A model for the acoustical transfer function of tissue

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A MODEL FOR THE ACOUSTICAL TRANSFER FUNCTION OF TISSUE

Iowa State University

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A model for the acoustical transfer function of tissue

by

Mohammad Jamil Mismar

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INTRODUCTION

Tissue identification by non-surgical techniques is preferred by patients and medical personnel for the detection of type and pathological state of tissue, i.e., benign and malignant tumors, cirrhosis of the liver, and ischemic myocardium. Although x-ray methods are commonly used, ultrasonic techniques can also be utilized for tissue differentiation. One technique of diagnostic ultrasound is the visualization of tissue interfaces within the body, non-invasively, by the pulse-echo method. The sequence (a-f) of Figure 1 shows the principle of the pulse-echo method. An ultrasonic transducer (probe) transforms short electrical pulses to mechanical vibrations, sound waves at a frequency typically in the megahertz range, that penetrate the body. At every interface or boundary between two distinct tissue layers, part of the incident wave is reflected due to differences in acoustical properties of the tissue layers. The reflected waves are transformed back to electrical energy by the same transducer. As a result, the recorded waveform on the oscilloscope shows the display of the amplitude of the reflected waves (ordinate) and the corresponding time (abscissa) that elapsed for the pressure wave to travel in each tissue layer. The reflected waves contain information about the acoustical properties of each tissue layer. The aim of tissue characterization research is to retrieve this information and classify the tissue.

Tissue characterization by ultrasonic techniques revolves around evaluating acoustical tissue parameters. These parameters that define
The sequence (a-f) shows the amplitude and residence time of the recorded echoes (Wells, 1969).

Figure 1. Principle of the pulse-echo technique
the ultrasonic tissue signature are the velocity and the attenuation coefficient of ultrasonic waves in tissue. An incident wave on any tissue layer is modified depending on these parameters. The wave modification is due to reflection and transmission at the boundary between two tissue layers and attenuation of the wave within each tissue layer.

A normally incident ultrasonic wave is divided into reflected and transmitted waves at the boundary between two distinct tissue layers. This process occurs due to the density difference of the two layers and the resultant propagation velocity difference of the ultrasonic waves in each tissue layer.

Attenuation of the traveling wave is due to absorption of the wave energy in tissue as well as geometrical diffraction of the ultrasonic wave. The mechanical vibrations of the tissue causes the propagating wave to convert the wave energy into thermal energy that is dissipated in tissue in the form of absorption loss. This energy loss is described by a tissue attenuation coefficient that depends on the type and thickness of tissue layer and the frequency of the vibrations. The geometrical diffraction process accounts for the loss of energy per unit area due to the wave divergence as it leaves the ultrasonic transmitter. The degree of wave divergence depends on the transmitter-receiver diameter, axial distance of tissue layer from the transducer, and the wavelength of the ultrasonic wave in the tissue.
Background

The acoustical tissue parameters and their effects on the ultrasonic signal have been the object of past and recent research. The velocity and the attenuation coefficient were measured by different techniques; furthermore, ultrasonic signal modification was analyzed theoretically and verified experimentally by several researchers. As a result, a number of models and techniques for tissue characterization were developed.

Velocity

The wave propagation velocity in tissue can be measured by pulse transit time and acoustic interferometric methods. The pulse transit time is measured by simply observing the time of flight of an ultrasonic wave over a known path length of a tissue layer (Wladimiroff, et al., 1975). The acoustic interferometric method employs continuous wave (CW) excitation of the transmitting transducer. As a result, a standing wave is created in the tissue sample. The wavelength of the given ultrasonic excitation frequency in the tissue part can be measured from the maxima and minima of the standing wave as the transducer moves away from the tissue sample (Goldman and Richards, 1954). The distance that is traveled by the transducer is equal to a half-wavelength for two consecutive maxima and minima. The propagation velocity can be calculated easily from the measured wavelength and the frequency of the signal from the following relation:

\[ v = f \lambda \]  
Eq. 1
where \( v \) = propagation velocity of the wave, cm/sec,
\[ f \] = frequency of CW employed, Hz,
\[ \lambda \] = wavelength of the wave in tissue, cm.

**Attenuation Coefficient**

The tissue attenuation coefficient can be evaluated by the pulse transmission method. This method requires measuring the amplitudes of several radio frequency (RF) pulses with and without the tissue sample between the transmitting and receiving transducers (Bauld and Schwan, 1974). The ratio of the two received signal amplitudes, with and without the sample, and the thickness of the sample are used to compute the attenuation coefficient per unit distance as a function of frequency. Mathematically,

\[
G(f, x) = A + \alpha(f, x) \quad \text{Eq. 2}
\]

where \( G(f, x) \) = amplitude ratio (with and without the sample), dB,
\( A \) = a constant describing the reflectivity of the sample, dB,
\( x \) = thickness of the sample, cm,
\( f \) = frequency of RF pulse, Hz,
\( \alpha \) = amplitude attenuation coefficient, dB.

Measuring the amplitude ratio for several samples of the same material, but of varying thickness, and plotting the ratio versus the sample thickness yields the amplitude attenuation coefficient per unit distance. The measurements are repeated using different excitation frequencies to determine the attenuation coefficient dependence on
frequency. The same analysis holds when an overdamped pulse is used instead of several RF pulses and the attenuation coefficient is calculated from the frequency spectrum of the received signals (Papadakis, 1976). The frequency analysis of the RF pulses is a convenient method of computing the attenuation dependence on frequency content of the ultrasonic signals.

**Signal Modification**

As mentioned previously, an ultrasonic wave incident on tissue samples is modified due to reflection, transmission, attenuation, and geometrical diffraction.

Reflection and transmission at the interface of two tissue layers can be described by the characteristic impedances of the tissue layers (Ahuja, 1979; Kinsler and Frey, 1950). The characteristic impedance is defined as the product of the wave velocity in and the density of tissue. Mathematically, the characteristic impedance is equal to

$$Z = DV$$

Eq. 3

where $Z$ = characteristic impedance, Kg/cm$^2$ - sec,

$D$ = density of medium, Kg/cm$^3$,

$V$ = propagation velocity, cm/sec.

The relation of the reflected and transmitted waves to the incident wave will be analyzed in the Model Development section of this dissertation.

Attenuation of ultrasonic waves in tissue is a complex process that involves several mechanisms contributing to the energy loss. These
mechanisms include: a) thermal or absorption loss, i.e., heat
dissipation in tissue; b) relaxation-process loss, i.e., energy loss due
to structural relaxation; and c) scattering process loss, i.e., energy
loss due to the non-homogeneous nature of the tissue (Bhatia, 1967;
Dunn, et al., 1969; Johnston, et al., 1979). Experimental evaluation of
attenuation by tissue samples indicates that these mechanisms can be
described by tissue attenuation coefficients. Figure 2 shows the
amplitude attenuation coefficient dependence on frequency as compiled by
Goldman and Heuter (1956) from the work of a number of investigators.
The ordinate is the amplitude attenuation coefficient divided by the
frequency \( (\alpha /f) \). The nearly constant value of \( (\alpha /f) \) for a given
tissue type indicates that the amplitude attenuation coefficient is
linearly dependent on frequency for the frequency range shown.

Geometrical diffraction is another form of energy loss due to wave
spreading as it propagates through tissue layers. Due to the complexity
of the diffraction effect on ultrasonic waves, the energy loss was
evaluated by numerical integration and the results were documented in a
tabular form (Benson and Kiyohara, 1974; Khimunin, 1972). Figure 3
shows graphically the effects of geometrical diffraction on the
amplitude and phase of the propagating pressure wave. As the pressure
wave travels in a medium, the amplitude is attenuated and the phase is
modified. These effects were evaluated by several investigators, as a
function of a dimensionless normalizing parameter \( S = \pi \lambda / r^2 \) to obtain a
universal curve for any transducer and medium. As the distance from the
transducer increases without limit, the loss approaches 6dB per
Figure 2. Acoustic amplitude attenuation coefficient for several mammalian tissues (Goldman and Heuter, 1956)
Loss equals amplitude attenuation, phase angle equals delayed phase difference (goes to a limit of $\pi/2$ radians), $x =$ distance from transducer, $\lambda =$ wavelength, and $r =$ radius of transducer (Papadakis, 1975).

Figure 3. Geometrical diffraction effects on traveling pressure wave
doubling of distance and the phase approaches a limit of \( \pi/2 \). Rhyne (1977) formulated an exact solution for geometrical diffraction by analyzing the pulse response of the transfer function of the pressure wave and media. The exact solution is difficult to implement since it includes several trigonometric and Bessel functions; however, a useful approximate form of Rhyne's solution will be developed in the Model Development section of this dissertation.

**Tissue Characterization**

Prevailing tissue characterization techniques are based on qualitative or quantitative evaluation of tissue signatures. Qualitative techniques depend on the interpretation of ultrasonic images of the examined areas; quantitative techniques are based on mathematical models that predict the relation of tissue physical and acoustical parameters to the ultrasonic signals modification.

Kobayashi (1979) examined the ultrasonic images (using the pulse-echo method) of several tumors in the female breast. The interpretation of these images was based on three conditions (boundary echoes, internal echoes and attenuation shadowing). Kobayashi concluded that these conditions were indicative of the tumor's pathological state. A benign tumor had an image of regular boundary echoes, absence of internal echoes, and lateral shadow. In contrast, a malignant tumor had an image of irregular boundary echoes, multiple internal echoes, and posterior shadow. However, Kobayashi found that necrotic fat regions had images comparable to malignant tumor images.
Taylor and Milan (1976) employed the pulse-echo technique to obtain the ultrasonic images of normal and abnormal spleens. As a result, the number and amplitude of the reflected signals from the internal structure were related to the pathological state of the spleen. An acute leukemic spleen had more internal reflected signals with larger amplitudes than a normal spleen. Furthermore, the number and amplitude of the internal reflected signals from a chronic inflammatory spleen exceeded those of normal and leukemic spleens. Consequently, Taylor and Milan suggested a method to quantitate their observation. A histogram of number versus amplitude of the reflected signals was prepared for each case and the mean amplitude was computed. As a result, these histograms only showed the trend of the expected observations and the mean amplitude had a large standard deviation in each case. Taylor and Milan concluded that this technique was useful in detecting gross abnormalities and it has less significance when the abnormalities were slight.

The Bragg diffraction principle was employed by Lele, et al. (1976) to characterize the internal structure of tissue. The Bragg diffraction condition for constructive interference is

\[ n \lambda = 2d \sin \theta \]  

Eq. 4

where \( \lambda \) = wavelength of ultrasonic signal in tissue,
\( n \) = integer number,
\( d \) = distance between adjacent scatters in tissue,
\( \theta \) = angle from the scattered signal path.
Lele, et al. measured the amplitudes of scattered signals from several tissue samples as the angle between the transmitting and receiving transducers was varied. The scattering profile (amplitude versus angle) was used to compute the average periodicity angle of successive amplitude maxima. These measurements were repeated at several different frequencies; consequently, the average periodicity angle was found as a function of frequency (frequency plot). Lele, et al. found that the frequency plot was more adequate than the scattering profile as a tissue signature. The repeatability of the scattering profile was not obtained; however, the average periodicity angle showed slight variation. Furthermore, most of the frequency plots had correlation coefficients between 0.8 and 0.93 with some exceptions that had a value about 0.5. Lele, et al. concluded that the frequency plot could be used as tissue signature and they proved that calf liver, calf muscle, and pig liver samples had distinct intercepts and slopes on the corresponding frequency plot.

Dines and Kak (1979) presented a model that related the integrated attenuation coefficient to the frequency shift of the ultrasonic wave spectra due to tissue presence. Mathematically, the formulation of the model is

$$y = \frac{(f_0 - f_r)}{2A} \quad \text{Eq. 5}$$

where $y = \sum_{n=1}^{i} \alpha_n X_n$, integrated attenuation coefficient of tissue,
$\alpha_n$ = amplitude attenuation coefficient of nth tissue layer,
$X_n$ = thickness of nth layer,
This simple model was developed by modeling the ultrasonic power spectrum by a Gaussian function. In order to implement the model for tissue characterization and imaging, Dines and Kak employed the pulse-transmission method and indicated that the technique was suitable for tomography of the female breast for cancer detection. The technique required two transducers (transmitter and receiver) to scan the examined area. The transmitted signals were measured by the receiver under two conditions. First, measurements were taken with the tissue to be examined placed between the two transducers. Second, measurements were taken without the tissue. Furthermore, Fourier analysis was used to obtain the center frequency of each received signal as required by their model (Equation 5). The integration attenuation coefficient as expressed in Equation 5 corresponded to all tissue layers that were in the path of the transmitted signal. Therefore, in order to reconstruct the image of the examined area and characterize tissue layers from the integrated attenuation coefficient, the measurements were repeated to make a 180° scan of the examined region. The scanning procedure was achieved by 18 projections at angular increments of 10° and each projection had 56 translational samples (2 mm sampling interval). Consequently, the received ultrasonic waveforms were analyzed to obtain...
18 x 56 = 1008 equations in the form of Equation 5. These equations with the corresponding direction of wave propagation were solved to give the integrated attenuation coefficient of each region. Dines and Kak proved the validity of their model by constructing the image of an excised dog heart from the integrated attenuation coefficient relation to the frequency shift. Although Dines and Kak's model is very adequate and simple, extensive measurements and equations are required to obtain the integrated attenuation coefficient of each region.

The Kuk, et al. (1979) model requires the use of the pulse-echo method. This model is used to estimate the attenuation coefficient of liver from the frequency spectrum of the reflected signals. Fourier analysis is used to compute the magnitude spectrum of the reflected signals to obtain the attenuation dependence on frequency. Furthermore, the boundaries of the liver tissue layers were modeled as linear filters with random zero-mean Gaussian impulse responses in order to account for geometrical effects of liver samples on ultrasonic signal modification. They tested the model by taking measurements from refrigerated and formalin-fixed liver samples. Their results of estimating the amplitude attenuation coefficient varied and they attributed the wide variation to the inadequate geometrical modeling of the liver samples.

The Levi and Keuwez (1979) model describes the attenuation dependence on frequency for a tissue mass using the pulse-echo method. Tissue mass by Levi and Keuwez's definition consists of one or more tissue layers, i.e., a uterine fibroma consists of normal muscle surrounding pathological tissues. The model requires measuring the
amplitudes of the reflected signals at two frequencies to obtain the differential attenuation coefficient of each examined mass. Mathematically, the formulation of the model is

$$\gamma = \frac{1}{2d} [A(f_1) - A(f_2)]$$  \hspace{1cm} \text{Eq. 6}$$

where $\gamma$ = differential attenuation coefficient, dB/cm,

$d$ = thickness of tissue mass, cm,

$A$ = amplitude ratio of the two reflected signals at the mass interfaces, dB,

$f_1, f_2$ = center frequencies of employed transducers, Hz.

Two narrow bandwidth transducers were used for each set of measurements (2 MHz and 4 MHz). Levi and Keuwez computed the differential attenuation coefficients (unique tissue mass signatures) of normal uteri, leiomyomas, and ovarian cysts, using in vivo measurements. They attributed the variation and dispersion of their results to the tumors' orientations with respect to the abdominal wall (oblique incidence) and geometrical diffraction effects on the measurements.

The above mentioned models are valid and useful for tissue characterization. Furthermore, the concepts of these models and their corresponding measurement techniques are representative of tissue classification methods that can be utilized for in vivo measurements. The qualitative techniques rely on individual expertise, i.e., the interpretation of the images is similar to a medical doctor's interpretation of an X-ray image. Consequently, there is a lack of accuracy and repeatability due to the subjective evaluation of the
qualitative techniques. The quantitative techniques are developed to allow for accurate and consistent tissue classification. Furthermore, the quantitative techniques measure one acoustical parameter (attenuation coefficient) or two signatures (intercept and slope of frequency plot) that are the basis of tissue characterization.

Research Objective

The objective of this research is to develop a model that predicts the acoustical transfer function of each tissue layer in a multilayered tissue mass by the pulse-echo technique. The acoustical transfer function is defined in this dissertation as the magnitude ratio of two reflected signals from the boundaries of the corresponding tissue layer. This model accounts for reflection and transmission at the boundary between two tissue layers and attenuation (due to both energy absorption and geometrical diffraction) within each tissue layer.

The model assumes normal incidence of ultrasonic signals that are generated by a circular transducer operating in the transmitting-receiving mode. Furthermore, it is assumed that each layer consists of one type of homogeneous tissue and has a thickness greater than the axial resolution of the ultrasonic signal, i.e., the spatial length of the pressure wave in tissue. This assumption requires tissue thickness greater than 2.0 mm for an overdamped signal of 1.2 μsec duration. Therefore, the model can be used in diagnostic ultrasound systems to determine, in vivo, the size and type of each tissue layer by recording the ultrasonic echoes from the tissues of any region to be examined in
the body. Tissue type and size can be determined from the acoustical tissue parameters that are implemented in the model to predict the tissue transfer function.

Tissue identification will be based on three acoustical parameters as tissue signatures: characteristic impedance, propagation velocity, and amplitude attenuation coefficient. Consequently, a more accurate tissue classification can be achieved by considering three acoustical parameters that distinctly characterize tissue type. Furthermore, the thickness of each tissue layer can be determined from the propagation velocity and residence time of the corresponding tissue layer.
MODEL DEVELOPMENT

The model has been developed to predict the effects of reflection and transmission on pressure waves normal to interfaces of tissues, absorption loss of energy in each tissue, and geometrical diffraction of pressure waves. This model holds for normal incidence of pressure waves that are generated from a circular ultrasonic transducer operating as a transmitter and receiver.

Governing Equations

Reflection and Transmission

The incident pressure wave at the interface between two tissues is divided into a reflected and a transmitted pressure wave. The following rules hold for normal incidence (Kinsler and Frey, 1950):

1. Acoustic pressures on the two sides of the interface are equal.

2. Particle vector velocities normal to the interface are equal.

These rules translate into the following relationships:

\[ P_I + P_R = P_T \]  \hspace{1cm} \text{Eq. 7}
\[ \vec{U}_I + \vec{U}_R = \vec{U}_T \]  \hspace{1cm} \text{Eq. 8}

where \( P \) = acoustic pressure, Kg/cm-sec²,
\( \vec{U} \) = particle vector velocity, cm/sec,
I = pertaining to incident wave,
R = pertaining to reflected wave,
T = pertaining to transmitted wave.
Solving these equations with the relation of particle velocity to pressure,

\[ \bar{u} = \pm \frac{P}{Z} \]  \hspace{1cm} \text{Eq. 9}

where \( Z \) = acoustical impedance of the medium (Kg/cm\(^2\)-sec), the following equations are obtained for the arrangement in Figure 4:

\[ \frac{P_{R1}}{Z_1} - \frac{P_{R1}}{Z_1} = \frac{P_{T2}}{Z_2} \]  \hspace{1cm} \text{Eq. 10}

\[ \frac{P_{R1}}{P_{II}} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]  \hspace{1cm} \text{Eq. 11}

\[ \frac{P_{T2}}{P_{II}} = \frac{2Z_2}{Z_2 + Z_1} \]  \hspace{1cm} \text{Eq. 12}

Equations 11 and 12 illustrate the relation of the reflected and transmitted waves to the incident pressure wave.

**Absorption Loss**

When a pressure wave travels in a medium, part of the wave energy is absorbed from the wave by the medium. The amplitude of the propagating wave decays exponentially with distance due to the energy loss (Bhatia, 1967; Dunn, 1965). The attenuation of the pressure wave is also a function of frequency and depends on the type of the medium (Bhatia, 1967; Goss, et al., 1978) as previously shown in Figure 2.
$P_o$ is the original incident wave from the transducer. $P_n$ is the pressure wave in nth medium. I = incident, R = reflected and T = transmitted.

Figure 4. Pressure wave propagation in multilayered media
Consequently, the following equations hold for the arrangement in Figure 4:

\[ P_{II} = P_0 \exp \left[ -x_1 \alpha_1(f) \right] \]  
\[ P_{I2} = P_{T2} \exp \left[ -(x_2-x_1) \alpha_2(f) \right] \]

where \( \alpha_1(f) \) is the amplitude absorption coefficient per unit distance and is a function of the frequency, \( f \), of the pressure wave in the \( i \)th medium.

**Geometrical Diffraction**

Geometrical diffraction is the divergence of the ultrasonic beam and causes spreading of the pressure wave; therefore, energy is lost as the pressure wave propagates. The attenuation due to geometrical diffraction has been analyzed by numerical integration for a circular transducer and the results have been tabulated by several researchers (Benson and Kiyohara, 1974).

The exact solution of geometrical diffraction (Rhyne, 1977) was simplified by the author and found sufficient to represent the amplitude attenuation of the pressure wave. The simplified version of Rhyne's formula is:

\[ \frac{P_x}{P_0} = \left[ \frac{\cos \left( \frac{2 \pi x}{a} \right) - J_0 \left( \frac{2 \pi x}{a} \right)}{\sin \left( \frac{2 \pi x}{a} \right) - J_1 \left( \frac{2 \pi x}{a} \right)} \right]^2 \left[ \frac{\sin \left( \frac{2 \pi x}{a} \right) - J_0 \left( \frac{2 \pi x}{a} \right)}{\cos \left( \frac{2 \pi x}{a} \right) - J_1 \left( \frac{2 \pi x}{a} \right)} \right]^{1/2} \]

where \( P_x \) = pressure wave amplitude at a distance \( x \) in medium,  
\( P_0 \) = pressure wave amplitude at the transducer face,  
\( J_0, J_1 \) = Bessel functions of the first kind,
Equation 15 was further simplified in order to obtain a less complicated solution, thus reducing the computation time devoted to evaluation of geometrical diffraction. Series expansion and term collection of the trigonometric and Bessel functions yield the following approximation:

\[
\frac{p_x}{p_o} \approx F(S) = \pi \exp\left[-\frac{1}{3}(\frac{S}{\pi})^2\right], \text{ for } S \geq \pi \tag{Eq. 16}
\]

where \( F(S) \) will be used in the analysis of the general model. Equation 16 is an adequate presentation of geometrical diffraction effect on the pressure wave amplitude. Figure 5 shows the percent error between Equation 16 and the tabulated results of geometrical diffraction by Benson and Kiyohara (1974). The error becomes negligible as the parameter \( S \) increases. The phase difference due to geometrical diffraction is assumed negligible; furthermore, the delayed phase of each reflected signal will be proven to be several orders of magnitude greater than the phase difference due to geometrical diffraction in the Model Verification section.

For the arrangement of Figure 4, the following relations hold:

\[
\frac{p_x}{p_o} = F(S_i) \tag{Eq. 17}
\]

\[
S_i = \sum_{n=1}^{i} \left( \frac{X_n - X_{n-1}}{r^2} \right) \frac{\lambda n}{r^2} = \sum_{n=1}^{i} \left( \frac{X_n - X_{n-1}}{r^2} \right) \frac{V n}{r^2} \tag{Eq. 18}
\]
Figure 5. Error introduced by approximating geometrical diffraction effect on pressure wave amplitude.
where $P_{II}$ = incident pressure wave at $i$th interface, Kg/cm-sec$^2$,
$P_0$ = incident pressure wave at the transducer face, Kg/cm-sec$^2$,
$x_i - x_{i-1}$ = thickness of $i$th medium, cm,
$\lambda_i$ = wavelength of $i$th medium, cm,
r = radius of the transducer, cm,
$V_i$ = velocity of sound in $i$th medium, cm/sec,
f = frequency, Hz.

General Model

The following theoretical analysis that predicts the modification of ultrasonic signals by multilayered media applies for normally incident ultrasonic pressure waves from a circular ultrasonic transmitter-receiver.

At the first interface (at $x=x_1$) of Figure 4, Equations 13 and 17 lead to the following relationship (the combined effects of attenuation and diffraction):

$$\frac{P_{II}}{P_0} = \exp[-x_1(jk_1+a_1)]F(S_1)$$

Eq. 19

where $k_1 = \frac{2\pi f}{V_1}$, the wavelength constant of the first medium,
$x_1 k_1$ = phase delay of incident wave at first interface.
$j = \sqrt{-1}$
Substituting Equation 19 into Equations 11 and 12 we obtain

\[
\frac{P_{R1}}{P_0} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \exp[-x_1(jk_1 + \alpha_1)]F(S_1) \quad \text{Eq. 20}
\]

\[
\frac{P_{R2}}{P_0} = \frac{2Z_2}{Z_2 + Z_1} \exp[-x_1(jk_1 + \alpha_1)]F(S_2) \quad \text{Eq. 21}
\]

At the second interface, the same analysis holds to give the following results:

\[
\frac{P_{R2}}{P_0} = \frac{2Z_2}{Z_2 + Z_1} \cdot \frac{Z_3 - Z_2}{Z_3 + Z_2} \exp[-x_1(jk_1 + \alpha_1) - (x_2 - x_1)(jk_2 + \alpha_2)]F(S_2) \quad \text{Eq. 22}
\]

\[
\frac{P_{R3}}{P_0} = \frac{2Z_2}{Z_2 + Z_1} \cdot \frac{2Z_3}{Z_3 + Z_2} \exp[-x_1(jk_1 + \alpha_1) - (x_2 - x_1)(jk_2 + \alpha_2)]F(S_2) \quad \text{Eq. 23}
\]

Equations 20 through 23 are the basic relations that are needed to obtain the acoustical transfer function of the second medium.

When the reflected signal returns to the surface of the ultrasonic transducer, all distances traveled by the ultrasonic waves are doubled and the ultrasonic waves are modified by transmission through the corresponding interface. In mathematical form, Equations 20 and 22 take the following forms (at \(x = 0\)):

\[
\frac{P_{R1}}{P_0} = R_1 \exp[-2x_1(jk_1 + \alpha_1)]F(2S_1) \quad \text{Eq. 24}
\]

\[
\frac{P_{R2}}{P_0} = R_2(1-R_1^2)\exp[-2x_1(jk_1 + \alpha_1) - 2(x_2 - x_1)(jk_2 + \alpha_2)]F(2S_2) \quad \text{Eq. 25}
\]

where \(R_1 = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i}\), reflection coefficient of \(i\)th interface.
The recorded voltage waveform is related to the reflected pressure wave by the transfer function of the transducer and the recording system as in Equation 26.

$$E_i = P_{Ri} H(f) \quad \text{Eq. 26}$$

where $E_i =$ recorded waveform of $i$th interface, volt,

$P_{Ri} =$ reflected pressure wave from $i$th interface, Kg/cm-sec$^2$,

$H(f) =$ transfer function of the system, volt-cm-sec$^2$/Kg.

The transfer function describes the transformation of the pressure wave to a voltage waveform by the transducer and the recording system. Piezoelectric ultrasonic transducers have a receiving constant of 0.025 Volt-cm-sec$^2$/Kg (Wells, 1969); however, any non-linearity due to frequency response of the system is considered in the transfer function.

Substituting Equation 26 into Equations 24 and 25, we obtain

$$E_1 = R_1 \exp[2x_1(jk_1+\alpha_1)]F(2S_1)H(f) \quad \text{Eq. 27}$$

$$E_2 = R_2(1-R_1^2)\exp[-2x_1(jk_1+\alpha_1)-2(x_2-x_1)(jk_2+\alpha_2)]F(2S_2)H(f) \quad \text{Eq. 28}$$

Equations 27 and 28 indicate the relation of the recorded voltage waveforms to the acoustical tissue parameters. The acoustical transfer function of the second medium is found by substituting Equation 16 into the ratio of $E_1$ and $E_2$ in Equations 27 and 28,

$$\frac{E_1}{E_2} = \frac{R_1}{R_2(1-R_1^2)} \exp[2L_2^2\alpha_2^2 - \frac{1}{3}\left(\frac{\pi}{2S_1}\right)^2 + \frac{1}{3}\left(\frac{\pi}{2S_2}\right)^2 \frac{S_2}{S_1}] \exp(j2L_2k_2) \quad \text{Eq. 29}$$
where \( L_i \) = the thickness of the \( i \)th medium, \( x_i = x_{i-1} \).

Following the same analysis, the voltage ratio of two consecutive echoes is evaluated theoretically by Equation 30:

\[
\frac{E_{i-1}}{E_i} = \frac{R_{i-1}}{R_i(1-R_{i-1})^2} \exp\left[2L_i \alpha_i - \frac{1}{3}(\frac{\pi}{2S_{i-1}})^2 + \frac{1}{3}(\frac{\pi}{2S_i})^2 \right] \frac{S_i}{S_{i-1}} \exp(j2L_i k_i)
\]

Eq. 30

Furthermore, the following relationships are substituted in Equation 30 to give the general model in Equations 35 and 36:

1. \( L_i = \frac{V_i T_i}{2} \)  
   Eq. 31
   where \( L_i \) = thickness of \( i \)th medium,
   \( T_i \) = residence time of pressure wave in \( i \)th medium,
   \( V_i \) = propagation velocity of pressure wave in \( i \)th medium,

2. \( \alpha_i = \alpha_{oi}^f \)  
   Eq. 32
   The amplitude absorption coefficient of tissue is assumed to be linearly dependent on frequency for \( f < 10 \) MHz, as shown previously in Figure 2,

3. Equation 18 can be rewritten as

\[
S_i = \sum_{n=1}^{i} \frac{T_n V_n}{2r^2 f}
\]

Eq. 33

4. \( k_i = \frac{2\pi f}{V_i} \); wavelength constant of \( i \)th medium,

5. Equation 30 is thus reduced to the following format:

\[
\frac{E_{i-1}}{E_i} = G_i' \exp(j\theta_i)
\]

Eq. 34
where \( G'_i = \text{magnitude ratio of } i\text{th medium transfer function}, \)

\( \theta'_i = \text{delayed phase difference of the transfer function}. \)

Therefore, the general model of the acoustical transfer function of tissue is,

\[
G'_i = k'_i \exp(k'_i f + k'_3 f^2)
\]

\( \theta'_i = 2\pi f T_i \)  

Eq. 35

Eq. 36

In dB notation, Equation 35 takes the form,

\[
G_i = k_i + k_2 f + k_3 f^2
\]

Eq. 37a

where \( G_i = \text{magnitude of the transfer function, dB}, \)

\[
k_{i} = 20 \log \left[ \frac{R_{i-l}}{R_i (1-R_{i-l})} \sum_{n=1}^{i} \frac{V_n T_n}{n^2} \right], \text{dB},
\]

Eq. 37b

\[
k_{2i} = 8.6859 V_i T_i \alpha_i, \text{dB/Hz},
\]

Eq. 37c

\[
k_{3i} = \frac{8.6859 \pi^2 r_i^4}{3} \left[ \left( \frac{1}{\sum_{n=1}^{i} V_n T_n} \right)^2 - \left( \frac{\sum_{n=1}^{i-l} V_n T_n}{\sum_{n=1}^{i} V_n T_n} \right)^2 \right], \text{dB/Hz}^2,
\]

Eq. 37d

\[
R_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i}.
\]

Eq. 37e

The general model development indicates that the acoustical transfer functions of a multilayered tissue mass can be characterized by measuring two consecutive echoes from the near and far boundaries of
each tissue layer. Since the ratio of the two echoes and the delayed phase difference are functions of frequency for each tissue layer, the tissue transfer function data can be obtained by calculating the magnitude and phase spectra of the reflected signals. An overdamped (low Q-factor) ultrasonic transducer can be used to improve axial resolution and to obtain a wide frequency bandwidth as shown in Figure 6. Improving the axial resolution will allow a more precise differentiation of tissue boundaries. A wide bandwidth allows collection of more data points as a function of frequency.

The reflection coefficient, R, of the first interface has to be determined. A water delay path in a flexible plastic bag (see Figure 7) of predetermined characteristic impedance will serve the following purposes:

1. providing knowledge of the reflection coefficient of the first interface,
2. satisfaction of Equation 13; \( S \geq \pi \),
3. extension of the examination field in tissue by reducing the multiple reflections from the first interface.

The general model differs from the aforementioned models (cf.p.10) in three ways. First, the proposed model evaluates three acoustical parameters (characteristic impedance, propagation velocity, and amplitude attenuation coefficient) and one physical parameter (thickness) of each tissue layer. The other models were based on evaluating one acoustical parameter (amplitude attenuation coefficient) or two signatures (intercept and slope of frequency plot). Second, the
acoustical parameters that can be evaluated by the proposed model are already compiled in literature. The intercept and slope of the frequency plot need to be measured and compiled for each tissue type and the uniqueness of the two signatures need to be proved. Finally, there is no need for separate measurements to evaluate the geometrical diffraction effect on the pressure wave. This general model describes the diffraction effect by a simple formula (Equation 16), unlike the Levi and Keuwez (1979) and Kuk, et al. (1979) models which do not consider geometrical diffraction. Considering these three points of difference, the proposed general model is more complete in describing the acoustical transfer function of tissue and characterizing tissue layers.
(A), (C) - high Q-factor transducer has under-damped response; the transducer is most sensitive at the natural frequency.  
(B), (D) - low Q-factor transducer has over-damped time response; axial resolution is enhanced with decreasing signal duration (Wells, 1969).

Figure 6. Effect of Q-factor on transducer response
Figure 7. Schematic of delay path
MODEL VERIFICATION

Experiments with both solid materials and tissue samples were conducted to verify the adequacy of the model in predicting the acoustical transfer function of these samples. Solid materials, Lucite® and rubber, were used to test the validity of the model over the different acoustical parameters of the materials. Furthermore, data were obtained from three positions across a tissue sample to test the repeatability of the measurements. The tissue sample consisted of the backfat and trapezius muscle of swine.

The model was verified on the basis of the statistical correlation between the predicted values and the measured data from each sample. Furthermore, the physical and acoustical parameters were computed on the basis of the proposed model.

Experimental Procedure

Samples of different materials were tested as shown in the arrangement shown in Figure 8. The solid samples were flat plates with dimensions of 100 x 100 mm. The thickness of the Lucite® sample was 6.1 mm and the rubber sample was 11.7 mm. The tissue sample had the dimensions of 70 x 70 mm and the thicknesses of the fat and muscle layers were about 10 mm to 15 mm.

The mechanical manipulator, rectilinear and angular manipulator, held the ultrasonic transducer in the required position. The ultrasonic
The transducer is excited by the pulse generator and the reflected signals from the sample are digitized by the analog-to-digital converter.

Figure 8. Experimental arrangement for data acquisition
transducer was an immersion type transducer (Panametrics\textsuperscript{1} 5.0MHz/0.25" diameter). The pulse generator excited the transducer with brief electrical pulses of 1.0 μs duration at a repetition rate of 300 Hz. As a result, the transducer transformed the electrical pulses to pressure waves. The ultrasonic pressure waves were partially reflected from the front and back surfaces of the sample, i.e. water-sample and sample-water interfaces. In the tissue sample case, the reflected pressure waves occurred at the water-fat, fat-muscle, and muscle-fat interfaces respectively. The reflected pressure waves were transformed back to electrical energy (voltage waveforms) by the same transducer. These voltage waveforms, analog signals, were response averaged 64 times in order to reduce the random noise in the signals. The amplitudes of the sampled signals were quantized to $\pm 256$ levels. The sampler was an analog-to-digital (A/D) converter, a Tektronix\textsuperscript{2} 7912AD Programmable Digitizer, operating at sampling rates of 102.4 MHz or 51.2 MHz. These sampling rates were adequate to digitize the reflected signals since the ultrasonic transducer had a center frequency of 5.0 MHz. The digitized data were stored in the memory of the computer, a Tektronix\textsuperscript{2} 4052. The pulse generator was also used for external triggering of the A/D converter. Consequently, an internally delayed signal set the reference time for the delayed phase of each echo.

\textsuperscript{1} Panametrics, Inc., Waltham, Massachusetts, 02154.
\textsuperscript{2} Tektronix, Inc., Beaverton, Oregon, 97075.
Data Analysis and Results

The digitized data of the reflected signals were gated separately by a rectangular window, i.e., data from each reflected signal were analyzed individually. Figures 9 through 11 show the original signal and the gated echoes of the Lucite® sample. The magnitude and phase spectra of each voltage waveform, reflected signals, were calculated by a 512 point Fast Fourier Transform (FFT). The software program in the computer used the data of each waveform and added trailing zeroes to complete the required 512 points. A 512 point FFT yielded a frequency resolution of 0.2 MHz for sampling rate of 102.4 MHz.

Figures 12 through 15 show the magnitude and delayed phase spectra of the reflected signals from the Lucite® sample. The magnitude spectra were used to obtain the acoustical transfer function of the sample, i.e., the magnitude ratio of the first to the second reflected signal (in dB notation). The delayed phase difference between the two reflected signals (see Figure 16) was used to calculate the residence time of the pressure wave in the sample. The residence time was obtained from the slope of the linear relation between the delayed phase difference and frequency as expressed in Equation 36.

Figures 17 through 21 show the acoustical transfer functions of the tested samples and the corresponding transfer functions that are predicted by the model (Equation 37). The constant parameters of Equation 37 were calculated using the acoustical parameters of each sample and the corresponding residence time as shown in Table 1. The constant acoustical parameters of the tissue sample vary with the
examined position due to the different thicknesses of the fat and muscle layers at each position.

The measured data were analyzed by a second degree polynomial regression in order to estimate the constant and frequency dependent parameters of each sample, i.e., the three parameters of the general model as expressed in Equation 37. The second degree polynomial regression was computed on the basis of the least squares estimators method (Walpole and Myers, 1978). The estimate of these three parameters and the corresponding measured residence time of each sample were used to compute the physical and acoustical parameters of the samples. Equation 37 shows the relationship of the model parameters and residence time to the physical and acoustical parameters of the sample. The results of this mathematical computation are shown in Table 2. Furthermore, Table 3 shows the acoustical parameters of each medium as published by several investigators.
The waveform is digitized at a sampling frequency of 51.2 MHz. The signals are from the front and back surfaces of the sample, water-Lucite\textsuperscript{®} and Lucite\textsuperscript{®}-water interfaces respectively.

Figure 9. The delayed voltage waveform of the reflected signals
A rectangular window gates the data points of the first echo only and suppresses the data outside the gate. The waveform corresponds to the water-Lucite® interface.

Figure 10. The voltage waveform of the first gated echo.
A rectangular window gates the data points of the second echo only and suppresses the data outside the gate. The waveform corresponds to the Lucite®-water interface.

Figure 11. The voltage waveform of the second gated echo
Figure 12. The magnitude/frequency spectrum of the first gated echo, from water-lucite interface.
Figure 13. The magnitude/frequency spectrum of the second gated echo, from Lucite®-water interface
Figure 14. The delayed phase spectrum of the first gated echo, from water-Lucite interface
Figure 15. The delayed phase spectrum of the second gated echo, from Lucite®-water interface
The residence time of the traveling wave is calculated from the slope of the linear relation between the delayed phase difference and frequency (Equation 36). The slope gives a residence time of 4.46 us in the Lucite® sample.

Figure 16. The delayed phase difference between the first and second echoes (water-Lucite®-water)
(●) - experimental evaluation of the acoustical transfer function from the magnitude spectra of the reflected signals and (——) - model prediction of the acoustical transfer function.

Figure 17. Acoustical transfer function of water-Lucite®-water interfaces
(●) - experimental evaluation of the acoustical transfer function from the magnitude spectra of the reflected signals and (—) - model prediction of the acoustical transfer function.

Figure 18. Acoustical transfer function of water-rubber-water interfaces
Experimental evaluation and model prediction, respectively, of the acoustical transfer function of (○), (—) - water-fat-muscle interfaces; and (●), (——) - fat-muscle-fat interfaces.

Figure 19. Acoustical transfer function of backfat and trapezius muscle of swine (first position)
Experimental evaluation and model prediction, respectively, of the acoustical transfer function of (○), (→) - water-fat-muscle interfaces; and (●), (—) - fat-muscle-fat interfaces.

Figure 20. Acoustical transfer function of backfat and trapezius muscle of swine (second position)
Experimental evaluation and model prediction, respectively, of the acoustical transfer function of (○), (—) - water-fat-muscle interfaces; and (●), (—) - fat-muscle-fat interfaces.

Figure 21. Acoustical transfer function of backfat and trapezius muscle of swine (third position)
Table 1. Acoustical parameters of the sample transfer function and residence time of the ultrasonic propagating wave

<table>
<thead>
<tr>
<th>INTERFACE</th>
<th>RESIDENCE TIME</th>
<th>ACOUSTICAL PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_{1-1}$</td>
<td>$T_1$</td>
</tr>
<tr>
<td>Water-Lucite®-Water</td>
<td>179.20</td>
<td>4.46</td>
</tr>
<tr>
<td>Water-Rubber-Water</td>
<td>135.90</td>
<td>14.08</td>
</tr>
<tr>
<td>(first position)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water-Fat-Muscle</td>
<td>92.20</td>
<td>18.20</td>
</tr>
<tr>
<td>Fat-Muscle-Fat</td>
<td>18.20</td>
<td>14.41</td>
</tr>
<tr>
<td>(second position)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water-Fat-Muscle</td>
<td>91.40</td>
<td>20.00</td>
</tr>
<tr>
<td>Fat-Muscle-Fat</td>
<td>20.00</td>
<td>12.50</td>
</tr>
<tr>
<td>(third position)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water-Fat-Muscle</td>
<td>88.60</td>
<td>20.60</td>
</tr>
<tr>
<td>Fat-Muscle-Fat</td>
<td>20.60</td>
<td>16.20</td>
</tr>
</tbody>
</table>

*For solid materials, attenuation coefficient is determined by the pulse-transmission method; for biological tissue, it is taken from Parry and Chivers (1979).
Table 2. Physical and acoustical parameters of samples as predicted by the general model (Equation 37)

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>PHYSICAL PARAMETER</th>
<th>ACOUSTICAL PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X (mm)</td>
<td>Z x 10^-2 (Kg/cm^2-sec)</td>
</tr>
<tr>
<td>Lucite®</td>
<td>6.05</td>
<td>3.32</td>
</tr>
<tr>
<td>Rubber</td>
<td>11.92</td>
<td>1.99</td>
</tr>
<tr>
<td>Fat</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First Position</td>
<td>13.25</td>
<td>1.38</td>
</tr>
<tr>
<td>Second Position</td>
<td>14.40</td>
<td>1.38</td>
</tr>
<tr>
<td>Third Position</td>
<td>14.94</td>
<td>1.37</td>
</tr>
<tr>
<td>Muscle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First Position</td>
<td>11.52</td>
<td>1.72</td>
</tr>
<tr>
<td>Second Position</td>
<td>9.94</td>
<td>1.68</td>
</tr>
<tr>
<td>Third Position</td>
<td>12.88</td>
<td>1.69</td>
</tr>
</tbody>
</table>
Table 3. Acoustical parameters of materials (Kaye and Laby, 1973; Parry and Chivers, 1979)

<table>
<thead>
<tr>
<th>Material</th>
<th>$Z \times 10^{-2}$ (Kg/cm$^2$-sec)</th>
<th>$V \times 10^{-5}$ (cm/sec)</th>
<th>$\alpha$ (dB/cm-MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lucite*</td>
<td>3.16</td>
<td>2.70</td>
<td>1.05$^a$</td>
</tr>
<tr>
<td>Rubber</td>
<td>1.92</td>
<td>1.60</td>
<td>3.72</td>
</tr>
<tr>
<td>Fat</td>
<td>1.38</td>
<td>1.44</td>
<td>0.65</td>
</tr>
<tr>
<td>Muscle</td>
<td>1.70</td>
<td>1.59</td>
<td>1.45</td>
</tr>
</tbody>
</table>

$^a$Measured by the pulse-transmission method.
DISCUSSION AND RECOMMENDATIONS

The focus of the research presented in this dissertation has been the development of an adequate model for tissue characterization. The need for a more versatile model was perceived after reviewing the literature on tissue characterization. Consequently, research was initiated in an attempt to formulate a more complete model. The basis of the analysis was ultrasonic wave modification by tissue; as a result, a model for the acoustical transfer function of tissue was developed.

Figures 17 through 21 show the acoustical transfer functions of the samples as computed from the measured data and the proposed general model. The general model correlates closely with the data of the transfer function within the corresponding frequency ranges. In order to satisfy the proposed formula, describing the geometrical diffraction process, the upper frequency was limited. In other words, the maximum frequency had to satisfy the minimum acceptable value of the universal parameters (S). In the case of biological tissue, the frequency range was limited in order to satisfy the linear dependence of amplitude attenuation coefficient on frequency as previously shown in Figure 2.

For the above mentioned figures, the theoretical values of the model were computed by using the acoustical parameters of the materials as published by several researchers. The acoustical parameters of biological tissue were taken from Parry and Chivers (1979) who compiled the values of velocity and the amplitude attenuation coefficient for several mammalian tissues, including tissues of swine. In the case of solid materials, data were taken from Kaye and Laby (1973). The
measured acoustical parameters of the solid samples agree with the corresponding data compiled by Kaye and Laby (1973) except for the amplitude attenuation coefficient of Lucite®. The measured amplitude attenuation coefficient of Lucite® is 1.05 dB/cm MHz; Kaye and Laby reported a value of 2.0 dB/cm-MHz.

The acoustical parameters, velocity and amplitude attenuation coefficient of each solid sample, were measured by the classical techniques (pulse transit time and pulse-transmission). These techniques were employed due to disagreement between the model prediction and the published value of the amplitude attenuation coefficient of Lucite® sample. The velocity of sound was determined from the measured residence time and the thickness of the sample. Using the pulse-transmission technique, measurements were taken with and without the sample between the transmitter and the receiver. The magnitude spectra of the two signals were calculated by a 512 point FFT. The slope of the magnitude ratio is a measure of the amplitude attenuation in the sample.

Based on Figures 17 through 21, it was concluded that the general model is a valid description of the acoustical transfer function of tissue. The main advantages of the proposed general model are twofold. First, a more accurate estimate of the reflection and the amplitude attenuation coefficients is achieved by considering geometrical diffraction. Second, a better decision about the type of tissue could be made since the model is based on three tissue acoustical parameters as signatures: reflection coefficient, velocity, and amplitude.
The proposed simplified formula for geometrical diffraction modifies the theoretical prediction of the acoustical parameters in two ways because it has two terms; one is constant and one is frequency dependent. The constant term modifies the value of the reflectivity of the sample boundaries. The frequency dependent term modifies the amplitude attenuation coefficient of the sample. For example, the reflection coefficient of fat-muscle-fat interface (first position) is modified by 15% due to geometrical diffraction. Also, the amplitude attenuation coefficient of the muscle layer is modified by 8.5% at 5 MHz as a result of geometrical diffraction. Consequently, for a more accurate estimation of the acoustical transfer function, geometrical diffraction must be considered. Geometrical diffraction is one factor which aids in accurately estimating tissue signatures. However, this general model also considers two other factors: the reflectivity at the tissue boundary and attenuation with tissue. Reflectivity and attenuation modify the acoustical signal and are directly affected by geometrical diffraction. Consequently, the mathematical presentation of these three factors will enable tissue identification from the three acoustical parameters: velocity, amplitude attenuation coefficient, and characteristic impedance.

Table 2 shows the computed acoustical and physical parameters of each sample. These acoustical parameters (predicted by Equation 37) agree with the published values of the parameters (see Table 3) and/or with the parameters that have been determined by the pulse-transmission
method. Furthermore, the physical parameter (thickness) is easily computed from the estimated propagation velocity and measured residence time. The computed thickness of each sample also shows a good agreement with the actual thickness of the corresponding sample.

The predicted acoustical parameters (characteristic impedance, propagation velocity, and amplitude attenuation coefficient) are the basis of tissue characterization. Although each position of the tissue sample has a different thickness, the predicted acoustical parameters of the fat and muscle layers show slight variation with position (see Table 2). Therefore, the three acoustical parameters serve as distinct tissue signatures.

In summary, two main ideas have been presented. First, geometrical diffraction has to be considered for a more accurate estimation of tissue parameters. Second, the general model is a valid presentation of the acoustical transfer function. Finally, it has been shown by one set of measurements that the three tissue acoustical parameters can be evaluated. Thus, a more accurate tissue characterization could be made.

On the basis of the analysis and results presented in this dissertation, two recommendations are offered for future research. The general model, as presented, could be adapted to diagnostic ultrasound systems. Also, consideration should be given to modify the model such that oblique incidence may be examined.
BIBLIOGRAPHY


