1987

Availability analysis of flexible manufacturing system

George Hanna Abdou

Iowa State University

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AVAILABILITY ANALYSIS OF FLEXIBLE MANUFACTURING SYSTEM

Iowa State University

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George Hanna Abdou

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Iowa State University
Ames, Iowa

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NOMENCLATURE

Latin symbols

A    steady state availability
A(t) instantaneous availability: probability that the system will be operating at time t
A(0,t) interval availability
DOUT the desired system output
E    system effectiveness
F    maximum number of failed components to maintain constant production rate
f    number of failed components in each group
H    row vector with elements (1 0 0 ... 0)
I    row vector with elements of initial operating condition
K    matrix to determine the row vector of steady state probability
k_{ac} constant that corresponds to the ath row and the cth column in the transition matrix M.
L    number of series and parallel groups in the system
M    transition matrix
N    number of failure modes
P    row vector with elements of steady state probability
P(s) Laplace transform of the transition probability vector
P(t) transition probability vector
PRATE constant representing system production rate
PRATE(t) expected production rate at time t of a system
Q    set of all system-states
q number of system-states
q_a general element of Q, can either be operating or failing state
Q_f set of failed system-states
Q_o set of operating system-states
q_o number of operating system-states
t implies time-point t; t > 0
u fraction utilization
w constant representing production rate
X_1 number of components in group 1
y Group of states No.
Z_1 subset of failure modes for components in group 1

\[ Z_1 = [f_1, f_2, \ldots, f_N] \text{ such that } \sum_{n=1}^{N} (f_n) \leq X \]

e.g. \( Z_2 = [3,0,1] \) shows that four components in group 2 are down, three components of failure mode 1, 0 of failure mode 2, and 1 of failure mode 3.

Greek Symbols
\( \tau_1(f) \) production rate distribution for parallel components in group 1
\( \lambda_{ni} \) failure rate of the nth failure mode of the ith component in series
\( \lambda_{rn} \) failure rate of the nth failure mode of parallel components during regular operation
\( \lambda_{hn} \) failure rate of the nth failure mode of parallel components during heavy operation
\( \mu_{ni} \) repair rate of the \( n \)th failure mode of the \( i \)th component in series

\( \mu_n \) repair rate of the \( n \)th failure mode of parallel components in either heavy or regular operation

**SUBSCRIPTS**

- \( c \) index for combined system
- \( i \) index for component number in series
- \( h \) implies heavy operation
- \( j_l \) index for number of components in parallel group \( l \)
- \( k \) implies state No. \( k \); \( k=1,2,\ldots,q \)
- \( l \) implies group No. \( l \); \( l=1,2,\ldots,L \)
- \( n \) implies failure mode No. \( n \); \( n=1,2,\ldots,N \)
- \( p \) index for parallel components
- \( S \) index for series components
INTRODUCTION AND LITERATURE REVIEW

Introduction

The term "FMS", Flexible Manufacturing System, is used in different ways in industry. However, according to Kearney and Trecker (Hall, 17), a major U.S. supplier of FMSs: FMS combines the existing technology of Numerical Control (NC) machine tool, automated material handling, and computer hardware and computer software to produce mid-volume and mid-variety of discrete parts.

The classification of a particular FMS results basically from its mode of operation as well as the properties of the three components above. According to the extent of use of the term "flexible", FMS can be classified into the following basic types (7):

1) Flexible Machining Cell (FMC): it consists of one CNC machine tool, interfaced with automated material handling. An articulated arm, robot or pallet changer is sometimes used to load and unload the machine tool.

2) Flexible Machining Systems (FMS): it is highly routing-flexible and product-flexible. It allows several routes for parts, with small volume production of each and consists of FMCs of different types of general purpose machine tools. Within FMS, the various kinds of material handling provide a wide range of flexibility.

3) Flexible Transfer Line (FTL): it is less process-flexible and less capable of automatically handling breakdowns. The layout of this
type is process-driven and the material handling is usually a carousel or conveyor.

The fields of application of FMC, FMS and FTL depend on the production quantity and other features of the parts to be produced. The performance of an FMS is frequently distorted by irregularities caused by components breakdowns. Breakdowns can result from one or a combination of five broad classes of failure in the system: mechanical, electrical, hydraulic, computer hardware and computer software.

The appropriate Markov model to study the availability of FMS is that describing the system as a discrete-state continuous-time Markov process. Numerical values of the appropriate availability are obtained by studying the different failure modes entered by the manufacturing system, as it evolves in time, and setting up and solving the mathematical models described in Chapter 2.

While the concept of availability is well known, the performance measures, such as production rate with lapse of time, are not so well determined. The formulas presented in Chapter 3, are developed by which the expected production rate can be determined after solving the system-state transition matrix.

Due to the numerical complexity of the method, a program in BASIC is designed to determine the transition matrix, steady-state availability and expected production rate. The transient behavior of the state probabilities and the performance measures are determined by a FORTRAN program which is executed under control of the BASIC code.
Problem Definition

The ideal performance measures in an FMS tend to be distorted by irregularities caused by machine breakdowns, tool failure, preventive maintenance, raw material quality and a variety of other short term interruptions. This is a significant problem since it affects the true productive capacity of the FMS.

Performance models of manufacturing systems subject to failures are basic tools to understand and predict accurately the behavior of the system to aid decision making. One of the most practical application of these models is in production schedule planning. Thus, optimum capacity planning can be determined as a result of the modeling. In addition, the time period that the predictive maintenance analyst is concerned with is the time between planned maintenance shutdowns during which components are cleaned, lubricated and adjusted, so that the system will continue to remain in the random failure period (constant hazard failures) the rest of its life. The system availability predicts the actual running time with respect to the scheduled operating time.

The Markovian models and the computer program presented in this research have been developed to analyze different types of FMS and to investigate realistic performance measures. Moreover, effects of desired system output on other system performance are analyzed.

Stochastic processes are used to define the Markovian models and the resulting probabilities are used to evaluate the system availability and consequently the system effectiveness. Table 1 (22) summarizes some of the common expressions of the time dependent and steady-state
availabilities of systems consisting of one, two, or three identical units that operate either in active parallel, or in standby without failures. Calculation of the general solution for time dependent availability can be quite complicated even when the number of states is only moderately large. Method of calculating the availability using a Markov approach was developed by decomposing the system into subsystem technique (1). In many cases, where instantaneous availability is not needed, the steady state availability and the mean time to failure were used to model a parallel system (8,18), and a group of series components (14).

Summary of Previous Research

Much of the previous published research concerning FMS has taken two basic directions: first, most of the recorded work attempts to apply models to evaluate FMS performance without considering the failure concepts. These models can be divided into five classes:

1. Static allocation models
   This type of model simply adds up the total amount of work distributed or assigned to each resource, and estimates the performance. It is static and simple. It ignores all dynamics, all interactions and all uncertainties.

2. Queueing network models
   These models tend to give reasonable estimates of performance. Although, they require relatively little input data, the output measures
### Table 1. Availability of systems comprised of identical units

<table>
<thead>
<tr>
<th>No. of Identical of</th>
<th>Type of system</th>
<th>Instantaneous Availability A(t)</th>
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<tr>
<td>component system</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>$\mu/(\mu+\lambda) + \lambda/(\mu+\lambda) e^{\mu t}$</td>
</tr>
<tr>
<td>2</td>
<td>Standby</td>
<td>$(2\mu / (2\mu^2 + 2\mu \lambda + \lambda^2) )$&lt;br&gt;$-\lambda^2 (s_2 s_1 t - s_1 s_2 t) / s_1 s_2 (s_1 - s_2)$</td>
</tr>
<tr>
<td></td>
<td>Active-parallel</td>
<td>$(\mu^2 + 2\mu \lambda ) / (2\mu^2 + 2\mu \lambda + \lambda^2)$&lt;br&gt;$-\lambda^2 (s_2 s_1 t - s_1 s_2 t) / s_1 s_2 (s_1 - s_2)$</td>
</tr>
<tr>
<td>3</td>
<td>Standby</td>
<td>$(6\mu^3 + 6\mu^2 \lambda + 3\mu \lambda^2) / (6\mu^2 + 6\mu \lambda + 3\mu^2 + \lambda^2)$&lt;br&gt;$+ \lambda^3 (s_2 s_3 (s_2 - s_3) e^{\mu t} - s_1 s_3 (s_1 - s_3) e^{2\mu t} + s_1 s_2 (s_1 - s_2) e^{3\mu t})$&lt;br&gt;$/s_1 s_2 s_3 (s_1 - s_2)(s_1 - s_3)(s_2 - s_3)$</td>
</tr>
<tr>
<td></td>
<td>Active-parallel</td>
<td>$(\mu^3 + 3\mu^2 \lambda + 3\mu \lambda^2 ) / (\mu + \lambda)^3$&lt;br&gt;$+ 6\lambda^3 (s_2 s_3 (s_2 - s_3) e^{\mu t} - s_1 s_3 (s_1 - s_3) e^{2\mu t} + s_1 s_2 (s_1 - s_2) e^{3\mu t})$&lt;br&gt;$/s_1 s_2 s_3 (s_1 - s_2)(s_1 - s_3)(s_2 - s_3)$</td>
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<td>$a_0=0$</td>
<td></td>
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<p>| | |</p>
<table>
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<tr>
<th></th>
<th></th>
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<tbody>
<tr>
<td>$a_1 = -(\lambda+\mu)$</td>
<td>$\mu/(\mu+\lambda)$</td>
</tr>
<tr>
<td>$a_2 = -\frac{1}{2}(2\lambda+3\mu) + (\mu^2 + 4\mu\lambda) \frac{1}{2}$</td>
<td>$(2\mu^2 + 2\mu\lambda)/(2\mu^2 + 2\mu\lambda + \lambda^2)$</td>
</tr>
<tr>
<td>$a_3 = -\frac{1}{2}(2\lambda+3\mu) -(\mu^2 + 4\mu\lambda) \frac{1}{2}$</td>
<td>$(2\mu^2 + 2\mu\lambda)/(2\mu^2 + 2\mu\lambda + \lambda^2)$</td>
</tr>
<tr>
<td>$a_4 = -2(\lambda+\mu)$</td>
<td></td>
</tr>
<tr>
<td>$a_5 = -(\lambda+\mu)$</td>
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$a_1$, $a_2$, and $a_3$ correspond to the three roots of

$\alpha^3 + a^2(3\lambda+6\mu) + a(3\lambda^2 + 9\mu\lambda + 11\mu^2) + (\lambda^3 + 3\mu\lambda^2 + 6\mu^2\lambda + 6\mu^2)$

$a_1$, $a_2$, and $a_3$ correspond to the three roots of

$(\mu^3 + 3\mu^2\lambda + 3\mu\lambda^2)/(\mu^3 + 3\mu^2\lambda + 3\mu\lambda^2 + \lambda^3)$

$\alpha^3 + a^2(6\lambda+6\mu) + a[11(\mu+\lambda)^2] + 6(\mu+\lambda)^3$
are average values, which assumes a steady state operation of the system. The earliest queueing network model of FMS was CAN-Q (32). The most recent developments in this area are priority mean value analysis (PMVA), (28), and mean value analysis of queues (MVAQ), (35). Some studies (31,33), proposed the use of the closed network of queues models. Other models of FMS (9) included the open queueing networks.

3. Discrete event simulation

Simulation is perhaps the most widely used computer-based performance evaluation tool for FMS. GPSS, SLAM and MAP/1 (27) are the main languages used in the simulation models. Although these models can be made very accurate, they cost too much in terms of programming time, input time to generate detailed data sets and computer running time. Thus, it is recommended to use queueing network models prior to conducting the more expensive simulation studies.

4. Perturbation analysis P/A

Perturbation is the observation of the detailed behavior of the system for one set of decision parameters. It is a new technique which has potential applications to both simulation and real-time operation of FMS. The modeling assumptions required by P/A are minimal, since it can work directly off real data. The main disadvantage of P/A is that it cannot predict accurately the effect of "large" changes in decisions (19).
5. Petri Nets

While in the past the main use of Petri Nets was to answer qualitative questions, recent advances in Petri Nets applied to FMS permit a dynamic, deterministic model of the system. However, there are still some questions about the efficiency of such models. Also, current models do not incorporate any uncertainty (13).

The second major direction is to evaluate the performance measures of an FMS with multiple components that are subject to failure and modeled as closed network of queues. A small number of studies have been carried out. These studies can be divided into academic and industrial.

Academic Studies

Some examples of the more recent practical studies are:

1. Vinod and John (40) investigated an FMS with two stage repair facility and presented a mathematical model that integrates queueing theory and integer programming to determine optimal capacities for repair facilities subject to preset availability requirements of the resources.

2. Vinod and Solberg (41) dealt with the approximate analysis and application of a closed queueing network to model the performance of multistage FMS with multiple server (machine) resources that are subject to resource failure.
3. Hitomi et al. (18) considered a manufacturing system in which two machines are arranged in parallel and investigated the variation of reliability and failure rate of cutting tools with lapse of time.

4. El Sayed and Turley (14) considered a two-stage transfer line with buffer storage where each stage has two failure modes, and presented the equilibrium probability equations for three repair policies.

Industrial Studies

A high proportion of availability studies performed in the manufacturing systems made use of reliability simulation. A study performed at IBM Federal System Division (16) developed a method of process generation that requires no event calendar under the assumptions of exponential independent failures and N-stage Erlang server. This method is implemented in Fortran code. Several measures of system effectiveness are evaluated, including reliability, availability, and mean time between failures.

Objectives of Research

This research presents a methodology that determines the time-dependent availability of flexible manufacturing systems. It provides performance models of manufacturing systems subject to failure and develops a methodology for assessing performance of these systems.

Because of the applied nature of this problem, every attempt has been made to investigate the problem within a realistic framework by
taking into account various technological considerations through a computer program that will help the industrial user to improve the performance of the manufacturing system.

The following objectives were established to meet the above requirements:

1. To study the FMS from the stand point of availability approach.
2. To determine the major critical component to system operation.
3. To develop a general Markovian model that describes different failure modes for the availability analysis of FMS.
4. To develop a computer program to carry out the above analyses and to evaluate system availability, component utilization, average production rate and system effectiveness.
5. To evaluate the effects of desired system output on other performance measures.
6. To examine the behavior of the system state probabilities under transient conditions over specified time interval.
7. To conduct sensitivity analysis on the results of the computer program and determine the optimum system capacity.
MATHEMATICAL MODEL AND ASSUMPTIONS

Introduction

For the availability modeling of FMS with many components and different failure modes, it is helpful to consider the system to be a collection of separate components and to study the random sequence of states entered by the manufacturing system, as it evolves in time. Movement between these states will be modeled using continuous time Markov chain models.

The use of Markov processes to model manufacturing systems imposes a few restrictions and limitations. One assumption is the independence of the different failure mechanisms. Another assumption is that the times to failure and the times to repair follow exponential distributions. Thus, the probability that a working unit will become nonoperational in a specified interval is independent of how it has been functioning (12).

The sensitivity of performance modeling of manufacturing systems to the assumption of exponentiality has been studied by several authors. Based on these studies, the following tentative conclusions can be drawn:

1. The assumption of exponentiality produces consistent results that are sufficiently accurate for practical applications. Particularly the results, demonstrated in the paper by Suri (34), have validated this approximation.

2. The exponential approximation overestimates the actual
throughput (15).

3. The relative throughput error decreases as the number of parts in the system increases (15,34).

4. The detailed error analysis is very useful to validate the accuracy of the exponential approximation (15,39).

However, the exponential assumption is valid for the failure events of many manufacturing problems, especially for those in which all components are properly burnt in and do not enter the wear-out region (4). Moreover, the exponential distribution is also valid for the repair time, since the manufacturing systems are designed so that those components which fail most frequently require less time to repair and vice versa (22).

Markov processes, used in this research, are stochastic processes describing movement between states of the process at times specified by the index. Each component will be in one of a discrete set of states at any point in time and so the state space of the process is discrete. Time is treated as continuous and failed states are not "absorbing"; that is, the time to repair the failed component and restore the system into an "up" state is included in the process.

There are several references (2,11,20,30) that describe methods for determining the possible states of a system, developing the system state transition matrix and solving the state equations to find the system availability. However, these methods do not describe how to use the system state transition matrix in calculating performance measures related to time interval between failures of a system required to
operate continuously.

Sections 2-4 contain background material and mathematical methodology about failure mode analysis, stochastic performance and availability. Then, the three Markovian models are described in the following sections.

Stochastic Performance

The dynamic behavior of FMS can best be described by a transition diagram among states, which represents the continuous-time discrete-state Markov processes. Each state specifies all possible combination of input and output transition rates. Then, the transition-rate matrix is obtained by inspection of the transition diagram. It can be found by first determining the off-diagonal elements based on the definitions of system states which is described in the next section. After all the off-diagonal elements are determined, the diagonal elements can be obtained and is equal to the negative sum of the remaining elements of the row.

Once the transition matrix is obtained, the set of differential equations relating the state probabilities of the system is found by using the state equation:

$$\frac{d P(t)}{dt} = M P(t) \quad \text{for } t > 0$$ (1)

The transition probability vector, $P(t)$, has $q$ elements and is given by,

$$P(t) = \{P_0(t) \ P_1(t) \ P_2(t) \ ... \ P_{q-1}(t)\}$$ (2)

$M$ is an $(q \times q)$ state transition (rate) matrix which has the following properties:
* It is a square matrix

* The sum of the elements in each column equals 0

* The diagonal elements are the values that correspond to the rates out of state. These values are negative.

* The elements of $M$ is the instantaneous transition rate between states.

The solution of the state equation is determined by solving the differential equations. This is typically done using of Laplace transforms. This method enables a first-order differential equation in terms of time to be converted into an algebraic equation in terms of Laplace transform variable $s$, while the inverse transform permits to convert to the opposite. Therefore, by taking the Laplace transform of the state equation, the following relation results,

$$\int \frac{d P(t)}{dt} = s P(s) - P(0)$$

(3)

The solution of the above equation is

$$(sI - M) P(s) = P(0)$$

or

$$P(s) = (sI - M)^{-1} P(0)$$

(4)

where $P(s)$ is Laplace transform of the transition probability

$s$'s are the roots of the characteristic equation given by the determinant $|sI - M| = 0$. In general, one eigenvalue must be zero and all others should be negative.

$I$ is an identity matrix

$M$ is the transition matrix

$P(0)$ is the initial condition of the system
Therefore, the matrix elements in $P(s)$ are calculated from the equation:

$$P_k(s) = \frac{-1}{\prod (s-s_i)}$$

where the $k$th element of the (cofactor) $[\text{cof}(sI - M)^T]_{k0}$, and $M_d$ is the matrix obtained by omitting the first column and the $k$th row of $(sI - M)^T$.

Then, the transition probability vector, $P(t)$, can be calculated numerically from the inverse Laplace transform. However, the Laplace transform method for a general transition matrix $M$ is very difficult to apply by hand when the number of states exceeds four. This is because the eigenvalues are the roots of a $q$-degree polynomial equation. For this reason, many methods were developed to solve first-order differential equations numerically. These methods are justified in a number of texts dealing with numerical analysis (5,30).

**Failure Mode Analysis**

Failure mode analysis is a systematic procedure for determining, evaluating and analyzing all potential failures in a manufacturing system. The term "component" will refer to a number of types of elements used in manufacturing systems, such as machine tools, material handling equipment, robots and pallet changer, etc. The term "failure mode" refers to the manner in which a component fails to meet the design intent; thus, mechanical, electrical, electronic, hydraulic and other
types of failures can be considered as failure modes. In FMS, failures may be categorized as:

1. Component failures. When the machine fails, it can detect, and display failure diagnostics. It can also give information for repair and maintenance to the operator. Component failures may be further categorized as: unexpected failures, scheduled maintenance and overload failures. This study considers both the unexpected and overload failures.

2. Operation failures. Examples of operation failures may include errors in supplying NC command data, selecting tools and specifying the cutting conditions.

A detailed description of the two categories and other possible sources of failure is illustrated in Appendix C. The results of the failure mode analysis are discussed in the case study.

The procedures for failure mode analysis, which are used in this research, are as follows:

1. Identify and list those component failures and combinations of component failure that cause any of the following to occur:
   a) Partial or complete system shutdown.
   b) Unacceptable performance of equipment.

2. Investigate each component in its potential failure modes for both regular and heavy operation described in Chapter 3.

3. Compute the frequency of each failure mode, the average failure rate and the average downtime to repair.

The results of this analysis could come from systems which used the
same type of equipment under similar operating conditions. It can then be used as input data for the application of the Markovian models described in the next section.

The failure rate for a specified time interval is defined as:

\[ \lambda = \frac{\text{number of failures}}{\text{total operating hours}} \]

and is expressed in terms of failures per hour. As an example, for the head indexer, 1488 hours of downtime have been experienced in 17 months. Downtime includes 209 total system interruptions, 149 caused by electrical failures, 49 for mechanical failures, and 11 for tool failures. Thus, the rates of three failure modes are 0.018, 0.006, and 0.004 failures per hour, respectively.

When one of the machines in active parallel fails, the other machines increases its production rate up to a 100 % utilization. The operation at this increased production rate is named as "heavy operation" and is described in Chapter 3.

Theoretically speaking, the failure rate for the heavy operation can be found from the mean time between failures which is given by,

\[ \theta = \int_{0}^{\infty} R(t) \, dt \]

To calculate the reliability function \( R(t) \), the availability formula given in Table 1 can be used with some exception. Consider a system of two components in active parallel, the corresponding equation can be applied, except that:
1. $R(\infty) = 0$, thus the left part of the equation is eliminated.

2. The eigenvalues $s_1$ and $s_2$ are different so that

$$s_1 s_2 = 2 \lambda_r \lambda_h$$

Thus, the reliability of the system is

$$R(t) = \frac{-s_2 x e^{s_1 t} + s_1 x e^{s_2 t}}{s_1 - s_2}$$

where

$$s_1 = -0.5 \left[ 2 \lambda_r + \lambda_h + \mu \right] + \left( 2 \lambda_r - \lambda_h \right)^2 + 2 \left( 2 \lambda_r + \lambda_h \right) + \mu^2 \right]^{0.5}$$

$$s_2 = -0.5 \left[ 2 \lambda_r + \lambda_h + \mu \right] - \left( 2 \lambda_r - \lambda_h \right)^2 + 2 \left( 2 \lambda_r + \lambda_h \right) + \mu^2 \right]^{0.5}$$

Therefore, the time between failures is given by

$$\Theta = \int_0^{\infty} R(t) \, dt = \frac{1}{s_1 - s_2} \left[ \frac{s_2}{s_1} - \frac{s_1}{s_2} \right]$$

$$= \frac{-(s_1 + s_2)}{s_1 x s_2}$$

Substituting $s_1$ and $s_2$ in the above equation, get

$$\Theta = \frac{2 \lambda_r + \lambda_h + \mu}{2 \lambda_r x \lambda_h}$$

For given values of $\Theta$, $\lambda_r$, and $\mu$, $\lambda_h$ can be obtained by:

$$\lambda_h = \frac{2 \lambda_r + \mu}{2 \Theta \lambda_r - 1}$$
Availability

Availability may be expressed and defined in three different ways as follows:

I) Pointwise (Instantaneous) Availability. The instantaneous availability, $A(t)$, for a given point in time, is the sum of probabilities of all the operating states at that given point in time. The system is up at time $t$ if a state is in $Q_o$, which is a subset of the state space $Q$. On the other hand, the system is down at time $t$, if a state is in $Q_f$, which is the complement of $Q_o$ in $Q$. The subset $Q_o$ depends on the structure of the system.

As $\lambda_i$ and $\mu_i$ are, respectively, the failure rates and the repair rates of components, they are positive quantities. Assuming there are enough repair crews, the availability of systems, comprised of single, series or parallel components, are derived and given in Table 2. Due to the difficulty of applying Laplace transform theory to a large number of states, the derivation of instantaneous availability was limited to two components for the parallel structure. A numerical comparison of the instantaneous availability is presented in Chapter 3.

II) Interval Availability

The interval availability, $A(0,T)$, is the expected proportion of the time interval from system initiation (time = 0) to time $t$ during which the system is working. Once the transition matrix is set up, the interval availability, which is:

$$A(0,T) = \frac{1}{T} \int_0^T A(t) \, dt$$
### TABLE 2. Availability of systems comprised of single, series or parallel components

<table>
<thead>
<tr>
<th>NO. of Components</th>
<th>Conditions</th>
<th>Type of System</th>
<th>No. of Failure Modes</th>
<th>A(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>single</td>
<td>single</td>
<td>1</td>
<td>((\mu/\mu+\lambda) + 0.1 \cdot e^{s_1t} / (\mu+\lambda))</td>
</tr>
<tr>
<td>1</td>
<td>single</td>
<td>single</td>
<td>2</td>
<td>((\mu_1+\mu_2)/s_1s_2 + (s_1+\mu_1) (s_2+\mu_2) e^{s_1t} / s_1(s_1-s_2) + (s_2+\mu_1) (s_2+\mu_2) e^{s_2t} / s_2(s_2-s_1))</td>
</tr>
<tr>
<td>1</td>
<td>single</td>
<td>single</td>
<td>N</td>
<td>[\prod_{i=1}^{N} \mu_i/s_i + 1/s_1 \cdot \prod_{i=1}^{N} (s_i+\mu_i) e^{s_1t} / \prod_{i=1}^{N} (s_i-s_1)]</td>
</tr>
<tr>
<td>2</td>
<td>series</td>
<td>1</td>
<td>((\mu_1\mu_2)/s_1s_2 + (s_1+\mu_1) (s_2+\mu_2) e^{s_1t} / s_1(s_1-s_2) + (s_2+\mu_1) (s_2+\mu_2) e^{s_2t} / s_2(s_2-s_1))</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>series</td>
<td>2</td>
<td>[\prod_{i=1}^{4} \mu_i/s_i + \sum_{j=1}^{4} \prod_{i=1}^{N} (s_j+\mu_j) e^{s_jt} / \prod_{i=1}^{N} (s_j-s_j)]</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>series</td>
<td>N ... N</td>
<td>[\prod_{i=1}^{N} \mu_i/s_i + \sum_{j=1}^{N} \prod_{i=1}^{N} (s_j+\mu_j) e^{s_jt} / \prod_{i=1}^{N} (s_j-s_j)]</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>parallel</td>
<td>1</td>
<td>((\mu^2+2\mu\lambda)/(\mu^2+2\mu\lambda-2\lambda^2) - [2\lambda^2 (s_2 e^{s_2t} - s_2 e^{s_2t}) / \mathcal{E}_{1,2} (s_2 e^{s_2t})])</td>
<td></td>
</tr>
</tbody>
</table>
n_{1,2} = -\frac{\sqrt{\lambda_1 + \lambda_2 + \mu_1 + \mu_2}}{\sqrt{4(\lambda_1 \lambda_2 \mu_2 + \mu_1 \mu_2)}}

s_{1,...,n} correspond to the roots of the N-degree polynomial equation

\prod_{i=1}^{N} \mu_i^2 s_i

s_{1,...,q} correspond to the roots of the q-degree polynomial equation

\prod_{i=1}^{q} \mu_i^2 s_i

s_1 = -2(\mu + \lambda), s_2 = (\mu + \lambda)

2 \frac{(\mu + 2\mu \lambda / (\mu + 2\mu \lambda + 2\lambda)}
or by approximation,

\[ A(0,T) = \frac{1}{T} \sum_{g=0}^{G} A(t x g) \]  \hspace{1cm} (6)

can be obtained simply by summing the solutions of the pointwise availability model for incremental value of \( t \). The number of increments \( G \) depends on the accuracy of the solution desired.

III) Steady-State Availability

The steady-state availability, \( A \), is the long term average fraction of time that a system will be in an "up" state performing its intended function (37). The steady state availability index is the limit of the pointwise availability as \( t \) goes to infinity, i.e., \( A \) is defined as,

\[ A = \lim_{t \to 0} A(t) \]  \hspace{1cm} (7)

Although the transition matrix \( M \) for the steady-state availability is identical to that for the pointwise availability, three factors should be considered:

1. The probabilities, \( P_i \), are constants; therefore, their first derivatives, \( P_i'(t) \), are equal to 0.
2. The elements of the vector \( P(0) \), which indicates the initial condition can be eliminated, since the steady state availability is affected by the initial condition of the system.
3. The sum of the transition probabilities at each interval of time is always equal to 1.
Thus, the state equation becomes:

\[ \mathbf{H} = \mathbf{K} \times \mathbf{P} \quad \text{or} \quad \mathbf{P} = \mathbf{K}^{-1} \times \mathbf{H} \]

where \( \mathbf{P} \) is the column vector of the state probabilities.

\( \mathbf{K} \) is the matrix obtained by omitting the last state in the matrix \( \mathbf{M} \) and adding a unit vector at the first row of the matrix. The probability of this state is equal to:

\[ 1 - \sum_{i=0}^{q-1} P_i \]

The corresponding equation to the row which was omitted can always be used as a check of the correctness of the algebra once the \( P_i \) are all calculated, by comparing the above value and the exact solution of the state equation including this state.

\( \mathbf{H} \) is a column vector equal to \((1 \, 0 \ldots 0)\).

The steady-state availability is then equal to the sum of the probabilities of the corresponding operating states:

\[ A^* = \sum_{i \in Q_o} P_i \quad (9) \]

where \( Q_o \) is the set of operating states.

Accordingly, the Markovian models described in the following section can be applied directly to systems that are physically in series, parallel, or a combination of the two, and indirectly to more complex systems provided that these systems are first decomposed into parallel/series arrangements.
The basic function of an FMS is to manufacture different families of parts (4-100 part numbers), each part requiring multiple (1-10) fixtured holdings or sequences and numerous operations per sequence. Thus, an FMS is conceptualized as in Figure 1. This configuration shows an FMS with one family of parts and two types of machining modules. It represents the physical relationship among the main components in FMS: load/unload stations, machine tools, material handling equipment that are controlled by a computer. The first sequence for this family of parts is scheduled for a system of one parallel group of machine type 1. The second sequence consists of two groups of parallel components. One group has two machines of type 1 and the other group has three machines of type 2.

*material handling equipment

☐ machining module type 1

▶ machining module type 2

FIGURE 1. A schematic diagram of FMS

Based on the above description, the FMS configuration can be represented by one of the three Markovian models, which are developed
and described in the following section. In each model, the system state is defined in terms of the failure modes affecting the availability of the system. Each state, \( q_a \), is an L-tuple \( \{Z_1, Z_2, \ldots, Z_L\} \) where \( Z_1 \) denotes the subset of failure modes in group 1. Each subset, \( Z_1 \), is ordered into:

\[
Z_1 = [f_1, f_2, \ldots, f_N]
\]

where \( f_n \) denotes the number of components failed by the \( n^{th} \) failure mode.

The three basic models to be considered are:

**Model A:** This model represents one or more flexible manufacturing cells connected in series, where the transfer of parts is performed by the worker between them. Associated with each component and its different failure modes, there are both a constant failure rate and a constant repair rate.

**Model B:** This model represents a manufacturing system with one group of identical components, operating in parallel and having the same operating conditions and consequently, the same types of failure. Associated with each failure mode, there is a constant failure and repair rate.

**Model C:** This model represents a typical FMS where groups of parallel machines are linked with the material handling equipment. By consequence, it is a combined case of models A and B.
Model A: Series system with one or more components

This model describes a system that has $S$ series components, each of which has $N_i$ different number of failure modes. For specified integer values of $S$ and $N_i$ (or $N$ if all $N_i$'s are equal), the system represented by this case has $q$ states:

$$q = 1 + \sum_{i=1}^{S} N_i$$

(10)

where $N$ is the number of failure modes.

Each component represents a group, i.e., $L=S$. A system state is represented by the unions of the subset of failure modes, i.e.,

$$q_a = \{Z_1 u Z_2 u ... u Z_L\}$$

(11)

and each $Z_i$ is represented by the subvectors:

$$Z_i = [f_1, f_2, ..., f_N]$$

(12)

where

$$f_n = \begin{cases} 1 & \text{if a component is down} \\ 0 & \text{if a component is up} \end{cases}$$

The system is down if one of the components fails by one of its failure modes. Thus, only a "1" shows in all subset and the remaining elements are 0. So, the system states are described as follows. State 0, corresponds to the case where all series components are running. States $\{(1,0,...,0), (0,1,0,...,0), ..., (0,0,...,1)\}$ correspond to the case where the $n^{th}$ failure mode of the $i^{th}$ component occurs and causes the entire system to shut down.

The transition matrix, $M$, for model A is of dimension $(q \times q)$. The partitioning of the matrix, is shown in Figure 2. The information
FIGURE 2. Partitioning of the transition matrix of model A.
needed to construct the matrix, particularly the constant $k_{ac}$, is summarized in Table 3.

TABLE 3. Variable information for Model A.

<table>
<thead>
<tr>
<th>Area</th>
<th>Description</th>
<th>$a$</th>
<th>$c$</th>
<th>$k_{ac}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>repair of 1 component</td>
<td>0</td>
<td>$(1,0,...,0)<em>{1}-(0,0,...,1)</em>{S}$</td>
<td>$\mu_{11}...\mu_{SN}$</td>
</tr>
<tr>
<td>II</td>
<td>failure of 1 component</td>
<td>$(1,0,...,0)<em>{1}-(0,0,...,1)</em>{S}$</td>
<td>0</td>
<td>$\lambda_{11}...\lambda_{SN}$</td>
</tr>
<tr>
<td>III</td>
<td>diagonal elements</td>
<td>$a = c$</td>
<td>$\sum_{a=0}^{q-1} k_{ac}$</td>
<td></td>
</tr>
</tbody>
</table>

The table contains values of a, c and $k_{ac}$ for the areas I, II and III. Each area has been defined in terms of the corresponding ranges for a and c. Area III corresponds to the diagonal elements of the transition matrix, i.e., $a = c$. For example, the variables information, shown in Table 2 have been applied to a series system with three components. Each of the first and third component has 3 failure modes and the second component has 2 failure modes, Figure 3. The transition diagram is shown in Figure 4 and the resulting transition matrix is shown in Figure 5.

![Figure 3](image)

FIGURE 3. A schematic pattern of a series system.
FIGURE 4. Transition diagram for the example in model A
<table>
<thead>
<tr>
<th>Component down No.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{ij} )</td>
<td>( \mu_{11} )</td>
<td>( \mu_{12} )</td>
<td>( \mu_{13} )</td>
<td>( \mu_{21} )</td>
</tr>
<tr>
<td>0 (0 0 0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (1 0 0)</td>
<td>( \lambda_{11} )</td>
<td>( -\mu_{11} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0 1 0)</td>
<td>( \lambda_{12} )</td>
<td>( -\mu_{12} )</td>
<td>( \lambda_{13} )</td>
<td>( -\mu_{13} )</td>
</tr>
<tr>
<td>(0 0 1)</td>
<td>( \lambda_{21} )</td>
<td>( -\mu_{21} )</td>
<td>( \lambda_{22} )</td>
<td>( -\mu_{22} )</td>
</tr>
<tr>
<td>(1 0)</td>
<td>( \lambda_{31} )</td>
<td>( -\mu_{31} )</td>
<td>( \lambda_{32} )</td>
<td>( -\mu_{32} )</td>
</tr>
<tr>
<td>(0 1)</td>
<td>( \lambda_{33} )</td>
<td>( -\mu_{33} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**FIGURE 5.** The transition matrix for the example in model A
Model B: System with one group of identical parallel components

This model represents a system that consists of \( X \) identical components in active parallel, each of which has \( N \) failure modes. The number of repair crews available and the repair policy will influence the transition matrix. For the description of this model, it is assumed that there are enough crews to have each failed component simultaneously under repair.

The system has \( q \) states,

\[
q = 1 + \sum_{i=0}^{X-1} \binom{N+i}{N-1}
\]  

(13)

and \( q_0 \), number of operating states is equal to,

\[
q_0 = 1 + \sum_{i=0}^{X-2} \binom{N+i}{N-1}
\]  

(14)

In this model, a system state \( q_a \) is \( N \)-tuple \( \{f_1f_2...f_N\} \) such that \( \sum_{n=1}^{N} f_n \leq X \). To simplify the analysis of the model, the states of the system can be described as follows:

**Group 0** State 0, corresponds to the case where all components in active parallel are running.

**Group 1** States \( \{(1,0,...,0),...,(0,0,...,1)\} \) correspond to the case where one component is failed by the \( n^{th} \) failure mode.

**Group 2** States \( \{(2,0,0,...,0),...,(0,0,0,...,2),(1,1,0,...,0),...,(0,...,\)
0.1,1)) correspond to the case where two of the X components are down.

Group X States {((X,0,...,0),...,(0,...,0,X),(X-1,1,0,...,0),...,
(0,0,...,1,X-1),(1,1,...,1))} correspond to the case where all X components have failed.

The transition matrix of this model can be subdivided into (1+2X) areas. The general procedure to find \( k_{\text{ac}} \) values for any area, is to consider the possible failures and repairs that can be made in one time for the states in the area. By comparing the two states, \( a \) and \( c \), the constant \( k_{\text{ac}} \) is obtained as the corresponding \( \lambda \) or \( \mu \) of the failure mode, not found in one of the two states. For example, consider the two states, \( a: "1,0,1" \) and \( c: "1,1,1" \), the constant \( k_{\text{ac}} \) is equal to \( \mu_2 \) and \( k_{\text{ca}} \) is equal to \( \lambda_2 \), since the failure mode 2 is zero in state \( a \).

The variables information shown in Table 4, have been applied to a system consisting of three parallel components, each of which has three failure modes. Figure 6 shows the transition diagram for such a system. The transition matrix, as shown in Figure 7, is divided into seven areas which have been labeled I,II,...,VII. The analysis of \( k_{\text{ac}} \) is conducted according to two possible values of \( f_n \): a) \( f_n = 1 \), b) \( 1 < f_n < X \). When \( f_n = 1 \), \( \lambda_{rn} \) is used to represent the failure rate of the \( n^{th} \) failure mode in the regular operation. But when \( 1 < f_n < X \), \( \lambda_{hn} \) is used to represent the failure rate of the \( n^{th} \) failure mode in the heavy operation. The "regular" and "heavy" operation are described in detail in Chapter 3.
TABLE 4. Variable information for model B

<table>
<thead>
<tr>
<th>Area</th>
<th>Description</th>
<th>a</th>
<th>c</th>
<th>( K_{ac} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>repair of 1 component</td>
<td>0</td>
<td>(1,0,0)-(0,0,1)</td>
<td>( \mu_1 - \mu )</td>
</tr>
<tr>
<td>II</td>
<td>failure of 1 component</td>
<td>(1,0,0)-(0,0,1)</td>
<td>0</td>
<td>3( \lambda_{r1} - 3\lambda_{rN} )</td>
</tr>
<tr>
<td>III</td>
<td>repair of 2 components</td>
<td>(1,0,0)-(0,0,1)</td>
<td>(2,0,0)-(0,1,1)</td>
<td>2( \mu_1 - 2\mu_N )</td>
</tr>
<tr>
<td>IV</td>
<td>failure of 2 components</td>
<td>(2,0,0)-(0,1,1)</td>
<td>(1,0,0)-(0,0,1)</td>
<td>2( \lambda_{h1} - 2\lambda_{hN} )</td>
</tr>
<tr>
<td>V</td>
<td>repair of 3 components</td>
<td>(2,0,0)-(0,1,1)</td>
<td>(3,0,0)-(1,1,1)</td>
<td>3( \mu_1 - 3\mu_N )</td>
</tr>
<tr>
<td>VI</td>
<td>failure of 3 components</td>
<td>(3,0,0)-(1,1,1)</td>
<td>(2,0,0)-(0,1,1)</td>
<td>( \lambda_{h1} - \lambda_{hN} )</td>
</tr>
<tr>
<td>VII</td>
<td>Diagonal elements</td>
<td>( a = c )</td>
<td>- ( \sum_{a=0}^{q} ) ( K_{ac} )</td>
<td></td>
</tr>
</tbody>
</table>

\( \lambda_{r1} \), \( \lambda_{rN} \), \( \mu_1 \), \( \mu_N \), \( \lambda_{h1} \), \( \lambda_{hN} \) are the failure rates of repair and components respectively.
<table>
<thead>
<tr>
<th>No. of components failed</th>
<th>J: 0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(J,0,0)</td>
<td>1 1,0,0</td>
<td>4 2,0,0</td>
<td>10 3,0,0</td>
<td></td>
</tr>
<tr>
<td>(0,J,0)</td>
<td>0 0</td>
<td>2 0,1,0</td>
<td>5 0,2,0</td>
<td>11 0,3,0</td>
</tr>
<tr>
<td>(0,0,J)</td>
<td>0 0</td>
<td>2 0,0,1</td>
<td>6 0,0,2</td>
<td>12 0,0,3</td>
</tr>
<tr>
<td>(1,1,0)</td>
<td>7 1,1,0</td>
<td>13 1,2,0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,0,1)</td>
<td>6 1,0,1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0,1,1)</td>
<td>6 0,1,1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,2,0)</td>
<td>13 1,2,0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,1,0)</td>
<td>14 2,1,0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,0,2)</td>
<td>15 1,0,2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0,0,1)</td>
<td>16 2,0,1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0,1,2)</td>
<td>17 0,1,2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0,2,1)</td>
<td>18 0,2,1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,1,1)</td>
<td>19 1,1,1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FIGURE 6. Transition diagram for the example in model B
FIGURE 7. Partitioning of the transition matrix of model B
Model C: Combined system

This model represents a manufacturing system that consists of \( L \) groups of parallel components and series components. The physical system for this model represents the combination of models A and B. The following description of this model is limited to two groups of parallel components and \( S \) series components (i.e., \( L=2+S \)). For specified integer values of \( S, X, N' \)'s the system represented by this model, has \( q \) different states.

\[
q = \left[ \prod_{l=1}^{2} \left( \sum_{j=0}^{X_l-1} C_{N_l}^{N_l+j+1} \right) \right] - \left( \prod_{l=1}^{2} C_{N_l}^{N_l+X_l-1} \right) + (\prod_{l=1}^{2} q_{01}) \times (\sum_{l=3}^{L} N_l)
\]

where \( q_{01} \) is the number of operating states of group 1, defined in relation (14).

The system state \( q_a \) is an \( L \)-tuple of the form

\[
\{(Z_1 \cap Z_2) \cap (Z_3 \cup Z_4 \cup \ldots \cup Z_L)\}
\]

where \( Z_1 \) and \( Z_S \) refer to the \( S \) series components, and \( Z_{S+1} \) to \( Z_L \) refer to the two parallel groups. Each subset \( Z_l \) is represented by the unions defined in the relation (12). The elements \( K_{ac} \) of the transition matrix can be determined by comparing the failure modes in the two states as described in model B. An illustrated application of this model is presented in Chapter 4.

To simplify the analysis of the model, the states of the system can be classified according to three scenarios:

a) States that correspond to component failures in either parallel
group, such that \( f_n \leq X - 1 \). These states and the elements of the transition matrix are similar to those described in model B. But the transition matrix for this part is subdivided into

\[
[1 + 2x(\sum_{i=1}^{2} X_i)] \text{ areas for the two parallel groups.}
\]

b) States that correspond to the combinations of component failures in the parallel groups, that do not cause the entire system to shut down. The number of states is equal to the product of \( q_0 \) of each parallel group. Consider a system consisting of two parallel groups, one has three components with two failure modes and the other has two components with three failure modes. The number of states is equal to,

\[
L = \prod_{l=1}^{2} q_0 = \left( \sum_{i=0}^{N_2+1} C \right) \times \left( \sum_{i=0}^{N_1+1} C \right) = (2xN_1+1) \times N_2 = 15
\]

where \( N_1 \) and \( N_2 \) are the number of failure modes in each group respectively. The system state \( q_a \) is defined as the first part in the relation (16), i.e. \( q_a = (Z_1 \cup Z_2) = \)

\[
((f_1 \cup \ldots \cup f_N)_1 \cup (f_1 \cup \ldots \cup f_N)_2), \quad \text{where } \sum_{n=1}^{N} f_n, \text{ in each group, is strictly less than } X. \text{ Consider the above example and given that one component in group 1 fails with failure mode 2, and two components in group 2 fail with failure mode 1, the system state will be } \{(0,1,0)_1 \cup (2,0)_2\}.\]
c) States that correspond to the failure of either all components in one parallel group or any series component. Thus, the transmission of production flow from an input point to an output point, is not possible. This case can be divided in two parts. The first part represents the failure of all components in any parallel group. Table 5 shows the number of possible states for the same example as in (b). Thus the number of states is equal to the sum for those states states marked by "*" in the table. The second part represents the states that can be reached from an operating states by the failure of any series component. The number of states of this part is equal to the number of operating states, $q_a$ multiplied by the sum of the number of failure modes for the series components. The values of $K_{ac}$ are based on the failure and repair rates of series components. Thus the elements, $K_{ac}$, follow a repeated pattern of $\mu_{11}, \ldots, \mu_{SN}$.

Table 5. Number of states for two groups of parallel components (one group has 2 components and the other has 3)

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l=2$, $N=2$ --&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$l=1$, $N=3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4*</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12*</td>
</tr>
<tr>
<td>2</td>
<td>6*</td>
<td>12*</td>
<td>18*</td>
<td></td>
</tr>
</tbody>
</table>

Based on the above analysis of the three Markovian models, a computer code is developed to carry out the methodology described in
this chapter to evaluate the performance measures described in the next chapter. A complete description of the computer code is provided in Appendix A. The computer program consists of two parts. The first part is the program AVAL, which is written in BASIC. It prepares the necessary transition matrix for the second part. It also computes all the performance measures, for the steady-state conditions, discussed in Chapter 3.

The second part is the program ODE, which is written in FORTRAN to perform the Markovian-process analysis. It also computes the system availability and system effectiveness as a function of time.
PERFORMANCE MEASURES

Performance measures discussed below include availability, production rate, component utilization, and system effectiveness.

Availability

It is a very essential performance measure because it deals with the requirements of both the operation and the repairs. The choice of availability measures requires consideration of whether the main penalty of system failures depends on the total duration of failures or the frequency of failures. If the total duration of failures is important, then the appropriate measure can be related to the availability of the system. If the frequency of failures is important, then the appropriate measure can be related to the MTBF (mean time between failures).

Numerical values of the appropriate availability can be obtained either by using simulation methods or by setting up and solving mathematical models. The system availability at time \( t \) is the sum of the probabilities that corresponds to the operating states at time \( t \).

\[
A(t) = \sum_{i \in Q_0} P_i(t)
\]

The above formula usually requires the solution of the system state equations, which can be obtained by running the computer program.

Table 6 summarizes the general formula of the instantaneous availability for the three Markovian models. A numerical comparison of
instantaneous and steady-state availability of different systems is shown in Table 7. The systems consist of one, two or three components that operate in series, in active parallel or in stand by. The computation of the two availabilities for the combined system shown at the end of the table, consists of one parallel group and a series component. This numerical comparison is based on the information of two types of machines with different failure and repair rates which are measured in the same units of time (these values are included in Chapter 4).

### TABLE 6. Instantaneous availability of the three Markovian models

<table>
<thead>
<tr>
<th>No. of machines</th>
<th>Model type</th>
<th>No. of failure mode</th>
<th>$A(t)$</th>
<th>$P_{0}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$A$</td>
<td>$N_1, \ldots, N_S$</td>
<td>$\sum_{i \in Q_0} P_i(t)$</td>
<td></td>
</tr>
<tr>
<td>$X$</td>
<td>$B$</td>
<td>$N$</td>
<td>$\sum_{i \in Q_0} P_i(t)$</td>
<td></td>
</tr>
<tr>
<td>$X_1$'s</td>
<td>$C$</td>
<td>$\tilde{N}_1$</td>
<td>$\sum_{i \in Q_0} P_i(t)$</td>
<td></td>
</tr>
</tbody>
</table>

Availability is a good performance measure for a single component or multiple components (in series) of a maintainable system. However, for a group of parallel components or parallel/series network, availability can not serve properly as a performance measure because each failure configuration has a different probability.
TABLE 7. Numerical comparison of instantaneous and steady-state availability

<table>
<thead>
<tr>
<th>No. of Machines</th>
<th>Type of Machine</th>
<th>Type of System</th>
<th>No. of Failure Modes</th>
<th>A(t) t=8</th>
<th>t=16</th>
<th>t=24</th>
<th>t=120</th>
<th>t=∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I</td>
<td>Single</td>
<td>2</td>
<td>.896</td>
<td>.841</td>
<td>.812</td>
<td>.773</td>
<td>.773</td>
</tr>
<tr>
<td>1</td>
<td>II</td>
<td>Single</td>
<td>2</td>
<td>.893</td>
<td>.865</td>
<td>.858</td>
<td>.855</td>
<td>.855</td>
</tr>
<tr>
<td>1</td>
<td>I</td>
<td>Single</td>
<td>3</td>
<td>.884</td>
<td>.820</td>
<td>.783</td>
<td>.706</td>
<td>.691</td>
</tr>
<tr>
<td>1</td>
<td>II</td>
<td>Single</td>
<td>3</td>
<td>.887</td>
<td>.857</td>
<td>.850</td>
<td>.853</td>
<td>.846</td>
</tr>
<tr>
<td>2</td>
<td>I's</td>
<td>Series</td>
<td>2</td>
<td>.804</td>
<td>.715</td>
<td>.673</td>
<td>.630</td>
<td>.630</td>
</tr>
<tr>
<td>2</td>
<td>II's</td>
<td>Series</td>
<td>2</td>
<td>.802</td>
<td>.750</td>
<td>.746</td>
<td>.746</td>
<td>.746</td>
</tr>
<tr>
<td>2</td>
<td>I,II</td>
<td>Series</td>
<td>2</td>
<td>.803</td>
<td>.737</td>
<td>.711</td>
<td>.683</td>
<td>.683</td>
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<td>2</td>
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<td>Series</td>
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<td>.783</td>
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<td>.630</td>
<td>.543</td>
<td>.528</td>
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<tr>
<td>2</td>
<td>II's</td>
<td>Series</td>
<td>3</td>
<td>.791</td>
<td>.746</td>
<td>.736</td>
<td>.732</td>
<td>.732</td>
</tr>
<tr>
<td>2</td>
<td>I,II</td>
<td>Series</td>
<td>3</td>
<td>.787</td>
<td>.714</td>
<td>.692</td>
<td>.625</td>
<td>.614</td>
</tr>
<tr>
<td>2</td>
<td>I</td>
<td>Active //</td>
<td>2</td>
<td>.988</td>
<td>.970</td>
<td>.955</td>
<td>.924</td>
<td>.923</td>
</tr>
<tr>
<td>2</td>
<td>II</td>
<td>Active //</td>
<td>2</td>
<td>.986</td>
<td>.975</td>
<td>.970</td>
<td>.967</td>
<td>.967</td>
</tr>
<tr>
<td>2</td>
<td>I</td>
<td>Standby</td>
<td>2</td>
<td>.994</td>
<td>.984</td>
<td>.975</td>
<td>.952</td>
<td>.951</td>
</tr>
<tr>
<td>2</td>
<td>II</td>
<td>Standby</td>
<td>2</td>
<td>.993</td>
<td>.986</td>
<td>.983</td>
<td>.981</td>
<td>.981</td>
</tr>
<tr>
<td>2</td>
<td>I</td>
<td>Active //</td>
<td>3</td>
<td>.989</td>
<td>.963</td>
<td>.934</td>
<td>.896</td>
<td>.876</td>
</tr>
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<td>2</td>
<td>II</td>
<td>Active //</td>
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<td>.984</td>
<td>.970</td>
<td>.963</td>
<td>.930</td>
<td>.889</td>
</tr>
<tr>
<td>2</td>
<td>I</td>
<td>Standby</td>
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<td>Standby</td>
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<td>.992</td>
<td>.983</td>
<td>.979</td>
<td>.959</td>
<td>.934</td>
</tr>
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<td>.798</td>
</tr>
<tr>
<td>1</td>
<td>II</td>
<td>Active //</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td>.758</td>
<td>.668</td>
</tr>
</tbody>
</table>

Utilization

The most important performance measure for an individual component is its utilization. There are several ways to define utilization. Normal industrial practice defines it as the fraction of time, over the long run, that a component is busy. However, for technical reasons, utilization can be defined as the long run average number of busy components in the parallel group. If the system has only one component, the two definitions are equivalent. If, on the other hand, the system has multiple components, utilization cannot be interpreted as a fraction, it may be larger than one. To obtain the average utilization per component, $U$, which can be interpreted as the fraction of time that each is busy, divide the desired system output $D_{OUT}$ by the total production rate of the parallel components, i.e., $U = D_{OUT}/(Xw)$, where $w$ is the production rate of each component.

Production Rate

Production rate is the average number of completed parts per unit time, and is denoted by $w$. The average production rate of each component is given by the reciprocal of the production time $t$,

$$w = 1/t$$

The net production rate which can be achieved from the system depends on the occurrence of various types of stoppages such as: breakdowns of machines and material handling, tool failures and maintenance, and the effect of these stoppages on the whole system.

The production rate of each component is denoted by $w$ and the
expected production rate at time $t$ is denoted by $PRATE(t)$ with the corresponding index for system structure. The following formulae express the production rate for the three models,
a) For $s$ dependent components (series),

$$PRATE(t) = P_0(t) \times w \quad \text{or} \quad PRATE(t) = A(t) \times w$$

where $w = \min \{w_1, \ldots, w_s\}$

b) For one group of parallel components:

In the process of performance analysis of this type of system, the following conditions are assumed:

1. The workload of the system is shared equally by all the components in this group. Thus the rate of flow of material out of the system must equal to the rate of flow into the system.

2. The components, in the same group, have the same operating conditions and consequently the same failure and repair rate for the different failure modes.

3. When all $X$ components in the group operate, it is referred to as "regular operation" and $PRATE$ is less than or equal to the desired system output. When $f$ out of $X$ components fail, the production rate of the remaining components can be increased instantaneously, up to 100% utilization and it is referred to as "heavy operation". Given the desired system output, $DOUT$, the maximum number of failed components, $F$, to maintain constant production rate, is determined as follows:

$$U = \frac{DOUT}{X \times w}$$

$$X \times w \times U = (X-F) \times w$$

so,

$$F = X \times (1-U)$$
The production rate distribution of a group of parallel components is given by

\[ PRATE(t) = \sum_{y=0}^{X-1} P_y(t) \times \tau_y(f) \]

where \( \tau(f) \) is the corresponding production rate for each failure state and it can be formulated as follows:

\[ \tau(f) = \begin{cases} \frac{(w/U)(X-f)}{F < f \leq X} \\ 0 & f = X \end{cases} \]

where \( \tau(f) = \begin{cases} \frac{(w/U)(X-f)}{0 < f \leq F} \\ 0 & f = X \end{cases} \)

for combined system, each series component represents a group, i.e. \( w_1 \) to \( w_S \) equal to \( \tau_1 \) to \( \tau_S \) respectively, and

\[ PRATE(t) = \sum_{y=0}^{q_0} P_y(t) \times \tau_y \]

where \( \tau \) can be interpreted as

\[ \tau = \min \{ \tau_1, \ldots, \tau_S, \ldots, \tau_L(f) \} \]

System Effectiveness

System effectiveness, \( E \), is defined as the probability that a system can successfully meet an overall operational demand within a given time when operated under specified conditions. System effectiveness is a term used in a broad context to reflect the system performance and may be expressed differently depending on the specific application. In this study the system effectiveness, \( E \), is defined as:

\[ E = \frac{PRATE}{DOUT} \]
CASE STUDY

The Physical System

The models developed in this research have been used to investigate state-of-the-art FMS for a manufacturer in state of IOWA. The FMS consists of 16 machine tools, five head indexers, and eleven machining centers, an integrated towline conveyor system controlled by the D.E.C. PDP 11-44 host computer, four load/unload stations, and the necessary equipment that is used in monitoring the quality of the output.

The horizontal 2-axis head indexers (H.I.) are used for precision boring and multi-spindle drilling and tapping operations. They use as many as 7 different spindles per operation. The vertical 3-axis CNC machining centers (M.C.), with each having a 69 tool capacity magazine, are used to do milling, drilling, boring, and tapping operations. They use 10 to 23 tools per operation. The computer-controlled towline conveyor uses identification codes on pallets to control the routing of parts to the right machine tool. The layout, as shown in Figure 8, provides two interconnected towline loops that serve the two rows of the CNC machines.

The load/unload station is heavy steel fabrication and is arranged to accept one 31 by 48 inches metal pallet at a time from the towline carts. Each load/unload station has pallet readers and one CRT/keyboard terminal.
FIGURE 9. System layout
FMS Operations

Fork lift trucks bring incoming castings to the load/unload stations. In the meantime, the towline carts arrive carrying fixtures on machining pallets, workers load parts on the fixtures, report part numbers and pallet codes at a CRT/computer terminal, and release the carts to the towline. These metal pallets remain on the carts after finished castings have been removed. The computer, then, routes the part to the machine and downloads the NC program to the machine.

After the palletized part arrives at the machine, a shuttle-mounted hydraulic cylinder actuates the transfer device that unloads and reloads a towline cart. But, first, a stop mechanism disengages the cart's tow pin from the in-floor conveyor chain, and holds the cart in proper alignment for the transfer.

Each part is usually machined on (1-2) machining centers and (1-2) head indexers. It also requires (2-6) fixtures (sequences) and (2-4) operations for each sequence. It is possible to machine all types of parts at once, with different operations being performed on all 16 machines.

The computer routes a part from any machine to the next one that is available to handle the part and selects CNC machines on a random basis, within the two categories of machines. After machining, carts pass through each machine for pick up on the opposite side and carry parts to the next machine or to the load/unload station.
Parts and their Characteristics

The finished products of this system are a family of eight large, heavy castings, used in drive-train assemblies. The FMS was purchased to produce a daily requirement of 218 pieces in three shifts. There are 30 different operations being performed at any one time in the system, on the two types of machining modules: five head indexers (machines 1, 2, 3, 9, and 10) and eleven machining centers (machines 4 to 8 and 11 to 16).

The cycle times range from 6 to 30 minutes per operation. Table 8 shows, for each family of parts, the part routings, the load/unload station number, the number of orientation fixtures and total process time in minutes. A schematic diagram for parts routings is illustrated in Figure 9. It is to be pointed out here that alternative routings are specified as it is allowed in practice.

In addition, set-up time for any of the parts is done simultaneously as the machines are tooled to run all parts and orientations. The average loading time of a part is 4.65 minutes and the average unloading time is 2.61 minutes. The pallet exchange time is very small, 30-45 seconds because there can be two parts on the shuttle at each machine. Although the complete determination of the performance measures await the solution of the transition matrix of the models, a few preliminary calculations yield some important information about the system. Table 9 summarize of the deterministic calculations.
TABLE 8. Family of parts information

<table>
<thead>
<tr>
<th>Family</th>
<th>Sequence</th>
<th>Machine</th>
<th>Load/ unload</th>
<th>No. of orientation</th>
<th>Total process</th>
<th>Daily demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>parts</td>
<td>number</td>
<td>parts sequence</td>
<td>number</td>
<td>station fixtures</td>
<td>time (min.)</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>L1-U2</td>
<td>3</td>
<td></td>
<td>18.201</td>
<td></td>
</tr>
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<td></td>
<td></td>
<td>L1-U2</td>
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<td></td>
<td>49.158</td>
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<td></td>
<td>L1-U2</td>
<td>6</td>
<td></td>
<td>68.264</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td></td>
<td>L1-U2</td>
<td>4</td>
<td></td>
<td>42.165</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>L1-U2</td>
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<td></td>
<td>34.542</td>
<td>37</td>
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<tr>
<td></td>
<td></td>
<td>L4-U3</td>
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<td></td>
<td>29.753</td>
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<tr>
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<td></td>
<td>L1-U2</td>
<td>4</td>
<td></td>
<td>60.684</td>
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<td></td>
<td>L4-U3</td>
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<td></td>
<td>87.129</td>
<td>16</td>
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<td>L4-U3</td>
<td>3</td>
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<td>49.246</td>
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<td>L4-U3</td>
<td>2</td>
<td></td>
<td>32.337</td>
<td>35</td>
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<td></td>
<td></td>
<td>L4-U3</td>
<td>3</td>
<td></td>
<td>28.386</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>L4-U3</td>
<td>5</td>
<td></td>
<td>47.721</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>L4-U3</td>
<td>2</td>
<td></td>
<td>28.405</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>L4-U3</td>
<td>3</td>
<td></td>
<td>40.545</td>
<td>21</td>
</tr>
</tbody>
</table>

*a"/" in this table implies "or".*
Family of parts No.

Figure 9. Schematic diagram for part routings
Figure 9. Continued
Example of Deterministic Calculations

A. Total production time:

Total process time per part = sum of process time of the different sequences for each part

Total load and unload time per part = (number of sequences/part) x (load+unload) time/sequence

Total pallet exchange time per part = (total number of operations/x (pallet exchange time/operation) / parts on the shuttle

Total production time = total process time + (load + unload) time + pallet exchange time

B. System effectiveness, assuming 100% availability, is equal to,

System Effectiveness = \frac{\text{Total production time in hours}}{\text{Total hours}}

= \frac{\text{Average time/part x No. of parts}}{(731.3x218) / (60x8)}

= \frac{\text{No. of machines x 24 hrs./day}}{16 x 24}

= .865 or 86.5%

C. Failure Data Analysis

An analysis was done in an attempt to define the failure modes associated with two types of machine modules. Mechanical and hydraulic failures were combined since mechanics handle both types of failures. Electrical and electronic failure were also combined, since electricians
repair both types of failures. The data needed for this analysis are given in Appendix D. Table 10 summarizes the results of this analysis. It indicates that 65.2% of the repair jobs, for the vertical machining center, were electrical in nature and these repairs accounted for 39.2% of the total downtime. It is obvious that the electrical failure was more critical than the other two types of failure. On the other hand, the average repair times for the three failure modes of the head indexer were reasonably close. In addition, it was pointed out that electrical failures account for a large share of downtime. Based on this discussion, further analyses were developed to define the nature of these problems.

TABLE 9. Summary of the deterministic calculations
(Note: all times are in minutes)

<table>
<thead>
<tr>
<th>Family of parts No.</th>
<th>Process time/part</th>
<th>Load/unload time/part</th>
<th>Total pallet exchange time</th>
<th>Total prod. time/part</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>135.622</td>
<td>21.78</td>
<td>2.625</td>
<td>160.027</td>
</tr>
<tr>
<td>2</td>
<td>76.707</td>
<td>14.52</td>
<td>1.875</td>
<td>93.102</td>
</tr>
<tr>
<td>3</td>
<td>177.566</td>
<td>21.78</td>
<td>3.375</td>
<td>202.721</td>
</tr>
<tr>
<td>4</td>
<td>81.583</td>
<td>14.52</td>
<td>1.875</td>
<td>97.978</td>
</tr>
<tr>
<td>5</td>
<td>28.386</td>
<td>7.26</td>
<td>0.750</td>
<td>36.396</td>
</tr>
<tr>
<td>6</td>
<td>47.721</td>
<td>7.26</td>
<td>0.750</td>
<td>55.731</td>
</tr>
<tr>
<td>7</td>
<td>28.405</td>
<td>7.26</td>
<td>1.125</td>
<td>36.790</td>
</tr>
<tr>
<td>8</td>
<td>40.545</td>
<td>7.26</td>
<td>0.750</td>
<td>48.555</td>
</tr>
<tr>
<td>Total</td>
<td>616.535</td>
<td>101.64</td>
<td>13.125</td>
<td>731.300</td>
</tr>
</tbody>
</table>
Application

To illustrate the methodology developed in Chapter 2, the time dependent-availability and other performance measures are calculated for the family of parts No. 5. The basic configuration, based on the description in Table 8, leads to the concept of treating this system as a collection of two groups of parallel components. Each group consists of identical components. The transition diagram of the system is illustrated in Figure 10. The analysis of the system is performed by the methodology developed for model C.

A list of all components and their regular and heavy failure rates, $\lambda_r$ and $\lambda_h$, as well as the repair rate $\mu$ for each component, is shown in table 11. The desired system output is 33 parts/day (1.375 parts/hr). For a numerical example of failure rate for heavy operation, consider a machining center with values of $\lambda_r$ and $\mu$ of .013 and .073, respectively. The mean time between failures is 200 hours. Therefore, the failure rate for heavy operation is equal to:

$$\lambda_h = \frac{2 \times 0.013 + 0.073}{2 \times 200 \times 0.013 - 1} = 0.025$$

Computer Model Usage

The computation procedures for this model have been coded in BASIC. The program was designed to determine the performance measures discussed in Chapter 3. The first part of the code, generates the system states and the transition matrix. The transition matrix is, then, transferred to the ODE program to compute the transition probabilities and production rate as a function of time.
TABLE 10. Results of failure analysis

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>% of the total jobs</th>
<th>% of the total downtime</th>
<th>Average repair rate</th>
<th>Failure repair rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Machining</td>
<td>Elect 65.2</td>
<td>39.2</td>
<td>13.71</td>
<td>.013</td>
</tr>
<tr>
<td>Center</td>
<td>Mech 9.1</td>
<td>26.3</td>
<td>24.04</td>
<td>.005</td>
</tr>
<tr>
<td>Head</td>
<td>Tool 25.7</td>
<td>34.5</td>
<td>30.00</td>
<td>.008</td>
</tr>
<tr>
<td>Indexer</td>
<td>Elect 71.3</td>
<td>64.3</td>
<td>6.43</td>
<td>.018</td>
</tr>
<tr>
<td></td>
<td>Mech 25.4</td>
<td>28.5</td>
<td>8.66</td>
<td>.006</td>
</tr>
<tr>
<td></td>
<td>Tool 5.3</td>
<td>7.2</td>
<td>9.68</td>
<td>.004</td>
</tr>
</tbody>
</table>

\(^a\) all times are in minutes.
\(^b\) all rates are in units/hr.

Table 11. List of failure and repair rates

<table>
<thead>
<tr>
<th>Resources</th>
<th>Production Rate/mach.</th>
<th>Failure Rate/(h^*)</th>
<th>Number of identical Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machining</td>
<td>1.05</td>
<td>1</td>
<td>0.013 0.025 0.073</td>
</tr>
<tr>
<td>Center</td>
<td>2</td>
<td>2</td>
<td>0.005 0.008 0.042</td>
</tr>
<tr>
<td>Head</td>
<td>0.70</td>
<td>1</td>
<td>0.018 0.023 0.154</td>
</tr>
<tr>
<td>Indexer</td>
<td>2</td>
<td>0.006 0.009 0.117</td>
<td></td>
</tr>
</tbody>
</table>

Steady State Solution

The output of program AVAL consists of three parts. The first part contains information concerning the states of the system. The states are divided into 11 clusters as shown in Table 12. The first 6 clusters contain the operating states with the number of components down. The last 5 clusters contain the failed states with the number of components down.
<table>
<thead>
<tr>
<th>No. of failed J1:0</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>machines J2:0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Z1  Z2

(J1.0) (J2.0)

(J1.0) (0, J2)

(0, J1) (J2.0)

(0, J1) (0, J2)

(J1.0) (1,1)

(0, J1) (1,1)

(1,1) (J2.0)

(1,1) (0, J2)

(1,1) (1,1)

(J1.0) (1,2)

(J1.0) (2,1)

(0, J1) (1,2)

(0, J1) (2,1)

FIGURE 10. Transition diagram of the case study
The failure modes shown in the output correspond to the states which are illustrated in the transition diagram. For example, state No. 10, \(((0,1)_1, (1,0)_2)\), shows that a component in group 1 has a failure mode 2 and a component in group 2 has a failure mode 1.

The second part of the output consists of two matrices. The transition matrix \(M\) and the matrix \(K\) for steady state. This part of the output is optional and it is printed for checking purposes.

The third part of the output contains information concerning system performance. The steady-state probabilities are printed first followed by the performance measures as shown in Table 13.

This output indicates that the proportion of time the system is available, is 92.6%. It also shows that each component is able to attain utilization of 65.5% and the expected production rate is 1.26 parts/hr. Thus the system effectiveness is equal to 0.915.
### TABLE 12. The first part of AVAL output (system states)

<table>
<thead>
<tr>
<th>Cluster No.</th>
<th>Number of States</th>
<th>State No.</th>
<th>Failure modes in group 1</th>
<th>Failure modes in group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
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<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>3</td>
<td>(1,0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>(0,1)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
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### TABLE 12. Continued

<table>
<thead>
<tr>
<th>Cluster No.</th>
<th>Number of States</th>
<th>State No.</th>
<th>Failure modes in group</th>
</tr>
</thead>
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<td></td>
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</tr>
</tbody>
</table>
### TABLE 13. The third part of AVAL output (performance measures)

<table>
<thead>
<tr>
<th>State No.</th>
<th>Probability</th>
<th>State No.</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>+ 0.3674</td>
<td>24</td>
<td>+ 0.0003</td>
</tr>
<tr>
<td>1</td>
<td>+ 0.1357</td>
<td>25</td>
<td>+ 0.0001</td>
</tr>
<tr>
<td>2</td>
<td>+ 0.0871</td>
<td>26</td>
<td>+ 0.0001</td>
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Steady state availability = 0.9256

Expected production rate = 1.2576

System effectiveness = 0.0146
The effect of DOUT on performance measures

Having determined the performance measures, the next step was to use the model to conduct a sensitivity analysis on the results of the computer program. To test the effect of the desired system output on the other performance measures, several runs of computer program were made with varying values of DOUT. Pertinent results of these runs are summarized in Table 14 and presented graphically in Figure 11.

TABLE 14. Results of sensitivity analysis

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As the figure indicates, the system effectiveness is not affected when DOUT increases up to 1 part/hr. Also, it is obvious that as DOUT increases, the utilization increases and the system effectiveness decreases. If the system is capable of processing 2.1 parts/hour, at a 100% utilization for each component (point c), the expected production rate of 1.397 parts/hr will be achieved. On the other hand, the system effectiveness, at point a, is equal to .926, and hence the actual production rate is .926 parts/hour.
FIGURE 11. Effect of DOUT on performance measures
However, the net production rate, PRATE, for the system is equal to the desired system output, DOUT, multiplied by the system effectiveness. Thus, PRATE could reach a maximum value of this product and then declines. This characteristic is evident from Figure 12. It can also be seen that the highest steady-state production rate, 1.14 parts/hour, is achieved at DOUT of 1.7 parts/hour and utilization of .809.

Critical component

The critical component can be determined for each system after computing the production rate for different failure configurations. Table 15 shows the output production rate of the application example for the different groups of states which represent the system failures. For example, cluster No. 5, represents the system in which one component failed in group 1 and two components failed in group 2. At cluster No.5, if one of the two machines in group 2 will be up, the system will be restored to cluster No. 4 in which the production is 1.375 parts/hr. On the other hand, if the machine in group 1 will be up, the system will be restored to cluster No. 3, in which the production rate is only 1.07 parts/hr. Therefore, the critical component in this case is the machine in group 2 (a machining center).
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<th>Machine failed in group 2</th>
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FIGURE 12. Relationship between DOUT and PRATE
Transient Solution

The output of program ODE consists of the transition probabilities, availability, production rate and system effectiveness as a function of time. Starting with the initial condition at time 0, the numerical values of the transition probabilities were determined for each successive point in time by step integration of the state equation. The same procedure was used three times, each time using different values of DOUT. The system is assumed to run at time 0, i.e.,

\[ P_0(0) = 1 \quad \text{and} \quad P_i(0) = 0 \quad \text{for} \quad i > 1 \]

Table 16 shows the transition probabilities and performance measures for the system till it reaches the steady state. The results provide a comparison between the behavior of the state probabilities. As intuitively expected, \( P_0 \) is "decreasing". The results validate this reasoning with a graph depicting \( P_0 \) as a function of time, Figure 13. As a basis of comparison, Figure 14 shows the behavior of certain of the transition probabilities for 1024 hours. The two lines \( P_1 \) and \( P_3 \) represents the behavior of the first failure mode in group 1 and 2, respectively. The line \( P_7 \) represents the behavior of state No. 7, in which two components in group 2 failed with failure mode 1. The line \( P_{15} \) represents the behavior of the combination of component failures in state No. 15.

The transition probabilities are normally affected by the failure and repair rates. By examining the behavior of \( P_1 \) and \( P_3 \), as shown in Figure 14, the value of \( P_1 \) increases from 0 at \( t=0 \) to the value .136 at
TABLE 16. Transient solution

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t=64 and then decreases to the value .135, while the value of $P_3$ increases to the value .147 at $t=16$ and then decreases to the value .126. From this figure, an evident effect of the repair rate is the subsequent reduction in the transition probabilities.

However, the ergodic theorem gives the conditions under which an average over time of a stochastic process will converge as the number of observed periods becomes large. In general, to estimate a mean value of a transition probability, a single observation of the entire process is needed, but over a sufficiently long time duration. Then, the steady-state probability can be used as an estimate of the constant mean. As can be expected, the behavior as $t \to \infty$ of the transition probabilities of Markov processes is similar to that of the steady-state.

Of more interest are other quantities derived from the transition probabilities, such as, probability of each cluster of operating states, system availability and system effectiveness.
FIGURE 13. Transient behavior of \( P_0 \)
FIGURE 14. Transient behavior of $P_1$, $P_3$, $P_7$, $P_{15}$
Table 17 shows the transition probabilities of the six clusters of operating states and the performance measures for three different values of \( D_{OUT} \). These results are illustrated in three graphs. The first graph, Figure 15, describes the transient behavior of these probabilities. As the figure indicates that \( P_3 \) and \( P_4 \) are equal between time 0 and 2 hours. Consider the critical component discussed in the steady state analysis and by examining the transient behavior of system states, we can conclude the following:

1. When the system is in state 12 or 15, an electrical repair for the Machining Center, could be needed till time 24. After this time, the need for this type of repair is less likely, since \( P_{10} \) is higher than \( P_5 \).

2. When the system is in state 13 or 16, a mechanical repair for the Machining Center is less likely, since both \( P_9 \) and \( P_{11} \) are higher than \( P_6 \).

3. When the system is in state 14 or 17, an electrical repair for the Head Indexer could be needed till time 2 and a mechanical repair for this machine could be needed till time 8.

As a result, the repair rate of mechanical failure for the Head Indexer should be increased up to time 8. The repair rate of electrical failure for the Machining Center should be increased up to time 24. To maintain optimum production rate by increasing the repair rate, two alternatives can be considered:

1. Improve repair methods.

2. Decreasing the response time which is a part of the repair time.
TABLE 17. Results of operating states

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DOUT = 1.375

| E1      | 1.00 | .99 | .98 | .97 | .94 | .90 | .89 | .89 | .89 | .89 | .89 |
| PRATE1  | 1.38 | 1.37 | 1.36 | 1.34 | 1.30 | 1.24 | 1.22 | 1.22 | 1.22 | 1.22 | 1.22 |

DOUT = 1.7

| E2      | 1.00 | .98 | .96 | .96 | .88 | .82 | .81 | .80 | .80 | .80 | .80 |
| PRATE2  | 1.70 | 1.66 | 1.63 | 1.57 | 1.49 | 1.40 | 1.37 | 1.36 | 1.36 | 1.36 | 1.36 |

DOUT = 1.9

| E3      | 1.00 | .95 | .91 | .86 | .80 | .74 | .72 | .72 | .72 | .72 | .72 |
| PRATE3  | 1.90 | 1.80 | 1.73 | 1.63 | 1.51 | 1.40 | 1.37 | 1.36 | 1.36 | 1.36 | 1.36 |
FIGURE 15. Transient behavior of operating states probabilities
The second graph, Figure 16 represents the transient behavior of system availability and system effectiveness. As the figure indicates, the variations of the system effectiveness as a function of time is significantly higher than that of system availability. Both measures reach the steady-state condition at time 128 hours with typical results to that of program AVAL. It is also apparent that E(t) is more sensitive to DOUT than that of availability measure. Thus, availability can not serve properly as a performance measure for this system.

The third graph, Figure 17, was derived showing PRATE as a function of time for three values of DOUT. It can be concluded that after 32 hours, PRATE is the highest for DOUT of 1.7 parts/hr. Therefore, the system can be loaded with 1.5 parts/hr. up to time 32 hours. Then, DOUT should be increased to 1.7 parts/hr. to maintain the daily production of 1.375 parts/hr.

Accordingly, the same analysis were applied to the remaining systems. Table 18 shows the different data sets for the 8 families of parts. The performance measures of these systems are summarized in Table 19. The results from the transient solutions are illustrated in a series of graphs that are plotted from the data generated by the computer program. These graphs are included in Appendices E, F AND G. Figures E.1-E.9 show the transient behavior of the group of operating states of the different systems. Figures F.1-F.13 represent the variations of A(t) and E(t) of each of the 14 systems, respectively. In Figures G.1-G.13, the systems treated in Figures F.1-F.13 are each subjected to three different values of DOUT, to show its effect on the
FIGURE 16. Transient behavior of $A(t)$ and $E(t)$
FIGURE 17. Transient behavior of PRATE
TABLE 18. Data sets for the 8 families of parts

(E: electrical, M: mechanical and T: tool failure)

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<td></td>
<td></td>
<td>(1.00)</td>
</tr>
<tr>
<td>2</td>
<td>3 M.C</td>
<td>.62</td>
<td>1.480</td>
<td>.81</td>
<td>.444</td>
<td>.604</td>
</tr>
<tr>
<td></td>
<td>1 H.I</td>
<td>1.60</td>
<td></td>
<td></td>
<td></td>
<td>(.542)</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>3 M.C</td>
<td>.70</td>
<td>1.375</td>
<td>.65</td>
<td>.915</td>
</tr>
<tr>
<td></td>
<td>2 H.I</td>
<td>1.05</td>
<td></td>
<td></td>
<td></td>
<td>(1.26)</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>3 M.C</td>
<td>.45</td>
<td>.960</td>
<td>.77</td>
<td>.535</td>
</tr>
<tr>
<td></td>
<td>1 H.I</td>
<td>1.25</td>
<td></td>
<td></td>
<td></td>
<td>(.513)</td>
</tr>
<tr>
<td></td>
<td>1 H.I</td>
<td>1.25</td>
<td></td>
<td></td>
<td></td>
<td>.25</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>3 M.C</td>
<td>.70</td>
<td>1.375</td>
<td>.65</td>
<td>.631</td>
</tr>
<tr>
<td></td>
<td>1 H.I</td>
<td>2.10</td>
<td></td>
<td></td>
<td></td>
<td>(.868)</td>
</tr>
</tbody>
</table>
other performance measures.

The results of this analysis, shown in Table 20, leads to the following:

1. DOUT of systems 1, 2, 6, 11, 12, 13 and 14 should be increased to the corresponding value in the table.

2. DOUT of systems 3, 4, 5, 7, 8, 9 and 10 should not change.

TABLE 20. Optimum DOUT

<table>
<thead>
<tr>
<th>System No.</th>
<th>Family of Parts No.</th>
<th>Sequence No.</th>
<th>Optimum DOUT</th>
<th>PRATE</th>
<th>E</th>
<th>UTL</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1.520</td>
<td>.875</td>
<td>.576</td>
<td>.46</td>
<td>.887</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1.000</td>
<td>.686</td>
<td>.686</td>
<td>.81</td>
<td>.897</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>.875</td>
<td>.492</td>
<td>.562</td>
<td>.97</td>
<td>.897</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
<td>1.540</td>
<td>1.019</td>
<td>.662</td>
<td>.85</td>
<td>.887</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>1.540</td>
<td>.665</td>
<td>.432</td>
<td>.86</td>
<td>.611</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>1</td>
<td>1.200</td>
<td>.670</td>
<td>.558</td>
<td>.60</td>
<td>.576</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>2</td>
<td>.670</td>
<td>.595</td>
<td>.888</td>
<td>.67</td>
<td>.887</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>3</td>
<td>.670</td>
<td>.314</td>
<td>.469</td>
<td>.96</td>
<td>.658</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>1</td>
<td>1.460</td>
<td>1.001</td>
<td>.686</td>
<td>.81</td>
<td>.887</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>2</td>
<td>1.460</td>
<td>.648</td>
<td>.444</td>
<td>.81</td>
<td>.604</td>
</tr>
<tr>
<td>11</td>
<td>5</td>
<td>1</td>
<td>1.700</td>
<td>1.402</td>
<td>.825</td>
<td>.81</td>
<td>.926</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>1</td>
<td>1.100</td>
<td>.558</td>
<td>.507</td>
<td>.88</td>
<td>.622</td>
</tr>
<tr>
<td>13</td>
<td>7</td>
<td>1</td>
<td>1.800</td>
<td>.936</td>
<td>.520</td>
<td>.86</td>
<td>.694</td>
</tr>
<tr>
<td>14</td>
<td>8</td>
<td>1</td>
<td>1.500</td>
<td>.743</td>
<td>.849</td>
<td>1.00</td>
<td>.694</td>
</tr>
</tbody>
</table>
Conclusion of the Application Example

According to the results discussed in this chapter, the desired system output for any family of parts, is not achieved. Therefore different values of DOUT should be applied for each family of parts. The values of PRATE, according to the optimum DOUT, are summarized in Table 21. Also listed are the maximum values of DOUT and current PRATE.

Thus, applying the optimum DOUT, increases the production rate by:

\[
\frac{5.731 - 5.361}{5.731} \times 100 = 6.5\% 
\]

This value is equivalent to an increase, in the daily production rate, of 9 parts/day.

<table>
<thead>
<tr>
<th>Family of Parts No.</th>
<th>Maximum DOUT</th>
<th>Current PRATE</th>
<th>Time t hrs.</th>
<th>Optimum DOUT before</th>
<th>Expected PRATE after</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.875</td>
<td>.492</td>
<td>16</td>
<td>.90</td>
<td>.70</td>
</tr>
<tr>
<td>2</td>
<td>1.540</td>
<td>.663</td>
<td>32</td>
<td>1.70</td>
<td>1.54</td>
</tr>
<tr>
<td>3</td>
<td>.670</td>
<td>.314</td>
<td>16</td>
<td>.70</td>
<td>.67</td>
</tr>
<tr>
<td>4</td>
<td>1.460</td>
<td>.648</td>
<td>32</td>
<td>1.80</td>
<td>1.46</td>
</tr>
<tr>
<td>5</td>
<td>1.375</td>
<td>1.280</td>
<td>32</td>
<td>1.50</td>
<td>1.70</td>
</tr>
<tr>
<td>6</td>
<td>.960</td>
<td>.513</td>
<td>1024</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>7</td>
<td>1.375</td>
<td>.868</td>
<td>1024</td>
<td>1.80</td>
<td>1.80</td>
</tr>
<tr>
<td>8</td>
<td>.875</td>
<td>.603</td>
<td>1024</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>Total</td>
<td>9.130</td>
<td>5.361</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
RESULTS OF ANALYSIS

The application problem given in Chapter 4, is an indication of the types of performance measures that can be explored with the aid of the Markovian models. It takes few minutes to prepare the data and run the program to test the different problems. The ability to obtain quick responses tends to encourage the analysis, which would in turn lead to a deeper understanding of the systems modeled.

It is apparent from this analysis, that the probability of failure with a higher repair rate, will reach a maximum level shortly after start up and then decreases as time increases. In addition, the effect of high component downtime with infrequent breakdowns, has a greater impact in the transition probabilities.

The variations of the system effectiveness as a function of time is significantly higher than that of system availability. It is also apparent that the system effectiveness is more sensitive to DOUT than of availability measure.

The results show that conducting the analysis, developed in this research, provides significantly higher production rate than the current one. The purpose of maximizing a production rate is to reduce the cost of manufacturing all families of parts.

Savings are realized through the reduction of system downtime. Indirectly, savings can also be gained by the reduction of station queues, resulting from any component failure. The gain of the system is the result of applying the optimum DOUT. Accordingly, PRATE for each family of parts could reach a maximum level.
The program code can be run in an IBM or compatible microcomputer. The following are the recommended procedures to be applied in the analysis of FMS. First, a failure mode analysis should be performed as discussed in Chapter 2. Second, the program code should be implemented in the computer. A sensitivity analysis is conducted and the optimum capacity planning will be displayed or printed for each sequence of family of parts. Third, the critical component will be determined for each system configuration.
CONCLUSIONS

Based on the analysis developed in this research, we can draw the following conclusions:

1. The analysis, adopted to the whole FMS and not to an individual component, explicitly demonstrates the effect of failures in modeling the performance of FMS.

2. The Markovian process was applied to the availability analysis of FMS, taking account of failure modes in regular and heavy operations.

3. Availability cannot serve properly as a performance measure for an FMS, since each failure configuration could have different probabilities.

4. The system effectiveness decreases with increasing DOUT. The production rate of a system reaches a maximum value of the product (DOUT x E), then it declines. The corresponding value of DOUT can be used as the optimum capacity planning for the system.

5. The critical component for each system can be determined after computing the production rate for each failure configuration.

6. The transient behavior of state probabilities and performance measures would provide management with optimum decisions in the analysis of FMS. Thus the managers of FMS should therefore have planning decisions that ensure system demand within the capacity of the system. Determining the transition probabilities, the managers could pay attention to those states with higher
probabilities. Also, they could expect when and which of the
failure modes is crucial.
7. The computer program, developed in this study, evaluate the time
dependent availability and average availability of an FMS described
by a Markov process. It also evaluate other performance measures,
such as, expected production rate, system effectiveness and average
component utilization. The program can be implemented in a
microcomputer, which determines the optimum capacity planning.

Further studies can be investigated, in particular the following:
1. Extension of the model to include bigger systems.
2. Alternative repair policies.
4. Developing control rules for an AGVS using the Markovian model B.
This study would not have been completed without the support and efforts of Professor J. C. Even, my major advisor and chairman of my Ph.D. committee. I take this opportunity to extend my profound gratitude and appreciation for his generous help and valuable guidance and for his patience, all of which have greatly influenced the completion of this study.

I would like to extend my appreciation and gratitude to Professor H. T. David for his help, valuable comments, suggestions and support. His assistance throughout this study is very much appreciated.

I wish to thank Dr. S. B. Vardeman for his constructive criticism and valuable suggestions which made this study possible.

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Special thanks are due Professor T. A. Barta and Professor V. A. Sposito. Their help in the final preparation of this dissertation is deeply appreciated.

Finally, to my wife Mary and my son Mark this is dedicated to make up in a small way for the patience and understanding required during the many hours spent in my research and in preparing this dissertation. Also to my parents and my wife's parents, my thanks too for their prayer and encouragement throughout my graduate program.
BIBLIOGRAPHY


APPENDIX A:

COMPUTER CODE FOR MARKOVIAN AVAILABILITY ANALYSIS

Introduction

This chapter is a brief description of the computer codes AVAL and ODE that evaluates the time dependent availability and all the performance measures described in Chapter 4, of a manufacturing system modeled by Markov process. A flow chart of the code is given in Figure 18. The methodology upon which the codes are based is presented in Chapters 2 and 3. The code consists of two parts:

1. AVAL which is written in BASIC, prepares the necessary transition matrix for the second part;

2. ODE which is written in FORTRAN, performs the Markovian availability analysis as a function of time.

Running the Program

The user can run the program either in an interactive mode or in a batch mode.

Interactive Mode

After the program has been loaded and the run command has been executed, an introduction to the program is displayed on the CRT. The user chooses the type of system from the menu. The program will then start to ask for input information such as the number of components of the system, the number of failure modes and the production rate for each component, the failure rate and the repair rate for each mode, and the
Program AVAl

Generate Set Q of Possible States

Partition of Set Q into $Q_0$ & $Q_e$

Generate Transition Matrix M as two Dim. Array

Generate Matrix K for Steady State

Generate Matrix B the inverse of K

$\text{AVAl} = \sum B(I,0)$

Generate Probability Vector P

$\text{PRATE} = \sum (P(I) \times W(I))$

Program ODE

Generate Transition Probability Vector P(t)

Calculate A(t), PRTAE(t), E(t)

FIGURE 18. Flow chart of computer program
desired system output. Next, a summary list will display the input information and give the user a chance to make changes if there are input errors.

If nothing is to be changed at this point, the program will then perform the necessary calculations for both steady state and transient solution. The output of each program is described later.

Batch Mode

In this mode, the first 10 lines in the program are reserved for multiple DATA statements. A separate DATA statement is used for each component. Each statement contains the following variables separated by commas. The order in which these variables should appear is:

System type 1:

Number of components in series,
number of failure modes of component 1,
production rate of component 1,

...,
...,
...,
number of failure modes of last component,
production rate of last component,
failure rate of mode 1 for component 1,
repair rate of mode 1 for component 1,

...,
...,
...,
failure rate of last mode for the last component,
repair rate of last mode for the last component.

System type 2:

Number of parallel components,
number of failure modes for each component,
production rate of each component,
failure rate of mode 1 for regular operation,
failure rate of mode 1 for heavy operation,
repair rate of mode 1,

...,
...,
...,
failure rate of last mode for regular operation,
failure rate of last mode for heavy operation,
repair rate of last mode.

System type 3:

Number of groups of parallel components (maximum 2),
number of components in series,
Number of components in group 1,
Number of failure modes in group 1,
production rate of each component in group 1,
number of component in group 2,
number of failure modes in group 2,
production rate of each component in group 2,
failure rate of mode 1 for regular operation,
failure rate of mode 1 for heavy operation,
repair rate of mode 1,

...,
...,
...,
failure rate of last mode for regular operation,
failure rate of last mode for heavy operation,
repair rate of last mode.
number of failure modes of series components,
number of failure modes of series 2,

...,
...,
...,
...,
number of last series component,
failure rate of mode 1 for series component 1,
repair rate of mode 1 for series component 1,

...,
...,
...,
...,
failure rate of last mode for the last series component,
repair rate of last mode for the last series component.

**IBM Compatible Microprocessors**

The program, although written on an IBM PC, was adapted to be compatible with a wide range of microprocessors such as Zenith, Kaypro
and Corona. In an effort to achieve greater accuracy, double precision variables have been used.

The procedure for the compatible microcomputers is slightly different. Thus, the two programs, AVAL and ODE, should be run separately, because of the limitation of GW BASIC of running an executable program inside BASIC.

Program AVAL

This part of the code written in BASIC, generates the set of all possible states of the system, partitions it into subset $q_0$ and $q_f$ and generates the transition matrix. In addition, this program calculates the following performance measures: steady state availability, average production rate, component utilization, and system effectiveness.

Subroutines

The program uses the following subroutines:

1. Subroutine Transition

   This subroutine consists of three parts based on the user's system structure. These are single component or component in series, components in parallel, and combined system. It also generates the states and the transition matrix.

2. Subroutine Factorial

   This subroutine calculates the number of states of each system configuration. For each number of failure modes $N$, with repeated removal of the same number permitted, the number of combinations formed by the removal of each $f$, number of components down, is calculated using the
following formula:

\[
\sum_{i=0}^{f-1} \binom{N+i}{N-1} \frac{1}{(N-1)! (N+2)! (N+f-1)!} \]

\[
\frac{N(N-1)}{2!} + \frac{N(N+1)(N+2)}{3!} + \cdots + \frac{N(N-1)\ldots(N+f-1)}{f!}
\]

3. Subroutine Inverse

This subroutine generates the inverse of the steady state matrix K. An identity matrix of the same size as K is added to the matrix. Then, row operations are performed. The first column of the inverse matrix is the steady state probability vector.

4. Subroutine Performance

This subroutine calculates the steady state availability, AVAL, the average production rate, PRATE, the average component utilization, U, and the system effectiveness, E.

Description of the Output

The program output consists of three parts. The first part contains information concerning the states of the system. For each possible system state, the number of operating states or the number of failed states and the total number of states are printed out. The states themselves are printed out and divided into groups. Each group contains either the operating states with the number of components "down" or the failed states with the number of components "down".
The second part of the output consists of two matrices: the transition matrix (M), and the steady-state matrix (K). This part of the output is printed for checking purposes and can be deleted. The transition matrix is automatically saved in an ASCII file under the name "Matrix".

The third part of the output contains information concerning system performance. The steady state probabilities are printed first, then the performance measures.

Source List of AVAL

Source list of AVAL and all other subroutines are given in Appendix B.

Application Problem

The output of the application problem is given in the CASE STUDY.

Dimensions Variables for AVAL

One-Dimensional array:

N(*) number of failure modes for each component.
LMDA(*) failure rate
MU(*) repair rate
W(*) production rate
SGS(\textasciitilde), ITS(\textasciitilde) string identifies the state
COMBS(\textasciitilde) string combines the left and right characters of other string.
UTL(*) component utilization in each parallel group
X(*) number of components in each parallel group
Two-dimensional array

M(\(*,\*) transition matrix
K(\(*,\*) steady-state matrix
B(\(*,\*) inverse of matrix K
RLMDA(\(*,\*) failure rate for regular operation
HLMDA(\(*,\*) failure rate for heavy operation
STATE(\(*,\*) number of states for each system configuration.

Note: * can be any integer.

Program ODE

This part of the code performs the solutions of system state equations and generates the transition probability vector P. In addition, the same performance measures as a function of time are calculated. The main program calls subroutine LSODA once for each point at which answers are desired.

Input Information

The input information for the ODE program is read from the ASCII file which is a part of AVAL output. The following variables are the input information for this part of the code.

NEQ Number of states
u system type
H number of groups of states
GRUP(*) number of states in each group of states (system configuration)
PR(*) production rate for each system configuration
Y(*) initial conditions
F(*,*) equivalent to the transition matrix

where * can be any integer as explained earlier.

Initial Conditions

In the ODE program, the initial conditions are denoted by the 1-dimensional vector array $Y(i)$ where $i$ denotes the system state and the time $t=0$ is already included at the beginning of the program. In the present program, the system is assumed to run at time $t=0$, i.e.,

$$Y(1) = 1 \text{ and } Y(i) = 0 \text{ for } i > 1$$

If different initial conditions are desired (to incorporate failures at $t=0$) the user can change the appropriate statements in the program (see listing of DATA statements).

Subroutine FEX

This subroutine, which is written in FORTRAN, defines the ODE system. The system is put in the first-order form

$$\frac{DY}{DT} = FEX(T,Y)$$

where $FEX$ is a vector-valued function of the scalar $T$ and the vector $Y$.

Subroutine FEX has the form;

```
SUBROUTINE FEX(NEQ,T,Y,YDOT)
DIMENSION Y(*),YDOT(*)
```

where $NEQ$, $T$ and $Y$ are inputs, and the array $YDOT:FEX(T,Y)$ is output. $Y$
and YDOT are arrays of length NEQ.

Output of ODE

The output of ODE consists of the transition probabilities, availability, production rate and system effectiveness as a function of time.

Source List of ODE

A source list of ODE is given in Appendix B.

Model Problem

The output of ODE for the application problem is given in the CASE STUDY.

Dimensioning Variables of ODE

One-Dimensional Array

\begin{itemize}
\item \textbf{Y(*)} \quad \text{Array of computed values of } Y(T)
\item \textbf{YDOT(*)} \quad \text{Array of the first derivatives of } Y(T)
\item \textbf{ATOL(*)} \quad \text{Absolute tolerance parameter (scalar or array of dimension NEQ)}
\item \textbf{RWORK(*)} \quad \text{Real work array of length at least } 22+\text{NEQ*MAX}(16,\text{NEQ}+9)
\item \textbf{IWORK(*)} \quad \text{Integer work array of length at least } 20+\text{NEQ}
\item \textbf{AVAL(*)} \quad \text{Availability as a function of time}
\item \textbf{PRATE(*)} \quad \text{Production rate as a function of time}
\end{itemize}

Two-Dimensional Array

\begin{itemize}
\item \textbf{F(*,*)} \quad \text{Transition matrix}
\end{itemize}
APPENDIX B:

SOURCE LIST OF COMPUTER PROGRAM

Source List of Program AVAL

1 CLEAR
3 DATA 2,2,15,25,.0127,.0245,.073,.005,.008,.0416
5 DATA 2,2,15,25,.0183,.023,.1544,.012,.03,.117
6 DATA
8 REM ******************************************************************************
10 REM INITIAL MESSAGE
15 REM******************************************************************************
20 PRINT "THIS PROGRAM DETERMINES THE TRANSITION MATRIX,"
25 PRINT "STEADY STATE AVAILABILITY AND PRODUCTION RATE"
30 PRINT "FOR DIFFERENT MANUFACTURING SYSTEMS AND"
35 PRINT "VARIOUS FAILURE MODES."
40 PRINT
45 PRINT
50 PRINT
55 PRINT " BY GEORGE ABDOU"
60 PRINT " IOWA STATE UNIVERSITY"
65 PRINT " AMES, IOWA"
70 REM******************************************************************************
75 REM EDITING SECTION
80 REM******************************************************************************
85 PRINT CHR$ (7)
90 PRINT "THE DIFFERENT SYSTEM'S STRUCTURES ARE:");U
95 PRINT " 1) A SINGLE COMPONENT OR SERIES COMPONENTS"
100 PRINT " 2) ONE GROUP OF PARALLEL COMPONENTS"
110 PRINT " 3) SERIES-PARALLEL NETWORK"
120 PRINT
130 PRINT
140 INPUT "THE SYSTEM TO BE ANALYZED IS OF STRUCTURE NO. ";U
150 IF U>3 OR U<1 THEN 85
160 ON U GOSUB 600,800,2000
170 REM******************************************************************************
180 REM PRINTING SECTION
190 REM******************************************************************************
191 ERASE M
192 OPEN "I","MATRIX"
193 INPUT #1,Q,U,H:Q=Q-1
194 FOR I=0 TO H:INPUT #1,GRUP(I),PR(I):NEXT I
195 DIM M(Q,Q)
196 FOR I=1 TO Q
198 FOR J=0 TO Q:INPUT #1,M(I,J):NEXT J
200 NEXT I
202 CLOSE #1
235 FOR J=0 TO Q
102

240 M(0,J)=1
260 NEXT J
263 IF Q < 30 THEN 277
265 CLS:PRINT "THE MATRIX K":PRINT TAB(10) "FROM STATE

.........../ Kac":PRINT "TO ":PRINT
266 FOR I=0 TO Q
267 K=0:ITS$(O)=" 0"
268 PRINT "STATE",I," |

270 IF M(I,J)=0 THEN 274
273 PRINT TAB(12) ITS$(J);" ":PRINT USING "+#.### ":M(I,J)
274 NEXT J
275 PRINT:NEXT I
276 GOTO 281
277 GOSUB 3050
281 GOSUB 360
282 AVL=B(0,0)
283 IF U=1 THEN PRATE=PR(0)*AVL:GOTO 320
285 CNT=1:SOM=0:PRATE=AVL*PR(0)
287 FOR R=1 TO H

288 PR0B=0:FOR J=CNT TO GRUP(R)+SOM:PROB=PROB+B(J,0):NEXT J
289 IF PR(R)>0 THEN AVL=AVL4-PR0B:PRATE=PRATE+PR(R)*PR0B
291 CNT=J:SOM=CNT-1:NEXT R
320 PRINT:PRINT TAB(5) "STEADY STATE AVAILABILITY= ";AVL
322 PRINT:PRINT TAB(5) "EXPECTED STEADY-STATE PRODUCTION RATE IN

UNITS/HR = "; PRATE
328 PRINT:PRINT TAB(5) "SYSTEM EFFECTIVENESS = "; PRATE/DOUT
330 SHELL "ODE.EXE"
350 END
360 REM ***********************************************

370 REM MATRIX INVERSION
380 REM ***********************************************
385 DIM B(Q,Q)
390 FOR J=0 TO Q
405 B(J,J)=1
410 NEXT J
412 FOR J=0 TO Q
415 FOR I=J TO Q
420 IF M(I,J)<0 THEN 430
425 NEXT I
430 FOR O=0 TO Q
435 Y=M(J,O)
440 M(J,O)=M(I,O)
445 M(I,O)=Y
450 Y=B(J,0)
455 B(J,0)=B(I,0)
460 B(I,0)=Y
465 NEXT O
470 T=1/M(J,J)
FOR O=0 TO Q
M(J,O)=T*M(J,O)
B(J,O)=T*B(J,O)
NEXT O
FOR L=0 TO Q
M(L,O)=M(L,O)+T*M(J,O)
B(L,O)=B(L,O)+T*B(J,O)
NEXT L
NEXT J
CLS
PRINT:PRINT " THE STEADY-STATE PROBABILITY VECTOR PI"
PRINT
PRINT "PRODUCTION RATE OF COMPONENT”;I;"IN UNITS/HR”;INPUT W(I):IF W(I)<MIN THEN MIN=W(I)
FOR J=1 TO N(I):Q=Q+1:PRINT "FAILURE RATE (IN UNITS/HR) OF COMPONENT”;I;"WITH FAILURE MODE”;J;PRINT LMDA(Q)
PRINT "REPAIR RATE (IN UNITS/HR) FOR COMPONENT”;I;"WITH FAILURE MODE”;J;PRINT MU(Q):STATE(Q,I)=J
NEXT J
CLS
PRINT " PROGRAM OUTPUT": FOR I=1 TO S: PRINT "AVERAGE UTILIZATION OF COMPONENT”;I;" = ";DOUT/W(I):NEXT I
CLS:PRINT TAB(S) "CLUSTER No. * 0 * ALL COMPONENTS UP ":PRINT TAB(S) "NUMBER OF STATES IN THIS CLUSTER = 1":GROUP(0)=1:PRINT(0)=MIN
FOR I=1 TO S: PRINT TAB(S*I) USING "#”;0: NEXT I
PRINT: PRINT: PRINT TAB(5) "CLUSTER No. * 1 * 1 COMPONENT DOWN"
PRINT TAB(5) "NUMBER OF STATES IN THIS CLUSTER = ", Q = GRUP(1) = 4: PR(1) = 0
FOR J = 1 TO Q: PRINT J; ": FOR I = 1 TO S: PRINT TAB(8 * I) USING 
"#"; STATE(J, I); : NEXT I: PRINT
NEXT J
PRINT: PRINT: PRINT
TTAL = 0
FOR J = 1 TO Q
M(0, J) = MU(J): M(J, 0) = LMDA(J)
TTAL = TTAL + M(J, 0)
M(0, 0) = -TTAL: M(J, J) = -M(0, J): NEXT J
L = S
U = 1: H = 1
OPEN "O", #1, "MATRIX"
FOR I = 0 TO H: PRINT #1, GRUP(I), PR(I): NEXT I
FOR I = 0 TO Q
FOR J = 0 TO Q: A# = M(I, J): PRINT #1, A#: NEXT J
NEXT I
CLOSE #1
PRINT " THE TRANSITION MATRIX M": PRINT: GOSUB 3000
RETURN
REM ************************************************************
CLEAR: DIM RLMDA(2, 5), HLMDA(2, 5), MU(2, 5), M(30, 30), COMBS(30)
DIM STATE(2, 5), W(2, 5), SG$(100), IT$(301
PRINT "HOW MANY COMPONENTS IN ACTIVE PARALLEL": INPUT X
PRINT "HOW MANY FAILURE MODES ASSOCIATED WITH EACH COMPONENT": INPUT N
L = 1: R = 1
FOR I = 1 TO N
PRINT "FAILURE RATE (IN UNITS/HR) OF MODE": I; "FOR REGULAR 
OPERATION": INPUT RLMDA(L, I)
PRINT "FAILURE RATE (IN UNITS/HR) OF MODE": I; "FOR HEAVY 
OPERATION": INPUT HLMDA(L, I)
PRINT "REPAIR RATE (IN UNITS/HR) OF MODE": I; "FOR EITHER 
OPERATION": INPUT MU(L, I)
NEXT I
PRINT: INPUT " DO YOU WANT TO CHANGE YOUR DATA [Y or 
N] ": US
IF LEFT$(US, 1) = "Y" OR LEFT$(US, 1) = "y" THEN 859
IF LEFT$(US, 1) = "N" OR LEFT$(US, 1) = "n" THEN 807
E = 0
GOSUB 1500
S = 1
FOR J=0 TO X
PRINT:PRINT TAB(5) "CLUSTER NO. * ";J:" * " COMPONENTS DOWN"
H=J:PR(J)=W(R,J):IF J=0 THEN PRINT TAB(5) "NUMBER OF STATES IN THIS
CLUSTER = 1":GOTO 871
PRINT TAB(5) "NUMBER OF STATES IN THIS CLUSTER = ";STATE(R,J)-
STATE(R,J-1):GRUP(J)=STATE(R,J)-STATE(R,J-1)
FOR I=S TO STATE(R,J)
IT$(I)=SG$(I):PRINT I",";TAB(9) SG$(I)
NEXT I
S=1+STATE(R,J)
NEXT J
GOSUB 873:GOTO 1125
REM ********************************************
REM ONE COMPONENT DOWN
REM ********************************************
FOR J=1 TO N
M(O,J+E)=MU(R,J)
M(J+E.O)=XR*RLMDA(R,J)
NEXT J
REM ********************************************
REM MORE THAN ONE COMPONENTS DOWN
REM ********************************************
FOR K=1 TO X-1
FOR I=STATE(R,K)+1 TO STATE(R,K+1)
FOR J=STATE(R,K-1)+1 TO STATE(R,K)
IF LEFT$(SG$(I),2*K)=SG$(J) THEN
Y=VAL(RIGHT$(SG$(I),2)):M(J+E,I+E)=(K+1)*MU(R,Y):M(I+E,J+E)=(X-
K)*HLMDA(R,Y)
NEXT J
NEXT I
NEXT K
RETURN
FOR I=0 TO STATE(R,X)
TTAL = 0
FOR J=0 TO STATE(R,X)
TTAL=TTAL+M(J,I)
NEXT J
M(I,I)=-TTAL
NEXT I
U=2
OPEN "O",#1,"MATRIX"
PRINT #1,O:PRINT:PRINT "CLUSTER No. No. OF STATES
RATE":FOR I=0 TO H:PRINT #1,GRUP(I),PR(I):PRINT I,GRUP(I),PR(I):NEXT I
FOR I=0 TO Q
1154 FOR J=0 TO Q;A#=M(I,J):PRINT #1,A#:NEXT J
1155 NEXT I
1156 CLOSE #1
1157 PRINT:PRINT:"THE TRANSITION MATRIX M":PRINT:GOSUB 3000
1160 GOTO 170
1500 REM **********************************************************************
1505 REM FACTORIAL/COMBINATION
1510 REM **********************************************************************
1515 Q=0
1520 FOR I=0 TO X
1525 A=N+I-1;G=N+I-1;C=N-1
1530 TK=G-C
1535 IF TK=0 THEN GRP=1:GOTO 1562
1540 A=A-1
1545 IF A=TK THEN 1555
1550 G=G*A:GOTO 1540
1555 GRP=G/C
1560 Q=Q+GRP
1561 STATE(R,I)=Q
1562 MOUT=X*WP
1563 UTL=DOUT/MOUT
1575 FR(R)=X*(1-UTL)
1580 IF I=<FR(R) THEN W(R,I)=DOUT:GOTO 1587
1587 NEXT I
1590 PRINT:PRINT:"PROGRAM OUTPUT":PRINT:"AVERAGE UTILIZATION"
1591 REM **********************************************************************
1592 REM * GENERATE SET OF POSSIBLE STATES *
1593 REM **********************************************************************
1594 FOR I=0 TO N
1595 SG$(I)=STR$(I)
1598 NEXT I
1601 FOR J=1 TO N
1604 SG$(J+I-1)=SG$(J)-(-SG$(J)
1607 NEXT J
1610 S=0
1613 FOR Z=1 TO N-1
1616 SG$(2*N+Z+S)=SG$(Z)+SG$(Z+1)
1619 IF Z+1>N-1 THEN 1628
1622 SG$(2*N+Z+1)=SG$(Z)+SG$(Z+2)
1625 S=S+1
1628 NEXT Z
1631 GRP=STATE(R,2)-STATE(R,1)
1634 FOR I=1 TO GRP
1637 SG$(STATE(R,2)+I)=SG$(1)+SG$(STATE(R,1)+I)
1640 NEXT I
1643 S=0
1646 FOR K=1 TO GRP
107

1649 IF LEFT$(SG$(STATE(R,1)+K),1) = "1" THEN 1658
1652 SG$(STATE(R,2)+I+S)=SG$(2)+SG$(STATE(R,1)+K)
1655 S=S+1
1658 NEXT K
1661 IF X=3 THEN SG$(STATE(R,X))=SG$(N)+SG$(N)+SG$(N)
1691 RETURN
2000 REM ****************************************************
2010 REM COMBINED SYSTEM
2011 REM ****************************************************
2012 DIM X(3),N(3),WP(3),UTL(3),COMBS$1000
2013 DIM RLMDA(3,10),HLMDA(3,10),MU(3,10)
2017 INPUT "HOW MANY GROUP OF PARALLEL COMPONENTS ";L
2018 INPUT "HOW MANY COMPONENTS IN SERIES ";SER:IF L=1 AND SER=0 THEN 800
2019 IF L=1 AND SER>0 THEN 2550
2023 FOR R=1 TO L
2024 PRINT "HOW MANY COMPONENTS IN GROUP" ;R; : INPUT X(R );PRINT "HOW MANY
2025 FAILURE MODE ASSOCIATED WITH EACH COMPONENT IN GROUP":R;:INPUT N(R)
2026 PRINT "PRODUCTION RATE (IN UNITS/HR) OF EACH COMPONENT IN
2027 GROUP";R;:INPUT WP(R):PRINT "DESIRED SYSTEM OUTPUT (IN UNITS/HR)";:INPUT
2028 DOUT
2029 FOR J=1 TO N(R)
2030 PRINT "FAILURE RATE (IN UNITS/HR) OF MODE";J;"FOR REGULAR
2031 OPERATION";:INPUT RLMDA(R,J)
2032 PRINT "FAILURE RATE (IN UNITS/HR) OF MODE";J;"FOR HEAVY
2033 OPERATION";:INPUT HLMDA(R,J)
2034 PRINT "REPAIR RATE (IN UNITS/HR) OF EITHER
2035 OPERATION";:INPUT MU(R,J)
2036 NEXT J
2037 NEXT R
2038 PRINT:INPUT " DO YOU WANT TO CHANGE YOUR DATA [Y or
2039 N]";US
2040 IF LEFT$(US,1)="N" OR LEFT$(US,1)="n" THEN 2039
2041 IF LEFT$(US,1)="Y" OR LEFT$(US,1)="y" THEN 2017
2049 REM ****************************************************
2050 REM DETERMINE SET OF POSSIBLE STATES *
2051 REM ****************************************************
2052 Y1=Y1+STATE(R,X):Y2=Y2*STATE(R,X):Y3=Y3*(STATE(R,X)-STATE(R,X-1))
2055 FOR R=1 TO L
2060 X=X(R):N=N(R):WP=WP(R)
2062 GOSUB 1500
2063 Y1=Y1+STATE(R,X):Y2=Y2*STATE(R,X):Y3=Y3*(STATE(R,X)-STATE(R,X-1))
2070 NEXT R
2078 E=0:H=0:PR(H)=DOUT
2080 FOR R=1 TO L
2082 P=1
2085 X=X(R):N=N(R):WP=WP(R)
2086 GOSUB 1591
2089 FOR I=1 TO STATE(R,X):GS$(R,I)=SG$(I):IT$(I+E)=GS$(R,I):NEXT I
2090 GOSUB 873
2094 FOR J=1 TO X-1:H=H+1:GRUP(H)=STATE(R,J)-STATE(R,J-1):PR(H)=W(R,J):
PRINT:PRINT TAB(5) "CLUSTER NO. * "H:" * ";J:"COMPOONENT OF GROUP 
","R:" ARE DOWN":PRINT TAB(5) "NUMBER OF STATES IN THIS CLUSTER = 
",";GRUP(H
2095 FOR I=P+E TO STATE(R,J)+E:PRINT I","; TAB(9) IT$(I):NEXT I
2097 P=1:STATE(R,J):NEXT J
2098 FOR I=STATE(R,X)+E TO STATE(R,X-1)+E+1 STEP-1
2100 FOR J=STATE(R,X-2)+F TO STATE(R,X-1)+E
2107 M(J,I)=0:M(I,J)=0
2110 NEXT J
2115 IT$(A)=IT$(I)
2117 A=A-1
2120 NEXT I
2125 E=E+STATE(R,X-1):F=E+1
2126 STATE(R,0)=0:V(0,0)=1
2127 FOR I=1 TO X(R)
2128 IF R=1 THEN V(0,I)=STATE(1,I)-STATE(1,I-1)
2129 IF R=2 THEN V(1,0)=STATE(2,1)-STATE(2,1-1)
2130 NEXT I
2135 NEXT R
2136 FOR I=1 TO X(I)
2137 FOR J=1 TO X(2):V(I,J)=V(0,I)*V(J,0):NEXT J
2138 NEXT I
2139 H=H+1
2140 Y=1:S=1:Z=1:T=1:D1=F
2142 PRINT:PRINT TAB(5) "CLUSTER NO. * ";H:" * ";S:"COMPOONENT OF GROUP 1
AND ";T:" COMPONENT OF GROUP 2 ARE DOWN
2143 PRINT TAB(5) "NUMBER OF STATES IN THIS CLUSTER = 
",";V(S,T)=GRUP(H)=V(S,T)
2144 GOSUB 3200:PR(H)=MIN
2145 FOR I=Y TO STATE(1,S)
2150 FOR J=Z TO STATE(2,T)
2155 IT$(F)=GS$(1,I)+GS$(2,J)
2156 PRINT F ","; TAB(9) IT$(F)
2157 F=F+1
2158 NEXT J
2159 NEXT I
2160 FOR P1=F-V(S,T) TO F-1
2161 FOR K1=1 TO STATE(1,S)
2162 IF LEFT$(IT$(P1),2)<>IT$(K1) THEN 2164
2163 O=VAL(RIGHT$(IT$(P1),2)):M(K1,P1)=MU(2,0):M(P1,K1)=X(2)*RLMDA(2,0)
109

2164 NEXT K1
2165 FOR L=K1 TO STATE(2,T)+K1-1
2166 IF RIGHT$(IT$(P1),2)<><IT$(L) THEN 2168
2167 0=VAL(LEFT$(IT$(P1),2)):M(L,P1)=MU(1,0):M(P1,L)=X(1)*RLMDA(1,0)
2168 NEXT L
2175 NEXT P1
2176 H=H+1:D2=F
2177 Z=J=T+1
2178 PRINT:PRINT TAB(5) "CLUSTER NO. * ";H;" * ";S;" COMPONENT OF GROUP
2179 AND ";T;" COMPONENT OF GROUP 2 ARE DOWN":PRINT TAB(5) "NUMBER OF
2180 STATES IN THIS CLUSTER = ";V(S,T):GRUP(H)=V(S,T):GOSUB

3200:PR(H)=MIN
2180 FOR I=Y TO STATE(1,S)
2185 FOR J=Z TO STATE(2,T)
2190 IT$(F)=GS$(1,I)+GS$(2,J)
2191 PRINT F","; TAB(9) IT$(F)
2194 F=F+1
2195 NEXT J
2196 D3=F
2197 NEXT I
2198 FOR P1=F-V(S,T) TO F-1
2199 IF X(2)=3 THEN 2201
2200 GOTO 2204
2201 FOR K=D1-V(2,0) TO D1-1:IF RIGHT$(IT$(P1),4)<><IT$(K) THEN 2203
2202 0=VAL(LEFT$(IT$(P1),2));M(K,P1)=MU(1,0):M(P1,K)=X(1)*RLMDA(1,0)
2203 NEXT K
2204 FOR L=D1 TO D1+V(l, 1)-1 : IF LEFT$(IT$(P1),2)<><LEFT$(IT$(L),2) THEN
2205 IF MIDS$(IT$(P1),3,2)=RIGHT$(IT$(L),2) THEN
2206 0=VAL(RIGHT$(IT$(P1),2)):M(L,P1)=2*MU(2,0):M(P1,L)=(X(2)-1)*HLMDA(2,0)
2207 NEXT L
2208 NEXT P1
2209 Y=I;Z=1:H=H+1
2210 PRINT:PRINT TAB(5) "CLUSTER NO. * ";H;" * ";S;" COMPONENT OF GROUP
2211 AND ";T;" COMPONENT OF GROUP 2 ARE DOWN":PRINT TAB(5) "NUMBER OF
2212 STATES IN THIS CLUSTER ";V(T,S):GRUP(H)=V(T,S):S=T:GOSUB

3200:PR(H)=MIN:S=1
2211 FOR I=Y TO STATE(1,T)
2212 FOR J=Z TO STATE(2,S)
2213 IT$(F)=GS$(1,I)+GS$(2,J)
2214 PRINT F","; TAB(9) IT$(F)
2215 F=F+1
2216 NEXT J
2217 NEXT I
2218 FOR P1=F-V(T,S) TO F-1
2219 IF X(1)>2 THEN 2221
2220 GOTO 2224
FOR K=STATE(1,1)+1 TO STATE(1,2): IF LEFTS(ITS(P1),4)<>LEFTS(ITS(K),4) THEN 2223
    2222 O=VAL(RIGHTS(ITS(P1),2)):M(K,P1)=MU(2,0):M(P1,K)=X(2)*RLMDA(2,0)
    2223 NEXT K
    2224 FOR L=D1 TO D1+V(1,1)-1:IF RIGHTS(ITS(P1),2)<>RIGHTS(ITS(L),2) THEN
    2225 IF MIDS(ITS(P1),3,2)=LEFTS(ITS(L),2) THEN O=VAL(LEFTS(ITS(P1),2)):
        M(L,P1)=2*MU(1,0):M(P1,L)=(X(1)-1)*HLMDA(1,0)
        O=VAL(MIDS(ITS(P1),3,2)):M(L,P1)=2*MU(1,0):M(P1,L)=(X(1)-1)*HLMDA(1,0)
    2226 IF LEFTS(ITS(P1),2)=LEFTS(ITS(L),2) THEN
        O=VAL(MIDS(ITS(P1),3,2)):M(L,P1)=2*MU(1,0):M(P1,L)=(X(1)-1)*HLMDA(1,0)
    2227 NEXT L
    2228 H=H+1
    2229 S=S+1:Z=J:IF S+1>X(1) AND T+1>X(2) THEN 2330
    2230 NEXT P1
    2231 NEXT P1
    2232 PRINT:PRINT TAB(5) "CLUSTER NO. * ";H;" * ";S;" COMPONENT OF GROUP 1 AND ";T;" COMPONENT OF GROUP 2 ARE DOWN":PRINT TAB(5) "NUMBER OF STATES IN THIS CLUSTER = ";V(S,T):GRUP(H)=Y(S,T):GOSUB 3200:PR(H)=MN
    2233 FOR I=Y TO STATE(1,S)
    2234 FOR J=Z TO STATE(2,T)
    2235 PRINT F","; TAB(9) ITS(F)
    2236 FOR P1=F-V(2,2) TO F-1
    2237 FOR L=D2 TO D2+V(l,2)-1:IF RIGHTS(ITS(P1),4)<>RIGHTS(ITS(L),4) THEN
    2238 IF MIDS(ITS(P1),3,2)=LEFTS(ITS(L),2) THEN O=VAL(LEFTS(ITS(P1),2)):
        M(L,P1)=2*MU(1,0):IF X(2)>2 THEN M(P1,L)=(X(1)-1)*HLMDA(1,0)
        M(L,P1)=2*MU(1,0):IF X(2)>2 THEN M(P1,L)=(X(1)-1)*HLMDA(1,0)
    2239 NEXT L
    2240 FOR K=D3 TO D3+V(2,1)-1:IF LEFTS(ITS(P1),4)<>LEFTS(ITS(K),4) THEN
    2241 IF MIDS(ITS(P1),5,2)=RIGHTS(ITS(K),2) THEN O=VAL(RIGHTS(ITS(P1),2)):
        M(K,P1)=2*MU(2,0):IF X(1)>2 THEN M(K,P1)=(X(2)-1)*HLMDA(1,0)
    2242 IF RIGHTS(ITS(P1),2)=RIGHTS(ITS(K),2) THEN O=VAL(MIDS(ITS(P1),5,2)):
        M(K,P1)=2*MU(2,0):IF X(1)>2 THEN M(K,P1)=(X(2)-1)*HLMDA(1,0)
    2243 NEXT L
    2244 NEXT P1
    2245 NEXT P1
    2246 PRINT:PRINT TAB(5) "CLUSTER NO. * ";H;" * ";S;" COMPONENT OF GROUP 1 AND ";T;" COMPONENT OF GROUP 2 ARE DOWN":PRINT TAB(5) "NUMBER OF STATES IN THIS CLUSTER = ";V(S,T):GRUP(H)=V(S,T):GOSUB 3200:PR(H)=MN
    2247 FOR I=Y+1 TO STATE(1,S)
    2248 FOR J=Z+1 TO STATE(2,T)
    2249 PRINT F","; TAB(9) ITS(F)
    2250 FOR P1=F-V(2,2) TO F-1
    2251 FOR L=D2+1 TO D2+V(l,2)-1:IF RIGHTS(ITS(P1),4)<>RIGHTS(ITS(L),4) THEN
    2252 IF MIDS(ITS(P1),3,2)=LEFTS(ITS(L),2) THEN O=VAL(LEFTS(ITS(P1),2)):
        M(L,P1)=2*MU(1,0):IF X(2)>2 THEN M(P1,L)=(X(1)-1)*HLMDA(1,0)
    2253 IF RIGHTS(ITS(P1),2)=RIGHTS(ITS(L),2) THEN O=VAL(MIDS(ITS(P1),3,2)):
        M(L,P1)=2*MU(1,0):IF X(2)>2 THEN M(P1,L)=(X(1)-1)*HLMDA(1,0)
    2254 NEXT L
    2255 NEXT P1
    2256 NEXT P1
    2257 IF T+1>X(2) THEN 2280
    2258 S=1:Y=1:T=1+1:Z=J
    2259 H=H+1
    2260 PRINT:PRINT TAB(5) "CLUSTER NO. * ";H;" * ";S;" COMPONENT OF GROUP 1 AND ";T;" COMPONENT OF GROUP 2 ARE DOWN":PRINT TAB(5) "NUMBER OF STATES IN THIS CLUSTER = ";V(S,T):GRUP(H)=V(S,T):GOSUB 3200:PR(H)=MN
2261 FOR I=1 TO STATE(1,S)
2262 FOR J=Z TO STATE(2,T)
2263 IT$(F)=GS$(1,I)+GS$(2,J)
2264 PRINT F".\"; TAB(9) IT$(F)
2265 F=F+1
2266 NEXT J
2267 NEXT I
2268 FOR P1=F-V(S,T) TO F-1
2269 IF L=D2 TO D2+V(S,2)-1: IF LEFT$(IT$(P1),2*S)<LEFT$(IT$(L),2*S)
2270 THEN 2273
2271 IF RIGHT$(IT$(P1),4)=RIGHT$(IT$(L),4) THEN 0=VAL(RIGHT$(IT$(P1),2))
2272 :M(L,P1)=MU(2.0):M(P1,L)=HLMDA(2.0)
2273 COMBS$(L)=MIDS$(IT$(P1),3,2)+RIGHT$(IT$(P1),3): IF COMBS$(L)=MIDS$(IT$
2274 (P1),3,4) THEN 0=VAL(MIDS$(IT$(P1),4,2)) :M(L,P1)=MU(2.0):M(P1,L)=HLMDA(2.0)
2275 NEXT L
2276 FOR L=F TO G: IF RIGHT$(IT$(P1),6) = RIGHT$(IT$(L),6) THEN 0=VAL(
2277 LEFT$(IT$(P1),2)):M(L,P1)=MU(1.0)
2278 NEXT L
2279 NEXT P1
2280 S=S+1
2281 Y=I:D2=D4:GOTO 2257
2282 IF S+1>X(1) THEN 2330
2283 Y=I:S=S+1:T=1:Z=1
2284 H=H+1
2285 PRINT:PRINT TAB(5) "CLUSTER NO. * \";H;\" * \";S;$" COMPONENT OF GROUP
2286 1 AND \";T;\" COMPONENT OF GROUP 2 ARE DOWN": PR INT TAB(5) "NUMBER OF
2287 STATES IN THIS CLUSTER = ":V(S,T):GRUP(H)=V(S,T):GOSUB
2288 3200:PR(H)=MIN
2289 FOR I=Y TO STATE(1,S)
2290 FOR J=Z TO STATE(2,T)
2291 IT$(F)=GS$(1,I)+GS$(2,J)
2292 PRINT F".\"; TAB(9) IT$(F)
2293 F=F+1
2294 NEXT J
2295 NEXT I
2296 T=T+1
2297 IF T+1>X(2) THEN 2330
2298 Z=J:GOTO 2290
2299 FOR R=2 TO 1 STEP-1
2300 H=H+1
2301 PRINT:PRINT TAB(5) "CLUSTER NO. * \";H;\" * \";X(R);\" COMPONENTS OF
2302 GROUP \";R;\" ARE DOWN": GOSUB 3200:PR(H)=MIN
2303 IF R=2 THEN V=V(X(R),0)
2304 IF R=1 THEN V=V(0,X(1))
2305 PRINT TAB(5) "NUMBER OF STATES IN THIS CLUSTER = ":V:GRUP(H)=V
112

2337 FOR I=1 TO V:PRINT F",";TAB(9) ITS(F):F=F+1:NEXT I
2340 NEXT R
2341 Q=V2
2360 FOR I=0 TO Q
2362 TTLA=0
2363 M(I,I)=0
2365 FOR J=0 TO Q
2370 TTLA=TTAL+M(J,I)
2380 NEXT J
2385 M(I,I)=-TTAL
2390 NEXT I
2391 U=3:MIN=0
2392 OPEN "O",#1,"MATRIX"
2393 PRINT #1,Q+1,U,H:PRINT:PRINT:PRINT "CLUSTER NO. No. OF STATES
2394 PRATE";FOR I=0 TO H:PRINT #1,GRUP(I),PR(I):PRINT I,GRUP(I),PR(I):
2395 NEXT I
2396 PRINT " THE MATRIX M";PRINT;PRINT;PRINT TAB(10) " FROM
2397 STATE ...../ Kac";PRINT " TO ":PRINT
2398 FOR I=0 TO Q
2400 K=0:IT$(0)=" 0"
2402 PRINT "STATE";IT$(I);"|"
2404 FOR J=0 TO Q
2405 IT$(0)=" 0"
2406 A#=M(I,J):PRINT #1,A#
2408 IF M(I,J)=0 THEN 2414
2412 PRINT TAB(12) IT$(J);":"PRINT USING "+.### ";M(I,J)
2414 NEXT J
2416 PRINT:NEXT I
2417 CLOSE #1
2500 RETURN
2550 CLEAR
2554 DIM RLMDA(1,5),HLMDA(1,5),MU(1,5),M(50,50),COMBS(30),ITS(50)
2562 DIM STATE(2,5),W(1,5),SG$(100),N(5),WP(5),LMDA(5),SMU(5),GATE(5)
2601 INPUT "HOW MANY COMPONENTS IN ACTIVE PARALLEL";X
2602 INPUT "HOW MANY FAILURE MODES ASSOCIATED WITH EACH COMPONENT";N
2603 INPUT "PRODUCTION RATE OF EACH COMPONENT IN UNITS/HR";WP
2604 L=1:INPUT "DESIZED SYSTEM OUTPUT IN UNITS/HR";DOUT
2605 FOR I=1 TO N
2606 PRINT "FAILURE RATE OF MODE No. ";I:"FOR REGULAR OPERATION,IN
2607 PRINT "FAILURE RATE OF MODE No. ";I:"FOR HEAVY OPERATION,IN
2608 PRINT "REPAIR RATE,IN UNITS/HR,FOR MODE No. ";I;"=";INPUT
2609 MU(L,I)
2609 NEXT I
2610 INPUT "HOW MANY COMPONENTS IN SERIES";SER:KM=0:MIN=DOUT
2611 FOR I=1 TO SER
2612 PRINT "NUMBER OF FAILURE MODES OF COMPONENT";I;:INPUT N(I):PRINT
2613 "PRODUCTION RATE OF COMPONENT";I;:INPUT WP(I):IF WP(I)<MIN THEN
2614 MIN=WP(I)
113

2613 FOR J=1 TO N(I):KM=KM+1:PRINT "FAILURE RATE OF COMPONENT";I;"WITH "
FAILURE MODE";J:: INPUT LMDA(KM):PRINT "REPAIR RATE OF COMPONENT";I;
"WITH FAILURE MODE ";J:: INPUT SMU(KM):GATE(KM)=J:NEXT J
2614 NEXT I
2615 PRINT:INPUT "DO YOU WANT TO CHANGE YOUR DATA [Y or"
N]":US
2616 IF LEFT$(US,1)="N" OR LEFT$(US,1)="n" THEN 2618
2617 IF LEFT$(US,1)="Y" OR LEFT$(US,1)="y" THEN 2601
2618 E=0:R=1
2619 GOSUB 1500
2620 S=1
2621 FOR J=0 TO X
2622 PRINT:PRINT TAB(5) "CLUSTER NO. *" ;J; " *" ;J; " COMPONENTS DOWN"
2623 H=J:PR(J)=W(R,J):IF J=0 THEN PRINT TAB(5) "NUMBER OF STATES IN THIS"
CLUSTER = 1":GOTO 2636
2624 PRINT TAB(5) "NUMBER OF STATES IN THIS CLUSTER =" ;STATE(R,J)-
STATE(R,J-1):GRUP(J)=STATE(R,J)-STATE(R,J-1)
2628 FOR I=S TO STATE(R,J)
2630 IT$(I)=SG$(I):PRINT I",";TAB(9) SG$(I)
2633 NEXT I
2634 S=1+STATE(R,J)
2635 NEXT J
2637 D9=I
2638 FOR J=1 TO N
2640 M(0,J+Q)=MU(R,J)
2642 M(J+Q,0)=X*RLMDA(R,J)
2644 NEXT J
2646 FOR K=1 TO X-1
2648 FOR I=STATE(R,K)-1 TO STATE(R,K+1)
2650 FOR J=STATE(R,K-1)+1 TO STATE(R,K)
2652 IF LEFT$(SG$(I),2*K)=SG$(J) THEN
Y=VAL(RIGHT$(SG$(I),2)):M(J+E,I+E)=MU(R,Y):M(I+E,J+E)=(X-K)*HLMDA
(R,Y)
2654 IF RIGHT$(SG$(I),2*K)=SG$(J) THEN Y=VAL(LEFT$(SG$(I),2)):M(J+E,I+E)
=MU(R,Y):M(I+E,J+E)=(X-K)*HLMDA(R,Y)
2656 COMBS(I)=LEFT$(SG$(I),2)+RIGHT$(SG$(I),2)
2658 IF COMBS(I)=SG$(J) THEN
Y=VAL(MIDS(SG$(I),3,2)):M(J+E,I+E)=MU(R,Y):M(I+E,J+E)=(X-K)*HLMDA
(R,Y)
2660 NEXT J
2662 NEXT I
2663 NEXT K
2666 H=H+1:PRINT:PRINT:PRINT TAB(5) "CLUSTER NO. * " ;H; " * 1 SERIES"
COMPONENT DOWN"
2668 PRINT TAB(5) "NUMBER OF STATES IN THIS CLUSTER =" ;KM
2670 FOR J=1 TO KM
2672 M(0,J+Q)=SMU(J):M(J+Q,0)=LMDA(J)
2674 PRINT D9",";TAB(9) GATE(J):ITS{STATE(R,X)+J}=STR$(GATE(J)):D9=D9+1
2676 NEXT J
FOR L=1 TO X-1
H=H+1:PRINT;PRINT:PRINT "CLUSTER NO. * ";H;" * 1 SERIES COMPONENT AND *";L;"PARALLEL COMPONENT DOWN"
PRINT TAB(5) "NUMBER OF STATES IN THIS CLUSTER = ";KM*(STATE(1,L)-STATE(1,L-1))
FOR K=C TO STATE(1,L)
C=STATE(1,L)+1
NEXT K
NEXT L
FOR I=0 TO Q
TTAL = 0
M(I,I)=0
FOR J=0 TO Q
TTAL=TTAL+M(J,I)
NEXT J
M(I,I)=-TTAL
NEXT I
U=3:MIN=0
OPEN "O",#1,"MATRIX"
PRINT #1,Q+1.U,H:FOR I=0 TO H:PRINT #1,GRUP(I),PR(I):NEXT I
FOR I=0 TO Q
FOR J=0 TO Q:A#=M(I,J):PRINT #1,A#:NEXT J
NEXT I
CLOSE #1
GOSUB 2900:GOTO 170
REM ************************************************************
REM * PRINTING THE TRANSITION MATRIX M *
REM ************************************************************
Q1=0:Q2=Q:IT$(0)=" 0"
IF Q2>Q THEN Q2=Q
PRINT:PRINT:PRINT
PRINT TAB(7):FOR K=Q1 TO Q2:PRINT USING " ";"STATE";:NEXT K
PRINT TAB(7):IF Q2<9 THEN FOR J=Q1 TO Q2:PRINT " ";:NEXT J:GOTO 3008
PRINT:PRINT:PRINT ";J:;NEXT J
PRINT TAB(7) "|----" :FOR K=1 TO 9*(Q2-Q1):PRINT "-";:NEXT K
FOR I=0 TO Q
IT$(0)=" 0"
PRINT:PRINT "STATE";I;"|
IF M(I,J)>0 THEN PRINT USING "+.#### ";M(I,J):GOTO 3020
PRINT " ";
NEXT J
115

3022 NEXT I
3024 Q1=J:IF Q-Q2=<10 AND Q-Q>0 THEN Q2=Q:GOTO 3003
3026 Q2=Q2+10:IF Q2<Q THEN 3003
3030 RETURN
3050 REM **********************
3055 REM * PRINTING MATRIX K *
3060 REM **********************
3063 CLS
3065 PRINT:PRINT "THE MATRIX K"
3070 PRINT:PRINT "The matrix K is obtained by deleting the last row and"
3075 PRINT "adding a vector of 1, the sum of probabilities at each time"
3080 PRINT "interval, in the first row of the matrix."
3100 Q1=0:Q2=9
3102 IF Q2>Q THEN Q2=Q
3103 PRINT:PRINT PRINT TAB(10);:FOR K=Q1 TO Q2:PRINT USING "\";"STATE":;
3104 PRINT TAB(7);:IF Q2>9 THEN FOR J=Q1 TO Q2:PRINT " ":NEXT J:
3105 FOR J=Q1 TO Q2:PRINT " ":NEXT J
3106 PRINT TAB(7);:IF Q2>9 THEN FOR J=Q1 TO Q2:PRINT " ":;J;:NEXT J:
3108 FOR J=Q1 TO Q2:PRINT " ":;J;:NEXT J
3109 PRINT TAB(7) "|-----"
3110 PRINT TAB(7) "v":PRINT " ":FOR K=1 TO 9*(Q2-Q1):PRINT "-":;NEXT K
3112 PRINT
3114 FOR I=0 TO Q
3115 IF I=0 THEN PRINT "SUM Pi= |":;GOTO 3118
3116 PRINT:PRINT "STATE";I;"|"
3118 FOR J=Q1 TO Q2:IF M(I,J)>0 THEN PRINT USING "+##.### ":M(I,J);;GOTO
3120
3121 PRINT " ";
3122 NEXT J
3124 Q1=J:IF Q-Q2=<10 AND Q-Q>0 THEN Q2=Q:GOTO 3103
3126 Q2=Q2+10:IF Q2<Q THEN 3103
3130 RETURN
3200 MIN=W(1,S)
3210 IF W(2,T)<MIN THEN MIN=W(2,T)
3230 RETURN
Source List of Program ODE

EXTERNAL FEX
DOUBLE PRECISION ATOL,RWORK,RTOL,T,TOUT,Y
DIMENSION Y(2550),ATOL(50),RWORK(2972),IWORK(70)
INTEGER SOM,CNT,H,GRP,U
OPEN(1,FILE='MATRIX')
READ(1,*) NEQ,U,H
IF (U.EQ.1) GO TO 7
DO 5 I=1,H+1
5 READ(1,*) GRUP(I),PR(I)
7 NNEQ = NEQ +(NEQ*NEQ)
READ(1,*) (Y(K),K=NEQ+1,NNEQ)
CLOSE (1)
Y(1)=1.0D0
DO 10 I=2,NEQ
10 Y(I)=0.0D0
T=0.0D0
TOUT=0.0D0
ITOL=2
RTOL=1.0D-4
DO 15 I=1,NEQ
15 ATOL(I)=1.0D-6
ITASK=1
ISTATE=1
ILOPT=0
LRW=2972
LIW=70
JT=2
DO 40 IOUT=1,11
   CALL LSODA(FEX,NEQ,Y,T,TOUT,ITOL,RTOL,ATOL,ITASK,ISTATE,
   1   ILOPT,RWORK,LRW,IWORK,LIW,JDUM,JT)
   AVL=Y(1)
   IF (U.GT.1) GO TO 24
   PRATE=H*AVL
   GO TO 37
24 CNT=2
   SOM=1
   PRATE=Y(1)*PR(1)
   DO 25 J=2,H
      PROB=0
      DO 26 K=CNT,GRUP(J)+SOM
         PROB=PROB+Y(K)
      26 IF (PR(J).EQ.0) GO TO 37
      AVL=AVL+PROB
      PRATE=PRATE+PR(J)*PROB
      CNT=K
      SOM=CNT-1

   GO TO 40
25 PROB=0
   DO 26 K=CNT,GRUP(J)+SOM
      PROB=PROB+Y(K)
   26 IF (PR(J).EQ.0) GO TO 37
   AVL=AVL+PROB
   PRATE=PRATE+PR(J)*PROB
   CNT=K
   SOM=CNT-1

   GO TO 40
CONTINUE
WRITE(*,31)T
FORMAT(1X,\,' AT T =',E12.5)
WRITE(*,32)(I,Y(I),I=1,NEQ)
FORMAT(1X,' Y(',I2,' ) = ',E12.5/)
WRITE(*,33)AVL
FORMAT(1X,10H AVL = ,E12.5)
WRITE(*,34)PRATE
FORMAT(1X,10H PRATE = ,E12.5)
IF (ISTATE .LT.0) GO TO 80
IF (IOUT.GT.1) GO TO 38
TOUT=TOUT+2.0D0
GO TO 40
TOUT=TOUT*2.0D0
CONTINUE
STOP
WRITE(*,90)ISTATE
FORMAT(1X,'ERROR HALT.. I STATE = ',I3)
STOP
C
SUBROUTINE FEX(NEQ,T,Y,YDOT)
DOUBLE PRECISION T,Y,YDOT
DIMENSION Y(3550),YDOT(50)
DO 100 I=1,NEQ
YDOT(I)=0.0D0
DO 150 J=1,NEQ
M = I*NEQ + J
150 YDOT(I)=YDOT(I)+Y(M) * Y(J)
CONTINUE
RETURN
END
APPENDIX C:
TYPICAL PROBLEMS IN FMS

This section discusses the possible sources of failure of an FMS, which consists of three basic modules: machine, material handling, and computer control, Figure 19. In addition, typical problems in the inspection module are included.

Machine Module

Machine tool errors either in size, shape, location, or surface finish of a feature of the part can be the result of one or a combination of five broad classes of failures in the manufacturing process: Mechanical, hydraulic, Electrical, Electronic, and Tooling. Mechanical and hydraulic failures can be combined since mechanics handle both types of failures. Electrical and electronic failures can also be combined for the same reason.

Mechanical failure

A classification, by which all possible failure modes could be included, consists of the location of failure and the process of failure. Each specific failure mode is then identified as a combination of one or more process together with a failure location. The two failure locations, each with subcategories, are:

1. Body type:
   * Head stock
   * Axis (X and Y)
FIGURE 19. Basic modules of an FMS
* Actuators (air motors, air cylinder)
* Bearings
* Drives (gear box, clutches, couples)
* Valves
* Filters (air, coolant, lube)
* Pumps (accumulators, intensifiers)
* Belts or chains
* Clamping

2. Surface type

* Fixture (clamp, locators, bushing plates, guide rails)
* Bed (column, swing)
* Actuators (feed screws)

The four processes of failure are:

1. Elastic and/or plastic deformation
2. Rupture or fracture
3. Vibrations
4. Material variation
   * Metallurgical
   * Chemical
   * Nuclear

The following list includes the most commonly observed failure modes of mechanical failure.

1. Force and/or temperature induced elastic deformation
2. Yielding
3. Ductile or brittle fracture
4. Fatigue (thermal, surface)
5. Corrosion (stress, cavitation, biological)
6. Wear (adhesive, abrasive)
7. Thermal shock
8. Radiation damage

**Electrical and electronic**

The failure locations are:

1. Control panel
2. Input devices (push buttons, tape recorder)
3. Output devices (servo's, programmable logic controller (PLC), printers)
4. Computer hardware (boards, modules, cathode ray tube, (CRT))
5. Computer software (part programs, patches offsets)
6. Relays (fuses, overloads)
7. Drives (transistor pack, SCR package)
8. Motor (AC, DC)

The following are examples of possible failure modes of electrical/electronic failure.

1. Breakdowns due to overdiffusion
2. Bad connections due to corrosion
3. Function loss and leak currents
4. Increased resistivity due to oxidation of the bonding surface
5. Thermal transients
6. Capacity-induced breakdowns
7. Change in the frequency bandwidth
Tooling

The two failure locations are:

1. Tool chain magazine
2. Automatic tool changer

Examples of possible failure modes are:

1. Thermal deformation of cutter (elastic, plastic)
2. Tool wear (built up edge)
3. Cracking
4. Deformation due to clamping (material variation)
5. Tool insert dimensional variations.

Material Handling Module

The two basic material handling systems (MHS) used in the U.S.A. for fully flexible machining systems are the AGVS and towline. Table 22 shows a comparison between the two types of material handling.

MHS module failure can be the result of one or a combination of broad classes of failures: mechanical, electrical/ electronic. Examples of possible sources of failure in each class are listed below.

Mechanical failure

1. Guide path (chain drives, switch gear, cams, jacks, pins and diverters)
2. Shuttles (rollers or moving cables)
3. Carrier (truck or cart, battery)
4. Probes (trigger)
5. Delivery and discharge (rollers or moving cables, hydraulic cylinder)

**Electrical/electronic failure**

1. guide path (cable, magnets, reflective tape or painted strip)
2. Probes (departure-sensing switches, entrance detector, code recorder)
3. Traffic control (computer hardware and software)
4. Signals transmitted (radio or infrared)
5. Incremental position recognition: after a temporary failure of the control system or the AGV is removed from the network, the truck can no longer find its way using incremental position recognition. The use of absolute digital position recognition is more expensive in terms of the number of magnets which have to be installed and in terms of the control design. However, it offers more reliability of transport operations and has to be provided in complex systems.

**System Control Modules**

Figure 20 illustrates a three-level computer control hierarchy.

1. A minicomputer provides the overall system or master FMS control. The control functions of the master module involves four categories: operational control, production control, traffic control, and data management.

2. The direct numerical control (DNC) module provides a numerical
FIGURE 20. Hierarchy of computer control in FMS
control program librarian and distributor function. It is an independent subsystem within the total FMS control system.

3. Computer numerical control (CNC) module is the local machine tool control that provides the direct servo control of the machine axis drives. It is devoted to communicate with the DNC system.

Examples of possible sources of failure are listed below.

1. Terminal, printer, audible or visual alarm system.
2. Data entry unit.
3. Post-processor.
4. APT converter and compiler.
5. Status display board
6. N/C "match coded" to the machine station.
7. Cathode Ray Tube (CRT).
8. Data Terminal Equipment (DTE), if the machine control is hardwired.
9. Intelligent terminal for CNC module.
10. Diagnostic communication system (DCS).
11. Tape puncher, tape reader and microprocessors.
12. Computer Hardware: fixed head disk, disk drives, multiplexers, boards or cards.
13. Software: In most FMSs, the first 6 months to 1 year of deployment are essentially a "shakedown cruise", during which errors are discovered and fixed by the users or through a software group which supports field operation from the development site (18,19). The following are typical examples of software failure (29, Chapter 5):
   a. Bad sector in a floppy disk or in removable cartridge disk
b. Wrong version of subroutine

c. Incompatible program with operating system or hardware

d. Design error

1. The series expansion used for a special mathematical function does not converge for certain values.
2. The THEN ELSE branches can be mistakenly interchanged in an IF statement.

e. Human error

The following are examples of human operator errors:

1. Mounting wrong disk on drive.
2. Entering wrong data or making typographical error.
3. Clearing all memory by mistake.
5. Forgetting the right sequence of commands on occasion because there are too many steps.
6. Not being able to react fast enough to enter control commands in an emergency situation.

f. System overload

1. Timesharing system designed to handle 24 terminals performs poorly when over than 20 terminals are connected.
2. The input module of text-editing cannot keep up with a very fast typist.
Inspection Module

The introduction of FMS technology and "unmanned" machining has compounded the accuracy problem, mainly, because finished parts are inspected elsewhere off the machine tool. The problem exists for producing a number of bad parts before corrective action can be taken, even if coordinate measuring machines (CMM) are included in the manufacturing system (38).

Avoidance of this problem requires a shift in philosophy from post-process part inspection to installation of inspection module and preventive maintenance, instead of measuring the part to see if the machine is functioning properly, measure, adjust and maintain the machine to assure that the part is manufactured properly.

Typical examples of problems in the inspection module are:

1. Part measurements using current methods are difficult and tedious. Simpler, less expensive and less time-consuming are needed. One recommendation to improve reliability is that contact gauges can be replaced with noncontact devices such as those using optical effects, eddy currents or capacitance-change methods. The noncontact gauges are not only less likely to wear and often more reliable, but they also allow higher rates of inspection.

2. Alignment and testing: more complicated part design will require the machine to move in more axes that at present.

3. Accuracy of geometry and surface finish: high accuracy machines will be required to produce higher performance products, less scraps and less inspection effort.
Suggestions for FMS Users

If a manufacturer is considering the installation or remodeling of an FMS, the following suggestions could be valuable tools. Therefore, the system designer can:

1. Specify the company needs and compare alternate systems for features offered and prices charged for these features.
2. Research the plan with existing installations.
3. Review the conditions and maintenance of the machines to be run under DNC.
4. Not to split the total vendor responsibility of the system, including the control interface connections.
5. Require that the system operate in the vendor's plant for at least 30 days before shipping.
6. Pay special attention to the machine tool interfaces and their effects as a valuable tool in detecting many potential problems, particularly on older NC machines.
7. Not to shortcut any computer power isolation or cabling since these may affect system reliability.
8. Set an agreed-to-performance standards as to system reliability, to determine when the system will be operative.
9. Connect one machine tool first and exercise the system before the installation of additional machines.
10. Allow training programmers, clerks, and NC maintenance crew during start-up period.
11. Expect and plan on some machine tool downtime while debugging the
12. Make sure that the system agreement includes a full maintenance contract for at least two years.

13. Keep detailed records about machine tool and other components failure to be able to track system progress and to take action accordingly.

### TABLE 22. Comparison between AGVS and towline system

<table>
<thead>
<tr>
<th>Definition</th>
<th>AGVS</th>
<th>Towline System</th>
<th>Proposed LIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description of carrier</td>
<td>Known as a truck. Three or four small hard-wheel trucks for assembly or random complex FMS.</td>
<td>Known as a cart. Four-wheel cart rolls on flat sheet ribbons embedded in the floor.</td>
<td>Represents one part of the motor (secondary) thus vehicle weight is less.</td>
</tr>
<tr>
<td>Power source</td>
<td>Most AGVS rely on lead-acid batteries (24V) for power to supply the drive and steering motor. They are recharged every 8-16 hours and exhausted after 1500 discharges.</td>
<td>The carts are powered by drop-forged, rivetless chain with telescoping action to facilitate the take-up mechanisms.</td>
<td>Uses a 3-phase AC source.</td>
</tr>
<tr>
<td>Speed</td>
<td>200 to 260 ft/min</td>
<td>120 to 150 ft/min -300 ft/min</td>
<td></td>
</tr>
<tr>
<td>Guide Path</td>
<td><strong>Magnetic Guidance</strong></td>
<td>The towchains slide in a 3&quot;x2.25&quot; track. Guide pins at the front and groove which is made into the floor surface. The wire is supplied by a pathway. The guide three-phase AC</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 22. Continued

<table>
<thead>
<tr>
<th>Definition</th>
<th>AGVS</th>
<th>Towline System</th>
<th>Proposed LIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>low-frequency generator that with transmitter allows optimum magnetic field. The AGVS scanning head, with 2 antennae, reads the instruction. given directly from wire to the truck via 10 digit keyboard. Permanent magnets are embedded in the floor at either side of the wire, to control reed contact underneath the trucks.</td>
<td>pin is raised or lowered by a cam-type mechanism mounted in the floor and actuated by computer control. Control is accomplished on a &quot;zone basis&quot;, which consists of a section of a chain, a stop blade low the pathway. and entrance/departure detector. If the zones serve an on/off shuttle, an in-position detector or/and a push bar are also in the zone. The zone length varies from 3 to 20 ft. Two types of stop zones are used: 1. accumulator stops to buffer part flow 2. precision stops to securely restrain the cart to prevent any movement. More recently magnetic coding, photoelecctrics, bar codes have alternatives to the mechanical probes.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Definition</td>
<td>AGVS</td>
<td>Towline System</td>
<td>Proposed LIM</td>
</tr>
<tr>
<td>------------</td>
<td>------</td>
<td>----------------</td>
<td>--------------</td>
</tr>
<tr>
<td>Delivery</td>
<td></td>
<td>A hydraulic cylinder mounted on the shuttle slides the part from one side to the other.</td>
<td>A hydraulic and cylinder mounted Discharge on the shuttle slides the part from one side to the other.</td>
</tr>
<tr>
<td>Accurate Stopping</td>
<td>Centering jacks on the truck are located on precision cones mounted on floor plates.</td>
<td>A hydraulically actuated ram holds cart in position to assure accurate stops.</td>
<td>Accurate stopping devices are eliminated.</td>
</tr>
<tr>
<td>Safety</td>
<td>1. The system software prevents a truck from entering into a segment of track that contain another truck by reducing the current behind the truck.</td>
<td>Only one cart is permitted to be in a zone at one time. The minicomputer, being notified that a cart has passed a zone departure sensor, will check if the second zone's stop blade must be raised to halt the cart because the third zone is occupied.</td>
<td>Either a safety bumper or a stop blade are eliminated.</td>
</tr>
<tr>
<td></td>
<td>2. Yellow caution signals flash when trucks move.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. A safety bumper extending 15&quot; from front and rear of the truck, prevent injury or damage to objects in its path.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Definition</td>
<td>AGVS</td>
<td>Towline System</td>
<td>Proposed LIM</td>
</tr>
<tr>
<td>------------</td>
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<td>--------------</td>
</tr>
<tr>
<td><strong>Pros</strong></td>
<td>1. Eliminates the risk of damage during transit. 2. Flexible with respect to breakdown and expansion. 3. Smaller battery size and lower charging costs. 4. Longer component life.</td>
<td>1. Low cost transporter. 2. Operates in normal environment of metal chips, coolant and oil. 3. High reliability. 4. Provides parts buffering between machines.</td>
<td>1. Low operating cost. 2. Less maintenance. 3. Gears are eliminated. 4. Accurate stopping devices are eliminated. 5. More flexible.</td>
</tr>
<tr>
<td><strong>Cons</strong></td>
<td>1. Requires good floor condition; floor should be concrete (not tar), smooth and dry. 2. The signal from the wire can be destroyed by steel sheets, mesh, or grating on or near the surface.</td>
<td>1. High cost per foot for extended runs. 2. Can be used only above a distance of approximately 300 ft. 3. Failure shuts down an entire zone.</td>
<td>1. Fixed or constant air gap must be maintained. 2. 90 degree turns must be gradual.</td>
</tr>
</tbody>
</table>
APPENDIX D:
ANALYSIS OF FAILURE DATA

This Appendix will present the information which have been received from the company and used for the analysis in the research.

1. A schematic layout of the FMS.

2. There actually 8 different part numbers with 30 different operations being performed at any one time in the FMS.

R79804  Power Shift Mechanical Front Wheel Drive Clutch Hsg.
R79805  Power Shift Clutch Hsg.
R79807  Power Shift Mechanical Front Wheel Drive Clutch Hsg.
R79808  Power Shift Clutch Hsg.

R70600  Power Shift Transmission Case
R70601  Power Shift Transmission Case with Mech Front Wheel Drive
R70396  Power Shift Transmission Case
R70397  Power Shift Transmission Case with Mech Front Wheel Drive

All part numbers are running at the same time. The following are the part routings:

R79804 - Load part in 1st fixture at load station 1. Part is processed through machine 7 or 8; then through machine 1; part is unloaded at unload station 2. There are 3 fixtures for this orientation.
Load part in 2nd fixture at load station 1. Part is processed through machine 1 or 2; then machine 4, 5 or 6; part is unloaded at unload station 2. There are 5 fixtures for this orientation.
Load part in 3rd fixture at load station 1. Part is complete and unloaded at unload station 2. There are 6 fixtures for this orientation.

R79805 - Load part in 1st fixture at load station 1. Part is processed through machine 4, 5 or 6; then through machine 1 or 2; part is unloaded at unload station 2. There are 4 fixtures for this orientation.
Load part in 2nd fixture at load station 1. Part is processed through machine 3; then machine 14, 15 or 16; part is complete and unloaded at unload station 2. There are 4 fixtures for this orientation.

R79807 - Load part in 1st fixture at load station 4. Part is processed through machine 7 or 8; then through machine 1; part is unloaded at unload station 3. There are 2 fixtures for this orientation.
Load part in 2nd fixture at load station 1. Part is processed through machine 1 or 2; then machine 4, 5 or 6; part is unloaded at unload station 2. There are 4 fixtures for this orientation.
Load part in 3rd fixture at load station 4. Part is processed through
machine 3; then through 9; then through 14, 15 or 16; part is complete and unloaded at unload station 3. There are 4 fixtures for this orientation.

R79808 - Load part in 1st fixture at load station 4. Part is processed through machine 4, 5 or 6; then through machine 1 or 2; part is unloaded at unload station 3. There are 3 fixtures for this orientation.

Load part in 2nd fixture at load station 4. Part is processed through machine 9; then through 14, 15 or 16; part is complete and unloaded at unload station 3. There are 2 fixtures for this orientation.

R70600 - Load part in fixture at load station 4. Part is processed through machine 9 or 10; then through 11, 12 or 13; part is complete and unloaded at unload station 3. There are 3 fixtures for this orientation.

R70601 - Load part in fixture at load station 4. Part is processed through machine 9; then 10; then through 11, 12 or 13; part is complete and unloaded at unload station 3. There are 5 fixtures for this orientation.

R70396 - Load part in fixture at load station 4. Part is processed through machine 10; then through 11, 12 or 13; part is complete and unloaded at unload station 3. There are 2 fixtures for this orientation.

R70397 - Load part in fixture at load station 4. Part is processed through machine 10; then through 11, 12 or 13; part is complete and unloaded at unload station 3. There are 3 fixtures for this orientation.

3. The FMS was purchased to produce a daily requirement of 109 clutch housings and 109 transmission cases. Because of the demand from dealers, the company is producing a varying percentage of the various part numbers.

4. The total process time in minutes:

<table>
<thead>
<tr>
<th>Part</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>R79804</td>
<td>18.201</td>
<td>49.158</td>
<td>68.264</td>
</tr>
<tr>
<td>R79805</td>
<td>42.165</td>
<td>34.542</td>
<td></td>
</tr>
<tr>
<td>R79807</td>
<td>29.753</td>
<td>60.684</td>
<td>87.129</td>
</tr>
<tr>
<td>R79808</td>
<td>49.246</td>
<td>32.337</td>
<td></td>
</tr>
<tr>
<td>R70600</td>
<td>28.386</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R70601</td>
<td>47.721</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R70396</td>
<td>28.405</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R70397</td>
<td>40.545</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(b) There is no setup time for any of the parts as the machines are tooled to run all parts and orientations.

(c) The average time to load either clutch housing and transmission case is 4.65 minutes. The average time to unload either clutch housing and transmission case from its fixture is 2.61 minutes.

(d) Because there can be 2 parts on the shuttle at each machine, the pallet exchange time is 30-45 seconds.

(e) There are 5 horiz. 2 axis head indexers (machine 1-2-3-9-10) that do boring and multi-spindle drilling and taping. There are 11 vert. 3 axis machining centers with each having a 69 tools capacity magazine that can do milling, drilling, boring and tapping.

An analysis of failure data was done in an attempt to define the types of failures associated with the two types of machine module. Tables D.1 and D.2 are associated with the machining center and tables D.3 and D.4 are associated with the head indexer. The time frame for these tables is 17 months and is summary of all emergency repair or unscheduled maintenance.

Analyzing the data for the machining center, Table D.1 shows that 65.2% of the repair job requests were electric in nature and these repairs accounted for 39% of the total downtime on that machine. And while only 9.1% of the requests were for tool failure, these failures accounted for 34%. Table D.2 shows the average for response time, repair time and total downtime. It is interesting to note that the electrical failure was more serious than the other two failure modes and that the large average repair time is for tool failure.

For the head indexer, Table D.3 shows that 71.3% of the emergency requests were for electricians, with failures accounting for 64.3% of total downtime. Table D.4 shows the averages for downtime and it should be pointed that the average for each failure mode is reasonably close.

The highest frequency of failures were occurring in the electrical control panel relays of the head indexers, while the part holding fixtures were responsible for a majority of the failures in the machining centers. Bed and tailstock failures, including both electrical and mechanical, have a relatively high frequency for both types of machine.
Table D.1. Summary of failure data of the machining center

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>No. of failures</th>
<th>% of total failures</th>
<th>Total downtime</th>
<th>% of total downtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical</td>
<td>114</td>
<td>65.2</td>
<td>1562.9</td>
<td>39</td>
</tr>
<tr>
<td>Mechanical</td>
<td>45</td>
<td>25.7</td>
<td>1081.8</td>
<td>27</td>
</tr>
<tr>
<td>Tool</td>
<td>16</td>
<td>9.1</td>
<td>1360.1</td>
<td>34</td>
</tr>
<tr>
<td>Total</td>
<td>175</td>
<td>100.0</td>
<td>4004.8</td>
<td>100</td>
</tr>
</tbody>
</table>

Table D.2. Summary of repair data of the machining center

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>No. of failures</th>
<th>Average Response time</th>
<th>Average repair time</th>
<th>Average downtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical</td>
<td>114</td>
<td>2.43</td>
<td>11.28</td>
<td>13.71</td>
</tr>
<tr>
<td>Mechanical</td>
<td>45</td>
<td>5.82</td>
<td>18.22</td>
<td>24.04</td>
</tr>
<tr>
<td>Tool</td>
<td>16</td>
<td>4.92</td>
<td>25.38</td>
<td>30.30</td>
</tr>
</tbody>
</table>

TABLE D.3. Summary of failure data of the head indexer

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>No. of failures</th>
<th>% of total failures</th>
<th>Total downtime</th>
<th>% of total downtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical</td>
<td>149</td>
<td>71.3</td>
<td>958.1</td>
<td>64.3</td>
</tr>
<tr>
<td>Mechanical</td>
<td>49</td>
<td>23.4</td>
<td>424.3</td>
<td>28.5</td>
</tr>
<tr>
<td>Tool</td>
<td>11</td>
<td>5.3</td>
<td>106.5</td>
<td>7.2</td>
</tr>
<tr>
<td>Total</td>
<td>209</td>
<td>100.0</td>
<td>1488.9</td>
<td>100</td>
</tr>
</tbody>
</table>

Table D.4. Summary of repair data of the head indexer

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>No. of failures</th>
<th>Average Response time</th>
<th>Average repair time</th>
<th>Average downtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical</td>
<td>149</td>
<td>1.46</td>
<td>4.97</td>
<td>6.43</td>
</tr>
<tr>
<td>Mechanical</td>
<td>49</td>
<td>2.12</td>
<td>6.53</td>
<td>8.66</td>
</tr>
<tr>
<td>Tool</td>
<td>11</td>
<td>2.50</td>
<td>7.17</td>
<td>9.68</td>
</tr>
</tbody>
</table>
APPENDIX E:

TRANSIENT BEHAVIOR OF OPERATING STATES
FIGURE E.1. System No. 1

FIGURE E.2. System No. 2, 4, 7 and 9
FIGURE E.3. System No. 3

FIGURE E.4. System No. 5
FIGURE E.7. System No. 10 Time

FIGURE E.8. System No. 12 Time
FIGURE E.9. System No. 13 and 14
APPENDIX F:

SYSTEM AVAILABILITY AND SYSTEM EFFECTIVENESS
FIGURE F.9. System No. 9

FIGURE F.10. System No. 10
FIGURE F.13. System No. 14
APPENDIX G:

EFFECTS OF DOUT ON PRATE
FIGURE G.3. System No. 3

FIGURE G.4. System No. 4

Time
FIGURE G.5. System No. 5

FIGURE G.6. System No. 6
FIGURE G.7. System No. 7

FIGURE G.8. System No. 8
Figure G.9. System No. 9. Time

Figure G.10. System No. 10. Time
FIGURE G.11. System No. 12

PRATE3
PRATE2
PRATE1

FIGURE G.12. System No. 13

PRATE3
PRATE2
PRATE1
FIGURE G.13. System No. 14