, b = 1.22 cm, 
d = 1.27 mm, w = 1.22 mm, $\varepsilon_r = 9.7$.)
Figure 14. Effective dielectric constant versus frequency.
(Upper curve: \(a = 1\) cm, \(b = 2\) cm, \(d = 0.635\) mm, \(w = 1.905\) mm; middle curve: \(a = b = 2\) cm, \(w = d = 1.27\) mm; lower curve: \(a = b = 1\) cm, \(w = d = 0.635\) mm.)
while the 50 Ω narrow substrate line results are within 4 percent. As before, some of the error could be due to misreading of the small graph from which this data was obtained.

Napoli and Hughes [46] used a voltage standing wave ratio technique to measure the characteristic impedance of 50 Ω (DC) microstrip lines on 0.025 and 0.05 inch alumina substrates. The dielectric constant of their substrate material was given as ε_r = 9.6. Our results are compared with their measurements in Figures 15 and 16. Both curves show good agreement at low frequencies. The comparison at high frequencies is not as good, but the experimental error will be greater for higher values of frequency. Krage and Haddad [42] point out that some of the assumptions made by Napoli and Hughes in interpreting their data allow a range of characteristic impedance values higher than those shown.

Krage and Haddad [42] considered the definition of characteristic impedance given in equation (91), but they used only the conduction portion of the total longitudinal current. This definition resulted in calculated results for characteristic impedance that increased with frequency. When our model is modified to neglect displacement current, the results obtained are in good agreement with those reported by Krage and Haddad. This is illustrated in Figures 17 and 18. The characteristic impedance predicted by our model is included for comparison.

The effective dielectric constant for the two cases considered by Krage and Haddad is shown in Figures 19 and 20. Again, the results
Figure 15. Characteristic impedance versus frequency. \((a = b = 1 \text{ cm}, w = d = 0.635 \text{ mm}, 
\varepsilon_r = 9.6.\)
Figure 16. Characteristic impedance versus frequency. \( (a = b = 1.5 \text{ cm}, w = d = 1.27 \text{ mm}, \epsilon_r = 9.6. ) \)
Figure 17. Characteristic impedance versus frequency. (a = 1 mm, b = 4 mm, d = 0.2 mm, $w = 0.1$ mm, $\varepsilon_r = 10$.)

ER = 10, $D=2W=0.2$ mm
Upper-strip I only.
Lower-dynamic soln.
Krage, Kadoao-theory-x
Figure 18. Characteristic impedance versus frequency. (a = 1 mm, b = 4 mm, w = d = 0.2 mm, $\epsilon_r = 10$.)
Figure 19. Effective dielectric constant versus frequency. \( (a = 1 \text{ mm}, b = 4 \text{ mm, } \\ d = 0.2 \text{ mm, } w = 0.1 \text{ mm, } \varepsilon_r = 10.) \)
Figure 20. Effective dielectric constant versus frequency. (a = 1 mm, b = 4 mm, w = d = 0.2 mm, $\varepsilon_r = 10$.)
are in good agreement with those obtained from our model.

The zero frequency values of characteristic impedance calculated here were compared with those predicted by Wheeler's [55] static theory. The results are summarized in Table IX. Our model is seen to give results that are in good agreement with Wheeler's.

Table IX. Comparison with Wheeler's Theory.

<table>
<thead>
<tr>
<th>Case</th>
<th>Geometric Parameters</th>
<th>Characteristic Impedance (Ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>w/d</td>
<td>(e_r)</td>
</tr>
<tr>
<td>Table IV</td>
<td>3</td>
<td>11.7</td>
</tr>
<tr>
<td>Table V (Figure 8)</td>
<td>1</td>
<td>11.7</td>
</tr>
<tr>
<td>Table VI</td>
<td>0.1</td>
<td>11.7</td>
</tr>
<tr>
<td>Figure 10</td>
<td>0.543</td>
<td>15.87</td>
</tr>
<tr>
<td>Figure 12</td>
<td>0.96</td>
<td>9.7</td>
</tr>
<tr>
<td>Figure 14</td>
<td>1</td>
<td>9.6</td>
</tr>
<tr>
<td>Figure 15</td>
<td>1</td>
<td>9.6</td>
</tr>
<tr>
<td>Figure 16</td>
<td>0.05</td>
<td>10</td>
</tr>
<tr>
<td>Figure 17</td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

The comparisons made in this and in the preceding section indicate that the model using the Yamashita current distribution with zero transverse current gives results that are in good agreement with others.
The effective dielectric constant predicted here is generally in excellent agreement with the results reported elsewhere. While the characteristic impedance doesn't show as good agreement, this is at least partially due to the difficulty in defining and measuring characteristic impedance for non-TEM transmission lines.

The computer time per frequency point necessary to evaluate the microstrip parameters using the model developed in this work is dependent on the number of points considered as well as on the number of terms summed in the series. This dependence occurs because of the fixed time necessary to evaluate the limit series and the other frequency independent terms, before beginning the calculation of the frequency dependent parameters.

Typically, 3,000 terms were summed in the limit series and 100 terms were included in the difference series. For these values, an IBM 360/65 computer required about three-fourths second per point to evaluate the normalized propagation constant, effective dielectric constant, phase velocity, characteristic impedance, and quasi-static characteristic impedance for 16 frequency points. Denlinger's [16] model required about 24 seconds per point and Krage and Haddad's [42] theory needed 15-20 seconds per point. Mittra and Itoh's [45] technique required 20-30 seconds on a CDC-620, which is about four times slower than the IBM 360/65.
D. Effect of Waveguide Size on Effective Dielectric Constant

In order to investigate the effect of the enclosing waveguide on the fundamental microstrip mode, the effective dielectric constant was calculated when the waveguide dimensions were varied. Figures 21, 22 and 23 illustrate the effect of changing the width of the guide when the height is fixed. Three values of frequency are considered for three strip width to dielectric substrate thickness ratios. Note that for higher values of frequency, the waveguide width can be made smaller without effecting the microstrip mode. Similarly, the guide width can be made smaller for wider strips. This is because at high frequencies, or for wide strips, the fields are more tightly bound to the conducting strips.

Figures 24 and 25 illustrate the effect of varying the waveguide height. The results for different strip width and frequencies are similar to those obtained when the guide width was varied, although the ratio of height to dielectric thickness can be made smaller than the ratio of guide width to strip width without effecting the microstrip mode. This occurs because in addition to the strip, the dielectric substrate acts to confine the field. Lower dielectric constant substrates would require larger waveguide dimensions than those indicated in Figures 21-25 in order to leave the microstrip mode undisturbed.

When the waveguide walls are close enough to the strip to disturb the fields, the current distribution on the strip will also be altered.
Figure 21. Variation of effective dielectric constant with waveguide width and frequency for w/d = 1. (a = 3.17 cm, w = d = 3.17 mm, $\varepsilon_r = 11.7$.)
Figure 22. Variation of effective dielectric constant with waveguide width and frequency for w/d = 0.25. (a = 3.17 cm, d = 3.17 mm, w = 0.7925 mm, \( \varepsilon_r = 11.7 \).)
Figure 23. Variation of effective dielectric constant with waveguide width and frequency for $w/d = 0.1$. ($a = 3.17$ cm, $d = 3.17$ mm, $w = 0.317$ mm, $\varepsilon_r = 11.7$.)
Figure 24. Variation of effective dielectric constant with waveguide height and frequency for $w/d = 1$. ($b = 3.17$ cm, $w = d = 3.17$ mm, $\varepsilon_r = 11.7$)
Figure 25. Variation of effective dielectric constant with waveguide height and frequency for w/d = 0.1. (b = 3.17 cm, d = 3.17 mm, w = 0.317 mm, $\varepsilon_r = 11.7$)
Since our model assumes a current distribution similar to that existing on open microstrip, we would expect poor results from our theory for this case. This conclusion is verified in Table X, where the results for the narrow strip considered earlier are compared to the results from Mittra and Itoh's theory when the side walls are ten times the strip width. Although the results are in good agreement at low frequencies, the error becomes huge at higher values of frequency. Therefore, our model is not useful for these cases.

Table X. Effective dielectric constant for a narrow strip in a narrow waveguide: \( a = 3.17 \) cm, \( b = d = 3.17 \) mm, \( w = 0.317 \) mm, and \( \varepsilon_r = 11.7 \).

<table>
<thead>
<tr>
<th>( f ) (GHz)</th>
<th>Yamashita Current</th>
<th>Maxwell Current</th>
<th>Mittra and Itoh</th>
<th>Yamashita, ( J_x = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.367</td>
<td>6.337</td>
<td>6.363</td>
<td>6.360</td>
</tr>
<tr>
<td>3</td>
<td>6.399</td>
<td>6.369</td>
<td>6.512</td>
<td>6.391</td>
</tr>
<tr>
<td>5</td>
<td>6.462</td>
<td>6.434</td>
<td>7.715</td>
<td>6.454</td>
</tr>
<tr>
<td>7</td>
<td>6.556</td>
<td>6.529</td>
<td>9.200</td>
<td>6.547</td>
</tr>
<tr>
<td>11</td>
<td>6.827</td>
<td>6.807</td>
<td>9.792</td>
<td>6.816</td>
</tr>
<tr>
<td>13</td>
<td>7.001</td>
<td>6.984</td>
<td>9.985</td>
<td>6.988</td>
</tr>
<tr>
<td>15</td>
<td>7.196</td>
<td>7.184</td>
<td>10.138</td>
<td>7.182</td>
</tr>
</tbody>
</table>
E. Waveguide Currents and Fields on the Dielectric Interface

The various currents flowing in the waveguide are illustrated in Figure 26 for the intermediate width strip. The total current represents the strip conduction current plus the displacement current, or the total wall current, since their magnitudes are identical. Note that the current in the ground plane is within 10 percent of the total current for frequencies above 2 GHz, and above 5 GHz they are virtually identical.

The currents for the wide and narrow strips were qualitatively similar. For wide strips at a given frequency, the displacement current was a smaller percentage of the total current than for narrow strips. This is logical since wide strips have a lower characteristic impedance than narrow ones.

Also, the ground plane current becomes virtually equal to the total current at much lower frequencies than for narrow strips. This follows from our earlier conclusion that the fields are more tightly bound to wide strips than to narrow ones.

The components of electric field intensity transverse to the dielectric interface are shown in Figures 27 and 28. These fields were derived for the non-zero transverse current case with the Yamashita longitudinal current. The fields were similar for the simplified model with zero transverse current.

Consideration of $E_y$ in Figure 27 helps explain the difficulty encountered when the boundary conditions were matched near the
Figure 26. Microstrip current, normalized to unity total current, versus frequency.
(a = b = 3.17 cm, w = d = 3.17 cm, $\varepsilon_r = 11.7$.)
Figure 27. Transverse electric field intensity on the dielectric interface.

\(a = b = 3.17\ \text{cm}, \ w = d = 3.17\ \text{mm}, \ \varepsilon_r = 11.7.\)
Figure 28. Longitudinal electric field intensity on the dielectric interface. 
\(a = b = 3.17 \text{ cm}, w = d = 3.17 \text{ mm}, \varepsilon_r = 11.7.\)
edge of the strip. In this region, $E_y$ becomes very large. This is a result of the large concentration of charge at the edge of the zero thickness conducting strip. Note that as the frequency increases, the fields move toward the strip.

F. Desk Calculator Results

The equations given in section III for propagation constant and characteristic impedance were programmed on a Hewlett Packard 9100B desk calculator. The results obtained were essentially the same as those presented earlier for the simplified model with zero transverse current.

The total time necessary to compute the effective dielectric constant using equation 115 was mainly dependent on the number of terms summed and the accuracy required. The convergence was fairly fast, even when the initial guess for $M^2$ was considerably off, and the effective dielectric constant was found to three significant figures after about ten iterations. When 30 terms were included in the series, each iteration took about 15 seconds.

The quasi-static characteristic impedance given by equations 116, 121 and 122 were found in one step. This procedure took about 5 seconds.
G. Higher Order and Evanescent Modes

A computer program (Appendix F) was developed that could efficiently separate the zeros from the infinities in the sum of the boundary condition series. This made it relatively easy to calculate the propagation constant for higher order and evanescent modes. A plot of the normalized propagation constant versus frequency for the fundamental and first six higher order modes for the intermediate strip width case considered earlier is given in Figure 29. Because of the cost involved, not enough data points were calculated to obtain a smooth curve, and the points are connected by straight lines. This plot is similar to that given by Corr and Davies [12].

A few values of the propagation constant calculated for evanescent modes were compared with those found using Mittra and Itoh's technique. This comparison is given in Table XI, with good agreement noted.

Table XI. Comparison of the normalized propagation constant for the first three evanescent modes with Mittra and Itoh's model:

$\alpha = b = 3.17 \text{ cm}, d = 3.17 \text{ mm}, \varepsilon_r = 11.7$ and $f = 2 \text{ GHz}$.

<table>
<thead>
<tr>
<th>w(mm)</th>
<th>3.17</th>
<th>0.317</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mittra and Itoh</td>
<td>This Theory</td>
</tr>
<tr>
<td>Theory</td>
<td>j2.0792</td>
<td>j2.0796</td>
</tr>
<tr>
<td>M</td>
<td>3.1369</td>
<td>j3.1367</td>
</tr>
<tr>
<td></td>
<td>j3.3676</td>
<td>j3.3636</td>
</tr>
</tbody>
</table>
Figure 29. Normalized propagation constant versus frequency for the fundamental and first six higher order modes. ($a = b = 3.17$ cm, $w = d = 3.17$ mm, $\varepsilon_r = 11.7$.)
Table XII. Variation of normalized propagation constant with frequency: \( a = b = 3.17 \text{ cm}, \ w = d = 3.17 \text{ mm}, \) and \( \varepsilon_r = 11.7. \)

<table>
<thead>
<tr>
<th>f(GHz)</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.764</td>
<td>2.786</td>
<td>2.847</td>
<td>2.906</td>
<td>2.961</td>
<td>3.011</td>
</tr>
<tr>
<td></td>
<td>j9.382</td>
<td>j4.582</td>
<td>j2.080</td>
<td>j1.129</td>
<td>j0.446</td>
<td>0.578</td>
</tr>
<tr>
<td>M</td>
<td>-</td>
<td>-</td>
<td>j3.137</td>
<td>j1.918</td>
<td>j1.235</td>
<td>j0.718</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>j3.364</td>
<td>j2.104</td>
<td>j1.419</td>
<td>j0.945</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>j5.154</td>
<td>j3.321</td>
<td>j2.360</td>
<td>j1.736</td>
</tr>
</tbody>
</table>

Table XIII. Variation of normalized propagation constant with substrate dielectric constant: \( a = b = 3.17 \text{ cm}, \ w = d = 3.17 \text{ mm}, \) and \( f = 2 \text{ GHz} \)

<table>
<thead>
<tr>
<th>( \varepsilon_r )</th>
<th>7.5</th>
<th>11.7</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.288</td>
<td>2.847</td>
<td>3.334</td>
</tr>
<tr>
<td>M</td>
<td>j2.092</td>
<td>j2.080</td>
<td>j2.069</td>
</tr>
<tr>
<td></td>
<td>j3.153</td>
<td>j3.137</td>
<td>j3.122</td>
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<tr>
<td></td>
<td>j3.358</td>
<td>j3.364</td>
<td>j3.367</td>
</tr>
</tbody>
</table>

Table XII gives the variation of the normalized propagation constant with frequency for the first five modes on the intermediate
width strip. Tables XIII and XIV show the variation of \( M \) with the
dielectric constant of the substrate for a constant frequency.

Table XIV. Variation of normalized propagation constant with
substrate dielectric constant: \( a = b = 3.17 \text{ cm}, \)
\( w = d = 3.17 \text{ mm}, \) and \( f = 4 \text{ GHz}. \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\varepsilon_r & 7.5 & 9.7 & 11.7 & 16 \\
\hline
M & 2.360 & 2.688 & 2.961 & 3.489 \\
& j0.491 & j0.469 & j0.446 & j0.381 \\
& j1.267 & j1.251 & j1.235 & j1.198 \\
& j1.419 & j1.419 & j1.419 & j1.418 \\
& j2.397 & j2.378 & j2.360 & j2.316 \\
\hline
\end{array}
\]

The subroutine used to evaluate the propagation constant in
Table XII was limited to \( |M| \leq 10 \). The blank spaces in the first
two columns represent values of \( M \) that were outside this range.
Note that there are two propagating modes for \( f = 5 \text{ GHz} \) in Table XII.
Also note that the evanescent modes in Tables XIII and XIV appear to
be relatively insensitive to the value of the dielectric constant.

The information presented in the preceding tables for the
intermediate width strip are repeated in Tables XV and XVI for the
narrow strip. Again, note that the normalized propagation constant
doesn't vary much with the substrate dielectric constant.
Table XV. Variation of normalized propagation constant with frequency:

\[ a = b = 3.17 \text{ cm}, \ d = 3.17 \text{ mm}, \ w = 0.317 \text{ mm}, \ \text{and } \varepsilon_r = 11.7. \]

<table>
<thead>
<tr>
<th>( f(\text{GHz}) )</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_r )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.5</td>
<td>2.634</td>
<td>2.644</td>
<td>2.675</td>
<td>2.704</td>
<td>2.735</td>
</tr>
<tr>
<td>j0.391</td>
<td>j4.597</td>
<td>j2.098</td>
<td>j1.145</td>
<td>j0.456</td>
<td></td>
</tr>
<tr>
<td>11.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>-</td>
<td>-</td>
<td>j3.156</td>
<td>j1.939</td>
<td>j1.254</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>j3.351</td>
<td>j2.086</td>
<td>j1.395</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>j5.159</td>
<td>j3.327</td>
<td>j2.630</td>
<td></td>
</tr>
</tbody>
</table>

Table XVI. Variation of normalized propagation constant with substrate dielectric constant: \( a = b = 3.17 \text{ cm}, \ d = 3.17 \text{ mm}, \ w = 0.317 \text{ mm}, \ f = 2 \text{ GHz}. \)

<table>
<thead>
<tr>
<th>( \varepsilon_r )</th>
<th>7.5</th>
<th>11.7</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.168</td>
<td>2.675</td>
<td>3.111</td>
<td></td>
</tr>
<tr>
<td>j2.106</td>
<td>j2.098</td>
<td>j2.092</td>
<td></td>
</tr>
<tr>
<td>j3.167</td>
<td>j3.156</td>
<td>j3.146</td>
<td></td>
</tr>
<tr>
<td>j3.343</td>
<td>j3.351</td>
<td>j3.354</td>
<td></td>
</tr>
</tbody>
</table>

A comparison of Tables XII and XV shows that the values of the propagation constant are relatively insensitive to the strip width as well as to the substrate dielectric constant. This indicates that the evanescent modes considered are waveguide modes rather than
microstrip modes. This conclusion is reinforced by the results presented in Table XVII. Here, the value of the normalized propagation constant is observed for four combinations of waveguide dimensions. While M for the fundamental microstrip mode remains nearly constant, its value for the evanescent modes decreases rapidly as the waveguide dimensions increase. This is typical of waveguide modes, since the mode cutoff frequency is lower for larger waveguides.

Table XVII. Variation of normalized propagation constant with waveguide dimensions: \( w = d = 3.17 \text{ cm}, \varepsilon_r = 11.7, \) and \( f = 3 \text{ GHz}. \)

<table>
<thead>
<tr>
<th>a (cm)</th>
<th>2.5</th>
<th>2.9</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>b (cm)</td>
<td>2.5</td>
<td>3.17</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<td>j4.275</td>
<td>j3.590</td>
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The transcendental equations given by Harrington [27, pp. 158-163] for the mode-propagation constants for an inhomogeneously filled waveguide were solved for \( f = 2 \text{ GHz} \) with the waveguide and substrate dimensions considered in Table XII. The values calculated for the three lowest order modes are \( j2.12, j3.18, \) and \( j3.34. \) It appears
that the evanescent modes found using this theory are these modes, slightly perturbed by the conducting strip.

Further study is needed to determine if the evanescent microstrip modes were not found because of the theory or because the search was not continued far enough. Several points were observed where the plot of the sum of the boundary condition series dipped toward zero without reaching it. The corresponding values of M may represent evanescent microstrip modes.
V. SUMMARY AND CONCLUSIONS

A hybrid-mode model for infinite microstrip in a waveguide has been developed. The form of both the transverse and longitudinal current density distributions were assumed in order to reduce the complexity and the resulting computation time necessary to solve the problem. It was found that the transverse current could be neglected for a wide range of microstrip width to dielectric thickness ratios with negligible effect on the results.

Two forms of the assumed longitudinal current were considered. They were found to give nearly equivalent results while one required considerably less computation time. When this form of the current was selected, the resulting model required considerably less computer time than comparable theories reported by other investigators.

The resulting simplified model was used to investigate various properties of microstrip. It was observed that for waveguide dimensions large compared to those of the microstrip, this model closely approximates the open microstrip results reported by other investigators. When the dimensions of the waveguide were made small compared to the microstrip, the assumed current was found to no longer approximate reality, and erroneous results were obtained.

The form of the characteristic impedance as a function of frequency was investigated. The quasi-static model was shown to result in values that are high when compared to the dynamic results.
predicted by this theory. Comparisons were made with theoretical and experimental results found in the literature, and the reason for a reported discrepancy was presented.

The transverse electric fields on the dielectric interface, and the current in the waveguide walls were investigated. It was shown that as the frequency increased, the fields concentrated around the strip, and the wall current tended to concentrate in the ground plane. The effect of the high concentration of charge on the edge of the strip was seen to result in large values for the transverse component of electric field intensity in this region. This made it difficult to match the boundary conditions near the edge of the strip for the assumed current distributions.

This model was used to find both propagating and evanescent modes in microstrip. The evanescent modes studied were found to represent waveguide modes rather than microstrip modes. Further study is needed to determine if the failure to find evanescent microstrip modes is due to a deficiency in this theory, or a failure to investigate enough modes.

A simplified model that could be programmed on a desk calculator was developed. It was noted that accurate dynamic results could be obtained in a reasonably short period of time with this model.
VI. LITERATURE CITED


VII. ACKNOWLEDGMENTS

Dr. R. E. Post has provided the author with a great deal of assistance and encouragement during the course of this work.

Mr. John Pavlat's help with computer programming problems is gratefully acknowledged. To many of the author's associates who provided helpful comments and useful advice during his graduate career, thanks are also due. Prominent among these are Dr. W. H. Childs, Dr. W. D. Swift, and Dr. D. M. Morton.

The thesis was typed by Susan Heitman.

Special thanks are due the author's wife, Cheryl, for her understanding patience and encouragement throughout his time in graduate school, and to the author's children, Tommy and Heather, who tried to understand when Dad couldn't stay home and play.

This work was partially done while the author was on a National Science Foundation graduate traineeship.
Rewrite (42)-(45) as

\[ 0 = (A'2) + c_{31} n + c_{32} \rightleftharpoons n + c_{33} n + c_{34} n \]  

\[ 0 = c_{31} n + c_{32} n + c_{33} n + c_{34} n \]  

\[ K_n = c_{41} n + c_{42} n + c_{43} n + c_{44} n \]  

where

\[ c_{11} = (\varepsilon_r - M^2) \cos \alpha d \quad c_{33} = \frac{n \pi}{b} \frac{M}{\varepsilon_{r_1}} \eta_o \sin \alpha d \]  

\[ c_{12} = (M^2 - \varepsilon r_2) \cos \beta L \quad c_{34} = -\frac{n \pi}{b} \frac{M}{\varepsilon_{r_2}} \eta_o \sin \beta L \]  

\[ c_{23} = \varepsilon_{r_2}(\varepsilon_{r_1} - M^2)\sin \alpha d \quad c_{41} = \frac{n \pi}{b} \frac{M}{\eta_o} \cos \alpha d \]  

\[ c_{24} = \varepsilon_{r_1}(M^2 - \varepsilon r_2)\sin \beta L \quad c_{42} = -\frac{n \pi}{b} \frac{M}{\eta_o} \cos \beta L \]  

\[ c_{31} = \alpha \sin \alpha d \quad c_{43} = -\alpha \cos \alpha d \]  

\[ c_{32} = \beta \sin \beta L \quad c_{44} = -\beta \cos \beta L \]
Solve (A.1) and (A.2) for \( A_n \) and \( C_n \), and use these to reduce (B.3) and (B.4) to two equations in two unknowns. Then

\[
A_n = \frac{\bar{H}_n - c_{12}B_n}{c_{11}}
\]

\[
= \frac{\bar{H}_n + (\varepsilon_{r2} - M^2) \cos \beta L B_n}{(\varepsilon_{r1} - M^2) \cos \alpha d}
\]

(A.6)

\[
C_n = -\frac{c_{24}}{c_{23}} D_n = -\frac{\varepsilon_{r1}(M^2 - \varepsilon_{r2}) \sin \beta L}{\varepsilon_{r2}(\varepsilon_{r1} - M^2) \cos \beta L} D_n
\]

(A.7)

and

\[
-\frac{\alpha \tan \alpha d}{(\varepsilon_{r1} - M^2)} \bar{H}_n = a_{11} B_n + a_{12} D_n
\]

(A.8)

\[
K_n = -\frac{n \pi M}{b \eta_0 (\varepsilon_{r1} - M^2)} \bar{H}_n = a_{21} B_n + a_{22} D_n
\]

(A.9)

where

\[
a_{11} = c_{32} - \frac{c_{12}c_{31}}{c_{11}}, \quad a_{12} = c_{34} - \frac{c_{24}c_{33}}{c_{23}}
\]

(A.10)

\[
a_{21} = c_{42} - \frac{c_{12}c_{41}}{c_{11}}, \quad a_{22} = c_{44} - \frac{c_{24}c_{43}}{c_{23}}
\]

(B_n and D_n can be found from (A.5), (A.8) and (A.9) as

\[
B_n = -\frac{\bar{H}_n [V_n \beta \cot \beta L + (\frac{n \pi}{b})^2 \Delta \varepsilon_{r}]}{U_n V_n (\varepsilon_{r2} - M^2) \cos \beta L}
\]

(A.11)
and
\[ D_n = \varepsilon_{r2} \frac{\frac{H_n}{n} \frac{n}{b} \frac{M}{n_0} U_n - K_n (V_n - M^2 U_n)}{U_n V_n (\varepsilon_{r2} - M^2) \sin \beta L} \quad (A.12) \]

where \( \Delta \varepsilon_r = \varepsilon_{r1} - \varepsilon_{r2} \) and
\[ U_n = \alpha \tan \alpha d + \beta \tan \beta L \quad (A.13) \]
\[ V_n = \varepsilon_{r2} \alpha \tan \alpha d + \varepsilon_{r1} \beta \tan \beta L \quad (A.14) \]
\[ W_n = \alpha \cot \alpha d + \beta \cot \beta L \quad . \quad (A.15) \]

Substituting into (A.6) and (A.7) results in
\[ A_n = \frac{\frac{H_n}{n} \frac{n}{b} \frac{M}{n_0} \frac{\cot \alpha d}{4 \pi^2} - (\varepsilon_{r1} - M^2) \cos \alpha d - K_n \frac{n}{b} M n_0 \Delta \varepsilon_r}{U_n V_n (\varepsilon_{r1} - M^2) \cos \alpha d} \quad (A.16) \]
and
\[ C_n = \frac{\frac{H_n}{n} \frac{n}{b} \frac{M}{n_0} U_n - K_n (V_n - M^2 U_n)}{U_n V_n (\varepsilon_{r1} - M^2) \cos \beta L} \quad . \quad (A.17) \]
IX. APPENDIX B:

FOURIER COEFFICIENTS FOR ASSUMED MICROSTRIP CURRENT

The Fourier coefficients for the longitudinal current density on microstrip are given by

\[ K_n = -\frac{1}{b} \int_{-b}^{b} J_z(y) \sin \frac{n\pi y}{b} \, dy. \]  \hspace{1cm} (39)

We have assumed \( J_z(y) \) is an odd function of \( y \), so the integrand in (39) is even. Then we can integrate from 0 to \( b \) and double the result.

With respect to \( y = \frac{b}{2} \), \( J_z(y) \) has even symmetry while \( \sin \frac{n\pi y}{b} \) is even for \( n \) odd and odd for \( n \) even. Therefore, the integral in (39) is zero for \( n \) even and four times the integral from \( \frac{b}{2} \) to \( b \) for \( n \) odd.

That is,

\[ K_n = \begin{cases} 
-\frac{4}{b} \int_{b/2}^{b} J_z(y) \sin \frac{n\pi y}{b} \, dy, & n \text{ odd} \\
0, & n \text{ even} 
\end{cases} \]  \hspace{1cm} (B.1)

Assume the \( J_z(y) \) is given by the Maxwell current distribution, so

\[ J_z(y) = \begin{cases} 
\frac{J_{zo}}{1 - \left(\frac{2y-b}{w}\right)^2}, & |2y-b| < w \\
0, & |2y-b| < w, 0 < y < b.
\end{cases} \]  \hspace{1cm} (64)
Make the change of variable \( r = \frac{2y-b}{w} \) and apply (B.1) to (64). The result is

\[
K_n = -\frac{2 J_{z_0} w}{b} \sin \frac{n \pi}{2} \int \limits_0^1 \frac{\cos \frac{n \pi w r}{2b}}{\sqrt{1 - r^2}} \, dr
\]

(B.2)

where \( n \) is odd only.

Jahnke and Emde [35, p.151] give the following equation for the \( p \)th order Bessel function.

\[
J_p(z) = \frac{2^{1/2} z^p}{\sqrt{\pi \Gamma(p+1/2)}} \int \limits_0^1 \cos z u (1-u^2)^{-1/2} \, du \tag{B.3}
\]

For the case \( p = 0 \), \( \Gamma(-1/2) = \Gamma(1/2) = \sqrt{\pi} \), and we have

\[
\int \limits_0^1 \cos \frac{n \pi w r}{2b} (1 - r^2)^{-1/2} \, dr = \frac{\pi}{2} J_0 \left( \frac{n \pi w}{2b} \right) \tag{B.4}
\]

so \( K_n \) is given by

\[
K_n = -\frac{\pi w J_{z_0}}{b} \sin \frac{n \pi}{2} J_0 \left( \frac{n \pi w}{2b} \right) . \tag{B.5}
\]

If \( J_z(y) \) is assumed to be that suggested by Yamashita [58],

\[
J_z(y) = \begin{cases} 
J_{z_0} \left( 1 + \frac{2y-b}{w} \right)^3 , & |2y-b| < w \\
0 , & |2y-b| > w, \ 0 < y < b 
\end{cases} \tag{66}
\]
and the same change of variables is made, $K_n$ is given by

$$K_n = -\frac{2wJ_z}{b} \sin \frac{n\pi}{2} \int_0^1 (1+r^3) \cos \frac{n\pi wr}{2b} \, dr.$$  \hspace{1cm} (B.6)

The result of this integration is

$$K_n = -\frac{32b^3 J_z}{\pi^3} \sin \frac{n\pi}{2} \frac{1}{4} \left\{ 6 + 3\left( \left( \frac{n\pi w}{2b} \right)^2 - 2 \right) \cos \frac{n\pi w}{2b} \ight.$$ \hspace{1cm} 

$$+ 2 \left( \frac{n\pi w}{2b} \right) \left[ \left( \frac{n\pi w}{2b} \right)^2 - 3 \right] \sin \frac{n\pi w}{2b} \right\}$$

$$= -\frac{32b^3 J_z}{\pi^3} \sin \frac{n\pi}{2} T_n \hspace{1cm} (B.7)$$

where $T_n$ is $n^{-4}$ times the term in brackets.

The Fourier coefficients for the transverse current density are given by

$$H_n = \frac{1}{b} \int_{-b}^b J_y(y) \cos \frac{n\pi y}{b} \, dy.$$  \hspace{1cm} (40)

$J_y(y)$ is an even function of $y$ and has odd symmetry about $y = \frac{b}{2}$.

Symmetry considerations similar to those used earlier result in

$$H_n = \begin{cases} \frac{4}{b} \int_{\frac{b}{2}}^{b+\frac{w}{2}} J_y(y) \cos \frac{n\pi y}{b} \, dy & , \ n \text{ odd} \\ 0 & , \ n \text{ even} \end{cases}$$  \hspace{1cm} (B.8)
The transverse current density is

\[
J_y(y) = \begin{cases} 
-j \frac{2 J_0}{w} (2y-b+w) & , \ -w < 2y-b < -\frac{w}{2} \\
\frac{2 J_0}{w} (2y-b) & , \ |2y-b| < \frac{w}{2} \\
\frac{2 J_0}{w} (2y-b-w) & , \ \frac{w}{2} < 2y-b < w \\
0 & , \ |2y-b| > w, \ 0 < y < b
\end{cases}
\tag{67}
\]

Straightforward integration leads to

\[
H_n = j \frac{32 b J_0}{\pi^2 w} \frac{1}{n^2} \sin \frac{n \pi}{2} \sin \frac{n \pi w}{4b} \left( \cos \frac{n \pi w}{4b} - 1 \right) 
\]

\[
= j \frac{32 b J_0}{\pi^2 w} \sin \frac{n \pi}{2} S_n 
\tag{B.9}
\]

where

\[
S_n = \frac{1}{n^2} \sin \frac{n \pi w}{4b} \left( \cos \frac{n \pi w}{4b} - 1 \right) .
\tag{B.10}
\]
X. APPENDIX C:

LIMIT SERIES

A. Boundary Condition Series

Consider the series necessary in the evaluation of the propagation constant. The boundary condition is expressed by equation (83), and the series are given in (79)-(82). \( \alpha \) and \( \beta \) can be obtained in terms of \( M \) from the separation equations (12)-(15). The result is

\[
\alpha = \sqrt{k_0^2 (\varepsilon_{r1} - M^2) - \left( \frac{n \pi}{b} \right)^2}
\]

\[
\beta = \sqrt{k_0^2 (\varepsilon_{r2} - M^2) - \left( \frac{n \pi}{b} \right)^2}
\]  

(C.1)
(C.2)

When \( n \) is large, \( \alpha \) and \( \beta \) are imaginary and approach \( j \frac{n \pi}{b} \). Note that as the frequency increases \( (k_0^2 \) increases) or for evanescent modes \( (M^2 < 0) \), a relatively larger number of \( \alpha \) and \( \beta \) terms will be real. For this situation, \( \alpha \) and \( \beta \) will approach \( j \frac{n \pi}{b} \) more slowly.

\( U_n \), \( V_n \) and \( W_n \) are given in equations (51)-(53). For large \( n \), the tangent and cotangent terms become imaginary hyperbolic tangents and cotangents and their magnitudes approach unity. Then

\[
U_n \to -2 \left( \frac{n \pi}{b} \right) \]

(C.3)

\[
V_n \to - \left( \frac{n \pi}{b} \right) (\varepsilon_{r1} + \varepsilon_{r2}) \]

(C.4)
When these limits are considered, the boundary condition series terms in (79)-(82) approach

\[ W_n \to 2 \left( \frac{\frac{n \pi}{b}}{b} \right) . \quad (C.5) \]

Thus, the boundary condition series terms in (79)-(82) approach

\[ S_{2n} \to - \frac{b}{\pi} T_n \sin \frac{n \pi}{2} \cos \frac{n \pi y}{b} \left( \varepsilon_{r1} + \varepsilon_{r2} \right) \quad (C.7) \]

\[ S_{3n} \to \frac{b}{\pi} \frac{S_n \sin \frac{n \pi}{2} \sin \frac{n \pi y}{b}}{\varepsilon_{r1} + \varepsilon_{r2}} \quad (C.8) \]

\[ S_{4n} \to \frac{b (\varepsilon_{r1} + \varepsilon_{r2} - 2M^2)}{2 \pi (\varepsilon_{r1} + \varepsilon_{r2})} T_n \sin \frac{n \pi}{2} \sin \frac{n \pi y}{b} \quad (C.9) \]

Note that for the Yamashita current assumption, \( T_n \) as expressed in equation (70) approaches

\[ T_n \to 2 \left( \frac{\frac{n \pi w}{2b}}{b} \right)^2 \sin \frac{n \pi w}{2b} \quad (C.10) \]

For the Maxwell current, \( T_n \) is \( J_0 \left( \frac{n \pi w}{2b} \right) \) and [27, p. 462],

\[ T_n \to 2 \left( \frac{b}{\pi n w} \right) \cos \frac{\pi}{4} \left( \frac{2n w}{b} - 1 \right) . \quad (C.11) \]

Therefore, the terms in (C.6)-(C.9) approach zero somewhat more slowly for the Maxwell current assumption than for the Yamashita current.
This would be expected because of the infinite current density at the edge of the strip in the Maxwell assumption.

For the simplified model used in the desk calculator solution, the boundary condition is

$$0 = \sum_{n=1}^{\infty} \frac{T_n(V_n - \frac{M^2 U_n}{W_n})}{\sum_{n=1}^{\infty} V_n W_n} \quad n \text{ odd}$$

(108)

When (C.3)-(C.5) are considered, the terms in this series approach

$$\frac{10w}{\pi} (1 - 2 \frac{M^2}{\varepsilon_{r1} + \varepsilon_{r2}}) \frac{T_n}{n}$$

(C.12)

where the substitution $b = 10w$ has been made.

B. Power Series

The power flowing down the microstrip is given by equations (95)-(98). When the limits discussed earlier are considered

$$K_{ln} \to \frac{1}{\varepsilon_{r1} + \varepsilon_{r2}} [- \frac{R_n S_n}{M} + T_n] [R k_o^2 \Delta \varepsilon_r n_s n_s$$

$$- T_n \left( \frac{n_n}{b} \right)^2 (\varepsilon_{r1} + \varepsilon_{r2})]$$

(C.13)
\[ K_{2n} = \frac{1}{2(n\pi/b)^2} \left[ \frac{RnS}{M^2} \left( -2\left(\frac{n\pi}{b}\right)^2 + k_o^2 (\varepsilon_{r1} + \varepsilon_{r2}) \right) \right] \]

\[ + 2 T_n \left(\frac{n\pi}{b}\right)^2 \left[ k_o^2 R_n S_n + T_n \left(\frac{n\pi}{b}\right)^2 \right] \quad (C.14) \]

\[ K_{3n} = \frac{1}{\varepsilon_{r1} + \varepsilon_{r2}} \left[ \frac{RnS}{M^2} \frac{n}{\varepsilon_{r1} + \varepsilon_{r2}} - T_n \right] R \Delta \varepsilon_{r} \quad (C.15) \]

When \( n \) is large,\( \cos \alpha d \) approaches \( \cosh \frac{n\pi d}{b} \) and as \( n \) increases, \( \cosh \frac{n\pi d}{b} \) approaches \( \frac{1}{2} \exp\left(\frac{n\pi d}{b}\right) \). Therefore, the terms in (95) involving \( \frac{1}{\cos^2 \alpha d} \) and \( \frac{1}{\cos^2 \beta L} \) numerators also become small. Then, for large \( n \), the terms in the expression for the power approach

\[ p_n \rightarrow \frac{b M}{4} \frac{n_o J_2 C_K}{\pi (\varepsilon_{r1} + \varepsilon_{r2})} \left[ 2 \frac{T_n^2}{n} \right] \]

\[ - \frac{R}{M^2} S_n T_n \left[ 2 - \frac{b^2}{2\pi^2} (\varepsilon_{r1} + \varepsilon_{r2} + 2M^2) \frac{k_o^2}{n^2} \right] \]

\[ - \frac{b^2 R^2 k_o^2}{\pi^2 M^2} \frac{S_n^2}{n} \left[ 1 - \frac{b^2}{2\pi^2} (\varepsilon_{r1} + \varepsilon_{r2}) \frac{k_o^2}{n} \right] \quad (C.16) \]
C. Current Series

The total current flowing down the microstrip (99) contains the series

$$\sum_{n=1}^{\infty} \frac{\sin \frac{n \pi}{2}}{n \nu \omega_n} \left[ \frac{n S_n U_n R + T_n (V_n - M^2 U_n)}{\alpha \tan \alpha d} \right]$$

$$+ \frac{\epsilon_{r1} (\sec \alpha d - 1)}{\alpha \tan \alpha d}$$

$$+ \frac{\epsilon_{r2} (\sec \beta L - 1)}{\beta \tan \beta L} \right].$$

(C.17)

For $n$ large, the secant terms rapidly approach zero, and the terms in this series approach

$$\frac{b^2}{\pi} \sin \frac{n \pi}{2} \left[ \frac{R S_n}{n^2} + \frac{\epsilon_{r1} + \epsilon_{r2} - 2M^2}{2} \frac{T_n}{n^3} \right].$$

(C.18)

The current in the waveguide walls is expressed in equations (103)-(106). The terms involving $I_{wt}$ and $I_{wb}$ approach zero because of the cosine terms in their denominators and

$$I_{ws} \rightarrow R k_o \frac{S_n}{b^2} + \frac{\pi^2}{2b^2} \frac{n T_n}{n^2}.$$  

(C.19)

Then, the terms in the wall current series approach

$$I_{wn} \rightarrow - \frac{2b}{\pi} J_{zo} C_K \sin \frac{n \pi}{2} \left( R k_o \frac{b^2}{n^2} \frac{S_n}{n} + \frac{T_n}{n} \right).$$

(C.20)
XI. APPENDIX D:

POWER AND CURRENT ON MICROSTRIP

A. Microstrip Power Calculation

Equation (93) gives the total power flowing longitudinally down the microstrip as

$$ P = \int_0^b \int_0^d \left( E_{x1} H_{y1}^* - E_{y1} H_{x1}^* \right) dx + \int_0^a \left( E_{x2} H_{y2}^* - E_{y2} H_{x2}^* \right) dx dy \quad (93) $$

where asterisks indicate the complex conjugate. When the fields are obtained from (54)-(55) and (57)-(58), the resulting products of electric and magnetic fields are,

$$ E_{x1} H_{y1}^* = F_{1n} \cos \alpha x \cos \alpha^* x \sin^2 \frac{n \pi y}{b} \quad (D.1) $$

$$ E_{y1} H_{x1}^* = F_{2n} \sin \alpha x \sin \alpha^* x \cos^2 \frac{n \pi y}{b} \quad (D.2) $$

$$ E_{x2} H_{y2}^* = F_{3n} \cos [\beta(a-x)] \cos [\beta^*(a-x)] \sin^2 \frac{n \pi y}{b} \quad (D.3) $$

$$ E_{y2} H_{x2}^* = F_{4n} \sin [\beta(a-x)] \sin [\beta^*(a-x)] \cos^2 \frac{n \pi y}{b} \quad (D.4) $$

where

$$ F_{1n} = \frac{\beta \tan \beta L \left( \frac{n \pi}{b} \frac{\mu}{\epsilon} \right) + M \eta_0 \frac{k}{\eta_0}}{v \frac{2}{n} w \cos \alpha d \cos \alpha^* d} \left[ -\left( \frac{n \pi}{b} \frac{\mu}{\epsilon} \right) \frac{k^2 M}{\eta_0 \Delta} \frac{1}{\epsilon} - \frac{\frac{\mu}{\epsilon}}{\eta_0} \Delta \frac{1}{\epsilon} \right] $$

$$ + K_n \left[ \beta \tan \beta L \frac{w}{n} + \left( \frac{n \pi}{b} \right)^2 \Delta \frac{1}{\epsilon} \right]^* \quad (D.5) $$
\[ F_{2n} = \frac{\left[ \frac{M}{\hat{n}} k_n^2 \mathbf{H}_n - \left( \frac{n \pi}{b} \right) K_n \right]^*}{V_n^2 W_n^2 \sin \alpha L \sin \alpha L^*} \left\{ \mathbf{H}_n \left[ \left( \frac{n \pi}{b} \right)^2 - k_o^2 V_n \right] \right\} \]

\[ + M \eta_o \left( \frac{n \pi}{b} \right) \mathbf{K}_n \mathbf{U}_n \} \]  

\[ (D.6) \]

\[ F_{3n} = \frac{-\alpha \tan \alpha d \left[ \left( \frac{n \pi}{b} \right) \mathbf{H}_n + M \eta_o \mathbf{K}_n \right]}{V_n^2 W_n^2 \cos \beta L \cos \beta L^*} \left\{ -\left( \frac{n \pi}{b} \right) k_o^2 \frac{M}{\eta_o} \Delta \varepsilon_r \mathbf{H}_n \right\} \]

\[ + \mathbf{K}_n \left[ -\varepsilon_r^2 \alpha \tan \alpha d \mathbf{W}_n + \left( \frac{n \pi}{b} \right)^2 \Delta \varepsilon_r \right]^* \]  

\[ (D.7) \]

\[ F_{4n} = \frac{\left[ \frac{M}{\hat{n}} k_n^2 \mathbf{H}_n - \left( \frac{n \pi}{b} \right) K_n \right]^*}{V_n^2 W_n^2 \sin \beta L \sin \beta L^*} \left\{ \mathbf{H}_n \left[ \left( \frac{n \pi}{b} \right)^2 - k_o^2 V_n \right] \right\} \]

\[ + M \eta_o \left( \frac{n \pi}{b} \right) \mathbf{K}_n \mathbf{U}_n \} \]  

\[ (D.8) \]

\( U_n, V_n, W_n \) are defined in equations (51)-(53). We need consider only products of electric and magnetic fields having the same mode number \( n \) because of the orthogonality of the normal modes [11, pp. 229-232].

Since we are considering lossless media, \( \alpha \) and \( \beta \) are either pure real or pure imaginary. The effect of an imaginary argument on a trigonometric function is summarized as follows. If \( u \) is a real quantity
\[
\sin j u = j \sinh u \quad (D.9)
\]
\[
\cos j u = \cosh u \quad (D.10)
\]

As a result, the products of sinusoidal functions in (D.2)-(D.8) are real and take the form \(\sin^2, \cos^2, \sinh^2, \) or \(\cosh^2\).

When the four integrations on \(x\) are performed, the expression for the power expressed in equation (93) becomes

\[
P_n = \frac{b}{4} \left\{ F_{1n} \left[ d + \frac{1}{\alpha} \sin \alpha d \cos \alpha d \right] + F_{3n} \left[ L + \frac{1}{\beta} \sin \beta L \cos \beta L \right] \right.
\]
\[
\left. - F_{2n} \left[ d - \frac{1}{\alpha} \sin \alpha d \cos \alpha d \right] + F_{4n} \left[ L - \frac{1}{\beta} \sin \beta L \cos \beta L \right] \right\} 
\]

(D.11)

where the subscript on \(P\) indicates the \(n\)th term in the summation for power. The upper term in brackets applies when \(\alpha\) or \(\beta\) are real, and the lower term when they are imaginary.

Let

\[
K_n = j \zeta_0 \Gamma_n \sin \frac{n \pi}{2} T_n \quad (D.12)
\]
where

\[ C_K = \begin{cases} \frac{-\pi w}{b} & \text{(Maxwell current)} \\ \frac{-32 b^3}{4 \pi^3} & \text{(Yamashita current)} \end{cases} \]  

and

\[ T_n = \begin{cases} J_0 \left( \frac{n \pi w}{2b} \right) & \text{(Maxwell current)} \\ \frac{1}{n} \left( 6 + 2 \left( \frac{n \pi w}{2b} \right) \left[ \left( \frac{n \pi w}{2b} \right)^2 - 3 \right] \sin \frac{n \pi w}{2b} \right) \\ + \left( \frac{n \pi w}{2b} \right)^2 \cos \frac{n \pi w}{2b} \right) & \text{(Yamashita current)} \end{cases} \]  


\[ H_n \text{ as given in (71) and (72) leads to} \]

\[ \bar{H}_n = j \frac{H_n}{\omega \varepsilon_0} \]

\[ = - J_{yo} \frac{32 b}{\pi^2 w} \sin \frac{n \pi}{2} S_n \]  

Define

\[ K_{ln} = \frac{W \varepsilon_0 \cos \alpha d \cos \alpha^* d}{M \eta_0} \frac{J_{zo}^2 C_K}{F_{ln}} \]  

\[ (D.15) \]
Equations (76) and (D.12)-(D.18) lead to the expressions for $K_{1n}$, $K_{2n} = K_{4n}$, and $K_{3n}$ given in (96)-(98).

\[
P_n = \frac{b M \eta_o J_zo}{4 W_n V_n} \left\{ \begin{array}{l}
K_{1n} = \frac{W_n V_n \sin \alpha d \sin \alpha^* d}{M \eta_o J_zo C_k^2} \quad F_{2n} \\
K_{2n} = \frac{W_n V_n \cos \beta L \cos \beta^* L}{M \eta_o J_zo C_k^2} \quad F_{3n} \\
K_{3n} = \frac{W_n V_n \sin \beta L \sin \beta^* L}{M \eta_o J_zo C_k^2} \quad F_{4n} \end{array} \right.
\]
Terms like $\sin \alpha d$ $\sin \alpha^* d$ are positive for $\alpha$ both real and imaginary, while $\sin^2 \alpha d$ is positive for $\alpha$ real and negative for $\alpha$ imaginary. Then, since $\cos \alpha d \cos \alpha^* d$ is always positive, equation (D.11) can be expressed as

$$P_n = \frac{b M \gamma_0 J_{2n}}{4 W_n V_n} \left[ \frac{K_{1n}}{V_n \cos^2 \alpha d} (d + \frac{1}{\alpha} \sin \alpha d \cos \alpha d) \right. \left. + \frac{K_{2n}}{W_n \sin^2 \alpha d} (d - \frac{1}{\alpha} \sin \alpha d \cos \alpha d) \right. \left. + \frac{K_{3n}}{V_n \cos^2 \beta L} (d + \frac{1}{\beta} \sin \beta L \cos \beta L) \right] .$$

(D.20)

Rearrangement of (D.20) leads to the expression for power given in equation (95).

Consider the terms in (D.20). Since terms like $\alpha \tan \alpha d$, $\alpha \cot \alpha d$, $\frac{\sin \alpha d}{\alpha}$ and $\cos \alpha d$ are always real, $U_n$, $V_n$ and $W_n$ are real. $J_y$ and $J_{zo}$ are either both real or both imaginary, so $C_K$ is real. $T_n$ and $S_n$ are real; therefore, $K_{1n}$, $K_{2n}$ and $K_{3n}$ are real quantities and the complex properties of $P$ will depend on $M$. This is as expected since the power is a lossless waveguide is real for propagating modes ($M$ real) and imaginary for evanescent modes ($M$ imaginary) [11, pp. 181-182], [27, p. 178].
B. Longitudinal Current on Microstrip

The expression to be integrated for total current down the guide is

\[
I = \int_{\frac{b-w}{2}}^{\frac{b+w}{2}} J_z(y) \, dy + j \omega \varepsilon_0 \left[ \int_{0}^{d} E_{z1} \, dx \right]_{r1}^{r2} + \int_{0}^{a} E_{z2} \, dy
\]

(94)

Consider the first integral. On the strip, \( J_z(y) \) is given by

\[
J_z(y) = J_{zo} \left( 1 + \frac{2y-b}{w} \right)^3
\]

(66)

for the Yamashita current assumption. This current density is an even function with respect to the center of the strip, \( y = \frac{b}{2} \). The integration on \( y \) can be taken from the center of the strip to the right edge and the result doubled. Make the change in variables \( u = \frac{(2y-b)}{w} \).

Then the conduction current is

\[
I_c = w J_{zo} \int_{0}^{1} (1 + u^3) \, du = 1.25 J_{zo}
\]

(D.21)

For the Maxwell current assumption,

\[
J_z(y) = \frac{J_{zo}}{\sqrt{1 - \left( \frac{2y-b}{w} \right)^2}}
\]

(64)

on the strip. When the change of variables suggested above is made, the first integral in (94) becomes
The value for $E_z$ in the second integral is obtained from (56). The displacement current term becomes

\[
I_d = \frac{\varepsilon_r}{\varepsilon_0} \sum_{n=1}^{\infty} \frac{[\bar{H}_n \frac{n\pi}{b} \frac{M}{\gamma_o} U_n - K_n (V_n - M^2 U_n)]}{V_n W_n} \int_0^\theta d\xi \int_0^\pi \sin \alpha \alpha \cos \alpha \, dx
\]

\[
+ \frac{\varepsilon_r^2}{\sin \beta L} \int_0^a \sin \beta (a-x) dx \sin \frac{n \pi y}{b} dy
\]

When the expressions for $\bar{H}_n$ and $K_n$ given in equations (76) and (D.12)-(D.14) are considered, the result expressed in (99) follows.

C. Longitudinal Current in the Waveguide Walls

The longitudinal current in the guide walls is given by (101) and (102) as

\[
I_w = \int_{\Gamma} \hat{n} \times \vec{H} \, dL
\]
When the line integral is broken into four segments, (D.24) becomes

\[ I_w = \int_0^a (H_x\bigg|_{y=b} - H_x\bigg|_{y=0}) \, dx + \int_0^b (H_y\bigg|_{x=0} - H_y\bigg|_{x=a}) \, dy \quad . \]  

(D.25)

The integration on \( x \) in (D.25) gives the side wall contribution to the current. Since the fundamental microstrip mode is even, symmetry requires the current in the side walls to be equal. Then,

\[ I_{w, \text{ sides}} = -2 \int_0^a H_x\bigg|_{y=0} \, dx \quad . \]  

(D.26)

When the value for \( H_x \) is obtained from (57), (D.26) becomes

\[ I_{w, \text{ sides}} = -2 \sum_{n=1}^{\infty} \frac{\left[ M - k_0^2 \frac{n\pi}{b} - \frac{n\pi}{b} \right]}{\omega_n} \left\{ \frac{1}{\sin \alpha} \int_0^d \sin \alpha x \, dx \right\} 
+ \frac{1}{\sin \beta L} \int_d^a \sin \beta (a-x) \, dx \}

= -2 \sum_{n=1}^{\infty} \frac{\left[ M - k_0^2 \frac{n\pi}{b} - \frac{n\pi}{b} \right]}{\omega_n} \left\{ \frac{\cos \alpha d - 1}{\alpha \sin \alpha d} \right\} 
+ \frac{\cos \beta L - 1}{\beta \cos \beta L} \} \quad . \]

(D.27)

The integration on \( y \) in (D.35) gives the top and bottom wall contribution to the total wall current. For the top,
\[ I_{w, \text{top}} = - \int_{0}^{b} H_y \left|_{x=a} \right. \, dy \quad . \tag{D.28} \]

\( H_y \) is given in equation (58), and

\[ I_{w, \text{top}} = - \sum_{n=1}^{\infty} \frac{1}{\nu_{\omega} n \cos \beta L} \left\{ \left[ -\frac{(n \pi)^2}{b} \right] \frac{k_o M}{\eta_o} \frac{H_n}{n} \Delta \varepsilon_r \right. \]
\[ \left. + K_n \left( -r_2 \alpha \tan \alpha d W_n + \frac{(n \pi)^2}{b} \Delta \varepsilon_r \right) \int_{0}^{b} \sin \frac{n \pi y}{b} \, dy \right\} \]
\[ = \frac{2b}{\pi} \sum_{n=1}^{\infty} \frac{1}{\nu_{\omega} n \cos \beta L} \left[ \left( \frac{n \pi}{b} \right)^2 \frac{k_o M}{\eta_o} \frac{H_n}{n} \Delta \varepsilon_r \right. \]
\[ \left. - K_n \left( -r_2 \alpha \tan \alpha d W_n + \frac{(n \pi)}{b} \Delta \varepsilon_r \right) \right] \quad . \tag{D.29} \]

Similarly,

\[ I_{w, \text{bottom}} = - \frac{2b}{\pi} \sum_{n=1}^{\infty} \frac{1}{\nu_{\omega} n \cos \alpha d} \left[ \left( \frac{n \pi}{b} \right)^2 \frac{k_o M}{\eta_o} \frac{H_n}{n} \Delta \varepsilon_r \right. \]
\[ \left. - K_n \left( r_1 \beta \tan \beta L W_n + \frac{(n \pi)}{b} \Delta \varepsilon_r \right) \right] \quad . \tag{D.30} \]

The total wall current is the sum of the expressions given in (D.27), (D.29) and (D.30). The result given in equations (103)-(106) follows when \( H_n \) and \( K_n \) from (76) and (D.12)-(D.14) are considered. For the reasons discussed earlier, the total longitudinal wall current is real.
XII. APPENDIX E:

MITTRA AND ITOH'S TECHNIQUE

A. SUMMARY OF RESULTS

Throughout this Appendix, Mittra and Itoh's [45] notation will be used. Their geometry is shown in Figure E.1. As in this work, Mittra and Itoh expressed the fields in terms of a pair of scalar potentials satisfying the Helmholtz equation and the appropriate boundary conditions on the waveguide walls and in the plane of symmetry. Their final result was an infinite set of homogeneous linear equations in terms of the unknown propagation constant. In order to solve this system, the determinant was set equal to zero.

The resulting equations are

\[
0 = \sum_{m=1}^{\infty} \left( k_0^\delta_{p m} - a_D^m - M_0^{K_0} \right) \bar{A}_m^{(e)}
\]

\[
- \sum_{n=1}^{\infty} \left( b_D^n + N_0 K_0 \right) \bar{A}_n^{(h)} , \quad p=1, 2, \ldots
\]  \quad (E.1)

\[
0 = \sum_{m=1}^{\infty} \left( -c_D^m - X_0 K_0 \right) \bar{A}_m^{(e)}
\]

\[
+ \sum_{n=1}^{\infty} \left( k_0^\delta_{q n} - d_D^n - Y_0 K_0 \right) \bar{A}_n^{(h)} , \quad q=1, 2, \ldots
\]  \quad (E.2)
Figure E.1. Microstrip geometry used by Mittra and Itoh.
Relationships between the parameters of the physical structure of Figure E.1 and the variables in equations (E.1) and (E.2) are summarized in the following paragraphs. \( \delta_{pm} \) is the Kronecker delta function and

\[
\frac{A}{k_n} = \frac{(n - \frac{1}{2}) \pi}{L} \quad (E.3)
\]

\[
\frac{-A_n^{(e)}}{A_n^{(h)}} = A_n^{(e)} \sinh \alpha_n^{(1)} d \quad (E.4)
\]

\[
\frac{-A_n^{(h)}}{A_n^{(h)}} = A_n^{(h)} \frac{\beta \alpha_n^{(1)}}{k_n} \sinh \alpha_n^{(1)} d \quad (E.5)
\]

\( A_n^{(e)} \) and \( A_n^{(h)} \) are two of the four unknown field potential coefficients, and

\[
\alpha_n^{(1)} = \sqrt{k_n^2 + \beta^2 - \epsilon k_o^2} \quad (E.6)
\]

\[
\alpha_n^{(2)} = \sqrt{k_n^2 + \beta^2 - k_o^2} \quad (E.7)
\]

\[
k_o = \omega \sqrt{\mu \epsilon_o} \quad (E.8)
\]

The fields were assumed to have z-dependence of \( e^{-j\beta z} \) and, as in our work, when \( \alpha_n^{(1)} \) and \( \alpha_n^{(2)} \) are imaginary, the hyperbolic functions must be replaced by appropriate trigonometric functions.

\( a_m, b_m, c_m, \) and \( d_m \) are found by solving equations (E.9) through (E.12).
\[ a_m = \hat{A}_m \left[ 1 - \frac{P_m (\beta) W(\beta) - T(\beta) Q_m (\beta)}{P(\beta) W(\beta) - T(\beta) Q(\beta)} \right] \quad (E.9) \]

\[ b_m = \hat{A}_m \frac{T_m (\beta) W(\beta) - T(\beta) W(\beta)}{P(\beta) W(\beta) - T(\beta) Q(\beta)} \quad (E.10) \]

\[ c_m = \hat{A}_m \frac{P(\beta) Q_m (\beta) - P_m (\beta) Q(\beta)}{P(\beta) W(\beta) - T(\beta) Q(\beta)} \quad (E.11) \]

\[ d_m = \hat{A}_m \left[ 1 - \frac{P(\beta) W(\beta) - T_m (\beta) Q(\beta)}{P(\beta) W(\beta) - T(\beta) Q(\beta)} \right] \quad (E.12) \]

where

\[ P_m (\beta) = \epsilon_r \frac{\alpha_m (1)}{\hat{A}_m} \coth \alpha_m (1) d + \frac{\epsilon_r - \beta^2}{1 - \beta^2} \frac{\alpha_m (2)}{\hat{A}_m} \coth \alpha_m (2) (h-d) \]

\[ + \beta^2 \frac{\hat{A}_m}{\alpha_m (2)} \frac{1 - \epsilon_r}{1 - \beta^2} \coth \alpha_m (2) (h-d) \quad (E.13) \]

\[ T_m (\beta) = \beta^2 \left[ \frac{\hat{A}_m}{\alpha_m (1)} \coth \alpha_m (1) d + \frac{\hat{A}_m}{\alpha_m (2)} \coth \alpha_m (2) (h-d) \right] \quad (E.14) \]

\[ Q_m (\beta) = \frac{\hat{A}_m}{\alpha_m (2)} \frac{1 - \epsilon_r}{1 - \beta^2} \coth \alpha_m (2) (h-d) \quad (E.15) \]

\[ W_m (\beta) = \frac{\epsilon_r - \beta^2}{1 - \beta^2} \frac{\hat{A}_m}{\alpha_m (1)} \coth \alpha_m (1) d + \frac{\hat{A}_m}{\alpha_m (2)} \coth \alpha_m (2) (h-d) \quad . \]

\[ (E.16) \]
\( \beta = \frac{\beta}{k_0} \) is the normalized propagation constant (\( M \) in this work), and

\[
P(\beta) = 2 \varepsilon_r \tag{E.17}
\]

\[
T(\beta) = 2 \beta^2 \tag{E.18}
\]

\[
Q(\beta) = \frac{1 - \varepsilon_r}{1 - \beta^2} \tag{E.19}
\]

\[
W(\beta) = \frac{\varepsilon_r - \beta^2}{1 - \beta^2} + 1 \tag{E.20}
\]

\( K_n \) is given by

\[
K_n = \frac{2}{L} \int_0^L \cos \frac{\hat{k}_1 x}{n} \sin \frac{\hat{k}_n x}{n} dx \left\{ 1 - \left[ \frac{\cos \frac{n x}{L}}{\alpha_1} \right]^2 \right\} \tag{E.21}
\]

where

\[
\alpha_1 = \frac{1}{2} (\cos \frac{n L}{L} - 1) \tag{E.22}
\]

\[
\alpha_2 = \frac{1}{2} (\cos \frac{n L}{L} + 1) \tag{E.23}
\]

\( D_{nm} \) is given by

\[
D_{nm} = \frac{2}{L} \int_0^L \cos \frac{n x}{2L} \left[ \sum_{m=1}^{\infty} \frac{P_m \sin q \theta - P_m \cos \theta}{mq \sin \theta} \right] \sin \frac{k_n x}{n} dx \tag{E.24}
\]
where $\theta$ is related to $x$ by

$$\cos \frac{\pi x}{L} = \alpha_1 + \alpha_2 \cos \theta, \ t \leq x \leq L, \ 0 \leq \theta \leq \pi . \quad (E.25)$$

$P_{mq}$ is defined by

$$\cos k_m x = \sum_{q=0}^{\infty} P_{mq} \cos q \theta . \quad (E.26)$$

The terms $M_m$ and $N_m$ are

$$M_m = - \frac{a_m}{I_h} \sum_{q=0}^{m-1} P_{mq} I_q \quad (E.27)$$

$$N_m = - \frac{b_m}{I_h} \sum_{q=0}^{m-1} P_{mq} I_q \quad (E.28)$$

where

$$I_h = \int_0^{\pi} \frac{d\theta}{1 - \alpha_1 - \alpha_2 \cos \theta}$$

and

$$I_q = \begin{cases} 
- \int_0^{\pi} \frac{\cos \theta \ d\theta}{0 1 - \alpha_1 - \alpha_2 \cos \theta} , & q = 0 \\
\int_0^{\pi} \frac{\sin q \theta \sin \theta \ d\theta}{0 1 - \alpha_1 - \alpha_2 \cos \theta} , & q = 1, 2, \ldots, (m-1).
\end{cases} \quad (E.29)$$
$X_n$ and $Y_n$ are found from

$$X_n = \frac{S_m - M Q I_n - (Q a_m - W c_m)}{S - W I_n} \quad (E.30)$$

$$Y_n = \frac{S_m' - N Q I_n - (Q b_m - W d_m)}{S - W I_n} \quad (E.31)$$

where

$$S = \sum_{n=1}^{\infty} \frac{\sin k\alpha t}{k_n} (W - W_n) K_n \quad (E.32)$$

$$S_m = \sum_{n=1}^{\infty} \frac{\sin k\alpha t}{k_n} \left[D_{nm} [(Q - Q_n) a_m - (W - W_n) c_m] + K_n (Q - Q_n) M_m \right] \quad (E.33)$$

$$S_m' = \sum_{n=1}^{\infty} \frac{\sin k\alpha t}{k_n} \left[D_{nm} [(Q - Q_n) b_m - (W - W_n) d_m] + K_n (Q - Q_n) N_m \right] \quad (E.34)$$

$$I_g = \frac{\alpha_2 L}{\sqrt{2} \pi^2} \int_0^{\frac{\pi}{2}} \ln \left[\frac{1 - \alpha_1 - \alpha_2 \cos \theta'}{1 - \alpha_1 - \alpha_2 \cos \theta'} \right] \frac{d \theta'}{\sqrt{1 - \alpha_1 - \alpha_2 \cos \theta'}} \quad (E.35)$$
\[ E_m \text{ is} \]
\[
E_m = \frac{\alpha_2 L}{\sqrt{2\pi^2}} \left( \sum_{q=1}^{m-1} P_{mq} J_q + P_{mo} J_0 \right) \tag{E.36}
\]

where
\[
J_0 = -\int_{0}^{\pi} \ln\left[ \frac{\sqrt{1-\alpha_1 \alpha_2 \cos \theta'} + \sqrt{1-\alpha_1 \alpha_2}}{\sqrt{1-\alpha_1 \alpha_2 \cos \theta'} - \sqrt{1-\alpha_1 \alpha_2}} \right] \frac{\cos \theta'}{\sqrt{1-\alpha_1 \alpha_2 \cos \theta'}} \, d\theta',
\]
\[
J_q = \int_{0}^{\pi} \ln\left[ \frac{\sqrt{1-\alpha_1 \alpha_2 \cos \theta'} + \sqrt{1-\alpha_1 \alpha_2}}{\sqrt{1-\alpha_1 \alpha_2 \cos \theta'} - \sqrt{1-\alpha_1 \alpha_2}} \right] \frac{\sin q \theta' \sin \theta' \, d\theta'}{\sqrt{1-\alpha_1 \alpha_2 \cos \theta'}}.
\tag{E.37}
\]

B. Computer Implementation

Mitra and Itoh [45] noted that good results were obtainable for propagating modes when only a single term was retained for each of the set of equations (E.1) and (E.2). This occurs because \( a_n, b_n, c_n, \) and \( d_n \) rapidly approach zero, and the series in (E.32)-(E.34) converge quickly.

For this case, only a few of the \( K_n, D_{nm} \) and \( P_{mq} \) terms expressed in (E.21), (E.24) and (E.26) need be evaluated. This reduces the work considerably, since the first few values of these expressions are readily evaluated analytically. For example,
\[ K_1 = \alpha_2 \]
\[ K_2 = \alpha_2 (1 + 2 \alpha_1) \quad \text{(E.38)} \]
\[ K_3 = \alpha_2 (4 \alpha_1^2 + 2 \alpha_1 - 1 + 2 \alpha_2^2) \]
\[ D_{11} = 0, \quad D_{12} = \alpha_2^2 \quad \text{(E.39)} \]
\[ D_{21} = -\alpha_2^2, \quad D_{22} = 2 \alpha_2^2 \]
\[ P_{10} = 1 \]
\[ P_{20} = 2 \alpha_1 - 1, \quad P_{21} = 2 \alpha_2 \quad \text{(E.40)} \]
\[ P_{30} = 4 \alpha_1^2 - 2 \alpha_1 - 1 + 2 \alpha_2^2, \quad P_{31} = 8 \alpha_1 \alpha_2 - 2 \alpha_2, \quad P_{32} = 2 \alpha_2^2 \]

Since we would like to use this technique to check the accuracy of our model when evanescent as well as propagating modes are considered, it is necessary to retain more than just two of the set of equations expressed in (E.1) and (E.2). The numerical solution becomes somewhat more difficult for this case since an analytical evaluation of \( K_n \), \( D_{nm} \), and \( P_{mq} \) becomes quite cumbersome when higher order values are needed. Therefore, numerical techniques have been developed to evaluate these expressions.
First, consider $P_{mq}$ as defined by equation (E.26). For a given value of $m$, (E.26) is

$$\cos \frac{k_m x}{\cos k_1 x} = P_m + P_{m1} \cos \theta + P_{m2} \cos 2\theta$$

$$+ \cdots + P_{m, m-1} \cos (m-1)\theta$$

(E.41)

where

$$\theta = \cos^{-1} \left( \frac{\frac{L}{\alpha_1} - \alpha_2}{\alpha_2} \right)$$

(E.42)

Equation E.41 requires that $m$ of the $P_{mq}$ terms be evaluated. These were found by substituting $m$ values of $x, t \leq x < L$, into (E.41). The resulting $m \times m$ matrix equation was solved for the $m$ values of $P_{mq}$ using standard numerical techniques.

$K_n$ and $D_{nm}$ once $P_{mq}$ is known, can be found by numerically integrating the expressions in (E.21) and (E.24). However, it was found that this approach required an excessive amount of computer time in order to attain reasonable accuracy, and an alternate technique was developed. Equation (E.24) can be expressed as

$$D_{nm} = P_{ml} B_{n1} + P_{m2} B_{n2} + \cdots + P_{n,m-1} B_{n,m-1} - P_{mo} A_n$$

(E.43)

where
\[ A_n = \frac{1}{L} \int_0^L \left[ \sin \left( \frac{(n-1) \pi x}{L} \right) + \sin \frac{n \pi x}{L} \right] \cos \theta \, dx \]

\[ = A_{n-1} - A_{n-2} + A_{n-3} - \cdots + (-1)^n A_1 \]

\[ + \frac{1}{L} \int_0^L \sin \frac{n \pi x}{L} \cos \theta \, dx \]  
(E.44)

and

\[ B_{nm} = \frac{1}{L} \int_0^L \left[ \sin \left( \frac{(n-1) \pi x}{L} \right) + \sin \frac{n \pi x}{L} \right] \sin m \theta \, dx \]

\[ = B_{n-1,m} - B_{n-2,m} + B_{n-3,m} - \cdots + (-1)^n B_{1m} \]

\[ + \frac{1}{L} \int_0^L \sin \frac{n \pi x}{L} \sin m \theta \, dx \]  
(E.45)

The integrals in (E.44) and (E.45) can be expressed as

\[ \frac{1}{L} \int_0^L \sin \frac{n \pi x}{L} \cos \theta \, dx = \frac{\alpha_2}{\pi} \int_0^\pi \frac{\sin \frac{n \pi x}{L}}{\sin \frac{\pi x}{L}} \cos \theta \, d\theta \]  
(E.46)

and

\[ \frac{1}{L} \int_0^L \sin \frac{n \pi x}{L} \sin m \theta \, dx = \frac{\alpha_2}{\pi} \int_0^{\pi} \frac{\sin \frac{n \pi x}{L}}{\sin \frac{\pi x}{L}} \sin \theta \sin m \theta \, d\theta \]  
(E.47)
Define $Z_{nq}$ such that

$$
\sin \frac{n\pi x}{L} = \sum_{q=0}^{\infty} Z_{nq} \cos q \theta.
$$

(E.48)

Then,

$$
\frac{\alpha_2}{\pi} \int_{0}^{\pi} \sin \frac{n\pi x}{L} \cos \theta \, d\theta = \begin{cases} 
0 & , \ n = 1 \\
\frac{\alpha_2}{2} Z_{n1} & , \ n \neq 1
\end{cases}
$$

(E.49)

and

$$
\frac{\alpha_2}{\pi} \int_{0}^{\pi} \sin \frac{n\pi x}{L} \sin \theta \sin m \theta \, d\theta = \begin{cases} 
\frac{\alpha_2}{4} [2Z_{n0} - Z_{n2}] & , \ m = 1 \\
\frac{\alpha_2}{4} [Z_{n,m-1} - Z_{n,m+1}] & , \ m \neq 1
\end{cases}
$$

(E.50)

$Z_{nq}$ can be evaluated in the same manner as $P_{nq}$. Then $D_{nm}$ can be found using (E.43)-(E.47).

The expression for $K_n$, equation (E.21), can be expressed as

$$
K_n = \frac{\alpha_2}{\pi} \int_{0}^{\pi} \frac{\sin (n-1)\pi x}{L} \, d\theta + \frac{\alpha_2}{\pi} \int_{0}^{\pi} \frac{\sin n\pi x}{L} \, d\theta = \alpha_2 (Z_{n-1,0} + Z_{n0})
$$

(E.51)
where \( Z_{oo} = 0 \). Then \( K_n \) can be easily found once \( Z_{nq} \) is known.

The remaining integrals were evaluated using numerical integration. The computation of \( I_h, I_g, \) and \( J_q \) is straightforward, but the expressions for \( I_g \) and \( J_o \) contain singularities at \( \theta = 0 \). As suggested by Mittra and Itoh, the range of integration was divided into two intervals, one from zero to \( \theta_p, \theta_p << 1 \), and the second from \( \theta_p \) to \( \pi \).

In the first interval \( \cos \theta \) was approximated by

\[
\cos \theta \approx 1 - \frac{\theta^2}{2}
\]  

inside the argument of the logarithm and by one otherwise. The resulting expressions were integrated analytically. The result for \( I_g \) is

\[
I_g' = \frac{L \theta_p}{2\pi \sqrt{-\alpha_1}} \left[ 2 + \ln \left( \frac{-16\alpha_1}{\alpha_2 \theta_p} \right) \right] \]  \hspace{1cm} (E.53)

and for \( J_o \),

\[
J_o' = \frac{-\theta_p}{\sqrt{-2\alpha_1}} \left[ 2 + \ln \left( \frac{-16\alpha_1}{\alpha_2 \theta_p} \right) \right] . \]  \hspace{1cm} (E.54)

The integration over the second interval can be evaluated numerically. The final values for \( I_g \) and \( J_o \) are found by adding the results from the two intervals.
XIII. APPENDIX F: COMPUTER PROGRAM LISTINGS

This section contains listings of the computer programs that were developed to evaluate the microstrip transmission line parameters. Many variations of the main programs are possible, and the ones listed should be thought of as representative samples of the ones used. For example, several of the subprograms calculate terms that can be used to check the convergence of the various series considered. The programs, as given, do not output these parameters.

It is only necessary to change the appropriate dimension statements in order to consider more frequency points than are possible with the given programs. The same change is necessary if more terms are desired in the summation of the various series, or if more propagation constants are wanted.

The main program for the calculation of the fundamental microstrip mode parameters using the Yamashita longitudinal current density with zero transverse current is given on page 143. The main program used to evaluate higher order propagating and evanescent modes using this model is given on page 145.

When both longitudinal and transverse current densities are considered, it is necessary to solve for the propagation constants and the characteristic impedance in two steps with the programs given. The main program for the propagation constant calculation is given on page 147, and the main program is listed on page 150 for the characteristic impedance. Only minor modifications to the main
programs and the appropriate subprograms are necessary to combine these two steps.

The evaluation of the microstrip parameters using Mittra and Itoh's [45] technique was done in three steps. The first step was to evaluate the various frequency independent terms. The main program is listed on page 152. Subroutine DRSGEA used in this procedure is a double precision real version of subroutine SGEA developed by Swift [51].

The second step was to examine the boundary condition determinant, separate the zeros from the infinities, and establish crude bounds on the zeros. Next, the propagation constants were evaluated to the desired accuracy. The main programs for these operations are given on pages 154 and 158.

The different subprograms are listed in the following order:

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THIS PROGRAM CALCULATES THE NORMALIZED PROPAGATION CONSTANT, EFFECTIVE DIELECTRIC CONSTANT, PHASE VELOCITY, CHARACTERISTIC IMPEDANCE, AND QUASI-STATIC CHARACTERISTIC IMPEDANCE AS A FUNCTION OF FREQUENCY FOR MICROWAVE. YAMASHITA'S LONGITUDINAL CURRENT AND ZERO TRANSVERSE CURRENT ARE ASSUMED.

DESCRIPTION OF INPUT PARAMETERS
A, B, D, W, ER - MICROSTRIP AND WAVEGUIDE PARAMETERS
MAXTMS - NUMBER OF TERMS SUMMED IN BOUNDARY CONDITION SERIES
INF - NUMBER OF TERMS IN LIMIT SERIES
FDELTA - DIFFERENCE BETWEEN FREQUENCY POINTS
IFPTS - NUMBER OF FREQUENCY POINTS

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
DTMY, DZME, DSM, DZO, DRTMI

IMPLICIT REAL*8(A-H, O-Z)
EXTERNAL DSM
COMMON A, B, D, W, ER, F, EL, W2UE
COMMON/CHECK/TERM, SUM
COMMON/PCHECK/SUMP, PTM, SUMDI, DITM
READ(5, 101) A, B, D, W, ER
READ(5, 102) MAXTMS, INF
F = 1.D0
CALL DTMY(MAXTMS, INF)
W2UEN = W2UE
WRITE(6, 103)
WRITE(6, 104) A, B, D, W, ER
WRITE(6, 105) MAXTMS, INF
WRITE(6, 106)
READ(5, 107) FDELTA, IFPTS
DO 1 IF = 1, IFPTS
F = DFLOAT(IF - 1)*FDELTA
W2UE = W2UEN*F*F
CALL DZMF(EM,EREFF,VP,ZO,ZOTEM,MAXTMS,IERP)
WRITE(6,108)F,EM,EREFF,VP,ZO,ZOTEM
IF(IERP.EQ.0) GO TO 1
WRITE(6,109)IERP
1 CONTINUE
101 FORMAT(D20.8/D20.8/D20.8/D20.8/D20.8/F8.5)
102 FORMAT(I5/I5)
103 FORMAT(*///,T26,' PROPAGATION CONSTANT, EFFECTIVE DIELECTRIC CONST.
*A, PHASE VELOCITY,'/T21,' CHARACTERISTIC IMPEDANCE, AND QUASI-STATIC CHARACTERISTIC IMPEDANCE FOR MICROSTRIP:'/T21,' ZERO TRANSVERSAL CURRENT AND YAMASHITA'S LONGITUDINAL CURRENT ASSUMED.')
104 FORMAT(*0 BOX HEIGHT =',1PD12.5/' BOX WIDTH =',1PD12.5/' DIELECTRIC THICKNESS =',1PD12.5/' STRIP WIDTH =',1PD12.5/' DIELECTRIC CONSTANT')
105 FORMAT('0 MAXTMS =',1P012.5/ ' DIELECTRIC THICKNESS =',1P012.5/ ' STRIP WIDTH =',1P012.5/ ' DIELECTRIC CONSTANT')
106 FORMAT('0 MAXTMS =',1P012.5/ ' BOX WIDTH =',1P012.5/ ' DIELECTRIC THICKNESS =',1P012.5/ ' STRIP WIDTH =',1P012.5/ ' DIELECTRIC CONSTANT')
107 FORMAT(D20.10/I3)
108 FORMAT(D20.10/I3)
109 FORMAT('0 ROOT PROBLEMS: IERP =',1P012.5/ ' STOP
END
C
C MAIN
C
C THIS PROGRAM CALCULATES NORMALIZED PROPAGATION CONSTANTS
C FOR BOTH PROPAGATING AND EVANESCENT MODES ON MICROSTRIP.
C YAMASHITA'S LONGITUDINAL CURRENT AND ZERO TRANSVERSE CURRENT
C ARE ASSUMED.
C
C DESCRIPTION OF INPUT PARAMETERS
C A,B,D,WIDTH,ER - MICROSTRIP AND WAVEGUIDE PARAMETERS
C EFF - FREQUENCY
C MAXRSM - NUMBER OF TERMS SUMMED IN BOUNDARY
C CONDITION SERIES FOR FINAL ROOT SEARCH
C (500 MAXIMUM)
C MINRSM - NUMBER OF TERMS SUMMED IN BOUNDARY
C CONDITION SERIES FOR INITIAL ROOT SEARCH
C (500 MAXIMUM)
C KTOT - MAXIMUM NUMBER OF MODES DESIRED (50 MAXIMUM)
C ENDFND - MAXIMUM MAGNITUDE OF EVANESCENT MODE
C PROPAGATION CONSTANTS
C STEP - SEARCH INTERVAL FOR HIGHER ORDER AND
C EVANESCENT MODES
C
C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C DTERMS,DRTFD,DRTMI,DSUM
C
C IMPLICIT REAL*8(A-H,O-Z)
C EXTERNAL DSUM
C DIMENSION RT(50),IER2(50),SUMO(50),AINF(50),SJMINF(50),SUMCK(50),
C *TERMCK(50)
C COMMON A,B,D WIDTH,ER,EFF,EL,PI,W2UE
C COMMON /DRT/RT,SUMO,AINF,SUMINF,TERMCK,,JJ,IER2
C READ(5,101)A,B,D,WIDTH,EFF,ER
C READ(5,102)MAXRSM,MINRSM,KTOT,ENDFND,STEP
C WRITE(6,103)A,B,D,WIDTH,ER
C WRITE(6,104)EFF
CALL DTERMS(MAXRSM)

DO 1 I=1,KTOT
    RT(I) = 0.00
    SUMCK(I) = 0.00
    TERMCK(I) = 0.00
    IER2(I) = 3
    AINF(I) = 0.00
    SUMINF(I) = 0.00
    SUMO(I) = 0.00
1 CONTINUE

CALL DRTFDR(KTOT,ENDFND,STEP,MAXRSM,MINRSM,KRE,IER3)

WRITE(6,105)
WRITE(6,106)(AINF(J),SUMINF(J),J=1, JJ)
WRITE(6,107)
WRITE(6,108)(RT(J),SUMO(J),IER2(J),SUMCK(J),TERMCK(J),J=1,KTJ)

DO 2 J=1,KTOT
    RTMAG = OABS(RT(J))
2 CONTINUE

101 FORMAT(D20.8/D20.8/D20.8/D20.8/D20.8/F8.5)
103 FORMAT(' BOX HEIGHT =',F8.5/' BOX WIDTH =',F8.5/' DIELECTRIC THICKNESS =',F9.6/' STRIP WIDTH =',F9.6/' DIELECTRIC CONSTANT =',F7.3)
104 FORMAT(/,' FREQUENCY EQUALS ',1PD9.2,/) 
105 FORMAT('0',T8,' M-LEFT',T22,' INFINITY(LEFT)' ,' )
106 FORMAT(OPF14.6,1PD20.6)
107 FORMAT('1',T14,' ROOT',T47,' SUM',T72,' IER',T93,' SUM CHECK',T113)
*  , ' TERM CHECK',/)
108 FORMAT('0',T10,OPF10.6,T40,1PD15.6,T70,I5,T90,1PD15.6,T110,1PD15.6)
109 FORMAT(/,' THE NUMBER OF PROPAGATING MODES IS',I3,' IER3 EQUALS',I3)
* S ',I3)
110 FORMAT(F20.8)
STOP
END
This program calculates the normalized propagation constant, effective dielectric constant, phase velocity, and ratio of transverse current to longitudinal current (Jyo/Jzo) as a function of frequency for microstrip with non-zero transverse current.

As written, Yamashita's longitudinal current is assumed. The changes necessary for Maxwell's longitudinal current are noted.

Description of input parameters:
- A, B, D, W, ER - microstrip and waveguide parameters
- Q - fraction of the distance from the center to the edge of the strip at which the boundary conditions are to be matched
- MAXTMS - number of terms included in the boundary condition series
- INF - number of terms included in the limit series
- FDELTA - frequency interval
- IFPTS - number of frequency points

Subroutines and function subprograms required:
- Yamashita current: DTERMT, DTSUM, DPFM, DRTMI
- Maxwell current: DTRMJT, DTSUM, DPFM, DRTMI, DBESJO

IMPLICIT REAL*8(A-H,O-Z)
EXTERNAL DTSUM
COMMON A,B,D,W,ER,F,EL,Q,W2UE,R,CK,MAXSUM
COMMON/CHECK/TM1,TM2,TM3,TM4,S1,S2,S3,S4,RCK
READ(5,101)A,B,D,W,ER
READ(5,102)Q,MXTMS,INF
READ(5,112)FDELTA,IFPTS
WRITE(6,103)
WRITE(6,104)A,B,D,W,ER
WRITE(6,105)Q,MAXTMS,INF
WRITE(6,106)
F = 1.DO
CALL DTERMT(MAXTMS,INF)
FOR MAXWELL'S CURRENT, REPLACE THE ABOVE STATEMENT WITH
CALL DTRMJT(MAXTMS,INF)
W2UEN = W2UE
DELTA = FDELTA*1.D9
DO 2 J=1,IFPTS
F = DFLOAT(J-1)*DELTA
W2UE = W2UEN*F*F
CALL DPFM(EM,EREFF,VP,CRATIO,MAXTMS,IERP)
IF(IERP.EQ.0) GO TO 1
WRITE(6,110)
WRITE(6,111)IERP
1 WRITE(6,107)F,EM,EREFF,VP,CRATIO,R
2 CONTINUE
101 FORMAT(D20.8/D20.8/D20.8/D20.8/F8.5)
102 FORMAT(F8.5/I5/I5)
103 FORMAT(/**/,T26,' PROPAGATION CONSTANT, EFFECTIVE DIELECTRIC CONSTANT','
*ANT, PHASE VELOCITY, NORMALIZED','T21,' PROPAGATION CONSTANT, AND CURRENT RATIO FOR INFINITE MICROSTRIP: NON-ZERO TRANSVERSE','T21,' CURRENT AND YAMASHITA"S LONGITUDINAL CURRENT ARE ASSUMED.',/**/)
104 FORMAT("OBOX HEIGHT =",1PD12.5/" BOX WIDTH =",1PD12.5/" DIELECTRIC CONSTANT"
* THICKNESS =",1PD12.5/" STRIP WIDTH =",1PD12.5/" DIELECTRIC CONSTANT"
*NT =",0PF6.3) MAIN 069
105 FORMAT("Q =",F6.3,", MAXTMS =",I3,", INF =",I5)
106 FORMAT(/**/,T3,"FREQUENCY",T17,"NCRM. PROP. CONST.",T40,"EFF.DIE. CON"
107 FORMAT("O",1PD10.2,T15,0PF13.3,T36,0PF13.5,T57,1PD15.6,T78,
* 1PD15.6,T100,1PD15.6) MAIN 074
110 FORMAT(//'O COMPUTATION TERMINATED BECAUSE OF DIFFICULTY IN FINDING MAIN 075
   *G THE ROOT' )
   MAIN 076
111 FORMAT('0  IERP = ',I5)
   MAIN 077
112 FORMAT(F8.5/I5)
   STOP
   MAIN 078
   END
   MAIN 079
   MAIN 080
THIS PROGRAM CALCULATES THE NORMALIZED PROPAGATION
CONSTANT, PHASE VELOCITY, CHARACTERISTIC IMPEDANCE,
AND QUASI-STATIC CHARACTERISTIC IMPEDANCE AS A FUNCTION
OF FREQUENCY FOR MICROSTRIP WITH NON-ZERO TRANSVERSE
CURRENT.

DESCRIPTION OF INPUT PARAMETERS

A, B, D, W, ER - MICROSTRIP AND WAVEGUIDE PARAMETERS
MAXTMS - NUMBER OF TERMS INCLUDED IN THE BOUNDARY
          CONDITION SERIES
INF - NUMBER OF TERMS INCLUDED IN THE LIMIT SERIES
F - FREQUENCY
EM2 - EFFECTIVE DIELECTRIC CONSTANT
R - CONSTANT PROPORTIONAL TO THE RATIO OF
    TRANSVERSE CURRENT TO LONGITUDINAL CURRENT

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED

YAMASHITA CURRENT: DTMPT, DTZO
MAXWELL CURRENT: DTMPTJ, DTZO, DBESJO

IMPLICIT REAL*8(A-H, O-Z)
COMMON A, B, D, W, ER, F, EL, W2UE
COMMON/PCHECK/SUMP, PTM, SUMDI, DITM
READ(5, 101) A, B, D, W, ER
READ(5, 102) MAXTMS, INF, NFPTS
WRITE(6, 103)
WRITE(6, 104) A, B, D, W, ER
WRITE(6, 105) MAXTMS, INF
WRITE(6, 106)
F = 1.0D0
CALL DTMPT(MAXTMS, INF)

FOR MAXWELL'S CURRENT, REPLACE THE ABOVE STATEMENT WITH
CALL DTMPTJ(MAXTMS,INF)

W2UE = W2UE
IFF = 1
DO 1 I=1,NFPTS
READ(5,150)F,EM2,R
IFF(F.GT.0)IFF=2
W2UE = W2UE*F*F
CALL DTZ0(EM2,R,Z0,MAXTMS)
EM = DSQRT(EM2)
GO TO (2,3),IFF
2 TZO2 = ZO*EM
3 ZOTEM = TZO2/EM
VP = 2.997925D8/EM
1 WRITE(6,175)F,EM,EM2,VP,Z0,ZOTEM
101 FORMAT(D20.8/D20.8/D20.8/D20.8/F8.5)
102 FORMAT(15/15/15)
103 FORMAT(/,,T26,' PROPAGATION CONSTANT, EFFECTIVE DIELECTRIC CONSTRAINT, EFFECTIVE PHASE VELOCITY, ^/T21,' Characteristic Impedance, and Quasi-Static Characteristic Impedance for Microstrip:^/T21,' Non-Zero Transversal Current and Yamashita's Longitudinal Current Assumed.')
104 FORMAT(' BOX HEIGHT =',1PD12.5,' BOX WIDTH =',1PD12.5,' DIELECTRIC THICKNESS =',1PD12.5,' STRIP WIDTH =',1PD12.5,' DIELECTRIC CONSTANT =',0PF6.3)
105 FORMAT(' MAXTMS =',14,' INF =',15)
106 FORMAT(/,,T3,' FREQUENCY',T17,' NORM. PROP. CONSTANTS',T40,' EFF. DIE. CONST. =',T60,' PHASE VELOCITY',T80,' =',T108,' ZO',T108,' ZOTEM')
150 FORMAT(D20.8,F20.10,D20.8)
175 FORMAT('0',1PD10.2,T15,0PF13.3,T36,0PF13.5,T57,1PD15.6,T78,
  * 1PD15.6,T100,1PD15.6)
STOP
END
THIS PROGRAM CALCULATES THE TERMS WHICH DEPEND ONLY ON THE
RATIO OF STRIP WIDTH TO WAVEGUIDE WIDTH IN MITRA AND ITOH'S
SOLUTION FOR MICROSTRIP.

DESCRIPTION OF INPUT PARAMETERS
T,EL - MICROSTRIP AND WAVEGUIDE PARAMETERS
THETAP - LOWER LIMIT ON SINGULAR INTEGRALS
NI1 - INTERVALS FOR 0-PI INTEGRATION (2500 MAXIMUM)
NI2 - INTERVALS FOR 0-PI/8 INTEGRATION (5000 MAXIMUM)
MQ - MAXIMUM NUMBER OF TERMS IN THE SERIES
(2MQ X 2MQ MATRIX)
IPUNCH - IPUNCH=0, NO PUNCHED OUTPUT
IPUNCH=1, PUNCHED OUTPUT

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
DSNQ,DRSGEA,DIEV,DPKD

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION VIQ(50),VJ(50),AK(50),P(50,50),D(50,50)
COMMON EL,T,PI,ALPHA1,ALPHA2
COMMON /PKD/C1,C2 /EQUIVI/AK,D,P
READ(5,101)T,EL,THETAP,NI1,NI2,MQ,IPUNCH
RATIO = T/EL
WRITE(6,102)T,EL,RATIO
WRITE(6,103)NI1,NI2,MQ,IPUNCH,THETAP
PI = 3.141592653D0
C1 = PI/EL
C2 = C1*5.D-1
ALPHA1 = 5.D-1*DCOS(C1*T) - 5.D-1
ALPHA2 = 1.D0 + ALPHA1
MQM1 = MQ - 1
CALL DIEV(VIH,VIQ,VJ,VIG,THETAP,NI1,NI2,MQM1)
WRITE(6,104)VIH,VIG
WRITE(6,105)

MAIN 001
MAIN 002
MAIN 003
MAIN 004
MAIN 005
MAIN 006
MAIN 007
MAIN 008
MAIN 009
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MAIN 028
MAIN 029
MAIN 030
MAIN 031
MAIN 032
MAIN 033
MAIN 034
MAIN 035
MAIN 036
MAIN 037
WRITE(6,106)(VJ(I),I=1,MQ)          MAIN 038
WRITE(6,107)                         MAIN 039
WRITE(6,106)(VIQ(I),I=1,MQ)         MAIN 040
CALL DPKD(MQ)                        MAIN 041
WRITE(6,108)                         MAIN 042
DO 1 I=1,MQ                          MAIN 043
WRITE(6,106)(P(I,J),J=1,I)           MAIN 044
1 WRITE(6,112)                        MAIN 045
WRITE(6,109)                         MAIN 046
DO 2 I=1,MQ                          MAIN 047
WRITE(6,106)(D(I,J),J=1,MQ)          MAIN 048
2 WRITE(6,112)                        MAIN 049
WRITE(6,110)                         MAIN 050
WRITE(6,106)(AK(I),I=1,MQ)          MAIN 051
IF(IPUNCH.NE.1) GO TO 3              MAIN 052
WRITE(7,111)((P(I,J),J=1,I),I=1,MQ),((D(I,J),J=1,MQ),I=1,MQ), MAIN 053
  * (AK(I),I=1,MQ),VIH,VIQ,(VJ(I),I=1,MQ),(VIQ(I),I=1,MQ)       MAIN 054
101 FORMAT(D20.8/D20.8/D20.8/I4/I4/I4) MAIN 055
102 FORMAT('IT =','1PD15.8'/'EL = ','1PD15.8'/'RATIO = ','1PD15.8') MAIN 056
103 FORMAT('ON1 =','I5'/'NI2 =','I5'/'MQ =','I5'/'IPUNCH =','I5'/'THETA' MAIN 057
  * =','1PD15.8)                                               MAIN 058
104 FORMAT('IVIH =','1PD20.8'/'OVIG =','1PD20.8)                MAIN 059
105 FORMAT('OVIJ(I):')                MAIN 060
106 FORMAT('OVIQ(I):')                MAIN 061
107 FORMAT('OVIQ(I):')                MAIN 062
108 FORMAT('OVIQ(I):')                MAIN 063
109 FORMAT('OVIQ(I):')                MAIN 064
110 FORMAT('OAK(I):')                 MAIN 065
111 FORMAT('IP4D20.10')              MAIN 066
112 FORMAT('0')                      MAIN 067
3 STOP                              MAIN 068
END                                 MAIN 069
MAIN

THIS PROGRAM CALCULATES THE DETERMINANT OF THE COEFFICIENT MATRIX FOR THE SET OF HOMOGENEOUS EQUATIONS ARRIVED AT IN MITTRA AND ITOH'S SOLUTION FOR MICROSTRIP. THE RESULTING OUTPUT CAN BE USED TO PUT BOUNDS ON THE SOLUTIONS TO THIS SET OF EQUATIONS.

DESCRIPTION OF INPUT PARAMETERS

T, E, H

DEE, ER - MICROSTRIP AND WAVEGUIDE PARAMETERS

M1 - NUMBER OF TERMS INCLUDED IN THE BOUNDARY CONDITION SERIES (LESS THAN OR EQUAL TO MQ)

M2 - NUMBER OF TERMS INCLUDED IN S, SM, AND SM' SERIES (LESS THAN OR EQUAL TO MQ)

MQ - SIZE OF INPUT ARRAYS (10 X 10 MAXIMUM)

P, D, AK, VIH, VIG, VJ, VIQ - FREQUENCY INDEPENDENT TERMS

R - TERM DIFFERENTIATING BETWEEN PROPAGATING (R=+1.D0) AND EVANESCENT (R=-1.D0) MODES

NINTS - NUMBER OF INTERVALS TO BE SEARCHED

IST - INTEGER VARIABLE DESCRIBING THE STARTING POINT FOR THE SEARCH INTERVAL

IEND - INTEGER VARIABLE FIXING THE END POINT FOR THE SEARCH INTERVAL

INC - INTEGER VARIABLE FIXING THE LENGTH OF THE SEARCH INTERVAL

CONST - SCALE FACTOR FOR IST, IEND, AND INC

F - FREQUENCY

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED

DET2, DTMT

IMPLICIT REAL*8(A-H, O-Z)

EXTERNAL DET2
DIMENSION AKN(10), AKN2(10), VEM(10), VIQ(10), VJ(10), AK(10), D(10, 10), P(10, 10), A1(10), A2(10), A3(10, 10)
COMMON ER, ERP1, ERM1, W2UE, HMD, DEE, C1, C2, C3, C4, C5, AKN, AKN2, VEM, VIQ, MAIN
COMMON/FNORM/TMNORM, INORM
READ(5, 101) T, EL, H, DEE, ER, M1, M2, MQ
DO 50 I = 1, MQ
DO 50 J = 1, MQ
50 P(I, J) = 0.00
READ(5, 107) ((P(I, J), J = 1, I), I = 1, MQ), ((D(I, J), J = 1, MQ), I = 1, MQ),
* (AK(I), I = 1, MQ), VIH, VIG, (VJ(I), I = 1, MQ), (VIQ(I), I = 1, MQ)
WRITE(6, 114) VIH, VIG
WRITE(6, 115)
WRITE(6, 116) (VJ(I), I = 1, MQ)
WRITE(6, 117)
WRITE(6, 118)
DO 111 I = 1, MQ
WRITE(6, 116) (P(I, J), J = 1, I)
111 WRITE(6, 122)
WRITE(6, 119)
DO 202 I = 1, MQ
WRITE(6, 116) (D(I, J), J = 1, MQ)
202 WRITE(6, 122)
WRITE(6, 120)
WRITE(6, 116) (AK(I), I = 1, MQ)
WRITE(6, 405)
WRITE(6, 103)
WRITE(6, 102) H, EL, DEE, T, ER
WRITE(6, 150) M1, M2, MQ
PI = 3.141592653D0
HMD = H - DEE
ERP1 = ER + 1.00
ERM1 = ER - 1.00
C0 = PI/EL
C1 = 5.0 -1/ERP1
C2 = ER*ERP1
C3 = ERM1*C1
C4 = 2.D0*ER
C5 = ER*C4
ALPHAl = 5.D-1*DCOS(CO*T) - 5.D-1
ALPHA2 = 1.D0 + ALPHA1
C6 = ALPHA2*EL/(1.4142135623731D0*PI**2)
C7 = 5.D-1*C0
VIG = EL*VIG
VIHI = 1.D0/VIH
DO 1 I=1,M2
  VEM(I) = 0.D0
1 A1(I) = 0.D0
  DO 3 M=1,M2
    DO 2 IQ=1,M
      VEM(M) = VEM(M) + P(M,IQ)*VJ(IQ)
    2 A1(M) = A1(M) + P(M,IQ)*VIQ(IQ)
      A1(M) = -A1(M)*VIHI
    3 VEM(M) = VEM(M)*C6
      DO 4 N=1,M2
        AKN(N) = C7*DFLOAT(2*N - 1)
        AKN2(N) = AKN(N)**2
        SKKI = DSINIAKN(N)*T)/AKN(N)
        A2(N) = SKKI*AK(N)
        DO 4 M=1,M2
          A3(N,M) = SKKI*D(N,M)
          W2UEN = (2.D0*PI/2.997925D8)**2
      4 W2UE = W2UEN*F*F
      READ(5,200)R,NINTS
      ITIMES = 0
    75 CONTINUE
      ITIMES = ITIMES + 1
      READ(5,201)IST,IEND,INC,CONST,F
      W2UE = W2UE*F**F
      WRITE(6,104)
      DO 5 IB=IST,IEND,INC
        BETA2 = DFLOAT(IB)*CONST
        BETA = DSQRT(BETA2*R)
        INORM = 1
      MAIN 075
      MAIN 076
      MAIN 077
      MAIN 078
      MAIN 079
      MAIN 080
      MAIN 081
      MAIN 082
      MAIN 083
      MAIN 084
      MAIN 085
      MAIN 086
      MAIN 087
      MAIN 088
      MAIN 089
      MAIN 090
      MAIN 091
      MAIN 092
      MAIN 093
      MAIN 094
      MAIN 095
      MAIN 096
      MAIN 097
      MAIN 098
      MAIN 099
      MAIN 100
      MAIN 101
      MAIN 102
      MAIN 103
      MAIN 104
      MAIN 105
      MAIN 106
      MAIN 107
      MAIN 108
      MAIN 109
      MAIN 110
      MAIN 111
DTB = DET2(BETA)
WRITE(6,106)F,BETA,BETA2,DTB
CONTINUE
IF(ITimes.EQ.NINTS) GO TO 300
WRITE(6,405)
GO TO 75
102 FORMAT('OBOX HEIGHT =',1PD12.5/' BOX HALF WIDTH =',1PD12.5/
* ' DIELECTRIC THICKNESS =',1PD12.5/' STRIP HALF WIDTH =',1PD12.5/
* ' DIELECTRIC CONSTANT =',0PF8.3,/
103 FORMAT('///',T26,' CRUDE ROOTFINDER FOR MITTRA AND ITOH''S TECHNIQUE
*E."
104 FORMAT('0  F',T20,' BETA',T40,' BETA2',T60,' DTB')
106 FORMAT(1PD10.2,1P3D20.8)
107 FORMAT(1P4D20.10)
114 FORMAT('1VH =',1PD20.8/'0VIG =',1PD20.8)
115 FORMAT('0VJ(I):')
116 FORMAT(1P6D20.8)
117 FORMAT('0VIQ(I):')
118 FORMAT('1P(I,J):')
119 FORMAT('1D(I,J):')
120 FORMAT('///','0AK(I):')
122 FORMAT('0')
150 FORMAT('0M1,M2,MQ: ',3I4)
200 FORMAT(F8.5/I3)
201 FORMAT(I10/I10/I10/D20.8/D20.8)
405 FORMAT('1')
300 STOP
END
MAIN 001

This program calculates the propagation constant for microstrip using Mittra and Itoh's technique. Upper and lower bounds must be placed on each propagation constant desired.

Description of Input Parameters

T, E, H, T, EL, H

DEE, ER - Microstrip and waveguide parameters

M1 - Number of terms included in the boundary condition series (less than or equal to MQ)

M2 - Number of terms included in s, SM, and SM' series (less than or equal to MQ)

MQ - Size of input arrays (10 X 10 maximum)

P, D, AK, VIH, VIG

VJ, VIQ - Frequency independent terms

ACC - Required accuracy of the root

R - Term differentiating between propagating (R=+1.0D0) and evanescent (R=-1.0D0) modes

IFPTS - Number of frequency points

F - Frequency

BETAL - Left bound on the normalized propagation constant

BETAR - Right bound on the normalized propagation constant

Subroutines and Function Subprograms Required

DET2, DTMT, DRTMI

Implicit Real*8(A-H, O-Z)

Real*4 ACC

External DET2

Dimension AKN(10), AKN2(10), VEM(10), VIQ(10), VJ(10), AK(10), D(10,10), MAIN 037
* P(10,10), A1(10), A2(10), A3(10,10)
COMMON ER, ERP1, ERM1, W2UE, HMD, DEE, C1, C2, C3, C4, C5, AKN, AKN2, VEM, VIG,
* A1, A2, A3, AK, D, R, M1, M2
COMMON/FNORM/TMNORM/INORM
READ(5,101) T, EL, H, DEE, ER, M1, M2, MQ
DO 50 I=1, MQ
DO 50 J=1, MQ
50 P(I,J) = 0.0
READ(5,107) (P(I,J), J=1, I), I=1, MQ), (D(I,J), J=1, MQ), I=1, MQ),
* (AK(I), I=1, MQ), VIH, VIG, (VJ(I), I=1, MQ), (VIQ(I), I=1, MQ)
WRITE(6,114) VIH, VIG
WRITE(6,115)
WRITE(6,116) (VJ(I), I=1, MQ)
WRITE(6,117)
WRITE(6,116) (VIQ(I), I=1, MQ)
WRITE(6,118)
DO 111 I=1, MQ
WRITE(6,116) P(I,J), J=1, I)
111 WRITE(6,122)
WRITE(6,119)
DO 202 I=1, MQ
WRITE(6,116) (D(I,J), J=1, MQ)
202 WRITE(6,122)
WRITE(6,120)
WRITE(6,116) (AK(I), I=1, MQ)
WRITE(6,105)
WRITE(6,103)
WRITE(6,102) H, EL, DEE, T, ER
WRITE(6,150) M1, M2, MQ
PI = 3.1415926530
HMD = H - DEE
ERPI = ER + 1.0
ERM1 = ER - 1.0
C0 = PI/EL
C1 = 5.0/ERPI
C2 = ER*ERPI
C3 = ERM1*C1
C4 = 2.00*ER
C5 = ER*C4
ALPHAI = 5.0-1*DCOS(C0*T) - 5.0-1
ALPHA2 = 1.00 + ALPHAI
C6 = ALPHA2*EL/(1.4142135623731D0*PI**2)
C7 = 5.0-1*CO
VIG = EL*VIG
VIHI = 1.00/VIH
DO 1 I=1,M2
  VEM(I) = 0.00
1   ALI(I) = 0.00
   DO 3 M=1,M2
   DO 2 IQ=1,M
     VEM(M) = VEM(M) + P(M,IQ)*VJ(IQ)
2   ALI(M) = ALI(N) + P(M,IQ)*VIQ(IQ)
3   ALI(M) = -ALI(M)*VIHI
   DO 4 N=1,M2
    AKN(N) = C7*DFLOAT(2*N - 1)
4    AKN2(N) = AKN(N)**2
    SKKI = DSIN(AKN(N)*T)/AKN(N)
    A2(N) = SKKI*AK(N)
    DO 4 M=1,M2
     A3(N,M) = SKKI*D(N,M)
4   READ(5,408)ACC,R
   WRITE(6,409)ACC
   WRITE(6,406)
   W2UEN = (2.00*PI/2.997925D8)**2
   DO 5 J=1,IFPTS
5   READ(5,407)F,BETAL,BETAR
   W2UE = W2UEN*F*F
   INORM = 1
   CALL DRTMI(BETA,VAL,DET2,BETAL,BETAR,ACC,35,IER)
   EREFF = BETA*8ETA
   VP = 2.997925D8/BETA
   WRITE(6,170)F,BETA,EREFF,VP,IER.
CONTINUE


FORMAT(* OBOX HEIGHT =',1PD12.5/' BOX HALF WIDTH =',1PD12.5/
  *' DIELECTRIC THICKNESS =',1PD12.5/' STRIP HALF WIDTH =',1PD12.5/
  *' DIELECTRIC CONSTANT =',1PD12.5)

FORMAT(/,T26,' PROPAGATION CONSTANT, EFFECTIVE DIELECTRIC CONSTM
  * ANT, AND PHASE VELOCITY FOR INFINITE MICROSTRIP: /T21,' MITTRA AND
  * ITOH'S TECHNIQUE.'/,/)

FORMAT(1P4020.10)
112 FORMAT(I2)

FORMAT('IVIH =',1PD20.8/'OVIG =',1PD20.8)

FORMAT('OVJ(I): ')

FORMAT('IP(I,J): ')

FORMAT('ID(I,J): ')

FORMAT('OAK(I): ')

FORMAT('O')

FORMAT('OM1,M2,MQ: ',3I4)

FORMAT('O',1PD10.2,T15,OPF13.3,T36,OPF13.5,T57,1PD15.6,T75,0P13)

FORMAT('I')

FORMAT(/,T3,' FREQUENCY',T17,' NORM. PROP. CONST.',T40,' EFF. DIE. CONSTM
  * ST.',T60,' PHASE VELOCITY',T75,' IER')

FORMAT(D20.8,2F20.8)

FORMAT(E20.8/F8.5)

FORMAT('OACCURACY ON THE PROPAGATION CONSTANT IS: ',1PE10.4)

STOP

END
SUBROUTINE DTMY(MAXTMS,INF)

DESCRIPTION OF PARAMETERS

MAXTMS - NUMBER OF TERMS SUMMED IN THE BOUNDARY CONDITION SERIES

INF - NUMBER OF TERMS IN THE LIMIT SERIES

THIS SUBROUTINE CALCULATES THE TERMS NECESSARY FOR THE EVALUATION OF THE PROPAGATION CONSTANT AND CHARACTERISTIC IMPEDANCE FOR MICROSTRIP. YAMASHITA'S LONGITUDINAL CURRENT AND ZERO TRANSVERSE CURRENT ARE ASSUMED.

SUBROUTINE DTMY(MAXTMS,INF)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION TN2P(I00),TNI1(I00),TNI2(I00),TN2(I00),TN(I00),G(I00)
COMMON A,B,D,W,ER,F,EL,W2UE
COMMON/ZSUM/G
COMMON/SUMCOM/TN,SS4,CT2,CT4
COMMON/ZTMS/TN2P,TNI1,TNI2,TN2,CP,SP1,CSI,CDI,SI2,HERP1,ERMI
PI = 3.14159300
VO = 2.997925D8
ETAO = 3.767D2
EL = A - D
W2UE = (2.D0*PI*F/VO)**2
ERMI = ER - 1.DO
ERP1 = ER + 1.DO
HERP1 = 5.D-1*ERP1
ERP1I = 1.DO/ERP1
CI = 1.25D0
CSI = W*CI
C1 = PI/B
C1I = 1.DO/C1
CT1 = C1I*C1I
CT2 = C1I*5.D-1
CT4 = C1I*ERP1I
CT5 = 2.DO*CT4
CTN = CT*EN
CCTN = DCOS(CTN)
SCTN = DSIN(CTN)
TNN = ENI4*(6.DO*(1.DO - CCTN) + CTN*(-6.DO*SCTN +
* CTN*(3.DO*CCTN + 2.DO*CTN*SCTN)))
TNIIN = TNN*ENI
SPI = SP1 + TNN*TNI1N
SI2 = SI2 +TNN*SPI2*ENI3
2 SS4 = SS4 + TNI1N
SP1 = CT5*SP1
SI2 = CT1*SI2
RETURN
END
SUBROUTINE DZME(EM, EREFF, VP, ZO, ZOTEM, MAXTMS, IERP)

THIS SUBROUTINE CALCULATES THE PROPAGATION CONSTANT, EFFECTIVE DIELECTRIC CONSTANT, PHASE VELOCITY, CHARACTERISTIC IMPEDANCE, AND QUASI-STATIC CHARACTERISTIC IMPEDANCE FOR MICROWAVE. ZERO TRANSVERSE CURRENT IS ASSUMED.

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
DSM, DRTMI, OZO

DESCRIPTION OF PARAMETERS
EM - NORMALIZED PROPAGATION CONSTANT
EREFF - EFFECTIVE DIELECTRIC CONSTANT
VP - PHASE VELOCITY
ZO - CHARACTERISTIC IMPEDANCE
MAXTMS - NUMBER OF TERMS IN THE BOUNDARY CONDITION SERIES
IERP - RESULTANT ERROR PARAMETER CODED AS FOLLOWS
IERP=0 - NO ERROR
IERP=1 - NO CONVERGENCE AFTER 40 STEPS IN SUBROUTINE DRTMI
IERP=2 - DSM(EM2L)*DSM(EM2R) LESS THAN ZERO NOT SATISFIED IN SUBROUTINE DRTMI
IERP=3 - NO PROPAGATING MODE FOUND

SUBROUTINE DZME(EM, EREFF, VP, ZO, ZOTEM, MAXTMS, IERP)
IMPLICIT REAL*8(A-H,O-Z)
COMMON A, B, D, W, ER, F, EL, M2UE COMMON/SUMTMS/MAXSUM IERP = 0 L = 0 STEP = ER*1.D-1 MAXSUM = MAXTMS/5 EM2L = ER TERML = DSM(EM2L)
1 EM2R = EM2L EM2L = EM2R - STEP
IF(EM2L.LT.0.D0) GO TO 4
TERMR = TERML
TERML = DSM(EM2L)
CKTM = TERML*TERMR
IF(CKTM.GT.0.D0) GO TO 1
MAXSUM = MAXTMS
2 CALL DRTMI(EM2,VAL,DSM,EM2L,EM2R,5.E-6,40,IER)
IERP1 = IER + 1
GO TO (7,3,5),IERP1
3 IERP = 1
GO TO 9
4 IERP = 3
GO TO 9
5 L = L + 1
IERP = 2
GO TO (6,9),L
6 EXPAND = STEP*2.D-1
EM2L = EM2L - EXPAND
EM2R = EM2R + EXPAND
IF(EM2R.GT.ER)EM2R = ER
GO TO 2
7 EREFF = EM2
EM = DSQRT(EM2)
VP = 2.997925D8/EM
CALL DZ0(EM2,Z0,MAXTMS)
IF(W2UE.GT.0.D0) GO TO 8
TZ02 = Z0*EM
8 ZOTEM = TZ02/EM
9 RETURN
END
FUNCTION DSM(EM2)

THIS FUNCTION SUBPROGRAM EVALUATES THE BOUNDARY CONDITION
SERIES FOR THE FUNDAMENTAL MODE ON MICROSTRIp. ZERO
TRANSVERSE CURRENT IS ASSUMED

FUNCTION DSM(EM2)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION TN(100),G(100)
COMMON A,B,D,W,ER,F,EL,W2UE
COMMON/ZSUM/G
COMMON/SUMCOM/TN,SS4,CT2,CT4
COMMON/SUMTMS/MAXSUM
COMMON/CHECK/STERM,SUM
CS1 = W2UE*(ER - EM2)
CS2 = W2UE*(1.D0 - EM2)
CS5 = CT2 - EM2*CT4
SUM = 0.D0
DO 3 N=1,MAXSUM
EN = DFLOAT(2*N - 1)
ENI = I.DO/EN
ALPHA2 = CS1 - G(N)
IF(ALPHA2.GT.0.D0) GO TO 1
ALPHA = DSQRT(-ALPHA2)
TAD = DTANH(ALPHA*D)
ATAD = -ALPHA*TAD
GO TO 2
1 ALPHA = DSQRT(ALPHA2)
TAD = DTANH(ALPHA*D)
ATAD = ALPHA*TAD
GO TO 2
2 BETA2 = CS2 - G(N)
BETA = DSQRT(-BETA2)
TBL = DTANH(BETA*EL)
BTBL = -BETA*TBL
ACAD = ALPHA/TAD
BCBL = BETA/TBL
UN = ATAD + BTBL
VN = ATAD + ER*BTBL
WN = ACAD + BCBL
STERM = TN(N)*(VN - EM2*UN)/(VN*WN) - ENI*CS5)
SUM = SUM + STERM
DSM = SUM + CS5*SS4
RETURN
END
SUBROUTINE DZ0(EM2, ZO, MAXTMS)

THIS SUBROUTINE EVALUATES THE CHARACTERISTIC IMPEDANCE OF MICROSTRIP. ZERO TRANSVERSE CURRENT IS ASSUMED.

SUBROUTINE DZ0(EM2, ZO, MAXTMS)
IMPLICIT REAL*8(A-H, O-Z)
DIMENSION TN2P(100), TNI1(100), TNI2(100), TN2(100), G(100)
COMMON A, B, D, W, ER, F, EL, W2UE
COMMON/ZSUM/G
COMMON/ZTMS/TN2P, TNI1, TNI2, TN2, CP, SPI, CSI, CDI, SI2, HERP1, ERM1
COMMON/PCHECK/SUMP, PTM, SUMDI, DITM

EM = DSQRT(EM2)
CS1 = W2UE* (ER - EM2)
CS2 = W2UF* (1.00 - EM2)
CS5 = HERP1 - EM2
SUMP = 0.00
SUMDI = 0.00
SPI2 = -1.00
DO 5 N=1, MAXTMS
EN = DFLOAT(2*N - 1)
GER = G(N)* ERM1
SPI2 = -SPI2
ALPHA2 = CS1 - G(N)
IF(ALPHA2, GT, 0.00) GO TO 1
ALPHA = DSQRT(-ALPHA2)
AD = ALPHA*D
TAD = DTANH(AD)
ATAD = -ALPHA*TAD
CAD2 = -(1.00/TAD)**2
IF(AD, GT, 0.00) GO TO 2
SECAD = 1.00/DCOSH(AD)
GO TO 2
1 ALPHA = DSQRT(ALPHA2)
AD = ALPHA*D
TAD = DTAN(AD)
ATAD = ALPHA*TAD
CAD2 = (1.0DO/TAD)**2
SECAD = 1.0DO/DCOS(AD)
BETA2 = CS2 - G(N)
IF(BETA2.GT.0.0DO) GO TO 3
BETA = DSQRT(-BETA2)
BL = BETA*EL
TBL = DTANH(BL)
BTBL = -BETA*TBL
CBL2 = -(1.0DO/TBL)**2
IF(BL.GT.50.0DO) GO TO 4
SECBL = 1.0DO/DCOSH(BL)
GO TO 4
BETA = DSQRT(BETA2)
BL = BETA*EL
TBL = DTAN(BL)
BTBL = BETA*TBL
CBL2 = (1.0DO/TBL)**2
SECBL = 1.0DO/DCOS(BL)
ACAD = ALPHA/TAD
BCBL = BETA/TBL
ATADI = 1.0DO/ATAD
BTBLI = 1.0DO/BTBL
UN = ATAD + BTBL
VN = ATAD + ER*BTBL
WN = ACAD + BCBL
VNI = 1.0DO/VN
WNI = 1.0DO/WN
VNNNI = VNI*WNI
PK1 = VNI*BTBL*TN2(N)*{ER*BTBL*WN +GER}
PK2 = -VNI*G(N)*TN2(N)*UN
PK3 = VNI*ATAD*TN2(N)*{ATAD*WN - GER}
PTM1 = D*(PK1 - PK2*CAD2)*SECAD*SECAD
PTM2 = EL*(PK3 - PK2*CBL2)*SECBL*SECBL
PTM3 = PK1/ACAD + PK2*(ATADI + BTBLI) + PK3/BCBL
PTM = VNNNI*(PTM1 + PTM2 + PTM3) - TN2P(N)
SUMP = SUMP + PTM
DITM1 = TN11(N)*(VN - EM2*UN)  
DITM2 = ER*ATADI*(SECAD - 1.0) + BTBLI*(SECBL - 1.0)  
DITM = SPI2*(VNWINI*DITM1*DITM2 - CS5*TNI2(N))  
5  SUMDI = SUMDI + DITM  
P = EM*CP*(SUMP + SP1)  
DI = W2UE*CDI*(SUMDI + CS5*SI2)  
TOTI2 = (CSI - DI)**2  
Z0 = P/TOTI2  
RETURN  
END
SUBROUTINE DRTMI

PURPOSE
TO SOLVE GENERAL NONLINEAR EQUATIONS OF THE FORM FCT(X) = 0
BY MEANS OF MUELLER'S ITERATION METHOD.

USAGE
CALL DRTMI (X,F,FCT,XLI,XRI, EPS, IEND, IER)
PARAMETER FCT REQUIRES AN EXTERNAL STATEMENT.

DESCRIPTION
OF PARAMETERS

X - DOUBLE PRECISION RESULTANT ROOT OF EQUATION
FCT(X) = 0.

F - DOUBLE PRECISION RESULTANT FUNCTION VALUE
AT ROOT X.

FCT - NAME OF THE EXTERNAL DOUBLE PRECISION FUNCTION
SUBPROGRAM USED.

XLI - DOUBLE PRECISION INPUT VALUE WHICH SPECIFIES THE
INITIAL LEFT BOUND OF THE ROOT X.

XRI - DOUBLE PRECISION INPUT VALUE WHICH SPECIFIES THE
INITIAL RIGHT BOUND OF THE ROOT X.

EPS - SINGLE PRECISION INPUT VALUE WHICH SPECIFIES THE
UPPER BOUND OF THE ERROR OF RESULT X.

IEND - MAXIMUM NUMBER OF ITERATION STEPS SPECIFIED.

IER - RESULTANT ERROR PARAMETER CODED AS FOLLOWS
IER = 0 - NO ERROR,
IER = 1 - NO CONVERGENCE AFTER IEND ITERATION STEPS
FOLLOWED BY IEND SUCCESSIVE STEPS OF
BISECTION,
IER = 2 - BASIC ASSUMPTION FCT(XLI)*FCT(XRI) LESS
THAN OR EQUAL TO ZERO IS NOT SATISFIED.

REMARKS
THE PROCEDURE ASSUMES THAT FUNCTION VALUES AT INITIAL
BOUNDS XLI AND XRI HAVE NOT THE SAME SIGN. IF THIS BASIC
ASSUMPTION IS NOT SATISFIED BY INPUT VALUES XLI AND XRI, THE
PROCEDURE IS BYPASSED AND GIVES THE ERROR MESSAGE IER=2.

SUBRoutines AND FUNCTION SUBPROGRAMS REQUIRED
THE EXTERNAL DOUBLE PRECISION FUNCTION SUBPROGRAM FCT(X)
MUST BE FURNISHED BY THE USER.

METHOD
SOLUTION OF EQUATION FCT(X)=0 IS DONE BY MEANS OF MUELLER'S
ITERATION METHOD OF SUCCESSIVE BISECTIONS AND INVERSE
PARABOLIC INTERPOLATION, WHICH STARTS AT THE INITIAL BOUNDS
XLI AND XRI. CONVERGENCE IS QUADRATIC IF THE DERIVATIVE OF
FCT(X) AT ROOT X IS NOT EQUAL TO ZERO. ONE ITERATION STEP
REQUIRES TWO EVALUATIONS OF FCT(X). FOR TEST ON SATISFACTORY
ACCURACY SEE FORMULAE (3,4) OF MATHEMATICAL DESCRIPTION.
FOR REFERENCE, SEE G. K. KRISTIANSEN, ZERO OF ARBITRARY
FUNCTION, BIT, VOL. 3 (1963), PP.205-206.

SUBROUTINE DRTMI(X,F,FCT,XLI,XRI,EPS,IEND,IER)
DOUBLE PRECISION X,F,FCT,XLI,XRI,XL,XR,FL,FR,TOL,TOLF,A,DX,XM,FM

PREPARE ITERATION
IER=0
XL=XLI
XR=XRI
X=XL
TOL=X
F=FCT(TOL)
IF(F)1,16,1
FL=F
X=XR
TOL=X
F=FCT(TOL)
IF(F)2,16,2
2 FR=F
   IF(DSIGN(1.0D0,FL)+DSIGN(1.0D0,FR))25,3,25
C
C BASIC ASSUMPTION FL*FR LESS THAN 0 IS SATISFIED.
C GENERATE TOLERANCE FOR FUNCTION VALUES.
3 I=0
   TOLF=100.*EPS
C
C START ITERATION LOOP
4 I=I+1
C
C START BISECTION LOOP
DO 13 K=1,IEND
   X=.50D0*(XL+XR)
   TOL=X
   F=FCT(TOL)
   IF(F)5,16,5
      IF(DSIGN(1.0D0,F)+DSIGN(1.0D0,FR))7,6,7
      INTERCHANGE XL AND XR IN ORDER TO GET THE SAME SIGN IN F AND FR
      TOL=XL
      XL=XR
      XR=TOL
      TOL=FL
      FL=FR
      FR=TOL
      TOL=F-FL
      A=F*TOL
      A=A+A
      IF(A-FR*(FR-FL))8,9,9
5 IF(DSIGN(1.0D0,F)+DSIGN(1.0D0,FR))7,6,7
C
C INTERCHANGE XL AND XR IN ORDER TO GET THE SAME SIGN IN F AND FR
6 TOL=XL
   XL=XR
   XR=TOL
   TOL=FL
   FL=FR
   FR=TOL
7 TOL=F-FL
   A=F*TOL
   A=A+A
   IF(A-FR*(FR-FL))8,9,9
8 IF(I-IEND)17,17,9
9 XR=X
   FR=F
C
TEST ON SATISFACTORY ACCURACY IN BISECTION LOOP

TOL=EPS
A=DABS(XR)
IF(A-1.00)11,11,10
10 TOL=TOL*A
11 IF(DABS(XR-XL)-TOL)12,12,13
12 IF(DABS(FR-FL)-TOLF)14,14,13
13 CONTINUE

END OF BISECTION LOOP

NO CONVERGENCE AFTER IEND ITERATION STEPS FOLLOWED BY IEND SUCCESSIVE STEPS IF BISECTION OR STEADILY INCREASING FUNCTION VALUES AT RIGHT BOUNDS. ERROR RETURN.

IER=1
14 IF(DABS(FR)-DABS(FL))16,16,15
15 X=XL
F=FL
16 RETURN

COMPUTATION OF ITERATED X-VALUE BY INVERSE PARABOLIC INTERPOLATION

A=FR-F
DX=(X-XL)*FL*(1.00+F*(A-TOL)/(A*(FR-FL)))/TOL
XM=X
FM=F
X=XL-DX
TOL=X
F=FCT(TOL)
IF(F)18,16,18

TEST ON SATISFACTORY ACCURACY IN ITERATION LOOP

TOL=EPS
A=DABS(X)
IF(A-1.00)20,20,19
19 TOL=TOL*A
20 IF(DABS(DX)-TOL)21,21,22
21 IF(DABS(F)-TOLF)16,16,22
C PREPARATION OF NEXT BISECTION LOOP

22 IF(DFSN(1.00,FL)+DFSN(1.00,FL))24,23,24
23 XR=X
   FR=F
   GO TO 4
24 XL=X
   FL=F
   XR=XM
   FR=FM
   GO TO 4

C END OF ITERATION LOOP

C ERROR RETURN IN CASE OF WRONG INPUT DATA

25 IER=2
   RETURN
   END
SUBROUTINE DTERMS(MAXTMS)

THIS SUBROUTINE CALCULATES THE TERMS NECESSARY FOR THE EVALUATION OF THE NORMALIZED PROPAGATION CONSTANTS.

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION TN(500),G(500)
COMMON A,B,D,WIDTH,ER,EFF,EL,PI,W2UE
COMMON /FTMS/TN,G,FSSUM
PI = 3.141593D0
V0 = 2.99792508
W = 2.DO*PI*EFF
W2UE = (W/V0)**2
EL = A - D
C1 = PI/B
C2 = 2.DO/(WIDTH*C1)
FSSUM = 0.00
DO 1 N=1,MAXTMS
EN = 2*N - 1
HN = C2/EN
HNR = 1.DO/HN
CHNR = DCOS(HNR)
SHNR = DSIN(HNR)
1 TN(N)=((6.DO*(1.DO-CHNR)*HN-6.DO*SHNR)*HN+3.DO*CHNR)*HN+2.DO*SHNR
DN(N) = (EN*C1)**2
FSSUM = FSSUM + TN(N)/EN**2
K = MAXTMS + 1
DO 2 N=K,3000
EN = 2*N - 1
HN = C2/EN
HNR = 1.DO/HN
CHNR = DCOS(HNR)
SHNR = DSIN(HNR)
FSTERM=(((6.DO*(1.DO-CHNR)*HN-6.DO*SHNR)*HN+3.DO*CHNR)*HN+2.DO*
* SHNR)/EN**2
2 FSSUM = FSSUM + FSTERM
RETURN
END
SUBROUTINE DRTFDR(KTOT,ENDFND,STEP,MAXRSM,MINRSM,KRE,IER3)

THIS SUBROUTINE DETERMINES WHETHER POINTS WHERE THE SUM OF
THE BOUNDARY CONDITION SERIES CHANGES SIGN REPRESENT ZEROS
OR INFINITIES. WHEN ZEROS ARE LOCATED, SUBROUTINE DRTWI IS
CALLED TO FIND THE ROOT.

SUBROUTINE DRTFDR(KTOT,ENDFND,STEP,MAXRSM,MINRSM,KRE,IER3)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION TN(500),G(500),RT(50),IER2(50),SJMO(50),AINF(50),
2 SUMINF(50),SUMCK(50),TERMCK(50)
COMMON A,B,D,WIDTH,ER,EFF,EL,PI,W2UE
COMMON /DRT/RT,SUMO,AINF,SUMINF,SUMCK,TERMCK,JJ,IER2
COMMON /FTMS/TN,G,FSSUM /STMS/R,CSUM,TERM,MAXSUM
RER = DSQRT(ER)
STEP2 = RER/15.DD
ENDRT = STEP2
L = 0
J = 0
JJ = 0
IEX = 0
IINF = 0
IER3 = 0
R = 1.DO
MAXSUM = MINRSM
EML = RER
SUML = DSUM(EML)
1
EMR = EML
SUMR = SUML
EML = EMR - STEP2
IF(EML.LT.ENDRT) GO TO 10
SUML = DSUM(EML)
12
CKS = SUML*SUMR
IF(CKS.GT.0.DO) GO TO 1
EML2 = R*EML**2
EMR2 = R*EMR**2
CL1 = ER - EML2
CL1 = ER - EMR2
CL2 = EML2 - 1.0D
CR2 = EMR2 - 1.0D
CL3 = W2UE*CL1
CR3 = W2UE*CR1
CL4 = -W2UE*CL2
CR4 = -W2UE*CR2
PII = 1.0D/PI
CPID = 2.0D*D*PII
CPIL = 2.0D*EL*PII
CC = B*DSQRT(CL3)*PII
NMAX = IDINT((CC + 1.0D)*5.0D-1)
IF(NMAX.EQ.0) GO TO 9
DO 8 N=1,NMAX
ALPHL2 = CL3 - G(N)
ALPHR2 = CR3 - G(N)
ALPHAL = DSQRT(ALPHL2)
TADL = DTAN(ALPHAL*0)
XL = 1.0D
IF(ALPHR2.GT.0.0D) GO TO 2
XR = -1.0D
ALPHAR = DSQRT(-ALPHR2)
TADR = DTANH(ALPHAR*0)
GO TO 3
2 XR = 1.0D
ALPHAR = DSQRT(ALPHR2)
TADR = DTAN(ALPHAR*0)
3 BETAL2 = CL4 - G(N)
BETAR2 = CR4 - G(N)
IF(BETAR2.GT.0.0D) GO TO 4
YR = -1.0D
BETAR = DSQRT(-BETAR2)
TBLR = DTANH(BETAR*EL)
IF(BETAL2.GT.0.0D) GO TO 5
YL = -1.0D
BETAL = DSQRT(-BETAL2)
DATE 111
DATE 110
DATE 109
DATE 108
DATE 107
DATE 106
DATE 105
DATE 104
DATE 103
DATE 102
DATE 101
DATE 100
DATE 99
DATE 98
DATE 97
DATE 96
DATE 95
DATE 94
DATE 93
DATE 92
DATE 91
DATE 90
DATE 89
DATE 88
DATE 87
DATE 86
DATE 85
DATE 84
DATE 83
DATE 82
DATE 81
DATE 80
DATE 79
DATE 78
DATE 77
DATE 76
DATE 75

MAXSUM = MAXSUM
II ENERGY = 0.00
GO TO (111.13.164,L)
10 L = L + 1
GO TO 1
17 MAXSUM = MIN Rum
GO TO (114.18.194), IF
14 L = IER +1
IF ((ENERGY) GT 0) GO TO 16
TERM(1) = TERM
SUMK(1) = SUMK
RUR(1) = SUMK
IEF = IEF + 1
RT(1) = RT(1) + 1
J = J + 1

CALL PRNTR(EM,EM,PSUM,PELM,REM,RELM,5.0,D-7-2.0,IER)
REM = ERM
RELM = ELM

MAXSUM = MAXSUM
9 CONTINUE
8
IF (KDEL) 15, 14, 8
KDEL = IGINT(CPID*ALPHA) - IGINT(CPID*ALPHA)
1 KDEL = IGINT(CPID*ALPHA) - IGINT(CPID*ALPHA)
IF(ALPHA2 * L.0.D0) GO TO 14
IF (KCA2.0.D0) GO TO 8

CRA = C0*apl
6 CR = C0*apl
4 QA = ALPHR2+ER*BTAL2+ALPHR*BTAL*YR*ER*TBL/R+TR+XS+LTR+LFR/R
3 TR = DORGAT (BTAL2)
2 BETA = BORGAT (BTAL2)
1 BETA = DORGAT (BTAL2)
5 YL = 1.00
4 VR = 1.00
Go To 6

CALL (BTAL2)
10 CALL (BTAL2)
EML = 0.00
SUMR = SUML
SUML = DSUM(0.00)
GO TO 12
13 R = -1.00
KRE = J
ENDRT = ENDFND
STEP2 = STEP
SUML = DSUM(EML)
GO TO 12
14 JJ = JJ + 1
AINF(JJ) = EML
SUMINF(JJ) = SUML
GO TO 1
18 IINF = IINF+1
GO TO (1,16),IINF
19 IEX = IEX+1
GO TO (21,21,16),IEX
21 EXPAND = 3.0-D-1*STEP2
REML = REML - EXPAND
REMR = REMR + EXPAND
GO TO 20
15 IER3 = 1
16 RETURN
END
FUNCTION DSUM(EM)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION TN(500),G(500)
COMMON A,B,D,WIDTH,ER,EFF,EL,PI,W2UE
COMMON /FTMS/TN,G,FSSUM /STMS/R,CSUM,TERM,MAXSUM

EM2 = R*EM**2
CS1 = ER - EM2
CS2 = EM2 - 1.D0
CS3 = W2UE*CS1
CS4 = -W2UE*CS2
CS5 = B*(CS1 - CS2)/(2.D0*PI*(ER + 1.DO))

CSUM = 0.DO
DO 5 N=1,MAXSUM
EN = 2*N - 1
ENI = 1.DO/EN
ALPHA2 = CS3 - G(N)
IF(ALPHA2.GT.0.DO) GO TO 1
X = -1.DO
ALPHA = DSQRT(-ALPHA2)
TAD = DTANH(ALPHA*D)
GO TO 2
1 X = 1.DO
ALPHA = DSQRT(ALPHA2)
TAD = DTANH(ALPHA*D)
GO TO 3
2 BETA2 = CS4 - G(N)
IF(BETA2.GT.0.DO) GO TO 3
Y = -1.DO
BETA = DSQRT(-BETA2)
TBL = DTANH(BETA*DL)
GO TO 4
3 \quad Y = 1 \cdot \text{DO}
\quad \text{BETA} = \text{DSORT(BETA2)}
\quad \text{TBL} = \text{DTAN(BETA*EL)}
4 \quad \text{QN} = \text{ALPHA2+ER*BETA2+ALPHA*BETA*(Y*ER*TBL/TAD+X*TAD/TBL)}
\quad \text{RN} = Y*BETA*CS1*TBL - X*ALPHA*CS2*TAD
\quad \text{TERM} = TN(N)*(RN/QN - CS5*EN1)*EN1
5 \quad \text{CSUM} = \text{CSUM + TERM}
\quad \text{DSUM} = \text{FSSUM*CS5 + CSUM}
\quad \text{RETURN}
\quad \text{END}
SUBROUTINE DTERMT(MAXTMS, INF)
FOR MAXWELL'S LONGITUDINAL CURRENT,
SUBROUTINE DTRMJT(MAXTMS, INF)

THIS SUBROUTINE EVALUATES THE TERMS NECESSARY IN THE
BOUNDARY CONDITION SERIES FOR MICROSTRIP WITH NON-ZERO
TRANSVERSE CURRENT.
AS WRITTEN, YAMASHITA'S LONGITUDINAL CURRENT IS ASSUMED.
THE CHANGES NECESSARY FOR MAXWELL'S LONGITUDINAL CURRENT
ARE NOTED.

SUBROUTINE DTERMT(MAXTMS, INF)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION S1T(100), S2T(100), S3T(100), S4T(100), G(100)
COMMON A, B, D, W, ER, F, EL, Q, W2UE, R, CK, MAXSUM
COMMON/TERMS/S1T, S2T, S3T, S4T, G, SS11, SS12, SS2, SS3, SS4, CT1, CT2, CT3,
  * CT4
  PI = 3.141593D0
  V0 = 2.997925D8
  EL = A - D
  W2UE = (2.D0*PI*F/V0)**2
  ERPII = 1.DO/(ER + 1.DO)
  C1 = PI/B
  C1I = 1.DO/C1
  CT1 = C1I*C1I
  CT2 = C1I*5.D-1
  CT3 = C1*ERPII
  CT4 = C1I*ERPII
  CT = C1I*W*5.D-1
  CQ = Q*CT
  CK = -4.D0*( (2.D0*C1I/W)**3 )/PI
  SS11 = 0.DO
  SS12 = 0.DO
  SS2 = 0.DO
  SS3 = 0.DO
ENI4 = ENI2*ENI2
CTN = CT*EN
CQN = CQ*EN
CCTN = DCOS(CTN)

FOR MAXWELL'S CURRENT, OMIT THE ABOVE STATEMENT

SCTN = DSIN(CTN)
CCQN = DCOS(CQN)
SCQN = DSIN(CQN)

TN = ENI4*(6.0*(1.0 - CCTN) + CTN*(-6.0*SCTN + CTN*(3.0*CCTN * + 2.0*CTN*SCTN))}

FOR MAXWELL'S CURRENT, REPLACE THE ABOVE STATEMENT WITH

CALL DBESJO(CTN, TN, 1.0-4, IER)

SN = ENI2*(5.0-1*SCTN - DSIN(5.0-1*CTN))
SNSCQN = SN*SCQN
SS11 = SS11 + EN*SNSCQN
SS12 = SS12 + SNSCQN
SS2 = SS2 + TN*SCQN
SS3 = SS3 + SN*CCQN
SS4 = SS4 + ENI*TN*CCQN

SS11 = CT3*SS11
SS12 = CT2*SS12
SS2 = CT4*SS2
SS3 = CT4*SS3
RETURN
END
FUNCTION DTSUM(EM2)

THIS FUNCTION SUBPROGRAM EVALUATES THE TWO BY TWO BOUNDARY
CONDITION DETERMINANT FOR MICROSTRIP WITH NON-ZERO
TRANSVERSE CURRENT.

FUNCTION DTSUM(EM2)
IMPLICIT REAL*8(A-H, O-Z)
DIMENSION SIT(100), S2T(100), S3T(100), S4T(100), G(100)
COMMON A, B, D, W, ER, F, EL, Q, W2UE, R, CK, MAXSUM
COMMON/TERMS/SIT, S2T, S3T, S4T, G, SS11, SS12, SS2, SS3, SS4, CT1, CT2, CT3,
* CT4
COMMON/CHECK/TM1, TM2, TM3, TM4, S1, S2, S3, S4, RCK
CS1 = W2UE*(ER - EM2)
CS2 = W2UE*(1.00 - EM2)
CS3 = CT1/EM2
CS4 = CT2*W2UE
CS5 = CT2 - EM2*CT4
S1 = 0.00
S2 = 0.00
S3 = 0.00
S4 = 0.00
DO 5 N = 1, MAXSUM
EN = DFLOAT(2*N - 1)
ALPHA2 = CS1 - G(N)
IF(ALPHA2.GT.0.00) GO TO 1
ALPHA = DSQRT(-ALPHA2)
TAD = DTANH(ALPHA*D)
ATAD = -ALPHA*TAD
GO TO 2
1 ALPHA = DSQRT(ALPHA2)
TAD = DTAN(ALPHA*D)
ATAD = ALPHA*TAD
2 BETA2 = CS2 - G(N)
IF(BETA2.GT.0.00) GO TO 3
BETA = DSQRT(-BETA2)
DTSU 001
DTSU 002
DTSU 003
DTSU 004
DTSU 005
DTSU 006
DTSU 007
DTSU 008
DTSU 009
DTSU 010
DTSU 011
DTSU 012
DTSU 013
DTSU 014
DTSU 015
DTSU 016
DTSU 017
DTSU 018
DTSU 019
DTSU 020
DTSU 021
DTSU 022
DTSU 023
DTSU 024
DTSU 025
DTSU 026
DTSU 027
DTSU 028
DTSU 029
DTSU 030
DTSU 031
DTSU 032
DTSU 033
DTSU 034
DTSU 035
DTSU 036
DTSU 037
TBL = DTANH(BETA*EL)
BTBL = -BETA*TBL
GO TO 4

3 BETA = DSQRT(BETA2)
TBL = DTAN(BETA*EL)
BTBL = BETA*TBL

4 ACAD = ALPHA/TAD
BCBL = BETA/TBL
UN = ATAD + BTBL
VN = ATAD + ER*BTBL
WN = ACAD + BCBL
VNWNI = 1.0/(VN*WN)
T1 = VNWNI*(G(N)*UN - W2UE*VN) - CT3*EN + CS4/EN
T23 = VNWNI*EN*UN - CT4
T4 = VNWNI*(VN - EM2*UN)*EN - CS5
TM1 = T1*S1T(N)
TM2 = T23*S2T(N)
TM3 = T23*S3T(N)
TM4 = T4*S4T(N)
S2 = S2 + TM2
S3 = S3 + TM3
S4 = S4 + TM4
S1P = CS3*(S1 + SS11 - W2UE*SS12)
S2P = -S2 - SS2
S3P = S3 + SS3
S4P = S4 + CS5*SS4
DTSUM = S1P*S4P - S2P*S3P
R = -S2P/S1P
RCK = -S4P/S3P
RETURN
END
SUBROUTINE DPFM(EM, EREFF, VP, CRATIO, MAXTMS, IERP)

THIS SUBROUTINE EVALUATES THE PROPAGATION PARAMETERS
FOR MICROSTRIP WITH NON-ZERO TRANSVERSE CURRENT.
AS WRITTEN, YAMASHITA'S LONGITUDINAL CURRENT IS ASSUMED.
THE CHANGES NECESSARY FOR MAXWELL'S LONGITUDINAL CURRENT
ARE NOTED.

SUBROUTINE DPFM(EM, EREFF, VP, CRATIO, MAXTMS, IERP)
IMPLICIT REAL*8(A-H, O-Z)
COMMON A, B, D, W, ER, F, EL, Q, W2UE, R, CK, MAXSUM
IERP = 0
L = 0
STEP = ER*1.D-1
MAXSUM = MAXTMS/4
EM2L = ER
TERML = DTSUM(EM2L)
1 EM2R = EM2L
EM2L = EM2R - STEP
IF(EM2L.LT.1.D0) GO TO 7
TERMR = TERML
TERML = DTSUM(EM2L)
CKTM = TERML*TERMR
IF(CKTM.GT.0.D0) GO TO 1
MAXSUM = MAXTMS
9 CALL DRTMI(EM2, VAL, DTSUM, EM2L, EM2R, 5.E-6, 30, IER)
IERP1 = IER + 1
GO TO (3, 4, 5), IERP1
4 IERP = 1
GO TO 6
7 IERP = 3
GO TO 6
5 L = L + 1
IERP = 2
GO TO (8, 6), L
8 EXPAND = STEP*2.D-1
EM2L = EM2L - EXPAND
EM2R = EM2R + EXPAND
GO TO 9
3 EREFF = EM2
EM = DSQRT(EM2)
VP = 2.997925D8/EM
CONST = (8/3.141593D0)**3/(W*W)
C FOR MAXWELL'S CURRENT, REPLACE THE ABOVE STATEMENT WITH
C CONST = (3.141593D0*W)**2/(32.D0*B)
C CRATIO = CONST*DSQRT(W2UE)/EM
6 RETURN
END
SUBROUTINE DBESJO(X,BJO,D,IER)

THIS SUBROUTINE IS A MODIFICATION OF SUBROUTINE BESJ FROM
THE IBM SCIENTIFIC SUBROUTINE PACKAGE.

DESCRIPTION OF PARAMETERS
X - ARGUMENT OF THE ZERO ORDER BESSEL FUNCTION
BJO - RESULTANT ZERO ORDER BESSEL FUNCTION
D - ACCURACY REQUIRED
IER - ERROR CODE
   =0 NO ERROR
   =1 REQUIRED ACCURACY NOT OBTAINED

SUBROUTINE DBESJO(X,BJO,D,IER)
IMPLICIT REAL*8(A-H,O-Z)
XI = 1.DO/X

COMPUTING STARTING VALUE OF M
IF(X.GE.5.D0) GO TO 1
MZERO = X + 6.D0
GO TO 2
1 MZERO = 1.4DO*X + 6.D1*XI
2 BPREV = 0.DO

SET UPPER LIMIT OF M
IF(X.GT.1.5D1) GO TO 3
MMAX = 2.D1 + 1.D1*X - X**2/3
GO TO 4
3 MMAX = 9.D1 + X*5.D-1
4 IER = 0
   DO 9 M=MZERO,MMAX,3
   SET F(M), F(M-1)
END
RETURN
I = 1
B = B0
IF (DBS(80-BPRED)\*LE\*DABS(D*B20)) GO TO 10
A = A
B0 = B0\*ALPHA
ALPHA = ALPHA + BK
BK = 2.0*DIM*XI - FM
A = A0 + BK
S = I + JT
JT = -JT
B0 = BK
IF (MK\*E\*I) GO TO 7
FM = BK
FM = FM
FM = 2.0*MK*FM\*XI - FM
MK = M\*K
K = 1+M2
M2 = M-2
JT = -JT
GO TO 6
JT = 1
IF (M2+Z*E\*G) GO TO 5
A = A0
FM = 0.0
FM = 1.0-28
SUBROUTINE DTMPT(MAXTMS,INF)

FOR MAXWELL'S LONGITUDINAL CURRENT,

SUBROUTINE DTMPTJ(MAXTMS,INF)

THIS SUBROUTINE COMPUTES THE TERMS NECESSARY FOR THE
EVALUATION OF THE CHARACTERISTIC IMPEDANCE FOR MICROSTRIP
WITH NON-ZERO TRANSVERSE CURRENT.

AS WRITTEN, YAMASHITA'S LONGITUDINAL CURRENT IS ASSUMED.
THE CHANGES NECESSARY FOR MAXWELL'S LONGITUDINAL CURRENT
ARE NOTED.

SUBROUTINE DTMPTCMAXTMS,INF)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION PNSN (100), TN2P(100), TNSNP(100), SN2P(100), SNI1(100),
* SNI2(100), TNI1(100), TNI2(100), TNIP(100), G(100), GI(100)
COMMON A,B,D,W,ER,F,EL,W2UE
COMMON/ZTMS/PNSN,TN2P,TNSNP,SN2P,SNI1,SNI2,TNI1,TNI2,TNIP,G,GI,
* SP1,SP2,SP3,SP4,SP5,S11,S12,ERM1,HERP1,CSI,CDI,CP
PI = 3.141593D0
VO = 2.997925D8
ETAO = 3.76702
EL = A - D
W2UE = (2.D0*PI*F/VO)**2
ERM1 = ER - 1.D0
ERP1 = ER + 1.D0
HERP1 = 5.D-1*ERP1
ERP11 = 1.D0/ERP1
CI = 1.25D0

FOR MAXWELL'S CURRENT, REPLACE THE ABOVE STATEMENT WITH
CI = PI*5.D-1

CSI = W*CI
C1 = PI/B
CII = 1.D0/C1
FOR MAXWELL'S CURRENT, OMIT THE ABOVE STATEMENT

CCIN = DCOS(CTN)
CCIN = CTEN
G(N) = 1.0D0(G(N))
G(N) = (EN*CT)**2
SP2 = -SP2

FOR MAXWELL'S CURRENT, OMIT THE ABOVE STATEMENT

EN4 = ENI*ENI2
EN3 = ENI*ENI2
EN2 = ENI*ENI
EN1 = I.DO/EN
DO I=N=I.MAX*MS
SIZ = 0.0D0
S11 = 0.0D0
S25 = 0.0D0
S40 = 0.0D0
S53 = 0.0D0
S32 = 0.0D0
S21 = 0.0D0
S10 = 0.0D0
CP = 2.5D0*ETA0*C**2
CO1 = 2.0D0*CI1*CK

CK = -CI*W

FOR MAXWELL'S CURRENT, REPLACE THE ABOVE STATEMENT WITH

CK = -3.0D1*(CI1/M)**3/Pi
CT = CI*W**5.0*1
CT5 = 2.0D0*CI3
CT4 = CI*CT3
CT3 = CI*ERP1
CT2 = CI*5.0*1
CT1 = CI*CI1


C

\[
\text{SCTN} = \text{DSIN(CTN)} \\
\text{TN} = \text{ENI4*(6.00*(1.00 - CCTN) + CTN*(-6.00*SCTN + CTN*(3.00*CCTN} \\
* + 2.00*CTN*SCTN))}
\]

C

FOR MAXWELL'S CURRENT, REPLACE THE ABOVE STATEMENT WITH

C

CALL DBESJO(CTN, TN, 1.D-4, IER)

C

\[
\text{SN} = \text{ENI2*(5.01*SCTN - DSIN(5.01*CTN))} \\
\text{TN2} = \text{TN*TN} \\
\text{SN2} = \text{SN*SN} \\
\text{TNSN} = \text{TN*SN} \\
\text{SP1TM} = \text{TN2*ENI} \\
\text{SP3TM} = \text{TNSN*ENI2} \\
\text{SP4TM} = \text{SN2*ENI} \\
\text{SP5TM} = \text{SN2*ENI2} \\
\text{SI1TM} = \text{SN*ENI2} \\
\text{SI2TM} = \text{TN*ENI3} \\
\text{SP1} = \text{SP1 + SP1TM} \\
\text{SP2} = \text{SP2 + TNSN} \\
\text{SP3} = \text{SP3 + SP3TM} \\
\text{SP4} = \text{SP4 + SP4TM} \\
\text{SP5} = \text{SP5 + SP5TM} \\
\text{SI1} = \text{SI1 + SPI2*SI1TM} \\
\text{SI2} = \text{SI2 + SPI2*SI2TM} \\
\text{TN2P(N)} = \text{SP1TM*CT5} \\
\text{SN2P(N)} = \text{SP4TM*CT4} \\
\text{SN11(N)} = \text{SN} \\
\text{SN12(N)} = \text{SI1TM*CT1} \\
\text{TN11(N)} = \text{TN*ENI} \\
\text{TN12(N)} = \text{SI2TM*CT1} \\
\text{TNIP(N)} = \text{TN} \\
\text{PNSN(N)} = \text{SN*EN} \\
\text{TNSNP(N)} = \text{TNSN*CT3} \\
\text{K} = \text{MAXTMS + 1} \\
\text{DO 2 N=K,INF} \\
\text{EN} = \text{DFLOAT(2*N - 1)}
\]
ENI = 1.D0/EN
ENI2 = ENI*ENI
ENI3 = ENI*ENI2
ENI4 = ENI2*ENI2

C FOR MAXWELL'S CURRENT, OMIT THE ABOVE STATEMENT
C
SPI2 = -SPI2
CTN = CT*EN
CCTN = DCOS(CTN)

C FOR MAXWELL'S CURRENT, OMIT THE ABOVE STATEMENT
C
SCTN = DSIN(CTN)
TN = ENI4*(6.D0*(1.D0 - CCTN) + CTN*(-6.D0*SCTN + CTN*(3.D0*CCTN
* + 2.D0*CTN*SCTN)))

C FOR MAXWELL'S CURRENT, REPLACE THE ABOVE STATEMENT WITH
C CALL DBESJD0(CTN,TN,1.D-4,IER)
C
SN = ENI2*(5.D-1*SCTN - DSIN(5.D-1*CTN))
SN2 = SN*SN
TNSN = TN*SN
SP1 = SP1 + TN*TN*ENI
SP2 = SP2 + TNSN
SP3 = SP3 + TNSN*ENI2
SP4 = SP4 + SN2*ENI
SP5 = SP5 + SN2*ENI2
SI1 = SI1 + SN*SPI2*ENI2
SI2 = SI2 + TN*SPI2*ENI3

2 SP1 = CT5*SP1
SP2 = CT5*SP2
SP3 = CT4*SP3
SP4 = CT4*SP4
SP5 = CT1*CT4*HERP1*SP5
SI1 = CT1*SI1
SI2 = CT1*SI2
SUBROUTINE DTZO(EM2,R,ZO,MAXSUM)

THIS SUBROUTINE EVALUATES THE CHARACTERISTIC IMPEDANCE OF MICROSTRIP FOR NON-ZERO TRANSVERSE CURRENT

SUBROUTINE DTZO(EM2,R,ZO,MAXSUM)
IMPLICIT REAL*8(A-H,0-Z)
DIMENSION PNSN(100),TN2P(100),TNSNP(100),SN2P(100),SNI1(100), SNI2(100),TNI1(100),TNI2(100),TNIP(100),G(100),GI(100)
COMMON A,B,D,W,ER,F,EL,W2UE
COMMON/ZTMS/PNSN,TN2P,TNSNP,SN2P,SNI1,SNI2,TNI1,TNI2,TNIP,G,GI,
* SP1,SP2,SP3,SP4,SP5,SI1,SI2,ERM1,HERP1,CSI,CDI,CP
COMMON/PCHECK/SUMP,PTM,SUMDI,DITM

EM = DSQRT(EM2)
CS1 = W2UE*(ER - EM2)
CS2 = W2UE*(1.D0-EM2)
CS3 = W2UE*HERP1
CS4 = W2UE*EM2 + CS3
CS5 = HERP1 - EM2
RK0 = W2UE*R
RKODER = RK0*ERM1
RM2I = R/EM2
RKOMI = RK0*RM2I
SUMP = 0.DO
SUMDI = 0.DO
SPI2 = -1.DO
DO 5 N=1,MAXSUM
EN = DFLOAT(2*N - 1)
GER = G(N)*ERM1
SPI2 = -SPI2
ALPHA2 = CS1 - G(N)
IF(ALPHA2.GT.0.DO) GO TO 1
ALPHA = DSQRT(-ALPHA2)
AD = ALPHA*D
TAD = DTANH(AD)
ATAD = -ALPHA*TAD

DTZO 001
DTZO 002
DTZO 003
DTZO 004
DTZO 005
DTZO 006
DTZO 007
DTZO 008
DTZO 009
DTZO 010
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DTZO 035
DTZO 036
DTZO 037
SUBROUTINE DSNQ(H,A,SUM,N)

THIS SUBROUTINE USES SIMPSON'S RULE TO INTEGRATE THE
FUNCTION DESCRIBED BY THE ARRAY A.

H - INTEGRATION INTERVAL
SUM - VALUE OF THE INTEGRAL
N - NUMBER OF POINTS (ODD)

SUBROUTINE DSNQ(H,A,SUM,N)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(5001)
NM1 = N-1
NM2 = N-2
SP1 = A(1) + A(N)
SP2 = 0.DO
SP3 = 0.DO
DO 1 J=2,NM1,2
  SP2 = SP2 + A(J)
1 CONTINUE
DO 2 K=3,NM2,2
  SP3 = SP3 + A(K)
2 CONTINUE
SUM = (SP1 + 4.DO*SP2 + 2.DO*SP3)*H/3.DO
RETURN
END
SUBROUTINE DRSGEA(N)  

C THIS SUBROUTINE SOLVES THE REAL LINEAR SYSTEM A*X=B WHERE  
C A = N BY N COMPLEX COEFFICIENT MATRIX (DESTROYED)  
C N = NUMBER OF EQUATIONS AND UNKNOWNS  
C B = N ELEMENT VECTOR (REPLACED BY SOLUTION VECTOR X)  
C X = N ELEMENT UNKNOWN VECTOR (SOLUTION)  
C THE METHOD USED IS GAUSS ELIMINATION WITH PARTIAL PIVOTING.  
C THE PIVOT ELEMENT IS THAT ELEMENT IN THE PIVOT COLUMN WITH  
C GREATEST NORM WHERE THE NORM USED IS  
C NORM(A) = |RE(A)| + |IM(A)|  

IMPLICIT REAL*8 (A-H,0-Z)  
DIMENSION A(50,50),B(50)  
COMMON/EQUIV3/A,B  
NP1=N+1  
NM1=N-1  

C FORWARD SOLUTION  
DO 50 J=1,NM1  
J1=J+1  
PNORM=0.00  
IMAX=J  

C SEARCH JTH COLUMN FOR PIVOT  
DO 11 I=J,N  
ANORM=DABS(A(I,J))  
IF(PNORM-ANORM.GE.0.00) GO TO 11  
10 PNORM = ANORM  
IMAX=I  
CONTINUE  

C INTERCHANGE ROWS IF NECESSARY  
IF(IMAX-J) 20,22,20  
20 DO 21 I=J,N  
SAVE=A(J,I)  
A(J,I)=A(IMAX,I)  
A(IMAX,I)=SAVE  
SAVE=B(J)  
B(J)=B(IMAX)  
21 CONTINUE  

50 CONTINUE  
END  

DRSG 001  
DRSG 002  
DRSG 003  
DRSG 004  
DRSG 005  
DRSG 006  
DRSG 007  
DRSG 008  
DRSG 009  
DRSG 010  
DRSG 011  
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DRSG 034  
DRSG 035  
DRSG 036  
DRSG 037
B(IMAX)=SAVE

DIVIDE PIVOT EQUATION BY PIVOT

22  RPIV = (1.0D0,0.0D0)/A(J,J)

DO 30 I=J,N

30  A(J,I)=A(J,I)*RPIV
    B(J) = B(J)*RPIV

ELIMINATE ELEMENTS BELOW DIAGONAL IN JTH COLUMN

DO 50 I=J1,N

SAVE=A(I,J)

DO 40 JJ=J,N

40  A(I, JJ)=A(I, JJ)-SAVE*A(J, JJ)
    B(I)=B(I)-SAVE*B(J)

CONTINUE

DO 50 JJ=J, N

CONTINUE

B(N)=B(N)/A(N,N)

BACK SUBSTITUTION

DO 60 I=1,NM1

IR=N-I

DO 60 J=I, I

60  B(IR)=B(IR)-A(IR, JC)*B(JC)

RETURN

END
SUBROUTINE DIEV(VIH, VIQ, VJ, VIG, THETAP, NI1, NI2, MQ)

DIEV 001
THIS SUBROUTINE EVALUATES THE INTEGRALS IH, IQ, J, AND IG
DIEV 002
USED IN MITTRA AND ITOH'S SOLUTION FOR MICROSTRIP.
DIEV 003

IMPLICIT REAL*8(A-H,0-Z)

DIMENSION VIQ(50), VJ(50), A1(5001), A2(5001), A3(2501)

COMMON EL, T, PI, ALPHA1, ALPHA2

COMMON /EQUIV1/ A1 /EQUIV2/ A2 /EQUIV3/ A3

C1 = 2.D0/ALPHA2 - 1.D0
C2 = DSQRT(1.D0/ALPHA2)
SQRT2 = 1.4142135620D0
SQRTA1 = DSQRT(-ALPHA1)

C3 = C2*SQRT2*SQRTA1
NPTS = NI1 + 1
DELTA = PI/DFLOAT(NI1)
TMD = -DELTA

DO 1 1 = 2, NI1
THETA = TMD + I*DELTA
CT = DCOS(THETA)
A1(I) = 1.D0/DSQRT(C1 - CT)
A2(I) = CT*A1(I)
C3A1 = C3*A1(I)
A3(I) = DLOG((1.D0 + C3A1)/(1.D0 - C3A1))
1
A1(1) = 1.D0/C3
A1(NPTS) = 1.D0/(SQRT2*C2)
A2(1) = A1(1)
A2(NPTS) = -A1(NPTS)
A3(NPTS) = DLOG((1.D0 + SQRTA1)/(1.D0 - SQRTA1))

CALL DSNQ(DELTA, A1, VIH, NPTS)
CALL DSNQ(DELTA, A2, VIQ(1), NPTS)
NST2 = 2*(NI1/16) + 1
NINT2 = NPTS - NST2
NPTS2 = NINT2 + 1

DO 2 I = 1, NPTS2
TMD = THETAP - DELTA
DO 9 I=1,NPTS
THETA = TMD + I*DELTA
CT = DCOS(THETA)
AA1 = 1.DO/DSQRT(C1 - CT)
C3A1 = C3*AA1
AA3 = DLOG((1.DO + C3A1)/(1.DO - C3A1))
A1(I) = AA3*AA1
A2(I) = CT*A1(I)
CALL DSNQ(DELTA,A1,VIG1,NPTS)
CALL DSNQ(DELTA,A2,VJ1,NPTS)
VIH = C2*VIH
VIQ(1) = -C2*VIQ(1)
VIQ(2) = C2*VIQ(2)
VJ(2) = C2*VJ(2)
C4 = THETAP*(2.DO*DOLOG(-1.6D1*ALPHA1/(ALPHA2*THETAP**2)))/C3
C5 = 1.DO/(SQRT2*C2*PI**2)
VIG = C5*(C4 + VIG1 + VIG)
VJ(1) = -C2*(C4 + VJ1 + VJ(1))
RETURN
END
SUBROUTINE DPKD(N)

THIS SUBROUTINE EVALUATES THE TERMS D, P, AND K FOR MITTRAH AND ITOH'S SOLUTION FOR MICROSTRIP.

SUBROUTINE DPKD(N)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(50),B(50,50),P(50,50),BP(50),BZ(50),AP(50,50),
* AZ(50,50),D(50,50),Z(50,52),AK(50)
COMMON EL,T,PI,ALPHA1,ALPHA2
COMMON /EQUIV1/AK,D,P /EQUIV2/A,B,BZ,AZ,Z /EQUIV3/AP,BP
COMMON/PKD/C1,C2
C3 = ALPHA2*5.D-1
C4 = C3*5.D-1
DELTA2 = EL - T
ALPH2I = 1.D0/ALPHA2
NM1 = N - 1
NP1 = N + 1
NP2 = N + 2
DO 1 I=1,N
Z(I,NP1) = 0.DO
Z(I,NP2) = 0.DO
DO 1 J=1,N
P(I,J) = 0.DO
DO 5 M=2,N
EM = DFLOAT(M)
DELTAM = DELTA2/EM
DO 3 I=1,M
API(I,1) = 1.DO
AZI(I,1) = 1.DO
XM = T + DELTAM*(I - 1)
THETAM = DARCOS((DCOS(C1*XM) - ALPHA1)*ALPH2I)
DO 2 J=2,M
AP(I,J) = DCOS((J-1)*THETAM)
1 Z(I,J) = 0.DO
P(I,1) = 1.DO
Z(I,1) = 1.DO
DO 5 M=2,N
EM = DFLOAT(M)
DELTAM = DELTA2/EM
DO 3 I=1,M
API(I,1) = 1.DO
AZI(I,1) = 1.DO
XM = T + DELTAM*(I - 1)
THETAM = DARCOS((DCOS(C1*XM) - ALPHA1)*ALPH2I)
DO 2 J=2,M
AP(I,J) = DCOS((J-1)*THETAM)
2  A(I,J) = A(P(I,J)
   C(Z) = C1*X(M)
   BZ(I) = DSIN(EM*CZ)/DSIN(CZ)
3  BP(I) = DCOS((EM - 5.*D-1)*C1*X(M))/DCOS(C2*X4)
   CALL DRSGCA(M)
   DO 4  II=1,M
   P(M,II) = BP(II)
   BP(II) = BZ(II)
   DO 4  II=1,M
4  AP(I,J,J) = A(I,J)
   CALL DRSGCA(M)
   DO 5  II=1,M
5  Z(M,II) = BP(II)
   DO 6  NN=1,N
   A(NN) = C3*Z(NN,2)
   B(NN,1) = C4*(Z.DO*Z(NN,II) - Z(NN,3))
   DO 6  M=2,N
6  B(NN,M) = C4*(Z(NN,M) - Z(NN,M+2))
   DO 7  NN=2,N
   NM = NP2 - NN
   NMM1 = NM - 1
   A(NM) = A(NM) + A(NMM1)
   DO 7  M=1,NM1
7  B(NM,M) = B(NM,M) + B(NMM1,M)
   DO 8  I=1,N
   DO 8  J=1,N
8  D(I,J) = -P(J,1)*A(I)
   DO 9  I=1,N
   DO 9  J=2,N
   JM1 = J-1
   DO 9  KK=1,JM1
9  D(I,J) = D(I,J) + P(J,KK+1)*B(KK)
   AK(I) = ALPHA2
   DO 10  I=2,N
10  AK(I) = ALPHA2*(Z(I-1,1) + Z(I,1))
   RETURN
END
FUNCTION DET2(BETA)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION AKN(10),AKN2(10),A1(10),A2(10),A3(10,10),VEM(10),EM(10),DET2 001
* EN(10),X(10),Y(10),AM(10),BM(10),CM(10),DM(10),D(10,10),AK(10), DET2 002
* SM(10),SMP(10),QMQ(10),WMW(10),A(20,20)
COMMON ER,ERP1,ERM1,W2UE,HMD,DEE,C1,C2,C3,C4,C5,AKN,AKN2,VEM,VIG, DET2 003
* A1,A2,A3,AK,D,R,M1,M2
COMMON /DT/A
COMMON/FNORM/TMNORM,INORM
BETA2 = R*BETA**2
B1 = ER - BETA2
B2 = 1.0D0 - BETA2
B3 = 1.0D0/B2
B4 = C1/B1
B5 = B1 + B2
B6 = ER*B5
B7 = 2.0D0*BETA2 - C2
B8 = B1*B5
B9 = ERM1*B4
B10 = BETA2*B9
B11 = BETA2*ERP1
B12 = C5 - B11
B13 = C4 - B11
B14 = -W2UE*B1
B15 = -W2UE*B2
B16 = W2UE*B8
B17 = B1/B2
Q = -ERM1*B3
W = B3*B5
QIG = Q*VIG

DET2 004
DET2 005
DET2 006
DET2 007
DET2 008
DET2 009
DET2 010
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```plaintext
DO 43 N=1,4
DO 6 M=1,4
1 S = AZIM(W) + (AZIM(W) - AIMAZ)*DA

EN(M) = AZIM(W) - AIMAZ

DO 6 M=1,4

CM(W) = 0

AT = 0

IF(ALPHA2*16.0*MOD(49,ALPHA2) + 8162) + 8162)

4 ALPHAS = DQRT(ALPHA2)

DO 5 TO 5

IF(ALPHA2*61.0*MOD(49,ALPHA2) + 8162)

3 ACAD = CALD/AFLHAL

2 ACAD = DQRT(ALPHA2)

1.000 TO 1.000

ACAD = CALD/AFLHAL

0.000 TO 0.000

ACAD = 0.000

IF(ALPHA2*61.0*MOD(49,ALPHA2) + 8162)

DO 3 TO 3

ACAD = 0.000

DO 2 TO 2

ACAD = 0.000

DO 1 TO 1

SWP(II) = 0.00

WM = 0.00

MIC = WM*WIC
```
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```
DW = A3(N,M)*WMW(N)
QK = A2(N)*QM(N)
SM(M) = SM(M) + DQ*AM(M) - DW*CM(M) + QK*EM(M)
6 SMP(M) = SMP(M) + DQ*BM(M) - DW*DM(M) + QK*EM(M)
XYTM = 1.D0/(S - WIG)
DO 7 M=1,M1
QEM = Q*VEM(M)
WEM = W*VEM(M)
X(M) = XYTM*(SM(M) - EM(M)*QIG - QEM*AM(M) + WEM*CM(M))
7 Y(M) = XYTM*(SMP(M) - EN(M)*QIG - QEM*BM(M) + WEM*DM(M))
DO 8 I=1,M1
DO 8 J=1,M1
IPM1 = I + M1
JPM1 = J + M1
A(I,J) = -AM(J)*D(I,J) - EM(J)*AK(I)
A(I,JPM1) = -BM(J)*D(I,J) - EN(J)*AK(I)
A(IPM1,J) = -CM(J)*D(I,J) - X(J)*AK(I)
8 A(IPM1,JPM1) = -DM(J)*D(I,J) - Y(J)*AK(I)
DO 9 I=1,M1
IPM1 = I + M1
A(I,I) = A(I,I) + AKN(I)
9 A(IPM1,IPM1) = A(IPM1,IPM1) + AKN(I)
IF(INORM.NE.1) GO TO 51
TNORM = 1.D0/A(1,1)
51 CONTINUE
DO 70 JJ=1,M1
DO 70 KK=1,M1
70 A(JJ,KK) = A(JJ,KK)*TNORM
M12 = 2*M1
CALL DMTM(DET2,M12)
IF(INORM.NE.1) GO TO 50
TMNORM = 1.D0/DABS(DET2)
INORM = 0
50 DET2 = DET2*TMNORM
RETURN
END
```
SUBROUTINE DTMT(DET,N)

THIS SUBROUTINE EVALUATES THE DETERMINANT OF A, WHERE A IS AN N X N MATRIX. THE METHOD USED IS UPPER TRIANGULARIZATION.

DIMENSION A(20,20)

IMPLICIT REAL*8(A-H,O-Z)

COMMON /DT/A

DET = A(1,1)

DO 2 I=2,N

IM1 = I - 1

DO 1 K=I,N

DPIV = A(K,IM1)/A(IM1,IM1)

DO 1 J=I,N

1 A(K,J) = A(K,J) - DPIV*A(IM1,J)

2 DET = DET*A(I,I)

RETURN

END