Estimation and economic analysis of protein and energy utilization by beef steers

Francis Michael Epplin
Iowa State University

Follow this and additional works at: https://lib.dr.iastate.edu/rtd

Part of the Agricultural and Resource Economics Commons, and the Agricultural Economics Commons

Recommended Citation
https://lib.dr.iastate.edu/rtd/7204

This Dissertation is brought to you for free and open access by the Iowa State University Capstones, Theses and Dissertations at Iowa State University Digital Repository. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.
INFORMATION TO USERS

This was produced from a copy of a document sent to us for microfilming. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the material submitted.

The following explanation of techniques is provided to help you understand markings or notations which may appear on this reproduction.

1. The sign or “target” for pages apparently lacking from the document photographed is “Missing Page(s)”. If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting through an image and duplicating adjacent pages to assure you of complete continuity.

2. When an image on the film is obliterated with a round black mark it is an indication that the film inspector noticed either blurred copy because of movement during exposure, or duplicate copy. Unless we meant to delete copyrighted materials that should not have been filmed, you will find a good image of the page in the adjacent frame.

3. When a map, drawing or chart, etc., is part of the material being photographed the photographer has followed a definite method in “sectioning” the material. It is customary to begin filming at the upper left hand corner of a large sheet and to continue from left to right in equal sections with small overlaps. If necessary, sectioning is continued again—beginning below the first row and continuing on until complete.

4. For any illustrations that cannot be reproduced satisfactorily by xerography, photographic prints can be purchased at additional cost and tipped into your xerographic copy. Requests can be made to our Dissertations Customer Services Department.

5. Some pages in any document may have indistinct print. In all cases we have filmed the best available copy.

University Microfilms International
300 N. ZEEB ROAD, ANN ARBOR, MI 48106
18 BEDFORD ROW, LONDON WC1R 4EJ, ENGLAND
EPPLIN, FRANCIS MICHAEL

ESTIMATION AND ECONOMIC ANALYSIS OF PROTEIN AND ENERGY UTILIZATION BY BEEF STEERS.

IOWA STATE UNIVERSITY, PH.D., 1975
Estimation and economic analysis of protein and energy utilization by beef steers

by

Francis Michael Epplin

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of DOCTOR OF PHILOSOPHY

Department: Economics
Major: Agricultural Economics

Approved:
Signature was redacted for privacy.

In Charge of Major Work
Signature was redacted for privacy.

For the Major Department
Signature was redacted for privacy.

For the Graduate College

Iowa State University
Ames, Iowa

1979
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Crude Protein System and Urea</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Previous Studies</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Objectives</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>The Experiment</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Natural phase</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Urea phase</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>THEORETICAL CONSIDERATIONS</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Production Theory</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Isoquant</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Time on feed</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Marginal rate of substitution</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>Price considerations and optimization</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Geometric analysis</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>Nutrition Theory</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Swine studies</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Protein utilization by ruminants</td>
<td>26</td>
</tr>
<tr>
<td>3</td>
<td>ESTIMATION OF RESPONSE</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Multicollinearity</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>Autocorrelation</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Stochastic Regressors</td>
<td>35</td>
</tr>
<tr>
<td>Topic</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>Production Function Estimation</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>Test for OLS bias</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>Polar coordinates</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>Rectangular coordinates</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>Time Requirement Analysis</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td>Natural growing</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>Natural finishing</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>Urea growing and finishing</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>CHAPTER IV. ECONOMIC ANALYSIS</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>Natural Growing</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>Least-cost feed</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Least-cost feed plus time</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Natural Finishing</td>
<td>64</td>
<td></td>
</tr>
<tr>
<td>Urea Growing</td>
<td>67</td>
<td></td>
</tr>
<tr>
<td>Urea Finishing</td>
<td>67</td>
<td></td>
</tr>
<tr>
<td>Natural Versus Urea</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>CHAPTER V. SUMMARY AND IMPLICATIONS</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td>Natural Phase</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td>Urea Phase</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>Limitations</td>
<td>76</td>
<td></td>
</tr>
<tr>
<td>Implications</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td>REFERENCES</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>84</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER I. INTRODUCTION

The goals of production economics as defined by Heady [16] center around the concept of efficiency. More specifically, one goal is to achieve efficient allocation of resources on individual farms as well as efficient use of resources from the viewpoint of the entire consuming economy.

The U.S. cattle feeding industry is often accused of being inefficient. During periods of food scarcity, some suggest that feeding "scarce" grains to livestock is inefficient and perhaps, inhumane. The argument is that many go hungry because meat, milk and eggs are being substituted for direct grain consumption. It has been estimated that about 2,000 pounds of grain must be fed to livestock to produce a sufficient quantity of meat and other livestock products to support an adult for one year. On the other hand, only 400 pounds of grain consumed directly is needed to fulfill the same need [10]. By this measure five times as many people could be fed on a grain diet as on a livestock diet. Critics utilize these numbers to question the efficiency of feeding livestock. On the surface, the livestock industry appears to be very inefficient.

However, not all animals consume grain. It has been estimated that 83% of the feed of beef cattle in the United States is in the form of pasture and other roughages [10]. Many of these roughages are grown on land not suitable for production of grains. Ruminants are very adept at converting very low quality roughages into livestock products. They also consume a sizeable quantity of crop by-products such as corn stalks and almond hulls.
Burroughs and his associates suggest a goal for future beef production is to produce high quality beef with nongrain diets [6].

Cattle feeding tends to reduce the magnitude of grain price fluctuations. When world crop shortfalls result in increased grain prices, the industry may reduce the grain that is fed by increasing the roughage portion of the ration, or reducing the time on feed, or both. On the other hand, in times of bumper crops feeders may divert more grain into beef rations. Thus, the net effect of feeding grain to cattle may be to reduce the magnitude of grain price fluctuations and, thereby, enhance the stability of grain production.

It might also be argued that the large U.S. market for feed grains has supported the grain-producing industry and provided incentives for genetic and technological advancements in grain production. Thus, several factors can be cited in defense of feeding grains to beef cattle.

One of the goals of cattle feeding is to formulate efficient rations. However, Ensminger reports that animal scientists may be recommending higher protein levels than are believed to be necessary for efficient performance [10]. Underfeeding protein costs more than overfeeding. Thus, many rations are formulated with excess protein. One goal of this study is to utilize data of a controlled experiment to estimate protein needs of feedlot steers such that efficient rations can be formulated.

Crude Protein System and Urea

The most widely used measure of protein content of feeds is the crude protein system. On the average, proteins are approximately 16% nitrogen.
However, they range from 15 to 18%. Since most nitrogen-containing feed components are proteins, the percent of crude protein in a feed is obtained by chemically determining the nitrogen content and multiplying by 6.25 (100/16 = 6.25). This is a crude measure of protein, hence, the system's name [7].

Not all nitrogen-containing materials in feeds are true proteins. But, nonprotein nitrogen (NPN) components are usually present in small amounts and ordinarily do not introduce a sizeable error into the crude protein system. To a certain extent ruminants can effectively use NPN-containing materials for meeting their protein needs [7].

In 1891 two German researchers, Zuntz and Hagemann, discovered the ability of ruminants to convert NPN compounds into useful nutrients [41]. German researchers continued to work with the compounds and by 1924, Morgan and his associates had shown that 30 to 40% of the protein in the ration of a sheep could be replaced by urea [10].

Urea is an NPN compound which results from a chemical combination of ammonia and carbon dioxide, both of which are products or by-products of an ammonia plant [31]. Feed grade urea contains 45% nitrogen. Thus, the crude protein content of urea is estimated to be 281% (45 x 6.25) [10].

United States scientists were generally skeptical of using urea in ruminant rations. However, by the late 1930s it was recognized that protein quality did not seem to be as important for ruminants as for nonruminants. Thus, scientific interest in the United States increased. Natural protein feeds became relatively more valuable during World War II. This provided additional incentive for investigating the value of urea in rations
By the early 1950s numerous experiments were underway at various U.S. agricultural experiment stations to study the value and use of urea as a natural protein substitute for beef animals [29].

The use of urea in beef rations has increased rapidly. Cullison [7] reports that by 1975, 500,000 tons of urea were being used in beef rations per year. This quantity of urea has a nitrogen equivalent of 3.2 million tons of soybean meal which would have been approximately one-fourth of the annual use of soybean meal in the United States.

Even though urea supplementation has become common, the process of urea utilization has not been completely understood. Nutritionists have generally recognized that urea feeding is subject to limitations which are not explicitly accounted for by the nitrogen content. The response of steers to a feed is biological and a function of the individual ability of the steer to obtain useful nutrient value from the feed as well as the chemical composition. Preston [33] notes that ground fence posts and corn grain may have the same gross energy value chemically but different useful energy values. The biological attributes of feeds are more difficult to determine than the chemical characteristics. Thus, chemical analysis has been and continues to form the basis for ration formulation. Even though it is generally recognized that the biological response of steers to nitrogen content of urea is different from their response to nitrogen from soybean meal, the crude protein system remains popular.

Burroughs and his associates contend that the crude protein system is adequate for rations which contain only natural proteins. However, they suggest that it is unsatisfactory for predicting performance when urea is
included in the ration [3]. They have proposed the metabolizable protein (MP) system as an alternative protein evaluation method which is especially useful for urea-containing rations.

Previous Studies

Several studies of alternative protein levels in beef feeding rations have been conducted. For example, Kay, Bowers and McKiddie [24] fed three natural protein rations to steers weighing in excess of 250 kg. The rations contained 11, 14 and 17% crude protein. They concluded that the 11% ration was sufficient for "maximum" gains.

Oltjen, Slyter and Wilson fed alternative rations containing 9.2, 14.0, 18.4, and 23.0% crude protein [32]. The three high protein rations contained urea. They concluded that the 14% ration resulted in the "fastest" rate of gain.

Satter and Roffler [35] suggest that steers weighing more than 317 kg need 11 to 12% crude protein rations or less.

The National Research Council [30] recommends that 8.5 to 10.9% crude protein is required for steers weighing 350 kg. The low requirement is for a maintenance diet and the 10.9% recommendation is for a "maximum" gain diet.

These studies and recommendations suggest that over a range, alternative protein-energy combinations can be fed. In other words, over some range different combinations of protein and energy can be fed to achieve equivalent amounts of gain. This implies that a protein-energy production surface exists. However, previous studies have stopped short of estimating
the protein-energy production function. Perhaps one reason for the reluctance to estimate protein-energy surfaces for beef steers and thus, the rate of substitution between energy and protein, is that nutritionists have been reluctant to speak in terms of energy "substituting" for protein and vice versa. For example, Lister [25] has argued that although excess protein can be converted into energy, energy cannot be substituted for protein. Thus, he concludes it is not logical to speak in terms of energy-protein substitution. While it is true that a ration containing no protein might be disastrous, over some range of energy and protein combinations, as reflected in various studies, output will be essentially the same. Thus, over this range it could be argued that energy and protein are substitutable.

Objectives

The general objectives of this study are to present a brief discussion of production and beef nutrition theory and to utilize data of a controlled experiment to estimate beef gain production functions from which efficient rations can be formulated.

The Experiment

The data which are analyzed in this experiment were obtained from a feeding trial conducted in 1976 by the Moorman Manufacturing Company of Quincy, Illinois. Two separate but similar feeding trials were conducted simultaneously. Rations of one of the experiments contained only natural protein sources. Steers in the other experimental pens received rations supplemented with urea. Thus, in this study the two phases of the experiment are referred to as natural phase and urea phase. Each experiment
contained twelve pens of eight steers. A total of 192 animals were included.

All quantities referring to feeds and ration protein percentages for the experiment are in terms of dry matter (DM). Dry matter is that part of the feed which is not water. It is calculated by determining the percentage of water and subtracting from 100.

Natural phase

The steers initially weighed 243 kg and were fed a growing ration for 41 days. The nonsupplement portion of the growing ration consisted of 17.5% corn grain and 82.5% corn silage fed in fixed proportions. The crude protein percentage during the growing period ranged from 11.1 to 15.5. (The experiment was designed to have the protein percentage range from 10 to 15 in one percent increments, however, this plan was not properly executed.) The steers gained 69 kg during the growing period as they finished the 41 days at 312 kg.

During the next 28 days the rations were gradually adjusted with increasing proportions of corn grain until the corn grain to corn silage ratio switched to 90% corn grain and 10% corn silage. After the adjustment period, corn grain and corn silage were again fed in fixed proportions. Thus, the experiment was essentially a two-stage feeding trial. According to Meiske, Goodrich and Crawford, beef feeding researchers agree that the most efficient use of corn grain and corn silage results when a high silage ration is fed to younger, growing cattle followed by a high corn grain ration during the finishing stage [26]. The average weight of the steers increased from 312 kg to 355 kg during the 28-day adjustment period.
The steers were finished from 355 kg to 461 kg on the high concentrate finishing ration which ranged from 12 to 16.6\% crude protein. Again, the design of the experiment which was to vary the protein percent from 10 to 15 in one percent increments was not properly executed. The finishing period lasted 106 days.

Interpolations of the feed consumption data to specific gain levels enabled plots of specific isoquants. These plots revealed that two of the twelve pens were obvious outliers. Data from these two pens were deleted prior to statistical estimation. In one pen, one steer was sick, lost weight and eventually died. The precise date of death was not recorded. It was not possible to separate the feed consumption of the seven healthy steers who shared the pen from that of the sick steer. Hence, all information from the pen was deleted. Data from the second outlier simply do not correspond with the other pens of the experiment. It is possible that some measurement of feed consumption or weight gain was not properly recorded. Data from the remaining ten pens were used for statistical estimation.

**Urea phase**

The design of the urea phase of the experiment was very similar to that of the natural phase. The primary difference was that these rations were supplemented with urea. The steers initially weighed 242 kg and finished the 41-day growing period at 308 kg. The protein supplement consisted of 10.92\% urea and 89.08\% soybean meal. The growing rations ranged from 11.5 to 16.3\% crude protein.
In the 28-day transition from the high corn silage to the high corn grain ration, the steers gained 44 kg. They started the 106-day finishing period at 352 kg and grew to 456 kg. The finishing rations ranged from 12.5 to 16.3% crude protein.
CHAPTER II. THEORETICAL CONSIDERATIONS

It is probably impossible to list all inputs involved in producing weight gain in beef steers. But, a simplification can be made by limiting experiments to a manageable number of variable inputs. For the present study, animals were randomly assigned to pens. Thus, the genetic potential is assumed to be fixed across pens. All pens were located in the same facility. Hence, environmental variables such as temperature and humidity, which may be very important in feedlot response, were fixed across pens. The composition of the nonsupplement portion of the rations was also fixed across pens. Since corn grain and corn silage were fed in fixed proportions during both the growing and finishing periods, the energy densities of the rations were nearly fixed across pens. The experiment was designed to evaluate the response of steers to rations containing alternative percentages of crude protein. Thus, the inputs which were permitted to vary across pens were supplement and corn (grain and silage) consumption.

Production Theory

Although other variables are important in producing beef gain this study is concerned primarily with the relationship between alternative levels of protein supplementation in rations. The variable inputs are the quantity of supplement and corn (silage and grain). Since corn silage and corn grain were fed in fixed proportions they can be considered as a single composite input. Thus, the beef gain production relationship for this study can be represented as a function of supplement and corn (grain and
silage) consumption with all other factors of production assumed to be included at fixed levels. The production function can be mathematically represented implicitly as

\[ \hat{\psi} = f(C, S) \]  

(2.1)

where:

\( \hat{\psi} \) = estimated kilograms of weight gain per steer;

\( C \) = kilograms (DM) of corn grain and corn silage consumption;

\( S \) = kilograms (DM) of supplement consumption; and

\( f \) = the functional form.

Variables other than supplement and corn (silage and grain) are either assumed to be fixed experimentally or beyond the control of the feeder. Hence, equation 2.1 ignores them.

Dillon [9] suggests that a useful simplifying theory of gain response has three basic assumptions. For the present study these assumptions can be summarized as follows.

(1) There is a continuous smooth causal relation between feed intake and weight gain, or alternatively, the first derivatives of equation 2.1, \( \frac{\partial \psi}{\partial C} \) and \( \frac{\partial \psi}{\partial S} \), should exist.

(2) The feed required per unit of gain is expected to increase at heavier weights, or alternatively, the marginal return per unit of feed is expected to decline \( \frac{\partial G}{\partial C} < 0 \) and \( \frac{\partial G}{\partial S} < 0 \).

(3) The marginal rate of substitution between feeds is decreasing and negative over the relevant range of economic problems. This assumption requires the isoquant to be convex to the origin if
the inputs are independent. Inputs are independent if they can be purchased and fed separately. For example, commodities such as corn and soybean meal are independent. But, nutrients such as energy and protein are not.

**Isoquant**

An isoquant represents all combinations of the two variable inputs that yield the same level of gain. Along the isoquant all inputs other than corn (grain and silage), supplement and time are assumed to be fixed. Time is a special variable. It is generally assumed that along an isoquant the gain will be achieved during the same "time period" [21]. Hence, even though a few more days may be required to achieve the gain level specified by an isoquant if the steers receive a low protein ration, the "time period" is assumed to be the same.

**Time on feed**

If time required is significantly different across rations and thus, along the isoquant, it would be important under some economic objectives and should be considered. Time may be incorporated into the analysis in several ways. It is possible to estimate fixed time isoquants. Along these isoquants the rate of gain, rather than gain, is held constant. Thus, the isoquants indicate the quantities of feeds or nutrients necessary to achieve a specified rate of gain. This approach has been used by Dent [8].

Another method for handling time, if it varies along the isoquant, is to estimate an additional function. For example, Heady, Roehrkkasse, Woods, Scholl, and Fuller [19] estimated consumption functions with one feed input
as a function of time, and the other inputs. Thus, time could be incorporated into the profit equation even though it was not held constant along the isoquant.

A similar but different method was used to incorporate time considerations into an analysis of swine rations conducted by Sonka, Heady and Dahm [37]. They estimated time on feed functions with time in days estimated as a function of the ration fed. Gain was fixed at specific levels and days required to achieve these levels were interpolated within weigh periods. For the present experiment a time on feed function can be estimated with days on feed as a function of the percent protein in the ration. Consider the following equation

\[ \hat{D} = h(K) \]  

(2.2)

where:

\( \hat{D} \) = the estimated number of days required to achieve a specified weight level;

\( K \) = the percent crude protein in the ration fed; and

\( h \) = the functional form.

This type of approach is adaptable to situations where the desired gain level can be specified. A new time on feed function is necessary for each isoquant. Thus, the approach by Heady et al. [19] is more general. However, in solving for least-cost rations, the desired weight level must be specified. Thus, the Sonka, Heady and Dahm [37] approach is used in this study when time on feed functions are needed.
Marginal rate of substitution

The production function can be solved for one input in terms of the other input and output. When output is fixed at a given level, a specific isoquant is defined. For example, the general isoquant equation for the production function of equation 2.1 is

\[ C = g(S, G_o) \quad \text{or} \quad S = g(C, G_o) \quad (2.3) \]

where:

\( g \) = the functional form; and

\( G_o \) = weight gain fixed at some arbitrary level.

A smooth continuous production function implies smooth gain isoquants. The slope of the isoquant is the marginal rate of substitution. This is an estimate of the rate at which one input substitutes for another. The marginal rate of substitution can be obtained by totally differentiating the production function.

\[ dG = \frac{\partial f}{\partial C} \, dC + \frac{\partial f}{\partial S} \, dS \quad (2.5) \]

Along the isoquant gain is constant. Thus, \( dG = 0 \) and it follows that

\[ \frac{dC}{dS} = -\frac{\frac{\partial f}{\partial S}}{\frac{\partial f}{\partial C}} \quad (2.6) \]

Furthermore, \( \frac{\partial f}{\partial S} \) and \( \frac{\partial f}{\partial C} \) are the marginal products of \( S \) and \( C \), respectively. Thus, the marginal rate of substitution and the slope of the isoquant is equal to the negative of the ratio of the marginal products.
Price considerations and optimization

The production function contains the physical information necessary to determine efficient rations. Additional necessary information is contained in the factor and product prices. For an individual producer in a competitive market, prices are assumed to be given. The individual has no control over prices. From the standpoint of society, prices are expected to transmit information on the relative scarcity and utility of the items. Incorrect prices will transmit wrong signals to producers and can distort decisions from efficiency.

Appropriate prices combined with the production function provide information for determining optimal rations. Beef producers may have many diverse objectives. For example, see Melton, Heady, Willham, and Hoffman [28]. However, one common objective is to formulate a least-cost ration subject to some output constraint. For example, the objective of a producer may be to minimize the cost of feed to achieve a certain level of gain.

The following Lagrangean function is a mathematical representation of the constrained cost minimization problem [21].

\[
\text{Minimize } L = P_c C + P_s S + \lambda \left[ G_o - f(C,S) \right] 
\]  

(2.7)

where:

- \( P_c \) = price per kilogram (DM) corn (silage plus grain);
- \( P_s \) = price per kilogram (DM) supplement;
- \( G_o \) = fixed level of gain;
- \( \lambda \) = Lagrangean multiplier; and
- other symbols are as previously defined.
The first-order or "necessary" condition for a minimum occurs when:

\[
\frac{\partial L}{\partial C} = p_c - \lambda \frac{\partial f}{\partial C} = 0 \quad (2.8)
\]

\[
\frac{\partial L}{\partial S} = p_s - \lambda \frac{\partial f}{\partial S} = 0 \quad (2.9)
\]

\[
\frac{\partial L}{\partial \lambda} = G_o - f(C, S) = 0 \quad \text{or} \quad (2.10)
\]

\[
\frac{p_C}{p_S} = \frac{\partial f/\partial C}{\partial f/\partial S} = \frac{\partial S}{\partial C} \quad \text{and} \quad (2.11)
\]

\[
G_o - f(C, S) = 0 \quad (2.12)
\]

Thus, the first-order condition requires that the ratio of marginal products should be equal to the ratio of the prices for the respective inputs subject to the constraint that the output, \(G_o\), is achieved. The second-order or "sufficient" condition for a minimum is satisfied if the isoquant is convex to the origin at the point where the first-order condition holds. This occurs if the relevant bordered Hessian determinant is negative [21].

\[
\begin{vmatrix}
-\lambda \frac{\partial^2 f}{\partial C^2} & -\lambda \frac{\partial^2 f}{\partial C \partial S} & -\frac{\partial f}{\partial C} \\
-\lambda \frac{\partial^2 f}{\partial S \partial C} & -\lambda \frac{\partial^2 f}{\partial S^2} & -\frac{\partial f}{\partial S} \\
-\frac{\partial f}{\partial C} & -\frac{\partial f}{\partial S} & 0
\end{vmatrix} < 0 \quad (2.13)
\]

If the second-order condition is satisfied, the point of tangency between the isoquant and the isocost line defined by the first-order condition is a solution to the least-cost ration problem.
Geometric analysis

The relationship between the isoquant and the isocost line is geometrically represented in Figure 2.1. The isoquant is represented by the convex to the origin line segment AC. The isoquant is bounded by the two ration lines which represent the extremes of the experimental rations. The slope of the line is equal to the negative of the ratio of the prices of feed 2 to feed 1. It is tangent to the isoquant at point B. The tangency requirement satisfies the first-order condition and the convex nature of the isoquant at the point of tangency satisfies the second-order condition. Thus, the optimal least-cost ration to feed to achieve the weight level represented by isoquant AC is determined by connecting point B with the origin. The two feeds should be fed in the proportion represented by the optimal ration line. If the feed price ratio changes, the tangency between the isoquant and the isocost line will change and a different ration will be optimal.

Not all estimated isoquant segments are convex to the origin. Figure 2.2 illustrates a concave isoquant over the range of experimental rations. The second-order conditions for a least-cost ration would not be met along the FG isoquant. The least-cost ration would be achieved either at one of the two extreme experimental rations or beyond the range of the experimental observations.

Another situation is represented by isoquant segment DE in Figure 2.3. It is upward sloping. The marginal product of feed 1 is negative (or beyond the ridge line) over the entire estimated segment. Ridge lines are ration lines which enclose the region in which the marginal products of both feeds
Figure 2.1. An illustration of a convex isoquant and linear isocost line

Figure 2.2. An illustration of a concave isoquant
are positive [21]. Since the marginal product of feed 1 is negative and the marginal product of feed 2 is positive, the entire isoquant segment DE is beyond the ridge line and, therefore, beyond the area of rational rations. A situation such as depicted in Figure 2.3 indicates that the rations fed contained an excessive amount of feed 1 or, perhaps, feed 1 is detrimental to growth.

![Diagram](image)

**Figure 2.3.** An illustration of an upward sloping isoquant

If situations such as those represented in Figures 2.2 and 2.3 occur, the standard methods for economic analysis via calculus cannot be followed. Economic textbooks generally limit their optimization discussions and techniques to isoquants which are convex to the origin. These isoquants result from "well-behaved" production functions. This is not to say that economics
cannot deal with nonconvex isoquants. It is straightforward to obtain solutions to upward sloping or concave isoquants if the inputs are independent. If the isoquant is strictly upward sloping from the horizontal axis, the optimal ration would include none of the feed represented on the vertical axis, since including it would increase total feed costs.

Nutrition Theory

A study by Heady, Carter and Culbertson reported in Heady and Dillon [18] includes estimated corn-protein substitution relationships with forage held constant at specified levels. The data used for the analysis were compiled from a number of feeding trials designed for other purposes. Hence, the authors suggest that the estimates should be used with caution.

A sizeable investment in terms of livestock, facilities, feeds, and labor is required to conduct experiments with beef cattle. This, in part, explains why there has been little empirical work on the estimation of these relationships with beef cattle.

Swine studies

A number of protein-energy substitution studies have been conducted with swine. One of the pioneering studies was conducted by Heady, Woodworth, Catron, and Ashton [17]. They estimated weight gain production functions with corn and soybean meal as variable inputs. The resulting isoquants were convex to the origin over the relevant range of rations.

More recent studies reported by Sonka, Heady and Dahm [37] and Heady, Sonka and Dahm [20] also used corn and a soybean meal base supplement as variable inputs. They also show convex to the origin isoquants. These
studies which estimate the corn grain-supplement isoquants have permitted
time to vary along the isoquant. Time has been introduced into the analysis
with time functions estimated to specific gain levels.

Fawcett [11] refers to the practice of estimating the gain response as
a function of the specific feed commodities as the "commodity feed school"
approach. Since the vast majority of swine producers in the United States
feed corn as the energy source and soybean meal as the supplemental protein
source it seems appropriate to use the commodities directly in the estima-
tion process. However, in other parts of the world, alternative feed com-
binations are more common. Dent [8] has estimated swine gain response
directly as a function of energy and crude protein. Fawcett refers to this
approach as the "nutritional school."

Time is fixed along the isoquants estimated by Dent [8]. Two isoquants
derived from his estimated production function are drawn in Figure 2.4.
With crude protein on the vertical axis and energy on the horizontal axis
these isoquants have a smooth arrow shape pointing toward the protein axis.
Fawcett, Whittemore and Rowland [12] have developed theoretical protein-
energy isoquants for swine. They also show arrow-shaped isoquants as graphed
in Figure 2.5. They suggest that the isoquants are biologically sound.
For high protein rations the isoquants are positively sloped (with protein
on the vertical axis). This is due to the energy cost of deaminating sur-
plus protein. Excess amino acids must be deaminated or excreted. This is
an energy-consuming activity. More energy is required to break up the ex-
cess protein than is yielded [11]. In the downward sloping part of the iso-
quant more feed is consumed to acquire the necessary amino acids. Thus,
Figure 2.4. Isogrowth curves for combinations of energy and protein for 1 lb and 1.25 lb of pig growth per day derived from a square root production function [8]

Figure 2.5. Isoquant functions for a 20 kg pig [12]
when protein is present in small proportions more energy must be consumed in order to acquire sufficient protein.

It is possible to convert a "commodity" isoquant into a "nutrient" isoquant. For example, the isoquant drawn in Figure 2.6 was derived from a quadratic production function estimated by Boggess, Olson, Heady, and Speer [2].

By converting the inputs of corn and soybean meal into crude protein and calories, the "nutrient" isoquant can be drawn as in Figure 2.7. The convex "commodity" isoquant converts into a U-shaped "nutrient" isoquant. Time is not held fixed along the isoquant in Figure 2.7. Thus, the isoquant of Figure 2.7 has a different shape than the isoquants of Figure 2.4. In the latter time is held fixed, but not in the former.

The low protein ration represented by F' requires more total energy and more total protein to yield 150 pounds of weight gain than any of the other rations fed. However, under some price scenarios the ration represented by F' could cost less than the ration represented by A' which requires less of both energy and protein. This result seems to conflict with economic theory. It does not. The inputs measured along the axes in Figure 2.7 are not independent. Most feeds contain both energy and protein. For example, Fawcett [11] contends that it is impossible to provide protein free of energy in a swine ration.

The actual shape of the isoquant in Figure 2.7 may be explained in part by the variance of time along the isoquant. More time is required to achieve the weight gain on the low protein percentage ration represented by F'. The additional time requires more days of maintenance and thus, more
Figure 2.6. Isoquant derived from a quadratic swine production function [2]

Figure 2.7. Energy-protein isoquant estimated from quadratic commodity production function [2]
of both inputs. Increasing the protein percentage in the ration (moving along the isoquant from F' to C') reduces both the energy and the protein required and, presumably, the time. From C' to A' along the isoquant, energy required declines but protein requirements increase.

Both the arrow-shaped isoquant of Figure 2.4 and the U-shaped isoquant of Figure 2.7 suggest that it is more difficult to conduct an economic analysis of nutrient isoquants relative to commodity isoquants. In other words, it is not possible to determine the optimal ration by equating the slopes of an isocost line with that of a convex isoquant when the isoquant is in terms of nutrients rather than actual commodities. Furthermore, it is not easy to determine the price of a unit of crude protein [39].

The energy and protein requirements at any point on the isoquant can be met by a large number of ration formulations. At each of these points one of the rations is preferred. The problem is to select the least-cost ration from among the group of least expensive rations at all points along the isoquant. Dent [8] selected several points along the isoquant and then used linear programming to select the optimal rations from the points examined. Townsley [38] demonstrated that since the production function used by Dent was quadratic, optimal rations could be obtained by parametric quadratic programming techniques.

To this point we have discussed energy-protein substitution in swine. The next section briefly explains the process of protein utilization by ruminants.
Protein utilization by ruminants

Figure 2.8 is taken from an article by Satter and Roffler [35]. It is a schematic summary of nitrogen utilization by the ruminant. The example diagram depicts a lactating cow consuming 40 pounds of ration dry matter containing 14% crude protein.

Figure 2.8. Schematic summary of protein utilization by ruminants [35]

When natural protein enters the rumen it follows one of two paths. Some of it passes undestroyed through the rumen. (This was estimated to be 40% for the example cow.) The remaining protein will be destroyed by microbial fermentation into ammonia. The percent of the true protein that escapes breakdown in the rumen depends on the source. For example, very little of the protein in milk escapes breakdown because it is highly destructible (digestible). On the other hand, corn proteins are less
susceptible to rumen microbial destruction. Hence, a greater percentage of
the protein in corn passes through the rumen into the intestine for diges­
tion. Some nutritionists have been experimenting with methods to "protect"
the natural protein from bacterial breakdown in the rumen. More of it would
pass through the rumen for digestion [5].

Ammonia which results from microbial fermentation can be used by
bacteria who reform it into bacterial protein. Production of bacterial pro­
tein depends on the number of bacteria present and how fast they are growing.
This, in turn, depends on how much feed energy (represented by TDN in Figure
2.8) is available in the rumen. Increasing the concentration of energy may
increase bacterial numbers. More bacteria can convert more ammonia into
bacterial protein. If the bacteria do not utilize all of the ammonia, it
will be converted into urea by the liver and excreted. Excess ammonia has
no value.

The metabolizable protein (MP) system If the bacterial population
is "large" and utilizing all of the ammonia resulting from the natural feeds,
the addition of urea may be useful. The MP system estimates the "urea
fermentation potential" (UFP) of rations [3, 4]. The system provides a
method for quantitatively evaluating the amount of urea that can be useful
in any given cattle ration. A positive UFP indicates the level to which
urea can be "usefully" fed. It is the estimated satisfactory level for
achieving maximum or near maximum formation of urea nitrogen into useful
microbial protein. Any urea fed in excess of the UFP is expected to result
in overflow ammonia. A negative UFP indicates that excess rumen ammonia is
expected to result from the ration. Additional ammonia from urea would be
useless for a ration with a negative UFP.
Since corn grain is relatively high in energy and yields a relatively small amount of ammonia, it has a positive UFP. The estimated UFP for corn grain is +11.8 g/kg. A dry matter kilogram of corn grain does not yield overflow ammonia and provides sufficient energy for bacteria to convert 11.8 g of urea into useful microbial protein. On the other hand, the UFP for soybean meal is -107.7 g/kg. Soybean meal yields more ammonia than can be converted via bacteria with the energy present in the unit of soybean meal.

**Limitation of urea phase** In the urea phase of the present study, corn grain, corn silage, soybean meal, and urea were fed. The UFP of the rations can be determined by multiplying the appropriate UFP by the quantity of feed fed and summing across the feeds. For example, during the 41-day growing period the eight steers of one of the pens were fed 247.35 kg of corn silage, 52.43 kg of corn grain, 35.07 kg of soybean meal, and 4.30 kg of urea. The UFP in g/kg of corn silage, corn grain and soybean meal is +6.4, +11.8 and -107.7, respectively [4]. Thus, the UFP of the ration is equal to

\[(247.35)(6.4) + (52.43)(11.8) + (35.07)(-107.7) = -1,575.4 \text{ g.}\]

The ration has no potential to convert urea into microbial protein. An ammonia overflow would result from the microbial destruction of the true protein in the ration. But, this particular ration also contained 4.3 kg of urea. Based on the MP system, this urea merely added to the ammonia overflow and may have resulted in ammonia toxicity. On this basis, the
supplement which contained urea and soybean meal during the growing period might have had a negative marginal product.

Six of the twelve pens of the urea growing phase received rations with a negative UFP. The two pens which received the least total feed to achieve 55 kg of weight gain during the growing period were the same two pens which received the least amounts of supplement. Perhaps, the rumens of "growing" steers are not developed to the point where they can process large quantities of energy which would be required to fuel a bacterial population sufficiently large to convert ammonia from urea into useful microbial protein.

None of the finishing rations fed to the steers of the urea phase contained a negative UFP. Four of the twelve pens were fed urea to levels less than UFP. The remaining eight pens were fed urea in quantities exceeding the UFP.
CHAPTER III. ESTIMATION OF RESPONSE

The statistical method used to estimate response functions from empirical data is regression analysis. The general regression model is of the form

\[ y = XB + e \]  

(3.1)

where:

- \( y \) = a \( n \times 1 \) vector of \( n \) observations on the dependent variable (e.g., kilograms of gain);
- \( X \) = a \( n \times m \) matrix of \( n \) observations on \( m \) independent variables (e.g., kilograms of feed intake);
- \( B \) = a \( m \times 1 \) vector of regression coefficients which relate \( y \) and \( X \);
- and
- \( e \) = a \( n \times 1 \) vector of stochastic (residual or error) terms.

The following set of assumptions is crucial for estimating the elements of the \( B \) vector.

1. \( E(e) = 0 \)
2. \( E(ee') = \sigma^2 I_n \)
3. The \( X \) matrix is a set of fixed numbers
4. \( x \) has rank \( m < n \).

Assumption (1) states that the elements of the \( e \) matrix are variables with zero expectation. The second assumption could be called a double assumption. The expected value of \( e \) multiplied by \( e \) transpose is assumed
to be a constant, $\sigma^2$, for all variances. This property is referred to as homoscedasticity. In addition, the off diagonal terms are assumed to be pairwise uncorrelated. Thirdly, the model assumes that the values of the $X$ matrix are known and fixed. In other words, an assumption is made that the components of the vectors of the $X$ matrix are not measured with error. If the variables of the $X$ matrix are stochastic they should be independent of $e$ such that $E(x'e) = 0$ [23]. If either of these two conditions hold, the variation in $e$ measured by $\sigma^2$ reflects the variation in $y$. The fourth assumption is that the number of observations must exceed the number of parameters to be estimated and that the vectors of the $X$ matrix are linearly independent. If these assumptions hold, best linear unbiased estimators may be obtained with ordinary least-squares (OLS) analysis.

OLS is a mathematical procedure which calculates the $B$ vector which minimizes the sum of squared residuals, $e'e$. The data of the present study may violate some of the four statistical assumptions. Three possible violations are multicollinearity, autocorrelation and stochastic regressors.

**Multicollinearity**

The fourth basic assumption of the OLS model is that the vectors of the $X$ matrix are linearly independent. If a linear dependency exists the OLS problem cannot be solved since solution to the problem requires inverting the $X'X$ matrix. This is the case of extreme multicollinearity which exists when some or all of the vectors of the $X$ matrix are perfectly collinear. It is also possible for two or more independent variables to be so highly correlated but less than perfectly, such that the $(X'X)^{-1}$ matrix
exists but it is difficult to distinguish the separate effects among the independent variables upon the dependent variable. On the other hand, it is possible to design experiments with no intercorrelation such that the effects of the different independent variables are strictly additive [13]. The major consequence of serious multicollinearity is that the standard errors of estimated regression coefficients may be increased as a result of the intercorrelation. In which case, estimated regression coefficients may not be statistically significantly different from zero not because the variable is unimportant, but due to the intercorrelation.

According to Johnston [23] there is no reason why collinearity should seriously bias the estimate of $\sigma^2$. He suggests that if the estimated parameters have an unsatisfactorily low degree of precision, "...we are in the statistical position of not being able to make bricks without straw" [23, p. 163]. When this happens, a new data set is needed. However, because of the cost of acquiring a new data set, if the $x'x$ matrix can be inverted such that the OLS problem can be solved, the results are generally accepted subject to the recognition that the standard errors of the estimated coefficients may be slightly exaggerated.

Autocorrelation

For the data of the present study repeated measurements were made on the same experimental units over time. The same group of animals was weighed periodically, usually in 28-day intervals, from the beginning to the end of the experiment to provide observations on feed inputs and weight gain. Hence, the feed consumption and gain observation measurements over time are not independent. The OLS residuals associated with these
measurements on the same animals over time may be correlated. This would violate part of the second assumption which is that the residuals should be pairwise uncorrelated. In addition, the variance estimates of the coefficients would be invalid as would statistical tests on the significance of the estimated coefficients.

The problem would not exist if different animals were used at each level of observation. But, this would add considerably to the cost of the experiment. Thus, statistical methods have been developed to correct for the problem. Steers with low rates of gain in one period generally compensate by gaining more in the subsequent period. These effects, commonly referred to as compensatory gains, are generally expected to dissipate in 10 to 14 days [27]. Thus, a first-order autoregressive scheme is assumed. This means that we assume that the residuals of adjacent periods are correlated but that residuals beyond adjacent periods are not correlated.

One statistical method which has been devised for obtaining valid estimates from a data set containing autocorrelation is to relax the second OLS assumption. For example, assumption (2) is that $E(ee') = \sigma^2 I_n$. This may be relaxed to $E(ee') = V$, where $V$ is a positive definite variance-covariance matrix of size $n \times n$. A nonsingular transformation matrix, $T$, of size $n \times n$ can be used to correct for autocorrelation. $T$ can be estimated such that $T'T = V^{-1}$ and $E(Tee'T) = TVT' = \sigma^2 I$.

An estimate of the $T$ or transformation matrix can be obtained by using the following steps.

(a) Regress steer gain on feed intake;

(b) Calculate $\hat{e}_i = e_i - \hat{G}_i$, where
\( \hat{e}_i \) = residual in period \( i \),
\( G_i \) = recorded gain in period \( i \),
\( \hat{G}_i \) = predicted gain in period \( i \), and
\( i = 1, 2, \ldots \text{ number of periods of observations per pen.} \)

(c) Calculate
\[
\hat{\rho} = \frac{\sum_{g} \sum_{t} \hat{e}_{g,t} \hat{e}_{g,t-1}}{\sum_{g} \sum_{t} \hat{e}_{g,t}^2}
\]
\( \hat{e}_{t-1} = 0 \) when \( t=1 \) for each pen,
\( t = 1, 2, \ldots, j, \)
\( j = \text{number of measurement observations,} \)
\( g = 1, 2, \ldots, k, \) and
\( k = \text{number of pens.} \)

(d) The transformation matrix for the pen is
\[
T_{j \times j} = \begin{bmatrix}
\sqrt{1 - \hat{\rho}^2} & 0 & 0 & \ldots & 0 \\
-\hat{\rho} & 1 & 0 & \ldots & 0 \\
0 & -\hat{\rho} & 1 & \ldots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & 1
\end{bmatrix}
\]

(e) A Kronecker product of an identity matrix of size \( k \times k \) and \( T_{j \times j} \) yields the \( T \) matrix for the data set.

The correlation between the residuals in the same period is \( 1.0, \hat{\rho} \) in adjacent periods and zero otherwise. Efficient regression coefficients can
be obtained by multiplying both the x matrix and the y vector by T and regressing Ty on Tx.

**Stochastic Regressors**

The third crucial assumption for OLS is that the x matrix is a set of fixed numbers. For the present study the x matrix contains the amount of feed fed to the steers. Actual feed intake was controlled by the steers and not the research workers. Also, feed fed per pen was recorded. Intake of individual animals was not recorded. Each animal in the pen was assumed to have consumed the same quantity. Thus, the regressors may contain a stochastic component. For example, consider the following equation.

\[ x = \bar{x} + w \]  \hspace{1cm} (3.2)

Assume that

\[ E(w) = 0 \text{ and } E(ww') = \sigma_w^2 I_n \]

where:

- \( x \) = a \( n \times m \) matrix of the measured or observed levels of the independent variables;
- \( \bar{x} \) = a \( n \times m \) matrix of the "systematic component" of \( x \); and
- \( w \) = a \( n \times m \) matrix of the "stochastic component" of \( x \) with mean equal to zero, variance of \( \sigma_w^2 \) and zero covariance.

Thus, if the x matrix contains a stochastic component the OLS model of equation 3.1 becomes

\[ y = (\bar{x} + w)B + e \]  \hspace{1cm} (3.3)
and since it cannot be assumed that $E(w'e) = 0$ nor that $E(x'e) = 0$, the use of OLS procedures may yield both biased and inconsistent estimates of $B$.

This situation is analogous to that of errors in variables in the sense that the independent variables of the regression equation may contain a stochastic component. This stochastic component could result in both biased and inconsistent regression coefficient estimates. Fuller [15] has developed a statistical test to determine if OLS techniques applied to a data set suspected of containing stochastic regressors will provide nearly unbiased estimators.

For the present study the test may be conducted by first determining which variables can be assumed to be fixed. The intercept, denoted by "1," the time on feed as measured by days, $D$, and the square of time, $D^2$, can be considered as fixed. The stochastic independent variables are measured observations on feed intake. For a linear model the variables $C$ and $S$ fall into this category. The logarithms of $C$ and $S$ denoted as $LC$ and $LS$ could be used for a nonlinear model. The next step is to regress the stochastic variables of the system on the fixed variables.

\[
\hat{C} = f(1, D, D^2) \quad (3.4)
\]
\[
\hat{S} = f(1, D, D^2) \quad (3.5)
\]

or for the logarithmic model

\[
\hat{LC} = f(1, D, D^2) \quad \text{and} \quad (3.6)
\]
\[
\hat{LS} = f(1, D, D^2) \quad (3.7)
\]
Let the estimated residuals from these equations be \( CR, SR, LCR, \) and \( LSR, \) respectively. The full models (\( fm \)) for the Fuller test are

\[
\hat{G} = f(1, C, S, CR, SR)
\]

and alternatively

\[
\hat{G} = f(1, LC, LS, LCR, LSR)
\]  \hspace{1cm} (3.9)

The reduced models (\( rm \)) are

\[
\hat{G} = f(1, C, S) \quad \text{and} \quad \hat{G} = f(1, LC, LS).
\]  \hspace{1cm} (3.10)

The \( F \) test is

\[
F_{w, n-u-w} = \frac{\frac{SSE_{rm} - SSE_{fm}}{w}}{\frac{SSE_{fm}}{n-u-w}}
\]  \hspace{1cm} (3.11)

where:

- \( SSE = \) sum of squares error;
- \( w = \) the number of stochastic variables in the full model;
- \( n = \) the total number of observations; and
- \( u = \) the number of nonstochastic variables in the full model.

If the calculated \( F \) is less than the tabled \( F, \) OLS estimates obtained from regression equations which contain the stochastic regressors will be nearly unbiased. If, on the other hand, at the specified level of significance the calculated \( F \) exceeds the table value for \( F, \) OLS procedures will yield biased estimates.
Production Function Estimation

For purposes of statistical estimation and subsequent economic analysis, the growing and finishing periods were examined independently for both the urea and natural phases of the experiment. Thus, the experiment yielded four data sets which are arbitrarily labeled natural growing, natural finishing, urea growing, and urea finishing.

Test for OLS bias

The Fuller F-test [15] was applied to all four data sets to check for OLS bias. In each case the test results indicate that the data will provide "nearly unbiased" estimators at the 0.01 level of probability. The actual calculated F values were $F_{2,15} = 0.79$, $F_{2,35} = 1.92$, $F_{2,19} = 4.32$, and $F_{3,41} = 2.36$ for natural growing, natural finishing, urea growing, and urea finishing, respectively. Thus, the independent variables (recorded measurements on feed consumption) are treated as though they do not contain a stochastic component. In other words, the data are treated as though they do not violate the third OLS assumption [36].

Conventional functional forms used for production function estimation include the Cobb-Douglas, quadratic and square root. The Cobb-Douglas functional form is somewhat restrictive in that if all power coefficients are positive it forces convex isoquants which become asymptotic to the input axes. Heady and Dillon [18] demonstrate that polynomials such as the quadratic and square root provide good estimates of the true surface over the data range because they are Taylor series expansions of the unknown true production function. However, these polynomial functional forms also have limitations. For example, a quadratic function will not permit
sigmoid isoquants and forces marginal products to be linear. These limitations may distort derived quantities. For an equivalent number of parameters, functional forms estimated in polar coordinates are less restrictive. Thus, production functions were estimated in polar coordinates to provide information on the shapes of various isoquants.

Polar coordinates

Any point in a rectangular coordinate system can be described as being some distance, \( d \), from the origin. Also, a line from the point to the origin forms an angle, \( Z \), with the horizontal axis. The points of a polar coordinate system are described in this manner. Since the present study contains only two variable inputs the data can be conveniently transformed into polar coordinates. Consider the following function.

\[
\hat{G} = f(d,Z)
\]  

where:

\( \hat{G} \) = estimated kilograms of weight gain per steer for the period;
\( d = (C^2 + S^2)^{1/2}; \)
\( Z = \text{arc tangent} \ (C/S); \)
\( C = \text{kilograms (DM) of corn (grain and silage) consumption for the period}; \) and
\( S = \text{kilograms (DM) of supplement consumption for the period}, \)

The unit of measure of the arc tangent function in the statistical package used to transform the data into polar coordinates is radians [1]. Thus, for the present study, the angle \( Z \) is measured in radians,
Many functional forms could be used for estimation purposes. However, the following quadratic polynomial was selected because of the ease in which isoquants can be derived.

\[ G = b_1 d + b_2 d^2 + b_3 dZ + b_4 d^2 Z + b_5 dZ^2 + b_6 d^2 Z^2 + e \quad (3.13) \]

where:

\[ b_1 - b_6 = \text{regression coefficients to be estimated.} \]

This equation is difficult to manipulate. But, it is possible to derive isoquants and represent them geometrically. The isoquant equation for this production function is as follows.

\[ d = \left[ -(b_1 + b_3 Z + b_5 Z^2) \pm \left( (b_1 + b_3 Z + b_5 Z^2)^2 \\
\quad + 4G(b_2 + b_4 Z + b_6 Z^2) \right)^{1/2} \right] \left[ 2(b_2 + b_4 Z + b_6 Z^2) \right]^{-1} \quad (3.14) \]

A specific isoquant can be derived by fixing \( G \) and solving for \( d \) at alternative values for the angle \( Z \) over the data range. It is then possible to represent the isoquant geometrically.

All four data sets were converted to polar coordinates. First-order autocorrelation coefficients were estimated for each. The data sets were transformed to correct for autocorrelation and the following quadratic polynomial production functions were estimated.
Natural Growing

\[ \hat{G} = 0.36130(d) - 0.00058(d^2) - 0.32274(dZ) + 0.00235(d^2Z) \]
\[ \text{(3.07)} \quad \text{(1.64)} \quad \text{(0.15)} \quad \text{(0.36)} \]
\[ + 2.26459(dZ^2) - 0.00936(d^2Z^2) \]
\[ \text{(0.26)} \quad \text{(0.35)} \]

\[ \text{MSE} = 4.88 \quad \text{df} = 14 \quad R^2 = 0.92 \]

Natural Finishing

\[ \hat{G} = 0.15284(d) - 0.00004(d^2) - 0.08952(dZ) + 0.00056(d^2Z) \]
\[ \text{(3.37)} \quad \text{(0.68)} \quad \text{(0.10)} \quad \text{(0.46)} \]
\[ + 1.05558(dZ^2) - 0.00288(d^2Z^2) \]
\[ \text{(0.27)} \quad \text{(0.55)} \]

\[ \text{MSE} = 9.82 \quad \text{df} = 34 \quad R^2 = 0.92 \]

Urea Growing

\[ \hat{G} = 0.45959(d) - 0.00088(d^2) - 2.82516(dZ) + 0.01088(d^2Z) \]
\[ \text{(4.68)} \quad \text{(3.59)} \quad \text{(1.13)} \quad \text{(1.74)} \]
\[ + 14.97581(dZ^2) - 0.05835(d^2Z^2) \]
\[ \text{(1.03)} \quad \text{(1.59)} \]

\[ \text{MSE} = 2.83 \quad \text{df} = 18 \quad R^2 = 0.91 \]

Urea Finishing

\[ \hat{G} = 0.13655(d) - 0.000008(d^2) + 0.64005(dZ) - 0.00072(d^2Z) \]
\[ \text{(2.80)} \quad \text{(0.11)} \quad \text{(0.44)} \quad \text{(0.34)} \]
\[ - 5.44869(dZ^2) + 0.00705(d^2Z^2) \]
\[ \text{(0.55)} \quad \text{(0.48)} \]

\[ \text{MSE} = 10.27 \quad \text{df} = 42 \quad R^2 = 0.98 \]
where:

- MSE = mean square error for the regression, \( (e' e / n - m) \);
- df = total number of observations minus the number of estimated parameters, \( (n - m) \);
- \( R^2 \) = percent of the variability in the transformed gain observations explained by the regression equation, \( [1 - (e' e / TCSS)] \) where:
  - TCSS = total corrected sum of squares; and
- t-values (absolute value) are listed in parentheses beneath the appropriate regression coefficient. The estimated standard errors for the coefficients can be obtained by dividing the regression coefficient by the t-value.

For each of the four estimated production functions the calculated \( R^2 \) indicates that over 90% of the variability of the transformed gain values is explained by the explanatory variables. The primary purpose of estimating the functions in polar coordinates was to prevent selection of functional forms in rectangular coordinates that could seriously restrict or bias the shape of the isoquant. Isoquants derived from equations 3.15, 3.16, 3.17, and 3.18 are graphed in Figures 3.1, 3.2, 3.3, and 3.4, respectively.

Figures 3.1 and 3.2 indicate that the natural growing and natural finishing data might be fit well with conventional functional forms in rectangular coordinates such as the quadratic. The isoquants of both are downward sloping over the data range. The 50 kg isoquant of Figure 3.2 is nearly linear. However, a quadratic functional form in rectangular coordinates could result in nearly similar isoquants and thus, should not seriously restrict the data.
The isoquants derived from production functions estimated from the urea phase of the experiment are not as "conventional" as those from the natural phase. Figure 3.3 indicates that the urea growing isoquant for gain equal to 50 kg is upward sloping. The 11.5% ration which contained the least percentage of crude protein of those fed would be optimal (least-cost) if time is ignored regardless of the relative prices of the inputs. This result is not surprising since data plots from interpolations of gain to 55 kg suggest an upward sloping isoquant. Further, the two pens which required the least total feed to achieve 55 kg of weight gain during the growing period for the urea phase, were the same two pens which received the least amounts
Figure 3.2. Isoquants derived from quadratic polynomial production function estimated in polar coordinates from natural finishing data

Natural Finishing
355 to 461 kg

Gain = 50 kg

Gain = 100 kg

16.6 % Ration

12 % Ration

Soybean Meal (kg DM)

Corn Grain plus Corn Silage (kg DM)
of the supplement. The MP theory suggests that feeding urea in excess of the UFP may result in excess ammonia production in the rumen and ammonia toxicity. Figure 3.3 does not conflict with the theory. Six of the twelve pens received growing rations estimated to have no potential to convert urea into useful microbial protein. But, they all received urea. Fitting these data to the Cobb-Douglas functional form in rectangular coordinates could seriously distort the shape of the isoquant since the Cobb-Douglas form would force convex to the origin isoquants if the marginal products are positive.

Figure 3.3. Isoquant derived from quadratic polynomial production function estimated in polar coordinates from urea growing data
Figure 3.4. Isoquants derived from quadratic polynomial production function estimated in polar coordinates from urea finishing data.
The isoquants of Figure 3.4 reflect the increased flexibility of polar coordinate estimation relative to rectangular coordinate estimation. The 50 kg isoquant is nearly upward sloping through the entire data range. But, the 100 kg isoquant is downward sloping. Again, conventional functional forms in rectangular coordinates may fail to capture these seemingly inconsistent shapes.

**Rectangular coordinates**

Since the production functions estimated in polar coordinates are difficult to manipulate, the four data sets were also fit in rectangular coordinates. A primary concern was to select functional forms such that derived isoquants would not seriously conflict with those derived from production functions estimated in polar coordinates.

**Natural growing** The Cobb-Douglas form yielded an acceptable statistical fit for the natural growing data set. The production function corrected for first-order autocorrelation is as follows. The natural logarithmic operator is designated by Ln.

\[
\hat{\text{LnG}} = 1.67844 + 0.36315\text{LnC} + 0.12994\text{LnS} \tag{3.19}
\]

\[
(8.87) \quad (7.93) \quad (3.98)
\]

\[\text{MSE} = 0.000996 \quad \text{df} = 17 \quad R^2 = 0.99\]

The t-values for equation 3.19 indicate that the estimated coefficients are statistically significantly different from zero at the 0.001 level of probability. The calculated $R^2$ indicates that 99% of the variability of the transformed LnG values is explained by the equation.
The 55 kg isoquant derived from equation 3.19 is graphed in Figure 3.5. It has the characteristic smooth convex-to-the-origin shape. It is less steeply sloped than its corresponding polar coordinate isoquant of Figure 3.1. Over the data range from the 11.1 to the 15.5% crude protein ration, the graph of the Cobb-Douglas isoquant suggests a greater degree of substitutability than the polar coordinate graph. For example, the graph suggests that over the data range the Cobb-Douglas function estimates that one unit of soybean meal will substitute for 2.89 units of corn (grain plus silage).

Figure 3.5. Isoquant derived from Cobb-Douglas production function estimated from natural growing data
But, the graph of the polar coordinate function estimates that one unit of soybean meal will replace only 1.67 units of corn (grain plus silage). Thus, the two are not identical. However, they do have similar shapes. Equation 3.19, the Cobb-Douglas production function, was selected for economic analysis of the natural growing rations.

**Natural finishing** The following quadratic production function was selected from among other functions estimated in rectangular coordinates to represent the natural finishing relationship.

\[
\hat{G} = -3.09548 + 0.15162C + 0.17954S - 0.000030^{(1.18)} - 0.001375^{(1.01)} + 0.00019CS^{(0.52)}
\]

\[
- 0.001375^{(1.01)} + 0.00019CS^{(0.52)}
\]

\[
MSE = 9.36 \quad df = 34 \quad R^2 = 0.92
\]

The 50 and 100 kg isoquants derived from this function are graphed in Figure 3.6. They very closely resemble the two isoquants derived from the polar coordinate function and graphed in Figure 3.2. Over the data range both estimates of the 100 kg isoquant suggest that an equivalent gain on the 12% ration requires 60 kg more of corn (grain plus silage) but 90 kg less of soybean meal than the 16.6% ration. Thus, the quadratic production function, equation 3.20, is used for economic analysis of the natural finishing data.

**Urea growing** Conventional functional forms resulted in very poor statistical fits for the urea growing data set. The modified square root function which follows resulted in the most acceptable statistical fit in terms of \( R^2 \) and \( t \)-values.
Figure 3.6. Isoquants derived from quadratic production function estimated from natural finishing data
\[
\hat{G} = 44.2341 + 0.9658c^{1/2} - 8.8816g^{1/2} + 0.5387\text{CS}^{1/2}
\]

\[(3.21) \quad (2.10) \quad (0.79) \quad (1.74) \quad (1.97)\]

\[
\text{MSE} = 2.89 \quad \text{df} = 20 \quad R^2 = 0.89
\]

The estimated coefficient on the square root of C is not statistically significantly different from zero at the 0.10 level of probability. But, the other three coefficients are significant at this level.

The 50 kg isoquant derived from equation 3.21 is graphed in Figure 3.7. It is upward sloping. The shape of the isoquant is very similar to the shape of its counterpart polar coordinate isoquant graphed in Figure 3.3.

Figure 3.7. Isoquant derived from modified square root production function estimated from urea growing data
Both estimates suggest that the 11.5% ration would be preferred over all of the other rations fed. Recall that the MP theory suggests that these rations contained excess urea which is detrimental to growth. Thus, over the data range, the marginal product of the supplement which contained urea is negative for the growing rations. This also reflects the problem inherent in the crude protein system which ignores everything but the nitrogen content of the feed.

Urea finishing For the urea finishing data set the square root production function which follows was selected over the Cobb-Douglas and quadratic estimates as most appropriate.

\[ \hat{G} = 21.9735 + 0.0280C - 0.1500S + 3.2374C^{1/2} \\
- 2.5051S^{1/2} + 0.2063(CS)^{1/2} \tag{3.22} \]

\[ \text{MSE} = 9.83 \quad \text{df} = 42 \quad R^2 = 0.98 \]

Although equation 3.22 yields a good fit in terms of $R^2$, the t-values indicate that the estimated regression coefficients for $C$ and $S$ are not statistically significantly different from zero. Thus, $C$ and $S$ were deleted and the following transformed modified square root production function was estimated.

\[ \hat{G} = -26.4721 + 4.3564C^{1/2} - 5.0396S^{1/2} + 0.2318(CS)^{1/2} \tag{3.23} \]

\[ \text{MSE} = 9.64 \quad \text{df} = 44 \quad R^2 = 0.98 \]

The modified square root production function, equation 3.23, yields a lower MSE than the square root production function, equation 3.22. In
addition, all estimated coefficients of the modified functional form are statistically significantly different from zero at the 0.01 level of probability. The null hypothesis that C and S do not contribute a statistically significant amount of information contingent upon the other variables being in the model could not be rejected. The actual calculated $F_{2,42} = 0.73$. Alternatively, the estimated regression coefficients for C and S in equation 3.22 are not statistically different from zero.

Two isoquants derived from the modified square root production function are graphed in Figure 3.8. The 50 kg isoquant is upward sloping. But, the 100 kg isoquant is downward sloping. Both isoquants are very similar to their counterparts derived from the polar coordinate production function and drawn in Figure 3.4. For the data range, the modified square root isoquant indicates slightly less substitutability along the 100 kg isoquant than the polar coordinate estimate.

Confidence limits Fuller [14] developed an iterative procedure for approximating confidence limits of derived isoquants. This technique was used to calculate 95% confidence limits for the isoquants graphed in Figures 3.6 and 3.8. It is not known if the "true" isoquant lies within the confidence limits. It either does or does not. The 95% confidence limits indicate that in repeated experimentation the derived confidence limits will contain the "true" isoquant for 95% of the experiments.

Time Requirement Analysis

To determine if additional time was required to achieve weight gain on the alternative rations an analysis of variance of the rate of gain
Figure 3.8. Isoquants derived from modified square root production function estimated from urea finishing data
across rations was conducted. For these tests, the experiment was assumed to be two replications on five treatments for the natural phase and six treatments for the urea phase. The treatments consisted of alternative levels of supplement per head per day. The tests were performed on the average daily gain across treatments. If the rate of gain does not vary across treatments one might conclude that the time required to achieve equivalent weight gain on alternative rations is not different in which case the cost of time may be ignored when comparing response across rations. If, on the other hand, there exists statistically significant differences in the rate of gain across treatments, time on feed functions are needed to incorporate the cost of time into economic analysis where appropriate.

**Natural growing**

The average daily gain ranged from 1.53 to 1.77 kg per day across the 10 pens for the 41-day natural growing period. The National Research Council of the National Academy of Sciences suggests that these are relatively high rates of gain and would be unlikely unless the animals exhibited compensatory growth [30]. The faster rates of gain were obtained with the higher protein rations. The data suggest that the rate of gain increased with the crude protein percentage over the range of rations fed. Furthermore, the rates of gain as measured at the 28-day observation exceeded those of the 41-day observation. This suggests that the steers may have experienced compensatory gains during the first 28 days.

An analysis of variance test of the rate of gain across the five treatments resulted in a calculated $F_{4,5} = 3.97$. The table value for
Thus, the null hypothesis of no difference in the rate of gain across treatments is rejected at the 0.10 level of probability and the alternative hypothesis of a difference in rate of gain across rations is accepted [40].

Since the rate of gain did vary across the natural growing rations an effort was made to fit an equation that could be used to estimate time on feed required to achieve a specific weight gain under alternative rations. Measurements on feed consumption and days were interpolated to a weight gain of 67 kg. It is difficult to select gain levels for data interpolations when measurements are taken at such wide intervals. Interpolation could distort relationships since the rate of feed intake and gain is assumed to be constant between measurement days. Thus, the interpolation is made to accommodate an estimate of the time required on alternative rations with the recognition that it is a crude approximation.

The time on feed was estimated as a function of the actual crude protein percent in the ration calculated from the interpolated feed quantities. Several functional forms were fit. The following function yielded the most significant t-values of those fit and was selected on this basis.

\[
\hat{D} = 95.91 - 155.52K^{1/2}
\]  
\( (3.50)  \quad (2.06) \)

\[
\text{MSE} = 22.97 \quad df = 8 \quad R^2 = 0.35
\]

where:

\( \hat{D} \) = estimated number of days required for 67 kg of weight gain; and

\( K \) = actual crude protein percent in the interpolated feed quantities required for 67 kg of weight gain.
Natural finishing

An analysis of variance of the rate of gain across the five treatments resulted in a calculated $F_{4,5} = 0.59$. Based on this test the rate of gain was not statistically different across rations in the finishing period. Thus, time on feed functions are not necessary for the finishing rations. The actual mean rates of gain for the 106-day finishing period were slightly lower (although not statistically significantly lower) for the pens which received the highest protein rations. The rate of gain was not increased by the increased percentage of crude protein in the higher protein rations.

Urea growing and finishing

The rates of gain were not statistically significantly different across treatments for either the urea growing or the urea finishing rations. The analysis of variance $F$ values with five and six degrees of freedom were 0.74 and 1.21 for the growing and finishing data sets, respectively. The actual rates of gain were slightly greater (although not statistically significantly greater) for those rations which received the least percentages of crude protein. For example, the two pens which received approximately 12.5% crude protein gained an average of 0.99 kg per day during the finishing period. But, the two pens which received approximately 16.2% crude protein gained only 0.93 kg per day. Eight of the twelve pens received urea in quantities exceeding the UFP during the finishing phase of the experiment. Excess urea is of no positive value and could be detrimental.
CHAPTER IV. ECONOMIC ANALYSIS

The preceding chapter contains statistical estimates of response functions. In this chapter an economic analysis of the estimated functions is presented.

Many producer objectives could be analyzed [28]. But, a common objective is to formulate a ration which will produce a specified quantity of gain at a minimum cost. This "least-cost" ration may be one which merely minimizes the total feed cost or one which minimizes the cost of time as well as the cost of feed. If the cost of time is ignored the problem can be formulated as a constrained cost minimization. Recall from Chapter II that the first-order condition for a minimum, or least-cost ration, is satisfied when $\frac{\partial C}{\partial S} = \frac{P_S}{P_C}$. The second-order condition is satisfied if the iso-quant is convex to the origin at the point where the first-order condition holds.

Natural Growing

The function selected to represent the natural growing data is of the form

$$\ln G = b_0 + b_1 \ln C + b_2 \ln S$$

or

$$\hat{G} = a C^{b_1} S^{b_2}$$

(4.1)

where:

$$b_0 = a.$$
Thus, \( \frac{\partial C}{\partial S} = \frac{b_2 C}{b_1 S} \) and at a minimum, if we let \( \frac{P_S}{P_c} = k \), \( \frac{b_2 C}{b_1 S} = k \). It follows that

\[
C = b_1 b_2^{-1} k S
\]  

(4.2)

Equation 4.2 is called an isocline equation [21]. It is a line which connects points of equal marginal rates of substitution, or equal slope, on successive isoquants. Hence, the isocline equation in conjunction with the appropriate isoquant denotes the optimal ration for any factor price ratio, \( k \). This can be accomplished by substituting the isocline, equation 4.2, into the production function, equation 4.1.

\[
S = \left( a^{-1} b_2 b_1^{-1} b_1 G \right)^{1/(b_1 + b_2)} 
\]  

(4.3)

From equation 4.3 the optimal least-cost ration can be determined for any relevant gain level at any soybean meal to corn (grain plus silage) price ratio. The graph in Figure 3.5 indicates that for the natural growing production function the second-order condition for a minimum will hold since the isoquant is convex to the origin. Mathematically, the second-order condition holds if \( \frac{\partial^2 C}{\partial S^2} > 0 \). Since

\[
C = \left( a^{-1} S b_2 G \right)^{(1/b_1)} 
\]  

(4.4)

\[
\frac{\partial^2 C}{\partial S^2} = a^{-1} b_1^{-2} b_2 (b_2 + b_1) G^{1/b_1} S \left( -b_2 + 2b_1 \right) / b_1
\]

Since \( a, b_1, b_2, G, \) and \( S \) are all \( > 0 \), \( \frac{\partial^2 C}{\partial S^2} > 0 \) and the second-order condition for a minimum will hold over the entire isoquant.
Based on annual prices from 1968 to 1977, the soybean meal to corn price ratio ranged from a low of 2.96 in 1975 to a high of 7.25 in 1973 [22]. (This is based on soybean meal price per hundred pounds and corn price per bushel. Both prices are on an as-fed basis.)

Least-cost feed

Isocline solutions were obtained for soybean meal to corn price ratios ranging from 2.8 to 7.4 in 0.2 increments. Results of this analysis for the natural growing production function are presented in Table 4.1. At higher soybean meal to corn price ratios the optimal least-cost ration contains less soybean meal and more corn. Rations containing more than 13.2% crude protein are not optimal under the price ratios examined when the cost of time is ignored.

For price ratios greater than 5.2, estimated optimal rations lie outside the range of the experimental data. For example, at a price ratio of 7.0, a ration containing 10.5% crude protein is estimated to be optimal. But, the lowest crude protein rations in the feeding trial contained 11.1% crude protein. Recommendations based on extrapolations beyond the experimental data range should be made with caution. The actual response to 10.5% crude protein rations cannot be determined from the experiment.

Least-cost feed plus time

The rate of gain was statistically significantly different across the natural growing rations. Equation 3.24 was estimated to reflect this difference in time required across rations. In addition to feed costs, producers have costs that vary directly with time. Included in this category
Table 4.1. Optimal crude protein percentages and quantities of feed required for growing steers from 243 to 310 kg under alternative soybean meal to corn price ratios

<table>
<thead>
<tr>
<th>Price ratio</th>
<th>Crude protein, optimal percent</th>
<th>Corn (kg)</th>
<th>Corn silage (kg)</th>
<th>Soybean meal (kg dry matter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.8</td>
<td>13.2</td>
<td>50.9</td>
<td>239.9</td>
<td>36.2</td>
</tr>
<tr>
<td>3.0</td>
<td>12.9</td>
<td>51.8</td>
<td>244.3</td>
<td>34.4</td>
</tr>
<tr>
<td>3.2</td>
<td>12.7</td>
<td>52.7</td>
<td>248.5</td>
<td>32.8</td>
</tr>
<tr>
<td>3.4</td>
<td>12.4</td>
<td>53.6</td>
<td>252.5</td>
<td>31.4</td>
</tr>
<tr>
<td>3.6</td>
<td>12.2</td>
<td>54.4</td>
<td>256.3</td>
<td>30.1</td>
</tr>
<tr>
<td>3.8</td>
<td>12.0</td>
<td>55.1</td>
<td>260.0</td>
<td>28.9</td>
</tr>
<tr>
<td>4.0</td>
<td>11.9</td>
<td>55.9</td>
<td>263.5</td>
<td>27.8</td>
</tr>
<tr>
<td>4.2</td>
<td>11.7</td>
<td>56.6</td>
<td>266.9</td>
<td>26.8</td>
</tr>
<tr>
<td>4.4</td>
<td>11.6</td>
<td>57.3</td>
<td>270.2</td>
<td>25.9</td>
</tr>
<tr>
<td>4.6</td>
<td>11.5</td>
<td>58.0</td>
<td>273.4</td>
<td>25.1</td>
</tr>
<tr>
<td>4.8</td>
<td>11.4</td>
<td>58.6</td>
<td>276.5</td>
<td>24.3</td>
</tr>
<tr>
<td>5.0</td>
<td>11.2</td>
<td>59.3</td>
<td>279.5</td>
<td>23.6</td>
</tr>
<tr>
<td>5.2</td>
<td>11.1</td>
<td>59.9</td>
<td>282.4</td>
<td>22.9</td>
</tr>
<tr>
<td>5.4</td>
<td>11.0</td>
<td>60.5</td>
<td>285.2</td>
<td>22.3</td>
</tr>
<tr>
<td>5.6</td>
<td>11.0</td>
<td>61.1</td>
<td>287.9</td>
<td>21.7</td>
</tr>
<tr>
<td>5.8</td>
<td>10.9</td>
<td>61.6</td>
<td>290.6</td>
<td>21.2</td>
</tr>
<tr>
<td>6.0</td>
<td>10.8</td>
<td>62.2</td>
<td>293.2</td>
<td>20.6</td>
</tr>
<tr>
<td>6.2</td>
<td>10.7</td>
<td>62.7</td>
<td>295.8</td>
<td>20.1</td>
</tr>
<tr>
<td>6.4</td>
<td>10.7</td>
<td>63.3</td>
<td>298.3</td>
<td>19.7</td>
</tr>
<tr>
<td>6.6</td>
<td>10.6</td>
<td>63.8</td>
<td>300.7</td>
<td>19.2</td>
</tr>
<tr>
<td>6.8</td>
<td>10.5</td>
<td>64.3</td>
<td>303.1</td>
<td>18.8</td>
</tr>
<tr>
<td>7.0</td>
<td>10.5</td>
<td>64.8</td>
<td>305.4</td>
<td>18.4</td>
</tr>
<tr>
<td>7.2</td>
<td>10.4</td>
<td>65.3</td>
<td>307.7</td>
<td>18.0</td>
</tr>
<tr>
<td>7.4</td>
<td>10.4</td>
<td>65.7</td>
<td>309.9</td>
<td>17.7</td>
</tr>
</tbody>
</table>

^aSoybean meal price per hundred pounds divided by corn price per bushel (as-fed) gives the price ratio. Corn silage price per ton (as-fed) is assumed to be 7.2 times the price per bushel of corn grain.

^bCrude protein percentages for soybean meal, corn grain and corn silage are assumed to be 51.5, 10.0 and 8.1, respectively.

^cDry matter content is assumed to be 89% for soybean meal and corn grain and 40% for corn silage.

^dRations with less than 11.1% crude protein are extrapolations from the experimental data.
are power, fuel and labor costs and interest on the investment in the
animals. Thus, the objective of some feeders may be to minimize the cost
of feed and time.

The time on feed equation was estimated specifically for 67 kg of
weight gain. Thus, the problem is to minimize $P_C + P_S + P_D$ subject to
the constraint of producing 67 kg of weight gain, where $P_d$ is the per day
cost. This problem is more difficult to solve with calculus than the pre­
vious problem. However, it is possible to select points along the 67 kg
isoquant and compare the costs along the isoquant. A good approximation
to the "exact" least-cost point was obtained by incrementing the quantity
of soybean meal by 0.1 kg, calculating the cost of feed and time and then
selecting the least-cost point from among those considered. A computer
program was formulated to determine the optimal rations for per day costs
of 10, 20, 30, and 40 cents. Results of this analysis are presented in
Table 4.2.

Soybean meal to corn price ratios were varied from 3.0 to 7.4 in 0.2
increments. In general, when the soybean meal-corn price ratio is high,
such as in 1973 when it was 7.25, including the cost of time does not alter
the optimal ration a great deal. Under these circumstances, the relative
cost of soybean meal outweighs the added costs resulting from extra time.
And, it is economical to reduce the crude protein percentage in the ration
even though it increases the time required to achieve the weight gain.
Alternatively, when the soybean meal-corn price ratio is relatively low as
in 1975, producers who wish to minimize the cost of feed and time may
Table 4.2. Optimal crude protein percentages for growing steers from 243 to 310 kg under a minimum cost of feed and time objective for alternative per day costs

<table>
<thead>
<tr>
<th>Price ratio</th>
<th>Crude protein, optimal percent</th>
<th>per day cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0c</td>
<td>10c</td>
</tr>
<tr>
<td>3.0</td>
<td>12.9</td>
<td>13.8</td>
</tr>
<tr>
<td>3.2</td>
<td>12.7</td>
<td>13.4</td>
</tr>
<tr>
<td>3.4</td>
<td>12.4</td>
<td>13.1</td>
</tr>
<tr>
<td>3.6</td>
<td>12.2</td>
<td>12.8</td>
</tr>
<tr>
<td>3.8</td>
<td>12.0</td>
<td>12.6</td>
</tr>
<tr>
<td>4.0</td>
<td>11.9</td>
<td>12.4</td>
</tr>
<tr>
<td>4.2</td>
<td>11.7</td>
<td>12.2</td>
</tr>
<tr>
<td>4.4</td>
<td>11.6</td>
<td>12.0</td>
</tr>
<tr>
<td>4.6</td>
<td>11.5</td>
<td>11.8</td>
</tr>
<tr>
<td>4.8</td>
<td>11.4</td>
<td>11.7</td>
</tr>
<tr>
<td>5.0</td>
<td>11.2</td>
<td>11.6</td>
</tr>
<tr>
<td>5.2</td>
<td>11.1</td>
<td>11.4</td>
</tr>
<tr>
<td>5.4</td>
<td>11.0&lt;sup&gt;c&lt;/sup&gt;</td>
<td>11.3</td>
</tr>
<tr>
<td>5.6</td>
<td>11.0</td>
<td>11.2</td>
</tr>
<tr>
<td>5.8</td>
<td>10.9</td>
<td>11.1&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>6.0</td>
<td>10.8</td>
<td>11.0&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>6.2</td>
<td>10.7</td>
<td>10.9</td>
</tr>
<tr>
<td>6.4</td>
<td>10.7</td>
<td>10.9</td>
</tr>
<tr>
<td>6.6</td>
<td>10.6</td>
<td>10.8</td>
</tr>
<tr>
<td>6.8</td>
<td>10.5</td>
<td>10.7</td>
</tr>
<tr>
<td>7.0</td>
<td>10.5</td>
<td>10.6</td>
</tr>
<tr>
<td>7.2</td>
<td>10.4</td>
<td>10.6</td>
</tr>
<tr>
<td>7.4</td>
<td>10.4</td>
<td>10.5</td>
</tr>
</tbody>
</table>

<sup>a</sup>Soybean meal price per hundred pounds divided by corn price per bushel (as-fed) gives the price ratio. Corn silage price per ton (as-fed) is assumed to be 7.2 times the per bushel price of corn grain.

<sup>b</sup>Crude protein percentages for soybean meal, corn grain and corn silage are assumed to be 51.5, 10.0 and 8.1, respectively.

<sup>c</sup>Rations outside the range of 11.1 to 15.5% crude protein are extrapolations from the experimental data.
economize by including relatively more soybean meal in the ration. This will increase the percentage of protein in the ration and reduce the time required to achieve 67 kg of weight gain.

The optimal least-cost ration for a feeder may be determined from Table 4.2. For example, if the cost of corn is $2.50 per bushel and the cost of soybean meal is $12.00 per hundred pounds, the appropriate price ratio is $12.00/$2.50 = 4.80. The optimal natural growing ration as shown in Table 4.2 contains 11.4% crude protein if the cost of time is ignored. If time costs are 30¢ per day, the fifth column of the table indicates that the optimal least-cost feed plus time ration contains 12.7% crude protein.

The National Research Council of the National Academy of Sciences reports that 250 kg steers gaining 1.3 kg per day require 12.7% crude protein rations [30]. This recommendation is reasonably consistent with the estimate obtained in the present study. During the natural growing period the steers gained more than 1.3 kg per day. The relatively high rate of gain was most likely a result of compensatory growth.

Natural Finishing

The time required to achieve equivalent gain was not statistically significantly different across the natural finishing rations. Thus, least-cost feed and least-cost feed plus time rations would be identical. Therefore, the cost of time need not be considered.

The quadratic production function, equation 3.20, was selected to represent natural finishing growth. Isocline solutions were obtained for the function and are reported in Table 4.3. Gain was fixed at 100 kg,
Table 4.3. Quadratic production function estimates of optimal crude protein percentages and quantities of feed required for finishing steers from 355 to 455 kg under alternative soybean meal to corn price ratios

<table>
<thead>
<tr>
<th>Price ratio(^a)</th>
<th>Crude protein,(^b)optimal percent</th>
<th>Corn kg dry matter</th>
<th>Corn silage kg dry matter</th>
<th>Soybean meal kg dry matter(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.8</td>
<td>12.4</td>
<td>646.1</td>
<td>71.8</td>
<td>47.8</td>
</tr>
<tr>
<td>3.0</td>
<td>12.2</td>
<td>651.8</td>
<td>72.4</td>
<td>44.2</td>
</tr>
<tr>
<td>3.2</td>
<td>12.0</td>
<td>657.6</td>
<td>73.1</td>
<td>40.7</td>
</tr>
<tr>
<td>3.4</td>
<td>11.8(^d)</td>
<td>663.7</td>
<td>73.7</td>
<td>37.2</td>
</tr>
<tr>
<td>3.6</td>
<td>11.6</td>
<td>669.9</td>
<td>74.4</td>
<td>33.9</td>
</tr>
<tr>
<td>3.8</td>
<td>11.5</td>
<td>676.2</td>
<td>75.1</td>
<td>30.7</td>
</tr>
<tr>
<td>4.0</td>
<td>11.3</td>
<td>682.7</td>
<td>75.9</td>
<td>27.6</td>
</tr>
<tr>
<td>4.2</td>
<td>11.1</td>
<td>689.4</td>
<td>76.6</td>
<td>24.5</td>
</tr>
<tr>
<td>4.4</td>
<td>10.9</td>
<td>696.1</td>
<td>77.3</td>
<td>21.6</td>
</tr>
<tr>
<td>4.6</td>
<td>10.8</td>
<td>702.9</td>
<td>78.1</td>
<td>18.7</td>
</tr>
<tr>
<td>4.8</td>
<td>10.6</td>
<td>709.8</td>
<td>78.9</td>
<td>16.0</td>
</tr>
<tr>
<td>5.0</td>
<td>10.5</td>
<td>716.8</td>
<td>79.6</td>
<td>13.3</td>
</tr>
<tr>
<td>5.2</td>
<td>10.4</td>
<td>723.9</td>
<td>80.4</td>
<td>10.7</td>
</tr>
<tr>
<td>5.4</td>
<td>10.2</td>
<td>731.0</td>
<td>81.2</td>
<td>8.2</td>
</tr>
<tr>
<td>5.6</td>
<td>10.1</td>
<td>738.1</td>
<td>82.0</td>
<td>5.8</td>
</tr>
<tr>
<td>5.8</td>
<td>10.0</td>
<td>745.3</td>
<td>82.8</td>
<td>3.4</td>
</tr>
<tr>
<td>6.0</td>
<td>9.9</td>
<td>752.5</td>
<td>83.6</td>
<td>1.1</td>
</tr>
</tbody>
</table>

\(^a\)Soybean meal price per hundred pounds divided by corn price per bushel (as-fed) gives the price ratio. Corn silage price per ton (as-fed) is assumed to be 7.2 times the price per bushel of corn grain.

\(^b\)Crude protein percentages for soybean meal, corn grain and corn silage are assumed to be 51.5, 10.0 and 8.1, respectively.

\(^c\)Dry matter content is assumed to be 89% for soybean meal and corn grain and 40% for corn silage.

\(^d\)Rations with less than 12% crude protein are extrapolations from the experimental data.
Thus, the optimal rations are for finishing steers from 355 to 455 kg. The soybean meal-corn price ratio was varied parametrically from 2.8 to 7.4 in 0.2 increments.

The natural finishing experimental rations ranged from 12 to 16.6% crude protein. For feed price ratios greater than 3.2, the estimated optimal rations lie beyond the range of experimental observations. The production function indicates that for price ratios greater than 6.0, such as those that prevailed in 1973 and 1977, soybean meal should not be included in the ration. The nonsupplement portion of the ration (90% corn grain: 10% corn silage) contained 9.81% crude protein. And, this mixture of grain and silage, devoid of soybean meal, would be least-cost for price ratios in excess of 6.0. These conclusions must be interpreted with care since they are based on extrapolations beyond the range of the experimental data.

In general, these results are consistent with National Research Council recommendations [30]. They recommend that a 400 kg steer be fed a 9.4 to 10.4% crude protein ration to gain 1.0 to 1.3 kg per day. Rouse [34] also suggests that a 408 kg steer may not need supplemental protein when fed a high corn grain ration. Unfortunately, all of the rations fed contained soybean meal. Thus, the expected response to rations containing only corn grain and corn silage must be based on data extrapolations.

The estimated production function does suggest that rations in excess of 12.5% crude protein would not be optimal under soybean meal-corn price ratios which have prevailed over the last decade. Subsequent experiments should be designed to study the lower crude protein percentage rations, perhaps from 9 to 13%.
Urea Growing

Both production functions estimated from urea growing data set, equations 3.17 and 3.21, indicate that the 50 kg isoquant is upward sloping over the data range. Data plots from interpolations of gain to 55 kg also suggest an upward sloping isoquant. In addition, the two pens which required the least total feed to achieve 55 kg of weight gain, were the same two pens which received the least amounts of supplement. The relatively poor response to the supplement probably resulted from excess ammonia. Six of the twelve pens received rations estimated to have no potential to convert urea into useful microbial protein. But, all the rations contained urea. The MP theory suggests that feeding urea in excess of the UFP may lead to excess ammonia production in the rumen and thus, ammonia overflow. The estimated production functions conform with the MP theory expectations.

Since the second-order conditions for a minimum do not hold along an upward sloping commodity isoquant, the least-cost ration from among those fed is the one which contained the least percentage of crude protein. In other words, an economic analysis of the estimated urea growing production functions yields a "corner point" solution at the 11.5% crude protein ration line. This is not to imply that the 11.5% crude protein ration should be recommended to feeders. More appropriately it might be said that the 11.5% ration was less detrimental to growth than the other rations.

Urea Finishing

A modified square root production function, equation 3.23, was selected to represent urea finishing response. The rate of gain was not
different across rations. Thus, isocline solutions will give both the least-cost feed and the least-cost feed plus time rations.

The supplement contained 10.92% urea and 89.08% soybean meal. The correlation estimate between the annual prices of soybean meal per hundred pounds and urea per ton was +0.92 for the period 1968 to 1977 [22]. Over the period the average price per ton of urea was 14.13 times the price per hundred pounds of soybean meal. Since the prices of the two factors which make up the supplement are highly correlated, the price per dry matter kilogram of supplement used in the solution procedure is as follows.

\[
P_s = 0.1092 \times \left( \frac{(P_{sbm} \times 14.13)}{(2000 \times 0.4536)} \right) \\
+ 0.8908 \times \left( \frac{P_{sbm}}{100 \times 0.4536 \times 0.89} \right)
\]

In this supplement price equation \( P_{sbm} \) is the price per 100 pounds of soybean meal as-fed. The dry matter content of urea and soybean meal is assumed to be 100 and 89%, respectively. One pound is equivalent to 0.4536 kg. This equation makes it possible to parameterize the price of soybean meal and solve for least-cost rations at alternative soybean meal-corn price ratios.

Solutions for the urea finishing rations are reported in Table 4.4. The experimental rations ranged from 12.5 to 16.3% crude protein. The optimal least-cost ration for a price ratio of 2.8 contains 11.5% crude protein. For all price ratios considered, the optimal rations contain less crude protein than the experimental rations. This suggests that of the rations fed in the experiment, the one containing the smallest proportion of crude protein is preferred. Since this ration did not require any more
Table 4.4. Modified square root production function estimates of optimal crude protein percentages and quantities of feed required for finishing steers from 351 to 451 kg under alternative soybean meal to corn price ratios

<table>
<thead>
<tr>
<th>Price ratio&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Crude protein&lt;sup&gt;b&lt;/sup&gt; optimal percent</th>
<th>Corn silage</th>
<th>Soybean meal</th>
<th>Urea (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>&lt;- kg dry matter&lt;sup&gt;c&lt;/sup&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.8</td>
<td>11.5&lt;sup&gt;d&lt;/sup&gt;</td>
<td>687.8</td>
<td>76.4</td>
<td>17.4</td>
</tr>
<tr>
<td>3.0</td>
<td>11.3</td>
<td>690.5</td>
<td>76.7</td>
<td>15.8</td>
</tr>
<tr>
<td>3.2</td>
<td>11.2</td>
<td>693.0</td>
<td>77.0</td>
<td>14.3</td>
</tr>
<tr>
<td>3.4</td>
<td>11.1</td>
<td>695.4</td>
<td>77.3</td>
<td>13.1</td>
</tr>
<tr>
<td>3.6</td>
<td>11.0</td>
<td>697.6</td>
<td>77.5</td>
<td>12.0</td>
</tr>
<tr>
<td>3.8</td>
<td>10.9</td>
<td>699.5</td>
<td>77.7</td>
<td>11.1</td>
</tr>
<tr>
<td>4.0</td>
<td>10.8</td>
<td>701.3</td>
<td>77.9</td>
<td>10.3</td>
</tr>
<tr>
<td>4.2</td>
<td>10.7</td>
<td>702.9</td>
<td>78.1</td>
<td>9.6</td>
</tr>
<tr>
<td>4.4</td>
<td>10.7</td>
<td>704.5</td>
<td>78.3</td>
<td>9.0</td>
</tr>
<tr>
<td>4.6</td>
<td>10.6</td>
<td>706.1</td>
<td>78.5</td>
<td>8.4</td>
</tr>
<tr>
<td>4.8</td>
<td>10.6</td>
<td>707.5</td>
<td>78.6</td>
<td>7.8</td>
</tr>
<tr>
<td>5.0</td>
<td>10.5</td>
<td>708.8</td>
<td>78.8</td>
<td>7.4</td>
</tr>
<tr>
<td>5.2</td>
<td>10.5</td>
<td>710.1</td>
<td>78.9</td>
<td>6.9</td>
</tr>
<tr>
<td>5.4</td>
<td>10.4</td>
<td>711.2</td>
<td>79.0</td>
<td>6.6</td>
</tr>
<tr>
<td>5.6</td>
<td>10.4</td>
<td>712.6</td>
<td>79.2</td>
<td>6.1</td>
</tr>
<tr>
<td>5.8</td>
<td>10.4</td>
<td>713.4</td>
<td>79.3</td>
<td>5.9</td>
</tr>
<tr>
<td>6.0</td>
<td>10.3</td>
<td>714.7</td>
<td>79.4</td>
<td>5.5</td>
</tr>
<tr>
<td>6.2</td>
<td>10.3</td>
<td>715.6</td>
<td>79.5</td>
<td>5.3</td>
</tr>
<tr>
<td>6.4</td>
<td>10.3</td>
<td>716.6</td>
<td>79.6</td>
<td>5.0</td>
</tr>
<tr>
<td>6.6</td>
<td>10.3</td>
<td>717.6</td>
<td>79.7</td>
<td>4.7</td>
</tr>
<tr>
<td>6.8</td>
<td>10.2</td>
<td>718.2</td>
<td>79.8</td>
<td>4.5</td>
</tr>
<tr>
<td>7.0</td>
<td>10.2</td>
<td>718.9</td>
<td>79.9</td>
<td>4.4</td>
</tr>
<tr>
<td>7.2</td>
<td>10.2</td>
<td>720.0</td>
<td>80.0</td>
<td>4.1</td>
</tr>
<tr>
<td>7.4</td>
<td>10.2</td>
<td>720.8</td>
<td>80.1</td>
<td>3.9</td>
</tr>
</tbody>
</table>

<sup>a</sup>Soybean meal price per hundred pounds divided by corn price per bushel (as-fed) gives the price ratio. Corn silage price per ton (as-fed) is assumed to be 7.2 times the price per bushel of corn grain. Urea price per ton is assumed to be 14.13 times soybean meal price.

<sup>b</sup>Crude protein percentages for soybean meal, corn grain, corn silage and urea are assumed to be 51.5, 10.0, 8.1 and 280.0, respectively.

<sup>c</sup>Dry matter content is assumed to be 89% for soybean meal and corn grain and 40% for corn silage.

<sup>d</sup>All estimated optimal rations contain less crude protein than the experimental rations and are extrapolations from the experimental data.
time to complete the 100 kg of gain, it would be optimal compared to the other rations fed for both the least-cost feed and the least-cost feed plus time objectives.

The MP theory suggests that urea in excess of the UFP is not beneficial. Under these circumstances, the rumen microbial population cannot use additional ammonia from urea. Eight of the twelve pens received urea in quantities exceeding the UFP.

Natural Versus Urea

The feeder may be confronted with the alternative of selecting between the all natural supplement or the soybean meal plus urea supplement. Based on the present study, the urea growing supplement resulted in a negative marginal product and thus, would not be recommended. But, the two finishing phases could be compared.

The urea phase steers weighed 351 kg at the start of the finishing period. The natural phase steers were slightly heavier. They weighed 355 kg. Thus, since the marginal gain per unit of feed decreases as animal weight increases, if the two 100 kg derived isoquants are compared, there will be a slight bias in favor of the lighter urea phase steers.

The hypothesis of no difference in rate of gain across the supplements during the finishing periods could not be rejected at a 0.10 level of probability. The calculated $t_{20} = 0.28$. Thus, time required to achieve a given amount of gain is not statistically significantly different across the supplements.

By ignoring the problem of different starting weights, which is expected to enhance the position of the urea phase supplement, the two
rations can be compared based on their respective 100 kg isoquants derived from the estimated production functions. Unfortunately, additional concessions must be made. Since all of the estimated optimal urea finishing rations contain a smaller percentage of crude protein than the rations which were fed, the urea phase isoquant must be extrapolated. This is less than an ideal situation for making producer recommendations. But, once these concessions are made, the two supplements can be compared.

Assume a soybean meal-corn price ratio of 2.8 and as-fed prices of $2.50 per bushel, $18.00 per ton, $7.00 per hundred pounds, and $98.92 per ton for corn grain, corn silage, soybean meal, and urea, respectively. The 12.4% crude protein natural finishing ration would be optimal and would cost $83.32. The optimal urea finishing ration would cost $83.11 and contain 11.47% crude protein. The urea supplement would cost $0.21 less, but this could easily be offset by the starting weight bias in favor of the urea supplement. Of course, none of the urea phase steers received an 11.47% ration. Hence, based on the present study, it is not known if the 11.47% crude protein urea finishing ration will result in performance indicated by the extrapolated isoquant.

An alternative method of comparing the two supplements would be to compare rations which were actually fed. The ration which contained the least percentage of crude protein would be optimal for the particular supplement under most relevant price ratios. The cost of these two rations can be compared. The 12% natural finishing ration would require 657.6 kg of corn grain, 73.1 kg of corn silage and 40.7 kg of soybean meal for 100 kg of gain. Based on the prices given, this ration would cost $84.42.
Alternatively, for 100 kg of gain the 12.5% urea finishing ration would require 673.5 kg of corn grain, 74.8 kg of corn silage, 28.0 kg of soybean meal, and 3.4 kg of urea. This ration would cost $83.44. If the price of urea is increased from $98.92 to $200.00 per ton, the ration cost increases by $0.38 to $83.82. Under this price scenario, if the starting weight bias in favor of the urea phase is ignored, the urea phase supplement is slightly less expensive. Since the natural finishing ration required more soybean meal, for higher soybean meal-corn grain price ratios the urea phase supplement is relatively less expensive. Thus, if the urea phase supplement had not contained soybean meal, as suggested by the MP system, it might be more economical. Unfortunately, the present study cannot be conclusive on this matter.
CHAPTER V. SUMMARY AND IMPLICATIONS

A beef feeding experiment was conducted to investigate the substitution between high energy feed, corn silage and corn grain, and supplement. The study was partitioned into two phases. Steers in the natural phase received a supplement of soybean meal. But, the urea phase steers received a supplement which contained both soybean meal and urea. Both phases were divided into a 41-day growing period and a 106-day finishing period. Production functions were estimated to represent natural growing, natural finishing, urea growing, and urea finishing. Production functions were estimated in polar coordinates to provide information on isoquant shapes. Then functional forms in rectangular coordinates that would permit isoquant shapes exhibited by the polar coordinate estimates were fit. The rate of gain was statistically significantly different across the natural growing rations. But, rates of gain were not different across the natural finishing, urea growing or urea finishing rations. Thus, a time on feed function was estimated for the natural growing data but not for the other three data sets.

Natural Phase

The steers initially weighed 243 kg. They were fed growing rations ranging from 11.1 to 15.5% crude protein and grew to 312 kg in 41 days. The rations were then adjusted from 17.5% corn grain and 82.5% corn silage to 90% corn grain and 10% corn silage. The steers were finished from 355 kg to 461 kg on rations which ranged from 12 to 16.6% crude protein.
Estimated optimal growing rations ranged from 10.4 to 13.2% crude protein for soybean meal-corn price ratios from 7.4 to 2.8. For price ratios greater than 5.2, estimated optimal rations contain less crude protein than any of the rations fed. Over a limited range, increasing the crude protein percent reduces the time required to achieve a given level of gain. Further, as per day costs increase, the optimal rations contain a higher percentage of crude protein. For example, if the soybean meal-corn grain price ratio is 5.0, the optimal least-cost ration contains 11.2% crude protein. But, if per day costs are calculated at 40 cents per day, the 13.2% ration would be optimal.

The National Research Council of the National Academy of Sciences [30] reports that a 250 kg steer gaining 1.3 kg per day requires a 12.7% crude protein ration. Thus, the results of the present study are consistent with the recommendations. However, the present study indicates that a producer may economize by reducing the soybean meal content of the ration during periods of high soybean meal to corn price ratios, and alternatively, increasing the soybean meal content during periods of low price ratios. Feeding a 12.7% ration under all price situations is not the most economical feeding strategy.

A quadratic production function was used to provide an estimate of natural finishing response. The rations fed ranged from 12.0 to 16.6% crude protein. For soybean meal-corn grain price ratios exceeding 3.2, estimated optimal least-cost rations contain less crude protein than the rations fed. For a price ratio of 6.0 or greater, the quadratic production function estimates that supplement should not be included in the ration.
The 10% corn silage-90% corn grain ration contains 9.81% crude protein and additional supplement is not needed. This finding is also consistent with National Research Council recommendations. They recommend that a 400 kg steer be fed a 9.4 to 10.4% crude protein ration to gain 1.0 to 1.3 kg per day. Unfortunately, all of the rations contained supplement and this conclusion must be based on extrapolation. Thus, we cannot conclude that steers performed well on rations devoid of supplement. However, a study of response to unsupplemented finishing rations is needed.

Urea Phase

In general, the urea growing rations resulted in poor performance. The steers did grow from 242 to 308 kg during the 41-day period. But, the production function estimates indicate a negative marginal product for supplement across the data range. Resulting isoquants, with supplement on the vertical axis, are upward sloping. Thus, the ration which contained the least amount of the supplement is preferred above all the rations fed under any factor price scenario. It was less detrimental than the other rations.

The urea finishing rations ranged from 12.5 to 16.3% crude protein. The marginal product of supplement was estimated to be negative across the data range at the 50 kg isoquant but positive at the 100 kg isoquant. For all soybean meal-corn grain price ratios considered, the least-cost ration contains a smaller percentage of crude protein than any of the rations fed. Thus, the 12.5% ration is preferred above all the other rations fed.
Limitations

Several limitations of the study should be noted. The steers were fed a high corn silage ration during the growing period and then switched to a high corn grain diet. Thus, the experiment was of a two-stage feeding system which is recommended by some nutritionists as the most efficient way to feed corn grain and corn silage [26]. But, rations which contain different energy feeds and, perhaps, different combinations of corn grain and corn silage may result in different protein utilization. Thus, it may not be appropriate to generalize the results of this study to feeding situations in which other energy feeds are fed.

An additional limitation of the study is that the range of crude protein percentages was relatively narrow. Also, optimal rations for many realistic price ratios were estimated to contain a smaller percentage of crude protein than any of the rations which were fed. Thus, an additional feeding trial is necessary which covers a lower range of crude protein percentage rations. The rations should extend down to one which includes only corn (grain and silage) and thus, no supplement.

A third limitation of the study is that the urea phase supplement contained both soybean meal and urea. Because of the large negative UFP of soybean meal, it does not seem to be wise to feed a supplement composed of both soybean meal and urea. This conclusion is consistent with that of Satter and Roeffler. They argue that either an all natural supplement or an all urea supplement should be fed. In addition, "... the common practice of mixing some NPN into commercial protein supplements, thus supplying
a mixture of plant protein and NPN, is not an appropriate way to supplement nitrogen for most feeding situations" [35, p. 16].

A fourth limitation of the present analysis is that the results are based on a single feeding trial. Although the natural phase results are reasonably consistent with National Research Council recommendations, more studies of a similar nature could increase the reliability of the statistical estimates.

Finally, the National Research Council suggests that as the rate of gain increases, rations should include a higher percentage of crude protein [30]. The present study did not test this recommendation. Thus, the results may not be as useful for steers growing at levels substantially different from those of the study.

Implications

One positive result of the present study is that the feeding of five or six alternative rations enables the estimation of a production function. By fixing inputs other than corn (grain plus silage) and supplement consumption, the analysis was simplified to that of looking at alternative combinations of the feeds and thus, crude protein percentages of the rations. This type of analysis permits estimation of precise optimal least-cost rations.

If the finishing rations fed in the experiment are indicative of the types of rations normally fed to cattle, then it is likely that excessive quantities of supplement are being fed. This would be consistent with Ensminger's hypothesis [10]. He contends that animal scientists may be
recommending higher protein levels than are believed to be necessary for efficient performance. Unfortunately, the crude protein content of the rations fed in the experiment was excessive. Most of the recommended least-cost natural finishing and all of the recommended least-cost urea finishing rations had to be based on extrapolations from the experimental data range. A researcher could feel much more comfortable about the recommendations if rations had been fed which contained smaller percentages of crude protein. Thus, a primary finding of this study is that another feeding trial is needed. The response to rations containing a smaller percentage of crude protein than those fed in this study should be investigated. One of the rations in this proposed experiment should consist of the high corn grain diet without additional supplement. In addition, based on the results of the present study and the MP system, the practice of mixing soybean meal with urea is questionable. Thus, for the subsequent experiment, the urea phase supplement should contain only urea. Future studies should consider the suggestions of the MP system with regards to urea supplementation.

For some price ratios, perhaps supplement should not be fed. For other price ratios a small quantity of urea may be optimal. Natural supplements such as soybean meal may not be necessary for economical finishing rations. Subsequent studies should test this hypothesis.

Finally, while the recommendations of the Natural Research Council may be good point estimates, feeders should recognize that more efficient rations may be obtained by adjusting the crude protein percentages in the ration as the relevant factor prices change. Nutritionists and economists
can and should provide producers with the information needed to select efficient rations.
REFERENCES


I wish to express my sincere appreciation to:

Shashanka Bhide,
Ray Bryan,
Maryellen Prendergast Epplin,
Earl O. Heady,
Roy Hickman,
M. Peter Hoffman,
Steve Johnson,
Donald Kaldor,
Bryan Melton,
Charles Meyer,
Vishnuprasad Nagadevara, and the
Moorman Manufacturing Company.