Horizontal advance of flooding irrigation water in relation to infiltration rate of soil

Mohamed Shaban Asseed

Iowa State University

Follow this and additional works at: https://lib.dr.iastate.edu/rtd

Part of the Agricultural Science Commons, Agriculture Commons, and the Agronomy and Crop Sciences Commons

Recommended Citation
Asseed, Mohamed Shaban, "Horizontal advance of flooding irrigation water in relation to infiltration rate of soil " (1966). Retrospective Theses and Dissertations. 5302.
https://lib.dr.iastate.edu/rtd/5302

This Dissertation is brought to you for free and open access by the Iowa State University Capstones, Theses and Dissertations at Iowa State University Digital Repository. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.
HORIZONTAL ADVANCE OF FLOODING IRRIGATION WATER
IN RELATION TO INFILTRATION RATE OF SOIL

by

Mohamed Shaban Asseed

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Soil Physics

Approved:

Signature was redacted for privacy.
In Charge of Major Work

Signature was redacted for privacy.
Head of Major Department

Signature was redacted for privacy.
Dean of Graduate College

Iowa State University
Of Science and Technology
Ames, Iowa

1966
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTRODUCTION AND OBJECTIVES</td>
<td>1</td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Objectives</td>
<td>2</td>
</tr>
<tr>
<td>REVIEW OF LITERATURE</td>
<td>3</td>
</tr>
<tr>
<td>Hydraulics of Surface Irrigation</td>
<td>3</td>
</tr>
<tr>
<td>Water Intake Characteristics of Soils</td>
<td>9</td>
</tr>
<tr>
<td>THEORY</td>
<td>15</td>
</tr>
<tr>
<td>Derivation of the Irrigation Advance Equation</td>
<td>15</td>
</tr>
<tr>
<td>Solution for the Infiltration Case, $y = Kt$</td>
<td>19</td>
</tr>
<tr>
<td>Solution for the Infiltration Case, $y = Et^a$</td>
<td>21</td>
</tr>
<tr>
<td>Solution for the Infiltration Case, $y = At^{1/2} + Bt$</td>
<td>22</td>
</tr>
<tr>
<td>EXPERIMENTAL EQUIPMENT AND PROCEDURE</td>
<td>32</td>
</tr>
<tr>
<td>The Irrigation Model</td>
<td>32</td>
</tr>
<tr>
<td>Construction of the Water Feeding System</td>
<td>35</td>
</tr>
<tr>
<td>Porous Mediums Used in the Model and Their Infiltration Characteristics</td>
<td>36</td>
</tr>
<tr>
<td>Test for Homogeneity of Packing of the Model with Porous Medium</td>
<td>42</td>
</tr>
<tr>
<td>Geometrical Verification and Fluid Characteristics</td>
<td>53</td>
</tr>
<tr>
<td>Arrangement for Taking Photographs</td>
<td>58</td>
</tr>
<tr>
<td>Experimental Procedure</td>
<td>58</td>
</tr>
<tr>
<td>Two Types of Experiments</td>
<td>59</td>
</tr>
<tr>
<td>Field Infiltration</td>
<td>60</td>
</tr>
<tr>
<td>RESULTS AND DISCUSSION</td>
<td>62</td>
</tr>
</tbody>
</table>
Horizontal Advance of Water in the Model with Glass Beads as Porous Mediums 62
Dimensionless Functions for the Horizontal Advance of Irrigation Water when the Infiltration Equation is \( y = E t^2 \) 89
Moisture Profiles in the Model during Infiltration 112
Horizontal Advance of Water in the Model when Ida and Webster Soils are Porous Mediums 114
Theoretical Horizontal Advance of Irrigation Water when the Infiltration Equation is \( y = A t^{1/2} + B t \) and the Parameters A and B are Determined under Field Conditions 137
Theoretical Horizontal Advance of Irrigation Water when the Infiltration Equation is \( y = K t \), as for Sand 165
SUMMARY AND CONCLUSION 178
BIBLIOGRAPHY 183
ACKNOWLEDGMENTS 188
APPENDIX 189
INTRODUCTION AND OBJECTIVES

Introduction

The pressure of human survival and the need for additional food supplies require a rapid expansion of irrigation agriculture throughout the world. Even though irrigation is of first importance in the arid and semi-arid regions of the earth, it is becoming increasingly important in humid regions.

Irrigation is defined as the application of water to soil for the purposes of supplying the moisture essential for plant growth. However, a broad and more inclusive definition is that irrigation is the application of water to the soil for any number of the following five purposes: (1) to supply the moisture essential for plant growth, (2) to provide crop insurance against short drought duration, (3) to cool or warm the soil and atmosphere, thereby making more favorable environment for plant growth, (4) to wash out or dilute salts in the soil, and (5) to soften tillage pans.

In all irrigation methods, except subirrigation, water is applied to the surface of land where it subsequently enters the soil, and is stored for later use by plants. Thus, the rate of entry of water into soil under field conditions, called intake rate, is of fundamental importance in surface irrigation.

Surface methods of irrigation pose a complicated problem in the hydraulics of unsteady, spatially variable flow with a free surface. Design of surface irrigation systems is now based largely on empirical criteria and limited theoretical development. Excessive irrigation wastes not only water, it also leaches water-soluble nutrients beyond the plant.
reach. High efficiency in application of irrigation water implies uniformity of depth of water applied at all points in a given field. Uniformity of depth applied is related to how water advances to a particular point and recedes from it. So the movement of water on the surface, as related to infiltration characteristics of soils is a major determinant of attainable efficiencies in water use.

Objectives

In planning this study the purpose was to determine the rate of horizontal advance of irrigation water as related to the infiltration characteristics of soils. The specific objectives were: (1) to design and operate an irrigation model in which the horizontal advance of water on the surface was to be determined for different porous mediums with different infiltration characteristics, and for different conditions of surface slope and roughness, (2) to compare mathematical solutions with model data, (3) to use dimensionless functions to represent the model data, and (4) to use field determined infiltration equations to develop theoretical curves for the horizontal advance of flooding irrigation water under field conditions. The theoretical curves are for different soil types, for different antecedent moisture content, and for different depths of surface storage.
REVIEW OF LITERATURE

Hydraulics of Surface Irrigation

The hydraulics of surface irrigation is complex and, consequently, not well understood. Nevertheless, good irrigation design depends upon its principles. Figure 1 shows a schematic diagram of border irrigation, illustrating the basic variables involved in the hydraulics of surface irrigation. They are as follows: (1) size of irrigation stream, (2) rate of horizontal advance on the surface of soil, (3) length of irrigation run and time required, (4) depth of flow or surface storage, (5) intake rate of soil, (6) slope of land surface, (7) surface roughness, (8) erosion hazard, (9) shape of flow channel, and (10) depth of water to be applied.

Approaches to the problem of the hydraulics of surface irrigation have been made by some investigators by the analysis of the rate of advance of the stream front down a border strip. Although this can be considered the only direct approach that has been made, it has been complicated by the fact that this is an unsteady spatially variable flow with a free surface, hence direct solution of this problem is not simple. Both Parker (1913) and Israelsen (1932) discussed the problem of surface irrigation and developed a formula for the rate at which water will cover a border strip. They applied the continuity equation to border flow, stating that all of the water applied to the border must either be in storage in the border or has infiltrated into the soil. This was stated mathematically as,

\[ q \, dt = C \, dx + ix \, dt \]
Figure 1. Schematic view of border irrigation illustrating the basic variables involved in the hydraulics of surface irrigation.
SHAPE OF FLOW CHANNEL
BORDER, FURROW SHAPE

SURFACE ROUGHNESS

EROSION HAZARD

SIZE OF STREAM

RATE OF ADVANCE

FLUID CHARACTERISTIC

DEPTH OF FLOW

SLOPE OF LAND

INTAKE RATE
where,

\[ q = \text{rate of application of water per unit width of border} \]

\[ \text{meter}^3/\text{meter} \]

\[ C = \text{depth of water in the border (meters)} \]

\[ i = \text{intake rate of soil (meter}^3/\text{meter}^2/\text{day} = \text{meter/day)} \]

\[ x = \text{horizontal distance (meters) water has advanced at time } t. \]

The above equation can be written as

\[ \frac{C \cdot dx}{q - ix} = dt \]

which can be solved by simple integration.

The solution may be given as,

\[ t = \frac{C}{i} \ln \frac{q}{q - ix}. \]

In obtaining this solution the assumption was made that the rate of infiltration \( i \) and the depth \( C \) of water on the surface of the border are constants throughout the irrigation period, which was not a realistic assumption. Isrealson, himself, pointed out, that the relation between the theoretical curves and the experimental data indicated that the rate of infiltration was not constant throughout the period of irrigation.

Lewis and Milne (1938) reviewed the early work on the problem of surface irrigation and put forward a quantitative approach which was a significant advance over that of previous investigators. The equation of Lewis and Milne will be derived later.

Shibata (1956) in Japan has proposed a mathematical expression for the rate of horizontal distance of advance of the irrigation stream as a function of the furrow input, furrow slope, distance down the furrow,
time, and variable coefficients depending on the soil type and furrow geometry. Shibata's expression is written as

\[ \log t = (1.608 - 0.106q) + sx(Ks - C) \]

where,

- \( s \) = slope of furrow in percent
- \( x \) = length of furrow in meters
- \( q \) = furrow input in liters per second
- \( t \) = time water requires to travel distance \( x \)
- \( K, C \) = coefficients depending on furrow input and soil characteristics.

The numbers 1.608 and 0.106 were empirically determined.

Philips (1952) has also proposed a logarithmic expression for the rate of horizontal advance of the irrigation stream applicable to soils of Australia. Philip's expression is written as:

\[ x = Aq^{0.72} s^{0.20} \log(1 + Bt) \]

where,

- \( x \) = length of furrow in feet
- \( q \) = furrow input in cubic feet per minute
- \( s \) = slope of the furrow expressed as a decimal
- \( t \) = time in minutes for water to travel distance \( x \)
- \( A, B \) = coefficients depending on soil type and furrow geometry.

In both expressions of Shibata and Philip, the coefficients are determined experimentally for each field.

Hall (1956) presented a numerical method of predicting the rate of horizontal advance of an irrigation stream in a border check. The
equations he obtained were based in general upon accepted laws and equa-
tions of hydraulics. Using the cumulative intake curve, the distance
advanced by the stream is computed for successive equal intervals of time.
However Hall's method is time-consuming, and when stream size is the de-
pendent variable, it can be found only by trial and error.

Beer (1957) using experimental data showed that in furrow irrigation
the irrigation stream advances through a field according to the equation

\[ T = Cd^m \]

where,

- \( T \) = time in minutes after the furrow input has been introduced
- \( d \) = the distance in feet down the furrows the irrigation stream
  has progressed in time \( T \).

However, Beer suggested that further data should be obtained on more soil
types to justify the assumption that two families of curves exist for the
determination of the coefficients \( C \) and \( m \).

Bishop (1961) and Christiansen et al. (1959) presented a relationship
between the intake rate of the soil and the length of the run in surface
irrigation with regard to the amount of water lost below the root zone
through deep percolation. A nomograph is included from which the percent-
age of loss by deep percolation can be estimated.

Bussett (1963) studied a flow system in which water flows at constant
rate into a wide rectangular channel whose bed will remove water at a rate
which is constant with time and distance. Water advanced down this channel
at a progressively slower rate until outflow from the bed was equal to the
inflow at the upstream end. Forward movement on the surface of the porous
medium then ceased and the resulting profile of surface water was fixed. A differential equation was developed to describe this terminal steady state profile. This equation was integrated numerically, and profiles were developed for a variety of flow conditions.

Smerdon and Hohn (1961), Smerdon (1963), and Smerdon and Glass (1964) developed equations which relate the water distribution along surface irrigation runs to three dimensionless ratios, representing time factors, distance along the run, and water application amounts. These equations take into account an assumed effect of the dynamic character of soil infiltration on the water distribution along the run. Curves relating water distribution efficiency to the infiltration function and a dimensionless time factor were also developed. The curves are useful in system design and evaluation from the standpoint of water distribution efficiency.

Water Intake Characteristics of Soils

A mathematical analysis of surface irrigation involves, as one of the principle variables, the intake rate of the soil. The intake rate of water by soils has been studied by numerous investigators. Kostiakov (1932), Lewis (1937), Criddle (1950), Schiff (1953), Aronocici (1954), Hansen (1956), Criddle et al. (1956), and others have shown that the intake rate decreases with time, and that it can be expressed by the simple empirical exponential equation

\[ I = kt^n \]

where,

\[ I = \text{the intake rate at any time} \]
\[ k = \text{a constant, representing the instantaneous intake rate at} \]
time \( t = 1 \)

\( n = \) an exponent which experimental data shows to be negative

with a value between -1 and 0.

Toksoz and Kirkham (1965) studied various processes of two dimensional infiltration under field conditions. They dug in the field, a simple open trench, the bottom of which was maintained ponded with a thin layer of water applied at the rate needed. Based on experimental data obtained on three different soils, the infiltration rate could be represented by the exponential relation, \( I = kt^n \). They found that the parameter \( k \) (of dimensions \( L/t^n \)) appears to depend on both texture and initial moisture content of the soil, while the parameter \( n \) is greater for two dimensional flow than for one dimensional flow. The dimensions of \( k \) depend on \( n \).

Philips (1957, 1958) proposed a theoretical equation with physical significance to describe the infiltration through a homogeneous porous medium. The equation proposed by Philip fits experimental data and is applicable to conditions where long periods are required to obtain the desired infiltration. Philip derived his equation from the diffusion equation. The diffusion equation may be written as

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left[ D(\theta) \frac{\partial \theta}{\partial x} \right] - \frac{\partial K(\theta)}{\partial x}
\]

where,

\( \theta = \) volumetric moisture content

\( K(\theta) = \) the capillary conductivity which is a function of the moisture content

\( x = \) the distance, positive downward

\( D(\theta) = \) the diffusivity which is a function of moisture content
and is defined as

\[ D(\theta) = K \frac{\partial \psi}{\partial \theta} \]

\[ \psi = \frac{p}{\rho g} \]

\( p \) = the pressure
\( \rho \) = density of water
\( g \) = the acceleration of gravity.

In the above, \( D(\theta) \) may also be defined as the flow per unit moisture gradient where the moisture is expressed as a volume percentage.

Equation 1 describes the vertical movement of water in a semi-infinite column of uniform soil where \( x \geq 0 \) and column is initially at a uniform moisture content \( \theta_n \), and subsequently has the plane \( x = 0 \) maintained at moisture content \( \theta_o \). The initial and boundary conditions governing equation 1 are

\[ \theta = \theta_n \quad t = 0 \quad x > 0 \]
\[ \theta = \theta_o \quad x = 0 \quad t > 0 \]  \hspace{1cm} (2)

For \( \theta_o > \theta_n \) equations 1 and 2 describe the phenomenon of infiltration into a vertical column. Philip has shown that the solution of 1 subject to 2 can be expressed in a power series of time, the coefficients of which are functions of the moisture content and are found as solutions to a series of ordinary differential equations by numerical methods.

The solution to 1 is in the form

\[ x = \phi t^{1/2} + \chi t + \psi t^{3/2} + \ldots \]  \hspace{1cm} (3)

with \( \phi = \phi(\theta) \), \( \chi = \chi(\theta) \), and \( \psi = \psi(\theta) \), etc. Series (3) converges so
rapidly that only a few terms are needed. Equation 3 can be used to
determine the moisture profile, cumulative infiltration, and the infiltra-
tion rate since the total change of moisture content in the semi-infinite
column equals the difference between the time integral of the flux at \( x = 0 \)
and at \( x = \infty \). This moisture accumulation is given by

\[
\int_{t=0}^{t=t} v_0 \, dt - \int_{t=t}^{t=\infty} v_\infty \, dt
\]

where \( v_0 \) is the flux at \( x = 0 \), \( v_\infty = \lim_{x \to \infty} v \), and \( t_{\alpha} \) is the end of the period of infiltration. The moisture accumulation is given by \( \int_{t=0}^{t=t} x \, d\theta \), then we have

\[
\int_{t=0}^{t=t} x \, d\theta = \int_{t=0}^{t=t} v_0 \, dt - \int_{t=0}^{t=t} v_\infty \, dt
\]

where in equation 3 \( t \) must now be put \( t=t_{\alpha} \).

The cumulative infiltration \( y \) is given by the integral \( \int_{t=0}^{t=t} v_0 \, dt \). Therefore we find

\[
y = \int_{t=0}^{t=t} v_0 \, dt = \int_{t=0}^{t=t} x \, d\theta + \int_{t=0}^{t=t} v_\infty \, dt
\]

But, since the moisture content \( x=0 \) is \( \theta_n \) and thus \( v_\infty \) does not change over
the time interval \( t = 0 \) to \( t = t_{\alpha} \), then \( v_\infty = K_n \), where \( K_n \) is the capillary
conductivity at moisture content \( \theta = \theta_n \); so we find

\[
\int_{t=0}^{t=t} v_\infty \, dt = \int_{t=0}^{t=t} K_n \, dt = K_n \int_{t=0}^{t=t} dt = K_n t_{\alpha}.
\]

Combining 6 and 7 we have
If equation 3 is integrated with respect to $\theta$, and $t$ is taken to be $t = t^\alpha$

$$\int_0^\theta x \, d\theta = t^{1/2} \int_0^\phi F + t^{3/2}_\alpha \int_0^\phi \chi + \ldots.$$  

(9)

where $t$ is held constant and thus brought outside the integral, and we have

$$\int_0^\theta x \, d\theta = \int_0^\phi \phi \, d\theta , \text{ etc.}$$

Combining 8 and 9 we obtain

$$y = t^{1/2}_\alpha \int_0^\phi F + t^{3/2}_\alpha \int_0^\phi \chi + \ldots.$$  

(10)

If $F$ is denoted by $A$ and $[\int + K_n]$ by $B$, and the series is truncated after the second term, we have after we drop the subscript $\alpha$ from $t$

$$y = At^{1/2} + Bt$$

(11)

which is the cumulative infiltration equation used in the present work.

Differentiation of 11 with respect to $t$ gives the expression

$$\frac{dy}{dt} = \frac{1}{2} At^{-1/2} + B .$$

(12)

Equation 12 gives the infiltration rate which is the flux at $x = 0$. The reliability of Philips' solution has been demonstrated in both the laboratory and field. Youngs (1957) conducted laboratory infiltration measurements on columns of dry porous materials and obtained results which agreed well with those computed by the analytical solution. Nielsen et al. (1961) obtained reasonable agreement between calculated moisture profiles and
those measured on two relatively uniform loess soils.

Discrepancies between measured and calculated results were explained on the basis of non-uniformity of the soil profile. Supported by both laboratory and field results, the analytical solution can be considered valid.
Derivation of the Irrigation Advance Equation

Figure 2 shows a diagram illustrating the problem. We are concerned with the advance of water down an irrigation check (confined rectangular area) when a constant amount of water, cm$^3$ sec$^{-1}$ per cm. of check width, is introduced at its head at time $t = 0$; so that with symbols as defined below, we have

$$Q = yt_x$$  \hspace{1cm} (13)$$

where $t_x$ = time for surface water to advance a distance $x$, $Q$ = total volume of water flows at the head of the check at time $t_x$, and $V$ = volume of water which flows per unit time at the head of the check.

Let $L$ be the width of the irrigation check. Then, by continuity, the total volume of water above and beneath the soil is

$$Vt_x = L \int_0^x (C + y)ds$$  \hspace{1cm} (14)$$

where,

- $C$ = the average depth of surface water and the assumption is taken that $C$ is a constant (see Phillips et al. 1964)
- $y$ = cumulative infiltration (cm$^3$/cm$^2$) at point $x = s$ at time $t$.

Now at the point D in Figure 2 we see that the length of time that the water has covered the surface up to this point is $t = t_x - t_s$ where $t_s$ is the time for the surface water to advance a distance $s$. Thus we see that $y$ in equation 14 is a function of $(t_x - t_s)$.

If we divide equation 14 by $L$ and write $y = y(t_x - t_s)$ we may obtain the expression
Figure 2. Depth of surface water $y$ that has entered vertically at point D into a unit area of soil when the point D is at a distance $s$ from the position 0 of irrigation water application and when the surface water has advanced a distance $x$. The value of $y$ is a function of $t_x - t_s$ where $t_x$ is the time for the surface water to advance the distance $x$ and $t_s$ is the time for the surface water to advance a distance $s$. $C$ is the average surface depth of irrigation water.
SOIL SURFACE
\[
\frac{V}{L} t_x = \int_0^x C \, ds + \int_0^x y(t_x - t_s) \, ds
\]
\[
= \int_0^x C \, ds + \int_0^x y(t_x - t_s) \frac{ds}{dt_s} \, dt_s
\]
\[
= \int_0^x C \, ds + \int_0^t y(t_x - t_s) \, x'(t_s) \, dt_s
\]  \hspace{1cm} (15a)

where we have used the notation

\[
x'(t_s) = \frac{ds}{dt_s}.
\]  \hspace{1cm} (15b)

If we now introduce a symbol \( q \) defined by

\[
q = \frac{V}{L}
\]

such that \( q \) is the amount of water which flows per unit width of check per unit time, then we may, after dropping the subscript \( x \) on \( t \), write equation 15a in the form

\[
qt = Cx + \int_0^t y(t - t_s) \, x'(t_s) \, dt_s
\]  \hspace{1cm} (16)

where \( x' = x'(t_s) \) is a function of \( t_s \).

Philips and Farrell (1964) stated that the identification of the argument of \( y \) as \( (t_x - t_s) \) is valid only if \( x \) is a monotonic increasing function of \( t \).

Physically we see that \( x \) is in fact a monotonic increasing function of \( t \) up to some value of \( t \) and some certain distance (if the irrigation run is long enough) when the soil absorbs all the applied water.

Equation 16 is general. We shall now apply it to some special cases.
Solution for the Infiltration Case, $y = Kt$

The cumulative infiltration equation $y = Kt$ describes the infiltration of irrigation water in a sand soil. Applying $y = Kt$ to equation 16 we find

$$q_t = C x(t) + \int_0^t K(t - t_s) x'(t_s) \, dt_s . \quad (17)$$

Differentiation of 17 with respect to $t$ (see books on advanced calculus) gives, after use of equation 15b, the expressions

$$q = \frac{dx}{dt} + \int_0^t K x'(t_s) dt_s + K(t - t) x'(t_s) - \text{zero}$$

$$= \frac{dx}{dt} + \int_0^t K x'(t_s) dt_s$$

$$= \frac{dx}{dt} + K x(t) - K x(0) .$$

Or, since $x(0) = 0$ the last expression gives

$$q = \frac{dx}{dt} + K x(t) . \quad (18)$$

Equation 18 is an ordinary linear differential equation, it can be written in the form

$$\frac{dx}{dt} + \frac{K}{C} x(t) = \frac{q}{C} . \quad (19)$$

An integration factor for equation 19 is

$$e^{\frac{K}{C} t} = e^{\frac{K}{C} t} .$$

Multiplying equation 19 by this integrating factor we find

$$\frac{K}{C} e^{\frac{K}{C} t} \frac{dx}{dt} + \frac{K}{C} e^{\frac{K}{C} t} x(t) = \frac{q}{C} e^{\frac{K}{C} t} . \quad (20)$$
which gives
\[
\frac{d}{dt} \left( x(t) e^{-\frac{Kt}{C}} \right) = \frac{q}{C} e^{\frac{Kt}{C}}. \tag{21}
\]
Integration of equation 21 gives
\[
x(t) e^{-\frac{Kt}{C}} = \frac{q}{K} e^{\frac{Kt}{C}} + C'. \tag{22}
\]
where \( C' \) is a constant of integration.

When \( t = 0 \) we have \( x = a \). Therefore we find
\[
C' = - \frac{a}{K}
\]
and equation 22 yields the solution
\[
x(t) = \frac{q}{K} \left( 1 - e^{-\frac{Kt}{C}} \right). \tag{23}
\]
Equation 23 gives, when \( t \) approaches infinity, the result
\[
\lim_{t \to \infty} x(t) = \frac{q}{K} \tag{24}
\]
which corresponds to Darcy's law being applied to the surface inflow into an infinitely deep homogeneous saturated soil over the whole distance \( x \).

For this last situation \( K \) in equation 24 is the Darcy law \( K \) for the soil and has dimensions length over time. Equation 24 shows that when we apply a constant amount of irrigation water \( q \) at the head of the irrigation check in sandy soils, the maximum distance that water will advance horizontally is \( q/K \), where \( K \) is the slope of the line when the cumulative infiltration \( y \) (cm\(^3\)/cm\(^2\)) is plotted against time, and \( q \) is the amount of applied irrigation water per unit width of the field per unit time.
Solution for the Infiltration Case, $y = E t^\alpha$

Let $E$ be a soil water entry coefficient and $\alpha$ a dimensionless parameter $0 < \alpha < 1$, then the solution of equation 16, when the cumulative infiltration is given by the equation $y = E t^\alpha$ is that worked out by Philips (1964) and we will discuss the validity of the solution. He used the Falting or Convolution theorem of the Laplace transformation to obtain the solution. Philips' solution is given in terms of an infinite series and is written as

$$\frac{x}{q} = \frac{t}{C} \sum_{n=0}^{\infty} \frac{(-E^\alpha t/C) [\Gamma(1 + \alpha)]^n}{\Gamma(2 + n\alpha)}$$

(25a)

where, $\Gamma$ represents the symbol for the Gamma function and is defined when $\alpha$ is an integer (but here $\alpha$ need not be an integer) by

$$\Gamma(1 + \alpha) = \alpha!$$

All other symbols are as before.

Equation 25a converges absolutely according to the ratio test (see Fulks, 1962, pp. 340, 349). If we apply the ratio test we will have

$$\frac{\frac{E t^\alpha \Gamma(1+\alpha)}{C\Gamma[2+(n+1)\alpha]} \cdot \Gamma(2+n\alpha)}{\frac{E \Gamma(1+\alpha) \cdot t^\alpha}{C} \cdot \frac{\Gamma(2+n\alpha)}{\Gamma(2+n\alpha+\alpha)}}$$

$$= \frac{E \cdot \Gamma(1+\alpha) \cdot t^\alpha}{C} \cdot \frac{1}{(2+n\alpha)^\alpha} \cdot \frac{(2+n\alpha)^\alpha \Gamma(2+n\alpha)}{\Gamma(2+n\alpha+\alpha)}$$

(25b)

but we have

$$\lim_{n \to \infty} \frac{1}{(2+n\alpha)^\alpha} = 0$$

and, proved in Hille (1959, p. 238, eq. 8.8.38), we have

$$\lim_{n \to \infty} \frac{(2+n\alpha)^\alpha \Gamma(2+n\alpha)}{\Gamma(2+n\alpha+\alpha)} = 1$$

so expression 25b$\to 0$ as $n \to \infty$. But unfortunately the computer does not
yield a practical value for equation 25a beyond a specific value of t. So equation 25a is not useful beyond specific value of t.

Although we have not sought a new solution of equation 25a which would be valid for large values of t, we have noticed in equation 25a that if we rearrange the equation as

\[
\frac{C_1}{q_t} = \sum_{n=0}^{\infty} \left[ \frac{(-Et^\alpha/C)(1 + \alpha)}{\Gamma(2 + \alpha)} \right]^{n}
\]

there appear dimensionless quantities \([C_1/(qt)]\) and \(Et^\alpha/C\) which we may plot against each other, and that we can do this even for large values of t if we use experimental value of \(C, x, q, t, E, t,\) and \(\alpha\). This observation is important because we can use our laboratory data to plot \(C_1/(qt)\) versus \(Et^\alpha/C\) and then use these graphs (similar to the way Grover and Kirkham (1964) did) to predict irrigation advance \(x\) and infiltration depth \(y\) in the field. We shall later see that to within experimental error the graph of \(qt/C_1\) versus \(Et^\alpha/C\) is a straight line over the range of values and conditions we used (but the straight line does not go through the origin).

Equations of the form \(qt/C_1 = a + b Et^\alpha/C\) were derived using our model experimental data and will be discussed in the Results and Discussion Section.

Solution for the Infiltration Case, \(y = At^{1/2} + Bt\)

The infiltration equation \(y = At^{1/2} + Bt\) has been enlarged in this thesis. The first term on the right-hand side of this infiltration equation, as Philips (1956) has shown, represents the contribution arising from capillarity, while the second term consists, essentially, of that arising from gravity.

The equation \(y = At^{1/2} + Bt\) was used by Philips (1964). Philips'
solution is not suitable for numerical computation so we obtain the solution in a different form as follows.

We use the following symbols (Churchill, 1941, 1958, 1960; Holl, Maple and Vinograde, 1959) for the Laplace transforms (where $L, F, f, s$ and $t$ have no relation to preceding symbols):

\[
\hat{L}(F) = \int_0^\infty e^{-st} F(t) \, dt = f(s)
\]

(27)

\[
F(t) = L^{-1}\{f(s)\}
\]

(28)

Applying the convolution theorem (Holl, Maple and Vinograde, 1959, pp. 43-49), and taking the Laplace transformation of equation 16, we get (with $q, s, x, x', y$ as in equation 16 the expression

\[
\frac{q}{s^2} = CL\{x\} + L\{x'\} L\{y\}
\]

\[
= CL\{x\} + [sL\{x\} - x(0)]L\{y\}
\]

\[
= CL\{x\} + sL\{x\} L\{y\}
\]

(29)

Since $x(0) = 0$ we see that equation 29 reduces to

\[
\frac{L\{x\}}{q} = \frac{1}{s^3L\{y\} + C s^2}
\]

(30)

in which we need a value of $L\{y\}$ where we now have by hypothesis the expression

\[
y = At^{1/2} + Bt
\]

From a handbook of Laplace transforms (as Holl, Maple and Vinograde, 1959, p. 148, third entry with $\alpha = 1$ and $\alpha = 1/2$) we find

\[
L\{y\} = AL\{t^{1/2}\} + BL(t)
\]

\[
= \frac{A\Gamma(1/2 + 1)}{s^{1/2} + 1} + B \frac{1}{s^2}
\]

(31)
But we have, when $x$ need not be an integer, the relation

$$\Gamma(x + 1) = x\Gamma(x), \quad \Gamma(1/2) = \pi^{1/2}$$

so that equation 31 becomes

$$L\{y\} = \frac{A\pi^{1/2}}{2} \frac{1}{s^{3/2}} + \frac{B}{s}$$

Combining equations 30 and 32, we have

$$\text{L}\{x\} = \frac{1}{s^3[\frac{1}{2}\pi^{1/2}A^{3/2} + \pi^{1/2}B^2 + \pi^{1/2}C^2]}$$

or

$$\text{L}\{x\} = \frac{1}{s^3[\frac{1}{2}\pi^{1/2}A^{3/2} + \pi^{1/2}B^2 + \pi^{1/2}C^2]}$$

To put equation 34 in an easy form for getting the inverse of the Laplace transform, two quantities are introduced:

$$\gamma = \frac{1}{4C} [\pi^{1/2}A + (\pi A^2 - 16 BC)^{1/2}]$$

$$\beta = \frac{1}{4C} [\pi^{1/2}A - (\pi A^2 - 16 BC)^{1/2}]$$

We now note the relation

$$\gamma \cdot \beta = \frac{1}{16C^2} [\pi A^2 - \pi A^2 + 16 BC] = \frac{B}{C}$$

and the relation

$$\gamma + \beta = \frac{1}{4C} [\pi^{1/2}A + (\pi A^2 - 16 BC)^{1/2} + \pi^{1/2}A - (\pi A^2 - 16 BC)^{1/2}]$$

$$= \frac{\pi^{1/2}A}{2C}$$
\[
\gamma - \beta = \frac{1}{4C} \left[ \pi^{1/2}A + (\pi A^2 - 16 BC)^{1/2} \right] - \frac{1}{4C} \left( \pi^{1/2}A \right) + \frac{1}{4C} \left( \pi A^2 - 16 BC \right)^{1/2}
\]

\[
= \frac{1}{2C} \left( \pi A^2 - 16 BC \right)^{1/2}. \tag{38b}
\]

Going back to equation 34, we write

\[
\frac{X}{q} = \mathcal{L}^{-1} \left\{ \frac{\gamma - \beta}{Cs[s + s^{1/2}(\gamma + \beta) + \gamma\beta]} \right\}
\]

\[
= \mathcal{L}^{-1} \left\{ \frac{\gamma - \beta + s^{1/2} - s^{1/2} - 2^{1/2}}{Cs[\gamma - \beta][s + s^{1/2} + s^{1/2} + \gamma\beta]} \right\}
\]

\[
= \mathcal{L}^{-1} \left\{ \frac{[s^{1/2} + \gamma] - [s^{1/2} + \beta]}{Cs(\gamma - \beta)[s^{1/2} + \gamma][s^{1/2} + \beta]} \right\}
\]

\[
= \mathcal{L}^{-1} \left\{ \frac{1}{C(Y - \beta)} \frac{1}{s(s^{1/2} + \beta)} - \frac{1}{s(s^{1/2} + \gamma)} \right\}. \tag{40}
\]

To get the inverse of the Laplace transform of equation 40, from equation 86, p. 290, 39th ed. of Handbook of Chemistry and Physics, we have
\[ L^{-1}\left\{ \frac{e^{-hs^{1/2}}}{s(s^{1/2} + a)} \right\} = -e^{ah} e^{2t} \text{erfc}(at^{1/2} + \frac{h}{2t^{1/2}}) + \text{erfc}(\frac{h}{2t^{1/2}}). \]  

(41)

In our case \( h = 0 \), so

\[ L^{-1}\left\{ \frac{a}{s(s^{1/2} + a)} \right\} = -e^{2t} \text{erfc}(at^{1/2} + 0) + 1 = 1 - e^{2t} \text{erfc} \left(\frac{at}{^2}\right). \]  

(42)

Equation 40 can be written as

\[ \frac{x}{q} = \frac{1}{(\gamma - \beta)c} L^{-1}\left\{ \frac{1}{\beta} \left[ \frac{1}{s(s^{1/2} + \beta)} - \frac{1}{\gamma} \right] \right\}. \]  

(43)

Now we can take the inverse of equation 43, so we have

\[ \frac{x}{q} = \frac{1}{(\gamma - \beta)c} \left[ \frac{1}{\beta} (1 - e^{\beta t} \text{erfc} \left(\frac{at}{^2}\right)) - \frac{1}{\gamma} (1 - e^{\gamma t} \text{erfc} \left(\frac{gt}{^2}\right)) \right]. \]  

(44)

Equation 44 is the solution for equation 33, it can be simplified further as follows:

\[ \frac{x}{q} = \frac{1}{(\gamma - \beta)c} \left[ \frac{1}{\beta} - \frac{1}{\beta} e^{\beta t} \text{erfc} \left(\frac{at}{^2}\right) - \frac{1}{\gamma} + \frac{1}{\gamma} e^{\gamma t} \text{erfc} \left(\frac{gt}{^2}\right) \right]. \]

\[ = \frac{1}{(\gamma - \beta)c} \left[ \frac{\beta - \gamma}{\beta \gamma} e^{\beta t} \text{erfc} \left(\frac{at}{^2}\right) - \frac{\gamma - \beta}{\gamma} e^{\gamma t} \text{erfc} \left(\frac{gt}{^2}\right) \right]. \]

\[ = \frac{1}{\beta \gamma c} \left[ (\beta - \gamma) e^{\beta t} \text{erfc} \left(\frac{at}{^2}\right) - (\gamma - \beta) e^{\gamma t} \text{erfc} \left(\frac{gt}{^2}\right) \right]. \]  

(45)

But from equation 37 we have \( \beta \gamma c = B \).

Then equation 45 takes the final form
\[
\frac{x}{q} = \frac{1}{B - i} \left[ 1 - \frac{1}{\gamma - \beta} [\gamma e^{\beta t} \text{erfc} \beta t^{1/2} - \beta e^{\gamma t} \text{erfc} \gamma t^{1/2}] \right]. \tag{46}
\]

Equation 46 is the solution for equation 34 when \( \gamma \) and \( \beta \) are both real numbers, but when \( c > \gamma A^2/(16B) \) both \( \gamma \) and \( B \) will be complex and will take the form

\[
\gamma = \frac{1}{4c} \left[ n^{1/2} A + i(16 B C - \Pi A^2)^{1/2} \right]; \tag{47}
\]

\[
\beta = \frac{1}{4c} \left[ n^{1/2} A - i(16 B C - \Pi A^2)^{1/2} \right]. \tag{48}
\]

But from complex variables, where \( i = (-1)^{1/2} \), if we define \( W(u) \) and \( W(u)^* \) by the expression

\[
W(u) = u(x,y) + iu(x,y)
\]

\[
W(u)^* = u(x,y) - iu(x,y)
\]

then \( W(u)^* \) signifies the conjugate of \( W(u) \) and equation 47 is the conjugate of equation 48, or we have the relation \( \gamma = \beta^* \).

Then our solution for \( c > \gamma A^2/(16B) \) becomes

\[
\frac{x}{q} = \frac{1}{B - i} \left[ \frac{1}{C_B} (1 - e^{\beta t} \text{erfc} \beta t^{1/2}) - \frac{1}{C_B} (1 - e^{\gamma^* t} \text{erfc} \gamma^* t^{1/2}) \right]. \tag{49}
\]

Equation 49 is very complicated, so by making use of complex variable theory, it was possible to derive a simple algebraic equation which is easy to use in calculation. From the experimental data, it was found that the condition \( C > \gamma A^2/(16B) \) is the one which is mostly encountered in the field, which makes the following derivation very useful.

We have

\[
\beta^* = u + iv \tag{50}
\]

\[
\beta = u - iv \tag{51}
\]

where \( u \) = the real part of \( \beta^* \), and \( v \) = the imaginary part. We let \( z \) and \( g \)
be defined by
\begin{align}
z &= i\beta t^{1/2} = (iu + v)t^{1/2} + -iz = \beta t^{1/2} \\
g &= i\beta^* t^{1/2} = (iu - v)t^{1/2} + -ig = \beta^* t^{1/2} \\
z^2 &= (i\beta t^{1/2})^2 = -\beta^2 t - z^2 = \beta^2 t \\
g^2 &= (i\beta^* t^{1/2})^2 = -\beta^2 t + -g^2 = \beta^2 t \\
[1 - e^{\beta^2 t} \text{erfc} \beta t^{1/2}] &= [1 - e^{-z^2} \text{erfc} (-iz)] \\
[1 - e^{\beta^2 t} \text{erfc} \beta t^{1/2}] &= [1 - e^{-g^2} \text{erfc} (-ig)].
\end{align}

So going back to equation 49 and substituting for equations 56 and 57 we will have
\begin{equation}
x = \frac{1}{\theta^* - \theta} \left\{ \frac{1}{\beta c} \left[ 1 - e^{-z^2} \text{erfc} (-iz) \right] - \frac{1}{\beta c} \left[ 1 - e^{-g^2} \text{erfc} (-ig) \right] \right\}. \quad (58)
\end{equation}

If we let
\begin{align}
w(z) &= e^{-z^2} \text{erfc} (-iz) \\
w(g) &= e^{-g^2} \text{erfc} (-ig)
\end{align}
equation 58 will take the form
\begin{equation}
x = \frac{1}{\theta^* - \theta} \left[ \frac{1}{\beta c} w(z) - \frac{1}{\beta c} w(g) \right]. \quad (59)
\end{equation}

But we have
\begin{equation}
\theta^* \theta = (u + iv)(u - iv) = u^2 - iuv + iuv - i^2 v^2 = u^2 + v^2. \quad (60)
\end{equation}
\[ \beta^* - \beta = u + iv - u + iv = 2iv \]  \hspace{1cm} (63)

Equation 61 becomes

\[
\frac{x}{q} = \frac{1}{(2iv)^C} \left\{ \frac{[u + iv][1 - w(z)] - [u - iv][1 - w(g)]}{u^2 + v^2} \right\}
\]

\[
= \frac{1}{(2iv)^C} \left\{ \frac{1 - R(w_z) + iI(w_z)}{u^2 + v^2} \frac{1 - R(w_g) - iI(w_g)}{u^2 + v^2} \right\} \hspace{1cm} (64)
\]

where

- \( R(\bar{w}_z) \) = real part of the function \( w(z) \)
- \( I(\bar{w}_z) \) = imaginary part of the function \( w(z) \)
- \( R(w_g) \) = the real part of the function \( w(g) \)
- \( I(w_g) \) = the imaginary part of the function \( w(g) \).

To establish the relation between \( R(w_z) \), \( R(w_g) \), \( I(w_z) \) and \( I(w_g) \) we proceed as follows:

\[
w(z) = w[t^{1/2}(v + iu)] = e^{-t(v + iu)^2} \int_{-\infty}^{\infty} e^{-t^2} dt \hspace{1cm} (65)
\]

\[
w(g) = w[t^{1/2}(-v + iu)] = e^{-t(-v + iu)^2} \int_{-\infty}^{\infty} e^{-t^2} dt \hspace{1cm} (66)
\]

\[
e^{-t(-v + iu)^2} = e^{-t(v^2 - u^2 - 2iuv)} \hspace{1cm} (67)
\]

\[
e^{-t(v + iu)^2} = e^{-t(v^2 - u^2 + 2iuv)} \hspace{1cm} (68)
\]

then

\[
e^{-t(-v + iu)^2} = e^{-t(v + iu)^2} \hspace{1cm} (69a)
\]

or

\[
e^{-t(-v + iu)^2} = \text{the conjugate of } e^{-t(v + iu)^2} \hspace{1cm} (69b)
\]

and
\[ \frac{2}{\pi} \int_{0}^{\infty} e^{-t^2} dt = \text{the conjugate of } \frac{2}{\pi} \int_{0}^{\infty} e^{-t^2} dt \quad (70). \]

But from complex variables we have,

\[ \frac{F}{FF^*} = \frac{F}{FF} \quad (71) \]

so from equations 65 to 71 we find

\[ [w(g)] = [w(z)]^* \quad (72) \]

or we find

\[ R(w_g) + i I(w_g) = R(w_z) - i I(w_z) \quad (73) \]

Then

\[ R(w_g) = R(w_z) \quad (74) \]
\[ I(w_g) = - I(w_z) \quad (75) \]

which is a very useful result. Applying equations 74 and 75 to equation 64 we will have

\[ \frac{\chi}{q} = \frac{1}{(2i\nu)^2} \left[ \frac{2iv - 2iuv(w_g) - 2ivR(w_z)}{u^2 + v^2} \right] \quad (76) \]

so our final algebraic equation is

\[ \frac{\chi}{q} = \frac{v[1 - R(w_z)] - u I(w_z)}{C \nu(u^2 + v^2)} \quad (77) \]

where

\[ u = R(\beta) = \frac{\pi^{1/2} A}{4C} \]
\[ v = I(\beta) = \frac{(16 BC - 4A^2)^{1/2}}{4C} \]

and \( w(z) = e^{-z^2} \text{erfc}(-iz) \), a function that is tabulated in Faddeyeva and Terentev (1961). To match the notation of Faddeyeva and Terentev with the notation used here we have
\[ X_F^{\text{in Faddeyeva}} = \frac{t^{1/2} \cdot (16BC - \Pi A)^{1/2}}{4C} \]

\[ Y_F^{\text{in Faddeyeva}} = \frac{t^{1/2} \cdot \Pi^{1/2} A}{4C} \]

So the values of \( X_F \) and \( Y_F \) are calculated for each given time \( t \) from the constant \( A \) and \( B \) which are given in the cumulative infiltration equation. Once we calculate \( X_F \) and \( Y_F \), values for \( \Re(\hat{w}_z) \) and \( \Im(\hat{w}_z) \) are obtained from Faddeyeva tables, and \( \frac{X}{q} \) can easily be obtained by equation 77.
EXPERIMENTAL EQUIPMENT AND PROCEDURE

The Irrigation Model

Figure 3 shows a schematic diagram of the model used in this study. The model was designed, so that the water stream penetrates the porous medium, and at the same time advances horizontally on the surface of the porous medium. The model was constructed from plexiglas, a transparent and craze-resistant plastic material. A model thickness of 1.9 cm. was obtained by use of triple thickness spacers of the same plexiglas material. These spacers were continuous strips of 2.54 cm. width along the ends of the model and along the bottom. The inside length of the model was 203 cm. and it was 28 cm. deep. The left end (Figure 3) of the model was shortened 1 cm. to allow an overflow from a simulated irrigation ditch which was 5 cm. deep and 4 cm. wide, affixed to the side of the model. The ditch was used to supply the model with a constant feeding quantity of water. Ten small holes for dye injection were drilled in the front of the model. They were 20 cm. apart and 8 cm. from the top. The dye used was potassium dichromate, as was used by Kirkham (1940). It colored the stream lines that developed when the water penetrated the porous medium. At the back of the model five rows of holes were drilled. In each row there were four holes, where rubber stoppers with capillary tubes were fitted. The rows were 40 cm. apart and the four holes in each row were five cm. apart. These holes were used for sampling for moisture determination at the end of each run. This set of holes plus another set of 20 holes in the bottom of the model allowed escape of the air during infiltration of water through the porous medium so back pressure would not
Figure 3. Schematic diagram of the model.
Q = CONSTANT
FRONT OF ADVANCING STREAM
SURFACE OF POROUS MEDIUM
HEAD DITCH
DITCH X
WET ZONE
SMALL HOLES FOR DYE INJECTION
(INSIDE WIDTH OF MODEL IS 19 cm.)
IMPERMEABLE 203 cm.

28 cm
3 cm
2 cm

develop due to air entrapment (see Youngs and Peck 1964).

In order to get soil moisture samples all at the same instant, from
the columns of vertical holes at the back of the model, five sampler
devices were constructed from plexiglas and brass tubes. Each of these
five sampler devices fitted a set of four vertical holes exactly and
twenty moisture samples were taken simultaneously at the end of an irri­
gation test.

Construction of the Water Feeding System

A system to provide simulated irrigation water at constant rate to
the head ditch (Figure 3) was constructed. The principal item of the
system was a cylinder (not shown in Figure 3) constructed from plexiglas.
At the bottom of the cylinder two holes were drilled and two capillary
tubes were fixed by rubber stoppers. The radius and length of the capil­
lar tubes were the same for a particular run. Different radius capillary
tubes were used for different runs, to obtain different rates of water
application. The length of the capillary tubes was 4 cm. and the radius
of them varied from one run to the other.

Actually the feeding system was designed on the basis of Poiseuille's
Law (1840) of capillary flow. This law in symbols is

\[ Q = \frac{r^4P}{8\pi L} \]

where

- \( Q \) = the quantity of fluid flowing per unit time across a
capillary tube
- \( r \) = the radius of the capillary tube
- \( P \) = the differences in pressure across the capillary tube
- \( L \) = the length of the tube
\[ n = \text{the viscosity of the fluid.} \]

By changing one of these variables in the equation, the rate of flow through the capillary tubes per unit time could be changed. In the present water feeding system different sets of capillary tubes of different radius were used; also the head of water in the cylinder could be changed, so \( P \) the difference in pressure across the capillary tubes could be changed. The required head of water in the cylinder was maintained constant by a "Mariotte" flask device.

Porous Mediums Used in the Model and Their Infiltration Characteristics

Porous mediums of different infiltration characteristics were used in the model. Three sizes of glass beads with an average diameter of 100, 50 and 28 μ were used. Two different soils, Ida silt loam and Webster clay loam, were used also. The glass beads are easy to work with and they are commercially available in convenient sizes. The individual beads are nearly spherical. The glass beads used in this study were purchased from the Minnesota Mining Company. Figure 4 shows the moisture release curves for the beads. The curves show that the major portion of water is held between zero and 100 cm. water tension for beads with an average diameter \( D \) of 100 μ; while for the other two sizes of beads the major portion of water is held between zero and 250 cm. water tension. The large rapid decrease in moisture content with increasing tension below the specific tensions, 100 or 250 cm. of water, for each of the three curves, is indicative of large pores which are readily drained by gravity. The flat portion of the curves above 250 cm. of water tension shows that a slight decrease in glass bead-water content causes a rapid increase in moisture
Figure 4. Moisture release curves for glass beads. D indicates the nominal diameter of the beads.
tension. The rapid decrease is attributed to the small range of pore size distribution: the beads are nearly the same size.

The Ida soil used in this investigation was obtained from the surface 10 inches at the Western Iowa Experimental Farm near Castana, Iowa, while the Webster soil was obtained from the surface 10 inches of Webster loam at the Agronomy Agricultural Engineering Farm, Ames. The soil was air dried, ground, passed through a 2 mm. sieve, mixed and stored until used. A routine laboratory analysis was made to determine texture, particle density and moisture release characteristics. The results are listed in Table 1, the moisture release curves are plotted in Figure 5.

Table 1. Characteristics of Ida and Webster Soils

<table>
<thead>
<tr>
<th>Property</th>
<th>Method</th>
<th>Soil</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ida</td>
</tr>
<tr>
<td>Texture</td>
<td>Hydrometer</td>
<td>47.6 sand (D=2-.02mm)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>26.8 silt (D=.02-.002mm)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20.6 clay (D&lt;.002mm)</td>
</tr>
<tr>
<td>Particle density</td>
<td>Picnometer</td>
<td>2.51 gm/cm³</td>
</tr>
<tr>
<td>Organic carbon</td>
<td>Wet combustion</td>
<td>1.69 cm/100 gm</td>
</tr>
<tr>
<td>Moisture release</td>
<td>Pressure cell</td>
<td>See Figure 5</td>
</tr>
</tbody>
</table>

See Methods of Soil Analysis (Black, 1965)

Cumulative infiltration data for each porous medium was obtained by isolating a section from the model, packing this section with glass beads or soil, ponding a layer of water on the surface and measuring with time, the volume of water that was absorbed vertically by the section of porous
Figure 5. Moisture release curves for Ida and Webster soils.
medium from the surface applied water (using the Mariotte device). The infiltration data were obtained this way to characterize the porous medium in the model for use when later "irrigation" runs were to be conducted. Precaution was taken when both the infiltration section and the whole model were packed with porous mediums. The infiltration data for different sizes of glass beads are shown in Figures 6 to 8. For glass beads the infiltration data fitted the equation \( y = E t^a \) where \( y \) is the cumulative infiltration in \( \text{cm}^3/\text{cm}^2 \), \( t \) is time in minutes, and \( E \) and \( a \) are the constants. The infiltration data for both Ida and Webster soil are shown in Figures 9 and 10. The cumulative infiltration is higher for the Ida soil than for the Webster soil. We see from the curves on Figures 9 and 10 that the cumulative infiltration from both soils fits Philip's infiltration equation, \( y = A t^{1/2} + Bt \), where \( y \) is the cumulative infiltration in \( \text{cm}^3/\text{cm}^2 \), \( t \) is time in minutes and \( A \) and \( B \) are the constants shown on the figures.

Test for Homogeneity of Packing of the Model with Porous Medium

With as large a model as the one used in this study it is desirable to assure that the packing of the model with porous medium is as homogeneous as possible throughout the whole model. To test for homogeneity of packing, the model was packed with porous medium and three locations (sections of the model) were chosen to run infiltration tests. Two locations were chosen close to both ends of the model, the third location was at the center. The model was packed twice, and the same way of packing was applied each time. Three infiltration tests were conducted each time the model was packed, making a total of six infiltration homogeneity tests.
Figure 6. Cumulative infiltration for initially air-dry 100-micron size glass beads.
GLASS BEADS

\[ D = 100 \mu \]

\[ y = E t^a \]

\[ K = 2.3 \text{(cm)}(\text{min})^{-0.656} \]

\[ a = 0.656 \]
Figure 7. Cumulative infiltration for initially air-dry 50 micron glass beads.
GLASS BEADS

$D = 50 \mu$

$y = Et^\alpha$

$K = 1.58 \text{(cm}\cdot\text{min}^{-0.568})$

$\alpha = 0.568$
Figure 8. Cumulative infiltration for initially air-dry 28 micron glass beads.
GLASS BEADS

\( D = 28 \mu \)

\[ y = E t^{a} \]

\( K = 1.69 (\text{cm})(\text{min})^{0.527} \)

\( a = 0.527 \)
Figure 9. Cumulative infiltration for initially air-dry Ida soil aggregates.
CUMULATIVE INFILTRATION (cm$^3$/cm$^2$)

IDA SOIL

$Y = A \sqrt{t} + B t$

A = 0.0605 (cm/kmin)$^{1/2}$
B = 0.0986 (cm/kmin)$^{-1}$

$\text{t (min.)}$

4  8  12  16
Figure 10. Cumulative infiltration for initially air-dry Webster soil aggregates.
WEBSTER SOIL

\[ y = At^{1/2} + Bt \]

\[ A = 0.48 \text{(cm)(min)}^{-1/2} \]
\[ B = 0.034 \text{(cm)(min)}^{-1} \]
Figures 11 and 12 show the results of these six infiltration tests. The fact that one curve could be drawn through all of the experimental points of the six infiltration tests, is an indication of the homogeneity of packing which could be achieved. These homogeneity tests were for the glass beads. Unfortunately there was not a large enough stock of the soil aggregates to make homogeneity tests for the Ida and Webster soils.

Geometrical Verification and Fluid Characteristics

When one works with models or permeameters, the size of the voids at the boundaries where the spherical beads contact the plane walls of the model might be different than the size of voids within the body of the porous medium. According to Franzini (1956) — see also McNeal and Reeve (1964) — when one uses spherical shot in a cylinder permeameter the effect on permeability of the large pores next to the wall where the packing is less dense will be negligible when the ratio $D/d$ is equal to 40 or more, where $D$ is the diameter of the permeameter and $d$ is the diameter of the shot. From the dimensions of the present model, if we assume that $D$ is equal to the width of the model, 1.9 cm, then the ratio $D/d$ will be very much higher than the critical value 40 for the three sizes of beads used in the model.

The model experiment was carried out in a constant temperature room in the Agronomy Basement, where the temperature, 25°C, was controlled within ± 1°C. Thus the fluid characteristics, viscosity and mass density would all have the same effect for all the experimental runs.
Figure 11. Results of three infiltration tests for homogeneity of glass bead packing at three different locations in the model; each circle point for a certain time $t$ represents a test for a different location in the model.
FIRST PACKING

GLASS BEADS

CUMULATIVE INFILTRATION (cm$^3$/cm$^2$)

$\begin{align*}
20.0 \\
15.0 \\
10.0 \\
5.0 \\
2.5 \\
0.0
\end{align*}$

$t$ (min.)

$\begin{align*}
2 \\
4 \\
6 \\
8 \\
10
\end{align*}$
Figure 12. Same as Figure 11, except for a second packing of the model with glass beads.
Arrangement for Taking Photographs

The glass beads reflect light when surrounded by air, but when surrounded by water the light is transmitted. So by using a black background in the model it was possible to record results by photography. A "speed graphic" camera with cable release was found to be satisfactory. The 3.25 x 4.25 inches film sheets used with this camera were large enough to use for contact prints and for enlargements. It was essential to have a good light source, but it must not heat the model. The light was arranged to avoid reflective glare from the surface of the model. Light objects back of the camera were avoided to prevent reflections on the surface of the model. The position of the camera with respect to the model must be absolutely fixed, so that the images on the film will be the same size. A concrete stand was established which worked well, as the camera could be bolted to the stand rigidly.

Experimental Procedure

During operation, the model was packed with beads or soil to provide the desired geometry. Care was taken to pack the model with the porous medium to obtain homogeneous packing as nearly as possible. The surface of the porous medium in the model was leveled. Crystals of potassium dichromate were placed at different positions to show the velocity and direction of stream lines. The feeding system was calibrated to give the required rate of simulated irrigation water for each run. The feeding system was then mounted on a stand and the constant head capillary tube apparatus was adjusted to supply the small ditch with the required rate of water for a run. Photographs were taken at several time intervals as
the water advanced horizontally on the surface of the porous medium. The zero time was the instance the water passes from the ditch to the porous medium. Enough photographs were obtained so that the entire range of the horizontal advance would be evaluated for each run. A clock running counterclockwise was mounted with the model to show the time lapsed between photographs.

Two Types of Experiments

There were two types of experiments:

Type 1 — Three groups of glass beads which have an infiltration equation of the form \( y = E_t^\alpha \) were used to provide the desired homogeneous medium. For each group size of beads runs were conducted, with the combination of 2 rates of application of simulated irrigation water, and for each rate 2 slopes in the model were used, 0 percent and 1 percent slope. For each slope, two different surface characteristics were used, one where the surface was level without any surface roughness, the other, where some means to create surface roughness were used. The surface roughness was obtained by spreading on the surface of the porous medium in the model large glass beads of 5 mm. diameter. Surface roughness was not applied with the finest size of glass beads. The objective of this experiment was to investigate the effect of the rate of application of water, slope of the surface, and the surface roughness on the rate of advance of the stream of water on the surface, when the porous medium is characterized by the infiltration equation of the type \( y = E_t^\alpha \).

Type II — In this experiment two different soils, Ida soil and Webster soil were used to provide the desired geometry in the model. The
infiltration equation for the two soils was of the form \( y = At^{1/2} + Bt \). For Ida soil two different rates of application of simulated irrigation water were used, and two different slopes in the model were used, 0 percent slope and 1 percent slope. For Webster soil one rate of application of simulated irrigation water was used, and two slopes in the model were used, 0 percent slope and 1 percent slope.

Field Infiltration

During the summer of 1962, an infiltrometer was used by research personnel of the soil and water conservation unit of the Agronomy Department (Edwards and others, 1964) to measure water infiltration in the field for three Iowa soils. The three soils were: Ida silt loam, Moody silt loam, and Grundy silt loam. Within each soil type measurements were made on a bare surface, representing a newly prepared corn seed bed, and on a well established bromegrass sod. These two surface conditions are referred to in subsequent figures as "corn" and "bromegrass". Three replications of each surface condition were established, and within each replication two representative sites were selected for measuring infiltration. At each site an initial run was made on dry soil and another run on moist soil. The infiltration equation of the form \( y = At^{1/2} + Bt \) gave the "best fit" to the observed infiltration values. These infiltration equations, obtained under the field conditions, were used to develop theoretical curves for the horizontal advance of the irrigation water of different rates for those field conditions. The curves were developed for the different soil types, for different antecedent moisture contents, and for different depths of
"surface storage" (water thickness on the soil surface). These curves should be useful in irrigation design. Examples for such usefulness are given in later sections.
RESULTS AND DISCUSSION

Horizontal Advance of Water in the Model with Glass Beads as Porous Mediums

For the glass beads and soils, the composite photographs of the model runs are given in Figures 62 to 85 of the Appendix (except for two runs where the data were read directly during the runs because photograph films were not available). From these photographs, graphs of the distance of horizontal advance of water on the surface of beads in the model versus time have been plotted. The zero time was the instant the water was applied on the surface of beads. For each size of glass bead there was an infiltration equation of the type \( y = Et^a \), where \( y \) is the accumulative infiltration in \( \text{cm}^3/\text{cm}^2 \), \( t \) is time in minutes, \( E \) and \( a \) are constants. These infiltration equations were determined for each size of beads by infiltration tests. The values of \( E \) and \( a \) obtained from the infiltration data were used in equation 25a to calculate the theoretical curve for each of the appropriate experimental runs in the model. In Figures 13 to 22 the solid lines are calculated by theory, while the scattered points are those obtained from the experimental runs. On each graph the geometry and the parameters which distinguish each experimental run are shown. The surface roughness and slope \( S \) — for small values of \( S \) — do not appear explicitly in the theory (but implicitly through the distance \( x \) — see Figure 2 — and the thickness of surface water \( C \)). Note that parts of the calculated curves of Figures 13 to 22 are solid lines. These lines correspond to theoretical portions of the curves for which the computer gave values for equation 25a. The dashed parts of the curves were drawn by extrapolation. For example, in Figure 15 we see that \( t = 7 \) minutes is in the dashed portion of the
Figure 13. Comparison of the experimental and theoretical horizontal distance of advance $x$ of the flooding irrigation water on the porous medium surface in the model, when glass beads are used; the dashed portion of the curve is not theoretical because the computer does not yield a practical value for equation 25a for values of $t$ in the dashed region. C(=1) the value averaged from individual values of $C$ for each location $x$ plotted. The value of $C$ is not critical. Theoretical curves for $C = 0.9$ and for $C = 1.3$ are shown for comparison.
\[ C = 0.9 \quad \triangle \quad \triangle \quad \triangle \]
\[ C = 1 \quad \text{THEORETICAL} \]
\[ C = 1.3 \quad \square \quad \square \quad \square \]
\[ \circ \quad \circ \quad \circ \quad \text{EXPERIMENTAL} \]

**Glass Beads**

- \( y = E t^a \)
- \( E = 2.3 \text{ (cm.)/(min.)} \)
- \( a = 0.656 \)
- \( C = 1.0 \text{ cm.} \)
- \( Q = 300 \text{ cm}^3/\text{min.} \)
- \( L = 1.9 \text{ cm.} \)
- \( q = \frac{Q}{L} = 157.89 \text{ cm}^2/\text{min} \)
- No surface roughness - \( S = 0\% \)
Figure 14. The same as Figure 13, except that the experimental conditions are as shown.
GLASS BEADS

$y = E t^a$

$E = 2.3 \text{ cm.}(\text{min.})^{0.656}$

$a = 0.656$

$C = 1.3 \text{ cm.}$

$Q = 300 \text{ cm.}^3/\text{min.}$

$L = 1.9 \text{ cm.}$

$q = Q/L = 157.89 \text{ cm.}^2/\text{min.}$

WITH SURFACE ROUGHNESS

$S = 0\%$

$X$, HORIZONTAL ADVANCE (cm.)

$t$ (min.)
Figure 15. The same as Figure 13, except that the experimental conditions are as shown.
GLASS BEADS

$y = E t^\alpha$

$E = 2.3 \text{ (cm.})(\text{min.})^{-0.656}$

$\alpha = 0.656$

$C = 1.2 \text{ cm.}$

$Q = 400 \text{ cm}^3/\text{min.}$

$L = 1.9 \text{ cm.}$

$q = Q/L = 210.52 \text{ cm}^2/\text{min.}$

NO SURFACE ROUGHNESS

$S = 0\%$
Figure 16. The same as Figure 13, except that the experimental conditions are as shown.
GLASS BEADS

$y = E_t^\alpha$

$E = 2.3 \text{ (cm.)(min.)}^{-0.656}$

$\alpha = 0.656$

$C = 1.2 \text{ cm.}$

$Q = 450 \text{ cm}^3/\text{min.}$

$L = 1.9 \text{ cm.}$

$q = Q/L = 236.84 \text{ cm}^2/\text{min.}$

WITH SURFACE ROUGHNESS

$S = 0\%$
Figure 17. The same as Figure 13, except that the experimental conditions are as shown.
GLASS BEADS

\[ y = E t^a \]

\[ E = 1.58 \text{(cm.)}(\text{min.})^{-0.568} \]

\[ a = 0.568 \]

\[ C = 1.1 \text{ cm.} \]

\[ Q = 200 \text{ cm.}^3/\text{min.} \]

\[ L = 1.9 \text{ cm.} \]

\[ q = Q/L = 105.26 \text{ cm}^2/\text{min.} \]

NO SURFACE ROUGHNESS

\[ S = 0\% \]
Figure 18. The same as Figure 13, except that the experimental conditions are as shown.
GLASS BEADS

\[ y = E t^\alpha \]

\[ E = 1.58 \text{(cm.)(min.)}^{-0.568} \]

\[ \alpha = 0.568 \]

\[ C = 1.3 \text{ cm.} \]

\[ Q = 210 \text{ cm}^3/\text{min.} \]

\[ L = 1.9 \text{ cm.} \]

\[ q = \frac{Q}{L} = 110.52 \text{ cm}^2/\text{min.} \]

WITH SURFACE ROUGHNESS

\[ S = 0\% \]
Figure 19. The same as Figure 13, except that the experimental conditions are as shown.
GLASS BEADS

$y = E t^a$

$E = 1.58 \text{ cm.}(\text{min.})^{-0.568}$

$a = 0.568$

$C = 1.1 \text{ cm.}$

$Q = 300 \text{ cm}^3/\text{min.}$

$L = 1.9 \text{ cm.}$

$q = Q / L = 157.89 \text{ cm}^2/\text{min.}$

NO SURFACE ROUGHNESS

$S = 0 \%$
Figure 20. The same as Figure 13, except that the experimental conditions are as shown.
GLASS BEADS
$y = E t^a$
$E = 1.58 \text{ cm.}(\text{min.})^{0.568}$
$a = 0.568$
$C = 1.3 \text{ cm.}$
$Q = 300 \text{ cm.}^3/\text{min.}$
$L = 1.9 \text{ cm.}$
$q = Q/L = 157.89 \text{ cm.}^2/\text{min.}$
WITH SURFACE ROUGHNESS $S = 0\%$
Figure 21. The same as Figure 13, except that the experimental conditions are as shown.
GLASS BEADS

\[ y = E t^\alpha \]

\[ E = 1.69 \text{(cm.)} (\text{min.})^{-0.527} \]

\[ \alpha = 0.527 \]

\[ C = 1.2 \text{ cm.} \]

\[ Q = 200 \text{ cm}^3/\text{min.} \]

\[ L = 1.9 \text{ cm.} \]

\[ q = Q/L = 105.26 \text{ cm}^2/\text{min.} \]

NO SURFACE ROUGHNESS

\[ S = 0\% \]
Figure 22. The same as Figure 13, except that the experimental conditions are as shown.
GLASS BEADS

\[ y = E t^a \]

- \( E = 1.69 \text{ cm.}(\text{min.})^{-0.527} \)
- \( a = 0.527 \)
- \( C = 1.0 \text{ cm.} \)
- \( Q = 300 \text{ cm}^3/\text{min.} \)
- \( L = 1.9 \text{ cm.} \)
- \( q = Q/L = 157.89 \text{ cm}^2/\text{min.} \)

NO SURFACE ROUGHNESS

\( S = 0\% \)
curve. That \( t = 7 \) minutes should be in the dashed portion of the curve means that the computer did not yield practical value for equation 25a when \( t = 7 \text{ min.} \) was used. When we use from Figure 15 the values \( t = T \text{ min.}, a = 0.656, C = 1.2 \text{ cm.}, E = 2.3 (\text{cm/min})^{-0.656} \), then this gives

\[
\frac{Et^a}{C} = \left[\frac{2.3}{1.2}\right]\left[7^{0.656}\right] \\
= \left[1.91\right]\left[3.593\right] \\
= 6.862.
\]

Thus we see that \( \frac{Et^a}{C} = 6.862 \) is not an appropriate value for equation 25a, so the computer does not yield a practical value for equation 25a when the value \( \frac{Et^a}{C} = 6.862 \) is used.

As shown on the graphs the experimental data fit the theoretical curves nicely for the different conditions which were studied with glass beads in the model. There was little difference between the calculated curve and the experimental points when there was surface roughness created by scattering 5 mm. glass beads on the surface of porous medium which was composed of finer beads, at the later stage of the run. We were not able to compare experimental data with theory for the experimental runs when we had a surface slope of 1 percent in the model. The thickness of surface water \( C \) was small and the divergence of the series of equation 25a did not allow us to make this comparison.

Figures 23 and 24 show the relation between the distance of advance and time for two identical runs except for the absence or presence of surface roughness. As shown from the graphs, the presence of surface roughness influences the advance of water on the surface when the surface was level (0 percent slope), but has negligible effect when the surface was
Figure 23. The relation between horizontal distance of advance, with and without surface roughness when the surface slope is 0 percent, and when glass beads are used; the curves connecting experimental points are not theoretical.
GLASS BEADS

\[ y = E t^a \]

\[ E = 1.58 \text{ (cm.) (min.)}^{-0.568} \]

\[ a = 0.568 \]

\[ Q = 300 \text{ cm}^3/\text{min.} \]

\[ L = 1.9 \text{ cm.} \]

\[ S = 0 \% \]

---

**X, HORIZONTAL ADVANCE (cm.)**

---

**t (min.)**

- ○ ○ ○ NO SURFACE ROUGHNESS
- □ □ □ WITH SURFACE ROUGHNESS
Figure 24. The relation between horizontal distance of advance, with and without surface roughness, when the surface slope is 1 percent; the curve connecting experimental points is not theoretical.
GLASS BEADS

\[ y = E t^\alpha \]

\[ E = 1.58 \text{ (cm.) (min.)} \]^{-0.568-}

\[ \alpha = 0.568 \]

\[ Q = 300 \text{ cm}^3 / \text{min.} \]

\[ L = 1.9 \text{ cm.} \]

\[ S = 1 \% \]

- ○ ○ ○ NO SURFACE ROUGHNESS
- □ □ □ WITH SURFACE ROUGHNESS
under 1 percent slope. This indicates that when we have surface roughness created by vegetation in the field, this will not greatly influence the advance of water on the surface when there is a slope on the surface. The magnitude of surface slope needed will depend on the magnitude of the roughness. When we have a slope of 1 percent on the surface, the surface storage or the average depth of water accumulated on the surface of the porous medium in the model was about 3/5 the amount accumulated when the surface was under 0 percent slope. This is a good indication of how the effect of surface conditions is taken care of in the theory by the term C, which is the average depth of surface storage. The surface roughness under the model condition had some effect on surface storage, too. This, in general, shows that the surface slope and roughness are represented by the term C which is the average depth of water on the surface in the theoretical equations.

The colored stream lines of potassium dichromate dye show the direction of the water movement within the porous medium and are seen as dark lines on the composite photographs in the Appendix. The dye shows that the direction of the movement of water in the porous medium is in the vertical direction at early time during the run, but at later times the direction of the stream lines deviates from the vertical and tend to move toward the direction of the moving water on the surface. This is an indication of some lateral movement which takes place during irrigation beside the vertical movement. This was expected since there is a hydraulic gradient both in the vertical and horizontal direction within the body of the porous medium.
Dimensionless Functions for the Horizontal Advance of Irrigation Water
when the Infiltration Equation is \( y = E t^a \)

The solution for the irrigation advance problem when the infiltration equation is given by \( y = E t^a \) may be written as

\[
X = \frac{t}{q} \sum_{n=0}^{\infty} \frac{[-E t^a/c][\Gamma(1 + a)]^n}{\Gamma(2 + na)}
\]

where all the terms are as defined before. This equation is cumbersome to solve since the computer does not yield practical values for large values of the quantity \( E t^a/c \). This equation can be rearranged as

\[
\frac{C}{q} = \sum_{n=0}^{\infty} \frac{[-E t^a/c][\Gamma(1 + a)]^n}{\Gamma(2 + na)}
\]

This form of the equation is dimensionless, in which the quantities \( C/q \) and \( E t^a/c \) are dimensionless. So the model data for glass beads were plotted in dimensionless functions as \( q_t/C \) vs \( E t^a/c \) in Figures 25 to 34. Ten figures for different conditions were plotted. Each figure is a result of two independent runs with the same experimental conditions, except that the amount of water applied for each run is different. The experimental points for the two independent runs fall essentially on the same curve for each graph. The data for the dimensionless functions fit straight lines for the different conditions of infiltration of the porous medium, different slopes and different surface roughness, which were studied in the model. Equations of the form

\[
\frac{q_t}{C} = a + b \frac{E t^a}{C}
\]

were derived for each condition studied. Since these are dimensionless
Figure 25. Values of the dimensionless functions of $\frac{qt}{Cx}$ versus $\frac{Et^\alpha}{C}$, calculated from the experimental runs in the model with glass beads; the quantity $t$ is the time for the water to advance on the surface a horizontal distance $x$. The graph is for two independent runs, where in one run the rate of water application was $Q_1 = 300 \text{ cm}^3/\text{min}$, while for the other run the rate of water application was $Q_2 = 400 \text{ cm}^3/\text{min}$. 
$$\frac{q^*}{x^*}C = 0.430 + 0.842 \frac{E \cdot t^\alpha}{C}$$

$E = 2.3 \text{ (cm.)} (\text{min.})^{-0.656}$

$\alpha = 0.656$

$Q_1 = 300 \text{ cm}^3/\text{min.}$

$Q_2 = 400 \text{ cm}^3/\text{min.}$

$L = 1.9 \text{ cm.}$

$S = 0 \%$

NO SURFACE ROUGHNESS

GLASS BEAD

$y = Et^\alpha$
Figure 26. The same as Figure 25, with different variables.
\[
\frac{q_t}{x_C} = 0.405 + 0.81 \frac{E_t^{\alpha}}{C}
\]

GLASS BEAD

\(E = 2.3 \text{ (cm.)}(\text{min.})^{-0.656}\)
\(\alpha = 0.656\)
\(Q_1 = 300 \text{ cm}^3/\text{min.}\)
\(Q_2 = 400 \text{ cm}^3/\text{min.}\)
\(L = 1.9 \text{ cm.}\)
\(S = 1 \text{ %}\)

NO SURFACE ROUGHNESS
Figure 27. The same as Figure 25, with different variables.
\[ \frac{q_t}{x_C} = 0.25 + 0.973 \frac{E_t^\alpha}{C} \]

**GLASS BEAD**

\[ y = E_t^\alpha \]

- \( E = 1.58 \text{(cm.) (min.)}^{-0.568} \)
- \( \alpha = 0.568 \)
- \( Q_1 = 200 \text{ cm}^3/\text{min.} \)
- \( Q_2 = 300 \text{ cm}^3/\text{min.} \)
- \( L = 1.9 \text{ cm.} \)
- \( S = 0 \% \)

**NO SURFACE ROUGHNESS**
Figure 28. The same as Figure 25, with different variables.
\[ \frac{q^t}{xC} = 0.410 + 0.800 \frac{E t^\alpha}{C} \]

GLASS BEAD

\[ y = E t^\alpha \]

\( E = 2.3 \text{ (cm.)(min.)}^{-0.656} \)

\( \alpha = 0.656 \)

\( Q_1 = 300 \text{ cm}^3/\text{min.} \)

\( Q_2 = 400 \text{ cm}^3/\text{min.} \)

\( L = 1.9 \text{ cm.} \)

\( S = 1 \% \)

WITH SURFACE ROUGHNESS
Figure 29. The same as Figure 25, with different variables.
\[
\frac{q_t}{x C} = 0.51 + 0.844 E t^\alpha
\]

\(Q_1= \bigcirc \bigcirc\)
\(Q_2= \bigtriangleup \bigtriangleup\)

GLASS BEAD
\(E = 2.3 \text{ (cm.) (min.)}^{-0.656}\)
\(\alpha = 0.656\)
\(Q_1 = 300 \text{ cm}^3/\text{min.}\)
\(Q_2 = 450 \text{ cm}^3/\text{min.}\)
\(L = 1.9 \text{ cm.}\)
\(S = 0 \%\)

WITH SURFACE ROUGHNESS
Figure 30. The same as Figure 25, with different variables.
\[ \frac{q^t}{xC} = 0.375 + 0.978 \frac{E^{\alpha}}{C} \]

GLASS BEAD

\[ y = Ef^{\alpha} \]

\[ E = 1.58 \text{ (cm.) (min.)} \]

\[ \alpha = 0.568 \]

\[ Q_1 = 210 \text{ cm}^3/\text{min.} \]

\[ Q_2 = 300 \text{ cm}^3/\text{min.} \]

\[ L = 1.9 \text{ cm.} \]

\[ S = 0 \% \]

WITH SURFACE ROUGHNESS
Figure 31. The same as Figure 25, with different variables.
\[ \frac{q_t}{x_C} = 0.45 + 0.745 \frac{Et^\alpha}{C} \]

- \( Q_1 = \bigcirc \bigcirc \)
- \( Q_2 = \triangle \triangle \)

**GLASS BEAD**

- \( y = Et^\alpha \)
- \( E = 1.58 \text{ (cm.) (min.)} \) \( -0.568 \)
- \( \alpha = 0.568 \)
- \( Q_1 = 200 \text{ cm}^3/\text{min.} \)
- \( Q_2 = 300 \text{ cm}^3/\text{min.} \)
- \( L = 1.9 \text{ cm.} \)
- \( S = 1 \% \)

NO SURFACE ROUGHNESS
Figure 32. The same as Figure 25, with different variables.
\[ \frac{q_t}{x_C} = 0.50 + 0.7745 \frac{E_t}{C} \]
Figure 33. The same as Figure 25, with different variables.
\[
\frac{q_t}{x_C} = 0.175 + 0.965 \frac{E t^\alpha}{C}
\]

GLASS BEAD

\( y = E t^\alpha \)

\( E = 1.69 \text{(cm.)}(\text{min.})^{-0.527} \)

\( \alpha = 0.527 \)

\( Q_1 = 200 \text{ cm}^3/\text{min.} \)

\( Q_2 = 300 \text{ cm}^3/\text{min.} \)

\( L = 1.9 \text{ cm.} \)

\( S = 0 \% \)

NO SURFACE ROUGHNESS
Figure 34. The same as Figure 25, with different variables.
\frac{q_t}{x C} = 0.2 + 0.80 \frac{E t^\alpha}{C}

Q_1 = \ circ \ circ
Q_2 = \ triangle \ triangle

GLASS BEAD

y = E t^\alpha

E = 1.69 (cm.)(min.)^{-0.527}

\alpha = 0.527

Q_1 = 200 \text{ cm}^3/\text{min.}

Q_2 = 300 \text{ cm}^3/\text{min.}

L = 1.9 \text{ cm.}

S = 1 \% 

NO SURFACE ROUGHNESS
functions, the equations may be used in irrigation design when the infiltration equation in the field is given in the form $y = E t^a$. Note that the experimental runs in the model were conducted using air-dry glass beads. But under the field condition, soil will be under different moisture levels. However, this does not affect the use of model dimensionless data, because the coefficient $E$ in the infiltration equation depends strongly on the initial moisture content of the soil in the field as Toksoz and Kirkham reported (1965). Since the quantity $E t^a/C$ is dimensionless, the value of $E$ obtained under field condition will reflect the effect of initial moisture content. For a numerical example to demonstrate the use of the dimensionless model data of Figures 25 to 34, let us take the following geometry for homogeneous soil in the field. Suppose we have an irrigation system with field values given as follows: at the head of the irrigation check, water is introduced at constant flow rate $q = 2.94$ ft$^2$/min (= total amount of irrigation water in cubic feet divided by the width of the field in feet); the average depth of flow is $C = .125$ ft.; the infiltration equation obtained under field condition is given by $y = E t^a$, where $E$ and $a$ are .0426 and .527 respectively; the field is taken to be level with negligible surface roughness. We now wish to solve for the position of the irrigation stream after 20 min., 40 min., 60 min., 80 min., and 100 min.

We can proceed as follows:

Condition I. $t = 20$ min.

We calculate $E t^a/C$
\[ E_t^{a/C} = \frac{(0.0426)(20)^{0.527}}{0.125} \]
\[ = (0.426)(4.85)/(0.125) \]
\[ = 0.2066/0.125 \]
\[ = 1.8948 \]

We now go to Figure 33 to read the value of \( q_t/C_x \) when \( E_t^{a/C} = 1.8948 \).

The equation of the straight line of Figure 33 is
\[ q_t/C_x = 0.175 + (0.965)(E_t^{a/C}) \]

Therefore, we have
\[ q_t/C_x = 0.175 + (0.965)(1.8948) \]
\[ = 2.00158 \]

Then we find
\[ x = \frac{(2.94)(20)}{(0.125)(2.00158)} \]
\[ = 235 \text{ ft.} \]

Condition II. \( t = 40 \text{ min.} \)

\[ E_t^{a/C} = \frac{(0.0426)(40)^{0.527}}{0.125} \]
\[ = (0.426)(6.99)/0.125 \]
\[ = 0.2977/0.125 \]
\[ = 2.382 \]

Therefore, we have
\[ q_t/C_x = 0.175 + (0.965)(E_t^{a/C}) \]
\[ = 0.175 + (0.965)(2.382) \]
\[ = 2.40363 \]

Then we find

...
Similarly

\[ x = (2.94)(40) \]

\[ = 381.4 \text{ ft.} \]

Then the position of the irrigation stream will be 235 ft., 381.4 ft., 468 ft., and 583.3 ft. after 20 min., 40 min., 60 min., and 100 min., respectively. This calculation of the position of the irrigation stream allows us to know how long water will pond on the surface at each location, and how much water will infiltrate into the soil. Table 2 shows such calculation for the chosen numerical example.

Moisture Profiles in the Model during Infiltration

As a further check of the theory, the moisture profiles were calculated and compared with the observed profiles in the model for the three sizes of glass beads. The calculated curves for the horizontal advance of the water on the surface of beads were used to determine the period that water was ponded on the bead surface at different horizontal distances from the source of the feeding ditch. Knowing the length of time that water was ponded on the bead surface at different locations, we could calculate the total volume of water which had infiltrated into the porous medium. We then used the calculated total volume of infiltrated water to calculate the position of the wet front, assuming all the pore spaces were saturated.
Table 2. Calculation of depth of water which penetrates the soil at different locations along the irrigation run for a chosen numerical example

<table>
<thead>
<tr>
<th>Distance s (see Fig. 2) along the irrigation run</th>
<th>Time $t$ since irrigation water was applied at the beginning of the irrigation run</th>
<th>Time $t$ that it took the water to reach the distance $s$</th>
<th>$t_s - t$ the period that water has been available for infiltration</th>
<th>Depth of surface water $y$ that has penetrated into the soil at the distance $s$ when the infiltration equation is $y = E(t_s)^{0.527}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>583 ft.</td>
<td>100 min.</td>
<td>100 min.</td>
<td>0 min.</td>
<td>$y = 0.0426(0)^{0.527} = 0$ ft.</td>
</tr>
<tr>
<td>543 ft.</td>
<td>100 min.</td>
<td>80 min.</td>
<td>20 min.</td>
<td>$y = 0.0426(20)^{0.527} = 0.2066$ ft.</td>
</tr>
<tr>
<td>468 ft.</td>
<td>100 min.</td>
<td>60 min.</td>
<td>40 min.</td>
<td>$y = 0.0426(40)^{0.527} = 0.2977$ ft.</td>
</tr>
<tr>
<td>381 ft.</td>
<td>100 min.</td>
<td>40 min.</td>
<td>60 min.</td>
<td>$y = 0.0426(60)^{0.527} = 0.3664$ ft.</td>
</tr>
<tr>
<td>235 ft.</td>
<td>100 min.</td>
<td>20 min.</td>
<td>80 min.</td>
<td>$y = 0.0426(80)^{0.527} = 0.426$ ft.</td>
</tr>
<tr>
<td>0 ft.</td>
<td>100 min.</td>
<td>0 min.</td>
<td>100 min.</td>
<td>$y = 0.0426(100)^{0.527} = 0.4825$ ft.</td>
</tr>
</tbody>
</table>
A comparison of the calculated location of the wetting front and the observed wetting front are shown in Figures 35 to 37. Each figure represents the comparison of the calculated and observed wetting front for one size group of glass beads and for different arbitrary chosen time increments. The graphs show an agreement between the calculated and observed position of the wetting front. We have to bear in mind that this comparison of the observed position of the wetting front with the calculated was possible only for glass beads because of the nearly uniform distribution of the moisture content of glass beads during the process of infiltration. This does not apply to soil, but if there were a gamma-ray measuring instrument (see Gurr 1962 and Davidson et al. 1963) to measure the moisture content at different locations in the model for different time increments, this could have been used to compare the calculated and observed position of the wet front using soil instead of glass beads in the model. This would be an interesting study when the gamma-ray equipments are available in the near future, but a thicker model than the one used in this study would then have to be used, because the thickness of the sample for measuring moisture content with the gamma-ray apparatus would have to be at least three centimeters to obtain a reliable measurement.

Horizontal Advance of Water in the Model when Ida and Webster Soils are Porous Mediums

It was desirable to conduct experimental runs in the model using actual soils obtained from the field to compare the theory with data from experiments under field soil conditions. Two types of soils were used. The Ida soil which is a fairly permeable soil, and the Webster soil, which
Figure 35. Observed and calculated depths of penetration, \( y/f \), of the wetting front, for different time increments.
GLASS BEADS

$y = E f^{-0.656}$

$E = 2.3 \text{ cm}^2 \text{ min}$

$\alpha = 0.656$

$Q = 300 \text{ cm}^3 \text{ min}$

$L = 1.9 \text{ cm}$

$S = 0\%$

WITH SURFACE ROUGHNESS
Figure 36. Same as Figure 35, with different variables.
THEORETICAL—EXPERIMENTAL

GLASS BEADS

\[ y = E t^\alpha \]

- \[ E = 1.58 \text{ cm.} \times \text{min.} \]
- \[ \alpha = 0.568 \]
- \[ Q = 200 \text{ cm}^3/\text{min.} \]
- \[ L = 1.9 \text{ cm.} \]
- \[ S = 0\% \]

NO SURFACE ROUGHNESS

\[ y/f, \text{ DEPTH OF PENETRATION (cm.)} \]

\[ s, \text{ HORIZONTAL DISTANCE (cm.)} \]
Figure 37. Same as Figure 35, with different variables.
\[ y = E t^{-0.527} \]

- \( E = 1.65 \text{ cm/min} \)
- \( \alpha = 0.527 \)
- \( Q = 200 \text{ cm}^3/\text{min} \)
- \( L = 1.9 \text{ cm} \)
- \( S = 0\% \)

**NO SURFACE ROUGHNESS**

**Theoretical** --- **Experimental**
is less permeable. The experimental runs for Ida and Webster soil were conducted using soil aggregates < 2 mm. diameter. The infiltration characteristics of the two soils are described by the equation

\[ y = At^{1/2} + Bt. \]

Figures 38 to 40 show the comparison between the experimental points and the calculated curves for the Ida soil. The calculated curves are obtained by equations 46 and 77. As the graphs show, the experimental points fit the calculated curves nicely, except the curve in Figure 40. In Figure 40 the experimental data deviate from the calculated curve at the late stage of the run, which might be due to unlevelness of the surface of soil in the model. The fit of the theoretical curves in Figures 38 to 40 to the experimental points shows that the theory takes into account the slope of the irrigation run through the average water thickness on the surface C.

(It is remembered for Figure 38, and other associated figures that the experimental points \((x,t)\) are those read from the photographs in the Appendix, where legends on the composite photographs are keyed to the legends in the figures and the times in the photographs are shown on the clock.)

Figure 41 shows the experimental data for two experimental runs with the same parameters, except that for one run the surface was at 0 percent slope, while for the other run the surface was at 1 percent slope. The rate of advance of water on the surface increased when the surface had a non-zero slope. It took the water stream 45 minutes to come to the end of the model \((x = 203 \text{ cm})\) when the slope was 1 percent, while it would take the same water stream about 70 minutes to come to the end of the model when the surface was level (or 0 percent slope). The surface
Figure 38. Comparison of the experimental and theoretical horizontal distance of advance \( x \) of the flooding irrigation water on the soil surface in the model, when Ida soil is used as porous medium.
Theoretical and experimental data for the horizontal advance of IDA soil are shown. Theoretical data are represented by a solid line, while experimental data are represented by circles. The equation for the theoretical data is given as:

\[ y = At^{1/2} + Bt \]

With:
- \( A = 0.605 \text{ cm.}(\text{min.})^{-1/2} \)
- \( B = 0.0986 \text{ cm.}(\text{min.})^{-1} \)
- \( C = 1.1 \text{ cm.}^3 \)
- \( Q = 50 \text{ cm.}^3/\text{min.} \)
- \( L = 1.9 \text{ cm.} \)
- \( q = Q/L = 26.32 \text{ cm.}^2/\text{min.} \)
- \( S = 0\% \)

The soil has no surface roughness. The graph shows the horizontal advance in centimeters over time in minutes.
Figure 39. Same as Figure 38, with different variables.
IDA SOIL

$y = At^{1/2} + Bt$

$A = 0.605 \text{(cm.)(min.)}^{-1/2}$

$B = 0.0986 \text{(cm.)(min.)}$

$C = 0.7 \text{ cm.}$

$Q = 80 \text{ cm}^3/\text{min.}$

$L = 1.9 \text{ cm.}$

$q = Q/L = 42.10 \text{ cm}^2/\text{min.}$

$S = 1\%$

NO SURFACE ROUGHNESS
Figure 40. Same as Figure 38, with different variables.
IDA SOIL

\[ y = At^{1/2} + Bt \]

- \( A = 0.605 \text{ (cm. X min.)}^{1/2} \)
- \( B = 0.0986 \text{ (cm. X min.)}^{-1} \)
- \( C = 0.6 \text{ cm.} \)
- \( Q = 50 \text{ cm.}^3/\text{min.} \)
- \( L = 1.9 \text{ cm.} \)
- \( q = Q/L = 26.32 \text{ cm.}^2/\text{min.} \)
- \( S = 1\% \)

NO SURFACE ROUGHNESS

**Theoretical**

**Experimental**
Figure 41. The relation between horizontal distance of advance with surface at 0 percent slope and 1 percent slope, using Ida soil.
IDA SOIL

$y = At^{1/2} + Bt$

$A = 0.605 \text{ cm.}(\text{min.})^{-1/2}$

$B = 0.0986 \text{ cm.}(\text{min.})^{-1}$

$Q = 50 \text{ cm}^3/\text{min.}$

$L = 1.9 \text{ cm}^3/\text{min.}$

NO SURFACE ROUGHNESS
storage or the depth of water on the surface when the surface of the model was at 1 percent slope was about 3/5 the surface storage when the surface was level.

Figures 42 and 43 show the data for the experimental runs using Webster soils. The solid lines are calculated by equations 46 and 77, while the scattered points are the experimental data. As it can be seen from the graphs the experimental data agree with the theory. Figure 44 shows the difference between two experimental runs with the same parameters, except for one run the surface was under 0 percent slope, while the other run had a surface slope of 1 percent. As the graph shows, the surface slope increased the rate of advance of the horizontal advance of water on the surface. The depth of surface storage when the surface slope was 1 percent was about 3/5 of the depth of the volume of surface storage (volume of water ponded on the surface) when the surface slope was 0 percent. From the experimental runs using Ida and Webster soils, it is concluded that the trend of the experimental data indicates good agreement between theory and experiment. The rate of horizontal advance of water is high at the early stage of the run, then decreases at later time. This phenomena takes place because at later time of the run the water will be covering a larger area, so the area accessible for infiltration is higher and more of the applied water infiltrates to the soil, therefore the rate of horizontal advance decreases at later time of the run. The depth of water on the surface in the model was a function of the surface condition, being smaller when we have the surface under slope than when the surface was level or at 0 percent slope. Under model condition the depth of
Figure 42. Comparison of measured and calculated horizontal distance of advance in the model, using Webster soil as porous media.
THEORETICAL — EXPERIMENTAL

WEBSTER SOIL

\[ y = At^{1/2} + Bt \]

- \( A = 0.48 \text{(cm.)(min.)}^{-1/2} \)
- \( B = 0.034 \text{(cm.)(min.)}^{-1} \)
- \( C = 1.1 \text{ cm.} \)
- \( Q = 35 \text{ cm}^3/\text{min.} \)
- \( L = 1.9 \text{ cm.} \)
- \( q = Q/L = 18.42 \text{ cm}^2/\text{min.} \)

NO SURFACE ROUGHNESS

- \( S = 0\% \)
Figure 43. Same as Figure 42, with different variables.
WEBSTER SOIL

\[ y = At^{1/2}Bt \]

\( A = 0.480 \text{(cm.)(min.)}^{-1/2} \)
\( B = 0.034 \text{(cm.)(min.)}^{-1} \)
\( C = 0.6 \text{ cm.} \)
\( Q = 35 \text{ cm}^3/\text{min} \)
\( L = 1.9 \text{ cm} \)
\( q = Q/L = 18.42 \text{ cm}^2/\text{min.} \)
\( S = 1\% \)

NO SURFACE ROUGHNESS
Figure 44. The relation between horizontal distance of advance with surface at 0 percent slope and 1 percent slope, using Webster soil.
WEBSTER SOIL

\[ y = A t^{1/2} + B t \]

- \( A = 0.480 \text{(cm.)(min.)}^{1/2} \)
- \( B = 0.034 \text{(cm.)(min.)}^{-1} \)
- \( Q = 3.5 \text{ cm}^3/\text{min.} \)
- \( L = 1.9 \text{ cm.} \)

NO SURFACE ROUGHNESS
storage when the surface was at 1 percent slope was about 3/5 of the surface storage when the surface was level or at 0 percent slope.

Theoretical Horizontal Advance of Irrigation Water when the Infiltration Equation is \( y = At^{1/2} + Bt \) and the Parameters A and B are Determined under Field Conditions

In this subsection we present graphs of theoretical solutions of the horizontal advance of irrigation water for different soil types and different initial moisture contents when the infiltration equation for the soils and their moisture contents is known from experimental tests to be

\[ y = At^{1/2} + Bt \]

In preparing the curves, the quantity C, the average surface thickness of water, was taken as a parameter and C was assigned arbitrary values 0.2, 0.3, 0.4, 0.5 and 0.7 ft. to represent families of curves for different field conditions.

Figures 45 to 56 show the theoretical families of curves obtained. The simplified algebraic equation \( y = At^{1/2} + Bt \) was the one used to develop these curves for most of the conditions. In these graphs the value of \( x/q \) is plotted versus the time \( t \), where \( x \) is the horizontal distance in feet or meters that the irrigation stream will advance in the field during surface irrigation, and \( q \) is the quantity of applied irrigation water per unit width of the field \( (\text{ft})^3/\text{ft} \) or \( \text{m}^3/\text{m} \). The parameter C, the average depth of surface storage, will depend on the surface slope and roughness. When the infiltration rate is high, as is the case in Figure 53 for Grundy soil, the rate of increase of the value \( x/q \) with time is small, and the family of curves are closer to each other. Figure 47 shows that \( x/q \)
Figure 45. Theoretical curves for the quantity $x/q$ versus $t$, for Ida silt loam soil, under field condition. The curves are developed for different values of $C$, using the field determined infiltration equation $y = At^{1/2} + Bt$, where $y$ is in ft. or m and $t$ is in hours and $A$ and $B$ have units as shown on the graph.
IDA SILT LOAM CORN PLOT DRY SOIL INFILTRATION EQ.

\[ y = A + \frac{B}{x^{1/2}} \]

\[ A = 0.2449 \text{ (ft./hr.)} \]
\[ B = 0.019 \text{ (ft./hr.)}^{-1} \]

- \( x = \frac{q}{p} \)

\[ q = \frac{1}{9.81} \text{ (m/s)} \]
\[ r = \frac{1}{9.81} \text{ (m/s)} \]

- \( A = 0.0761 \text{ (m/hr.})^{1/2} \)
\( B = 0.0058 \text{ (m/hr.)}^{-1} \)

- \( C = 0.2 \text{ ft.} = 0.061 \text{ m} \)
\( C = 0.3 \text{ ft.} = 0.091 \text{ m} \)
\( C = 0.4 \text{ ft.} = 0.122 \text{ m} \)
\( C = 0.5 \text{ ft.} = 0.1525 \text{ m} \)

- \( A = 0.0761 \text{ (m/hr.)}^{1/2} \)
\( B = 0.0058 \text{ (m/hr.)}^{-1} \)

- \( C = 0.2 \text{ ft.} = 0.061 \text{ m} \)
\( C = 0.3 \text{ ft.} = 0.091 \text{ m} \)
\( C = 0.4 \text{ ft.} = 0.122 \text{ m} \)
\( C = 0.5 \text{ ft.} = 0.1525 \text{ m} \)
Figure 46. Legend same as Figure 45. See the figure for the parameters.
IDA SILT LOAM
CORN PLOT
MOIST SOIL
FIELD INFILTRATION EQ.
y = At^{1/2} + Bt
A = 0.1215 (ft.) (hr.)^{-1/2}
B = 0.0059 (ft.) (hr.)^{-1}

\[ x = \frac{q}{q} \]
\[ t (\text{hr.}) \]

C = 0.2 ft. [0.061 m]
C = 0.3 ft. [0.0915 m]
C = 0.4 ft. [0.122 m]
C = 0.5 ft. [0.1525 m]
C = 0.7 ft. [0.2135 m]

A = 0.037 (m)(hr.)^{-1/2}
B = 0.0018 (m)(hr.)^{-1}
Figure 47. Legend same as Figure 45. See the figure for the parameters.
IDA SILT LOAM
BROME-GRASS PLOT
DRY SOIL
FIELD INFILTRATION EQ.
\[ y = A t^{1/2} + B t \]
A = 0.1529 (ft.) (hr.)^{-1/2}
B = 0.044 (ft.) (hr.)^{-1}

C = 0.2 ft. (0.061 m)
C = 0.3 ft. (0.0915 m)
C = 0.4 ft. (0.122 m)
C = 0.5 ft. (0.1525 m)
C = 0.7 ft. (0.2135 m)

<table>
<thead>
<tr>
<th>t (hr.)</th>
<th>x (ft.)</th>
<th>q (ft./hr.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.6</td>
<td>6.6</td>
</tr>
<tr>
<td>2</td>
<td>5.3</td>
<td>13.1</td>
</tr>
<tr>
<td>3</td>
<td>7.6</td>
<td>19.7</td>
</tr>
<tr>
<td>4</td>
<td>9.8</td>
<td>26.2</td>
</tr>
<tr>
<td>5</td>
<td>12.1</td>
<td>32.2</td>
</tr>
</tbody>
</table>
Figure 48. Legend same as Figure 45. See the figure for the parameters.
IDA SILT LOAM
BROME-GRASS PLOT
MOIST SOIL
FIELD INFILTRATION EQ.
\[ y = At^{1/2} + Bt \]
\[ A = 0.12234 \text{ (ft.)(hr.)}^{-1/2} \]
\[ B = 0.0723 \text{ (ft.)(hr.)}^{-1} \]

\[ x \left( \frac{\text{ft.}}{\text{ft}^2} \right) \]

\[ t \text{ (hr.)} \]

C = 0.2 ft. \[= 0.061 \text{ m} \]
C = 0.3 ft. \[= 0.0915 \text{ m} \]
C = 0.4 ft. \[= 0.122 \text{ m} \]
C = 0.5 ft. \[= 0.1525 \text{ m} \]
C = 0.7 ft. \[= 0.2135 \text{ m} \]
Figure 49. Legend same as Figure 45. See the figure for the parameters.
MOODY SILT LOAM
CORN PLOT
DRY SOIL
FIELD INFILTRATION EQ.
\[ y = A t^{1/2} + B t \]
\[ A = 0.2842 \text{(ft.)(hr.)}^{-1/2} \]
\[ B = 0.019 \text{(ft.)(hr.)}^{-1} \]

- \[ C = 0.2 \text{ ft.}[=0.061 \text{ m}] \]
- \[ C = 0.3 \text{ ft.}[=0.0915 \text{ m}] \]
- \[ C = 0.4 \text{ ft.}[=0.122 \text{ m}] \]
- \[ C = 0.5 \text{ ft.}[=0.1525 \text{ m}] \]
- \[ C = 0.7 \text{ ft.}[=0.2135 \text{ m}] \]
Figure 50. Legend same as Figure 45. See the figure for the parameters.
MOODY SILT LOAM
CORN PLOT
MOIST SOIL
FIELD INFILTRATION EQ.
\[ y = A t^{1/2} + B t \]
\[ A = 0.1292 \text{ (ft.)} (\text{hr.})^{-1/2} \]
\[ B = 0.004 \text{ (ft.)} (\text{hr.})^{-1} \]

\[ A = 0.0394 \text{ (m.)} (\text{hr.})^{-1/2} \]
\[ B = 0.00122 \text{ (m.)} (\text{hr.})^{-1} \]
Figure 51. Legend same as Figure 45. See the figure for the parameters.
MOODY SILT LOAM
BROME-GRASS PLOT
DRY SOIL
FIELD INFILTRATION EQ.
\[ y = At^{1/2} + Bt \]
\[ A = 0.1324 \text{ (ft.)(hr.)}^{-1/2} \]
\[ B = 0.019 \text{ (ft.)(hr.)}^{-1} \]

\[ A = 0.0345 \text{ (m.)(hr.)}^{-1/2} \]
\[ B = 0.0058 \text{ (m.)(hr.)}^{-1} \]

Graph showing infiltration rates for different values of \( C \): 0.2 ft. (0.061 m), 0.3 ft. (0.0915 m), 0.4 ft. (0.122 m), 0.5 ft. (0.1525 m), 0.7 ft. (0.2135 m).

Units: 
- \( x \) is in ft. or m.
- \( q \) is in ft/hr or m/hr.
- \( t \) is in hr.
Figure 52. Legend same as Figure 45. See the figure for the parameters.
MOODY SILT LOAM
BROME-GRASS PLOT
MOIST SOIL
FIELD INFILTRATION
EQ.

\[ y = At^{1/2} + Bt \]

- \( A = 0.0311 \text{ (m)}(\text{hr})^{-1/2} \)
- \( B = 0.0229 \text{ (m)}(\text{hr})^{-1} \)
- \( A = 0.0993 \text{ (ft.)(hr.)}^{-1/2} \)
- \( B = 0.0751 \text{ (ft.)(hr.)}^{-1} \)

Diagram showing infiltration rates for different values of \( C \): 0.2 ft (0.061 m), 0.3 ft (0.0915 m), 0.4 ft (0.122 m), 0.5 ft (0.1525 m), 0.7 ft (0.2135 m).
Figure 53. Legend same as Figure 45. See the figure for the parameters.
GRUNDY SILT LOAM
CORN PLOT
DRY SOIL
FIELD INFILTRATION EQ.
\[ y = A t^{1/2} B t \]
\[ A = 0.327 \text{(ft.)}^{1/2} \text{(hr.)}^{-1} \]
\[ B = 0.0064 \text{(ft.)} \text{(hr.)}^{-1} \]

The graph shows the infiltration of water into the soil over time for different values of \( C \), which represent the depth of the soil layer.

- \( C = 0.2 \text{ ft.} = 0.061 \text{ m} \)
- \( C = 0.3 \text{ ft.} = 0.0915 \text{ m} \)
- \( C = 0.4 \text{ ft.} = 0.122 \text{ m} \)
- \( C = 0.5 \text{ ft.} = 0.1525 \text{ m} \)
- \( C = 0.7 \text{ ft.} = 0.2135 \text{ m} \)
Figure 54. Legend same as Figure 45. See the figure for the parameters.
GRUNDY SILT LOAM
CORN PLOT
MOIST SOIL
FIELD INFILTRATION EQ.

\[ y = At^{1/2} Bt \]

\[ A = 0.10487 \text{(ft.})(\text{hr.})^{-1/2} \]
\[ B = 0.05033 \text{(ft.})(\text{hr.})^{-1} \]

\[ t \]

\[ A = 0.0319 \text{(m.})(\text{hr.})^{-1/2} \]
\[ B = 0.0153 \text{(m.})(\text{hr.})^{-1} \]
Figure 55. Legend same as Figure 45. See the figure for the parameters.
GRUNDY SILT LOAM
BROME-GRASS PLOT
DRY SOIL

FIELD INFILTRATION EQ.

\[ y = At^{1/2} + Bt \]

- \( A = 0.3905 \text{(ft.)/(hr.)}^{1/2} \)
- \( B = 0.0233 \text{(ft.)/(hr.)}^{-1} \)

- \( C = 0.2 \text{ ft.} \) (0.061 m)
- \( C = 0.3 \text{ ft.} \) (0.0915 m)
- \( C = 0.4 \text{ ft.} \) (0.122 m)
- \( C = 0.5 \text{ ft.} \) (0.1525 m)
- \( C = 0.7 \text{ ft.} \) (0.2135 m)

- \( E = 26.224 \)
- \( E = 19.668 \)
- \( E = 13.113 \)
- \( E = 6.556 \)
Figure 56. Legend same as Figure 45. See the figure for the parameters.
GRUNDY SILT LOAM
BROME-GRASS PLOT
MOIST SOIL
FIELD INFILTRATION EQ.

\[ y = A t^{1/2} + B t \]

- \( A = 0.1725 \text{(ft.)(hr.)}^{-1/2} \)
- \( B = 0.007 \text{(ft.)(hr.)}^{-1} \)

- \( C = 0.2 \text{ ft.} = 0.061 \text{ m} \)
- \( C = 0.3 \text{ ft.} = 0.0915 \text{ m} \)
- \( C = 0.4 \text{ ft.} = 0.122 \text{ m} \)
- \( C = 0.5 \text{ ft.} = 0.1525 \text{ m} \)
- \( C = 0.7 \text{ ft.} = 0.2135 \text{ m} \)

\( A = 0.0536 \text{(m.)(hr.)}^{-1/2} \)
\( B = 0.00214 \text{(m.)(hr.)}^{-1} \)
increases rapidly with time because the infiltration rate of the soil is low. Therefore, more water runs on the surface and covers a larger surface area.

The family of curves of Figures 45 to 56 could be used for other soil types, if their measured cumulative infiltration equations are the same as the ones for which these theoretical curves are developed.

Figures 45 to 56, as we shall later show in a numerical example, can be used in irrigation design to determine the maximum allowable length of run, which is the longest distance in which the maximum allowable irrigation stream can produce nearly uniform distribution of water in the soil to a certain depth without much deep seepage loss, or without the leaching of water-soluble nutrients beyond the plants reach. The irrigation problem is that on a field being irrigated, by surface or gravity methods, inefficient use of irrigation water usually shows up in poor yields at the upper and lower parts of the field. The yields at the upper and lower parts of the field are low because the upper part loses nutrients by erosion and leaching and is kept too wet for good growth, while the lower part receives too little water to supply ample moisture to growing plants. So Figures 45 to 56 enable us to set up a good irrigation design, where uniformity of the depth of application of water is achieved at nearly all the parts in the irrigated field. For a numerical example to demonstrate the use of data of the set of figures the following example will be considered.

Suppose we have a Moody silt loam soil to be irrigated and from moisture determination it is found that 6.75 inches of water are needed to
refill the root zone; that is, we need 6.75 inches of water to seep into the soil at the end of our irrigation run. What length of the irrigation run (length of check) do we now need to get the 6.75 inches at the end of the run and a minimum depth (more than the 6.75 inches) at the beginning of the run (head of the check)?

Step 1: We solve first for the time needed to put 6.75 inches by direct vertical infiltration into soil. The infiltration equation for Moody silt loam soil (see Figure 51) is

\[ y = 0.1324t^{1/2} + 0.019t \]

where \( y \) is in ft. and \( t \) in hours.

If we take \( y = 6.75 \) inches \( = 0.562 \) ft., we get the equation

\[ 0.562 = 0.1324t^{1/2} + 0.019t \]

which is the same as

\[ 0.019t + 0.1324t^{1/2} - 0.562 = 0 \]

We now use the quadratic formula to solve for \( t^{1/2} \) and find

\[ t^{1/2} = 3.0 \text{ hrs.} \]

which gives the value

\[ t = 9 \text{ hours}. \]

Now, from Griddle (1950), Griddle and Davis (1951) and Griddle et al. (1956), the rule in irrigation is that the water stream should reach the lower end of the irrigation system in \( 1/4 \) the time needed to refill the root zone with water. Therefore, in our example the time needed for the irrigation stream to reach the lower end of the irrigation check is

\[ 9/4 = 2.25 \text{ hours}. \]
Now, in our curves, Figures 46-58, we have for several values of average surface water thickness $C$ values of $q/x$ versus $t$ and not $x$ versus $t$, but we still know for this soil of Figure 51 that $t$ must be 2.25 hrs.
and we shall choose $C$ as being

$$C = 0.4 \text{ ft}$$

for our example. Therefore from Figure 52 we can read

$$x/q = 4.2 \left( \text{ft/ft}^2 \right).$$

For the Moody soil in Iowa if a suitable value of $q$ which does not cause erosion hazard is

$$q = 162 \left( \text{ft}^3/\text{ft} \right)/\text{hr}.$$

Therefore the design length $x$ should be

$$x = (162)(4.2).$$

$$= 680 \text{ ft}.$$

If we had chosen a different value of $q$ we would have obtained different values of $x$.

Continuing with our example we see that on the upper end of the field the irrigation water will be on the soil surface

$$9 + 2.25 = 11.25 \text{ hrs.},$$
while at the lower end the surface will be wet with surface water for 9 hrs. We can now exactly calculate how much water will go to the soil at the upper end of the field in the 11.25 hours by the infiltration equation

$$y = 0.1324t^{1/2} + 0.019t$$
in which we use the value $t = 11.25 \text{ hours}$. The result is

$$y = 0.658 \text{ ft} = 7.89 \text{ inches}$$
so 7.89 inches will be absorbed at the upper end of the field, while 6.75
inches will be absorbed at the lower end of the field (which is at a distance equal to 680 ft.) during the total time of irrigation.

The advantage of the approach in this example is that we need not measure in the field \( t \) vs. \( x \) for different rates of applications \( q \). Instead we measure the infiltration rate and we then set up a design value for \( q \).

If water beyond that needed to refill the root zone is needed for salinity control, this amount of water must be taken into account (by increasing the depth of water infiltration at the far end of the irrigation run).

We can further calculate the position of the irrigation stream for each time increment. This calculation of the position of the irrigation stream allows us to know how long water will pond on the surface at each location, and how much water will infiltrate into the soil. Table 3 shows an example of this calculation.

Theoretical Horizontal Advance of Irrigation Water When the Infiltration Equation is \( y = kt \), as for Sand

In sandy soils, similar to the ones in newly developed areas in Egypt, the infiltration characteristics of the soil can be described by the equation, \( y = Kt \), where \( y \) is the cumulative infiltration in \( (\text{ft}^3/\text{ft}^2) \) of in \( (\text{m}^3/\text{m}^2) \), \( t \) is the time in hours that water has been available on the surface and \( K \) is the slope of the straight line when \( y \) is plotted versus \( t \), just equal to the Darcy law \( K \) for the sand if capillary effects are negligible.

In Egypt most of the soil in the newly developed area is under
Table 3. Calculation of depth of water which penetrates the soil at different locations along the irrigation run for a chosen numerical example

| Distance s (see Fig. 2) along the irrigation run | Time \( t \) since irrigation water was applied at the beginning of the irrigation run | Time \( t \) that it took the water to reach the distance \( s \) | \( t - t_0 \) the period that water has been available for infiltration | Depth of surface water \( y \) that has penetrated into the soil at the distance \( s \) when the infiltration equation is 
\[
y = A(t^{1/2} + Bt^2 + B(t - t_0)^{1/2})
\]
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>680 ft.</td>
<td>135 min.</td>
<td>135 min.</td>
<td>0 min.</td>
<td>0 ft.</td>
</tr>
<tr>
<td>600 ft.</td>
<td>135 min.</td>
<td>118 min.</td>
<td>17 min.</td>
<td>0.0755 ft.</td>
</tr>
<tr>
<td>500 ft.</td>
<td>135 min.</td>
<td>87 min.</td>
<td>48 min.</td>
<td>0.1330 ft.</td>
</tr>
<tr>
<td>400 ft.</td>
<td>135 min.</td>
<td>64 min.</td>
<td>71 min.</td>
<td>0.1667 ft.</td>
</tr>
<tr>
<td>300 ft.</td>
<td>135 min.</td>
<td>47 min.</td>
<td>88 min.</td>
<td>0.1880 ft.</td>
</tr>
<tr>
<td>200 ft.</td>
<td>135 min.</td>
<td>23 min.</td>
<td>112 min.</td>
<td>0.2168 ft.</td>
</tr>
<tr>
<td>100 ft.</td>
<td>135 min.</td>
<td>13 min.</td>
<td>122 min.</td>
<td>0.2276 ft.</td>
</tr>
<tr>
<td>0 ft.</td>
<td>135 min.</td>
<td>0 min.</td>
<td>135 min.</td>
<td>0.2413 ft.</td>
</tr>
</tbody>
</table>
irrigation. Water in this area is the dominant factor for agricultural expansion, so it should be used efficiently to decrease loss by deep seepage. The solution for the irrigation advance problem when the infiltration equation is $y = Kt$ was used to develop theoretical families of curves describing the rate of advance of irrigation stream under this sandy soil condition. Figures 57 to 61 show the theoretical curves where the quantity $x/q$ was plotted versus $t$, where $x$ is the horizontal distance of advance of the irrigation stream and $q$ is the rate of application of irrigation water per unit width of the irrigated check per unit time. The theoretical curves are for different values of $K$ and for different depths of surface water thickness $C$. These graphs should be useful in irrigation design to calculate the maximum allowable length of irrigation run for minimum deep seepage loss.

The solution for the irrigation advance when the infiltration equation is $y = Kt$ is (see equation 23)

$$x(t) = \frac{Q}{K} [1 - e^{-(K/C)t}]$$

and the maximum distance of advance is given (see equation 24) by the expression

$$\lim_{t \to \infty} x(t) = \frac{q}{K}$$

This shows, as was noted earlier, that when we apply a constant amount of irrigation water $q$ at the head of the irrigated check, then the maximum distance of advance is equal to the ratio $q/K$ regardless of the depth of surface storage $C$. 
Figure 57. Theoretical curves for the quantity $x/q$ versus $t$ for sandy soil. The curves are developed for different values of $C$, assuming the field infiltration equation is $y = Kt$. 
\[ y = Kt \]

SANDY SOIL

\[ K = 0.05 \text{(ft.)(hr.)}^{-1} \]
\[ = 0.0015 \text{(m)(hr.)}^{-1} \]
Figure 58. Legend same as Figure 57. See the figure for the parameters.
SANDY SOIL

\( y = Kt \)

\( K = 0.1 \text{(ft.)(hr.)}^{-1} \)

\( = 0.03 \text{(m)(hr.)}^{-1} \)
Figure 59. Legend same as Figure 57. See the figure for the parameters.
Figure 60. Legend same as Figure 57. See the figure for the parameters.
SANDY SOIL

\[ y = Kt \]
\[ K = 0.2 \text{(ft.) (hr.)}^{-1} = 0.06 \text{(m) (hr.)}^{-1} \]

- \[ C = 0.1 \text{ ft.} \approx 0.030 \text{ m} \]
- \[ C = 0.2 \text{ ft.} \approx 0.061 \text{ m} \]
- \[ C = 0.3 \text{ ft.} \approx 0.0915 \text{ m} \]
- \[ C = 0.4 \text{ ft.} \approx 0.122 \text{ m} \]
- \[ C = 0.5 \text{ ft.} \approx 0.1525 \text{ m} \]
Figure 61. Legend same as Figure 57. See the figure for the parameters.
SANDY SOIL

\[ Y = Kt \]
\[ K = 0.25 \text{ (ft/hr)}^{-1} \]
\[ = 0.075 \text{ (m/hr)}^{-1} \]

\[ C = 0.1 \text{ ft} \]
\[ C = 0.2 \text{ ft} \]
\[ C = 0.3 \text{ ft} \]
\[ C = 0.4 \text{ ft} \]
\[ C = 0.5 \text{ ft} \]

\[ \frac{(\frac{b}{x})}{(\frac{f_t}{t})} \]
SUMMARY AND CONCLUSION

Mathematical equations describing the horizontal advance of the irrigation stream on the surface of soil were derived and discussed for different types of infiltration equations corresponding to different known field conditions. Complex variable theory was applied to transform certain complicated forms of solutions, not useable in practical work, to solutions in algebraic form that could easily be used in irrigation design. A laboratory model for testing the theory was made. The model had glass beads or soil aggregates as porous medium, water as fluid and walls of transparent plexiglas through which water on the surface of the porous medium or in the porous medium could be photographed.

Flooding irrigation water was introduced at one end of the model and, by photographs, followed in time. Potassium dichromate dye was injected in the porous medium to trace the direction of the stream lines when the water moved within the body of the porous medium. The horizontal advance of the water on the surface of beads was studied for different sizes of glass beads with different infiltration characteristics. The infiltration equation for the beads was found to be \( y = Et^a \), where \( y \) is the cumulative infiltration, \( t \) is the time since infiltration started and \( E \) and \( a \) are coefficients.

A comparison between experimental data obtained with the model and the calculated position of the water on the surface and below the surface by theory shows a good agreement between theory and experiment.

The equation giving the mathematical solution for the horizontal advance of water on the surface of porous medium when the infiltration
equation is \( y = E t^a \) does not give practical values by the computer for
large values of \( t \). This equation therefore does not allow for the compari-
on of the model data with theory for the whole range of time used in the
experimental runs. But from the theory, for values of time when the equa-
tion was valid, it was seen that certain dimensionless quantities could be
graphed to advantage both for small and large values of time \( t \). The ad-
vantage was that the graphs could be used for irrigation design problems
based on field data over a wide range of field conditions.

The dimensionless quantities plotted are \( qt/Cx \) on the vertical axes
and \( E t^a/C \) on the horizontal axis where

\[
q = \text{the rate of application of irrigation water in}
\]
\[
(\text{ft}^3/\text{ft}) \text{ or } (\text{m}^3/\text{m})
\]
\[
t = \text{the time in hours}
\]
\[
C = \text{the average depth of surface water in ft., or m.}
\]
\[
x = \text{the horizontal distance of the advance of the irriga-
tion stream on the surface of soil in ft., or m.}
\]
\[
E = \text{constant which has the dimension (ft)(hr)^{-a}, or}
\]
\[
(\text{m})(\text{hr})^{-a}
\]
and

\[
a = \text{constant which is dimensionless.}
\]

The dimensionless quantity \( qt/Cx \) was plotted versus the dimensionless
quantity \( E t^a/C \) and it was found to within experimental errors that the
model data fell on straight lines of the form

\[
qt/Cx = a + b E t^a/C
\]
where \( a \) and \( b \) were constants depending on the surface roughness, surface
slope and the infiltration characteristics of the porous medium. Because
these equations are dimensionless, they can be used under field conditions to calculate the horizontal distance of advance of the irrigation stream on the soil surface as a function of time and also the depth of penetration of irrigation water at any distance along the irrigation run. Thus, the equations help to avoid improper use of water.

The results of a sample calculation for a soil having an infiltration equation \( y = 0.0426t^{0.527} \) (\( y \) = depth of water penetration in ft., \( t \) = time in minutes) shows that if an irrigation stream of amount 2.94 ft\(^3\) per ft width of irrigation run per minute is introduced at the top of a field of the soil in question, then the advance of the irrigation stream on the soil surface will be 235 ft. at 20 min., 381 ft. at 40 min., 468 ft. at 60 min., 543 ft. at 80 min. and 583 ft. at 100 min. Furthermore, the depth of penetration of the water into the soil at these several distances after the irrigation stream has been turned on for the 100 minutes is: at zero distance, 0.4825 ft. of water penetration; at 235 ft. distance, 0.426 ft.; at 381 ft. distance, 0.3664 ft.; at 468 ft. distance, 0.2977 ft.; at 543 ft. distance, 0.2066 ft.; and at 583 ft. distance, 0 ft.

Theoretical water infiltration profiles were calculated for different sizes of glass beads and for different time increments and were compared with the observed infiltration profiles in the model and a good agreement was obtained.

Ida and Webster soils were obtained from the field and sieved aggregates < 2 mm. diameter from these soils were used in the model. Experimental and theoretical horizontal advance of the water on the surface of the soils was in good agreement when the infiltration equation used was
\[ y = At^{1/2} + Bt \] where \( y \) is the depth of surface water that has infiltrated into the soil, \( t \) is the time since infiltration started and \( A \) and \( B \) are coefficients that depend on the soil and its initial moisture content. From experimental model runs with glass beads and soils, it was found that the term \( C \) (which is the average depth of surface water during a run up to a horizontal distance \( x \) of advance of surface water) accounts for surface conditions such as slope and roughness. When the surface of the porous medium was at 1 percent slope the value of \( C \) was about \( 3/5 \) of the value for 0 percent slope. In order to apply the theory to field soils which had an infiltration equation of the type \( y = At^{1/2} + Bt \), where the parameters \( A \) and \( B \) are determined under different soil conditions, a family of curves for different values of \( C \) were developed, and were applied in some field irrigation problems. The results of a sample calculation for a soil having an infiltration equation \( y = 0.132t^{1/2} + 0.019t \) (\( y \) = depth of water penetration in ft, \( t \) = time in hours) shows that if an irrigation stream of amount 162 ft\(^3\) per ft. width of irrigation check per hour is introduced at the top of a field of the soil in question, then the advance of the irrigation stream on the soil surface will be 100 ft. at 13 min., 200 ft. at 23 min., 300 ft. at 47 min., 400 ft. at 64 min., 500 ft. at 87 min., 600 ft. at 118 min. and 680 ft. at 135 min. Furthermore, the depth of penetration of the water into the soil at these several distances after the irrigation stream has been turned on for the 135 minutes is: at zero distance, 0.2413 ft. of water penetration; at 100 ft. distance, 0.2276 ft.; at 200 ft. distance, 0.2168 ft.; at 300 ft. distance, 0.1880 ft.; at 400 ft. distance, 0.1667 ft.; at 500 ft. distance, 0.1330 ft.; at 600 ft.
distance, 0.0755 ft.; and at 680 ft. distance, 0 ft.

For soils, such as sand, the infiltration equation is of the type \( y = Kt \). This equation was used and theoretical families of curves were developed for different values of \( C \) and for different values of \( K \). The theoretical curves for the "sand" equation should be useful for agricultural application in arid and semi-arid regions where sand is a major soil type. The limiting distance of advance of water for a surface applied irrigation stream on a sandy soil is equal to \( q/K \) where \( q \) and \( K \) are as have been defined.
BIBLIOGRAPHY


Faddeyeva, V. N. and Terentev, N. M. 1961. Tables of values of the functions \( w(z) \) for a complex argument (English translation). Pergamon Press, New York, N.Y.


Parker, P. 1913. The control of water. D. Van Nostrand Co., New York, N.Y.

Philips, J. R. 1952. A quantitative approach to the hydraulics of primary flow furrow irrigation. Unpublished mimeographed paper presented at irrigation course, Yanco Exp. Farm, Deniliquin, N.S.W., Australia. Yanco Exp. Farm, Deniliquin, N.S.W., Australia.


ACKNOWLEDGMENTS

The author thanks Dr. Don Kirkham, his major professor, for suggesting the thesis problem and for his guidance and assistance. I have thoroughly enjoyed studying under the guidance of Dr. Don Kirkham. The author acknowledges the help of his graduate committee. Appreciation is also expressed to the graduate students in the Soil Physics Section for their cooperation and help. The author thanks his wife, Mady Asseed, for her encouragement during the course of the work.
Figure 62. Composite photographs of the model showing the change of the position of the water stream on the surface of the porous medium with time. The photographs show too the movement of the wet front within the porous medium for each time increments. The dark lines are the stream lines which are colored with potassium dichromate. The description of each condition is shown with the photographs. The clock runs counterclockwise.
GLASS BEADS

$y = 2.3^{0.656}$

$L = 1.9\, \text{cm}$

SLOPE = 1\%

$Q = 300\, \text{cm/min}$ WITH SURFACE ROUGHNESS
Figure 63. See legend for Figure 62.
GLASS BEADS

$y = 2.3 t^{0.656}$

$t = t_N - t_S$

$Q = 300 \text{ cm}^3/\text{min}$

$L = 1.9\text{cm}$

SLOPE = 1%

NO SURFACE ROUGHNESS
Figure 64. See legend for Figure 62.
GLASS BEADS

\[ y = 2.31^{10.636} \]
\[ t = t_2 - t_1 \]
\[ Q = 400\text{cm/min} \]

L = 1.9cm

SLOPE = 0%

NO SURFACE ROUGHNESS
GLASS BEAD

\[ y = 2.310656 \]
\[ L = 1.9 \text{ cm} \]
\[ SLOPE = 0\% \]
\[ Q = 300 \text{ cm}^3/\text{min} \]

WITH SURFACE ROUGHNESS
Figure 66. See legend for Figure 62.
GLASS BEADS

$y = 2.31^{0.656}$

$L = 1.9 \text{ cm}$

$t = t_e - t_0$

$Q = 400 \text{ cm}^3/\text{min}$ WITH SURFACE ROUGHNESS

SLOPE = 1\%
Figure 67. See legend for Figure 62.
GLASS BEAD

\[ y = 2.3t^{0.636} \]

\[ t = t_f - t_s \]

\[ 0 = 450 \text{ cm}^3/\text{min} \]

L = 1.9 cm

SLOPE = 0%

WITH SURFACE ROUGHNESS
Figure 68. See legend for Figure 62.
GLASS BEAD

\[ y = 2.3 \times l^{0.856} \]

\[ t = t_e - t_s \]

\[ Q = 400 \text{ cm}^3/\text{min} \]

\[ L = 1.9 \text{ cm} \]

SLOPE = 1%

NO SURFACE ROUGHNESS
Figure 69. See legend for Figure 62.
GLASS BEADS

\[ y = 1.58 \pm 0.568 \]

\[ t = t_b - t_s \]

\[ Q = 210 \text{ cm}^3/\text{min} \]

\[ \text{SLOPE} = 0\% \]

WITH SURFACE ROUGHNESS
Figure 70. See legend for Figure 62.
GLASS BEAD

\[ y = 5.8y^{0.58} \]

\[ m \cdot 5 - 1 \]

Q=200cm^2/min  NO SURFACE ROUGHNESS

L=1.9 cm  SLOPE +1%
Figure 71. See legend for Figure 62.
GLASS BEADS

$y = 1.581^{0.568}$

$\frac{1}{t} = \frac{1}{t_s} - \frac{1}{t_b}$

$L = 1.9 \text{ cm}$

SLOPE = 0%

$Q = 200 \text{ cm}^3/\text{min}$

NO SURFACE ROUGHNESS
Figure 72. See legend for Figure 62.
GLASS BEADS

\[ y \times 1.56 \times 1.9^{0.56} \]
\[ l = 1.9 \text{ cm} \]
\[ f = 0.5 \]
\[ SLOPE = 1\% \]
\[ Q = 200 \text{ cm}^3/\text{min} \]
WITH SURFACE ROUGHNESS
Figure 73. See legend for Figure 62.
GLASS BEADS

\[ y = 1.58 t^{0.508} \]

L = 1.9 cm

\[ t = t_0 - t_f \]

Q = 300 cm³/min

NO SURFACE ROUGHNESS

SLOPE = 0%
Figure 74. See legend for Figure 62.
GLASS BEAD

\[ y = 1.581^{0.568} \]

\[ t = t_x - t_g \]

\[ Q = 300 \text{cm}^3/\text{min} \]

L = 1.9 cm

SLOPE 1%

WITH SURFACE ROUGHNESS
Figure 75. See legend for Figure 62.
Figure 76. See legend for Figure 62.
GLASS BEADS
L 1.53em
SLOPE = 1%
NO SURFACE ROUGHNESS
Figure 77. See legend for Figure 62.
GLASS BEADS

\[ y = 1.69 t^{-0.527} \]

\[ L = 1.9 \text{ cm} \]

\[ t = t_i - t_e \]

\[ Q = 300 \text{ cm}^3/\text{min} \]

SLOPE = 0%

NO SURFACE ROUGHNESS
Figure 78. See legend for Figure 62.
NO SURFACE ROUGHNESS

SLOPE = 1\%  
L = 1\text{.9 cm}

Y = 1.6910337

GLASS BEADS
Figure 79. See legend for Figure 62.
GLASS BEAD
Diam: 1.9mm
No Surface Roughness
Figure 80. See legend for Figure 62.
IDA SOIL

\[ y = 0.60505 + 0.0986t \]

\[ t = t_a - t_b \]

\[ Q = 80 \text{ cm}^3/\text{min} \]

\[ Q = 80 \text{ cm}^3/\text{min} \]

NO SURFACE ROUGHNESS

SLOPE = 1%
Figure 81. See legend for Figure 62.
IDA SOIL

\[ y = 0.608 + 0.0986 \]  

\[ L = 1.9 \text{ cm} \]

\[ f = l_1 - l_2 \]

SLOPE <0%

\[ Q = 50 \text{ cm} \text{rain} \]

NO SURFACE ROUGHNESS
Figure 82. See legend for Figure 62.
IDA SOIL

\[ y = 0.605t^{0.5} + 0.0986 \]

\[ t = t_x - t_s \]

\[ Q = 50 \text{ cm}^3/\text{min} \]

\[ L = 1.9 \text{ cm} \]

SLOPE = 1%

NO SURFACE ROUGHNESS
Figure 83. See legend for Figure 62.
WEBSTER SOIL

$Y = 0.480^{+0.0341}_{-0.0344}$

$L = 1.9 \text{ cm}$

$s = \frac{\text{L}}{2}$

$q = 35 \text{ cm}^2/\text{min}$

SLOPE = 0\%
NO SURFACE ROUGHNESS
Figure 84. See legend for Figure 62.
WEBSLER SOIL

\[ \begin{align*}
Y &= 0.4801^2 + 0.341 \\
L &= 1.9 \text{ cm} \\
l &= l_{\text{ave}} \\
o &= 35 \text{ cm}^2/\text{min} \\
\text{SLOPE} &= 1\% \\
\text{NO SURFACE ROUGHNESS}
\end{align*} \]
Figure 85. Photographs showing the infiltration tests which were conducted to test for homogeneity of backing of the porous medium in the model.