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A theoretical study of parton fragmentation and evolution into hadrons

Alexander H. Ng

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A THEORETICAL STUDY OF PARTON FRAGMENTATION AND EVOLUTION INTO HADRONS

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I. INTRODUCTION

Jetlike structure in multiparticle hadron production has recently been observed in hard processes, for instance, in high energy electron-positron annihilation, deep inelastic lepton-nucleon scattering (DIS) and large transverse momentum ($P_T$) hadron-hadron collisions (1), as illustrated in Fig. 1.1. This phenomenon is predicted by a fundamental theory of strong interactions, quantum chromodynamics (QCD), which is an asymptotically free field theory promising to describe the basic gauge dynamical properties of the interaction of an octet of massless vector gluons with a triplet or bigger multiplet of nearly massless or massive spin 1/2 quarks (2). In this theory, the strength of interaction depends on the distance scale, such that when the parton scattering occurs within a small(large) distance, the interaction becomes weaker(stronger).

In the framework of the QCD quark-parton model, hadrons are believed to consist of fundamental constituents, i.e., quarks and gluons; therefore, any hadronic interaction can be described in terms of the interaction of these elementary quanta. The inclusive production of a hadron jet in a hadron-hadron collision, $h_1 + h_2 \rightarrow \text{hadron jet} + X$, where $X$ stands for all the undetected particles in the final state, can be viewed as a three-fold process. First, the initial
Fig. 1.1  Schematic of jet production in the reactions: (a) NN collisions, (b) $e^+e^-$ annihilations and (c) IN interactions.
parton states are identified within the incident hadrons, $h_1$ and $h_2$. Then, one of the energetic partons in each hadron enters into a small region where a violent interaction takes place via the exchange of colored gluons or fusion process. Such a basic hard scattering at the parton level can be calculated in perturbative QCD. Finally, the perturbed partons emerge from the interaction region and fragment into jets of hadrons as a consequence of the confinement mechanism.

One may define a jet as a well-collimated cluster of energetic particles produced with large $q_L$ and limited $q_T$ which are, respectively, the momentum parallel to the jet axis and momentum transverse to this axis, the jet axis being the direction of the emitted parton or of the spectator partons immediately after the hard collision. Such jets of particles can be produced in a wide class of reactions. A possible general hadronic interaction is shown in Fig. 1.1(a). Four jets are produced in the final state, yielding what could be called the trigger jet, away jet, beam jet and the target jet. The first two jets result from the hard collision and give the dominant feature of the large-$P_T$ data, $P_T$ being the momentum component perpendicular to the incident beam direction. The latter two spectator jets, either coming from a quark or diquark, are responsible for the low-$P_T$ multihadron production in the beam and target fragmentation.
regions. In deep inelastic lepton-nucleon scattering and electron-positron annihilation, one would observe two hadron-jet events in lowest order QCD as shown in Fig. 1.1(b) and 1.1(c). If a fast parton, produced from any interaction, does not remember its origin, then according to the factorization hypothesis one may naturally assume that the non-perturbative effects in particle production, or hadronization, can be separated in time from the hard scattering process. And we may conjecture that the parton jet fragmentation is universal for all processes. A test of the universality of parton jet fragmentation could be carried out, for instance, by measuring single jet multiplicity in various reactions.

Experiments to search for the Gell-Mann and Zweig quarks (3) with fractional charge, have been done for many years, but there has been no confirmed detection in the laboratory (4). If quarks are permanently confined in hadrons and cannot be observed as free particles, then observation of the jet of hadrons into which a quark fragments can be the best indirect approach to test for the existence of quarks. The properties of the final state hadron jet could give valuable information on the hadronization mechanism, on the non-perturbative effects in the parton cascade into observable hadrons, and on the strong forces that bind partons into the original hadron and into the produced hadrons.
Certainly, some difficult questions might be raised; for instance, how do partons carrying an additional color degree of freedom rearrange themselves into color singlets in the hadron jet? Such a hidden quantum number - color, was originally introduced in order to satisfy quark statistics, and consequently the prediction of the decay rate of $\pi^0 \to 2\gamma$, the branching ratio for $\tau \to \bar{\nu}_\tau e^- \nu_e$, the ratio $R = \frac{\sigma(e^+e^\to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$, Drell-Yan cross sections and the triangle anomalies. But the leakage of color remains an unsolved problem theoretically and experimentally. In fact, despite the acceptibility of the color hypothesis, the experimentalists find it very difficult to perform a direct test on any color quantum numbers of quarks. In spite of the lack of any direct tests, our belief in the existence of colored dressed quarks has grown strong enough to make QCD seem very convincing, with color playing a role as a source of parton interactions in analogy with quantum electrodynamics (QED) where the electric charge acts as the source of photons for the electromagnetic interaction.

In contrast with QED, in QCD the quark-gluon coupling strength tends asymptotically to zero as the momentum transfer squared $Q^2$ increases. Thus, a fast quark in a state of high $Q^2$ can easily radiate gluons, which subsequently turn into quark-antiquark pairs and form a parton shower. In the end of such a cascade, quark confinement becomes dominant and
forces hadronization to take place as time progresses. Since the parton fragmentation into hadrons is a non-perturbation effect, the process of hadronization is presently incalculable in perturbative QCD. To understand the non-perturbative effects associated with hadronization it is important to obtain information about the strong coupling constant $\alpha_s(Q^2)$, or equivalently the QCD scale parameter $\Lambda$, which is related to the strong coupling constant by

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)}, \quad (1.1)$$

in lowest order. Here, $\beta_0 = 11 - 2N_f/3$ is derived from the light-quark and gluonic loop corrections to the effective coupling constant; $N_f$ is the number of quark contributions to the vacuum polarizations. Recent measurements of the scale parameter $\Lambda$ have shown $\Lambda$ to be around 100 MeV/c (5,6) with preference for smaller values in recent the experimentals (7); but, the value also depends on the theoretical model.

Since jetlike structure observed in the multihadron production is believed due to the parton fragmentation, in order to investigate this jet structure in the fragmentation process, models have been introduced to allow comparisons between theoretical predictions and experimental results for a wide class of hadronic processes in high-$P_T$ and low-$P_T$
physics. Based on the naive quark-parton model, various fragmentation models have been developed to describe the production of hadron jets in a detailed manner, for example, the Field-Feynman (FF) cascade model (8,9) and color flux tube model (10,11). To date, at energies accessible with present accelerators, the FF model has been relatively successful in reproducing the jet data. It is widely used by experimentalists to evaluate the corrections due to the effects of fragmentation and to their detector bias. Theorists have used it for making comparisons with other models. In this thesis, we shall present a detailed study of jet structure by using Monte Carlo technique and employing the FF model.

Scale breaking effects, such as shifting the structure function \( F_2(x, Q^2) \) of a nucleon towards the small x region for growing \( Q^2 \), were observed long ago (12). Recently, similar effects for the fragmentation functions appear to have been detected in high energy e^+e^- annihilation (13,14). This gives considerable impetus for modifying the fragmentation models used in the past. For instance, the FF model was used to describe the fragmentation of quarks into mesons as a recursive type cascade, characterized by several phenomenological parameters, all independent of \( Q^2 \).

An early attempt to explain the scaling violation in the parton distribution functions in terms of \( Q^2 \) dependence
inside the nucleon was made by Kogut and Susskind (KS) (15). Later in 1977, Altarelli and Parisi (AP) (16) reformulated the KS recursion equation into an integro-differential equation which describes the quark and gluon distributions as follows,

\[
\frac{\partial q_i(x,t)}{\partial t} = \frac{\alpha_s(t)}{2\pi} \int \frac{dy}{y} \left[ P_{qq}(x/y) q_i(y,t) + P_{qG}(x/y) G(y,t) \right]
\]

(1.2)

\[
\frac{\partial G(x,t)}{\partial t} = \frac{\alpha_s(t)}{2\pi} \int \frac{dy}{y} \left[ P_{Gq}(x/y) \sum_{i=1}^{2N_f} q_i(x,t) + P_{GG}(x/y) G(y,t) \right]
\]

(1.3)

where the variable \( t \) is related to the QCD scale parameter by \( t = \ln(g^2 / \Lambda^2) \), \( \alpha_s(t) \) is the running coupling constant and \( N_f \) is the number of quark flavors as defined earlier in Eq. (1.1). \( G(y,t) \) and \( q_i(y,t) \) are the probability functions for finding a gluon and a quark of type \( i \), respectively, with momentum fraction \( y \) of the nucleon. The Ps are the AP splitting functions, for instance, \( P_{Gq}(x) \) is the probability that quark \( q \) splits into gluon \( G \) and quark with momentum fractions \( x \) and \( 1-x \), respectively. To solve the AP integro-differential equations (1.2) and (1.3), which are known as the master equations, requires phenomenological input such as quark and gluon distribution functions measured
in DIS at a reference energy $Q_0^2$. These equations also have been modified to study the $Q^2$ dependence of the structure function of the photon (17,18).

In analogy to equations (1.2) and (1.3), Owens (19) and Uematsu (20) both have independently applied AP type equations to the phenomenon of parton jet evolution deriving analogous integro-differential equations for quark and gluon fragmentation functions,

$$\frac{dD_q^h(z,t)}{dt} = \frac{\alpha_s(t)}{2\pi} \int \frac{dz'}{z'} [P_{qq}(z',z) D_q^h(z',t) + P_{Gq}(z',z) D_G^h(z',t)]$$

(1.4)

$$\frac{dD_G^h(z,t)}{dt} = \frac{\alpha_s(t)}{2\pi} \int \frac{dz'}{z'} [P_{qG}(z',z) \sum_{i=1}^{2N_f} D_{q_i}^h(z',t) + P_{GG}(z',z) D_G^h(z',t)]$$

(1.5)

where $D_q^h(z,t)$ and $D_G^h(z,t)$ are the fragmentation functions specifying the probabilities of finding hadron $h$ with momentum fraction $z$ of the initial quark and gluon, respectively. The functions $P$ are the same AP splitting functions used in Eqs. (1.2) and (1.3).

A diagrammatic interpretation of Eqs. (1.4) and (1.5) is shown in Fig. 1.2: One may observe a hadron $h$ fragmenting from a quark $q$ right after the emission of a gluon in Fig. 1.2(a) or fragmenting from a gluon which is radiated by the
Fig. 1.2 Owens-Uematsu type diagrams contributing to the evolution equation for the single particle fragmentation.
quark $q$ in Fig. 1.2(b). Since a gluon may decay into a $q\bar{q}$ pair, one may observe a hadron fragmenting from either quark in Fig. 1.2(c). There is no QED analog to Fig. 1.2(d), in which a gluon splits into two gluons and one of them fragments into hadron $h$. This phenomenon results from the non-Abelian (SU(3)$_\text{COLOR}$) nature of QCD.

As mentioned above, in the FF cascade model, the fragmentation functions have been parametrized to satisfy a description of the available experimental data with $Q^2 = 4 \text{ GeV}^2/c^2$ (8,9). Thus, there is no $Q^2$ dependence in the FF parton fragmentation functions. One simple approach to introduce a $Q^2$ dependence is to evolve a parton from $Q^2 = 4 \text{ GeV}^2/c^2$ to some large $Q^2$ by employing the Owens-Uematsu evolution equations. However, a lot of complex structure in parton jet fragmentation is still oversimplified by the FF fragmentation process, in which a meson is formed either by combining an incoming valence quark with a sea antiquark created from the vacuum or by combining a sea quark with a sea antiquark. Although the true dynamical picture of the transition from perturbative parton states to non-perturbative hadron final states is at present still unclear, the appropriate approach is to understand the relation of high-$P_T$ physics to low-$P_T$ physics in which the dynamical picture of the non-perturbative hadronization can be improved.
Low-$P_T$ physics has been thought to be more difficult to describe in terms of partons due to the small momentum transfer in the soft hadronic process than its counter-part (high-$P_T$ physics) in the hard scattering process where perturbative QCD is applicable. Various dynamical models, such as the quark fragmentation model (8,9) and quark recombination model (21), are motivated by the successful predictions of the parton model in high-$P_T$ physics and they also claim to be successful in describing the experimental data.

In the soft hadronic process, for instance, $p + p \rightarrow M + X$, where $M$ denotes a fast meson which can be chosen as $\pi^+$ for illustration. The formation of a $\pi^+$ can be described by many models of which we use two, the quark fragmentation model and the quark recombination model.

In the fragmentation model, the inclusive cross section for $p + p \rightarrow \pi^+ + X$ is given by

$$\frac{1}{\sigma} \frac{d\sigma^{\pi^+}}{dx} = \int \frac{dy}{y} F_u(y) D^{\pi^+}_u\left(\frac{x}{y}\right)$$

(1.6)

where $F_u(y)$ is the probability density for finding a $u$ quark within the proton with momentum fraction $y$ and $D^{\pi^+}_u(y)$ is the fragmentation function. With $(1-x)$ behavior as given by the quark counting rules (22,23), Eq. (1.6) behaves like
(l-x)^5 (24), whereas experiments show a power of three (25), instead. This steep, incorrect (l-x)^5 behavior can be logically understood in terms of the convolution of two probability densities, \( F \) and \( D \), and an alternative approach to this model with milder \( x \) dependence is required.

This different approach based on the recombination model was introduced by Das and Hwa (21) to give the inclusive distribution of \( \pi^+ \) in pp collisions as

\[
\frac{x}{\sigma} \frac{d\sigma}{dx} = \int d\lambda_1 d\lambda_2 \ F_{ud}(\lambda_1, \lambda_2) \ R_{ud}^{\pi^+}(\lambda_1, \lambda_2; x) \quad (1.7)
\]

where \( F_{ud}(\lambda_1, \lambda_2) \) is the joint distribution function for finding two quarks, \( u \) and \( d \) within the proton with momentum fraction \( \lambda_1 \) and \( \lambda_2 \), respectively; and, \( R_{ud}^{\pi^+}(\lambda_1, \lambda_2; x) \) is the probability of quarks \( u \) and \( d \) recombining into meson \( \pi^+ \) at \( x \). In the recombination model description of soft processes, the detected meson is assumed to carry most of the longitudinal momentum of a fast valence quark and a slow sea quark in such a way that the momentum fraction of the meson is approximately equal to that of the valence quark, i.e., \( x \sim \lambda_1 \). Thus, the momentum distribution of the \( \pi^+ \) is correlated to the momentum distribution of the valence \( u \) quark in the proton. According to the quark counting rule, the inclusive cross section gives a \((l-x)^3\) behavior, which is
a power of 2 less than previous model and is in good agreement with the experimental results.

From the above two models, we see that there are two basic components of which one is common for the inclusive cross section in the soft hadronic processes. The common component is the quark distribution function $F$, and the second one, which characterizes the non-perturbative process, is the fragmentation function $D$ in the quark fragmentation model or the recombination function $R$ in the quark recombination model.

The main objective of this work is to study a mechanism in which a parton fragments and evolves into hadron jet. Such a mechanism is formulated with two ingredients: (i) a generalized quark distribution function; and (ii) a recombination idea. Instead of single-quark or two-quark distribution functions $F$, a more general multiparton distribution function has been formulated by Kuti and Weisskopf (KW) (26) and has been applied in the recombination model by some authors (27), recently. However, the KW formulation is unable to account for the scale dependence of the inclusive-particle distributions observed in $e^+e^-$ annihilation (13). A general approach, which includes scale breaking effects in the multiparton distribution function, was introduced by Konishi, Ukawa and Veneziano (KUV) (28) who present a simple QCD algorithm, i.e., Jet Calculus in the
leading-logarithmic approximation (LLA), to provide rules for constructing the multiparton distribution function $D_{i}a_{1}a_{2}...a_{n}(x_{1},x_{2},...,x_{n};Q^{2},Q_{0}^{2})$. It is a joint momentum distribution function for $n$ partons at $Q_{0}^{2}$ resulting from the evolution of an initial parton $i$ at $Q^{2}$.

The outline of this thesis is as follows. In Chapter II, we study jet structure in detail by using Monte Carlo technique and employing the FF model. To illustrate this technique, the general features of hadron jets will be generated in the QCD Compton process, wherein a quark emits a gluon after interacting with an incident photon. Chapter III presents our formalism and the essential results of the perturbation calculation in leading-logarithmic order of single-particle and two-particle inclusive spectra in a model which unites the machinery of the KUV jet calculus (28) with the ideas of the quark recombination model (21); and we compare our model with Willen's FF model on the effect of scaling violation. Finally, Chapter IV is devoted for our conclusions.
II. MONTE CARLO SIMULATION OF JET PRODUCTION

The existence of jets, especially those produced by gluons, has been an elusive concept to prove and establish experimentally. This chapter will serve to illustrate how jets behave in reactions and how their deduction from hadron distributions is the source of their elusiveness, even in a reaction that should allow direct study of gluon jets.

With Monte Carlo calculations one can conveniently describe the structure of hadron jets in large transverse momentum events by splitting the complex process of jet fragmentation into two manageable regions. The first region at short distances is where perturbative QCD applies and in the other at long distances one relies heavily on phenomenological parametrizations. To be definite, we would like to describe qualitative and quantitative features of QCD jet fragmentation in the QCD Compton process to provide a possible test of perturbative QCD.

Many studies of large-\(P_T\) phenomena based on the QCD quark parton model have been made of gluon bremsstrahlung processes in DIS or in \(e^+e^-\) annihilation (6). Another gluon production process which may provide a test of perturbative QCD is the QCD Compton effect, \(\gamma q \rightarrow qg\), where the incoming real photon transfers its entire energy to the final state quark and gluon jets at high transverse momenta (29). The
point-like component of the photon entering this reaction produces final state characteristics distinct from the hadronlike component, especially at the edges of available phase space, i.e., at high transverse momenta, high fractional parton momenta, etc., where the typical vector-meson component of the photon is sharply cut off. The particular advantage of the QCD Compton effect over gluon bremsstrahlung derives from kinematics. In the QCD Compton effect, as viewed from the photon-quark center-of-mass (c.m.) frame, there is nothing going forward in the initial photon direction to mask the produced high-$P_T$ parton jets as there normally would be in hadron produced reactions. The spectator diquark jet is of relatively low momentum and travels in the backward c.m. direction. There are other potentially interesting hard scattering processes that have the same kinematical advantage, for example, the inverse QCD Compton effect $qg \rightarrow q\gamma$ (direct photon production) or photon-gluon fusion $\gamma g \rightarrow q\bar{q}$. In this analysis, we shall concentrate on the QCD Compton effect and simulate complete events resulting from the hadronization of the final state quarks and gluons $\gamma N \rightarrow (q\text{-jet}) + (g\text{-jet}) + (\text{di-quark-jet})$.

A. The Model

The invariant jet cross section for the QCD Compton effect (29) as depicted in Fig. 2.1(a) is
Fig. 2.1 Diagrams depicting the QCD Compton process: (a) photon-quark collision in the center of momentum frame indicating the final state quark, gluon and diquark jets, and (b) photon + quark $\rightarrow$ gluon + quark with the subprocess kinematical variables, \( \hat{s}, \hat{t}, \hat{u} \) labelled.
(a) DI-QUARK JET

QUARK-JET

NUCLEON

(b) PHOTON

GLUON-JET

γ

^s

q_i

u

q_f

G
\[ E \frac{d\sigma}{d^3p} (\gamma P \rightarrow \text{jet} + X) = \frac{1}{\pi} \sum_i x_i f_i(x_i, Q^2) \frac{2}{2 - x_i e^{-Y}} \frac{d\sigma_i}{dt} \quad (2.1) \]

where \( E \) is the c.m. energy, \( p \) the momentum of the detected particle, and \( f_i(x_i, Q^2) \) gives the momentum density distribution of quark \( i \) with momentum fraction \( x_i \) inside the proton. The differential cross section for the QCD Compton subprocess with variables as labelled in Fig. 2.1(b) is

\[ \frac{d\sigma_i}{dt} (\gamma q_i \rightarrow q q_i) = \frac{-8\pi e_i^2 \alpha_s(Q^2)}{3 s} \left( \frac{1}{u} + \frac{1}{s} \right) \quad (2.2) \]

with \( e_i = \) fractional electric charge carried by quark \( i \), \( Y \) is the gluon c.m. rapidity, \( \alpha = 1/137 \), and

\[ x_i = \frac{X_i * e^{-Y}}{2 - X_i * e^{-Y}} \quad \quad X_i = \frac{2 * P_{T_i}}{s} \]

\[ \alpha_s(Q^2) = \frac{12 * \pi}{(33 - 2N_f) * \ln \left( \frac{Q^2}{\Lambda^2} \right)} \]
where \( \Lambda = 0.30 \) GeV/c and we adopt Feynman's (30) definition of \( Q^2 \).

Our Monte Carlo program (31) simulates complete QCD Compton events in photon-nucleon interactions in the photon-nucleon center-of-mass frame. We neglect the effective parton masses and initial parton Fermi motion inside the nucleon. We also impose restrictions on the kinematical range of variables \( Q^2 \) and the momentum fraction \( x_i \) of the proton carried by parton \( i \), \( Q^2 > 1 \) GeV\(^2\)/c\(^2\), \( 0.04 < x_i < 0.99 \). For the \( Q^2 \) dependent fractional momentum distributions of quarks in the target nucleon we use the Buras and Gaemers parametrization (32).

Without much loss in physical significance of our Monte Carlo simulation results, we may assume that the valence diquark system in the proton, which is the spectator to the hard photon-quark scattering, is a valence antiquark (i.e., 3). It moves backwards, opposite to the direction of the incoming photon beam, whereas the two large \( P_T \) jets (quark and gluon jets) move apart from each other at angles large compared to the initial photon direction.
The photon-nucleon reactions we examine produce three color systems moving separately in the center-of-mass frame of reference. In this section, we employ the standard prescription of Field and Feynman (9) to simulate the subsequent fragmentation of the color systems resulting from the confinement mechanism. We treat the radiated gluon as a quark-antiquark pair of definite flavor. The original quark of this pair carrying some fraction of the gluon momentum moves along the fragmentation axis and generates a color field in which new quark-antiquark pairs evolve. The original quark then fragments into a "leading" meson (which is not necessarily the fastest meson in the "string") by combination with an antiquark from a newly created $q\bar{q}$ pair in the color field. The original quark transfers a portion of its momentum to the remaining quark of this new $q\bar{q}$ pair, which then cascades further in the same manner until all the available energy and momentum are exhausted. The last "primary" meson is a combination of a new quark and the "spectator" antiquark (di-quark). With a similar procedure, a second string of primary mesons is created in such a way that the first meson has the antiquark of the gluon and the last meson contains the final state quark of the QCD Compton process $\gamma q \rightarrow gq$.

The momenta of the new quarks (antiquarks) transverse to the fragmentation axis are distributed according to a
Gaussian, \( \frac{d\sigma}{dP_T^2} \propto e^{-P_T^2/2\sigma_q^2} \), where \( \sigma_q = 350 \text{ MeV/c} \) (33). The longitudinal momenta are distributed according to the scaling forms of the fragmentation function proposed by Field and Feynman (9,10).

We utilize a recent measurement of the relative probability for generating strange quark-antiquark pairs, \( P_s \), in the color field as compared to \( uu(\bar{d}\bar{d}) \) quark pairs, \( P_s / P_u \) (\( P_s / P_d \)), and take the probabilities to be \( P_u = P_d = 0.44, P_s = 0.12 \), where we have neglected charmed particle production (\( P_u + P_d + P_s = 1 \)) (34).

Since the experimental data for the production ratio between primary mesons in different spin states are not accurate enough, we assume that vector and pseudoscalar mesons are produced with equal probability. This is not inconsistent with the experimental results (35).

Finally, the three-jet final states, fully simulated by our Monte Carlo program, are Lorentz transformed into the laboratory system along the z-direction which is defined by the incoming photon. Because we neglected the parton masses and transverse motion in the nucleon this transformation is satisfactory, though only approximately correct.

**B. Results and Discussion**

We have generated 3000 QCD Compton events with different
kinematical selections for each c.m. energy studied. In the following, we shall study the invariant jet cross sections and general characteristics of these events.

The cross section for the reaction $\gamma p \rightarrow \text{gluon jet} + X$ is estimated to be between 10 and 20 nb with the gluon jet at $90^\circ$ in the overall center-of-mass frame. Figure 2.2 shows the theoretical prediction of the gluon jet cross section at the gluon c.m. rapidity $Y = 0$, which falls smoothly with the gluon transverse momentum $P_T$. Dependence on the incident photon energy is shown in Fig. 2.3, where we plot the invariant cross section for $\sqrt{s}$ of 9.7, 15.6, 19.4 and 27.4 GeV. A $P_T^{-4}$ behavior is seen at $\sqrt{s} = 19.4$ GeV. The curve labelled "VMD" shows the "hadron-like" behavior of the photon as described by the Vector Meson Dominance model which leads to a steeply falling cross section as a function of $P_T$ (36). The rapidity distribution at different fixed $P_T$ values are shown in Fig. 2.4. Concentration of produced hadrons in the jet is clearly seen at positive rapidities for each $P_T$.

The fragmenting quark should be dominantly a $u$-quark because of the electromagnetic coupling, and thus one would expect that the positively charged mesons would reflect the original quark flavor. In Fig. 2.5, we show the $P_T$-distribution, $d\sigma / dP_T$, for all the charged particles excluding $\pi^+$ mesons and for $\pi^+$ mesons only. No significant differences are observed, as expected. Note that to make
Fig. 2.2 Theoretical prediction for the inclusive gluon transverse momentum $P_T$ distribution in $\gamma N$ interaction at $\sqrt{s} = 19.4$ GeV with the gluon rapidity equal zero.
$\gamma p \rightarrow (\text{Gluon Jet}) X$

$\sqrt{s} = 19.4$ GeV

$\gamma = 0$
Fig. 2.3 Predictions of the invariant gluon-jet cross section at various energies: ....... for $\bar{s} = 9.7$ GeV, .--. for $\bar{s} = 15.6$ GeV, - - - for $\bar{s} = 19.4$ GeV and ----- for $\bar{s} = 27.4$ GeV. The vector meson dominance contribution is the curve labelled by VMD.
$\gamma p \rightarrow \text{(Gluon Jet)} \times \quad 0 < Y < 0.1$

$\sqrt{s} \ (\text{GeV})$

- 27.4
- 19.4
- 15.6
- 9.7

$E \frac{d^3\sigma}{dp^3} \ (\text{cm}^2 \ \text{GeV}^2 \ \text{c}^3)$

$P_T \ (\text{GeV} \ \text{c}^1)$
Fig. 2.4 Theoretical predictions for gluon-jet rapidity
distributions at the energy $\sqrt{s}$ of 19.4 GeV and
transverse momentum $P_T$ of 1, 2 and 4 GeV/c.
\( \gamma p \rightarrow (\text{Gluon Jet}) X \)

\( \sqrt{s} = 19.4 \text{ GeV} \)

- \( P_T = 1. \text{ GeV/c} \)
- \( P_T = 2. \text{ GeV/c} \)
- \( P_T = 4. \text{ GeV/c} \)
Fig. 2.5 Predictions of the $P_T$ distributions for the final state hadrons: (i) unshaded area for all charged particles without including $\pi^+$ mesons and (ii) shaded area for $\pi^+$ mesons only, at the energy $\sqrt{s}$ of 19.4 GeV. The gluon jets are restricted to $0 \leq Y \leq 1$ and $2 \leq P_T \leq 6.5$ GeV/c.
Mesons only

All Charged particles without $\pi^+$ Mesons

$\frac{d\sigma}{dp_T}$ (cm$^2$ GeV$^{-1}$ c)/0.1 GeV$^{-1}$

$10^{30}$

$10^{31}$

$10^{32}$

$10^{33}$

$P_T$ (GeV/c)
absolute normalization possible we have restrictions, 
0 \leq Y \leq 1, 2.0 \leq p_T \leq 6.5 \text{ GeV/c},\text{ on the final state gluon.}

The expected increase in the ratio of $\pi^+$ to $\pi^-$ mesons in 
the forward hemisphere as a function of Feynman x-values and 
the hadron transverse momenta relative to the photon 
direction can be seen in Fig. 2.6.

The average charged particle multiplicity $\langle n_{\text{ch}} \rangle$ has been 
reported to have an energy dependence much stronger than $\ln s$ 
in various experiments, such as $e^+e^-$ annihilations and $p\bar{p}$ 
collisions (37). Our Monte Carlo generated QCD Compton 
events exhibit the same behavior for the average multiplicity 
of final state hadrons as a function of $\sqrt{s}$, illustrated in 
Fig. 2.7. However our prediction for the absolute charged 
multiplicity is much less than the naively expected value, an 
effect we discuss shortly.

We show the energy dependence of the invariant mass 
distribution in Fig. 2.8(a) and (b). The total invariant 
mass in the forward c.m. hemisphere increases faster with 
energy than the corresponding mass in the backward 
hemisphere. Characteristics of the gluon jet can also be 
studied by means of the energy flow outside an angular cone 
about the gluon jet axis in the overall center-of-mass frame. 
Our prediction for the energy flow (38), as shown in Fig. 
2.9, rises sharply at small angles and shows little change as 
the size of the angular cone increases. The dependence of
Fig. 2.6 Predictions for the ratio $\pi^+/\pi^-$ (a) as a function of $X_F$ and (b) as a function of $P_T$ in the forward c.m. hemisphere: ......for $\sqrt{s} = 15.6$ GeV, --- for $\sqrt{s} = 19.4$ GeV and --- for $\sqrt{s} = 27.4$ GeV
Fig. 2.7 Predictions for the mean multiplicities of (a) $\pi^+$ mesons, (b) $k^+$ mesons, (c) neutral particles and (d) all charged particles vs $s$. Note that $\langle \pi^- \rangle$ and $\langle k^- \rangle$ are not shown as they are very similar to $\langle \pi^+ \rangle$ and $\langle k^+ \rangle$, respectively.
MULTIPLICITY \(<n>\)

\[
\begin{array}{c}
\text{vs (GeV)} \\
8 & 10 \\
20 & 30
\end{array}
\]

\[
\begin{array}{c}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16
\end{array}
\]

(a) \hspace{1cm} (b)
Fig. 2.8(a) Predictions for the total invariant mass distribution for the final state hadrons in the c.m. forward and backward hemisphere: (i) unshaded area for particles with $y > 0$ and (ii) shaded area for particles with $y < 0$, at $\sqrt{s} = 15.6$ GeV. The gluon jets are restricted to $0 \leq Y \leq 1$ and $2 \leq P_T \leq 6.5$ GeV/c. Here $M^2$ is defined as $\left( \sum_{i=1}^{n} p_i \right)^2$ for final state hadrons. (b) Same as (a) except $\sqrt{s} = 19.4$ GeV.
Hadron Masses at $y < 0$

Hadron Masses at $y > 0$

$\sqrt{s} = 15.6$ GeV

$2 \leq p_t \leq 6.5$

$0 \leq y \leq 1$

$\frac{d\sigma}{dW}(\text{cm}^2 \cdot \text{GeV}^{-2})/0.25 \text{ GeV}^2$ BIN WIDTH

$M(\text{GeV}^2)$
Fig. 2.8(b) Total invariant mass distribution at $\sqrt{s} = 19.4$ GeV.
Fig. 2.9 Predictions for the angular energy flow outside of the cone about the gluon jet axis in the overall c.m. frame: (i) \ldots for \sqrt{s} = 9.7 \text{ GeV}, (ii) \ldots for \sqrt{s} = 15.6 \text{ GeV}, (iii) \ldots for \sqrt{s} = 19.4 \text{ GeV} and (iv) \ldots for \sqrt{s} = 27.4 \text{ GeV}. 
$\sqrt{s}$ in GeV

- 27.4
- 19.4
- 15.6
- 9.7

Energy Flow outside of the Cone
Gluon Jet
Axis in CM Frame

$\frac{d\langle E \rangle}{d\lambda}$ (GeV Degree$^2$)
the rate of energy flow on the incoming photon energy is also illustrated in this figure.

With Monte Carlo simulation, complete QCD Compton events were generated to study the final state production of 3 jets in detail. Subsets of this analysis can be compared with the work of other authors who examine one jet. Such invariant jet cross sections, as for the $\gamma N \rightarrow (\text{jet}) + X$ reactions, have been studied theoretically by Owens (36) and other authors (39). Our simulation results for the invariant gluon jet cross section at various energies appear to have the same behavior predicted by these previous authors, (see Fig. 2.3). As the energy increases, the invariant gluon jet cross sections become flatter allowing us to isolate the VMD contribution from the QCD Compton processes at $P_T > 4.5$ GeV/c (36). The hadron-like component of the photon does not give any important background at such large $P_T$ values due to the faster decrease of the parton distribution functions in the target nucleon with increasing parton fractional momentum.

The accumulation of hadrons produced in the gluon jet at c.m. rapidities $Y > 0$ is seen in Fig. 2.4. Such behavior is also seen by plotting the invariant mass distribution, $d\sigma / dM$ vs $M$ for the final state hadrons in both forward and backward hemispheres as shown in Fig. 2.8. Without experimental data, it is reasonable to assume that there is a
50% probability for finding a gluon jet and 50% probability for finding a quark jet among the isolated jets in the laboratory. From the phenomenological point of view, the gluon jet is expected to contribute more to the average multiplicity than the quark jet, since a color octet has 3/2 the color charge of the color triplet. Thus, we should see more hadrons in the forward hemisphere, than expected for the hadronlike photon-nucleon processes. Furthermore, as the energy increases, the leading order QCD term becomes a dominant contribution in the photon production processes, which is demonstrated by the strong increase in the number of final state hadrons in the forward cms hemisphere relative to the backward hemisphere.

There is no obvious difference between the $P_T$-distribution of all charged particles without including $\pi^+$ mesons with that due to $\pi^+$ mesons only, as shown in Fig. 2.5. The ratio $R = \pi^+ / \pi^-$ of $\pi^+$ mesons to $\pi^-$ mesons produced in the forward c.m. hemisphere increases with increasing $X_F$ and $P_T$, as shown in Fig. 2.6, an effect illustrating the dominance of u-quark jets. The relatively slow increase of $R$ as a function of $X_F$ or $P_T$ can be understood in terms of our prior assumption of equal probability for formation of $u\bar{u}$ and $d\bar{d}$ pairs in the color field during the fragmentation process.

Final particle multiplicities are shown in Fig. 2.7. We
do not show the average multiplicities of $\pi^-$ mesons and $K^-$ mesons, because they are almost identical with those of $\pi^+$ mesons and $K^+$ mesons, respectively. For $\pi^0$- multiplicities, we roughly expect the relation $\langle n_{\pi^0} \rangle = (\langle n_{\pi^+} \rangle + \langle n_{\pi^-} \rangle) / 2$ (isospin symmetry) to hold. The relative number of $K$ mesons just reflects the experimental value of the ratio $P_s / P_u$.

From Fig. 2.7, we conclude that there is a strong increase in the average charged and neutral multiplicities, making the average charged multiplicity a convenient quantity to measure experimentally. For pure three-jet events we have made conservative estimates (based on completely independent jet production) of the average charged particle multiplicity at $\sqrt{s} = 27.4$ GeV, which turns out to be within the range $20 < \langle n_{ch} \rangle < 30$. However, the generated QCD Compton events predict $\langle n_{ch} \rangle = 13.77$, which is lower than such a naive estimate. This is to be expected within our present knowledge because, at such an energy three jets will not be entirely independent and must be correlated by their mutually sharing the available energy.

Fig. 2.9 shows the characteristics of the gluon jet in the overall center-of-mass frame, which can be used to differentiate it from quark jets. The slow variation in these curves at angles greater than $20^0$ is expected from the uncertainty principle which limits the transverse momenta of hadrons fragmenting from the gluon jet. From the
phenomenological results, we used the Gaussian distribution with $\sigma_q = 350$ MeV/c to generate the transverse momenta of the final hadrons.

When higher energy experimental data on photon reactions at the Tevatron becomes available, it should be possible to make considerable progress with the work presented here as the starting point. We stress that even though, in our simulation, we ignore effective parton masses, parton Fermi motion inside the nucleon and treat the diquark as an antiquark, we should get reasonably reliable results. Our work suggests that one can identify the jet by studying the $P_T$ distributions, mass distributions, multiplicities, and energy flow. Some information relevant to the kinematic constraints can also be obtained from the simulation method to aid the experimental set-up of the detectors for the study of $\gamma N$ reactions. The photon-nucleon data expected from experiments possible with the Tevatron should hopefully be able to isolate signals from the $\gamma q \rightarrow qq$ subprocess. However, such Tevatron studies will need considerable statistics to be able to yield explicit information on the individual jets.
III. JET FRAGMENTATION IN PERTURBATIVE QCD

A. Formalism

1. Single particle

A general expression for the fragmentation functions for parton $i$ into $n$ hadrons will be constructed from KUV jet calculus which describes the evolved partons at the perturbative QCD (leading-logarithmic) level and recombination ideas based on wave function information for hadronization at the end of the QCD evolution. The fragmentation function for parton $i$ to fragment into hadrons $h_1, \ldots, h_n$ is

$$D_i \rightarrow h_1 \ldots h_m (\bar{x}_1, \ldots, \bar{x}_m; Q^2, Q_0^2)$$

$$= \int_0^1 \prod_{j=1}^n \frac{dx_j}{x_j} F_i \rightarrow a_1 \ldots a_n (x_1, \ldots, x_n; Q^2, Q_0^2)$$

$$\times h_1 \ldots h_m (x_1, \ldots, x_n; \bar{x}_1, \ldots, \bar{x}_m),$$

where $F_i \rightarrow a_1 \ldots a_n$ gives the $n$-parton joint longitudinal momentum distribution within a jet. Equation (3.1) describes an initial parton $i$ "kicked" or perturbed to a large off-shellness $Q^2$, which subsequently evolves into $n$ partons.
at low off-shellness $Q_0^2$, such that partons $a_1, \ldots, a_n$ carry longitudinal momentum fractions $x_1, \ldots, x_n$ of the initial parton $i$, respectively. $R^m_{a_1 \ldots a_n}$ is the probability for parton $a_1$ at $x_1$, ..., and $a_n$ at $x_n$ to recombine into final-state hadrons $h_1, \ldots, h_m$ at $\bar{x}_1, \ldots, \bar{x}_m$, respectively, with $m \leq n/2$. Note that repeated indices denote a summation over all possible combinations of the intermediate parton states, i.e., $u, \bar{u}, d, \bar{d}, s, \bar{s}$ and gluon, in the SU(3)$_{\text{flavor}}$ framework.

With KUV jet calculus accounting for an infinite sum of QCD leading-log-order contributions, we can quickly evaluate the longitudinal momentum distribution of two and three partons in a jet after doing the integration of relative transverse momenta of two partons at the intermediate vertices. For instance, an expression for the probability for finding partons $a_1$ and $a_2$ with momentum between $x_1, x_1 + dx_1$ and $x_2, x_2 + dx_2$, respectively, from the jet-initiating parton $i$, is

$$F_i \rightarrow a_1a_2(x_1, x_2; Q^2, Q_0^2) = \int \frac{Q^2 dk^2}{Q_0^2} \frac{\alpha_s(k^2)}{2\pi} \int_{x_1+x_2}^1 d\xi K_{ji}(\xi; Q^2, k^2) \int_{x_1}^{\xi-x_2} \frac{dz}{\xi} \hat{P}_{j+b_1b_2}(z)$$

$$\times \frac{x_1}{z} K_{a_1b_1}(\frac{x_1}{z}; k^2, Q_0^2) \frac{x_2}{\xi-z} K_{a_2b_2}(\frac{x_2}{\xi-z}; k^2, Q_0^2).$$
Figure 3.1 illustrates this equation with the branching $j \rightarrow b_1 b_2$ and evolution $b_1 \rightarrow a_1$, $b_2 \rightarrow a_2$ obviously indicated. The KUV parton propagator $K_{ab}(x; k^2, Q_o^2)$ describes the probability that parton $b$ at $k^2$ evolves down to some energy reference scale $Q_o^2$ into parton $a$ with momentum fraction $x$; the $\hat{P}_{j-ab}$'s are the Altarelli-Parisi splitting functions at the various QCD vertices.

For the joint longitudinal momentum distribution of three partons $a_1$, $a_2$, and $a_3$ with momentum between $x_1$, $x_1 + dx_1$, $x_2$, $x_2 + dx_2$ and $x_3$, $x_3 + dx_3$, respectively, from the jet-initiating parton $i$, we have an expression given by

$$F_i \rightarrow a_1 a_2 a_3(x_1, x_2, x_3; Q^2, Q_0^2)$$

$$= \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_{x_1+x}^{1} d\xi K_{ji}(\xi; Q^2, k^2) \int_{\xi}^{x} \frac{dz}{\xi} \hat{P}_{j-b_1 b_2}(\frac{z}{\xi})$$

$$\times \frac{x_1}{z} K_{a_1 b_1} \left( \frac{x_1}{z}; k^2, Q_0^2 \right) \int_{Q_0^2}^{k^2} \frac{dk'}{k'^2} \frac{\alpha_s(k'^2)}{2\pi} \int_{z}^{x} \frac{dx'}{x} \hat{P}_{b_2 \rightarrow c_1 c_2} \left( \frac{x'}{z} \right)$$

$$\times K_{b_2' b_2} \left( \frac{x'}{z}; k^2, k'^2 \right) \int_{x_2}^{x'-x_3} \frac{dz}{x'} \hat{P}_{b_2' \rightarrow c_1 c_2} \left( \frac{z'}{x'} \right)$$

$$\times K_{a_2 c_1} \left( \frac{x_2}{z}; k^2, Q_0^2 \right) \frac{x_3}{x'-z} K_{a_3 c_2} \left( \frac{x_3}{x'-z}; k^2, Q_0^2 \right).$$

A possible specific application is the calculation of
Fig. 3.1 Jet calculus diagram for producing partons $a_1$ and $a_2$ with momentum fractions $x_1$ and $x_2$ of the initial parton $i$ with momentum $l$. The intermediate parton labels are $b_1$, $b_2$, $j$ and the remaining letters are the momentum fraction labels of the partons.
the hadron spectrum in a QCD-predicted gluon jet (40). An essential characteristic of QCD distinguishing it from the naive quark-parton model is the prediction of jets derived from the initial creation of a gluon quantum, the gluon being a massless, flavorless and neutral vector parton. Detailed calculations of gluon fragmentation into a meson exists in the literature (20,41,42,43). Instead of the meson distribution in a gluon jet (see Ref. 43), we would like to consider the baryon distribution, for example, proton P or lambda hyperon Λ, in a gluon jet. By substituting Eq. (3.3) into Eq. (3.1), we express the gluon fragmentation into baryon B as

\[ D_g \rightarrow B(x; Q^2, Q_0^2) \]  

\[ = \int \frac{dx}{x} \frac{3}{2} \int_0^{Q_0^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_0^1 d\xi K_{1g}(\xi; Q^2, k^2) \]

\[ \times \int_{x_1}^{x} \frac{dz}{z} \int_{x_1}^{x'} \frac{dz'}{z'} \int_{x_2}^{x_3} \frac{dx'}{x'} K_{1b_1 b_2}(\xi) \frac{x_1}{z} K_{a_1 a_1}(\frac{x_1}{2}; k^2, Q_0^2) \int_{x_2}^{x_3} \frac{dz'}{x'} \]

\[ \times \frac{\alpha_s(k', 2)}{2\pi} \int_{x_2}^{x_3} \frac{dx'}{x'} K_{b_1 b_2}(\xi - z; k^2, k'^2) \int_{x_2}^{x_3} \frac{dz'}{x'} \]

\[ \times \frac{\alpha_s(k'^2)}{2\pi} \int_{x_2}^{x_3} \frac{dz'}{x'} K_{c_1 c_2}(x') \frac{x_2}{x'} K_{a_2 c_1}(\frac{x_2}{z'}; k'^2, Q_0^2) \frac{x_3}{x' - z'}, \]
In the above expression, we employ the variable of KUV, 
\[ y = \frac{1}{2\pi b} \ln(1 + \alpha_0 b \ln(k^2 / \Lambda^2)) \], where \( 12\pi b = 27 \) with the assumption of SU(3) flavor and color symmetry (44); \( \Lambda \) and \( \alpha_0 \) in \( \Lambda^2 = \Lambda^2 \exp(-1 / \alpha_0 b) \) are constants determining the scale and strength of the QCD coupling 
\[ \alpha_s(Q^2) = \frac{1}{(\ln(Q^2 / \Lambda^2))} \]. Here, we take \( \alpha_0 = 10 \) as used by MJL (45). The function \( R_{a_1 a_2 a_3}^B(x_1, x_2, x_3; x) \) represents the probability that partons with momentum fractions \( x_1 x_2 x_3 \) will make up and recombine into a baryon \( B \) with momentum fraction \( x \). By changing the integration variable \( k^2 \) and \( k'^2 \) to \( y \) and \( y' \), respectively, the baryon fragmentation function is written as

\[
D_g \to_B (x; Y, Y_0)
= \int_0^1 \prod_{n=1}^3 \frac{dx_n}{x_n} R_{a_1 a_2 a_3}^B(x_1, x_2, x_3; x) \int_Y^{Y_0} dy \int_{x_1 + x'}^{1} \frac{d\xi}{\xi^2}
\times K_{jg}(\xi; Y, y) \int_{x_1}^{1-x'/\xi} \frac{dz}{z(1 - z)} P_{j \to b_1 b_2}(z) K_{a_1 b_1}(\frac{x}{z}, y, Y_0)
\]
After a few steps of algebra, the baryon fragmentation function can be simplified to

\[ D_g \rightarrow B(x; Y, Y_0) \]

\[ = \int_0^1 \frac{dx}{x} \sum_{\varphi=1}^n R^B_{a_1 a_2 a_3} (x_1, x_2, x_3; x) \left[ \int \cdots \right] R^B_{a_1 b_1} (w_1; y', Y_0) \]

\[ \times K_{a_2 c_1} (w_2; y', Y_0) K_{a_3 c_2} (w_3; y', Y_0) \hat{P}_{b_2 \rightarrow c_1 c_2} (z') \]

\[ \times K_{b_2 b_2} (z'; y, y') \hat{P}_{j \rightarrow b_1 b_2} (z) K_{j g} (\xi; Y, y) \delta(x_1 - z\xi w_1) \]

\[ \times \delta(x_2 - z' w_2) \delta(x_3 - (1 - z') x' w_3) \delta(x' - x'' \xi (1 - z)) \]

\[ = \int_0^1 D_{a_1 a_2 a_3, g} (x_1, x_2, x_3; Y, Y_0) R^B_{a_1 a_2 a_3} (x_1, x_2, x_3; x) \]

\[ \times \prod_{\eta=1}^3 dx \eta \]

where the delta functions \( \delta \) express momentum conservation in
terms of the momentum fraction variables $\xi$, $z$, $w_1$ and $w_2$. These quantities are shown in Fig. 3.2 for clarity. An integration over the region of parton evolution from $Y_0$ to $Y$ was performed. \[ \int \ldots \] \( dz \ldots dz' \) denotes \( dydy'dzdz'dx'dx''dw_1dw_2dw_3 \).

As the parton evolution ends up at $y = Y$, there are quarks $q$, antiquarks $\bar{q}$ and gluon $g$ generated in the parton shower. For single baryon inclusive production in a jet, only four distinguishable terms would contribute in the fragmentation function, such that

\[ D_{g \to B}(x; Y, Y_0) = \sum_{i=0}^{3} D^{(i)}(x; Y, Y_0). \quad (3.7) \]

The first term $D^{(0)}$ corresponds to the zero-gluon or $qqq$ recombination contribution, and the next three terms are respectively $gqq$, $gqg$, and $ggg$ contributions to baryon production. They are expressed as follows

\[ D^{(0)}(x; Y, Y_0) \]

\[ = \int D_{q_1q_2q_3} g(x_1, x_2, x_3; Y, Y_0) R^B_{q_1q_2q_3}(x_1', x_2', x_3'; x) \]

\[ \times \prod_{\eta=1}^{3} dx_\eta, \]
Fig. 3.2 Diagram depicting evolution in a gluon jet of four momentum squared $Q^2 (Y \sim \ln \ln Q^2)$ down to $Q_0^2$ where partons $a_1 a_2 a_3$ are recombined via the recombination process $R$ to yield the final baryon $B$ of momentum fraction $x$. The intermediate partons $b_1, b_2, b_2', c_1, c_2$ with their momentum fraction values are indicated.
\[
\int_{(\epsilon \leftrightarrow \Omega)} (x : \epsilon^3) \quad \epsilon B \epsilon B \epsilon B \epsilon B \quad ^a_\mathcal{D} (\epsilon \Lambda : x) (\epsilon) \quad \mathcal{D}
\]

\[
\int_{(\epsilon \leftrightarrow \Omega)} (x : \epsilon^3) \quad \epsilon B \epsilon B \epsilon B \epsilon B \quad ^a_\mathcal{D} (\epsilon \Lambda : x) (\epsilon) \quad \mathcal{D}
\]

\[
\int_{(\epsilon \leftrightarrow \Omega)} (x : \epsilon^3) \quad \epsilon B \epsilon B \epsilon B \epsilon B \quad ^a_\mathcal{D} (\epsilon \Lambda : x) (\epsilon) \quad \mathcal{D}
\]
Note that we label the final partons now as $q_1$, $q_2$, $q_3$ instead of $a_1$, $a_2$, $a_3$ as in Fig. 3.2 as $q_1$, $q_2$, and $q_3$ denote the quarks that end in the baryon. When one or more of the final partons is a gluon, the momentum conserving approach of Chang and Hwa (42), by inserting $\bar{P}(z)$, is used to convert each gluon into quark-antiquark pairs from which the quark enters the baryon. The function $\bar{P}(z) = |z^2 + (1 - z)^3 / 2$ is the probability that a gluon turns into a quark-antiquark pair of momentum fraction $z$ and $1-z$.

In Eq. (3.7), written in the form of integrals (as in Eq. (3.8), (3.9), (3.10) and (3.11) for $D^{(0)}$, $D^{(1)}$, $D^{(2)}$ and $D^{(3)}$), we may depict the jet calculus kernel $D_{q_1q_2q_3}^g$ in jet calculus diagram form as in Fig. 3.3. There are eight different forms for such diagrams giving gluon fragmentation into quarks uud or uds. This contrasts markedly with the result given in Ref. 45 where 23 different forms contribute to uud(proton) production and 18 forms contribute to uds(lambda) production from u quarks. (We note that the intermediate solid lines in each case are summed over $u$, $\bar{u}$, $d$, $\bar{d}$, $s$, $\bar{s}$ and $g$.)

The function $R^B_{a_1a_2a_3}$ giving the probability for partons $a_1$, $a_2$, $a_3$ to form the final baryon with fraction $x$ is
Fig. 3.3 Irreducible jet calculus diagrams for the evolution of a gluon into final u, d, and s quarks and, as represented to the right of the vertical line, into final u, u, and d quarks.
where \( r = 1 \) and \( R = 27 \) are used explicitly in our QCD calculation according to the justification given by Lepage-Brodsky (46) and MJL.

Following the moment approach of JLSW (47), we define the moments of the baryon fragmentation functions, of the Altarelli-Parisi splitting functions and of the QCD-vertex functions, respectively, as

\[
D_i \rightarrow B(n; Y, Y_0) = \int_0^1 dx \ x^n \ D_i \rightarrow B(x; Y, Y_0), \quad (3.13)
\]

\[
A_{ba}^m = \int_0^1 dz \ z^m \ \hat{P}_{a \to bc}(z), \quad (3.14)
\]

and

\[
\hat{P}_{a \to bc}^m, n = \int_0^1 dz \ z^m \ (1 - z)^n \ \hat{P}_{a \to bc}(z). \quad (3.15)
\]

For simplicity, we introduce, \( 0 \) and \( L \), such that
\[ O(\alpha, \beta, \gamma, \delta, \mu; \{y\}, Y_0) \]  

\[ = K_{q_1 b_1}^\alpha (y, Y_0) K_{q_2 c_1}^\beta (y', Y_0) K_{q_3 c_2}^\gamma (y', Y_0) K_{b_2' b_2}^\delta (y, y') \times K_{j g}^H (y, y) \hat{P}_{\alpha, \delta}^{\alpha, \delta} \hat{P}_{\beta, \gamma}^{\beta, \gamma} \]  

\[ L^1(\alpha, \beta, \gamma) = A_{q_1 g_1}^\alpha + A_{q_2 g_2}^\beta + A_{q_3 g_3}^\gamma, \quad (3.17) \]  

\[ L^2(\alpha, \beta, \gamma) = A_{q_1 g_1}^\alpha A_{q_2 g_2}^\beta + A_{q_1 g_1}^\alpha A_{q_3 g_3}^\gamma + A_{q_2 g_2}^\beta A_{q_3 g_3}^\gamma, \quad (3.18) \]  

and

\[ L^3(\alpha, \beta, \gamma) = A_{q_1 g_1}^\alpha A_{q_2 g_2}^\beta A_{q_3 g_3}^\gamma. \quad (3.19) \]  

Then, the expression for the moments of the baryon fragmentation function can be written as

\[ D_g \to B(n; Y, Y_0) \]  

\[ = \sum_{i=0}^{3} D^{(i)}(n; Y, Y_0) \]  

\[ = \sum_{i=0}^{3} \sum_{m=0}^{n-3} \sum_{r=0}^{m} \frac{\Gamma(n - 4)}{\Gamma(n - m - 4) \Gamma(m - r - 1) \Gamma(r - 1)} \]
\[ x \int_{Y_0}^{Y} dy \int_{Y_0}^{Y} dy' \, E^1(\{n'\}; \{y\}, Y_0) \]

with

\[ E^0(\{n'\}; \{y\}, Y_0) = 0(n-m-2,m-r+1,r+1,m+2,n;\{y\}, Y_0), \quad (3.21) \]

\[ E^1(\{n'\}; \{y\}, Y_0) = L^1(n-m-2,m-r+1,r+1) \tilde{0}, \quad (3.22) \]

\[ E^2(\{n'\}; \{y\}, Y_0) = L^2(n-m-2,m-r+1,r+1) \tilde{0}, \quad (3.23) \]

and

\[ E^3(\{n'\}; \{y\}, Y_0) = L^3(n-m-2,m-r+1,r+1) \tilde{0}. \quad (3.24) \]

Here, \( \{n'\} \) stands for \( n, m \) and \( r \) and \( \{y\} \) stands for \( y \) and \( y' \);
\( \tilde{0} \) denotes the parton labels \( j, b_1, b_2, b'_2, c_1 \) and \( c_2 \) as shown in Fig. 3.2. \( \tilde{0} \) is obtained from \( 0 \) by making a change of quark label "q" to gluon label "g" in the KUV propagator \( K_{qa} \) so that correspondence in indices exists with the associated anomalous dimension \( A_{qg} \), for instance \( A_{q2g2} \). We present in Appendix A the basic ingredients for our calculation, namely, the moments of the KUV propagators and QCD-vertex functions and the anomalous dimensions.
2. Two particles

In our experience with jet calculus applications as discussed for baryon production above (see Appendix B for meson production), the number of KUV propagators $K$ entering the kernel $F$ has profound influence on the $Q^2$ dependence at large, but not infinite, $Q^2$. In our model for the two-particle fragmentation function $D_{1-h_1h_2}$ for the inclusive production of two mesons, two sets of $q'q$ pairs are generated in the QCD jet. This requires seven KUV propagators in the jet calculus, each propagator evolving in $A-P$ fashion at the parton level. The folding together of these seven elementary dependences in moment space leads to a quite complicated structure for the $Q^2$ evolution of for the $D_{1-h_1h_2}$.

We write the two-particle fragmentation function $D_{1-M_1M_2}$ from Eq. (3.1) for parton 1 to produce meson $M_1$ and $M_2$ with momentum fractions $\bar{x}_1$ and $\bar{x}_2$, respectively, when 1 carries momentum 1,

$$D_{1 \rightarrow M_1M_2}(\bar{x}_1, \bar{x}_2; Q^2, Q_0^2)$$  \hspace{1cm} (3.25)
This expression describes evolution of an initial parton $i$ with off-shell mass $Q^2$ fragmenting into four partons $a_1$, $a_2$, $a_3$, and $a_4$ with, respectively, longitudinal momentum fractions $x_1$, $x_2$, $x_3$, and $x_4$ of parton $i$; and $R_{a_1a_2a_3a_4}$ is the recombination function, which specifies the probability of recombining four partons at $x_1$, $x_2$, $x_3$, and $x_4$ into two observed mesons, $M_1$ and $M_2$.

The probability $F$ to find four partons in the parton shower has two topologically distinct diagrams for the QCD branching process, as shown in Fig. 3.4, and we write it as a sum of two contributions,

$$ F_i \rightarrow a_1 a_2 a_3 a_4 (x_1, x_2, x_3, x_4; Q^2, Q_0^2), $$

(3.26)

$$ = F^{[a]} ([x]; Q^2, Q_0^2) + F^{[a]} ([x]; Q^2, Q_0^2). $$

where $[a]$ and $[x]$ stand for $a_1$, $a_2$, $a_3$, $a_4$ and $x_1$, $x_2$, $x_3$, $x_4$, respectively. The parton distribution for diagram I is

$$ F^{[a]} ([x]; Q^2, Q_0^2) $$

(3.27)

$$ = \int \frac{Q^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_1^{x_1 + x_'} d\xi \, K_{jj}(\xi; Q^2, k^2) \int_{x_1}^{\xi - x_'} \frac{dz}{\xi} \hat{P}_{j-b_1 b_2} \left( \frac{z}{\xi} \right) $$

$$ \times \frac{x_1}{\xi} K_{a_1 b_1} \left( \frac{x_1}{z}; k^2, Q_0^2 \right) \int \frac{Q^2}{k^2} \frac{\alpha_s(k'^2)}{2\pi} \int_{x_2 + x_1}^{\xi - z} \frac{dx'}{\xi - z}. $$
Fig. 3.4 Jet calculus diagrams I and II for producing partons $a_1$, $a_2$, $a_3$, and $a_4$ with momentum fractions $x_1$, $x_2$, $x_3$, and $x_4$ of the initial parton $i$ with momentum 1. The parton labels are $a$, $b$, $c$, ... $j$ and all remaining letters are the momentum fraction labels of the partons. The diagram can be looked upon as proceeding from left to right in the variable $y$ and the vertices are labelled by their $y$ value.
\[ x \cdot K_{a_2 c_1, \xi} \left( \frac{x}{\xi - z'} ; k^2, k' \right) = \int_{x_2}^{x_1} \frac{dx_{14}}{x} \cdot \hat{F}_{b_2 c_1, \xi} \left( \frac{z'}{x'} \right) \frac{x_2}{z'} \]

\[ \times K_{a_3 e_1, \xi} \left( \frac{x_3}{\xi - z'} ; k^2, k'' \right) \int_{x_3}^{x_{14}} \frac{dz_{14}}{x_{14}} \cdot \hat{F}_{c_2 e_2, \xi} \left( \frac{z''}{x_{14}} \right) \frac{x_3}{z''} \]

\[ \times K_{a_4 e_2, \xi} \left( \frac{x_4}{\xi - z''} ; k^2, k'' \right) \int_{x_4}^{x_{14}} \frac{dx_{14}}{x} \cdot \hat{F}_{f_{b_2 b_1}, \xi} \left( \frac{z}{x_{14}} \right) \frac{x_4}{z} \]

and for diagram II is

\[ F_{II}^{[a]}(\{x\}; Q^2, Q'^2) \]

\[ = \int_{Q'^2}^{Q^2} \frac{d\xi}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_{x'}^{x_{14}} d\xi \; K_{j_1} (\xi; Q^2, k^2) \int_{x_{14}}^{x_{14}} \frac{dz}{x_{14}} \cdot \hat{F}_{j_{b_1 b_2}, \xi} \left( \frac{z}{x_{14}} \right) \]

\[ \times \int_{Q'^2}^{Q^2} \frac{d\xi}{k''^2} \frac{\alpha_s(k''^2)}{2\pi} \int_{x_{14}}^{z} \frac{dx_{14}}{z} \cdot K_{b_2} \left( \frac{x_{14}}{z} ; k^2, k''^2 \right) \]

\[ \times K_{b_2 b_1, \xi} \left( \frac{x_{14}}{z} ; k^2, k''^2 \right) \]
We next substitute Eq. (3.26), (3.27) and (3.28) into (3.25) and change the integration variables \( k^2, k'^2 \) and \( k''^2 \) to \( y, y' \) and \( y'' \). The fragmentation of parton \( i \) into two mesons, \( M_1 \) and \( M_2 \), is

\[
D_i \to M_1 M_2 (x_1', x_2'; Y, Y_0) \tag{3.29}
\]

\[
= D_I (x_1, x_2; Y, Y_0) + D_{II} (x_1, x_2; Y, Y_0)
\]

\[
= \int_0^1 \frac{d\xi}{\xi} \int_{\eta=1}^{M_2 M_2} d\eta \alpha_{a_1 a_2 a_3 a_4} (x_1, x_2, x_3, x_4; \bar{x}_1, \bar{x}_2) \int_{Y_0}^{Y} dy \int_0^1 \frac{d\xi}{\xi^2}
\]

\[
\times K_{ji} (\xi; Y, y) \int_{\frac{1-x'}{\xi}}^{1-\frac{x'}{\xi}} dz \frac{z}{z(1-z)} \hat{P}_{j \to b_1 b_2} (z) K_{a_1 b_1} (\frac{z}{z_0}; y, Y_0)
\]

\[
\times K_{a_2 c_2} (\frac{x}{x''}; k''^2, Q_0^2) \int_{\xi''}^{\xi_0} d\xi'' \frac{d\xi''}{\xi''} \frac{d\xi}{\xi''}
\]

\[
\times K_{b_2 c_2} (\frac{x}{x''}; k''^2, Q_0^2) \int_{\xi''}^{\xi_0} d\xi'' \frac{d\xi''}{\xi''}
\]
\[
\times \int_{y_0}^{y} dy' \int_{x_2+x_1^v}^{x} \xi(1-z') \frac{dx'}{x_2+x_1^v} K_{b_2' b_2} \left( \frac{x'}{(1-z')x}; y', y' \right) \int_{x_2}^{1} \frac{dz'}{z'(1-z')}
\]

\[
\times \hat{p}_{b_2' c_1 c_2} (z') K_{a_2 c_1} \left( \frac{x_2}{z', x}; y', y_0 \right) \int_{y_0}^{y'} dy'' \int_{x_3+x_4}^{x' (1-z')} \frac{dx''}{x_1^v z''(1-z'')}
\]

\[
\times K_{c_2' c_2} \left( \frac{x_1^v}{(1-z')x''}; y', y'' \right) \int_{x_3}^{1} \frac{dz''}{x_1^v z''(1-z'')} \hat{p}_{c_2' e_1 e_2} (z'')
\]

\[
\times K_{a_3 e_1} \left( \frac{x_3}{z'' x''}; y'', y_0 \right) K_{a_4 e_2} \left( \frac{x_4}{(1-z'') x_1^v}; y'', y_0 \right)
\]

\[
+ \int_{0}^{1} \frac{4}{x} R_{a_1 a_2 a_3 a_4} (x_1, x_2, x_3, x_4; \bar{x}_1, \bar{x}_2) \int_{y_0}^{y} dy
\]

\[
\times \int_{x'-x_1^v}^{x} \frac{d\xi}{x^v} K_{j_1} (\xi; y, y) \int_{x_1^v}^{1} \frac{dz}{x(1-z')} \hat{p}_{j' b_1 b_2} (z) \int_{y_0}^{y'} dy''
\]

\[
\times \int_{x_1+x_2}^{x_2} \frac{dx''}{x_1^v} K_{b_2' b_1} \left( \frac{x''}{z''}; y, y'' \right) \int_{x_1^v}^{1} \frac{dz''}{z''(1-z'')}
\]
After several steps of algebra, we obtain

\[ D_1 \rightarrow M_1 M_2 (\mathcal{K}_1, \mathcal{K}_2; \mathcal{Y}, \mathcal{Y}_0) \]

\[ = \int \frac{1}{\Pi} \prod_{\eta=1}^4 \frac{1}{\mathcal{M}_1 \mathcal{M}_2} \mathcal{R}_{\mathcal{a}_1 \mathcal{a}_2 \mathcal{a}_3 \mathcal{a}_4} (x_1, x_2, x_3, x_4; \mathcal{K}_1, \mathcal{K}_2) \left\{ \left[ \begin{array}{c} 1 \\ \vdots \\ 1 \end{array} \right] \right\} \]

\[ \times \mathcal{K}_{a_1 b_1} (w_1; \mathcal{Y}, \mathcal{Y}_0) \mathcal{K}_{a_2 c_1} (w_2; \mathcal{Y}, \mathcal{Y}_0) \mathcal{K}_{a_3 e_1} (w_3; \mathcal{Y}, \mathcal{Y}_0) \]

\[ \times \mathcal{K}_{a_4 e_2} (w_4; \mathcal{Y}, \mathcal{Y}_0) \hat{P}_{b_1 c_2} (z) \mathcal{K}_{c_1 c_2} (z); \mathcal{Y}, \mathcal{Y}) \]

\[ \times \delta(x_1 - w_1 z \xi) \delta(x_2 - w_2 z \xi) \delta(x_3 - w_3 z) \delta(x_4 - w_4 z) \]

\[ \times \delta(x - w_1 z \xi) \delta(x - w_2 z \xi) \delta(x - w_3 z) \delta(x - w_4 z) \]

\[ \delta(x - w_1 z \xi) \delta(x - w_2 z \xi) \delta(x - w_3 z) \delta(x - w_4 z) \]

\[ \times \delta(x - w_1 z \xi) \delta(x - w_2 z \xi) \delta(x - w_3 z) \delta(x - w_4 z) \]

\[ \times \delta(x - w_1 z \xi) \delta(x - w_2 z \xi) \delta(x - w_3 z) \delta(x - w_4 z) \]
\[ + \left[ \int \ldots \right] K_{a_1 c_1}(w_1; y'', Y_0) K_{a_2 c_2}(w_2; y'', Y_0) \]

\[ \times K_{a_3 c_1}(w_3; y', Y_0) K_{a_4 c_2}(w_4; y', Y_0) \hat{P}_{b_2 \rightarrow c_1 c_2}^{y''} \]

\[ \times \hat{P}_{b_2 \rightarrow c_1 c_2}^{y'} K_{b_2 b_1}(z') \]

\[ \times \delta(x_{1V} - x''z\xi) \delta(x' - x''(1-z)\xi) \]

\[ \times \delta(x_1 - w_1z''x_{1V}) \delta(x_2 - w_2(1-z'')x_{1V}) \delta(x_3 - w_3z'x') \]

where

\[ [f_1 \ldots ] = \int dydy'dy''d\xi dzdz'dz''dx'dx''dx_{1V}dx_{2V} dw_1 dw_2 dw_3 dw_4 \]

and similarly for \([f_2 \ldots ]\) except that the upper limit of the integration on \(y''\) is replaced by \(y\), as shown explicitly in Fig. 3.4.

As in the earlier application, the total contributions to a parton fragmenting into two mesons can be decomposed into a sum of five distinguishable terms corresponding to various final parton states, such that

\[ D_i \rightarrow M_{12}(\bar{x}_1, \bar{x}_2; Y, Y_0) \]

(3.31)
\[ D_1(\bar{x}_1, \bar{x}_2; Y, Y_0) + D_{11}(\bar{x}_1, \bar{x}_2; Y, Y_0) \]

\[ = \sum_{i=0}^{4} \left\{ D^{(i)}_1(\bar{x}_1, \bar{x}_2; Y, Y_0) + D^{(i)}_{11}(\bar{x}_1, \bar{x}_2; Y, Y_0) \right\}. \]

The first term \( D^{(0)} \) corresponds to the zero-gluon or qgqq recombination contribution, and the next four terms are respectively gqqq, ggqq, gggq and gggg contributions to two-meson production. We left out the subscripts I and II in the following expression to indicate that they are identical in form for both \( D_1 \) and \( D_{11} \).

\[ D^{(0)}(\bar{x}_1, \bar{x}_2; Y, Y_0) \]  

\[ = \int D_{q_1q_2q_3q_4} J(x_1, x_2, x_3, x_4; Y, Y_0) \]

\[ \times \frac{M_1 M_2}{q_1 q_2 q_3 q_4} (x_1, x_2, x_3, x_4; \bar{x}_1, \bar{x}_2) \sum_{\eta=1}^{4} d\eta, \]

\[ D^{(1)}(\bar{x}_1, \bar{x}_2; Y, Y_0) \]

\[ = \int D_{q_1q_2q_3q_4} J(x_1, x_2, x_3, x_4; Y, Y_0) \]
\[
\begin{align*}
\mathcal{D}^{(2)}(\overline{x}_1, \overline{x}_2; Y, Y_0) \\
= \int D_{g_1}g_2g_3g_4 \, i(x_1, x_2, x_3, x_4; Y, Y_0) \\
\times R_{q_1q_2q_3q_4}(\xi_1, \xi_2, \xi_3, \xi_4; \overline{x}_1, \overline{x}_2) \prod_{n'=1}^{2} \overline{g}_{n'}q_{n'} \bar{q}_{n'}(z_{n'}) \\
\times \delta(\xi_1 - z_1, \xi_2) \, d\xi_1\, dz_1 \prod_{n=1}^{4} d\eta_{n} + (1 \leftrightarrow 2) + (1 \leftrightarrow 3) \\
+ (1 \leftrightarrow 4),
\end{align*}
\]
Next, we would like to consider an alternative form of the recombination function for crystalizing four partons to two observed mesons (see Appendix B for a general approach to the recombination of two partons to single meson). In any given experiment involving measurements on two hadrons, one might measure the momentum distribution of two hadrons separately or treat them as a single system in a hadron jet. With the lack of a theoretical description for the hadronization mechanism in the non-perturbative regime, we may write the function \( R \) in Eq. (3.25) either in terms of
two-parton functions used in inclusive single meson production as by JLSW such that

\[
R_{a_1a_2a_3a_4}(x_1, x_2, x_3, x_4; \bar{x}_1, \bar{x}_2) = \sum_{\text{Perm}(i,j,k,l)} R_{a_ia_j}(x_i, x_j; \bar{x}_1) R_{a_ka_l}(x_k, x_l; \bar{x}_2),
\]

or in terms of a four-parton function, given as

\[
R_{a_1a_2a_3a_4}(x_1, x_2, x_3, x_4; x) = R'(\frac{x_1x_2x_3x_4}{x^4}) \delta(x_1 + x_2 + x_3 + x_4 - x),
\]

where \(x_r\) is the fraction of the momentum of parton \(i\).

The function \(R^M_{ab}\) in Eq. (3.37)

\[
R^M_{a_1a_j}(x_i, x_j; \bar{x}) = R(\frac{x_i x_j}{x^2}) \delta(x_i + x_j - \bar{x})
\]

is symmetric and linear in \(x_i\) and \(x_j\) as expected from
justification given by Migneron et al. (45) and Van Hove's (48) arguments based on rate considerations; we refer the reader to the literature (42,47,49,50) for further details.

Eq. (3.38) represents a recombination of four partons to a cluster of two mesons with a total longitudinal momentum fraction $\bar{x}$ and some undetected particles. This choice of recombination provides us with a less sensitive way to describe the correlation of two mesons and hence, we decide to choose the first one in our QCD calculation, though both could be justifiably used from the phenomenological point of view.

For notational convenience, we again define the following quantities

$$O_1(\alpha, \beta, \gamma, \delta, \mu, \nu, \rho; \{y\}, Y_0)$$

$$= K_1^{\alpha} (Y, Y_0) K_2^{\beta} (y', Y_0) K_3^{\gamma} (y'', Y_0) K_4^{\delta} (y'', Y_0)$$

$$\times P_{c_1 e_1}^{y, \delta} K_{c_2 e_2}^{\mu} (y''; y') P_{b_1 b_2}^{\gamma, \mu} K_{b_1 b_2}^{\nu} (Y, y') P_{b_1 b_2}^{\alpha, \nu} K_{ji}^{\rho} (Y, y);$$

$$O_{II}(\alpha, \beta, \gamma, \delta, \mu, \nu, \rho; \{y\}, Y_0)$$
\[ L^1(\alpha, \beta, \gamma, \delta) = A^\alpha_{q_1g_1} + A^\beta_{q_2g_2} + A^\gamma_{q_3g_3} + A^\delta_{q_4g_4}, \quad (3.42) \]

\[ L^2(\alpha, \beta, \gamma, \delta) = A^\alpha_{q_1g_1}A^\beta_{q_2g_2} + A^\alpha_{q_1g_1}A^\gamma_{q_3g_3} + A^\alpha_{q_1g_1}A^\delta_{q_4g_4} + A^\beta_{q_2g_2}A^\gamma_{q_3g_3} + A^\beta_{q_2g_2}A^\delta_{q_4g_4}, \quad (3.43) \]

\[ L^3(\alpha, \beta, \gamma, \delta) = A^\alpha_{q_1g_1}A^\beta_{q_2g_2}A^\gamma_{q_3g_3} + A^\alpha_{q_1g_1}A^\beta_{q_2g_2}A^\delta_{q_4g_4} + A^\alpha_{q_1g_1}A^\gamma_{q_3g_3}A^\delta_{q_4g_4} + A^\beta_{q_2g_2}A^\gamma_{q_3g_3}A^\delta_{q_4g_4}, \quad (3.44) \]

and

\[ L^4(\alpha, \beta, \gamma, \delta) = A^\alpha_{q_1g_1}A^\beta_{q_2g_2}A^\gamma_{q_3g_3}A^\delta_{q_4g_4}. \quad (3.45) \]

It is straightforward, but exceedingly time consuming,
to do the numerical calculations in moment space where the double moment transformation is defined as

\[ D_i \rightarrow M_{12}(n, m; Y, Y_0) \] (3.46)

\[ = \int d\bar{x}_1 d\bar{x}_2 \bar{x}_1^n \bar{x}_2^m \theta(1 - \bar{x}_1 - \bar{x}_2) D_i \rightarrow M_{12}(\bar{x}_1, \bar{x}_2; Y, Y_0). \]

Then, we have

\[ D_i \rightarrow M_{12}(n, m; Y, Y_0) \]

\[ = \int d\bar{x}_1 d\bar{x}_2 \bar{x}_1^n \bar{x}_2^m \theta(1 - \bar{x}_1 - \bar{x}_2) \left\{ D_i(\bar{x}_1, \bar{x}_2; Y, Y_0) \right\} \]

\[ + D_{II}(\bar{x}_1, \bar{x}_2; Y, Y_0) \}

\[ = \sum_{i=0}^{4} \left\{ D_i^{(i)}(n, m; Y, Y_0) + D_{II}^{(i)}(n, m; Y, Y_0) \right\} \]

\[ = \sum_{i=0}^{4} \sum_{k=1}^{3} R_1 R_2 \sum_{\alpha} \sum_{\beta} \left( \begin{array}{c} n-2 \\ p \end{array} \right) \left( \begin{array}{c} m-2 \\ q \end{array} \right) \]

\[ \times \left\{ \int_{Y_0}^{Y} dy \int_{Y_0}^{y} dy' \int_{Y_0}^{y'} dy'' E_i^{i,k}(\{n'; \{y\}, Y_0) \right\} \]

\[ + \int_{Y_0}^{Y} dy \int_{Y_0}^{y} dy' \int_{Y_0}^{y} dy'' E_{II}^{i,k}(\{n'; \{y\}, Y_0) \right\}. \] (3.47)
Here, \( \{n'\} \) stands collectively for \( n, m, p, \) and \( q; \) \( \{y\} \) stands for \( y, y', \) and \( y''; \) \( \Omega \) denotes the labels of all the intermediate parton lines in the jet diagram. The coefficients \( R_1 \) and \( R_2 \) are the normalization factors which are determined by the experiment. The functions \( E \) are explicitly given in Appendix C.

B. Results and Discussion

1. Baryon production in a gluon jet

With the known moments of the KUV propagators, \( K_{ab} \), and QCD splitting functions, \( \hat{P} \) (see Appendix A), we are ready to calculate the inclusive baryon spectrum from the moments of the gluon fragmentation function \( D_{g\rightarrow B}(n; Y, Y_0) \). We adopt Yndurain's method (51) or the generalized Bernstein polynomial method given by Willen (52) for inversion from moment \( (n) \) space to \( x \) space. Hence, the single moment inversion is given as

\[
D(x_{N,M}) = \frac{(n+1)}{M!} \sum_{r=0}^{N-M} \frac{(-1)^r}{r!(N-M-r)!} D(M+r)
\]

with \( x_{N,M} = (M+1)/(N+2) \), for \( 0 < M < N \).

The values of the parameters \( Q_0^2 \) and \( A \) are used and determined in the same manner as in MJL; \( Q_0^2 \) is the value of...
\( Q^2 \) at the end of the evolution when the quarks crystalize into hadrons. This should be large enough so that the QCD coupling constant \( \alpha_s(Q_0^2) \) is reasonably small to allow perturbation ideas to still work; MJL concluded that \( Q_0^2 \) should lie between 1 and 5 GeV\(^2\)/c\(^2\). One may note that, few years ago, the measurement of the QCD scale parameter \( \Lambda \) in the deep inelastic leptonucleon scattering was found within the range 500 and 700 MeV/c (53,54). However, this scale parameter has significantly shifted to a lower range between 50 and 300 MeV/c, due to improvement in experimental accuracy and theoretical understanding (5,6,55,56,57). Representative values in these ranges will be used in our calculation of single- and two-particle distributions in parton jets.

In Fig. 3.5, we show explicitly the different contributions of the single-baryon fragmentation function \( D_{1-B}(x; Y, Y_0) \) in the sense of Eq. (3.7) (i.e., the qqq, gqq, gqq and ggg terms) that contribute to a gluon jet. These same terms are shown for the u-quark jet for comparison purposes, again at \( Q^2 = 33^2 \) GeV\(^2\)/c\(^2\). The major differences between the two cases are seen to come from the \( a_1^2 a_2 a_3 = gqq \) and ggg contributions. In the gluon jet, the contributions of each term are directly related to the number of gluons in the final state, such that \( D^{(0)} < D^{(1)} < D^{(2)} < D^{(3)} \). Therefore, the dominant contribution to \( D_{1-B} \) comes from the ggg term for the entire kinematical region. However, this
Fig. 3.5  Fragmentation functions $D$ for (a) $u$ quark $\rightarrow$ proton and (b) gluon $\rightarrow$ proton versus momentum fraction $x$ showing the partial contributions from three gluon (ggg), two gluon plus quark (gg), one gluon plus two quark ($g$) and zero gluon (qqq) terms at $Q^2 = 33^2 \text{ GeV}^2/c^2$, $Q^2_0 = 5 \text{ GeV}^2/c^2$ and $\Lambda = 50 \text{ MeV}/c$. The term with zero gluon (qqq) at the end of gluon-jet evolution $Q^2_0$ is not shown because it is very small.
\( K = 5 \text{ GeV}^2 \)
\( \Lambda = 0.05 \text{ GeV} \)

\( D(x) \)

\( q^2 = 33^2 \text{ GeV}^2 \)
\( Q_0^2 = 5 \text{ GeV}^2 \)
\( \Lambda = 0.05 \text{ GeV} \)
pattern is not the same in the quark jet. The slope of the ggg term in the quark jet falls off faster than the other terms as $x \to 1$, which contrasts with the gluon jet where all terms have roughly the same slope. In Fig. 3.5(b), the ggq term is smaller than the ggg term by a factor between 5 and 6 at all $x$, whereas in Fig. 3.5(a) the ggq term gives the dominant contribution at large $x$. This dominance at large $x$ illustrates a phenomenon known as the leading-quark effect which will be discussed shortly. Here, we note that the direct evolution in the quark and gluon jets of three quarks, i.e. $a_1 a_2 a_3 = q_1 q_2 q_3$, makes a very small contribution in baryon production.

Recently, the leading-particle effect has been studied in some detail in hadronic physics, by M. Basile et al. (58). They have defined a quantity to measure the leading-hadron effect as

$$L = \frac{\int_{x_1=0.4}^{x_2=0.8} H(x) \, dx}{\int_{x_1=0.4}^{x_0=0.2} H(x) \, dx},$$

(3.49)

where $H(x)$ is the inclusive single-particle cross section, and the ranges of $x_1$ are chosen to minimize the effect of the central production and of diffractive production. They show that the leading effect present in hadron-hadron and lepton-hadron reactions has the value $L \gtrsim 0.5$ and in $e^+e^- \to$
hadrons without such leading effect has $L < 0.5$. From their analysis, they conclude that the leading-particle phenomenon is correlated with definite quantum numbers, such as spin and flavor, which are transferred from the initial state hadron to the final state hadron via the constituents of the initial hadron.

Similarly to the hadronic physics example (57), our study of parton fragmentation shows a corresponding leading-particle effect in jet calculus (QCD) theory, where the distribution of charged particles falls off toward $x = 1$ faster in gluon jets than in quark jets. A simple scaling form of the gluon fragmentation function which has the predicted $x \to 0$ and $x \to 1$ limiting behavior is given by Brodsky (59),

$$D_G \to \text{hadron}(x) \propto \sum_i (1 - x) D_i \to \text{hadron}(x),$$

where $i$ is summed over all quarks and antiquarks.

In Fig. 3.6, we present our results for $D_{u\to p}$ and $D_{g\to p}$ for $u$-quark and gluon fragmentation into protons, both as functions of longitudinal momentum fraction $x$ and moment number $n$. We note that moments are given for $n$ values between 3 and 11 (60). This limited range in $n$ resulting strictly from machine limitations leads directly to the restricted range in $x$ as calculated via Eq. (3.48), namely,
Fig. 3.6 Comparisons of (a) fragmentation functions $D$ for $u$ quark → proton and gluon → proton versus momentum fraction $x$ and (b) moments of the fragmentation functions $D$ versus moment number $n$ at $q^2 = 33^2 GeV^2/c^2$, $Q_0^2 = 5 GeV^2/c^2$ and $\Lambda = 50$ MeV/c
\[ \begin{align*} \text{(a)} & \quad (0) \\ \text{(b)} & \quad Q^2 = 33^2 \text{ GeV}^2 \\
\quad Q^2 & = 5 \text{ GeV}^2 \\
\quad \Lambda & = 0.05 \text{ GeV} \end{align*} \]
\[ x_{N,M} = \frac{(M+1)}{(N+2)}, \] displayed in our figures. The moments for gluon fragmentation fall off faster than those for u-quark fragmentation reflecting the fact that it is more difficult for a gluon to materialize as a proton at large \(x\) than it is for a u-quark which can be a leading quark. This large-\(x\) behavior follows entirely from the dominance of the \(ggq\) term in Eq. (3.7). We see in Fig. 3.6 that there is a sharp peak for both u-quark and gluon jets at small \(x\), which is the same phenomenon observed in either quark or gluon fragmenting to a single pion due to the soft gluon bremsstrahlung and lack of heavy quark masses (42). The analogous curves for \(u \rightarrow \Lambda\) and \(g \rightarrow \Lambda\) are shown in Fig. 3.7; the general trends are similar to those in Fig. 3.6 for proton production. The differences follow from the fact that the initial u quark has two possible ways to be a leading quark in proton production.

What would be the consequence if one turns off SU(3) symmetry in the gluon jet cascade? In the previous chapter, we used a measurement of SU(3) symmetry violation in deep inelastic scattering from Ammosov et al. (34). They have measured the relative size \(\lambda\) of the strangeness suppression in pair creation in quark fragmentation to be 0.27. This suppression factor \(\lambda\) is about the mean of values used in the Field and Feynman cascade model (9) and those measured in the electroproduction experiment (61). Malhotra and Orava have
Fig. 3.7 Comparisons of (a) fragmentation functions $D$ for $u$ quark $\rightarrow$ lambda hyperon and gluon $\rightarrow$ lambda hyperon versus momentum fraction $x$ and (b) moments of the fragmentation functions $D$ versus moment number $n$ at $Q^2 = 33^2/c^2$, $Q_0^2 = 2 \text{GeV}^2/c^2$ and 200 MeV/c
made a complete analysis of the experimental data including heavy particle decays finding an average strangeness suppression factor \( \langle \lambda \rangle = 0.29 \) (62). However, a moderately weak suppression in the rate of \( ss \) pair creation from the vacuum has been reported by ABCDHW Collaboration (63). This SU(3) symmetry violation is presumably due to the mass difference of the strange s and non-strange u(d) quarks; the mass effect impedes the tunneling of the s quark from the vacuum relative to u or d quarks.

In this chapter, we consider an equal rate of \( uu, dd \) and \( ss \) pair creations and neglect other flavors which are beyond the strange-quark threshold. One may recall that the KUV formulation of jet calculus (28), which even with improvement (64), has, for calculational simplicity, been symmetric in flavors. Therefore, strangeness suppression has not been incorporated and our calculations are implicitly symmetric between u, d and s quarks. The effect of mass in strange baryon production relative to the non-strange baryon production in quark jets has been noted by MJL. A sharper peak for \( D_{u\rightarrow\Lambda} \) than \( D_{u\rightarrow p} \) at \( x \rightarrow 0 \) and similar predictions at \( x \rightarrow 1 \) are found. Furthermore, with SU(3) symmetry lambda hyperon (uds) production is higher than the proton (uud) production by a factor between 4/3 and 2 over the \( x \) range in electron-positron annihilation.

We have extended the QCD calculation for \( \Lambda \) hyperon and
proton production to the case of a gluon jet. The results are shown in Fig. 3.8 for the parameters $Q^2_0 = 5 \text{ GeV}^2/c^2$ and $\Lambda = 50 \text{ MeV}/c$. Without a mass suppression factor $\Lambda$ hyperons are more copious than protons in a gluon jet. The factor of two between the fragmentation of the gluon into $\Lambda$ and $p$ is due to SU(3) flavor symmetry. In practical terms, the number of possible ways to pick out uds sets of quarks from the dominant ggg term in Eq. (3.7) is twice that for picking out uud sets of quarks. This is the same factor found by MJL for relative $\Lambda$ and $p$ baryon production in quark jets at small momentum fraction $x$ where the ggg term dominates; however, in gluon jets, the factor of two is independent of value $x$. The constant enhancement in gluon jets follows from two facts: (i) the same irreducible diagrams in Fig. 3.3 apply to both lambda hyperon and proton productions; and (ii) the ggg term provides the dominant contribution over the entire $x$ region. This counting enhancement compensates partly for the strange-quark mass suppression effect and experiments on gluon jets should consequently find that protons are not more than a factor of two more abundant than lambda hyperons.

We close this section with some graphs illustrating the sensitivity of our calculation to the values of $Q^2_0$ and $\Lambda$. In Fig. 3.9(a), we show moments of the gluon fragmentation into protons at $Q^2 = 900 \text{ GeV}^2/c^2$ with $\Lambda = 50 \text{ MeV}/c$ for $Q^2_0$ values of 1 and 5 $\text{ GeV}^2/c^2$. As $n$ decreases the curve for
Fig. 3.8 Comparison between momentum fraction $x$ times fragmentation functions $D$ for gluon $\rightarrow$ lambda hyperon and gluon $\rightarrow$ proton at $q_0^2 = 5$ GeV$^2$/c$^2$, $Q^2 = 33$ GeV$^2$/c$^2$ and $\Lambda = 50$ MeV/c.
$Q^2 = 33^2 \text{ GeV}^2$
$Q_0^2 = 5 \text{ GeV}^2$
$\Lambda = 0.05 \text{ GeV}$
Fig. 3.9  Graphs (a) showing the $Q_0^2$ sensitivity of the moments of the fragmentation functions $D$ for gluon $\rightarrow$ proton at $Q^2 = 30^2$ GeV$^2$/c$^2$ and $\Lambda = 50$ MeV/c; for case I, $Q_0^2 = 5$ GeV$^2$/c$^2$ and for case II, $Q_0^2 = 1$ GeV$^2$/c$^2$, and (b) showing the sensitivity of the moments of the fragmentation functions $D$ for gluon $\rightarrow$ $\Lambda$ hyperon to the parameters $Q_0^2$ and $\Lambda$. The curves are calculated for $Q_0^2 = 5$ GeV$^2$/c$^2$ and $\Lambda = 50$ MeV/c and 200 MeV/c, respectively, the dashed curve corresponding to case II. Case III is calculated for $Q_0^2 = 1.5$ GeV$^2$/c$^2$ and $\Lambda = 100$ MeV/c, producing a curve very close to II.
\[ D(n) \]

(a) $Q = 30 \text{ GeV}^2$

$Q_0^2 = 5 \text{ GeV}^2$

$\Lambda = 0.05 \text{ GeV}$

$g \rightarrow p$

(b) $Q^2 = 33^2 \text{ GeV}^2$

$Q_0^2 = 5 \text{ GeV}^2$

$\Lambda = 0.05 \text{ GeV}$

$g \rightarrow \Lambda$
$Q_0^2 = 1 \text{ GeV}^2/c^2$ increases faster than that for $Q_0^2 = 5 \text{ GeV}^2/c^2$. This reflects the fact that the evolution occurs over a longer range of $y = \ln \ln Q^2$ for $Q_0^2 = 1 \text{ GeV}^2/c^2$. It is probably of more interest that the two curves are not dramatically different. The effect of varying $\Lambda$ is illustrated in Fig. 3.9(b), where curves are plotted for $Q^2 = 33^2 \text{ GeV}^2/c^2$. Cases I and II both have $Q_0^2 = 5 \text{ GeV}^2/c^2$ with $\Lambda$ spanning the "accepted" range 50 to 200 MeV/c (56), respectively. The larger scale value $\Lambda = 200 \text{ MeV/c}$ gives larger results for small moments. The curve labelled III with $Q_0^2 = 1.5 \text{ GeV}^2/c^2$ and $\Lambda = 100 \text{ MeV/c}$ is included as an example to show that $Q_0^2$ and $\Lambda$ compensate considerably; the curves for cases II and III are nearly identical.

2. Meson production in quark and gluon jets

Based on the QCD leading-logarithmic approximation, KUV jet calculus predicts the n-parton fragmentation function, or probability for finding a final states of n partons with off-shell mass $Q_0^2$, as branching products from an initial parton with off-shell mass $Q^2$. Unlike the case of single hadron production in quark jets, for $n = 4$, the branching process $F_{1 \rightarrow a_1 a_2 a_3 a_4}$ in Eq. (3.26) takes place in two different configurations as shown in Fig. 3.4. The appearance of two possible configurations indicates the complexity resulting when attempting to study final states with increasing numbers.
of partons. Therefore, calculations involving higher multiplicity may provide insight into the nature of strong interaction.

In our calculation of two-meson production in parton jets, the purely non-perturbative effect is contained in $M_1 M_2$ of Eq. (3.25). We should point out that at the present time, the dynamical picture of hadronization cannot be described without any model dependence. Hence, we would like to follow the MJL approach by incorporating the wavefunction form-factor information (65) into the KUV jet calculus to recombine quarks and antiquarks to be able to predict two-particle fragmentation functions. A study of the relative importance of different hadronization models will be present in Section III.3.

In order to study the two-meson fragmentation function in longitudinal momentum space, we apply the generalized Bernstein polynomial method (52) to invert the double moment fragmentation function $D_{1-M_1 M_2} (n,m; Y, Y_0)$ of Eq. (3.47):

$$
\hat{D}(x_N, k_1, x_N, k_2) = \frac{(N+2)!}{k_1! k_2!} \sum_{m=0}^{N-k_1-k_2} \frac{(-1)^m}{(N-k_1-k_2-m)!} \sum_{n=0}^{m} \frac{D(k_1+m-n, k_2+n)}{n!(m-n)!}
$$

(3.50)

and $x_{N, k_i} = (k_i + 1)/(N + 3)$, for $0 \leq k_i \leq N$ and $0 \leq k_1 + k_2 \leq N$. 


In Fig. 3.10, we see the two-pion fragmentation function $x_1 x_2 x D(x_1, x_2; Y, Y_0)/R$ at our input values of the parameters $Q^2 = 3.2$ GeV$^2$/c$^2$, $Q_0^2 = 1.0$ GeV$^2$/c$^2$ and $L = 0.3$ GeV/c, plotted vs $x_2$ by choosing the value $x_1 = 0.176$. Notice that the QCD scale parameter $\Lambda$ is labelled as L in our figures and $R$ denotes the product of the recombination parameters $R_1$ and $R_2$. We also show five components of $D_{u\rightarrow \pi^+ \pi^-}$ of Eq. (3.31), which correspond to five distinct parton states generated by the KUV jet calculus with four-parton combination into two pions, namely, $qqqq$, $gqqq$, $ggqq$, $gggq$ and $gggg$ in Fig. 3.10(a) and Fig. 3.10(b). It is clear that the curves of $D^{(0)}$ and $D^{(3)}$ given in Eqs. (3.32) and (3.35) remain as the minimum and maximum contributions, respectively, in both figures, and form an envelope which contains the distributions of $D^{(1)}$, $D^{(2)}$ and $D^{(4)}$. One can see that the relative distributions of overall ten components of two-pion production in u-quark jet show more complicated than that of four components of single-proton production in u-quark jet, (see Fig. 3.5).

In Fig. 3.11, we show a plot of $x_1 x_2 x D(x_1, x_2; Y, Y_0)/R$ vs $x_2$ at same input values. We can see that the curve of fragmentation function $D_{\Pi}$ is relatively higher than that of fragmentation function $D_{\Pi}$ in a wide range of $x_2$, which indicates that the dominated branching process in u-quark jets is due to the second configuration shown in Fig. 3.4.
Fig. 3.10(a) Fragmentation function $D$ for $u$ quark $\to \pi^+\pi^-$
times momentum fractions $x_1x_2$ and divided by
recombination parameter $R = R_1R_2$ versus momentum
fraction $x_2$ showing the partial contributions from
four gluon ($gggg$), three gluon plus quark ($3glq$),
two gluon plus two quark ($2g2q$), one gluon plus
three quark ($lg3q$) and zero gluon ($qqqq$) terms at
$q^2 = 33^2 \text{ GeV}^2/c^2$, $q_0^2 = 1 \text{ GeV}^2/c^2$, and $\Lambda = 0.2$
GeV/c with momentum fraction $x_1 = 0.176$ for
configuration I. (b) Same as (a) except for
configuration II.
\[ B = 33.0 \text{ GEY} \]
\[ QD = 1.0 \text{ GEY} \]
\[ L = 0.3 \text{ GEY} \]
\[ x_1 = 0.176 \]

\[ \text{U} \rightarrow \text{p} + \text{p}^- \]

\[ \text{TOTAL} - 1 \]

\[ 0.10 \quad 0.20 \quad 0.30 \quad 0.40 \quad 0.50 \quad 0.60 \quad 0.70 \quad 0.80 \]

\[ x_2 \]

\[ x_1 x_2 \text{ mod } (x_1, x_2) / R \]
Fig. 3.10(b) Fragmentation function for configuration II.
Fig. 3.11 Comparison between momentum fractions $x_1x_2$ time fragmentation functions $D$ and divided by recombination parameter $R = R_1R_2$ versus momentum fraction $x_2$ for configuration I and configuration II at $Q^2 = 33^2 \text{ GeV}^2/c^2$, $Q_0^2 = 1 \text{ GeV}^2/c^2$, and $\Lambda = 0.3 \text{ GeV}/c$ with momentum fraction $x_1 = 0.176$. 
Q = 33.0 GEV
Q0 = 1.0 GEV
L = 0.3 GEV
X1 = 0.176

U → π⁺π⁻
TOTAL - I+II
TOTAL - I
TOTAL - II
Even though in our model the KUV jet calculus predicts parton evolution to two mesons via two types of configurations, it would be difficult to measure these two processes independently due to such a wide range of $x_2$-distribution of fragmentation function $D_{II}$.

From Figs. 3.12(a)-3.12(c), we show the leading-quark effect in our calculations of $\pi^+(u\bar{d})\pi^-(d\bar{u})$ fragmentation functions $D_{i\rightarrow\pi^+\pi^-}(x_1,x_2,Y,Y_0)$ at $Q^2 = 33^2$ GeV$^2$/c$^2$. The contribution of $1q3g$ component appears to be dominate in the u-quark and d-quark jets, as shown in Fig. 3.12(a) and 3.12(b), respectively. This leading effect can be understood in terms of the flow of quantum number from the original quark to final state hadron within the quark jet. It is consistent with the MJL prediction of leading-quark effect for single baryon production in quark jets. In Fig. 3.12(c), we see that the two-pion fragmentation function $D_{s\rightarrow\pi^+\pi^-}(x_1,x_2,Y,Y_0)$ receives substantial contribution from the component of four-gluon state. In this fragmentation of the s quark, $\pi^+\pi^-$ production requires the recombination of four sea quarks generated by the gluon bremsstrahlung; whereas in the u- or d-quark fragmentation, $\pi^+\pi^-$ production requires the recombination of one valence quark, which is favored in the KUV jet calculus formalism, and three sea quarks. Because of the suppression of quark flavor changing in the KUV parton propagators $K_{ij}$, we expect the $1q3g$
Fig. 3.12(a) Fragmentation function $D$ for $u$ quark $\rightarrow \pi^+\pi^-$ times momentum fractions $x_1x_2$ and divided by recombination parameter $R = R_1R_2$ versus momentum fraction $x_2$ showing the partial contributions from four gluon (gggg), three gluon plus quark (3glq), two gluon plus two quark (2g2q), one gluon plus three quark (lg3q) and zero gluon (qqqq) terms at $Q^2 = 33^2$ GeV$^2$/c$^2$, $Q_0^2 = 1$ GeV$^2$/c$^2$, and $\Lambda = 0.3$ GeV/c with momentum fraction $x_1 = 0.176$. (b) Same as (a) except for $d$ quark $\rightarrow \pi^+\pi^-$. (c) Same as (a) except for $s$ quark $\rightarrow \pi^+\pi^-$. 
Fig. 3.12(b) Fragmentation function for d quark → π⁺π⁻.
Fig. 3.12(c) Fragmentation function for s quark → $\pi^+\pi^-$. 
contribution to be less dominate in s-quark jet. An alternative form of recombination function $R(x_1, x_2, x_3, x_4, \bar{x})$ given in Eq. (3.38) for measuring a cluster of two mesons in a jet should not change our prediction of leading-quark effect for $q \to \pi^+\pi^-$ production. In the SU(3) flavor symmetry, our model predicts identical distributions of $\pi^+\pi^-$ production in u-quark and d-quark jets, and similarly in d-quark and û-quark jets, s-quark and s-quark jets. From Figs. 3.13(a) through 3.13(c), we see that a similar leading-quark effect arises for $q \to \pi^+k^-$ production, in which "favored" two-particle production contains the quantum number of the original quark, for instance, $u$ and $s \to \pi^+(u\bar{d})k^-(s\bar{u})$; whereas the leading effect is suppressed for $d \to \pi^+(u\bar{d})k^-(s\bar{u})$ which becomes "unfavored" two-particle production in quark jet.

In Fig. 3.12(c), our calculation shows that the distribution of the $D^{(4)}$ component of s-quark fragmentation function $D_{s \to \pi^+\pi^-}(x_1, x_2, Y, Y_0)$ given in Eq. (3.36) is overwhelmingly above the other four components, $D^{(0)}$, $D^{(1)}$, $D^{(2)}$ and $D^{(3)}$ in an order of magnitude 10. However, we would like to show that a much stronger enhancement of the $D^{(4)}$ component can be seen in gluon jet fragmentation $D_{g \to \pi^+\pi^-}(x_1, x_2, Y, Y_0)$, as shown in Fig. 3.14. According to the non-Abelian QCD prediction, the probability for a gluon to emit another gluon is about 9/4 times the probability for a
Fig. 3.13(a) Fragmentation function $D$ for $u$ quark $\rightarrow \pi^+ k^-$ times momentum fractions $x_1 x_2$ and divided by recombination parameter $R = R_1 R_2$ versus momentum fraction $x_2$ showing the partial contributions from four gluon ($g g g g$), three gluon plus quark ($3 g l q$), two gluon plus two quark ($2 g 2 q$), one gluon plus three quark ($l g 3 q$) and zero gluon ($q q q q$) terms at $Q^2 = 33^2$ GeV$^2$/c$^2$, $Q_0^2 = 1$ GeV$^2$/c$^2$, and $\Lambda = 0.3$ GeV/c with momentum fraction $x_1 = 0.176$. (b) Same as (a) except for $d$ quark $\rightarrow \pi^+ k^-$. (c) Same as (a) except for $s$ quark $\rightarrow \pi^+ k^-$. 
$Q = 39.0 \text{ GEV}$

$Q_0 = 1.0 \text{ GEV}$

$L = 0.3 \text{ GEV}$

$X_1 = 0.176$

$U \rightarrow P + K^-$

$TOTAL - I+I$

$GGGG$

$1835$

$2426$

$3016$

$QQQ Q$

$0.10 \quad 0.20 \quad 0.30 \quad 0.40 \quad 0.50 \quad 0.60 \quad 0.70 \quad 0.80$
Fig. 3.13(b) Fragmentation function for d quark → π⁺k⁻.
Fig. 3.13(c) Fragmentation function for s quark → π⁺K⁻.
Fig. 3.14 Fragmentation function $D$ for gluon $\rightarrow \pi^+\pi^-$ times momentum fractions $x_1x_2$ and divided by recombination parameter $R = R_1R_2$ versus momentum fraction $x_2$ showing the partial contributions from four gluon (gggg), three gluon plus quark (3glq), two gluon plus two quark (2g2q), one gluon plus three quark (1g3q) and zero gluon (qqqq) terms at $Q^2 = 33^2$ GeV$^2$/c$^2$, $Q_0^2 = 1$ GeV$^2$/c$^2$, and $\Lambda = 0.3$ GeV/c with momentum fraction $x_1 = 0.176$. 
$Q = 33.0 \text{ GEV}$

$Q_0 = 1.0 \text{ GEV}$

$L = 0.3 \text{ GEV}$

$n = 0.176$

$G \rightarrow \pi^+\pi^-$

TOTAL - I+II

$GGG$

$1835$

$2025$

$3015$

$3333$
quark to emit a gluon. It is clearly due to the fact that color charge of the SU(3) octet is 3/2 times that of the SU(3) triplet. Therefore, one may consider the suppression of the leading-quark effect in gluon or "unfavored" quark jet fragmentation as a result of the domination of three-gluon coupling $g \rightarrow gg$. A similar phenomenon has been given in our previous discussion of single baryon production in gluon jet (see Fig. 3.5).

We plot in Fig. 3.15 our results of two-pion production in u-quark, d-quark, s-quark and gluon jets vs $x_2$ at the same input values of the parameters $Q^2$, $Q_0^2$ and $\Lambda$. By using the isospin invariant property, one may qualitatively predict that the distributions of $u \rightarrow \pi^+\pi^-$ and $d \rightarrow \pi^+\pi^-$ are most likely to be the same everywhere in the $x$ space, and it exactly agrees with our model prediction quantitatively as shown in the above figure. We see that the behavior of the distribution of $\pi^+\pi^-$ production in gluon jet is softer as $x_2 \rightarrow 1$ and steeper as $x_2 \rightarrow 0$ than that in u-quark or d-quark jet, which is expected from the QCD prediction. This is analogous to the discussion of the behaviors of fragmentation $D_{g\rightarrow p}$ and $D_{u\rightarrow p}$ as shown in Fig. 3.6. Finally, we show that the contribution of "unfavored" $\pi^+\pi^-$ production in s-quark jet is comparably lower in Fig. 3.15.

In Fig. 3.16, we show the strangeness suppression effect in terms of $\pi^+\pi^-$ and $\pi^+k^-$ productions in singlet quark jet
Fig. 3.15 Comparison between momentum fractions $x_1x_2$ time fragmentation functions $D$ and divided by recombination parameter $R = R_1R_2$ versus momentum fraction $x_2$ for $u$ quark $\rightarrow \pi^+\pi^-$, $d$ quark $\rightarrow \pi^+\pi^-$, $s$ quark $\rightarrow \pi^+\pi^-$ and gluon $\rightarrow \pi^+\pi^-$ at $Q^2 = 33^2 \text{ GeV}^2/c^2$, $Q_0^2 = 1 \text{ GeV}^2/c^2$, and $\Lambda = 0.3 \text{ GeV}/c$ with momentum fraction $x_1 = 0.176$. 
Fig. 3.16  Comparison between momentum fractions $x_1x_2$ time
fragmentation functions $D$ and divided by
recombination parameter $R = R_1R_2$ versus momentum
fraction $x_2$ for $u$ quark $\rightarrow \pi^+\pi^-$, $u$ quark $\rightarrow \pi^+k^-$,
gluon $\rightarrow \pi^+\pi^-$ and gluon $\rightarrow \pi^+k^-$ at $Q^2 = 33^2$ GeV$^2$/c$^2$,
$Q_0^2 = 1$ GeV$^2$/c$^2$, and $\Lambda = 0.3$ GeV/c with momentum
fraction $x_1 = 0.176$. 
\( Q = 33.0 \text{ GEV} \)
\( Q_0 = 1.0 \text{ GEV} \)
\( L = 0.3 \text{ GEV} \)
\( x_1 = 0.176 \)

TOTAL - 1+II
QUARK \( \rightarrow \) PI+PI-
QUARK \( \rightarrow \) PI+K-
GLUON \( \rightarrow \) PI+PI-
GLUON \( \rightarrow \) PI+K-

\( x_1 \times x_2 \times D(x_1, x_2) / A \)

\( x_2 \)
and in gluon jet. The singlet quark fragmentation functions are defined as

\[ D^{\text{SINGLET}} \rightarrow M_1 M_2 (x_1, x_2; Y, Y_0) = \sum_i D_i \rightarrow M_1 M_2 (x_1, x_2; Y, Y_0) \]

(3.51)

where the sum in Eq. (3.51) runs over all quarks and antiquarks in our jet calculus model.

We see that the curve of quark \( \rightarrow \pi^+\pi^- \) is equal to that of quark \( \rightarrow \pi^+k^- \) as \( x_2 \rightarrow 0 \) and is higher by a factor of 1.5 as \( x_2 \rightarrow 0.75 \). A quite different result, as compared with the curves plotted for gluon \( \rightarrow \pi^+\pi^- \) and gluon \( \rightarrow \pi^+k^- \), is also shown in Fig. 3.16. Naively, one may expect the \( x \rightarrow 0 \) limiting behavior to be the same for both \( \pi^+\pi^- \) and \( \pi^+k^- \) productions in singlet quark jet (or in gluon jet) due to soft gluon bremsstrahlung at small \( x \) region. Let us consider, for instance, Fig. 3.12(a), we see that among those five components which contribute to the two-meson fragmentation function introduced in Eq. (3.31), only the \( 1q3g \) and \( 2q2g \) components show some dominant features at the small \( x \). However, we can be convinced ourselves that the 4-gluon component is indeed the dominant contributor as \( x \rightarrow 0 \) by extrapolating the result of fragmentation function \( D^{(4)} (x_1, x_2; Y, Y_0) \) given in Eq. (3.36).

Even though with the assumption of SU(3) flavor independence of the propagator \( K_{ij} \) and QCD vertices \( \hat{P}_{a \rightarrow bc} \)
in the KUV jet calculus formalism, there exhibits a higher probability of producing $\pi^+\pi^-$ pairs than $\pi^+k^-$ pairs in singlet quark jet at large $x$. We expect this, because it is most probable to produce a final state quark-antiquark pair with antiflavors than with different flavors in gluon cascade, as shown in Fig. 3.17. In the u-quark jet, the $\pi^+\pi^-$-pair production is a recombination of a valence up-quark and three sea quark, namely down, anti-down and anti-up quarks, whereas the $\pi^+k^-$-pair production has, instead of down quark, a strange quark in its final state. This phenomenon is demonstrated by our jet-calculus calculation, such that in Fig. 3.12(a) we see the contribution to the $\pi^+\pi^-$ fragmentation function arising from the 1q3g and 3qlg components; however, in Fig. 3.13(a) we see the contribution to the $\pi^+k^-$ fragmentation function arising from the 1q3g and 2q2g components.

The two curves gluon $\rightarrow \pi^+\pi^-$ and gluon $\rightarrow \pi^+k^-$ in Fig. 3.16 predicted from our calculations are identical, because (i) the 4-gluon component is a dominant contribution in both cases and (ii) the numbers of ways to assign four quarks to the gluons for $\pi^+\pi^-$ and $\pi^+k^-$ pairs are equal. However, if the strangeness suppression is taken into account, one may expect our predictions of the $\pi^+k^-$ production in quark and gluon jets to be small roughly by a suppression factor $\lambda = 0.29$ relative to the $\pi^+\pi^-$ production.
Fig. 3.17 Diagrams depicting the contributions of gluon cascade into (a) $q\bar{q}$ final parton state and (b) $qq'$ final parton state to the evolution equation for two particle fragmentation functions.
The inclusive two-particle cross section in high energy electron-positron annihilation is given as

\[
\frac{1}{\sigma_{\text{TOT}}} \frac{d\sigma}{dx_1 dx_2} \bigg|_{h_1 h_2} = \frac{3}{2} \sum_i e_i^2 D_i \to h_1 h_2 (x_1, x_2; Q^2, Q_0^2), \quad (3.52)
\]

where \( e_i^2 \) is the square of quark charges and the sum runs over all quark flavors. \( \sigma_{\text{TOT}} \) is the total cross section defined as

\[
\sigma_{\text{TOT}} = \frac{6}{9} \left( \frac{4\pi \alpha^2}{s} \right). \quad (3.53)
\]

By using charge conjugation and isospin invariance, one can easily obtain the following results,

(i) for \( h_1 = \pi^+ \) and \( h_2 = \pi^- \)

\[
\frac{1}{\sigma_{\text{TOT}}} \frac{d\sigma}{dx_1 dx_2} \bigg|_{\pi^+ \pi^-} \approx \frac{5}{6} \left( D_u^{\pi^+ \pi^-} + D_d^{\pi^+ \pi^-} \right) + \frac{1}{3} D_s^{\pi^+ \pi^-}, \quad (3.54)
\]

and

(ii) for \( h_1 = \pi^+ \) and \( h_2 = K^- \)
\[
\frac{1}{\sigma_{\text{TOT}}} \frac{d\sigma}{dx_1 dx_2} \bigg|_{\pi^+k^-} \approx \frac{5}{6} \left( D_u^\pi^+k^- + D_s^\pi^+k^- \right) + \frac{1}{3} D_d^\pi^+k^-.
\] (3.55)

We show in Figs. 3.18(a) and 3.18(b) our jet calculus predictions of the strangeness suppression in
\[e^+e^- \rightarrow (\pi^+\pi^-)_{\text{jet}} + X\] and \[(\pi^+k^-)_{\text{jet}} + X,\] respectively, at \[Q^2 = 33^2 \text{ GeV}^2/c^2.\] Note that the subscript "jet" denotes two mesons measured in the same jet. We see that the inclusive distributions of the \(\pi^+k^-\) production in \(e^+e^-\) annihilation are much steeper than that of the \(\pi^+\pi^-\) production as \(x_1 \rightarrow 1\) which is expected as a result of the second dominant contribution arising from \(2q2g\) component.

It seems natural for one to ask a question of how to distinguish quark jets of different flavor. Berge, et al. have measured the jet net charge in DIS lepton-nucleon scattering to determine the quark origin of the hadron jets (66). Furthermore, recent study in hadron-hadron interaction at CERN-ISR center of mass energy \(\sqrt{s} = 62 \text{ GeV}\) has shown an alternative way to identify the quark origin of the trigger jet by measuring the ratio of opposite charge densities of secondary particles with respect to the trigger particle in the trigger jet (63),
Fig. 3.18(a) Predictions of the inclusive two-particle cross section $d\sigma/dx_1 dx_2/R$ for $e^+e^- \rightarrow (\pi^+\pi^-)_{\text{jet}} + X$ versus momentum fraction $x_2$ with momentum fractions $x_1$ of 0.176 through 0.706 at $Q^2 = 33^2$ GeV$^2$/c$^2$, $Q^2_0 = 1$ GeV$^2$/c$^2$ and $\Lambda = 0.3$ GeV/c.

Notice that $R = \sigma_{\text{TOT}} R_1 R_2$. (b) Same as (a) except for $e^+e^- \rightarrow (\pi^+k^-)_{\text{jet}} + X$.
Fig. 3.18(b) Inclusive cross section for $e^+ e^- \to e^+ e^- (K^+ K^-)_\text{jet}$. 

Legend:
- $Q = 9.0 \text{ GeV}$
- $X = 0.0 \text{ GeV}$
- $Z = 1.0 \text{ GeV}$
- $Y = 3.0 \text{ GeV}$
and their results are consistent with the predicted quark origin of hadron jets. In our jet calculus model, we show in Fig. 3.19 our predictions of the short range charge compensation of the $\pi^+\pi^-$ and $\pi^+\pi^+$ productions in the "favored" and "unfavored" parton jets. We see that the "favored" quark jets, for instance, $u$-quark $\rightarrow \pi^+\pi^-$ and $u$-quark $\rightarrow \pi^+\pi^+$, are behaving substantially in a distinct way than the "unfavored" quark jets, $s$-quark $\rightarrow \pi^+\pi^-$ and $s$-quark $\rightarrow \pi^+\pi^+$ at large $x$ limit. This is due to the domination of the triple-gluon coupling in the "unfavored" quark jets. We plot in the above figure the distributions of the $\pi^+\pi^-$ and $\pi^+\pi^+$ productions in gluon jets and show the similarity of the charge compensation in the $s$-quark jet and gluon jet.

We present in Fig. 3.20(a) the inclusive two-particle cross section in the reaction $\nu p \rightarrow \mu^- + (h_1 h_2)^{\text{jet}} + X$

$$\frac{1}{\sigma_{\text{TOT}}} \frac{d\sigma}{dx_1 dx_2} \left| h_1 h_2 \right. \rightarrow D_u \rightarrow h_1 h_2(x_1', x_2'; Q^2, Q_0^2).$$ (3.57)

and in Fig. 3.20(b) the inclusive two-particle cross section in the reaction $\bar{\nu} p \rightarrow \mu^+ + (h_1 h_2)'^{\text{jet}} + X$ at $Q^2 = 33^2 \text{ GeV}^2/c^2$
Fig. 3.19 Comparison between momentum fractions $x_1 x_2$ time fragmentation functions $D$ and divided by recombination parameter $R = R_1 R_2$ versus momentum fraction $x K_2$ for $u$ quark $\rightarrow \pi^+ \pi^-$, $u$ quark $\rightarrow \pi^+ \pi^+$, $s$ quark $\rightarrow \pi^+ \pi^-$, $s$ quark $\rightarrow \pi^+ \pi^+$, gluon $\rightarrow \pi^+ \pi^-$ and gluon $\rightarrow \pi^+ \pi^+$ at $Q^2 = 33^2 \text{GeV}^2/c^2$, $Q_0^2 = 1 \text{GeV}^2/c^2$, and $\Lambda = 0.3 \text{GeV}/c$ with momentum fraction $x_1 = 0.176$. 
Fig. 3.20(a) Predictions of the inclusive two-particle cross section $d\sigma/dx_1 dx_2/R$ for the reactions: (i) $\nu p \to \mu^- + (\pi^+ \pi^-)_\text{jet} + X$ and (ii) $\nu p \to \mu^- + (\pi^+ \pi^+)_\text{jet} + X$ versus momentum fraction $x_2$ at $Q^2 = 33^2 \text{ GeV}^2/c^2$, $Q_0^2 = 1 \text{ GeV}^2/c^2$, and $\Lambda = 0.3 \text{ GeV}/c$ with momentum fraction $x_1 = 0.176$. Notice that $R = \sigma_{TOT}^{R_1R_2}$. (b) Same as (a) except for the reactions: (i) $\bar{\nu} p \to \mu^+ + (\pi^+ \pi^-)_\text{jet} + X$ and (ii) $\bar{\nu} p \to \mu^+ + (\pi^+ \pi^+)_\text{jet} + X$. 
$Q = 33.0 \text{ GEV}$

$Q^2 = 1.0 \text{ GEV}$

$L = 0.3 \text{ GEV}$

$x_1 = 0.176$
Fig. 3.20(b) Inclusive cross sections for $\bar{\nu}_p$ scattering.
where $\sigma_{\text{TOT}}$ is the total cross section defined in Eq. (3.53). Note that the subscript "jet" denotes the two particles $h_1$ and $h_2$ measured in the same jet; or, in other words, they are measured in the current fragmentation region. The two particles $h_1$ and $h_2$ are either $\pi^+$ and $\pi^-$, or $\pi^+$ and $\pi^+$. Eqs. (3.57) and (3.58) are motivated by the validity of the factorization hypothesis for the inclusive one-particle cross section as reported by Berge et al. (66). We show that in Fig. 3.20(a) there are two inclusive distributions of the $\pi^+\pi^-$ and $\pi^+\pi^+$ productions in the "favored" quark jet. In Fig. 3.20(b), we plot two curves such that one is for the $\pi^+\pi^-$ production in the "favored" quark jet and the other is for the $\pi^+\pi^+$ production in the "unfavored" quark jet. From these results, we expect that when the two-particle data are available from the neutrino experiment it may provide a good test of our jet calculus model in the predictions of charge compensation effect in quark jets and quark origin of hadron jets, and may strengthen our believes in the factorization hypothesis.

As pointed out by MJL that the rate of change of the single particle fragmentation function with $Q^2$ depends
somewhat on the input values of the parameters $Q_0^2$ and $\Lambda$. For instance, the integration variable $y$ has a lower and an upper limits ranging from $y_0 \propto \ln \ln(Q_0^2/\Lambda^2)$ to $y \propto \ln \ln(Q^2/\Lambda^2)$. The effects of varying the parameters $Q_0^2$ and $\Lambda$ have been demonstrated in Fig. 3.9. Furthermore, they have concluded that $Q_0^2$ should lie between 1 and 5 GeV$^2$/c$^2$ for single particle production in quark jet and prefered to use small values of the parameter $Q_0^2$ which is close to the QCD scale parameter $\Lambda$ in order to resemble the Owens-Uematsu equations. Since we adopt MJL approach in our jet calculus model for two-particle fragmentation functions, we let the input value of the parameter $Q_0^2$ to be 1 GeV$^2$/c$^2$ and choose the value of the QCD scale parameter $\Lambda$ to be 300 MeV/c. A limitation on the value of the parameter $\Lambda$ used in single particle fragmentation function to be the same value used in two-particle fragmentation function is recently claimed by Puhala (67) due to the momentum sum rule (28)

$$
\Sigma \int_0^{1-x_1} dx_2 \int_0^{x_2} D_{i-h_1} (x_1, x_2; y, y_0) = (1-x_1) D_{i-h_1} (x_1; y, y_0).
$$

(3.59)

Even though he argues that to use same value of the parameter $\Lambda$ in different processes is unjustified, we believe that our predictions of the inclusive two-particle cross sections in $e^+e^-$ annihilation and $\nu\bar{\nu}$ scattering with the same value of $\Lambda$ will remain as general features of our jet calculus model.
At our input values of $Q_0^2 = 1 \text{ GeV}^2/c^2$ and $\Lambda = 0.3$ GeV/c, we show in Fig. 3.21(a) and 3.21(b) our prediction of fragmentation functions for a singlet quark $\rightarrow \pi^+ \pi^-$ and a gluon $\rightarrow \pi^+ \pi^-$, respectively, given in Eq. (3.30), with the values of $Q^2$ increasing from $33^2 \text{ GeV}^2/c^2$ to $1000^2 \text{ GeV}^2/c^2$. One can see a strong $Q^2$ dependence in our jet calculus model. The figures show scaling violation in two-particle fragmentation functions. There is an increase in contribution at small $x$ and rapid fall off at large $x$ as $Q^2$ increases. This is a general feature expected in perturbative quantum chromo- dynamics, due to the fact that a larger parton cloud will be created as $Q^2$ increases. Such a strong $Q^2$ dependence may serve the purpose of contrasting our model with the cascade model as discussed in the following section.

3. Recombination model vs. cascade model

An alternate model for $D_i \rightarrow h_1 h_2$ proposed by Willen (52) requires three KUV propagators which yields a considerably different $Q^2$ dependence than does our model. Willen's two-particle fragmentation function illustrated in Fig. 3.22 has moments given by

$$D_i \rightarrow M_1 M_2 (n, m; Y, Y_0)$$

(3.60)
Fig. 3.21(a) Fragmentation function $D$ for singlet quark $\rightarrow \pi^+\pi^-$ times momentum fractions $x_1x_2$ and divided by recombination parameter $R = R_1R_2$ versus momentum fraction $x_2$ at $Q^2$ ranging from $33^2$ GeV$^2$/c$^2$ to $10^6$ GeV$^2$/c$^2$, $Q_0^2 = 1$ GeV$^2$/c$^2$, and $\Lambda = 0.3$ GeV/c with momentum fraction $x_1 = 0.176$. (b) Same as (a) except for gluon $\rightarrow \pi^+\pi^-$. 
$Q_0 = 1.0 \text{ GEV}$

$Q = 33.0 \text{ GEV}$

$L = 0.3 \text{ GEV}$

$Q = 1000.0 \text{ GEV}$

$X_l = 0.176$
Fig. 3.21(b) Fragmentation function for gluon → π⁺π⁻.
Fig. 3.22 Diagram for an alternate model (52) to describe the fragmentation function for parton i into two mesons $M_1$ and $M_2$. Jet calculus is used to generate the momentum fraction distribution of parton a and b (at $q_0^2 = 4 \text{ GeV}^2/c^2$) which fragment into mesons $M_1$ and $M_2$, respectively, via the empirical distributions indicated by the dark blobs.
where \( D_{a \rightarrow h}^{n}(Y_{0}) \) is a phenomenological fragmentation function giving an empirical probability that hadron \( h \) will materialize from parton \( a \) when \( a \) has off-shellness \( Q_{0}^{2} \). In Eq. (3.47) the \( q \bar{q} \) decay products of any gluons among \( a_{1}, a_{2}, a_{3}, a_{4} \) make up the final state hadrons whereas in Eq. (3.60) gluons among \( a \) and \( b \) lead to hadrons via \( D_{g \rightarrow h} \), with \( D_{i \rightarrow h} \) taken from Feynman and Field (9), where \( i = q \) or \( g \). Thus, \( Y_{0} \) or \( Q_{0}^{2} \) have different interpretations in Eq. (3.47 and 3.60).

Chang and Hwa (42) have recently argued that a significant improvement can be made in their predictions of single charged-pion inclusive distribution at small \( x \) in \( e^{+}e^{-} \) annihilation with a modified recombination function which contains a phenomenological prescription (68) of resonance formation and decay. They show in Fig. 17 of Ref. 42 that the curve with resonance contribution is higher than that without resonance contribution by a factor of 4 at \( x = 0.1 \) and 1.3 at \( x = 0.5 \), and both are identical as \( x \rightarrow 1 \). However, this may not provide a great impact in our study of \( Q^{2} \) dependence of fragmentation functions without resonance contributions between two different models.
Since there is not yet a theoretical prescription of resonance formation and decay in the recombination picture and evidence from experiment (63) shows that less than 5% of leading particles in the trigger jet come from decays of neutral vector mesons, no attempt will be made to account for the effect of resonance decay in our model. Besides, we will make comparisons of ratios of fragmentation functions at different $Q^2$ values to bring out the features illustrating scaling violation, so that many of the uncertainties in the input to our model will be kept at a minimum and the only parameter ($R$) in Eq. (3.37 and 3.39), which is typically (42,47,49,50) determined by experimental data or crude theoretical arguments, will cancel. Hence, the comparison we make of the dominant contribution ($a_1a_2a_3a_4 = qggg$) in Eq. (3.47) with that of Willen given in Eq. (3.60) is independent of parameter adjustment.

The calculation of all double moments is extremely tedious, requiring, e.g., the evaluation of 47,250 jet calculus diagrams for the subdiagrams of type I given in Fig. 3.9. Thus, the total of jet calculus diagrams is close to $10^5$ for type I+II together. The inversion of double moments into momentum fraction space to reconstruct $D(x_1,x_2)$ is done by a method developed by Willen (52) from the standard (42,47,49,50) single moment approach. Due to VAX computer limitations only $n, m$ values to 14 could be calculated,
limiting $x_i$ to values between 0.176 and 0.765. The problem was made tractable for our present results by making a modification of the leading-quark approximation (45,69). Only these diagrams are evaluated which have the initial quark $i$ as one of the final partons, $a_1$, $a_2$, $a_3$, or $a_4$. The strictly leading-quark approximation would have quark $i$ present at all stages of the evolution; on the basis of possible experiment (69) this is a reasonable region of phase space in which to do calculations. Despite this restriction the VAX CPU time requirements were two days for the results we now discuss, based only on the dominant jet calculus configuration, $a_1 a_2 a_3 a_4 = qggg$. Note that from the previous discussion, in the $(\pi^+ \pi^-)$ final states the $qqgg$ term becomes more like the single quark term at the kinematical boundary, $x_1 + x_2 = 1$. Furthermore, the inclusion of the $qqgg$ term adds less than a 1% change in the scale breaking effects.

The results of numerical calculations of ratios of the fragmentation functions, Eq. (3.25), for different $Q^2$ are given in Tables I and II, which are plotted in Fig. 3.23 and 3.24, respectively. Table I(a) contains these ratios for a u-quark fragmenting into $\pi^+ \pi^-$ mesons at $Q^2 = 33^2$ GeV$^2$/c$^2$ to that at $Q^2 = 5^2$ GeV$^2$/c$^2$, i.e., $D_{u-\pi^+\pi^-}(x_1,x_2;Q^2=33^2\text{GeV}^2/c^2)/D_{u-\pi^+\pi^-}(x_1,x_2;Q^2=5^2\text{GeV}^2/c^2)$, which is defined as $\rho$ for convenience. Table I(b) lists the corresponding ratios for $Q^2 = 10^6$ GeV$^2$/c$^2$ to those for $Q^2 = 5^2$ GeV$^2$/c$^2$. The entries
Table I

Ratio of fragmentation functions $\rho(x_1, x_2; Q^2) = D(x_1, x_2; Q^2, Q_0^2 = 4 \text{ GeV}^2/\text{c}^2) / D(x_1, x_2; 5 \text{ GeV}^2/\text{c}^2, 4 \text{ GeV}^2/\text{c}^2)$ for fragmentation of the $u$ quark into $\pi^+\pi^-$. Table (a) is for $Q^2 = 33^2 \text{ GeV}^2/\text{c}^2$ and Table (b) for $Q^2 = 10^6 \text{ GeV}^2/\text{c}^2$. These results are calculated for the diagrams shown in Fig. 3.4.

(a)

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</table>
Table II

Ratio of fragmentation functions for the diagram shown in Fig. 3.22 $p(x_1, x_2; Q^2) = D(x_1, x_2; Q^2, Q^2 = 4 \text{ GeV}^2/\text{c}^2) / D(x_1, x_2; 5 \text{ GeV}^2/\text{c}^2, 4 \text{ GeV}^2/\text{c}^2)$ for fragmentation of the u quark into $\pi^+\pi^-$ according to the model of Willen (52). Table (a) is for $Q^2 = 33^2 \text{ GeV}^2/\text{c}^2$ and Table (b) for $Q^2 = 10^6 \text{ GeV}^2/\text{c}^2$.

### Table (a)

<table>
<thead>
<tr>
<th>$x_2^+$</th>
<th>0.176</th>
<th>0.235</th>
<th>0.294</th>
<th>0.353</th>
<th>0.412</th>
</tr>
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<td>$x_1$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.176</td>
<td>1.920</td>
<td>1.694</td>
<td>1.550</td>
<td>1.441</td>
<td>1.351</td>
</tr>
<tr>
<td>0.235</td>
<td>1.719</td>
<td>1.511</td>
<td>1.375</td>
<td>1.271</td>
<td>1.183</td>
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<tr>
<td>0.294</td>
<td>1.597</td>
<td>1.398</td>
<td>1.266</td>
<td>1.163</td>
<td>1.075</td>
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<tr>
<td>0.353</td>
<td>1.505</td>
<td>1.313</td>
<td>1.183</td>
<td>1.080</td>
<td>0.991</td>
</tr>
<tr>
<td>0.412</td>
<td>1.427</td>
<td>1.240</td>
<td>1.111</td>
<td>1.006</td>
<td>0.914</td>
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<td>0.471</td>
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<td>1.040</td>
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<td>0.836</td>
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<td>0.529</td>
<td>1.281</td>
<td>1.097</td>
<td>0.965</td>
<td>0.853</td>
<td>0.751</td>
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<td>0.588</td>
<td>1.203</td>
<td>1.017</td>
<td>0.881</td>
<td>0.764</td>
<td>0.659</td>
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<td>0.927</td>
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<td>0.706</td>
<td>1.012</td>
<td>0.821</td>
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<td>0.000</td>
<td>0.000</td>
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<tr>
<td>0.765</td>
<td>0.888</td>
<td>0.000</td>
<td>0.000</td>
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<td></td>
<td>0.471</td>
<td>0.529</td>
<td>0.588</td>
<td>0.647</td>
<td>0.706</td>
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<td>1.271</td>
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<td>1.049</td>
<td>0.970</td>
<td>0.881</td>
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Table II (cont.)

(b)

\[ x_2 \rightarrow \begin{array}{cccccc}
0.176 & 0.235 & 0.294 & 0.353 & 0.412 \\
\end{array} \]

\[ x_1 \]
\[ \downarrow \]
\[ \begin{array}{cccccc}
0.176 & 2.329 & 1.829 & 1.542 & 1.341 & 1.184 \\
0.235 & 1.886 & 1.469 & 1.226 & 1.054 & 0.920 \\
0.294 & 1.640 & 1.269 & 1.049 & 0.892 & 0.769 \\
0.353 & 1.464 & 1.125 & 0.922 & 0.775 & 0.659 \\
0.412 & 1.320 & 1.006 & 0.816 & 0.677 & 0.566 \\
0.471 & 1.191 & 0.898 & 0.718 & 0.585 & 0.478 \\
0.529 & 1.067 & 0.792 & 0.621 & 0.494 & 0.391 \\
0.588 & 0.942 & 0.684 & 0.522 & 0.402 & 0.000 \\
0.647 & 0.811 & 0.571 & 0.420 & 0.000 & 0.000 \\
0.706 & 0.671 & 0.454 & 0.000 & 0.000 & 0.000 \\
0.765 & 0.523 & 0.000 & 0.000 & 0.000 & 0.000 \\
\end{array} \]
<table>
<thead>
<tr>
<th></th>
<th>0.471</th>
<th>0.529</th>
<th>0.588</th>
<th>0.647</th>
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</tbody>
</table>
Fig. 3.23(a) Contour plot of ratio of fragmentation functions for $u$ quark $\rightarrow \pi^+\pi^-$ shown in Fig. 3.4

$$p(x_1, x_2; Q^2) = \frac{D(x_1, x_2; Q^2, Q_0^2 = 4 \text{ GeV}^2/c^2)}{D(x_1, x_2; 5 \text{ GeV}^2/c^2, 4 \text{ GeV}^2/c^2)}$$ at $Q^2 = 33^2 \text{ GeV}^2/c^2$. (b) Same as (a) except at $Q^2 = 10^6 \text{ GeV}^2/c^2$
Fig. 3.23(b) Ratio of fragmentation functions for u quark $\rightarrow \pi^+\pi^-$ at $Q^2 = 10^6$ GeV$^2$/c$^2$. 
Fig. 3.24(a) Contour plot of ratio of fragmentation functions for $u$ quark $\to \pi^+\pi^-$ according to the model of Willen (52) shown in Fig. 3.22 $p'(x_1, x_2; Q^2) = D(x_1, x_2; Q^2, Q_0^2 = 4 \text{GeV}^2/c^2) / D(x_1, x_2; 5 \text{GeV}^2/c^2, 4 \text{GeV}^2/c^2) \text{ at } Q^2 = 33^2 \text{ GeV}^2/c^2$. (b) Same as (a) except at $Q^2 = 10^6 \text{ GeV}^2/c^2$. 
Fig. 3.24(b) Ratio of fragmentation functions for $u$ quark $\rightarrow \pi^+\pi^-$ at $Q^2 = 10^6$ GeV$^2$/c$^2$. 
are in matrix form with \( x_1 \) labelling rows and \( x_2 \) columns. The parameter \( Q_0^2 \) is taken as \( 4 \text{ GeV}^2/c^2 \), consistent with Migneron et al. (45) in the jet calculus approach. It is also consistent with the value \( ^{2}Q_0^2 = 4 \text{ GeV}^2/c^2 \) at which the empirical fragmentation functions given in Ref. 9 used by Willen (52) were evaluated; these two \( Q_0^2 \)'s have somewhat different meanings but comparison is most logical by choosing the same \( Q_0^2 \) values. The corresponding ratios at these same \( Q^2 \) values for Willen's approach are listed in Tables II(a) and II(b), respectively. The ratios in Table I are considerably larger than those in Table II and the \( q^2 \) dependence is much stronger. Another way to illustrate this is to take the difference in \( p \) values between the largest and smallest \( x \) values; i.e., \( \Delta p = p(0.18,0.18) - p(0.18,0.77) \) differences are listed in Table III. These numbers indicate a large difference in correlations between this model and Willen's and also indicate scaling is broken in both models in the correlations that exists. The two columns in Table III would be identical if no \( Q^2 \) dependence existed, and the violation is a factor of 5 bigger in the present model than in Willen's.

In Table III, we also show \( \Delta p \) in Willen model with resonance contributions, which is not significantly different from that without resonance decay. However, we believe that even though we have neglect the effect of resonance formation
Table III

Differences of the ratios listed in Tables I and II, \( \Delta \rho(0^2) = \rho(0.18, 0.18; 0^2) - \rho(0.18, 0.77; 0^2) \), for the two models described in Fig. 3.4 and 3.22, respectively, for the inclusive fragmentation, \( u \rightarrow \pi^+\pi^- \).

<table>
<thead>
<tr>
<th>Model</th>
<th>( \Delta \rho(0^2 = 33^2) )</th>
<th>( \Delta \rho(0^2 = 10^6) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>present model</td>
<td>20.59</td>
<td>117.63</td>
</tr>
<tr>
<td>Willen model</td>
<td>1.039</td>
<td>1.813</td>
</tr>
<tr>
<td>Willen model with resonance contributions</td>
<td>0.904</td>
<td>1.535</td>
</tr>
</tbody>
</table>
and decay, our results showing strong $Q^2$ dependence should remain fairly the same.

C. Appendix A: Definitions of Various Moments

In this appendix, we list the anomalous dimensions $A_i$ and moments of the KUV parton propagators $K$ and the QCD-vertex functions $P$, which have been used extensively throughout our calculations of $Q^2$ dependence of single- and two-particle fragmentation functions in jet calculus model.

1. Anomalous dimensions

Moments of the Altarelli-Parisi splitting functions \((16,28,70)\) are defined as

$$A_{ab}^n = \int_0^1 dx \, x^n \, P_{a \to bc}(x),$$ \hspace{1cm} (3.61)

$$A_{qq}^n = \frac{4}{3} \left[ - \frac{1}{2} + \frac{1}{(n+1)(n+2)} - 2 \sum_{j=1}^{n} \frac{1}{j+1} \right],$$ \hspace{1cm} (3.62)

$$A_{Gq}^n = \frac{4}{3} \frac{n^3 + 3n + 4}{n(n+1)(n+2)},$$ \hspace{1cm} (3.63)

$$A_{qG}^n = \frac{n^3 + 3n + 4}{2(n+1)(n+2)(n+3)},$$ \hspace{1cm} (3.64)

$$A_{GG}^n = 3 \left[ - \frac{1}{2} + \frac{2}{n(n+1)} + \frac{2}{(n+2)(n+3)} - 2 \sum_{j=1}^{n} \frac{1}{j+1} \right].$$ \hspace{1cm} (3.65)
2. KUV parton propagators

The KUV parton propagators are obtained by solving the evolution equations of the nonsinglet quark and gluon fragmentation functions \((19,20,70)\), and moments of the propagators are

\[
K_{qjqi}^n(y, y') = \exp\left[\lambda_{qjqi}^n(y-y')\right] \delta_{ij} \tag{3.66}
\]

\[
+ \frac{\sigma^+_n\left(\exp\left[\lambda_{qjqi}^n(y-y')\right] - \exp\left[\lambda_{qq}^n(y-y')\right]\right)}{6\Sigma^n}
\]

\[
+ \frac{\sigma^-_n\left(\exp\left[\lambda_{qq}^n(y-y')\right] - \exp\left[\lambda_+^n(y-y')\right]\right)}{6\Sigma^n},
\]

\[
K_{Gqi}^n(y, y') = \frac{\exp\left[\lambda_+^n(y-y')\right] - \exp\left[\lambda_-^n(y-y')\right]}{6\Sigma^n}, \tag{3.67}
\]

\[
K_{qiG}^n(y, y') = \left(-\sigma^+_n\right)\frac{\exp\left[\lambda_+^n(y-y')\right] - \exp\left[\lambda_-^n(y-y')\right]}{\Sigma^n}, \tag{3.68}
\]

and

\[
K_{GG}^n(y, y') = \frac{\sigma^+_n \exp\left[\lambda_+^n(y-y')\right] - \sigma^-_n \exp\left[\lambda_-^n(y-y')\right]}{\Sigma^n}, \tag{3.69}
\]

where

\[
\Sigma^n = \sigma^+_n - \sigma^-_n.
\]
3. **OCD-vertex functions**

Double moments of the vertex functions are defined by

\[
\sigma_n^\pm = \left( \frac{A_{GG}^n - A_{qq}^n}{A_{qq}^n} \right) \pm \left[ \left( \frac{A_{GG}^n - A_{qq}^n}{A_{qq}^n} \right)^2 + \frac{24 A_{Gq}^n A_{Gq}^n}{12 A_{Gq}^n} \right]^{1/2},
\]

\[
\lambda_n^\pm = A_{qq}^n + 6 A_{Gq}^n \sigma_n^\pm.
\]

### D. Appendix B: Single-Meson Fragmentation Functions

For completeness and comparison purposes, we examine the inclusive breakup of a parton into a single meson. Following
Eq. (3.1) and (3.2), we have for fragmentation of parton i into meson M.

\[ D_i \rightarrow M(x; Q^2, Q_0^2) \quad (3.75) \]

\[
= \int_0^1 \frac{2}{\Pi} \frac{dx_i}{x_i} \int_{Q_0^2} \frac{dQ^2}{Q_0^2} \frac{d\alpha_s(k^2)}{2\pi} \int_{x_1+x_2}^1 d\xi K_{ji}(\xi; Q^2, k^2) \\
\times \int_{x_1}^{\xi-\frac{x_2}{z}} \frac{dz}{\xi} \hat{P}_{j \rightarrow b_1 b_2}(z) \frac{x_1}{z} a_1 b_1(\frac{x_1}{z}; k^2, Q_0^2) \frac{x_2}{\xi - z} \\
\times K_{a_2 b_2}(\frac{x_2}{\xi - z}; k^2, Q_0^2) R_{a_1 a_2}^M(x_1, x_2; x).
\]

We change the integration variable \( k^2 \) to \( y \) as follows,

\[ D_i \rightarrow M(x; y, Y_0) \quad (3.76) \]

\[
= \int_0^1 \frac{2}{\Pi} \frac{dx_i}{x_i} R_{a_1 a_2}^M(x_1, x_2; x) \int_y^{Y_0} dy \int_{x_1+x_2}^1 d\xi K_{ji}(\xi; y, y) \\
\times \int_{\frac{x_1}{z(1-z)}}^{\frac{x_2}{\xi}} \frac{dz}{\xi} \hat{P}_{j \rightarrow b_1 b_2}(z) K_{a_1 b_1}(\xi z; y, Y_0) \\
\times K_{a_2 b_2}(\frac{x_2}{(1 - z); y, Y_0}).
\]
With momentum conservation explicitly displayed, we further simplify to

\[ D_i \rightarrow M(x; Y, Y_0) \]  
\[ = \int_0^1 \frac{2}{x_2} \sum_{\eta=1}^{\infty} dx_\eta \ dx_2 \ d\xi \ dz \ R_{a_1 a_2}^M(x_1, x_2; x) \ K_{a_1 b_1} \ (w_1; y, Y_0) \]
\[ \times K_{a_2 b_2} \ (w_2; y, Y_0) \ \hat{D}_{j-1 b_2 b_2} \ (z) \ K_{ji} \ (\xi; Y, y) \ \delta(x_1 - \xi z w_1) \]
\[ \times S(x_2 - \xi (1 - z) w_2) \]
\[ = \int_0^1 D_{a_1 a_2, i} \ (x_1, x_2; Y, Y_0) \ R_{a_1 a_2}^M \ (x_1, x_2; x) \sum_{\eta=1}^{\infty} dx_\eta. \]

The single meson fragmentation function may then be decomposed into the qq, gq and gg terms and written as

\[ D_i \rightarrow M(x; Y, Y_0) = \sum_{j=0}^{2} D^{(j)}(x; Y, Y_0), \]  
\[ D^{(0)}(x; Y, Y_0) \]
\[ = \int D_{q_1 q_2, i} \ (x_1, x_2; Y, Y_0) \ R_{q_1 q_2}^M \ (x_1, x_2; x) \sum_{\eta=1}^{\infty} dx_\eta. \]
The meson recombination function is given by
\[ R_{a_1a_2}^M(x_1, x_2; x) = R^r \left( \frac{x_1 x_2}{x^2} \right)^r \delta(x_1 + x_2 - x) \] (3.82)

where the value \( r = 1 \) can be given solid theoretical basis from the meson form-factor work of Lepage and Brodsky (65). The normalization factor \( R^r \) can be determined by the following constraint
\[ \frac{1}{x} \int_0^1 R_{a_1a_2}^M(x_1, x_2; x) \, dx_1 \, dx_2 = 1, \] (3.83)

which gives
Next, we may transform Eq. (3.77) to moment space by

\[ D_i \rightarrow M(n; Y, Y_0) = \int_0^1 dx x^n D_i \rightarrow M(x; Y, Y_0). \]  

(3.84)

We define the quantities \( O \) and \( L^i \) for notational convenience, such that

\[ O(\alpha, \beta, \gamma; y, Y_0) \]

\[ = K_1^{\alpha} q_1 b_1 (y, Y_0) K_2^{\beta} q_2 b_2 (y, Y_0) K_{j_1}^{\gamma}(y, y) \hat{a}_{j_1}^{\alpha, \beta} \]

\[ L^1(\alpha, \beta) = \hat{a}_{q_1}^{\alpha} + \hat{a}_{q_2}^{\beta}, \]

(3.85)

\[ \text{and} \]

\[ L^2(\alpha, \beta) = \hat{a}_{q_1}^{\alpha} \hat{a}_{q_2}^{\beta}. \]

(3.86)

(3.87)

Then, Eq. (3.84) is written as

\[ D_i \rightarrow M(n; Y, Y_0) \]

\[ = \sum_{i=0}^{2} D^{(i)}(n; Y, Y_0) \]  

(3.88)
\[ E^i({n'}; y, Y_0) = L^i(n-m-r, m+r) \tilde{0}, \quad (3.90) \]

and

\[ E^2({n'}; y, Y_0) = L^2(n-m-r, m+r) \tilde{0}. \quad (3.91) \]

In Eq. (3.88) \( \Omega = \{j, b_1, b_2, q_1, q_2\} \), \({n'}\) stands for \( n \) and \( m \), and \( \tilde{0} = 0(n-m-r, m+r, n; y, Y_0) \).

**E. Appendix C: Definitions of Functions \( E^i, k \)**

In this appendix, the explicit forms of the functions \( E \) from Eq. (3.47) are displayed.

1. Functions \( E^i_1 ({n'}; \{y\}, Y_0) \)

\[ E^0_1 = O^1_1(n-p-1, p+1, m-q-1, q+1, m, m+p+1, n+m; \{y\}, Y_0), \quad (3.92) \]

\[ E^0_2 = O^2_1(n-p-1, m-q-1, p+1, q+1, p+q+2, m+p+1, n+m; \{y\}, Y_0), \quad (3.93) \]

\[ E^0_3 = O^3_1(n-p-1, m-q-1, q+1, p+1, p+q+2, m+p+1, n+m; \{y\}, Y_0), \quad (3.94) \]
\[ E_{I}^{1,1} = L_{I}^{1}(n-p-1,p+1,m-q-1,q+1) \tilde{\theta}_{I}^{1} \]  

\[ E_{I}^{1,2} = L_{I}^{1}(n-p-1,m-q-1,p+1,q+1) \tilde{\theta}_{I}^{2} \]  

\[ E_{I}^{1,3} = L_{I}^{1}(n-p-1,m-q-1,q+1,p+1) \tilde{\theta}_{I}^{3} \]  

\[ E_{I}^{2,1} = L_{I}^{2}(n-p-1,p+1,m-q-1,q+1) \tilde{\theta}_{I}^{1} \]  

\[ E_{I}^{2,2} = L_{I}^{2}(n-p-1,m-q-1,p+1,q+1) \tilde{\theta}_{I}^{2} \]  

\[ E_{I}^{2,3} = L_{I}^{2}(n-p-1,m-q-1,q+1,p+1) \tilde{\theta}_{I}^{3} \]  

\[ E_{I}^{3,1} = L_{I}^{3}(n-p-1,p+1,m-q-1,q+1) \tilde{\theta}_{I}^{1} \]  

\[ E_{I}^{3,2} = L_{I}^{3}(n-p-1,m-q-1,p+1,q+1) \tilde{\theta}_{I}^{2} \]  

\[ E_{I}^{3,3} = L_{I}^{3}(n-p-1,m-q-1,q+1,p+1) \tilde{\theta}_{I}^{3} \]  

\[ E_{I}^{4,1} = L_{I}^{4}(n-p-1,p+1,m-q-1,q+1) \tilde{\theta}_{I}^{1} \]  

\[ E_{I}^{4,2} = L_{I}^{4}(n-p-1,m-q-1,p+1,q+1) \tilde{\theta}_{I}^{2} \]  

\[ E_{I}^{4,3} = L_{I}^{4}(n-p-1,m-q-1,q+1,p+1) \tilde{\theta}_{I}^{3} \]
2. Functions $E_{II}^{i,k}(n',\{y\},\gamma)$

\[ E_{II}^{0,1} = O_{II}^{1}(n-p-1,m-q-1,q+1,n,m,n+m;\{y\},\gamma), \]  
\[ (3.107) \]

\[ E_{II}^{0,2} = O_{II}^{2}(n-p-1,m-q-1,p+1,q+1,n+m-p-q-2,p+q+2,n+m;\{y\},\gamma), \]  
\[ (3.108) \]

\[ E_{II}^{0,3} = O_{II}^{3}(n-p-1,m-q-1,q+1,p+1,n+m-p-q-2,p+q+2,n+m;\{y\},\gamma), \]  
\[ (3.109) \]

\[ E_{II}^{1,1} = L^{1}(n-p-1,p+1,m-q-1,q+1) O_{II}^{1}, \]  
\[ (3.110) \]

\[ E_{II}^{1,2} = L^{1}(n-p-1,m-q-1,p+1,q+1) O_{II}^{2}, \]  
\[ (3.111) \]

\[ E_{II}^{1,3} = L^{1}(n-p-1,m-q-1,q+1,p+1) O_{II}^{3}, \]  
\[ (3.112) \]

\[ E_{II}^{2,1} = L^{2}(n-p-1,p+1,m-q-1,q+1) O_{II}^{1}, \]  
\[ (3.113) \]

\[ E_{II}^{2,2} = L^{2}(n-p-1,m-q-1,q+1,p+1) O_{II}^{2}, \]  
\[ (3.114) \]

\[ E_{II}^{2,3} = L^{2}(n-p-1,m-q-1,p+1,q+1) O_{II}^{3}, \]  
\[ (3.115) \]

\[ E_{II}^{3,1} = L^{3}(n-p-1,p+1,m-q-1,q+1) O_{II}^{1}, \]  
\[ (3.116) \]

\[ E_{II}^{3,2} = L^{3}(n-p-1,m-q-1,p+1,q+1) O_{II}^{2}, \]  
\[ (3.117) \]

\[ E_{II}^{3,3} = L^{3}(n-p-1,m-q-1,q+1,p+1) O_{II}^{3}, \]  
\[ (3.118) \]
\[ E_{II}^{4,1} = L_4^4(n-p-1,p+1,m-q-1,q+1) \bar{0}_{II}^1 , \] (3.119)

\[ E_{II}^{4,2} = L_4^4(n-p-1,m-q-1,p+1,q+1) \bar{0}_{II}^2 , \] (3.120)

\[ E_{II}^{4,3} = L_4^4(n-p-1,m-q-1,q+1,p+1) \bar{0}_{II}^3 . \] (3.121)
IV. CONCLUSIONS

In the first part of this work, we used the technique of Monte Carlo simulation to conduct a detailed investigation of jetlike structures which are expected to arise in hard scattering processes, and subsequently fragmentation of energetic partons in hadron-hadron collision, deep inelastic lepton-nucleon scattering or electron-positron annihilation. We believe from this work that the present status of QCD Monte Carlo simulation techniques is developed enough to lead to new depth in the understanding of jet phenomena. Our work indicates that one can identify jets by studying the $P_T$ distributions, mass distributions, multiplicities, and energy flow. Some information relevant to the kinematical constraints can also be obtained from the simulation method to aid the experimental set-up of the detectors for the study of QCD in $\gamma N$ reactions in future experiments. The photon-nucleon data expected from experiments possible with the Tevatron should be able to isolate the signals from the $\gamma q \rightarrow gq$ subprocess.

In the second and major part of the present work, a theoretical QCD model based on summing leading-logarithmic expressions was developed to study inclusive single-particle and two-particle production in parton jets. This work was done to gain understanding of the behavior of quark and gluon
fragmentation functions, emphasizing longitudinal momentum and $Q^2$ dependence. We have given a general prescription for obtaining multiparton distribution functions from jet calculus combined with bound state form factor (called the recombination function) information in a manner similar to the functions used by Migneron, Jones and Lassila for baryon production. For example, for two-particle production, this recombination function as given by our prescription yields the product of binding probabilities for two mesons. With these jet calculus multiparton distribution functions and recombination functions, we have determined the quark and gluon fragmentation functions without the use of any phenomenological input parton (quark or gluon) fragmentation functions at some lower energy scale $Q_0^2$. Qualitatively, the quark and gluon fragmentation functions have similar limiting behavior at $x \to 0$ and $x \to 1$ (59).

Our predictions for single baryon inclusive spectra in gluon jets can be tested in annihilation of heavy quark-antiquark bound states into three gluons. In fact, the upsilon $b\bar{b}$ states near 10 GeV may (71) be massive enough to test our predicted $x$ independence of the lambda hyperon / proton ratio (72). A gluon jet of such energies may not be of high enough initial $Q^2$ to satisfy the quark mass independence assumption implicit in our use of jet calculus. But, the physical ideas on which our calculations are based
seem compelling enough that we would be seriously worried if the $\Lambda / p$ ratio were not constant in the upsilon region. The constant certainly will not be 2 because of the mass suppression expected for strange quark production.

Our model incorporates the power and elegance of jet calculus with perhaps, still primitive, recombination model ideas in a manner which satisfies momentum conservation by the method of Ref. 42. Any gluon appearing among the final jet partons $a_1a_2a_3$ in Eq. (3.6) is converted to quark-antiquark pairs. Since the three gluon term leads to a $q_1\bar{q}_2\bar{q}_2q_3\bar{q}_3$ final set of recombining quarks, there will be equal numbers of baryons and antibaryons in $x$ regions dominated by this ggg term. Therefore, a gluon jet will have equal distributions for $\Lambda$ and $\Lambda$ versus $x$, or for $p$ and $\bar{p}$ versus $x$, etc. A quark jet will show such equality at small $x$ only and at large $x$ the leading quark effect will cause the baryon with the same sign of baryon number as the initial quark to dominate.

We have demonstrated that the KUV jet calculus predicts two complex and distinctive internal branching processes for quark and gluon breakup into two mesons, which dominate entirely the inclusive two-meson spectra. The predictions of competitive models (52, 67, 70) are significantly different; these have only one jet diagram for a parton branching into two hadrons. Consequently, in these models phenomenological
input is required for boundary conditions in the $Q^2$ (evolution) differential equations (19,20) and the majority of the dynamics is put in by hand without being predicted.

We find that there are common features in both single-particle and two-particle inclusive spectra, for instance, the effects of leading quark, strangeness suppression and scale breaking, which we expect to be general features of all multihadron fragmentation functions. Our predictions of the inclusive two-meson cross sections in high energy electron-positron annihilation and deep inelastic lepton-nucleon scattering may provide further tests of the universality of fragmentation functions for time-like and space-like processes and determine the validity of the factorization hypothesis. Finally, the strong $Q^2$ dependence of our model may provide an explicit way for gaining understanding of the hadronization mechanism and differentiating it from other models.
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