An asset/liability management model of a Federal Intermediate Credit Bank

Ann Marie Hackert

Iowa State University

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Hackert, Ann Marie, Ph.D.
Iowa State University, 1987
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An asset/liability management model
of a Federal Intermediate Credit Bank

by

Ann Marie Hackert

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
DOCTOR OF PHILOSOPHY
Major: Economics

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Department

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For the Graduate College

Iowa State University
Ames, Iowa

1987
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CHAPTER I. INTRODUCTION AND PROBLEM STATEMENT

Introduction

This dissertation examines asset/liability management for an individual Federal Intermediate Credit Bank. As an institution of the Farm Credit System, the Federal Intermediate Credit Bank (FICB) raises funds in national money and capital markets to finance agricultural lending activity. In the past, the use of national money markets assured access to a liquid, relatively low cost source of funds. The future cost and availability of those funds depends on the financial management of the institutions of the Farm Credit System (FCS).

Although recent declines in the farm economy have focused attention on the default experience of commercial banks and institutions of the Farm Credit System, interest rate risk remains a fundamental problem. Variable rate loan pricing contributes to the financial stress of borrowers during periods of increasing interest rates causing repayment problems. Because of the structure of FICB balance sheets, declining rates erode interest earnings and capital.

The current operations and financial condition of the Farm Credit System and Federal Intermediate Credit Banks is presented prior to development of the problem statement. Next, the specific objectives of this study are reviewed. This chapter concludes with an overview of the dissertation.
Overview of FCS Financial Condition and Operations

Activities of FCS institutions

Farm Credit System institutions are borrower-owned financial cooperatives. Federal Intermediate Credit Banks provide funds to local Production Credit Associations (PCAs) that in turn make short- and intermediate-term loans to agricultural producers. Federal Land Banks specialize in real estate lending. Banks for Cooperatives make loans to eligible agricultural cooperatives. These institutions are regulated and examined by the Farm Credit Administration.

Lending is divided between institutions on a functional basis. Each type of institution in the Farm Credit System specializes in a different category of agricultural lending based on the ultimate use of the funds by the borrower. Lending is also restricted on a geographic basis. The United States is divided into districts limiting institution operations to specific geographic regions.

Equity for the Farm Credit System is raised through retaining earnings and requiring borrowers to own stock. The FCS raises debt funds by selling agency securities in national money markets. The securities are consolidated System-wide bonds and notes. These securities are the shared liability of Farm Credit System banks.

Dealers, brokerage houses and dealer banks represent the primary market for FCS issues. Well established secondary
markets exist for FCS securities. The federal government does not guarantee the securities, but investors have perceived FCS bonds and notes as default free resulting in moderate spreads above treasury bill yields (Irwin, 1985).

The financial condition of FCS borrowers

The financial stress of agricultural producers has a direct effect on institutions with a high proportion of their portfolio invested in farm loans. An increasing number of defaults and the existence of nonperforming loans has contributed to the deteriorating financial condition of both commercial banks specializing in farm lending and institutions of the Farm Credit System.

In the 1970s lenders willingly financed expansion based on the expectation of continued increases in commodity prices and appreciation in land values. Producers purchased land and equipment with an increasing reliance on debt financing. Table 1.1 indicates debt to asset ratios for United States agricultural producers for 1984 through 1986.

A high degree of financial leverage does not necessarily translate into default but indicates potential problems if cash flows or sales decline. Leverage ratios measure the extent to which a firm relies on borrowed money to finance their operations. Leverage magnifies the effect of increases or decreases in operating income on net income.
Table 1.1: Debt to asset ratios for U.S. agricultural producers\textsuperscript{a}

<table>
<thead>
<tr>
<th>Debt Ratio</th>
<th>1984</th>
<th>1985</th>
<th>1986</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-40 %</td>
<td>79.30</td>
<td>78.10</td>
<td>78.70</td>
</tr>
<tr>
<td>40-70%</td>
<td>11.10</td>
<td>11.60</td>
<td>12.70</td>
</tr>
<tr>
<td>70-100%</td>
<td>6.60</td>
<td>7.30</td>
<td>4.60</td>
</tr>
<tr>
<td>Over 100%</td>
<td>3.00</td>
<td>3.00</td>
<td>4.00</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Johnson et al. (1985; 1986).
The ratios in Table 1.1 may not indicate the extent of possible default and financial problems for two reasons. First, the implicit assumption that assets are liquid and can be sold to repay loans is not correct. Land, buildings and equipment are not liquid assets. In addition, the market value of these assets has declined along with farm income. Second, the debt ratios understate the problem if debt is priced at variable rates. Additional risk exists if interest expense is a variable cost that does not decline as income declines.

**Legislative response to problems**

The overall financial condition of the FCS is closely linked with that of the agricultural sector. Earnings of FCS institutions have declined as the conditions of producers deteriorated. Table 1.2 indicates the extent of the problems. The key financial ratios listed for 1984 through 1986 illustrate a declining financial position of FCS institutions.

In 1985 and 1986 Congress responded to the financial problems of the FCS by amending the Farm Credit Act. The objective of the legislation was to halt the deteriorating condition of the System by adding flexibility to bank operations. The regulatory authority, the Farm Credit Administration, obtained additional powers to solve the problems of the FCS.
Table 1.2: Selected ratios for the Farm Credit System\(^a\)

<table>
<thead>
<tr>
<th>Ratio</th>
<th>1986</th>
<th>1985</th>
<th>1984</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return (loss) on average assets</td>
<td>(2.55)%</td>
<td>(3.35)%</td>
<td>0.43%</td>
</tr>
<tr>
<td>Return (loss) on average capital</td>
<td>(26.06)</td>
<td>(16.07)</td>
<td>3.17</td>
</tr>
<tr>
<td>Net interest income as a percentage of average earning assets</td>
<td>1.08</td>
<td>1.72</td>
<td>1.76</td>
</tr>
<tr>
<td>Net charge offs to average loans</td>
<td>2.06</td>
<td>1.48</td>
<td>.53</td>
</tr>
<tr>
<td>Allowance for loan losses to loans</td>
<td>6.24</td>
<td>4.57</td>
<td>1.66</td>
</tr>
<tr>
<td>Capital to assets</td>
<td>8.05</td>
<td>10.48</td>
<td>13.60</td>
</tr>
<tr>
<td>Debt to capital</td>
<td>11.43</td>
<td>8.54</td>
<td>6.35</td>
</tr>
</tbody>
</table>

\(^a\)Federal Farm Credit Banks Funding Corporation (1987).
As part of the 1985 legislation, the Farm Credit Administration was granted the power to issue cease-and-desist orders, level civil penalties and remove bank officers and directors. These powers are similar to those available to commercial bank regulators. The review and oversight activities of the regulators are enhanced by their ability to enforce changes.

The Farm Credit Capital Corporation was created to assist weaker institutions with financial assistance from stronger institutions. In the past, the FCS has not operated as a single entity but as a decentralized group of institutions. Some institutions in the FCS remain financially sound. The extent of individual bank problems depends on the type of loans they have made or their geographic location. The Capital Corporation was created to assist weaker institutions by assessing stronger banks with fees.

Additional powers allowed the Capital Corporation to purchase nonperforming assets and assist the management of problem institutions. Although the aggregate financial condition of the entire System suggests that sufficient capital is available to ensure survival, healthy institutions have resisted funding weaker banks. Legal challenges have prevented the flow of capital between geographic regions (Pine, 1987).
The 1985 legislation did not solve all the problems of FCS institutions so the 1986 Amendments were passed to add flexibility to interest rate and capital management. The board of directors at each bank are now allowed to set interest rates on loans as long as they consider the effect of their decision on capital adequacy. Prior to this change, the Farm Credit Administration set lending rates. This provision allows regions and institutions to respond to their unique and changing conditions.

Accounting changes were also part of the 1986 legislation. Institutions are now allowed to keep one set of books for regulatory purposes and another using generally accepted accounting principles. On the liability side of the balance sheet, institutions can amortize the cost of their long-term debt over twenty years. Asset accounting changes permit twenty-year amortization of loan losses greater than one-half of one percent of total loans.

Accounting changes on both the assets and liability side represent short run solutions that will not solve the inherent problems. The actual financial condition of the institutions is unchanged but reported differently in the accounting statements regulators require. Institutions will still be required to keep one set of books based on generally accepted accounting principals showing current losses in the current period. If banks carry loan losses and debt costs
over a twenty year period, their long term financial condition will reflect current problems. The change seems to delay rather than solve the issue.

Congress is currently reviewing proposals to provide financial assistance to the Farm Credit System. The 1985 and 1986 Amendments have not prevented further declines in earnings and capital. The current objective of legislators is to provide funds to prevent bankruptcy of the System and implement management and structural changes to insure future solvency.

Problem Statement

The deteriorating financial condition of borrowers is not the only factor contributing to the problems of the Farm Credit System. Asset/liability management decisions played a key role in determining the current financial condition of institutions. Although the issue of the financial stress of individual producers is important, the focus of this dissertation is on risk exposure that could have been controlled through the use alternative management activities.

The issues of interest rate risk and asset/liability management present several significant problems for the current and future financial health of FCS institutions. The general assumption is that Federal Intermediate Credit Banks have effectively managed their interest rate exposure through the use of variable rate loan pricing. This assumption
requires further review. This section develops the relationships between interest rate risk, default risk, capital adequacy and the overall exposure of FICBs.

**Overall institution risk**

Federal Intermediate Credit Banks face risk from several sources. The financial structure of the institutions represents one source of risk. The institutions are highly levered meaning that they finance most of their asset acquisitions with debt funds. The use of financial leverage magnifies the affect of changes in operating income on net income. A high leverage ratio does not indicate a problem but represents the potential for losses if earnings decline. The existence of leverage requires the effective management of the other risks faced by the institution.

Federal Intermediate Credit Banks do not hold a diversified loan portfolio. This exposes institutions to risk because declines in a single sector of the economy effects the entire loan portfolio. In contrast, commercial banks may specialize in one type of lending activity but hold other kinds of loans in their portfolio. The commercial bank can invest in consumer loans, commercial loans, real estate loans and agricultural loans. A Federal Intermediate Credit Bank holds a loan portfolio representing eighty to ninety percent of all assets. The only loan category is agricultural loans.
Alternative lending activities are not available to FCS institutions. In addition, the division of the United States into geographic regions further restricts bank activity. Problems in the agricultural sector are not dispersed equally throughout geographic regions. Geographic restrictions further limit diversification opportunities.

Liquidity risk is another source of exposure for the typical financial intermediary. The institutions of the Farm Credit System have been able to purchase liquidity with their access to national money and capital markets. The size of their activities and frequency of borrowing make them a major money market participant. In the past, investors have perceived agency debt securities as a relatively safe investment and required low risk premiums. With the publicized problems in the financial condition of the FCS, yield spreads increased from a few basis points above the treasury bill rate to almost 100 basis points. Spreads dropped after the passage of the Farm Credit Act Amendment of 1985, but remained above their formerly low levels (Moran, 1986).

Institutions of the Farm Credit System still have ready access to the money market but it is important to note that the cost of those funds reflects the overall risk of the System. The System continues to repay debt, but investor perceptions of future potential problems is reflected in the
higher required risk premium. If overall risk increases to the point that default seems imminent or if Congress fails to pass legislation to fund losses, the System's access to money and capital markets could disappear.

Another source of risk is the mismatch of cash flows between asset returns and liability costs. In an effort to minimize interest rate risk, Federal Intermediate Credit Banks have priced loans on an average cost basis. The rate charged borrowers changes each month based on the new value of total average liability costs. If repricing of assets and liabilities does not occur at the same time, variable rate loan pricing will not eliminate institution's interest rate exposure. The next section discusses the continuing problem of interest rate risk management at Federal Intermediate Credit Banks.

The problem of incorrectly specifying interest exposure

When determining the interest rate exposure of an institution, it is important not just to consider loan pricing terms and maturities in isolation but the relationship between asset and liability accounts. Focusing on one side of the balance sheet fails to recognize the relationships between the pattern of returns, costs and earnings.

If each loan is repriced as often as the liability that funds it, the institution is assured that the spread between
costs and returns is maintained. When assets and liabilities are not repriced at the same point in time, the institution is exposed to interest rate risk. The kind of interest rate change that will trigger losses depends on the exact nature of the mismatch between assets and liabilities.

Financial institution interest rate risk management is based on the concept of the "gap" between assets and liabilities. The gap measures the difference between rate sensitive assets and rates sensitive liabilities. If the institution holds more rate sensitive liabilities than assets, increases in interest rates result in a decline in earnings because there is a lag between the time when higher costs are incurred and when they are passed on the borrowers. If assets are more rate sensitive then liabilities interest, rate declines are more quickly reflected in decreasing returns than in liability costs.

The extent of interest rate exposure depends on the magnitude of the difference between rate sensitive assets and liabilities on the balance sheet. Chapter III reviews the financial theories that offer alternative methods of measuring the gap. The cash flow characteristics of assets and liabilities on the FICB balance sheet are not exactly matched. It is important to explicitly measure interest rate risk rather than assuming variable rate pricing eliminates exposure.
Variable rate loan pricing and producer stress

An additional problem exists with the use of variable rate pricing. This method of interest rate risk management eliminates exposure at the institution level by passing the risk on to loan customers. This is an acceptable method of interest rate risk management if borrowers will have sufficient cash flows to meet costs regardless of future increases in interest rates.

Santomero (1983) suggests that the overall risk of an institution may not be reduced through the use of variable rate loan pricing. If changes in interest costs and borrower income are not positively correlated then an increase in the interest rate can result in cash flows that are not sufficient to service the debt. In this situation, the institution increases the probability that borrower default occurs while reducing the institution's interest rate risk.

Studies quantifying the extent of this problem do not exist, but a review of loan costs and farm incomes indicates that producer income was declining during the recent period of increasing and high interest rates. During periods of high interest rates, the cash flows required to service variable rate loans reached levels unanticipated at the time the funds were borrowed. The FICB use of average cost pricing means that during periods of declining borrowing
costs, loan rates continue to reflect the existence of higher cost long term debt.

It is important to examine the effects of monthly loan repricing. The existence of a positive or negative gap must be quantified in order to determine the extent of interest rate exposure and the kind of interest rate environment that would result in losses. Examining the maturity structure of the liability side of the balance sheet may indicate that a longer loan repricing period more effectively matches assets and liabilities.

**Interest rate risk hedging for the agricultural producer**

Even if the borrower income changes are not correlated with interest rate changes, risk management is still possible through the use of hedging techniques. One alternative would be borrower use of financial futures markets. Several factors limit the ability of the agricultural borrower to hedge their own interest rate exposure. In order to enter futures markets margin requirements must be met. Once a producer enters the futures market to hedge the activities, gains and losses are marked to market on a daily basis. The agricultural producer may find it difficult to raise the cash to meet initial margin requirements and subsequent margin calls.

There is not a financial futures instrument designed specifically to hedge agricultural loans. The size of an
individual operator's loan may not be sufficiently large to enter the futures market where contract sizes are $1 million when the underlying instrument is a short-term discount security and $100,000 when the underlying instrument is a long-term coupon instrument. A high level of financial expertise and cash commitment would be required before financial futures would represent a realistic hedging tool for the average agricultural producer.

The problem of capital adequacy and interest rate risk

A commercial bank requires adequate capital in order to establish confidence on the part of depositors and investors. Regulators make minimum capital requirements in order to manage the risk position of institutions. Ultimately, capital represents a cushion so the bank can continue operation in the event of unexpected losses.

The issue of capital adequacy is also of importance to institutions of the Farm Credit System. The 1986 Amendments mandate consideration of the capital position when directors determine loan rates. Loan losses have eroded the capital position of FCS institutions causing concern that they will be unable to meet additional losses.

Capital adequacy is important for another reason. Because the FCS raises debt funds in national money and capital markets, its financial condition affects the price required to compensate investors for possible default risk.
Declines in capital adequacy ratios coupled with increases in debt ratios and loan losses signal the market that financial problems exist, thus increasing risk premiums.

Interest rate risk is closely related to the issue of capital adequacy. Changes in interest rates result in changes in the discounted present value of assets and liabilities. There is an inverse relationship between changes in market values of financial assets and liabilities and the direction of the interest rate change.

Since net worth equals the difference between the value of assets and liabilities, any change in interest rates affects the value of net worth. Unless the change in the value of assets exactly equals liabilities, net worth can increase or decrease depending on the magnitude of the balance sheet mismatch and the direction of the interest rate change.

Ratio analysis indicates the relationships between the accounting value of balance sheet categories. Using book value to measure capital understates possible risk exposure. It is important to consider the affect of interest rate changes on the market value of capital.

A firm with a negative market value of net worth is technically insolvent and unable to repay creditors. Two alternatives exist in the case of the Farm Credit System. One possibility is that investors holding FCS bonds would not
receive their principal and interest payments. Bondholders include commercial banks, institutions and individual investors. FCS default would have repercussions throughout the financial system.

Another alternative is government financed assistance. The amount of funding required would be dependant on the value of capital. Loan losses have already eroded the value of capital. Further losses due to interest rate risk exposure compound the problem.

Objectives

The general objective of this study is to investigate the asset/liability management practices of a Federal Intermediate Credit Bank. A micro model of an FICB is constructed using data from the Omaha bank. In order to examine the issues and problems discussed in the previous section, the following objectives are specified.

1. The first objective of this study is to develop a quadratic programming model of an FICB in order to evaluate the overall investment opportunity set available to the institution. Previous research has examined the liability side of the balance sheet but interest rate risk requires analysis of both assets and liabilities.

2. The second objective is to quantify the interest rate exposure of the Omaha FICB. Rather
than assume that variable rate loan pricing eliminates all risk, this study examines the actual structure of the balance sheet in order to quantify the relationship between assets and liabilities.

3. The final objective is to examine alternative interest rate risk management activities available to the FICB. Quantifying the exposure of the institution will indicate if risk has been eliminated from the balance sheet. If interest rate exposure still exists on the balance sheet then the model developed to measure interest rate risk will be used to review the options available to the bank under the current legislative restrictions.

Organization of the Dissertation

Chapter II reviews portfolio theory and its application to the asset and liability activities of financial institutions. Determining the opportunity set under conditions of uncertainty and selecting the optimal portfolio represent the two basic issues addressed. The chapter considers the limitations of the theory and some methods for solving the problems discussed. The chapter also discusses the application of the theory to financial intermediaries and examines quantitative studies modeling the asset and liability activities of banks.
The theory of interest rate risk measurement and management is developed in Chapter III. Methods of quantifying interest rate risk for the asset portfolios of individual investors and the balance sheets of financial intermediaries are considered. Several interest rate risk management techniques are also developed. In addition, empirical studies of the theoretical concepts are reviewed.

Chapter IV provides a general description of the activities of the Farm Credit institutions and Federal Intermediate Credit Banks. A review of the literature examines both research on federal agency debt management and studies pertaining to the specific problems of Farm Credit institutions.

Chapter V outlines the methods and procedures used to examine FICB asset/liability management. A quadratic programming model is designed to examine the opportunity set for an institution selecting a portfolio of assets and liabilities on the basis of risk and expected return. The results of this analysis are presented and provide information on overall bank activities.

In Chapter VI, a monthly balance sheet model is constructed to measure and quantify the relationships between the cost of funds and the returns from loans. This chapter utilizes the theoretical interest rate risk management tools
developed in Chapter III to examine the balance sheet of the Omaha FICB.
CHAPTER II. PORTFOLIO THEORY AND FINANCIAL INTERMEDIATION

This chapter presents the theoretical framework used to model and analyze the management activities of the Omaha Federal Intermediate Credit Bank. Portfolio theory along with financial intermediation are reviewed in this chapter. Chapter III provides a review of the theory of interest rate risk management and measurement. Chapter IV presents a description of the Farm Credit System and reviews studies modelling Federal Intermediate Credit Bank risk management and debt selection decisions.

Portfolio Theory

Portfolio theory analyzes the selection of an optimal combination of assets for a risk averse investor. This optimal or best choice depends on consumer preferences and the characteristics of the opportunity set. The opportunity set describes the available investment alternatives. The preferences of investors are represented by their trade-offs between risk and return as depicted by indifference curves.

First, the characteristics of the opportunity set under uncertainty are developed. Then the theory of investor preferences is reviewed and combined with the opportunity set in order to discuss optimal portfolios. This section concludes by discussing solutions to the problems of applying the theory to investor and firm decisions.
The Opportunity Set Under Uncertainty

Measuring risk and return

The mean and standard deviation are two parameters commonly used to describe uncertain events. The mean or expected return is a measure of the average outcome. Measures of risk or dispersion indicate the difference between the actual observation and the average. The variance is the sum of the squared deviations around the mean and the standard deviation is the square root of the variance.

For a single asset, the expected return is the probability-weighted sum of all the possible returns. This is represented mathematically as:

\[ E(R) = \overline{R} = \sum_{i=1}^{N} P_i R_i \]

\( E(R) = \overline{R} \) = expected return or mean
\( P_i \) = probability of return \( i \)
\( R_i \) = return for outcome \( i \)

The risk of a single asset is measured by calculating either the variance or standard deviation. The equation for variance is:

\[ \text{VAR}(R) = E [R_i - E(R)]^2 \]

\[ = \sum_{i=1}^{N} P [R_i - E(R_i)]^2 \]

\[ = \sigma^2 \]

The standard deviation is:

\[ \sigma = \sqrt{\sigma^2} \]
Opportunity sets for a combination of assets involve interrelationships among the components. The expected return for a group or portfolio of assets is the weighted average of the individual returns. Mathematically:

\[ \bar{R}_p = \sum_{i=1}^{N} X_i E(R_i) \]

\( \bar{R}_p \) = Expected return of the portfolio

\( X_i \) = proportion of the portfolio invested in security \( i \)

\( E(R_i) \) = expected return of security \( i \)

The risk of a combination of assets is not merely the sum of the risk of the assets held in isolation. The variance of a portfolio is developed below for the two-asset case. The variance of a two-asset portfolio is:

\[ \text{VAR}(R_p) = \sigma_p^2 = \chi_1^2 \sigma_1^2 + 2\chi_1 \chi_2 \text{C}_{12} + \chi_2^2 \sigma_2^2 \]

Where:

\( \text{VAR}(R_p) \) = the variance of the portfolio

\( \sigma_p^2 \) = the variance of return for security 1

\( \sigma_2^2 \) = the variance of return for security 2

\( \chi_1 \) = the proportion of the portfolio held in security 1

\( \chi_2 \) = the proportion of the portfolio held in security 2

The term \( \text{C}_{12} \) is the covariance of securities 1 and 2.

Intuitively, the covariance measures the relationship between asset returns. The equation for the covariance is;
(2.5)  \[ C_{12} = R_{12} \sigma_1 \sigma_2 \]

\( C_{12} \) = the covariance of assets 1 and 2
\( R_{12} \) = the correlation coefficient between asset 1 and 2
\( \sigma_1 \) = the standard deviation of return for asset 1
\( \sigma_2 \) = the standard deviation of return for asset 2

The value of the correlation coefficient is between -1.0 and +1.0. A value of -1.0 indicates perfect inverse correlation. If the value of the correlation coefficient is zero then returns are independent. A correlation coefficient of +1.0 represents perfect correlation.

The interrelationship of the asset returns in a portfolio means that the total portfolio risk may be less than the sum of the individual assets. With perfect negative correlation, it is possible to construct a portfolio in which all risk is eliminated. If returns are independent, risk can be reduced almost to zero by diversification. If the correlation coefficient is +1.0, diversification does not reduce risk.

For n-number of securities the analysis is accomplished utilizing matrix algebra. The equation for return is;

(2.6)  \[ E(R_p) = [E(R_1) E(R_2) ...] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix} = R'X \]

Expected return is an (1xN) row vector multiplied by the (Nx1) column vector of asset proportions. The variance
equation in matrix notation for the two variable case is:

\[
(2.7) \quad \text{VAR}(R_p) = [x_1 \ x_2] \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x'Sx
\]

The variance-covariance matrix = S. This naturally extends to the n vector and nxn covariance matrix.

The Markowitz efficient frontier

Efficient portfolios in the Markowitz framework provide the highest return for a given level of risk or offer the least risk for a given return. Tracing out efficient portfolios in risk return space yields a frontier of minimum risk portfolios at selected rates of return.

In developing the concept of efficient frontiers, Markowitz (1959) made certain assumptions concerning the securities to include in the choice set. He allowed only nonnegative weights on assets and excluded both riskless assets and perfect negative correlation between assets. Based on these constraints, Figure 2.1 illustrates an efficient frontier. Points below the frontier are not efficient. Compare points A and B representing two different portfolios of risky assets. Portfolio A offers a lower expected return for the same risk as portfolio B. If an investor is made better off by more return and worse off by more risk, then portfolio B would be preferred to portfolio A. B offers a higher return for the same level of risk.
Figure 2.1: The Efficient Frontier
The efficient frontier is drawn without including investor preferences. A later section on investor choice under uncertainty indicates how an individual selects a specific portfolio on the efficient frontier. As drawn, the efficient frontier represents only a set of opportunities with certain restrictive characteristics.

Tracing out the trade-off between risk and return to obtain the efficient frontier in the opportunity set is a mathematical programming problem. The efficient set is found by minimizing risk subject to some selected value of return. Mathematically the problem is:

\[(2.8) \quad \text{Minimize } \sigma_P^2 \text{ subject to } E(R_P) = K\]

\[\sigma_P^2 = \text{portfolio variance}\]

\[E(R_P) = \text{portfolio return}\]

\[K = \text{a constant, a selected level of return}\]

The solution to equation 2.8 yields the proportions of the assets held in an efficient portfolio with a given rate of return. This problem can be solved using quadratic programming because the choice variable in the objective function contains only squared terms and the constraints are all linear. The frontier is obtained by minimizing equation 2.4 subject to a return constraint. Mathematically:

\[(2.9) \quad \text{Min} \left( \sigma_P^2 = x_1^2 \sigma_1^2 + 2x_1x_2 \sigma_1 \sigma_2 + x_2^2 \sigma_2^2 \right)\]

Subject to: \[E(R_P) = x_1 E(R_1) + x_2 E(R_2) = K\]
Solving the problem at various rates of return or values for $K$ in equation 2.9 traces out an efficient frontier. It is important to note a fundamental characteristic of both the Markowitz model and its modifications. The opportunity sets are concave in risk return space. The shape of the efficient frontier is determined by the covariance effect. If the efficient frontier were convex, there would exist portfolios which could be combined to form new portfolios with a higher return at each level of risk.

The Markowitz choice set reflects information about the securities. His restrictions on the assets that excluded borrowing, short sales and investment at a risk free rate prompted further development and extension of the basic model. Later models modify the Markowitz model by changing some of the basic assumptions.

**Modifications of the Markowitz model**

Tobin (1965) introduced borrowing and lending at a riskless rate, $R_w$. His model also excluded other risky assets with negative weights. The riskless asset is assumed to have a standard deviation of zero with covariance terms also equal to zero. The equation of the efficient frontier is a straight line and is found by solving the problem in 2.9 when one asset has a variance of zero.

The existence of riskless borrowing and lending at the same rate simplifies portfolio analysis. In Figure 2.2a, the
riskless rate is represented by the return, \( R_p \), on the vertical axis. \( W, X \) and \( Y \) all represent points along an efficient frontier drawn on the basis of the Markowitz framework.

The riskless asset and a risky portfolio selected from the efficient frontier yield a linear combination of risk and return drawn as a ray through the riskless rate (\( R_f \)) and the risky portfolio. Figure 2.2a illustrates three possible combinations of risky portfolios and the riskless asset.

Portfolio \( X \) combined with the riskless asset dominates the choice of a portfolio \( W \) and the riskless asset. The ray through \( R_p \) and \( X \) offers alternatives with higher expected returns for each level of risk than the ray through \( R_f \) and \( W \).

If the security with the riskless rate is combined with a risky portfolio so that a ray through \( R_p \) is tangent to the Markowitz frontier, the highest expected return for a given level of risk is attained.

Risk averse investors, those preferring more return and less risk, will always select a portfolio containing the risky portfolio \( Y \) regardless of the specific form of their utility function. Those who are more risk averse would place their funds in a combination of the riskless asset and the risky portfolio. These combinations are located along line segment \( R_f Y \).
Figure 2.2a: The efficient set with a riskless asset

Figure 2.2b: The efficient set with borrowing and lending at different rates
Investors exhibiting less risk aversion would borrow at the riskless rate and invest those funds in the risky portfolio Y. Line segment YZ represents the case in which borrowing at the riskless rate occurs.

This modification has significant implications. Riskless borrowing and lending at the same rate, combined with risky assets yields the specific risky portfolio that all risk averse investors will select regardless of the form of their utility function. The degree of risk aversion determines if borrowing or lending occurs at the riskless rate.

Brennan (1971) modifies the assumption of borrowing and lending at the same rate of interest. If riskless borrowing costs are greater than lending costs, two separate rays are drawn for the respective rates. Figure 2.2b represents the model with unequal rates. The efficient frontier is represented by the linear segment R_L, the concave segment L_B and the linear segment of B_C.

The implication of this model is that only risky portfolios along the Markowitz efficient frontier represented by the segment L_B would be selected by risk averse investors. The difference between the borrowing and lending rate determines the length of the segment along which risk averse individuals select portfolios.

Black (1972) develops a model inclusive of short sales. Investors can take a negative position in an asset or sell
securities short using the proceeds to purchase other securities. Selling a security short is equivalent to borrowing or issuing a liability to raise funds. An investor selling something he does not own is borrowing from a third party to make delivery. Eventually, the short position must be closed out by repaying borrowing or purchasing the security.

Assuming all assets are risky, the efficient frontier with short sales resembles Figure 2.3a. Portfolios exist with infinite rates of return because investors can sell short to buy securities with high expected returns. The efficient set is the upper half of the curve represented by the solid line.

Dyl (1975) adds further modifications to the basic model in order to include short sales with a margin requirement. Imposing the margin is equivalent to the addition of a new set of risky assets negatively correlated with the existing assets. The total portfolio variance and subsequently the shape of the efficient frontier reflects the modification. In Figure 2.3b, Dyl's efficient frontier is graphed in risk return space. The minimum variance portfolio represented by point A consists of a long and short position in the same security. With the offset of positive and negative returns the zero variance portfolio offers a zero expected return. Point B represents the maximum return portfolio. If portfolios contain securities with negative returns, the
Figure 2.3a: The efficient set with short sales

Figure 2.3b: The efficient set with margined short sales
positive returns from short sales in this model could shift the maximum return point to the left of that in a Markowitz model.

The efficient frontier between points A and B would dominate the Markowitz model as a result of the covariance effect introduced with margined short sales. The efficient frontier shifts to the left indicating a better trade off between risk and return at each point.

The Markowitz model and its extensions share the characteristic of a concave opportunity set. For the two assets case, drawing the efficient frontier is an algebraic exercise. Examining the characteristics of the choice set when there are n assets involves computational complexity. In addition, the short sales and negative asset weights of the modified models must be included.

The next section discusses methods used to mathematically construct the efficient frontier. The data required in the analysis are reviewed along with alternative solution techniques. Delineating the choice set is half of the portfolio problem. The methodology of deriving information about portfolios precedes the section on utility that discusses transforming information on choices into decisions.

Mathematical portfolio analysis

Mathematical portfolio analysis involves use of either calculus or quadratic programming. The problem is to
determine the weights or proportions of the securities in the efficient portfolio. The required inputs include estimates of expected return for the assets, the standard deviation or variance of the rates of return and the covariances between the assets examined.

Differential calculus or quadratic programming offer solution techniques for problems involving several securities. The problem is to find the minimum variance portfolio at selected levels of return.

The calculus method involves minimizing a Lagrangian objective function. The general formulation of this problem is to minimize;

\[
\text{VAR} (R_P) = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_{ij}
\]

There are two constraints. First, the desired expected return must be achieved. In addition, the proportions of assets must sum to one. Mathematically:

\[
\sum_{i=1}^{n} X_i E(r_i) - E^* = 0
\]

\[
E^* = \text{the desired expected return}
\]

\[
\sum_{i=1}^{n} X_i - 1 = 0
\]

For the n-asset case the minimum risk portfolio is obtained by solving a system of N + 2 linear equations. The Lagrangian objective function with a return constraint is;
In order to find the minimum risk portfolio it is necessary to set $\delta Z/\delta X_i = \delta Z/\delta = 0$ for $i=1,...,n$ and $j=1, 2$.

In matrix notation the system of equations is represented by;

\[(2.14) \quad C X = K\]

- $C$ = the coefficient matrix
- $X$ = the weights
- $K$ = a vector of constants

Taking the inverse of the coefficients matrix yields the solution of the vector of asset proportions that minimizes variance. Mathematically:

\[(2.15) \quad C^{-1} CX = C^{-1} K\]

- $I X = C^{-1} K$
- $X = C^{-1} K$

An alternative solution procedure is to maximize return at each level of risk. The objective is to maximize;
\[ (2.16) \quad E (r_p) = \sum_{i=1}^{n} X_i E (r_i) \]

\[ \quad - \text{VAR} (r_p) = - \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j \sigma_{ij} \]

Subject to:

\[ \quad \sum_{i=1}^{n} X_i = 1 \]

The computational complexity of solutions derived from the use of calculus restricts its use in multi-security portfolio analysis. Markowitz (1956) developed an algorithm to solve the portfolio problem. A QP algorithm minimizes the quadratic objective function tracing out the efficient frontier for different values of \( E(r_p) \). The program calculates the weights of the securities in the portfolio. The slope of the efficient frontier represents the tradeoff between risk and return.

There are advantages to using the QP solution method. The most significant advantage is that QP allows inequality constraints. Solutions with the calculus method are limited to equality constraints. With QP, upper and lower bounds on investments can be included in the problem.

For example, a firm may face regulatory constraints on the proportion of a portfolio held in a particular security. This constraint can be readily modelled using QP. For the case of regulatory portfolio constraints, the unconstrained frontier can be compared to the constrained model indicating
the affect of regulations on the choice set. This is of particular interest when the intent of regulation is to influence the riskiness of a firm's activities.

Both QP and the calculus method allow solution of problems involving short sales and leverage. The use of QP provides computational advantages because the complexity of a model with multiple assets and constraints makes solution with calculus difficult.

Portfolio analysis involves the use of mathematics to solve for the weights of securities yielding the minimum risk at various levels of return. The QP algorithm offers a flexible method for analysis of multi-security portfolios and realistic inequality constraints. Conducting comparative static experiments for alternative regulatory constraints, new asset or liability products and environmental conditions allows policy makers to review the risk return characteristics of the choice set and determine the weights of activities to include for efficient portfolios.

This discussion of the efficient frontier does not indicate how a decision-maker selects a particular portfolio. Mapping an efficient frontier in risk-return space allows comparison of opportunity sets. Changing the allowable activities or constraints may shift the efficient frontier to the left of its initial position in risk return space. The result is a higher return at each level of risk. If risk is a
characteristic the investor seeks to avoid or minimize and higher levels of expected return are desirable, the new efficient frontier dominates other choice sets.

Individual investor preferences and selection of optimal portfolios are reviewed next along with the limitations and problems involved in modeling consumer preferences.

Portfolio Theory and Investor Choice

Portfolio theory rests on several assumptions about consumer behavior. The following assumptions are typically made:

1. Investors maximize the expected utility of terminal wealth.
2. Investors are risk averse.
3. Investors have a single period planning horizon.
4. Investors select the optimal portfolio based on the mean and the standard deviation of returns.

These assumptions restrict an individual's choice to a point on the efficient frontier. Investors select the portfolio with the lowest risk at a given rate of return. Decisions based on the risk parameter would result in the selecting the portfolio at the desired level of risk that offered the highest rate of return.

These assumptions require further explanation. The next section clarifies the theoretical foundations of choice theory.
along with the restrictions introduced for decisions within the constraints of portfolio theory.

Choice Under Uncertainty

Axioms of choice under uncertainty

Utility theory provides the framework for investor selection of the optimal portfolio. A basic set of rules initially developed by Von Neumann and Morgenstern (1947) explains the behavior of individuals when future outcomes or events are uncertain. The rules are reviewed in several sources but the following is based on Henderson and Quandt (1980).

1. Comparability. A and B denote risky alternative outcomes. An investor is able to state a preference or compare the outcomes. Given a choice, the investor indicates preference for A compared to B or B to A. Or the individual may be indifferent between the two choices.

2. Transitivity. A, B and C denote three alternative risky outcomes. Investors are consistent in ranking preferences. If A is preferred to B and B to C, then it follows that A is preferred to C.

3. Continuity. For an individual preferring A to B and B to C a subjective probability, P, exists such that B is indifferent to a lottery that yields A.
with a probability of $P$ and $C$ with a probability of $1-P$.

4. Independence. An individual is indifferent between two alternatives, $A$ and $B$. Outcome $Z$ represents some third risky outcome. It is possible to construct a gamble in which the individual has a subjective probability, $P$, for outcome $A$ and $1-P$ for outcome $Z$. If $A$ is indifferent to $B$ then the investor will be indifferent between the two gambles structured as follows:

- $A$ with a probability $P$ and $Z$ with a probability $1-P$ or;
- $B$ with a probability $P$ and $Z$ with probability $1-P$

5. Unequal probabilities. Assume that $A$ is preferred to $B$. Let $L_1$ represent a lottery with a probability of $P_1$ for outcome $A$ and $1-P_1$ for outcome $B$. Lottery $L_2$ has a probability, $P_2$, of outcome $A$ and $1-P_2$ for outcome $B$. If $A$ is preferred to $B$, a prospect with $P_1 > P_2$ is preferred. The consumer prefers $L_2$ to $L_1$ only if $P_2 > P_1$.

6. Compound probabilities. The alternative selected does not depend on the number of steps required to obtain the outcome. Decisions are based on the total or conditional probability of an event.
Lottery \( L_1 \) is a lottery with a probability \( P_1 \) of outcome \( A \) and \( 1-P_1 \) for \( B \). Lottery \( L_2 \) has a probability \( P_2 \) of \( L_3 \) and \( 1-P_2 \) for \( L_4 \). \( L_3 \) and \( L_4 \) are lotteries described as follows:

- \( L_3 \) with probability \( P_3 \) of \( A \) and \( 1-P_3 \) for \( B \)
- \( L_4 \) with probability \( P_4 \) of \( A \) and \( 1-P_4 \) for \( B \)

Then \( L_1 = L_2 \) if \( P_1 = P_2P_3 + (1-P_2)P_4 \). Given \( L_2 \), the probability of obtaining \( L_3 \) is \( P_3 \). The probability of \( L_4 \) is \( P_2P_3 \). The probability of \( L_4 \) is \( 1-P_2 \). The probability of obtaining \( A \) with the lottery is \( L_4 \) and the probability of obtaining \( A \) with \( L_2 \) is the sum of the two probabilities.

These axioms provide a minimum set of conditions consistent with the rational behavior of individuals. Based on these rules, it is possible to rank outcomes based on their respective levels of utility. Expected utility theory is derived from these basic conditions and the basic properties of utility functions.

**Expected utility**

If an individual obeys the rules of choice under conditions of uncertainty, a utility function exists reflecting preferences for alternative possible outcomes. The utility of each choice is determined by the expected value of the utility of the possible outcomes.
This utility function has three basic properties. Although these characteristics are reviewed in several sources, the following discussion is based on the framework of Anderson, Dillon and Hardaker (1977).

The first property of the expected utility function depends on the individual's ability to compare alternative outcomes. A and B represent risky outcomes with a subjective probability for outcome A of P and probability 1-P for outcome B. If A is preferred to B then the utility of A exceeds the utility of B. Individuals prefer outcomes with greater utility. Their objective is to maximize the expected utility.

The second property of the utility function is that the utility of the risky outcomes is the expected value of utility. From statistics, the expected value is calculated as follows:

\[ E(X) = \sum_{i=1}^{n} P_i X_i \]

\( E(X) \) = the expected value of X
\( P_i \) = the probability of the ith outcome
\( X_i \) = the alternative outcomes

The utility of an outcome is the weighted average of the utility of each of the alternatives possible. For example, the utility of a lottery with a probability P for outcome A and 1-P for outcome B is represented as;

\[ U(L) = P U(A) + 1-P U(B) \]

\( U(.) \) = the utility of the outcome
The expected utility is the sum of the expected utilities. If choices are discrete and utility is a function of wealth then the expected utility function is represented by:

\[ U(W) = E[U(W)] = \sum_{i=1}^{n} P_i U(W_i) \]

The final property is that a linear transformation of the expected utility function will rank choices identically to the initial utility function. Comparing utility between individuals is meaningless. Altering an individual's utility function with a linear transformation does not alter their ranking of individual choices but would change the values and alter rankings between individuals.

**Attitudes towards risk**

The axioms of consumer choice and expected utility theory under provide a general framework for making rational decisions. Selection of an optimal portfolio requires information in addition to preferences towards wealth. In order to determine the optimal portfolio, it is necessary to specify the exact functional form of investor preferences. Attitudes towards risk determine the shape of utility functions.

Risk averse, risk neutral and risk seeking attitudes represent the three possible alternatives. All investors are assumed to prefer more wealth to less regardless of their
attitude toward risk. This preference for more wealth is indicated by a positive marginal utility of wealth. The second derivative, $U''$, focuses on attitudes towards risk taking.

Risk aversion means that an individual will reject a fair gamble. A numerical example illustrates the principal. Assume an investor has a logarithmic utility function and that the expected wealth from a gamble is given by the following equation:

$$E(W) = 0.6(7) + 0.4(35) = 18.2$$

Where 0.6 and 0.4 are the probabilities of the $7 and $35 outcome respectively.

The expected utility of $18.2 received with certainty is 2.9 which is calculated by taking the log of the expected value. The expected utility of the gamble is given by the sum of the probability weighted utilities from each of the outcomes.

$$E[U(W)] = 0.6 U(7) + 0.4 U(35) = 2.504$$

The risk averse individual receives greater utility from the certain $18.2 than from the gamble with an expected value of $18.2. The shape of the preference or utility function implied by risk aversion is illustrated in Figure 2.4.

Risk averse individuals exhibit decreasing marginal utility of wealth represented by a value of $U'' < 0$. The shape of the utility function is concave. The expected value of the
Figure 2.4: A utility function for a risk averse investor
gamble is represented by the straight line segment. The utility of expected wealth, receiving the $18.2 with certainty, is greater than the expected utility of wealth.

A risk neutral individual is indifferent between a fair gamble and a certain outcome. This implies constant marginal utility of wealth. The preference function as illustrated in Figure 2.5a is a straight line. \( U_m \), the second derivative is zero.

A risk loving attitude is diagramed in Figure 2.5b. The utility function exhibits increasing marginal utility of wealth or a value of \( U_m > 0 \). The risk loving investor will engage in a fair gamble.

**Indifference curve analysis**

Indifference curves are derived from the underlying utility function. Each of the alternative attitudes towards risk implies a particular shape for the respective indifference curve.

The slope of the indifference curve is the ratio of the marginal utilities or the marginal rate of substitution. If utility is a function of wealth, expanding the value of expected utility via a Taylor series yields the following:

\[
E(U) = U(\bar{W}) + U_1(\bar{W}) E(W-\bar{W}) + \left[ U_2(\bar{W})/2! \right] E[(W-\bar{W})^2] + \sum U(\bar{W}_i)/i! E[(W-\bar{W}_i)]
\]

The term \( U(\bar{W}) \) is the value of the utility function evaluated at its expected value. \( E[(W-\bar{W})^2] \) is zero because
Figure 2.5a: Risk neutral utility function

Figure 2.5b: Risk loving utility function
$E(W) - E(\bar{W})$ equals zero. $E[(W - \bar{W})^2]$ is the variance of wealth or $\sigma_W^2$.

Using the Taylor series expansion of the investor's utility function and assuming derivatives beyond the second are approximately equal to zero results in the following partial derivatives of utility with respect to risk and return:

\begin{equation}
\frac{\delta E(U)}{\delta W} = U_1(\bar{W}) + \sigma_W^2 U_2(\bar{W})/2
\end{equation}

\begin{equation}
\frac{\delta E(U)}{\delta \sigma_W} = \sigma_W U_2(\bar{W})
\end{equation}

The slope of the indifference curve is the marginal utility of return divided by the marginal utility of risk. The value of $U_\omega$ is assumed to equal zero. Assuming more wealth is preferred to less implies a positive value for $U_1$, the marginal utility of wealth. Since $U_\omega < 0$ for a risk averse investor, the ratio of the marginal utilities is negative. Figure 2.6a illustrates the shape of the indifference curve for risk averters.

Since $U_\omega$ is equal to zero for a risk neutral individual, the indifference curve is a straight line as illustrated in graph 2.6b. The risk loving investor has indifference curves of the shape as shown in 2.6c.
Figure 2.6: Indifference curves
Measuring risk aversion

Pratt (1964) and Arrow (1971) examine how preferences change as wealth changes. There are two ways to measure the changes. One method is to determine the total dollar change in risky asset holdings for changes in wealth. The other is to review the proportional amounts held in risky assets as wealth changes.

Absolute risk aversion (ARA) is represented by the following equation where \( U'(W) \) and \( U''(W) \) are respectively the first and second derivative of wealth.

\[
(2.25) \quad \text{ARA} = -\frac{U''(W)}{U'(W)}
\]

The first derivative of absolute risk aversion, \( \text{ARA}'(W) \), describes how risk averse investors respond to changes in wealth. If \( \text{ARA}'(W) \) is positive, increasing absolute risk aversion exists meaning fewer total dollars are held in risky assets as wealth increases. Constant absolute risk aversion with \( \text{ARA}'(W) \) equal to zero indicates an investor holds the same amount of risky assets as wealth increases. A value for \( \text{ARA}'(W) \) less than zero suggests decreasing absolute risk aversion where increases in wealth increase the amount of risky assets held.

Relative risk aversion (RRA) is the absolute risk aversion measure multiplied by wealth. Mathematically:

\[
(2.26) \quad \text{RRA} = -\frac{W U''(W)}{U'(W)}
\]
Interpretation of the sign of the first derivative of the expression is similar to the previous discussion of ARA excepts holdings of risky assets are in percentage terms.

Empirical studies attempting to determine investors' absolute and relative risk aversion have not been conclusive but suggest that decreasing absolute risk aversion describes investor preferences.

Blume and Friend (1975) using data on the financial characteristics of consumers determine that investors exhibit constant relative risk aversion. They conclude that absolute risk aversion is decreasing as wealth increases. Lease, Lewellen and Schlarbaum (1974) examine survey data of brokerage customers. They find that investors display both decreasing and absolute relative risk aversion.

Although the studies are flawed because they rely on survey data that compare different individuals rather than the actions of one individual as wealth changes, a general assumption made in portfolio analysis is that investors exhibit decreasing absolute risk aversion.

**Mean-variance choice criteria**

Before considering the use of utility analysis in the selection of the optimal portfolio, it is necessary to examine if choices should be evaluated on the basis on the mean and variance of returns. If individuals maximize the expected utility of wealth their choice set is described by the mean
and variance of returns under two alternative conditions: if returns are normally distributed or if the expected utility function is quadratic. Under both circumstances, selection of mean-variance efficient portfolio maximizes expected utility.

Wealth at the end of the holding period depends on returns earned on investments. The returns are random variables. If the returns are normally distributed the mean and variance completely describe the shape of the probability distribution.

Choices based on the mean and variance will also maximize expected utility if the individual's utility function is quadratic. A quadratic utility function is represented by the following equation:

\[ U(W) = W - bW^2 \]

\( U(W) \) = the utility of wealth
\( b \) = a constant

Taking the expected value of utility yields the following:

\[ E[U(W)] = E[U(W)] + b(E[U(W)]^2) + b \sigma_W^2 \]

\( U_1 = 1 - 2bW \)
\( U_2 = -2b \)

There are two restrictions on the form of this utility function. First, risk aversion requires a negative second derivative. The value of \( b \) must be greater than zero. The problem with this assumption is that it results in function
that exhibits increasing absolute risk aversion. Individuals would invest a smaller dollar amount in risky assets as wealth increases. Second, restriction that $W < \frac{1}{2}b$ is also necessary to assure that the marginal utility of wealth is always greater than zero.

Research concerning the use of mean-variance analysis has analyzed several issues. Mossin (1973) and Anderson, Dillon and Hardaker (1977) have developed quadratic functions that approximate the desirable properties of decreasing absolute risk aversion.

Alternative utility functions including logarithmic and power functions have been proposed. These functional forms have first and second derivatives consistent with decreasing absolute risk aversion. In addition, relative risk aversion is constant for these alternative utility functions.

Mean-variance choice without specifying the utility function

It is necessary to specify the exact form of the utility function in order to solve for an optimal portfolio. Expected utility will be maximized by selecting a mean-variance efficient portfolio along the efficient frontier if returns are normally or the utility function is quadratic.

The two conditions that allow choice on the basis of the mean and variance are restrictive and involve the problems discussed in the previous section. If a quadratic utility function is not used then the returns must be normally
distributed in order to make choices on the basis of the mean and variance.

Research conducted by Levy and Markowitz and (1979) and Kroll, Levy and Markowitz (1984) suggests that investors basing decisions on mean-variance criteria select portfolios that approximate utility maximization even if both the assumptions of normally distributed returns or a quadratic utility function are violated. Since the 1984 research builds on the 1979 study, the later results are discussed in the following section.

Kroll, Levy and Markowitz (1984) select points along an efficient frontier and calculate the expected utilities of portfolios at those points. Several forms of power and logarithmic utility functions were used for the calculations. The authors selected the portfolio that maximized expected utility for each of the alternative utility functions.

Then the authors select a portfolio by calculating the expected utility for a specific utility function considering all feasible portfolios and not just those located along the efficient frontier. The objective is to compare decisions based on the mean and variance with those based explicitly on maximization of an expected utility function.

The results of the study indicate that selecting a portfolio by directly maximizing utility very closely approximates choices made on the basis of selecting the best
mean-variance efficient portfolio. The results were unchanged for the case of leveraged portfolio.

The authors suggest a practical application of their results. An investment advisor without knowledge of the specific form of an investor's utility function can solve the problem by calculating the mean-variance efficient set of portfolios. The investor can then select from the efficient set without significant loss of welfare from the situation in which a choice is made to directly maximize utility.

The optimal portfolio

If the utility function is specified then selection of an optimal portfolio is illustrated by Figure 2.7. Indifference curves $U_1$, $U_2$, and $U_3$ are preferences for a risk averse investor. The indifference curves represent combinations of risk and return for which the investor is indifferent. Higher indifference curves imply the individual is better off. For example, $U_3$ offers greater return at each level of risk compared to $U_1$.

The efficient frontier is analogous to the budget constraint because it represents the possible combinations of choices available. The point of tangency between indifference curve $U_2$ and the efficient frontier is the optimal portfolio. This is the best the investor can do given the shape of the indifference curve and the efficient frontier.
Figure 2.7: The optimal portfolio
$U_3$ is unattainable. There are no efficient opportunities that are available along this indifference curve. There are many possible portfolios associated with $U_1$. Two of those combinations, A and B, are along the efficient frontier. $U_3$ is the highest attainable indifference curve possible given the efficient set. Point O offers greater utility than available at point A or B or any of the points along $U_1$ below the efficient frontier. $U_3$ provides portfolios with greater return at each level of risk.

The marginal rate of substitution is the slope of the indifference curve representing tradeoffs between risk and return. The slope of the efficient frontier is the marginal rate of transformation between risk and return. The optimal portfolio is that point at which the marginal rate of substitutions equals the marginal rate of transformation.

Even if the specific form of the utility function is not known or if returns are not normally distributed portfolio theory approximates choices based on maximizing expected utility. If the investor is presented with an efficient frontier and makes choices based on a choice set that is mean-variance efficient then decisions approximate selection of an optimal portfolio.

The next section reviews financial intermediation and the advantages and services offered in comparison to direct interaction between net savers and net borrowers in the
economy. The application of portfolio theory to financial intermediation is developed and quantitative studies of institution activities are reviewed.

Financial Intermediation

The first part of this section presents a general description of the activities and basic functions of financial intermediaries. Financial intermediaries use debt and deposit funds to invest in loans and securities. This activity represents indirect finance. The role of financial intermediaries in the financial system is defined by the advantages they provide that are not available from the direct interaction of borrowers and investors.

The application of portfolio theory to financial intermediation is reviewed along with empirical models that attempt to incorporate risk into quantitative management decisions. Several studies of bank and intermediary activity provide the framework for analysis of Federal Intermediate Credit Bank activities.

Advantages of financial intermediaries

Financial intermediaries channel funds from surplus spending units to deficit spending units in the economy. Direct finance matches a borrower and lender without the intervention of an intermediary. Indirect investment occurs when a net saver purchases the liability of a financial
institution. The financial institution in turn invests or lends funds to the deficit spending units.

Indirect investment offers several advantages to both the surplus spending units and the deficit spending units. Campbell (1982) delineates the services provided by financial intermediaries. These include:

1. Risk reduction through diversification. The first section of this chapter reviewed theory examining risk in a portfolio framework. In order to diversify, an individual investor must pay transaction costs and have sufficient funds to purchase a wide variety investments. A financial intermediary can pool the funds of many small investors and achieve the benefits of diversification.

2. Maturity intermediation. Because of borrower and depositor preferences, financial intermediaries have traditionally engaged in maturity intermediation. Assets were repriced less frequently than deposits or other liability funds. Intermediaries are exposed to interest rate risk if cash flows from assets and liabilities do not match.

This activity exposes the institution to interest rate risk. If interest rates are stable or can be predicted with certainty, the bank can profit
from interest rate positions. With unstable and unpredictable interest rates, institutions face the possibility that assets will not be repriced as often as liabilities resulting in negative cash flows or declines in the market value of balance sheet categories.

3. Intermediaries engage in activities that reduce the costs of contracting between market participants. There are costs involved in both writing and monitoring contracts. Standardized contracts between borrowers and lenders are required in an auction market. Intermediaries can write contracts tailored to individual borrowers. In addition, the intermediary processes the skill and resources to monitor borrowers more effectively than individual lenders.

4. Information production. If net savers in the economy lent money directly to borrowers, a complete credit analysis would be necessary in order to reduce the risk of default. Small savers would need to undertake a quantitative analysis of borrowers and interpret the results of their study in order to determine the safety of the loan proposition. Intermediaries perform this function. They undertake credit analysis and monitor the financial
health of borrowers. Depositors at the bank do not have detailed information on the loan and investment portfolio. In addition, firms needing large sums of money do not need to seek out numerous small lenders and provide them with detailed financial analysis in order to receive funds.

5. Financial intermediaries offer other services to their customers. Depositors benefit from liquidity and safety of principal is assured because of deposit insurance.

Categories of financial intermediaries

The balance sheets of intermediaries contain financial assets purchased for their own accounts. Asset acquisitions are financed with deposits, purchased money and debt. The differences involve their sources and uses of funds. Each of the categories of institutions faces different regulatory constraints limiting the allowable asset and liability activities. Edmister (1986) reviews the three categories of institutions. These include depository, contractual and conduit intermediaries. The following review of the kinds of institutions provides a basic description of the difference in their activities.

Commercial banks, savings and loans, credit unions and mutual savings banks are all financial institutions that finance investments with depository funds, purchased money and
other borrowing. Assets include bonds and loans. The largest category is loans. Loan categories include commercial, real estate, consumer, and agricultural loans. A significant liability is the deposit category. Another source of funds is the money market.

Insurance companies and pension funds are contractual intermediaries. Insurance companies collect and invest premiums paid by policyholders. Assets held include bonds, stocks and loans to policyholders. Earnings of contractual intermediaries are tied to interest rates.

Pension funds are contractual intermediaries because they insure the risk of living beyond wage-producing years. Pension funds function as intermediaries because they receive regular amounts from individuals which they invest long term to meet future obligations.

Conduit intermediaries exist to channel funds into specific sectors of the economy. They include the federally sponsored agencies, investment companies and real estate investment trusts. Investment companies invest customer funds in bonds and stock. Real estate investment trust assets include mortgages, constructions financing notes and real estate equity ownership.

Federally sponsored credit agencies channel funds to housing, education and agriculture. They are privately held organizations raising funds in national money and capital
markets. Their assets are loans made to specific sectors of the economy. Liquidity exists due to their access to national money and credit markets and investor perceptions that the federal government will guarantee their liabilities in the case of default.

**Risks of financial intermediation**

Financial intermediaries are exposed to three kinds of risk: default risk, liquidity risk and interest rate risk. The focus of default risk is the asset side of the balance sheet. There exists a chance or probability that a loan customer will be unable to make repayment. Institutions also hold bonds representing investment assets. Credit or default risk is also associated with the stream of returns from holding bonds.

Depository institutions require liquidity to meet loan demand or deposit withdrawals. Traditionally, liquidity existed if the institution held assets which could be easily sold for cash. Management emphasis in the last decade has shifted to liquidity through the use of liability management. When funds are required for asset acquisition or to meet loan demand they are purchased in the money market. The money market is a short-term source of funds priced on the basis of supply and demand and future expectations of interest rates.

Increased reliance on purchased liquidity requires careful analysis of interest rate risk. An intermediary locks
in a positive spread between the return on assets and the cost of liability funds. If liabilities are repriced before assets and interest rates increase, the spread deteriorates. The increased volatility of interest rates since the late 1970s and the use of liability funds priced at market rates has focused attention on techniques available to manage interest rate risk.

The risks of banking are closely related. The institution that uses variable rate loans to manage interest rate exposure passes that risk on to the borrowing customer and faces potential increases in defaults. Liquidity can be purchased but the cost is increased price volatility. Profits and income are tied to interest rates so completely eliminating that risk can lower earnings.

Institution management considers the overall risk position of the firm. Financial intermediaries are highly levered organizations meaning that the majority of their funds are purchased in the money market, borrowed from depositors and raised by issuing debt securities. A typical bank has a capital to asset ratio of six percent. The small amount of owner equity means minimal capital is available to fund operating losses. The potential for losses is determined by the overall combined risk of the bank.

This discussion of risk indicates the intuitive inter-relationship between credit, interest rate and liquidity risk.
Although this dissertation focuses on interest rate risk, it is important to recognize that selection of a particular management strategy depends not just on management risk preferences but also on the overall risk faced by the institution.

For example, Federal Intermediate Credit Banks have limited diversification in their asset portfolio. The types of loans and the geographic regions in which they make loans restricts their ability to hold a balanced loan portfolio. This means that the issue of interest rate risk is even more important to these institutions. Unlike a commercial bank holding a portfolio of loans to consumers, agriculture, real estate and business firms, the Federal Intermediate Credit Banks make loans only to the farm sector. As a consequence, these institutions must carefully monitor and measure other risks.

The following section reviews the theoretical framework of financial intermediary activity and is followed by results from quantitative studies of intermediaries. The issue of interest rate risk is developed in the next chapter along with the tools for its measurement and management.

Risk Aversion Models of Financial Intermediaries

One approach to modeling bank activity has been to explicitly recognize risk within the framework of portfolio theory. Uncertainty is incorporated into the theory of the
banking firm by assuming that the objective of the institution is maximize expected utility rather than maximize returns. The basic model is discussed after a brief review of its shortcomings.

Review of theoretical portfolio models

The portfolio models of financial intermediaries are based on the Markowitz-Tobin analysis. Financial intermediaries are modeled as a collection of assets and liabilities. The assets have positive weights and the liabilities are negative. The asset rates of return and liability costs are stochastic. The objective is to select the minimum variance portfolio of assets and liabilities at alternative rates of return tracing out an efficient frontier.

Pyle (1971) does not consider real resource costs or liquidity risk. He develops a three-security model of a financial intermediary based on the objective of expected utility maximization. The securities include a riskless security, a loan and a deposit account. The interest rates for the securities are uncertain.

The firm's profits are represented by the following function:

\[
\pi = i_{RF} X_{RF} + i_{L} X_{L} + i_{D} X_{D}
\]

\[
= X_{L}(i_{L} - i_{RF}) + X_{D}(i_{D} - i_{RF}) + i_{RF} K
\]
Subject to;
\[ \sum X_i = K \]
- \( i_{RF} \) = the risk free interest rate
- \( i_L \) = the loan rate
- \( i_D \) = the deposit rate
- \( X_{RF} \) = the proportion of the portfolio in the riskless security
- \( X_L \) = the proportion of the portfolio in loans
- \( X_D \) = the proportion of the portfolio in deposits
- \( K \) = capital

Intermediation occurs if there is a positive spread between asset returns and liability costs. The greater the correlation between loan and deposit rates and the higher the yield spread, the greater the expected profit. The main criticism of the model is its failure to include resource costs.

Hart and Jaffee (1974) assume that the institution is restricted in the kinds of assets and liabilities that it can hold. They also include a reserve constraint to model liquidity requirements.

The intermediaries net worth position is an important component of this model. The highly levered position of financial intermediaries differs from the case of individual investors with net worth representing a large proportion of the right-hand side of the balance sheet. The authors assume that if net worth is a fixed dollar amount then that amount equals zero. If net worth is allowed to vary it is treated as if it were any other liability. Finally, the authors consider
an a constraint in which capital must be some specific amount as determined by regulators.

The objective of the firm is maximization of expected utility. Absolute nonincreasing risk aversion is specified. No risk free security is included based on the assumptions that cash is not risk free and the maturity of deposits does not match liabilities.

The study concludes that the efficient frontier is linear. In addition the authors obtain the following results: holding an asset is positively related to its expected yield while liability holdings are inversely related to yield and holdings of assets and liabilities are inversely related to the standard deviation.

These general models have results consistent with Markowitz-Tobin modified models. The shortcomings of the portfolio models are considered in the next section. These problems are important to consider when modelling commercial bank activity but do not apply to Federal Intermediate Credit Banks.

Problems with the portfolio model of institution behavior

There are two basic shortcomings of portfolio models of financial intermediation. First, real resource costs are excluded. As part of their activities, commercial banks facilitate transactions in the economy. An elaborate system of accounts must be maintained and the exchange of funds
between parties in the financial system must be facilitated. This activity involves real resource expenditures in the form of personnel, equipment and record keeping.

The second problem is that the portfolio model of the bank firms excludes rate-setting behavior on the liability side of the balance sheet. The implicit assumption of the portfolio models is that banks are price takers in the deposit market and can select any quantity of funds needed at the market rate. Banks face uncertain fund availability. In addition, banks have the ability to exercise rate setting power now that regulation Q no longer caps allowable interest rates on deposits.

Although these issues are important for commercial banks, it is realistic to apply a portfolio model to nondepository intermediaries. Institutions in the Farm Credit System do not engage in the transaction function or fund asset acquisition through the use of deposit funds. This means that the exclusion of real resource costs associated with depository functions more closely models the actual activities of the Farm Credit Institutions.

The second shortcoming of portfolio models is also not applicable to Farm Credit System institutions. The funds to finance assets are raised in national money markets in which the FCS institutions are recognized as active and frequent participants. The Farm Credit System can raise all the funds
it needs at the going market price. The System competes for funds with other borrowers in the money market including the Treasury, corporations and other organizations with agency status.

The next section reviews the results of quantitative models of financial intermediaries. The studies utilize quadratic programming to generate sets of bank portfolios. **Empirical studies of bank portfolio models**

Several studies use Markowitz portfolio analysis to solve for the optimal balance sheet proportions of assets and liabilities. The objective of the institution is minimizing the variability of returns on equity at alternative selected rates of return. Use of the quadratic programming algorithm and actual institution data results in delineation of an efficient frontier. Each point along the frontier represents a different set of weights for the various asset and liability categories.

Model construction can include regulatory constraints, risks, competitive constraints, asset management, liability management, liquidity management, interest rate risk management and hedging activities. A brief review of quadratic programming models of financial intermediaries indicates the scope of the studies.

The initial requirement is estimation of a variance-covariance matrix of returns and costs. Constrained and
unconstrained solutions are considered in order to conduct comparative static experiments for alternative management activities, environmental conditions and regulatory policies.

Robison and Barry (1977) model a representative institution in order to identify portfolios with minimum variance for alternative levels of expected wealth over a three-month period. The model considers only the asset side of the balance sheet solving for relative weights of safe and risky assets along an efficient frontier.

The objective of the research is to consider the ability of rural financial institutions to manage loan and liquidity risks. The bank activities include lending, security purchases, deposits, borrowing and paying taxes and dividends. Net earnings are added to wealth.

A riskless asset, three-month bonds is included with a zero covariance. The variance-covariance matrix represents returns for the alternative investment activities. After selecting an efficient E-V solution, the quadratic programming model is used to consider the affect of changes in variance, deposit feedback and interest rates.

The results indicate that an increase in deposit costs shifts the efficient frontier to the right. With constant absolute risk aversion, the only affect is a decline in wealth. Decreasing absolute risk aversion results in a decline in the holdings of risky assets.
An increase in the rate of return on agricultural loans shifts the efficient frontier to the left. With constant absolute risk aversion, holdings of risky assets increase and the amount of safe assets declines.

The deposit feedback effect occurs when loan customers place the proceeds of the loan in a deposit account at the lending bank. Reduced deposit feedback of real estate loans causes declines in both real estate and agricultural loans.

Increased variance in the agricultural loan portfolio results in a shift of the efficient frontier to the right. Constant absolute risk aversion results in the purchase of fewer risky assets.

The objective of the research is to develop an analytical framework for considering the response of risk-averse bankers to changes in policy variables. The use of quadratic programming indicates changes in the efficient frontier for possible policy alternatives. This methodology allows consideration of the relationships between policy or environmental changes and subsequent affects on the risk and return conditions for the institution.

J. C. Francis (1978) compares constrained and unconstrained frontiers for small-, medium- and large-sized commercial banks. Comparisons are made about the asset/liability management techniques of bank managers.
Francis builds the model assuming owners minimize risk at various alternative rates of return. Cash, government bonds, mortgages, installment loans and business loans represent assets on the balance sheet. Liabilities include demand deposits, savings deposits and certificates of deposit. Cash is a riskless asset with a zero return and variance.

Quadratic programming is used to generate a set of efficient frontiers for small-, medium- and large-sized banks. With the unconstrained efficient frontier, funds were raised almost exclusively with deposit funds due to their low cost. The bank held a diversified portfolio of assets that included mortgage, installment and business loans.

Constraints are modeled to reflect liquidity risk. This forces the banks to hold the nonearning asset cash. A competitive constraint is also added based on the assumption that the institutions have limited availability of deposit funds.

Efficient frontiers generated are compared to actual institution portfolio selections. Only large banks are on or near their efficient frontier. Francis concludes that either small- and medium-sized institutions are inefficiently managed or have limited opportunities for diversification.

Paulson (1979) studies liability management. Instead of obtaining liquidity from the sale of assets, it is purchased on the money market. Interest rate risk management is modeled
through the use of a variable-rate loan asset. E-V frontiers for fixed and variable rate loan pricing policies are generated and compared.

The study incorporates measures of liquidity risk into a quadratic programming model of an individual institution. Liability management activities are introduced into the basic model allowing the purchase of money to fund fixed rate lending. The use of liability management shifts the efficient frontier to the left.

When variable rate lending is introduced, the efficient frontier shifts to a lower variance position while maintaining the level of expected return. The model assumes that price changes for variable rate loans is perfectly correlated with changes in the cost of funds. This means that loans are repriced each time the cost of funds changes.

The study concludes that variable rate loan pricing increases fund availability in a rural loan market. The issue of changes in the customer's interest rate risk position is not addressed. In addition, alternatives that more realistically model variable rate loan terms and result in correlation coefficients less than one are not considered.

Barnard (1982) reviews the affect of deposit rate deregulation on financial intermediaries. The study assumes that the effect of deregulation of deposit rates will be an increase in the variability of the cost of those funds. The
objective of the study is to examine alternative time periods under several alternative scenarios; lower deposit feedback, increased costs of funds and lower reserve requirements, less elastic loan demand, increased correlation between nonloan asset returns and liability costs, increased correlation between loan asset returns and liabilities and increased rates of returns for loans.

The model results evaluate management responses to deregulation. Since regulation Q placed a ceiling on the interest rates that banks could pay depositors, deregulation affects the cost of liability funds. Possible responses to deregulation include greater correlation between asset and liability activities, use of variable rate loans and increases in the rates on loans.

Three time periods, 1977, 1980 and 1987 are modeled. It is assumed that additional sources of money market funds become available over time and these liabilities are priced at market rates. A comparison of the basic models in the different time periods indicates that the efficient frontier for the 1980 model intersects the 1977 or base year model. The later time period provides greater risk efficiency at higher expected rates of return. The author suggests that this result occurs because of the increased covariances between returns and costs.
The 1987 model is to the left of the other time periods indicating higher variance at each level of expected return. This occurs due to the increased rate sensitivity of liability funds.

Asset/liability management is modeled for the investment portfolio by maturity matching. As a result, the bank holds greater nonloan relative to loan assets. This results holds for each of the time periods considered.

To model variable rate lending the author increases the correlation coefficient between loans and rate sensitive liabilities. Variable rate pricing results in greater lending activity and reductions in both expected income and variance.

The final alternative considered involves increasing loan rates as a response to increased interest rate risk. The results of models incorporating this alternative indicate greater lending activity, higher expected returns and greater variability.

The major problem with this study is the method of modelling interest rate risk management. The correlation coefficients are altered arbitrarily without reference to the theoretical literature concerning interest rate risk. The study does not consider the affect of reduced interest rate risk on default risk. No interest rate risk management activities are modeled other than the use of variable rate loans.
Drabenstott and McDonley (1982) use a QP model of a representative institution to examine management use of futures markets to hedge interest rate risk. They conclude that hedging improves performance as measured by income, return on equity or portfolio size.

In order to include futures they assume that hedging reduces 80 percent of the variability of funds costs. There is no attempt to calculate hedge ratios and evaluate a trading program based on the theoretical material on hedging covered in the next chapter. A more appropriate test of hedging requires clearly specifying the objective and statistically reviewing the resulting changes in risk and return over the hedging period. Their study presupposes that the hedge successfully reduces variance.

Quadratic programming models offer the opportunity to examine changes in regulatory constraints, management practices and the kinds of assets and liabilities included on the balance sheet. The next chapter covers the topic of interest rate risk management. Although the empirical studies reviewed in this section provide some insight into interest rate risk management, it is necessary to discuss current theories concerning the issue of interest exposure measurement and management.
CHAPTER III. INTEREST RATE RISK THEORY

Interest rate risk exists if unanticipated changes in interest rates affect the discounted present value or market value of assets and liabilities. This chapter considers theories of interest rate determination in order to develop the problems associated with forecasting interest rates. Interest rate risk measurement and management from the perspective an individual investor is reviewed before the theory is modified for the case of financial intermediaries.

An inverse relationship exists between the direction of the interest rate change and the change in the market value of a financial asset. If future changes in the interest rate are predicted accurately at the time the asset is purchased, future rate changes are already accounted for in the price of the asset. Any actual or anticipated change in interest rates from that embodied in the current interest rate structure results in a bond market price response. Uncertainty as to the direction, magnitude and timing of future interest rate changes results in unanticipated changes in the price of a financial asset (bond).

Financial intermediaries are exposed to interest rate risk on both sides of the balance sheet because they finance the purchase of financial assets by borrowing from depositors and the money market. The income of a financial institution depends on the spread between returns and costs. In
addition, the net worth of a financial intermediary is subject to fluctuation as interest rate changes affect its market value.

The General Level of Interest Rates

Interest rate theory is divided into two components. First, theories explaining the general level of interest in the economy are presented. Then the relationship between maturity and yield, term structure theory, is added to the analysis of interest rate determination.

Inflation premium theory

The inflation premium theory proposed by Fisher (1930) offers an explanation of short run nominal interest rates. The role of inflationary expectations is explicit in this model. The nominal interest rate is given by the following equation:

\[ i = i_R + \Delta P^E \]

\( i \) = the nominal rate of interest
\( i_R \) = the real rate of interest
\( \Delta P^E \) = the expected change in the price level

The nominal rate of interest is the sum of the real rate and expected changes in the price level or expected inflation. The existence of the inflation premium is logically explained by recognizing that investments made at the present time result in payments in some future time
period. If inflation occurs, the purchasing power of the dollars received in payment are eroded.

The real rate of interest is a default free rate of return which Fisher assumed was constant so forecasting nominal interest rates rests with forecasting expectations as to the future level of inflation. It is important to note that this variable is not the historical level of inflation but that which is expected to occur in the future. The expression $\Delta P^E_t$ is the anticipated or expected future change in the price level over the time period under analysis.

The role of inflation in interest rate determination is widely accepted. The problem with Fisher's theory is that it requires a forecast of expectations before it provides a forecast of interest rates. The manner in which expectations are formed along with their relationship to actual historical rates must be quantified.

**Loanable funds theory**

This theory postulates that the nominal rate of interest in the economy is attained by the intersection of the supply and demand for loanable funds. Consumers, business and government are the market participants whose behavior influences the shape of the supply and demand curves. The magnitude of shifts in supply and demand in response to changes in the models variables determines the new equilibrium rate of interest.
Consumer demand for loanable funds is assumed inelastic. If individuals and households base lending decisions on monthly payment size or maturity then their demand depends on income. Government demand for loanable funds is also inelastic. Business borrowing is assumed responsive to interest rates. Higher interest rates eliminate certain projects when interest rates increase if firms use discounted cash flow analysis to evaluate investment decisions.

The demand curve as drawn in Figure 3.1 slopes down to the right and is the sum of all three sectors. The supply of loanable funds depends on saving, dishoarding and money creation in the banking system. The intersection of supply and demand determines the quantity and price of loanable funds.

The three components of the supply curve require further explanation. Household saving depends on the level of income. Research indicates that a wealth effect influences the level of saving. Interest rate increases cause a decline in the wealth of net savers due to a lower present value worth for their financial assets. Net borrowers with fixed rate financial liabilities benefit from rate increases.

The demand and supply of money is included in the model in the form of hoarding and dishoarding. If the demand for money exceeds supply then the volume of loanable funds
Figure 3.1: Interest rate determination with loanable funds theory
declines. If individuals have funds in excess of demand for
money, loanable funds increase.

Through the multiple expansion of deposits or money
creation in the banking system, the supply of loanable funds
increases. With a greater volume of loanable funds, the
supply curve shifts and equilibrium is at a lower rate of
interest.

Accurately predicting future interest rates using
loanable funds requires a model that includes all relevant
sectors and variables. The behavioral response of
individuals and business to changes must be carefully
developed. These relationships may change over time thus
invalidating forecasts based on historic data.

Liquidity preference theory

The demand and supply of money determines the interest
rate in this model. Keynes (1936) developed the theory in
order to explain short run changes in interest rates. This
theory is based on several behavioral assumptions explaining
the reasons for holding money instead of bonds. The motives
for holding money include the transactions, precautionary and
speculative motives.

Money is used for transactions purposes. Cash inflows
and outflows occur at different points in time. Individuals
hold an inventory of cash to pay for purchases of goods and
services. Transactions are known expenses that occur between cash payments.

The precautionary motive explains cash holdings based on unexpected expenses or costs. Uncertainty about future required cash outflows means individuals hold some quantity of cash to pay surprise expenses.

The speculative motive determines proportions of the two financial assets, money and bonds, held by consumers. Expectations about future interest rates influence the amount of wealth held as either money or bonds. If a decline in the interest rate is anticipated, bond prices will increase and investors will prefer to hold bonds instead of cash. If individuals anticipate increases in the interest rate it implies that bond prices will fall. Under those circumstances, money holdings will increase relative to bonds.

The shape of the demand curve in Figure 3.2 is determined by the speculative motive based on the assumption that the precautionary and transaction holdings depend on the level of income. The supply curve is a vertical line based on the assumption that the monetary authority exercises complete control over the money supply.

This theory describes short run interest rate determination and requires a complete understanding of expectations in order to make accurate forecasts. Demand for
Figure 3.2: Interest rate determination with liquidity preference theory
credit and influences on the money supply other than government control are excluded.

**Rational expectations**

This theory explains interest rate determination by focusing on the formation of expectations and the efficient operation of money and capital markets. The existence of unanticipated information leads to changes in interest rates. Forecasting interest rates requires a complete model of the market expectation formation.

Rose (1986) reviews the basic assumptions of rational expectations. The theory is based on the following six assumptions:

1. Market participants establish a probability distribution of future security prices and interest rates. All available information is used to assess the future values of security prices and interest rates.
2. Changes in interest rates are correlated with unanticipated events and information.
3. There are no unexploited opportunities to earn above normal profits.
4. The serial correlation between interest rates and security yields in successive time periods is zero.
5. Information, transaction and storage costs are zero.

6. Market participants exhibit rational economic behavior.

The final assumption merits further discussion. It implies that the objective of market participants is profit maximization. Those active in the financial markets do not ignore information that is important for forming a forecast about future rates. If money and capital markets are efficient, all market participants have access to and use all the information that has an influence on yields.

This theory implies that historical interest rates are not necessarily an indication of future rates. Market expectations determine the observed interest rate. The current rate will change only if new information becomes available and market participants anticipate an effect on future rates. Any variables that imply a change in the future interest rate environment are used to form expectations. For example, bond prices would respond to information on the future course of monetary policy prior to any actual changes in money supply or action on the part of the monetary authority.

Studies by Mishkin (1978) and Phillips and Pippenger (1976) seem to support the theory of rational expectations. These studies found that past changes in the interest rate are
not significantly related to the current returns on stocks and bonds. In another study, Rozeff (1974) found that past macroeconomic information is not significantly related to the current interest rate or changes in the interest rate. The studies imply that historical information and events do not explain the current values of financial variables.

The four theories reviewed provide alternative explanations of the general level of interest rates. Although the theories draw different conclusions as to the importance of different factors in interest rate determination, expectations seem to be a component in each of the alternatives. The next section reviews the relationship between the observed yield for a particular security and the time that remains until that security matures.

The Term Structure of Interest Rates

Forecasting the general level of interest rates is only part of the problem. There is more than a single interest rate in the economy. Different securities and loans are priced at different rates. Taxes, default risk and call features all influence the return investors require. Another important aspect of bond yields is the relationship between yields and the time remaining until a security matures, term to maturity.

Yield curves are drawn by comparing the interest rates for bonds that differ only in the time remaining until their
maturity. There is more than one theory that attempts to explain the shape of the yield curve. The yield curve is drawn at a point in time and so represents the best available information for that particular instant. The exact information content of a yield curve depends on which theory actually represents the relationship between yield and maturity.

Figure 3.3 graphically depicts the relationship between the time remaining to maturity and the yield or interest rate for a bond. The yield curve has been drawn with an upward slope. Yields are plotted on the vertical axis and time to maturity on the horizontal axis. Yields are nominal annual rates of return. Bond prices and yields in the *Wall Street Journal* for bonds of similar risk can be used to draw a yield curve for a particular point in time.

The graph illustrates an upward sloping yield curve. A general interpretation of an upward sloping yield curve is that rate increases are anticipated. A specific interpretation of the shape of a yield curve depends on theoretical relationships.

The following section discusses three theories of the term structure of interest rates. Beginning with unbiased expectations, the relationships between current long term rates or spot rates for bonds with different maturities and forward one year rates in future periods are developed.
Figure 3.3: An upward sloping yield curve
Unbiased expectations

This theory assumes investors consider two choices; holding one long term bond over their planning horizon or a series of one year bonds. If the one year strategy offers a higher return, investors will buy one year bonds to capture the yield advantage. When all market participants recognize this opportunity, excess demand will bid up prices and yields will decline until equilibrium is achieved between holding period returns for long and short bonds.

The assumption that equilibrium exists between returns for long and short term bonds is shown by the following equation:

\[ i_N = \frac{N}{\sqrt{(1+i_1)(1+i_{E2})\ldots(1+i_{En})}} - 1 \]

\( i_N \) = the spot rate: The yield of a bond with \( N \) years to maturity.

\( i_{En} \) = the forward rate on a one-period bond expected in period \( n \)

In year one the expected one period rate equals the actual one period rate so \( i_1 \) equals \( i_{E1} \). The variable \( i_N \) represents the spot rate or the rate of return for a bond with \( N \) years to maturity. This equation is a mathematical representation of the term structure and indicates that the shape of yield curve depends on expectations about future one-period rates. The yield curve slopes up if rate increases are expected, is flat if constant rates are assumed and slopes down for declining rates.
Examining the shape of the yield curve provides information on forward interest rates. Equation 3.3 can be used to solve for a forward rate. The equation for calculating the forward rate is:

\[ i_{EN} = \frac{(1+i_N)^N}{(1+i_{N-1})^{N-1}} \]

The shape of the yield curve implies a series of one year forward rates. The values of \( i_N \), the rate of interest for a series of \( N \) period bonds, in equation 3.4 are taken from the yield curve. The calculated value of \( i_{EN} \) is the market forecast of the one year rate expected to prevail in year \( N \).

**Liquidity premium theory**

This theory attempts to incorporate consumer preferences for liquidity. The assumption is that consumers prefer shorter maturities and require a premium in order to invest for terms of increasing length. There is not indifference between the stated maturity of securities. Decisions are no longer based solely on holding period yields.

The current long term rate is the geometric average of the forward rates and liquidity premium required for investments with different maturities. The longer the term of the bond, the higher the liquidity premium. The mathematical representation of the theory is as follows:
\( (3.5) \quad i = N \sqrt{1+\frac{i_1}{1+E_{2}+L_2}} \ldots \left(1+i_{EN}+L_N\right) - 1 \)

- \(i_N\) = the spot rate
- \(i_{EN}\) = the forward rate in period \(N\)
- \(L_N\) = the liquidity premium in period \(N\)

Now the interpretation of the yield curve changes. The existence of the liquidity premium introduces a positive bias to the shape of the curve. Even if future rates are expected to remain constant, the observed yield curve has an upward slope. In order to make decisions based on the anticipated interest rate environment as derived from the yield curve, knowledge of both forward rates and liquidity premiums is required.

**Market segmentation**

This theory hypothesizes that liquidity considerations are not the only influence on investor maturity preferences. The supply and demand of securities at various maturities determines the shape of the yield curve.

Investors are not a homogeneous group. They include individuals, firms and institutions with differing planning horizons. If investors try to match the maturity of assets and liabilities, demand depends on individual preferences for securities with different maturities.

Borrowers issuing securities supply maturities based on their own pattern of cash flows and financing needs. These
firms try to match repayment with the positive cash flow generated by operations or a project.

The net result is that bonds with different maturities are not perfect substitutes. The shape of the yield curve becomes even more difficult to interpret because any point represents the equilibrium of supply and demand at that particular maturity.

The above discussion indicates that no one theory offers an explanation of interest rate determination. The methods used to forecast interest rates depend on specifying a theoretical model, collecting necessary data and understanding the changes occurring in model relationships over time. These theories can indicate general trends for future interest rates. Borrowers and investors need more specific information. Information on the magnitude, timing and direction of rate changes is required to eliminate the interest rate risk of actual or anticipated asset and liability holdings.

**Forecasting interest rate changes**

Interest rates prior to the late 1970s were relatively stable. Several factors combined to increase interest rate volatility and focus attention on the problem of bond price risk associated with rate changes. Inflation, the U.S. government deficit, the internationalization of the macro economy and the shift of Federal Reserve policy from an
interest rate to a money supply target affected interest rates.

If the magnitude, direction and timing of interest rate changes can be accurately predicted, borrowers and investors can make current decisions knowing the future pattern of returns and costs. The clear advantages of predicting interest rate changes are tempered by problems that limit accuracy.

Each theory presented in the previous section suggests a different mathematical model to predict future interest rates and requires collecting different types of data for statistical analysis.

If the theory of efficient markets correctly explains the determination of market prices and interest rates then current prices and rates represent the market's best guess as to the future pattern of changes. A more accurate forecast than the market's requires access to better information than the market possesses.

Economists continue to forecast interest rates using various econometric models. Rose (1984) examines methods analysts use to forecast rates. Econometric models use the Fisher effect, expectations, income and liquidity to develop the mathematical framework for analysis. Although the models are constructed differently, it is possible to consider some of the general components they contain.
Monetary policy and its influence on credit availability is one component of the models. Interest rates are modeled to respond to monetary policy and the public's inflationary expectations.

McNees (1981) indicates that different models have had different degrees of success depending on which variable is being forecast. Some models do better at forecasting the short run while others perform better in the long run. The poor performance of models in signalling major turning points in the economy and their inability to deal with external shocks has resulted in borrower and investor dissatisfaction in model predictions.

The models are complex requiring use of economic data that may not accurately measure the variables needed for forecasts. The actual economic environment is even more complex than the models and subject to political and social shocks and relationships that current mathematical models are unable to duplicate.

Other forecasting approaches are built on the foundations of the loanable fund theory and estimate the supply of and demand for loanable funds based on data contained in the Flow of Funds Accounts published each month in the Federal Reserve Bulletin.

Some of the methods include both analysis of supply and demand factors in debt markets using the framework of market
segmentation theory and review of market forecasts embodied in the yield curve.

Each of these forecasting methods involve inherent problems. Estimation of market supply and demand requires accurate models of the determinants of supply and demand, accurate data for those variables and correct estimation of future rates from historical data. Expectations-based forecasts require correctly modelling the method in which expectations are formed and the way information is used to form rate predictions. Expectations models indicate the markets best guess and do not necessarily provide an accurate prediction of actual future rates.

The limitations of forecasting techniques and the large losses possible through incorrectly anticipating rate changes led to the development of interest rate risk management techniques to insulate the investor, borrower or financial intermediary from interest rate risk. A forecast of general trends does not provide sufficient information for many financial decisions.

Before interest rate risk management techniques can be used, the risk must first be identified and quantified. The next section discusses measurement of interest rate risk and the use of these measures in developing management strategies.
Bond portfolio interest rate risk

The investor planning to purchase and hold a bond that has just been issued until it matures will realize the yield promised on day one. If the bond is sold before maturity, its value and the realized rate of return depend on the difference between the interest rate on the purchase and sale date. Fong (1980) reviews three alternative measures of risk: standard deviation, bond sensitivity and duration.

The previous chapter covering portfolio theory provides the theoretical framework for the first method of analysis. The first technique uses the standard deviation and the covariance of returns to analyze asset allocation. An efficient frontier is generated in which involves combinations of bonds that provide minimum risk at the selected level of returns.

The standard deviation of a bond will exhibit systematic changes due to the time remaining to maturity, call features and conversion features. Other methods of bond analysis have been developed to include consideration of the effect that the time remaining until maturity has on changes in the price of the bond.

The first of these alternatives is bond sensitivity analysis. This method calculates returns under several possible interest rate environments. The sensitivity analysis
is conducted by comparing the probability that some target return will not be attained based on management's selected interest rate scenarios. This specification of risk requires identifying all relevant possible outcomes and their associated probabilities.

Using the standard deviation or sensitivity analysis to measure bond price risk provides a portfolio manager with some insight into the effect of rate changes on bond value. The problem with these methods is that they fail to recognize that an investor has two sources of return from buying and holding a bond.

If a bond offers periodic coupon payments, those cash flows can be reinvested at the prevailing market interest rate. Over time, the accumulated interest earned from reinvesting coupons will be greater the higher the interest rate. The reinvestment effect refers to changes in the accumulated future value of coupon payments. A rate decline would result in compounding at a lower rate of interest causing a decrease in the interest income earned by reinvesting coupon payments.

If an investor's bond matures at the end of the planning horizon, the par or face value of the bond is received. If time remains to maturity at the end of the planning horizon, the investor will need to sell the bond. Interest rate changes that occur after the bond was purchased cause price
appreciation or depreciation. This change is the capital or price effect.

If rates increase, investors can reinvest coupons at the new higher interest rate and accumulate interest earnings at a higher rate. The price effect works in the opposite direction. The discounted present value of a bond, its market price, falls when interest rates increase. If the interest rate increases between the time of purchase and the time the bond is resold, the rate of return falls because the price of the bond falls. Although an investor reinvests the cash flows from the bond at a higher rate, that benefit is offset by the declining market value of the asset.

The return an investor earns over the holding period will depend on both the rate earned from reinvesting coupons and the difference between the price paid for the bond and its selling price. Since the two effects work in opposite directions, researchers sought a method of measuring the responsiveness of bond prices to changes in interest rates to explicitly include both the coupon and reinvestment effect.

Duration is an alternative measure of interest rate risk that recognizes both the price and reinvestment cash flows that accrue to the bondholder over time. Coupon interest payments reinvested over time have a future value that increases directly with interest rates. This contrasts with
the inverse relationship between the price of a bond and interest rates.

Duration was developed by several researchers in the 1930s and 1940s in an attempt to find a better explanation for the relationship between changes in the interest rate and changes in the price of a bond. The time remaining to maturity which was used to describe bond price changes in response to interest rate changes considers only the influence of maturity on price. The cash flows a bond offers, coupon and principal payments, will also affect the discounted present value price when the interest rate changes.

Frederick Macaulay (1938) developed a mathematical measure of a bond's life as an alternative to the time remaining to maturity. Using the number of years remaining to maturity to explain observed changes in the prices of bonds does not consider the cash flows obtained by reinvesting coupon payments. Macaulay sought a measure to explicitly include cash flows from both coupon and principal payments.

Mathematically, the Macaulay duration is represented by the following:

\[
D_1 = \frac{\sum_{t=1}^{n} S_t \frac{t \cdot (1+i)^{-t}}{P_0}}{n}
\]

where:
The term to maturity for a bond is the time remaining until the final payment is made. Duration considers the final payment at maturity and also takes into account the coupon cash flows that occur prior to maturity. Duration is the weighted time to maturity for a bond where the weights are the present values of each coupon and principal payment.

Inclusion of coupon interest payments means the value of duration is less than the term to maturity. Zero coupon bonds that offer no payments other than principal at maturity have a duration equal to the term to maturity. Duration is the amount of time, weighted in terms of present value, to recover the cost of a bond.

Duration can also be characterized as an elasticity. Duration is the change in bond price for a change in the interest rate. Mathematically:

\[
D = \frac{dP_o/P_o}{di/i}
\]

\(P_o = \text{the price of the bond}\)
\(i = \text{the yield to maturity}\)

Bond price sensitivity depends on the size of the coupon rates and the time remaining to maturity. The duration measure recognizes the role of discounted cash flows in
determining the price of a bond. The numerical example calculated in Table 3.1 uses the Macaulay duration and indicates the computational procedure and provides insight into the duration equation. The bond in the example has a term to maturity of four years. Since the bond offers semiannual interest payments, there are eight six month periods remaining until the bond matures.

The first column lists the number of periods remaining to maturity. The second column lists the cash flows occurring in each time period. A coupon interest payment of $35 is received every six months. The final payment consists of an interest payment and payment of principal. The discount factor for each period is listed in column three. Column four is the discounted present value of each of the individual cash flows. In the last column the discounted present value is multiplied by the time period in which the cash flow occurs. The duration for the bond is 3.54 years. The duration or the weighted average life of the bond is less than the number of years remaining to maturity.

From the example it is apparent that the duration calculation in tabular form is time consuming. There are several alternative computational methods. Use of a computer program or spreadsheet model represents one possible method. Published tables exist for duration and are available from the Financial Publishing Company (1985).
Table 3.1: Duration calculation for a semiannual bond

<table>
<thead>
<tr>
<th>Period (t)</th>
<th>Cash flow (CF)</th>
<th>Interest factor 1/(1+i)^n</th>
<th>Present value of CF = PVCF</th>
<th>PVCF x t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$35</td>
<td>.9524</td>
<td>33.33</td>
<td>33.33</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
<td>.9070</td>
<td>31.75</td>
<td>63.50</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
<td>.8638</td>
<td>30.23</td>
<td>90.69</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>.8227</td>
<td>28.79</td>
<td>115.16</td>
</tr>
<tr>
<td>5</td>
<td>35</td>
<td>.7835</td>
<td>27.42</td>
<td>143.95</td>
</tr>
<tr>
<td>6</td>
<td>35</td>
<td>.7462</td>
<td>26.12</td>
<td>164.52</td>
</tr>
<tr>
<td>7</td>
<td>35</td>
<td>.7107</td>
<td>24.87</td>
<td>174.09</td>
</tr>
<tr>
<td>8</td>
<td>1035</td>
<td>.6768</td>
<td>700.48</td>
<td>5603.88</td>
</tr>
</tbody>
</table>

Bond price = 899.99

Duration = 6389.08/899.99 = 7.099/2 = 3.54
Macaulay also formulated a computational equation for duration that allows easier calculation of the measure the formula is:

\[ D = \frac{R - QR + T(1 + Q - QR)}{R - 1 - Q - QR} \]

- \( R = 1 + \frac{i}{m} \)
- \( i \) = the yield to maturity or interest rate
- \( m \) = the number of times compounding occurs per period
- \( T = m \times n \)
- \( n \) = the number of periods
- \( Q = \frac{V}{C/m} \)
- \( C \) = the size of the periodic coupon payment
- \( V \) = the face or maturity value of the bond

The last step in using this equation is to divide the final result by \( m \) in order to indicate duration in annual terms. Without this final step, duration is measured in the same units as \( m \). This formula can only be used if the periodic cash flows are equal over time. Unequal period payments requires discounting each individual cash flow separately.

It is unrealistic to assume that investors hold one bond. The management techniques discussed in the next section use one measure of duration but involve interest rate risk management for a portfolio of bonds. The duration for a portfolio of bonds is the weighted duration. The weights are the market values or the present value prices of the bonds. Mathematically:
Other researchers developed further analysis of bond price changes using duration. Hicks (1939) is credited with developing duration as an elasticity. Samuelson (1945) first applied duration to both sides of the balance sheet calculating the sensitivity of financial institution net worth to interest rate changes. Redington (1952) used the same set of equations to try and measure changes in the market value of insurance company assets and liabilities.

Duration has a long history but application of the technique and further theoretical development is limited to the last decade. Interest rate volatility in the 1970s prompted the rediscovery of the theory.

Implicit assumptions of Macaulay duration

As with any theoretical model, certain simplifying assumptions are made as the concepts are developed. These assumptions are:

1. Bonds are default and option free.
2. The objective of investors is return maximization at some chosen level of risk.
3. Changes in all interest rates are perfectly correlated.
4. The stochastic process is known.
Assumption four means that the specific form of the change in the movement of the yield curve is known. If actual realized forward rates in time \( t \) do not equal those anticipated at time \( t=0 \), the yield curve shifts. Macaulay duration assumes a parallel shift of a flat yield curve. A flat yield curve implies that there is no liquidity premium on one year rates in future time periods. Forward rates must be constant in each future time period in order to obtain a flat yield curve.

There are several ways in which the duration measure can be incorporated into a risk management program. Those alternatives are presented in the next section.

**Managing Interest Rate Risk**

**Naive duration strategy**

Hopewell and Kaufman (1973) relate changes in bond prices or price volatility to changes in yield to maturity and duration. Using the Macaulay equation and differentiating price with respect to the interest rate yields the following for relatively small changes in the interest rate:

\[
\frac{dP}{P} \approx - D \frac{di}{i}
\]

This equation is commonly rewritten in the literature as:

\[
\% \Delta P \approx - D \Delta i
\]

The inverse relationship between interest rates and bond prices holds. If rates fall, the sign on the interest rate is
negative and price appreciation results. Interest rate increases cause the price of a bond to decline.

The value of D, duration, can be interpreted as an index number or a factor of proportionality. For a given change in the interest rate, bonds with larger durations exhibit greater percentage changes in their price. An investor making decisions among alternative bonds could use this one measure, duration, to compare risks.

Since the percentage change in the price of the bond equals the negative value of duration times the rate change, a naive management strategy would be to select the bond with the smallest duration in order to minimize risk. A bond with a duration of 10 years compared to a bond with a duration of 5 years will exhibit a greater percentage change in price for a given interest rate change. If interest rates increase, the bond with a duration of ten years will fall in price by a greater amount than the five year bond. Selecting the bond with the smallest duration reduces price variability.

This strategy is a very simplistic specification of risk. Investors buy, hold and sell bonds to meet certain objectives. They may invest to meet future liability payments or have planned consumption goals. This means that investors have planning horizons. If they can buy and hold a zero coupon bond with a maturity equal to their horizon they will not face interest rate risk. Duration measures can be incorporated
with information on investor planning horizons to provide a range of alternative management strategies. These alternatives are discussed in the next section.

The duration measure also explains some fundamental bond price relationships. First, bonds with lower coupons exhibit greater price volatility. The higher the coupon interest payment, the smaller the value of duration. Intuitively, larger coupon payments decrease the present value weighted cash flow.

Second, duration explains the relationship between maturity and the price volatility of a bond. The longer the term to maturity, the higher the value of n in the duration equation. As maturity increases the present value of distant cash flows declines.

Some of the problems of duration as a measure of risk result from the Macaulay's assumptions. Bonds are assumed to be noncallable and default free. Relaxing these assumption requires reformulating the duration measure.

Second, the Macaulay duration is based on the assumption of small, one-time parallel shifts of a flat yield curve. The duration measure needs to be respecified if interest rate changes involve a different stochastic process.

In addition, the Macaulay duration measure assumes that the stochastic process of rate changes is known with certainty. If the stochastic process is incorrectly
identified, the model contains stochastic process risk. This would require inclusion of an error term in the equation for the expected return of a bond. A further discussion of these issues occurs in the review of empirical studies.

**Immunizing to manage interest rate risk**

Selecting the bond with the smallest duration fails to recognize that investors have a planning horizon or some time frame related to their need to meet liability outflows. Duration measures can be coupled with investor planning horizons to yield a hedged position free from interest rate risk.

Immunization is based on the effect of interest rate changes on the value of the two cash flows received from a bond. Over time the investor receives coupon interest payments which are reinvested at the interest rate currently prevailing in the economy. The investor reinvests the coupon payments until the end of the planning horizon is reached and the bond is sold. Higher interest rates result in the accumulation of higher reinvestment income.

If the bond matures at the end of the planning horizon, the price received will equal its face or maturity value. If, however, the bond is sold at the end of the planning horizon and has time remaining until it matures, the price the investor receives will depend on both the number of years to
maturity and any changes in the prevailing interest rate over the holding period.

If the current interest rates for bonds of similar risk are greater than the coupon interest rate the bond offers then the price of the bond is less that its face value or the bond sells at discount. If the interest rate prevailing in the economy, the yield to maturity, is less than the coupon rate the bond sells at a premium; its price is greater than the par or maturity value.

Because of the coupon or reinvestment effect and the capital or price effect, investors benefit regardless of the direction of interest rate change. However, the investor also suffers either a price or a reinvestment loss regardless of the direction of the rate change. Immunization is a technique that sets the dollar magnitude of losses exactly equal to the dollar magnitude of gains for any interest rate change. The objective is to attain a certain target rate of return even if unanticipated interest rate changes occur.

In Table 3.2, an investor selects a bond with a duration equal to the planning horizon. This sets the capital or price effect equal to the reinvestment effect regardless of changes in the interest rate. This is a passive strategy meaning that both the risk of loss and the chance to gain from unanticipated rate changes are eliminated. In this example
Table 3.2: Immunization for a four year planning horizon

<table>
<thead>
<tr>
<th>Years to maturity</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield to maturity</td>
<td>12%</td>
</tr>
<tr>
<td>Annual coupon rate</td>
<td>12%</td>
</tr>
<tr>
<td>Par value</td>
<td>$1,000</td>
</tr>
<tr>
<td>Bond price</td>
<td>1,000</td>
</tr>
<tr>
<td>Duration</td>
<td>4</td>
</tr>
</tbody>
</table>

Bond price and coupon values for a 200 basis point increase in interest rates occurring immediately after purchase.

Effect of reinvesting coupon payments

- Coupon future value with no rate change $573.52
- Coupons future value at 14% rate 590.53
  - Total added investment income 17.10

Price effect

- Bond price at t=4 with no rate change 1000.00
- Bond price at t=4 at 14% interest rate 982.46
  - Total decline in bond price (17.54)

Net cash flow effect at the end of year four 0.00
the investor purchases a bond at the time it is issued and holds it until the end of the planning horizon.

If the interest rates do not increase, coupon payments in the example are reinvested at a rate of twelve percent. An investor would accumulate $573.52 by saving or reinvesting the coupon payments over the planning horizon. A two percent interest rate increase allows reinvestment of the coupon payments at a higher rate of interest. The accumulated value at the end of the planning horizon is $17.10 higher after the rate increase.

The bond does not mature at the end of the planning horizon so the investor sells the bond at the end of the fourth year. The market price of the bond is equal to the par or face value if the interest rate is unchanged from that in effect on the date the bond was issued. If the interest rate increases, the discounted present value or market price of the bond declines. The price of the bond falls by $17.54 if interest rates increase by 200 basis points.

The illustrated strategy eliminates all interest rate risk. Selecting a bond with a duration equal to the planning horizon is equivalent to the purchase of a zero coupon bond that matures at the end of the planning horizon. On day one, the investor locks in the rate of return that will be realized at the end of the planning horizon. Regardless of the change
in interest rates, the coupon and price affects are offsetting.

**An active duration strategy**

One criticism of immunization is the assumption that the objective of the investor is to eliminate all risk. Active strategies recognize investor tradeoffs between risk and return. The duration measure can also be used to undertake active bond strategies. Selecting a bond with a duration greater than the planning horizon results in a capital effect that exceeds the reinvestment effect. If an interest rate decline were expected and the investor is willing to accept some interest rate risk, the appropriate strategy is to select a bond with a duration greater than the planning horizon. If a bond is selected with a duration less than the planning horizon the magnitude of the reinvestment effect exceeds that of the capital effect and rate increases result in net profits.

The mathematical formulation for the active bond strategy based on the formulation suggested by Bierwag, Kaufman and Toeys (1983a) recognizes that some investors are willing to accept greater risk to earn a higher expected return. Investors with expectations of future interest rate changes that differ from forecasts embodied in the initial yield curve select bonds with durations greater than or less than their
planning horizon, depending on the expected direction of the change in rates.

The expected return on a bond is given by Bierwag, Kaufman and Toevs as:

\( E(R_j) = a + R_{ph} + p \left( ph \cdot (R - R_{q, ph}) \right) + e \)

- \( a \) = extra market return \( a=0 \) if markets are efficient
- \( E(R_j) \) = expected rate of return on bond \( j \)
- \( ph \) = planning horizon
- \( R_{ph} \) = risk-free rate of return over the planning horizon
- \( R_{q, ph} \) = predicated rate of return on a reference bond with a duration equal to \( q \)
- \( p \) = proportionality factor that depends on the stochastic process
- \( e \) = error term \( e=0 \) if the stochastic process of the change in interest rates is correctly identified

Equation 3.12 is used by the authors to derive efficient risk-return frontier. Mathematically:

\( E(R_j) = R_{ph} + C \cdot \sigma(R_j) \)

- \( \sigma \) = the expected standard deviation of \( R_j \)
- \( C \) = a parameter related to the probabilities of different values of \( R_j \)

The greater the difference between the planning horizon and the duration of a bond, the greater \( E(R_j) \) and \( \sigma \). If interest rates are expected to increase then a risk taking investor would select a bond with a duration less than the planning horizon. The greater the difference between the value of duration and the planning horizon, the greater the magnitude of the dollar difference between the price and reinvestment effect.
If an interest rate decline is anticipated, then durations greater than the planning horizon are selected. The larger the difference between the value of duration and the planning horizon the greater the risk and possible return.

An investor's selection along a frontier traced out by the risk-return tradeoff will depend on both the certainty of the interest rate forecast and the investor's utility function. Risk averse investors will immunize against interest rate risk. Risk taking investors will accept more interest exposure if they are confident of their interest rate forecast.

Contingent immunization

The objective of contingent immunization is to earn a rate of return greater than the immunized yield while limiting the potential losses from incorrect forecasts of future interest rates. The technique was developed by Lebowitz and Weinberger (1982) and combines an active and an immunization strategy.

If the investor immunized the portfolio on day one, the rate earned would be the immunized return. Rather than lock in the immunized return at the beginning of the investment planning horizon, an investor selects the lowest acceptable return for the holding period. This minimum value represents the floor return.
Based on an interest rate forecast, the investor selects a bond with a duration different from the planning horizon. If an interest rate increase is expected the bond duration selected is greater than the planning horizon. If rates increase, the value of the reinvestment effect exceeds the price effect. The investor profits by earning a return higher than the immunized return possible on day one of the investment planning period.

Over time, the investor re-evaluates the potential realized return for the planning horizon. That return depends on the interest rate changes already occurring and those expected to occur in the time remaining until the end of the planning period. An unfavorable change in the interest rate causes a decline in the return that can be realized over the investment period. If it seems that the realized return could fall below the floor set at the onset of the planning horizon, an immunization strategy is triggered.

The investor makes a limited bet on the future direction of interest rate changes over the course of the planning horizon. The accepted loss is the difference between the immunized and floor return. The investor gains the opportunity to profit from the upside risk while limiting potential downside losses. The investor selects the acceptable amount of possible losses and risk based on individual preferences.
Dedicated bond portfolios

This strategy recognizes that investors purchase assets in order to fund future liabilities. The dedicated bond portfolio matches the duration of a liability with the duration of an asset. This equalizes the price sensitivity of both sides of the balance sheet for changes in the interest rate.

The concept of a dedicated bond portfolio can be illustrated in the model developed by M. S. Grove (1974) to illustrate the relationship between the duration of assets and liabilities and changes in the firm's net worth. His model of the balance sheet is as follows:

\begin{align*}
\text{(3.14)} & \quad A_0 = \sum_{t=1}^{\infty} \frac{C_t}{(1+i_0)^t} \\
& \quad A_0 = \text{the current value of the firm's assets in } t=0 \\
& \quad C_t = \text{the cash inflow in time } t \text{ from the firm's existing assets} \\
& \quad i_0 = \text{the current interest rate} \\
\text{(3.15)} & \quad L_0 = \sum_{t=1}^{\infty} \frac{E_t}{(1+i_0)^t} \\
& \quad L_0 = \text{current value of the firm's liabilities in } t=0 \\
& \quad E_t = \text{the cash outflow of payments in time } t \text{ from existing liabilities} \\
\text{(3.16)} & \quad V_{0L} = \sum_{t=1}^{\infty} C_t \\
& \quad V_{0L} = \text{the current value of the firm's equity in time } t=0 \\
& \quad \text{and;} \\
& \quad V_{0L} = A_0 - L_0
\end{align*}
For an instantaneous change in interest rates the change in \( V_{oL} \) with respect to the change in \( i_0 \) is as follows:

\[
(3.17) \quad \frac{dV_o}{di_0} = \sum_{t=1}^{\infty} \frac{(C_t - E_t)/(1+i_0)^t}{1-\frac{1}{(1+i_0)^t}} + 1
\]

Using the Macaulay duration equation, the asset and liability durations are defined as:

\[
(3.18) \quad D_A = \sum_{t=1}^{\infty} \frac{tC_t/(1+i_0)^t}{A_0}
\]

\[
(3.19) \quad D_L = \sum_{t=1}^{\infty} \frac{tE_t/(1+i_0)^t}{L_0}
\]

These definitions of duration are substituted back into equation 3.17 to yield the following:

\[
(3.22) \quad \frac{dV_o}{di_0} = \frac{1}{(1+i_0)} \left[ D_{L0} - D_{A0} \right]
\]

A change in the interest rate will not affect \( V_{oL} \), the value of the firm's equity if the following condition exists:

\[
(3.21) \quad \sum_{t=1}^{\infty} \frac{t(C_t - E_t)}{(1+i_0)^t} = 0
\]

or:

\[
(3.22) \quad D_L^L_0 = D_A^A_0
\]

If the duration of the assets equals the duration of liabilities, the change in net worth for a change in interest rates equals zero. The firm's balance sheet is immunized. Net worth will not decline (increase) if interest rates
increase (decline). An immunized position represents a risk averse strategy. Both losses and possible gains from interest rate changes are hedged.

**Specifying the stochastic process**

As previously defined, the stochastic process describes the way that the term structure of interest rates changes. Bierwag, Kaufman and Toevs (1983b) discuss alternative duration measures. The Macaulay measure of duration is based on the assumption of a one time shift of a flat yield curve. Other measures of duration can be formulated for other assumptions about term structure changes. The next section reviews empirical studies that suggest the appropriate measure of duration to use in risk management. In this section, a brief description of the alternatives is provided.

Unless interest rate changes for all rates in the term structure are perfectly correlated, then more than one factor determines the shape of the yield curve. Using the Macaulay measure of duration in an immunization strategy will not eliminate all interest rate risk under these circumstances. It is possible to describe different kinds of changes in the yield curve and formulate an alternative duration measure and immunizing strategy for each. The implicit assumption is that a better measure of the pattern of term structure changes will allow implementation of duration strategies with improved performance.
At the initial date or the beginning of the planning horizon the term structure of interest rates is represented as follows:

\[(3.23) \quad i_N(0, t), \quad t = 1, 2, 3 \ldots N\]

\[i_N = \text{the term structure of interest rates} \]
\[t=0 = \text{the initial date}\]

The forward or one period rates in future time period will not be the same as the current one year rate. Forward rates will change, but those forward rates are embodied in the current term structure of interest rates. The term structure of interest rates could shift randomly in which case the new term structure of interest rates is:

\[(3.24) \quad i_N'(0, t), \quad t = 1, 2, 3 \ldots N\]

If the changes in spot rates for all terms to maturity in the portfolio are perfectly correlated, the shifts in the term structure can be described by on factor. The factor, \(s\), is a random variable indicating the amount and of the term structure shift. Three alternatives are possible.

In the Macaulay duration, the term structure is assumed flat implying constant forward rates. One interest rate, the yield to maturity, represents the entire term structure of interest rates. The Macaulay duration as developed in equation 3.6 describes the responsiveness of bond price to changes in the yield to maturity.

If the term structure is not flat, the yield curve drawn at a point in time implies a series of different forward
rates. In an additive change, the term structure shift is represented by;
\[
(3.25) \quad i_{N}'(0,t) = i_N(0,t) + s
\]
The duration calculation when term structure changes occur due to an additive process is, \(D_a\) in the literature, is calculated by;
\[
(3.26) \quad D_a = \sum_{t=1}^{n} \frac{S_t}{P_0} \frac{t[1+i_N(0,t)]^{-t-1}}{P_0}
\]
where:
- \(S_t\) = the cash flows from the bond
- \(t\) = the time period in which the cash flow occurs
- \(P_0\) = the present value price of the bond

The Fisher-Weil duration measure, \(D_f\), is used if the yield curve shift is multiplicative. The relationship between the old and new term structures is;
\[
(3.27) \quad 1 + i_N(0,t) = s[1 + i_N(0,t)]
\]
The new duration measure is given by the following equation:
\[
(3.28) \quad D_f = \sum_{t=1}^{n} \frac{S_t}{P_0} \frac{t[1+i_N(0,t)]^{-t}}{P_0}
\]

Figure 3.4 illustrates interest rate changes for alternative stochastic processes. Figure 3.4a represents the Macaulay duration, 3.4b an additive stochastic process and 3.4c shows a multiplicative stochastic process.
\[ \frac{\Delta s}{\Delta t} = 0 \]

![Graph (a)](image)

For an upward sloping term structure:
\[ \frac{\Delta s}{\Delta t} > 0 \]

For a downward sloping term structure:
\[ \frac{\Delta s}{\Delta t} < 0 \]

![Graph (b)](image)

![Graph (c)](image)

Figure 3.4: Alternative stochastic processes
Immunizing a bond portfolio requires use of the correct duration measure and determining the correct duration measure requires information on the type of yield curve shift that could occur in the future. If the wrong assumption is made at time zero concerning the term structure shifts, the portfolio is exposed to interest rate risk due to a misspecification of the stochastic process.

Management techniques to reduce interest rate risk exposure are designed to replace decisions based on forecasts. The problem is that correct choice of a duration measure also requires a forecast or knowledge of the kind of shift in the yield curve that will occur. The parallel shift of a flat yield curve allows use of the Macaulay duration to measure exposure and implement risk management. Sloped yield curves with parallel or additive shifts require use of other duration measures.

A hypothetical situation illustrates the investor’s problem. At time t=0 an investor selects a bond with a duration equal to the planning horizon in order to immunize returns from interest rate changes. If the bond duration has been calculated based on the Macaulay duration, the investor will be exposed to interest rate risk if changes in the term structure of interest rates differ from the assumed one-time shift of a flat yield curve. Other kinds of yield curve
changes require use of an alternative duration measure in order to immunize.

After development of duration and hypothetical management techniques, research focused on measuring the performance of bond portfolios based on the use of alternative duration equations. The ultimate test of the various measures rests with their performance. Although it is realistic to assume that changes in the term structure may be more complex than Macaulay's specification, the real issue is how well the strategies perform relative to one another. The objective of the research studies reviewed in the next section is to address these issues.

Empirical research

There are two basic approaches utilized by researchers investigating the merits of duration as a risk management tool. One set of studies simulates portfolio results for alternative duration strategies based on estimates of the term structure of interest rates. The results from the simulation are compared to term structure strategies. Regression studies are conducted to examine the characteristics of changes in the term structure of interest rates. The purpose of these studies is to examine which duration measures capture the important factors involved in interest rate changes.

The duration of a zero coupon bond equals its maturity. If an investor has a known planning horizon, selecting a bond
with a maturity equal to the planning horizon eliminates interest rate risk as long as the zero coupon bond is held to maturity. The return earned over the holding period equals that implied in the discount on day one.

Using the duration measure allows the investor to select a portfolio of coupon bonds with a duration equal to the planning horizon. The objective is to match the characteristics of a zero coupon bond with a maturity equal to that of the investment period.

In the duration studies, investment strategies are simulated based on estimated term structures from different sources. Although the data, time periods and strategies considered differ among studies, it is possible to reach some general conclusions based on empirical results. If the Macaulay measure performs as well as more complex measures, then the problem of estimating future term structure changes is solved.

Each of the studies uses one of two sources for historical bond price data. The Durand series is available for prime corporate bonds. The Center for Research in Security Prices (CRSP) provides data for U.S. Treasury securities since the 1930s. These two kinds of bonds would meet the requirement of the Macaulay assumption that there is no default risk.
Fisher and Weil (1971) presented the first major simulation study using the Durand data for 1925-1968. They construct several hypothetical bond portfolios using the measure of duration, $D_m$, as previously defined.

The objective of their analysis is to compare holding period yields for portfolios with coupon bonds to the return that would have been earned with a buy and hold strategy for a zero coupon bond with a maturity equal to the planning horizon. At the beginning of the planning horizon a zero coupon bond has some promised yield, $r_p$, that will equal the realized holding period yield, $r_r$, over the planning horizon.

Two bonds are included in the immunizing portfolio. Planning horizons for 5, 10 and 20 years are considered. At the end of each year reinvestment occurs and a new portfolio is selected with a duration equal to the remaining time to the end of the planning horizon.

Immunization is compared to a maturity strategy in which the investor purchases a coupon bond with a maturity equal to the planning horizon. The results of the strategies are compared based on the performance measure of the difference between promised return at the beginning of the planning horizon and realized return at the end of the period.

Their results suggest that an immunization strategy outperforms a maturity strategy. Performance is the ability of the investor to lock in a holding period at the beginning of
the planning horizon and actually obtain that return over

time.

Bierwag, Kaufman, Toeys and Schweitzer (1981) conduct a
simulation extending the work of Fisher and Weil. Their
experiment uses the same data, the Durand series, and a
performance measure to evaluate risk management success.

Their study extends the analysis through 1978 and
includes additional duration measures. The Fisher-Weil
duration is tested along with the simple Macaulay duration and
measures that represent a multiplicative stochastic process.
The duration strategies are compared to a simple maturity
strategy, a rollover strategy and a long-bond strategy.

In the rollover strategy, the portfolio is constructed
entirely of one-period bonds. At the end of each year when
the bonds mature they are reinvested for the next one year
period. The long bond alternative involves selecting a 20-
year bond with a maturity that declines over the planning
horizon so 10 years remain to maturity at the end of the
planning horizon.

The authors reach several important conclusions based on
comparing results of the duration strategies with each other
and with the maturity strategies. The study indicates that
the Macaulay measure performs as well as more complex duration
measures. The duration strategies also outperform the
maturity strategies.
Using both the Macaulay and Fisher-Weil duration measures, Ingersoll (1983) constructs three alternative immunizing portfolios for a 5-year planning horizon using CRSP data. The three alternatives he considers are:

1. The bullet portfolio composed of two bonds. One bond has a duration slightly shorter than five years and the other bond has a duration slightly greater than five years.

2. The barbell portfolio also contains two bonds but those with the shortest and the longest duration are selected.

3. To construct the diversified ladder portfolio, Ingersoll minimizes $\sum w_i^2$ subject to the constraints $\sum w_i = 1$ and $\sum w_i D_i = 5$. The variable $w$ represents the weights of the various securities.

The nonduration strategies used for comparison include the simple maturity strategy initially used by Fisher and Weil and a strategy in which a 2-year bond is held with cash flow reinvestment in another 2-year bond every six months. This reinvestment strategy is also repeated for a 10-year bond.

The target yield to which the strategies are compared is a bond with a Macaulay duration equal to the planning horizon. The yields from each strategy are compared to their target yield based on the root mean squared differences (RMSD) calculated as follows:
The results suggest that a maturity strategy performs as well as a duration strategy. Both the duration and maturity strategies perform better than those in which a 2- or 10-year bond is reinvested every six months.

Although the results indicate no advantage for a duration strategy, Ingersoll's method of selecting bonds for the immunized portfolios may have been the problem. It is possible that the maturity portfolio has a duration approximately equal to that of the duration strategy. Another possible explanation rests with the use of bid/ask quotes rather than actual prices to estimate the term structure.

Bierwag, Kaufman and Toeves (1983a) repeat the simulations of their 1981 study using CRSP data for the period 1957-1974. They estimate the term structure using a method to account for the effect of income taxes on yields rather than the procedure Ingersoll used. The results of this study indicate duration strategies perform better than maturity strategies.

Brennan and Schwartz (1983) use CRSP data to estimate term structures and then compare immunization strategies using single and two-factor models. They also examine a five year planning horizon. They use a ladder method to construct their portfolios. A laddered portfolio consists of approximately

\[
(3.29) \quad \text{RMSD} = \sqrt{\frac{1}{N} \sum_{t=1}^{N} (r_t - \bar{r})^2}
\]

\[\text{N = the number of observations}\]
\[\text{r = the strategy yield}\]
\[\text{r_t = target yield}\]
equal proportions in the alternative maturity categories. The conclusions of the study suggest that using the single factor specification of the change in the term structure is appropriate.

Babbel's (1983) objective is to adjust duration for stochastic process risk. Using regression analysis he estimates a process of rate changes. Babbel uses a five year planning horizon and compares various duration strategies to each other and to a simple maturity strategy. He draws two conclusions. First, the maturity strategy does not perform as well as a duration strategy. Second, the less complex single-factor duration measure performs as well as those attempting to account for the stochastic process of term structure changes.

Nelson and Schaefer (1983) consider a multifactor process for changes in the term structure of interest rates in addition to one and two factor models. Their two factor model performed worse than the single factor Fisher and Weil model for the 10 year planning horizon. For the five year horizon, each of the single or multifactor models offered similar results.

Regressions studies attempt to explain the characteristics of the stochastic process underlying shifts in the yield curve or explain the actual relationships between the change in the price of a bond and the interest rate.
Figure 3.5 illustrates the relationship between the log of the bond price and the log of the interest rate. The Macaulay measure results in a linear relationship between bond price and interest rate changes when the actual relationship is represented by the curved line. Multiple factor models should trace out a line closer to the true relationship. The Macaulay measure will equal the true measure only at the point of tangency.

Although studies by Gultekin and Rogalski (1984), Brennan and Schwartz (1983) and Nelson and Schaefer (1983) suggest that the addition of more factors leads to a closer approximation of the true relationship, the general conclusion of the empirical studies is that the Macaulay duration performs reasonably well when used in an immunization strategy. More complex measures better explain the relationship between bond price and interest rate changes but do not significantly improve portfolio performance.

This leads to the important conclusion that the Macaulay duration with its relatively simple computational requirements can appropriately be used to formulate a risk management strategy. Estimations of term structure changes are not required and the problem of forecasting the future pattern of rate changes in order to implement a risk management program is solved.
Figure 3.5: Duration and bond price relationship
Additional issues

Duration requires complete information on the cash flows from assets and liabilities. Calculating the duration of a portfolio containing bonds with call provisions or equity requires data on expected cash flows. The key issue is the volume of information needed to calculate duration. This problem is solved by bond managers using the information and computational capacity of computers.

Another important aspect of cash flows that is not addressed by the duration literature is the assumption that bonds are default free. This does not describe securities other than the government and high grade corporate debt studied in the simulations.

Finally, the use of duration techniques to manage interest rate risk relies on cash market purchase and sale of financial instruments. There are limitations in the portfolio manager's ability to readjust the portfolio in the cash market. The financial futures markets offer an alternative to techniques that depend on cash market transactions.

The next section discusses the use of financial futures to manage interest rate risk. After an overview of futures markets, alternative hedging strategies are discussed. The section concludes with a review of the empirical studies of alternative hedging techniques.
Interest Rate Risk Management with Financial Futures

An overview of futures markets

A futures contract is an agreement to buy or sell a commodity or financial instrument on a specified future date. Futures contracts are standardized agreements with specifications set by the exchange on which the contract trades. The contracts are for a homogeneous, carefully described commodity or financial instrument with the time and method of delivery specified.

Futures market prices are determined in an auction market by open outcry. Market participants post a margin with gains and losses marked-to-market in the margin account on a daily basis. A clearing organization or clearinghouse records transactions and stands as a counter party for every trade. Traders in the market who buy and sell futures are insulated from default risk because the clearinghouse is the opposite for each trade.

Market participants include hedgers and speculators. Speculators trade in order to profit from price volatility. Speculators take a position in the futures market without holding a corresponding cash market position. The presence of speculators adds liquidity to the market.

Hedgers trade in the futures markets to offset an existing or anticipated cash market position. Changes in cash
market prices are hedged with price changes in the futures market that move in the opposite direction.

A market participant can enter the futures market as a buyer (long) or a seller (short). Market positions are liquidated by an offsetting transaction. The actual delivery of the commodity or financial instrument is usually not the objective of traders. Delivery is at the option of the trader and the existence of that opportunity links the prices in cash and futures markets.

For the hedger, the objective is to take a position in futures market that will result a price change in the opposite direction from the cash market price change. Brief examples illustrate the circumstances that would require both a long and short hedge.

A hedger enters the financial futures market selling contracts if an interest rate increase will result in cash market losses. For example, an investor planning to sell a bond in the future would be adversely affected by an interest rate increase because that interest rate change causes a decline in the market value of the security. A firm needing to borrow money in the future faces the possibility of higher borrowing costs if rates increase. The sale of a futures contract results in offsetting gains when the position is liquidated by a futures contract purchase. The inverse relationship for price and interest rates holds for futures
prices. If rates increase, the contract is sold for an amount less than it cost to purchase.

Long positions are used to hedge the situation in which an interest rate decline would result in cash market losses. For example, a pension fund manager planning the future cash market purchase of bonds will pay an increased price for the debt securities if rates fall. A long position in the futures market results in gains if interest rates fall since the position is liquidated by the sale of contracts at a price higher than the purchase price.

With a perfect hedge, the dollar amount of the cash market price change is offset exactly by the change in the value of the futures contract in the opposite direction. A perfect hedge usually is not possible because futures contracts have specifications and delivery dates that are not exactly the same as the cash market financial instrument or commodity.

Cross hedging is the use of a futures instrument with characteristics that do not exactly match the cash market instrument. The futures instrument chosen for the cross hedge should have a price pattern that correlates closely with that of the cash market instrument.

Traditional hedging theory developed by Working (1953) suggests the objective of hedgers is to minimize risk. Under
this assumption, the existence of a hedge provides insurance against adverse price movements in the cash market.

Traditional theory with its emphasis on risk aversion does not recognize that the objective of the firm may be better described by profit maximization. Working (1962) and Telser (1986) emphasize the use of hedging to minimize costs or as a low-cost alternative to a planned spot market transaction.

Portfolio hedging theory views futures market transactions within the framework of the hedger's holding of a portfolio of cash market securities. Powers and Vogel (1984) suggest that traders using this method recognize tradeoffs that exist between risk and return. Hedging decisions are made by comparing expected returns for a hedged and unhedged portfolio.

Traditional hedging theory implies cash market positions exactly offset with a futures position. With the portfolio hedge, decisions to hedge or not depend on expected returns. This suggests that positions in the cash market are not necessarily exactly offset. The cash portfolio may be only partially offset if the possibility of sufficient returns from an unhedged position exists. Hedging reduces risk at the cost of reduced returns.
Alternative Hedge Ratio Calculations

The hedger in the futures market has several steps to follow in order to institute the hedge. First, the hedger must decide on the objective of the hedge. Then the individual hedger must identify the asset or liability exposure being hedged. A financial intermediary or corporation with interest rate exposure in both assets and liabilities must examine the net balance sheet position to avoid an inappropriate macro position.

The next step is determining the number of contracts to buy or sell. The general procedure for calculating the number of contracts is to find $N$ where:

$$N = \frac{V \times HR}{F}$$

- $N$ = the number of contracts to buy or sell
- $V$ = dollar value of the cash market liability or asset being hedged
- $F$ = dollar value of the futures contract used to hedge
- $HR$ = hedge ratio

Several methods exist for calculating the value of $HR$, the hedge ratio. Hedge ratio calculations attempt to account for the variance between the prices of cash and futures instruments. The objective is to select the number of contracts that will match the changes in value of the cash and futures market instrument. The next section reviews methods of hedge ratio estimation.
Naive method

The underlying assumption in this method is that the cash instrument and the futures instrument have exactly the same characteristics and so respond to changes in interest rates on a dollar for dollar basis. Drabenstott and McDonley (1984) provide the equation for the naive hedge ratio:

\[(3.31) \quad HR = \frac{M_c}{M_f} \times C\]

HR = hedge ratio
\(M_c\) = maturity value of the cash market asset or liability being hedged
\(M_f\) = maturity value of the futures contract used to hedge
\(C\) = correlation coefficient of the cash and futures market prices

The number of contracts is then calculated by inserting the number obtained from equation 3.28 into 3.27. If \(M_c = M_f\) and \(C\) equals +1.0, then the number of contracts is determined by dividing the value of the cash market instrument by the dollar value of the futures market contract. The only calculation required is \(C\), the value of the correlation coefficient. This method ignores any differences in the contracts other than dollar values and maturities. Other factors should be considered when calculating the hedge ratio.

Conversion factors

The detailed specifications of the futures contract may differ from the characteristics of the cash market instrument. For example, the treasury bond contract traded on the Chicago
Board of Trade calls for delivery of a $100,000 bond with an 8% coupon. The price of a bond with a different size coupon would change by more or less than the 8% coupon bond for a change in interest rates. Conversion factors account for relative difference in price volatility when coupons differ from the contract specifications.

To calculate the conversion factor, the price of a bond is compared to the price at which it would yield eight percent based on its current term to maturity. If the cash market instrument has a coupon that differs from the 8% specified for the futures contract, changes in interest rates will result in unmatched cash and futures market price movement patterns.

**Basis point method**

The basis point model Schwartz, Hill and Schneeweis (1986) develop adjusts the hedge ratio for use by those undertaking a cross hedge. The model assumes the objective is to offset the change in the dollar value of the cash instrument with the change in the value of the futures contract. The calculation this hedge ratio is:

\[
(3.32) \quad HR = \left( \frac{BVC_s}{BVC_{cd}} / CF_{cd} \right) \times B
\]

- **HR** = hedge ratio
- **BVC_s** = dollar value change per basis point of the cash security to be hedged
- **BVC_cd** = dollar value change per basis point of the cheapest to deliver security
- **CF_cd** = conversion factor for the cheapest to deliver
- **B** = relative yield change volatility of cash to cheapest to deliver security
Price changes for the cash security are compared to the cheapest-to-deliver security is estimated by the relative basis point values. The basis point value represents the dollar change in a $100,000 security for a 1 basis point change in the interest rate. The change in the price of the futures contract is measured by $BVC_{cd}/CF_{cd}$.

The actual cash instrument being hedged and the cheapest-to-deliver security may respond differently to changes in interest rate. $B$ is an estimate of the yield volatility of the cash compared to the cheapest-to-deliver security obtained by regression.

**Minimum variance hedge ratio estimation**

**Regression method** Based on modern portfolio theory, the regression hedge ratio offers explicit consideration of both risk and return. To examine this alternative model the mathematical concepts are developed following the framework of Figlewski (1986). The application of the portfolio technique was originally developed by Edington (1979) and his empirical tests of the theory are discussed in the next section.

At the beginning of the holding period the price of the cash asset is $P_0$. The price or value of that security at the end of the holding period is denoted by $P_1$. $P_1$ is uncertain at $t = 0$. The mean of the variable $P_1$ is denoted by $\bar{P}_1$ and the standard deviation of $P_1 - P_0$ equals $\sigma_0$. 
The price of futures contracts at the beginning of the holding period is denoted by $F_0$. $F_1$ represents the ending price of the futures contract which is not known at $t = 0$. $\bar{F}_1$ is the mean of $F_1$ and $\sigma_f$ the standard deviation of the difference between the price at the beginning and end of the holding period.

Assume a hedge against increasing interest rates, a short hedge with $h$ representing the number of futures contracts required per unit of cash position in order to hedge. The variable $C_{\pi}$ represents the correlation coefficient between cash and futures prices.

The profit from the position is represented mathematically by:

$$\pi = (P_1 - P_0) - h(F_1 - F_0) \tag{3.33}$$

with the expected profit given as:

$$E(\pi) = (\bar{P}_1 - P_0) - h(\bar{F}_1 - F_0) \tag{3.34}$$

The variance and standard deviation of profits are:

$$\sigma^2_{\pi} = \sigma_P^2 + h^2\sigma_f^2 - 2 C_{\pi} h \sigma_P \sigma_f \tag{3.35}$$

$$\sigma_{\pi} = \sqrt{\sigma_{P}^2 + h^2\sigma_{f}^2 - 2 C_{\pi} h \sigma_{P} \sigma_{f}} \tag{3.36}$$

Various possible combinations of risk and expected return are generated by inserting various values of $f_c$ into the equations for expected return and standard deviation.

Taking the derivative of $\sigma_{\pi}$ with respect to $h$ and setting it equal to zero yields the value of $f_c$ that minimizes risk.
The value of $h^*$ represents the optimal hedge ratio. Mathematically the result is as follows:

$$h^* = \frac{\text{cov} (P_1 - P_0, F_1 - F_0)}{\text{Var} (F_1 - F_0)}$$

$h^*$ = the risk minimizing number of contract units

or alternatively:

$$h^* = \frac{C_{fc} \sigma_p}{\sigma_f}$$

Substituting equation 3.38 into equation 3.36 yields the minimum risk hedge portfolio. Mathematically:

$$\sigma_{\pi} (\text{min}) = \sigma_p \sqrt{1 - \frac{C_{fc}^2}{\sigma_f^2}}$$

Using the risk-minimizing hedge does not completely eliminate risk unless the value of the correlation coefficient equals +1.0 or -1.0. The higher the correlation of the futures contract price with the cash market price, the greater the risk reduction.

The computational procedure for estimating the number of futures contracts needed to result in the risk minimizing hedge involves estimation via use of linear regression. The data requirements include both the past price changes of the cash and the futures market instrument. Period to period price changes are entered for the relevant historical time period. Figlewski suggests use of data for the year and a
half prior to the hedge period. Month-to-month price changes are used in estimating an equation of the form:

$$P = a + bF$$  

$$P = \text{past change in price of cash market instrument being hedged}$$  

$$F = \text{past change in price of futures market instrument yields}$$  

$$b = \frac{\text{cov}(P,F)}{\text{var}(f)}$$  

$$b = \text{the hedge ratio to minimize the standard deviation}$$

In applying regression analysis to hedging additional statistics generated by commonly used statistical packages provide further information on hedging effectiveness. The value of $R^2$ gives the fraction of the cash position variance eliminated by the hedge. Mathematically:

$$R^2 = \frac{C_{fc}^2}{(\text{cov}(P,F))^2/(\text{var}(P))(\text{var}(F))}$$

In addition, the standard error of the regression generated from the statistical package indicates the difference between the regression line and $P$, the past price changes of the cash market instrument being hedged. In a futures hedge, the standard deviation of the regression residuals represent the random price changes in the cash position not offset by the futures hedge.

In addition to the information content of the statistics generated along with the hedge ratio, this method offers a
link to modern portfolio theory in which both risk and return are recognized and their tradeoffs play a role in investor decisions.

It is important to note the assumptions of the model. First, the futures contract selected for the regression should be that which has the highest correlation with the cash security. Second, use of OLS regression is appropriate only if the relationship between the change in futures and the change in cash prices is linear. Finally, the historical data used in this model should have variance and covariance characteristics consistent with current and expected conditions.

Duration method Kolb and Chiang (1981) applied duration theory to hedge ratio estimation. Duration is assumed to better describe the relationship between the change in price of a cash and futures market security. The hedge ratio is derived as follows:

\[
HR = \frac{\Delta P}{\Delta F}
\]

where:

HR = hedge ratio
\(\Delta P\) = the change in the value of the bond to be hedged
\(\Delta F\) = the change in the value of the futures contract

The change in the dollar value of both the cash and futures instruments is given by the following duration equations:
(3.44) \( \Delta P = -D_C P \Delta i_C \)

(3.45) \( \Delta F = -D_f F \Delta i_f \)

with:

- \( D_C \) = the duration of the cash instrument
- \( P \) = the price of the asset expected on the planned termination date
- \( \Delta i_C \) = the change in yields for the cash market instrument
- \( D_f \) = the duration of the futures instrument
- \( F \) = the price of the contract on the planned termination date
- \( \Delta i_f \) = the change in yield on the instrument underlying the futures contract

Durations for futures contracts are computed for the delivery date of the contract. The duration of the futures contract is calculated based on the characteristics of the underlying contract. For example, the treasury bill futures contract specifies delivery of a bill with 90 days to maturity. Since this is a zero coupon discount security the duration of the futures contract equals its maturity.

An implicit assumption underlying the model is that the futures contract and the asset being hedged are both subject to the same stochastic process risk. The model assumes proportional changes in yields for the cash instrument and the cheapest-to-deliver bond underlying the futures contract.

It is also important to note that futures price changes are marked to market with daily settlement. Futures losses involve cash payments while the cash instrument price changes are not realized until the bond is sold.
Basis risk

A futures market hedge cannot eliminate all the risk from a cash market position. Basis risk has an effect on hedging effectiveness and will reduce the value of the minimum risk hedge ratio. The basis is the difference between the cash and futures market yields. Hedging exchanges absolute price risk for basis risk. Basis risk still remains unless the hedge ratio calculation method completely accounts for all the factors that influence the difference between cash and futures prices. The size of the risk minimizing hedge ratio is reduced if basis risk exists.

Designing a hedging strategy requires recognition of the influence of basis risk. Basis risk will be more important if the cash security being hedged has a different credit rating or maturity than the futures market, if the cash security is not trading in a liquid market or if the hedge period is less than a week.

The naive and conversion factor methods of calculating hedge ratios implicitly assume that the cash and futures market response to changes in the interest rate are perfectly correlated. If the characteristics of the cash market security match that of the underlying instrument of the futures contract then this is a realistic assumption. The minimum risk hedge ratios account for differences between the cash and futures market instruments by analyzing the factors
that influence relative price changes. Naive hedging strategies may appear successful if the conditions for a perfect hedge exist. A minimum risk hedge ratio should be used if cross hedging is required or if all the conditions of a perfect hedge are not met.

**Empirical Analysis of Hedging**

**Performance of the naive hedge ratio**

Draper and Hoag (1983) test five alternative strategies for hedging a portfolio of bonds. They develop a simulation based on CRSP data for spot prices. Both futures and options are considered. The study covers the period from 1960-1979 during which futures and options were not traded on exchanges. The authors simulate a price series for the futures contracts for time periods before they were actually traded on the futures exchanges.

The study tests the following alternatives:

1. long fixed-income bonds
2. long call options and bond futures
3. long call options and short futures
4. long bonds and short futures
5. long bills and short or long bill futures

The authors use a naive hedge ratio rather than the portfolio or duration method. Hedges are placed for a period of six months. The futures strategies use maturities of 6-,
12-, 18- and 24-months. Three alternative option strike prices are estimated and used.

The statistical performance measures used to determine hedging results are the mean, standard deviation, minimum and maximum returns. Hedging in this study improved the return and reduced risk when cash market bond prices were declining.

**Tests of the regression method**

Edington (1979) is credited with developing the minimum risk hedge ratio or portfolio hedging model. His research examined the hedging performance of the Government National Mortgage Association Collateralized Deposit Receipt (GNMA CDR) and the treasury bill contract used to hedge cash market positions. The Edington portfolio contains no riskless asset. Cash and futures positions are not substitutes. Cash market decisions are made and then the investor decides what portion of the risk to hedge.

The performance measure is the percent reduction in variance given by the following equation:

\[
(3.46) \quad e = 1 - \frac{\text{Var}(H^*)}{\text{Var}(U)}
\]

\(e = \text{variance} \quad \text{Var}(H^*) = \text{minimum variance hedged portfolio} \quad \text{Var}(U) = \text{variance from unhedged position}\)

Reviewing both two- and four-week hedges, Edington estimates the minimum variance hedge ratio for the two contracts. The resulting effect on variance indicates that the size of the risk minimizing hedge ratio and the
performance of the hedge varied according to the length of time the position was hedged and maturity date of the contract used.

The results represent a significant modification of traditional hedging theory in which the hedge ratio was determined by the total dollar cash position rather than the pattern of price movements between cash and futures market instruments. The objective of this technique is to select the number of contracts needed to offset price variability in the cash market with futures price changes.

Franckle (1980) uses Edington's data to reexamine the effectiveness of the minimum risk hedge ratio. Franckle recognized that as the hedge position is held over two or four weeks, the days remaining to maturity of the cash market instrument decline. This maturity effect in turn influences the cash market price because the discounted present value is affected by the number of days remaining to maturity. At the time the hedge is lifted, the cash instrument has 76 days to maturity for a two-week hedge or 62 days for the four week hedge. The futures instrument has a constant maturity of 90 days based on the contract specifications of T-bill futures.

The fewer the days remaining to maturity, the lower the price volatility of the discount bond. The issue of price volatility requires an adjustment to the hedge ratio. The maturity effect influences the covariances of the cash and
futures instrument. This affects both the value of \( e \), the variance, and the minimum risk hedge ratio.

After adjusting the hedge ratio for the maturity and price effects, Franckle examines the performance measure, \( e \). The results indicate an increased effectiveness of two-week hedges. This research provided an explanation for Edington's results indicating that the two-week T-bill hedges were ineffective.

**Tests of the duration method**

Price variability is also the focus of the duration method. In an effort to better explain the relationship between changes in cash and futures prices, duration theory was applied to hedge ratio analysis. Empirical studies of this method examine the effectiveness of hedges placed to manage the risk exposure of fixed-income portfolios. The results of duration hedges are compared to both naive hedge ratios and the minimum risk hedge ratio.

The studies take two approaches. One category of studies involves placing a hedge using historical data and comparing results between alternative methods or unhedged positions. Other studies simulate an interest rate environment and review performance for alternative strategies.

Kolb and Chiang (1981) introduce the duration hedging strategy. The model offers a better method of determining the number of contracts needed to hedge interest rate risk. Their
The key issue is that the objective of a hedge is to offset price sensitivity of a cash market instrument with a futures market contract. Duration offers a measure of price sensitivity that incorporates the influences of maturity, coupons and interest rate changes on the prices of securities.

The study reviews a hypothetical T-bill and T-bond hedge. Determining the duration for a discount security is simplified because no coupon payments are received. In the case of T-bonds which make coupon payments, conversion factors must be applied in order to create equivalency between cash and futures instruments. Using examples of a bill and bond hedge, the authors compare their results to a naive strategy. Their results indicate that smaller losses or errors occur with the duration strategy.

Although the study does not offer a rigorous statistical test of several alternatives, it is important to note that this technique seeks to minimize the variance of the hedge over its entire life. The duration method represents a modification of the portfolio strategy and maintains the same objective of explaining cash and futures price changes. Instead of using regression analysis to explain the relationship between cash and futures prices, the duration measure is used to model relative price sensitivity for interest rate changes.
Gay, Kolb, and Chiang (1983) construct a sample of 250 bonds selected at random from the Wall Street Journal quotations on the final trading day of 1978. Pairs of dates are selected at random for 1979-1980 with the first date representing the day the hedge is placed and the second day the date the hedge is closed.

The hedger plans on purchasing a bond with $1 million at the end of the hedging period. The authors compare a naive hedging strategy in which the number of contracts is calculated based only on the dollar amount of the cash market investment. The second naive strategy uses the market price rather than the face value of the bond. The final hedge ratio is calculated using duration theory. All of the hedging strategies are compared to an ex post perfect hedge.

The duration strategy performs better than the two naive hedges. With duration, 68.69 percent of wealth is hedged. The authors suggest three possible problems preventing a perfect hedge with the duration strategy: errors due to change in the basis, the length of the hedging period and mismatches between maturity and cash flow characteristics for the cash market instrument and the futures contract.

In an attempt to explain the problems, the authors regress the dollar error on the change in basis over the life of the hedge. The hedge length issue is analyzed with the same methodology as is the coupon and maturity problem.
Results of the regression indicate that 75 percent of the error is explained by the four variables.

Gay, Kolb and Chiang conclude that the duration strategy represents an effective method to estimate the hedge ratio to cross hedge corporate bonds using T-bond futures.

Landers, Stoffels, and Seifert (1985) examine cross hedging using the duration strategy. The authors select 110 industrial and utility bonds for the period 1978-1981. The sample consists of grade B or better securities. Twelve portfolios of 100 bonds of each of ten different issues are constructed. Five of the portfolios consist of a random sample of various bonds. The remaining portfolios are structured to meet specified yield, maturity or quality characteristics.

Two different planning horizons, a two- and four-year period, are studied. Investors could hedge either the beginning value or the ending value of the portfolio. Hedge ratios based on duration are rounded to an integer to model realistic management decisions. The coupon payments are reinvested.

This study presents significant results. The duration hedge prevents 80 percent of the loss in value suffered by unhedged portfolios. The duration hedging strategy is not a perfect hedge but it eliminates a significant proportion of the risk. The authors suggest that the quality ratings of
some of the bonds selected might explain the results. Since duration is based on the assumption of default free securities and default risk affects the required rate of return, this seems a reasonable explanation.

Chance (1986) estimates the term structure of interest rates and constructs a portfolio of a single government bond and a futures contract using the duration method. Monthly revision occurs with cash flow reinvestment and futures positions marked to market. Bond price data are obtained from the CRSP tapes.

Various holding periods are considered. They range from a series of one year periods to a five year period. Analysis of hedging performance is based on several statistics. The target return along with the mean, maximum and minimum of realized return are calculated. The mean difference between the target and realized return, the standard deviation and the t-statistic are also presented.

The overall performance for the hedged compared to the unhedged position indicates that the immunized portfolios yield smaller variation from target returns. The results are consistent for various planning horizons and subsamples of the data.

Toevs and Jacob (1986) compare the regression-based portfolio strategy to a duration strategy. The study covers the time period from 1982-1984. The minimum variance hedge
ratio is estimated using Edington's methodology. The purpose of the study is to determine which method of hedge ratio estimation provides the best explanation of the relationship between changes in cash and futures prices. Both are minimum variance hedge ratios, but the duration strategy considers factors in addition to past price changes when determining price sensitivity.

The study identifies four kinds of hedges and clearly specifies the objective of the hedger. The possible types of hedges include:

1. The weak form cash hedge minimizes price variance for an asset portfolio that will be held for an indefinite time period. Interest rate increases reduce the price of the asset holdings, so short hedges are used.

2. The strong form cash hedge involves an investor with a known planning horizon. Immunization is the strategy used to minimize portfolio variance.

3. The strong form anticipatory hedge involves the receipt of cash at a known time period that will be invested in a financial instrument. The objective of this hedge is to minimize the variance of the acquisition cost so a long hedge is undertaken.

4. The weak form anticipatory hedge occurs when an asset will be purchased on some unspecified date.
The objective is to achieve a minimum variance rate of return for a specific holding period.

A simple historical hedge is constructed in which a trader has a $10 million cash position in U.S. Treasury bonds paying a 12.75 percent coupon and maturing on November 15, 2010. Five methods of constructing a hedge are compared and contrasted along with two variations. The major techniques are dollar value matching, conversion factor method, regression method using actual prices, regression using price level changes and a duration method. Daily hedges are constructed from June 24, 1982 to January 1, 1984.

In order to undertake variance analysis, the authors compute the hedged and unhedged portfolio returns for each day. Next they average the two series using a ten-day moving. Then they compute the variances of the ten-day average returns over the entire period. The final step is to find the percentage of the variance of the unhedged positions reduced by the futures position. The study also calculates 30-day moving averages.

Using a futures contract with similar cash flow characteristics results in a 96 percent reduction in the variance for the 30-day moving average and a 93 percent reduction for the 10-day average. The regression strategies perform as well or better than the duration strategy. The naive strategies also perform well in this simple example.
All the strategies perform well because the cash and futures market instruments are closely related. It is possible to obtain a nearly perfect hedge because bond and futures contract have very similar characteristics. The superior performance of the duration hedge ratio is demonstrated in the examples where dissimilar cash and futures instruments are involved. The authors analyze maturity mismatch and quality mismatch examples.

Table 3.3 gives the results from the study. In each instance the duration strategy outperforms the alternatives. This study is important because it clarifies the relationship between the minimum risk hedge ratio calculated using regression and that developed using duration measures. Since duration includes additional factors that influence the price pattern of cash and futures, it offers greater reduction of risk exposure than the regression method that considers historical price relationships.

The general conclusions of the empirical studies have important consequences for interest rate risk management. Research suggests that the duration method provides better results than naive or regression hedge ratio estimates. As part of a program of portfolio management, use of the futures markets can provide a flexible opportunity for interest rate risk management. The tests indicate the practical application of duration theory. Although duration does not provide an
Table 3.3: Results of Toevs and Jacob's Study

<table>
<thead>
<tr>
<th></th>
<th>10-day Moving average</th>
<th>30-day Moving average</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cash security: U.S. Treasury 13% of 1990</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Dollar Matching</strong></td>
<td>44%</td>
<td>68%</td>
</tr>
<tr>
<td><strong>Conversion Factor</strong></td>
<td>50</td>
<td>72</td>
</tr>
<tr>
<td><strong>Change in Price Regression</strong></td>
<td>78</td>
<td>84</td>
</tr>
<tr>
<td><strong>Price Level Regression</strong></td>
<td>76</td>
<td>86</td>
</tr>
<tr>
<td><strong>Duration</strong></td>
<td>80</td>
<td>87</td>
</tr>
<tr>
<td><strong>Duration</strong></td>
<td>77</td>
<td>88</td>
</tr>
<tr>
<td><strong>Curvilinear Price Regression</strong></td>
<td>79</td>
<td>89</td>
</tr>
</tbody>
</table>

| **Cash security: GMAC 8's of 2007** |                       |                       |
| **Dollar Matching**   | 78%                   | 89%                   |
| **Conversion Factor** | 75                    | 80                    |
| **Change in Price Regression** | 75               | 80                    |
| **Price Level Regression** | 75               | 86                    |
| **Duration**          | 78                    | 88                    |
| **Duration**          | 78                    | 89                    |
| **Curvilinear Price Regression** | 75               | 86                    |

| **Cash security: Tenneco 8 3/8s of 2002** |                       |                       |
| **Dollar Matching**   | 64%                   | 65%                   |
| **Conversion Factor** | 64                    | 65                    |
| **Change in Price Regression** | 64               | 69                    |
| **Price Level Regression** | 64               | 69                    |
| **Duration**          | 64                    | 70                    |
| **Duration**          | 64                    | 70                    |
| **Curvilinear Price Regression** | 64               | 70                    |
Financial Intermediary Interest Rate Risk

Financial intermediaries are exposed to interest rate risk if the maturities of assets and liabilities are mismatched. Interest rate risk affects both the market value of assets and liabilities and the spread between the cost of funds and the earned rate of return. In the past, financial institutions financed asset acquisition with the use of low-cost deposit funds. In the early 1960s banks had profitable loan opportunities but insufficient funds to make the investments. Banks solved the shortage of funds by purchasing money on the money market. The funds were used to finance profitable loans and meet liquidity needs, but the cost of money market funds was subject to the forces of supply and demand.

Maturity mismatch existed because short-term money market funds were used to finance longer-term loans and investments. If liabilities are repriced at market rates more often than assets, the interest spread can narrow or may even become negative. In the volatile rate environment, theoretical interest rate risk measurement and management techniques were applied to bank balance sheet management.
This section discusses the alternatives available to financial intermediaries for interest rate risk analysis. Gap management, duration matching and financial futures are the alternative methods used by commercial banks to minimize the negative affect of adverse interest rate changes. The bank can still provide maturity intermediation while minimizing the risks inherent in that activity.

Commercial banks are examined prior to developing alternatives for analysis of Federal Intermediate Credit Banks. FICBs engage in restricted interest rate risk management. The method used is variable rate loan pricing. Although interest rate risk can be managed by passing the risk on to borrowers, FICBs could better serve borrower-owners and decrease possible default risk by adopting the use of an institution hedging method.

Commercial banks owned by stockholders take calculated interest rate risks in order to leverage their earnings. Federal Intermediate Credit Banks owned by the borrowers would seem to have an even greater incentive to manage interest rate risk at the institution level to limit the risk exposure of their borrowers. The risks inherent in the undiversified loan portfolio of FICBs suggests other bank risks; interest rate, liquidity and credit risk should be minimized.

The next section describes the objectives of asset/liability management and is followed by development of
the theoretical measurement and management of interest rate risk for financial intermediaries.

Asset/Liability Management and Interest Rate Risk

Interest rate risk management is an important objective of asset/liability management. In order to maximize return and minimize risk as measured by the variance of profits, the institution should coordinate liquidity, interest margin and capital management.

Ideally, an institution could change the proportion of various asset and liability categories or adjust the maturities and pricing terms as dictated by evolving market conditions. With complete flexibility to restructure the balance sheet, the institution could manage its risk exposure even without perfect knowledge about future interest rate conditions.

The bank's flexibility is restricted for several reasons. The bank does not have complete control over the proportions of various balance sheet categories. The size of the loan portfolio depends on current demand and previous loan commitments. The bank can influence demand with pricing terms, but recognizes the need to service customers throughout the business cycle.

The deposit accounts on the liability side of the balance sheet are another example of a balance sheet category that is only partially controllable. The bank's depositors respond to
economic conditions and pricing terms. Prices and maturities depend on competitors' policies and general economic conditions.

The bank limits its flexibility with decisions on prices and maturities in asset and liability categories. Writing a 20-year fixed-rate mortgage locks the institution into a contractual interest rate over the life of the loan. If the bank funds the loan with liabilities repriced more frequently, then the institution is exposed to interest rate risk.

Customers, market conditions and bank management decisions are not the only factors limiting balance sheet flexibility. Commercial bank activities are also restricted by bank regulation. Although deregulation in the 1980s eliminated interest rate ceilings and allowed banks to offer additional products, regulators still examine institutions reviewing their financial condition and management practices.

Managing and Measuring Interest Rate Risk

Loan pricing terms

Variable rate loan pricing is one method of managing interest rate risk. Both consumer and business loans are priced on this basis. Although this management technique apparently solves the problem of interest rate risk, further analysis indicates potential problems. The issues are the borrower's ability to repay the loan, the method and frequency
of price adjustment, and refinancing or loan repayment that occurs when interest rates decline.

First, consider the interaction between interest rate and default risk. Variable rate lending reduces the interest exposure of the financial institution by passing the risk on to the borrowers. The ability of the borrower to manage interest rate risk determines if the bank merely trades one risk for another. Unless the borrower can hedge the risk or has cash flows sufficient to meet unanticipated rate changes, credit risk increases.

The financial institution may share the interest rate risk with borrowers in several ways. If the institution finances the loan portfolio with money market funds, management may decide to reduce the variability of money costs through the use of a futures hedging program. Borrowers have the option to institute their own hedging program, but may not have the expertise or margin requirements needed to institute a hedging program.

The frequency and rate of adjustment also influences the extent to which interest rate risk is shared between a borrower and lender. The loan may be repriced monthly, quarterly, annually or at other intervals. If assets and liabilities are not repriced on the same date, the institution faces interest rate risk during the interval between the change in liability costs and change in the loan price.
Using an interest rate cap limits the magnitude of potential interest rate changes. The bank then shares the risk of rate increases with borrowers. Negative amortization may be used in conjunction with interest rate caps or if loan payments adjust less frequently than the actual funding costs. The unpaid monthly interest expense is added to the loan balance. This practice solves the interest rate risk problem but may increase default risk.

Finally, when borrowers convert from variable to fixed rates as interest rates decline, the institution is exposed to potential risk if rates subsequently increase. Although the bank may be financing the loan with lower cost funds, maturity mismatch could result in losses when interest rates return to higher levels.

The extent of rate exposure must be quantified in order to use available management techniques. The calculation of the gap ratio or dollar amount of gap offers one measure of risk.

**Traditional gap management**

The traditional gap management model measures the differences between the dollar amounts of assets and liabilities sensitive to interest rate changes. Calculation of either a dollar gap or a gap ratio is made and used to measure the exposure of the bank to interest rate risk.
In order to calculate a gap the bank must determine the relevant time period to consider. If for example, that period is one year, then those assets or liabilities which are repriced within that time period would be categorized as rate sensitive. The bank might decide to calculate the gap for various time periods beginning with asset and liability categories that will be repriced within the next 30 days.

The bank needs to decide if it will actively manage gap to improve the interest spread or try to preserve the current margin. If the bank decides to pursue the active strategy, an interest rate forecast is required. Different types of yield curves require different gap management strategies.

The equation for calculating the dollar gap is:

\[ (3.47) \quad \text{Gap} = \text{RSA} - \text{RSL} \]

\[ \text{Gap} = \text{dollar gap} \]
\[ \text{RSA} = \text{dollar amount of rate sensitive assets for the time period under consideration} \]
\[ \text{RSL} = \text{dollar amount of rate sensitive liabilities for the time period under consideration} \]

If the dollar gap equals zero then rate sensitive assets equal rate sensitive liabilities. This means that regardless of the interest rate changes within the time frame considered, the bank is insulated from risk. If the dollar gap is positive, then rate sensitive assets exceed rate sensitive liabilities. With an upward sloping yield curve, this strategy has a positive affect on the interest spread because some variable rate assets have been financed with fixed rate
liabilities. If the yield curve slopes downward, this strategy results in a deteriorating spread.

When rate sensitive liabilities exceed rate sensitive assets the dollar gap is negative. A negative gap is a profitable strategy if the yield curve slopes down because some of the fixed rate assets are being financed with liabilities with declining costs. An upward sloping yield curve with a negative gap results in a declining interest margin.

An alternative calculation is the gap ratio. The gap ratio is calculated as follows:

\[(3.48) \quad \text{Gap ratio} = \frac{RSA}{RSL}\]

\(RSA\) = dollar amount or proportion of rate sensitive to fixed rate assets

\(RSL\) = dollar amount or proportion of rate sensitive to fixed rate liabilities

If the gap ratio equals one, rate sensitive assets exactly equal rate sensitive liabilities. This means fixed rate assets are financed with fixed rate liabilities and the variable rate categories are repriced at the same time. A gap ratio greater than one corresponds to the positive gap discussed previously. A gap ratio less than one corresponds to the discussion of negative gap.

The typical gap position of a commercial bank is fewer rate sensitive assets than liabilities or a negative gap. The size of the gap depends on the bank's reliance on purchased money and the type of lending the institution specializes in.
Savings and loan association funding home mortgages with short term money market funds faced financial problems during periods of increasing interest rates. Interest rate risk eroded the capital position of the institutions. Federal Intermediate Credit Banks have a positive gap position with greater rate sensitivity in assets than in liabilities. These institutions would face capital erosion in periods of declining interest rates.

Figure 3.6a illustrates traditional gap management. The shaded area represents the positive gap. This means the bank finances some rate sensitive assets with fixed rate liabilities. If interests rates increase over the gapping period under analysis, interest income increases as the yields from rate sensitive assets increase. A positive gap results in losses when interest rates fall.

Gap management can be active or passive. Passive gap management occurs if the dollar gap is zero or the gap ratio equals one. When interest rates change in either direction, assets and liability cash flows change in the same proportion. If the bank has a reliable interest rate forecast then it might undertake an active strategy setting up a positive gap if rate increases are expected or a negative gap if rate declines are anticipated.

There are limitations to gap management. The flexibility to adjust the balance sheet with cash market transactions is
Figure 3.6a: Positive gap

Figure 3.6b: Negative gap
restricted. Customer maturity preferences affect loan terms and the size of deposit balances. Regulations represent additional constraints on asset and liability activities. Some of the assets do not trade in liquid secondary markets representing an additional limitation. Further problems exist.

Problems with gap management

There are several problems with the use of gaps to measure and manage interest rate risk. First, assets and liabilities are classified as rate sensitive regardless of when they are repriced during the period of time selected for the gap computation. Toevs (1983) gives the example of a bank selecting one year as the gap period. If assets are repriced on day 30 and liabilities on day 360, the gap ratio would equal one or there would be a zero dollar gap even though the institution would be exposed to risk between day 30 and day 360.

One solution to this problem is the selection a shorter time periods for analysis within the initial chosen time frame of a year. Thirty day, ninety day or other time periods could be selected. These periods are known as maturity buckets. The remaining problem is that even the selection of a thirty day bucket for analysis can cause distortion of the actual exposure to interest rate risk.
Another problem is that this measure of risk implicitly assumes that interest rate changes for assets and liabilities are of the same magnitude. The literature deals with the problem by suggesting that the volatility of the interest rates considered vary in constant proportion to some benchmark rate. Baker (1978) develops the gap model by considering the calculation of proportionality constants for various account categories.

In spite of improvement to the gap management model, problems remain. The focus of gap management is the net interest margin or the spread. If the objective of the bank firm is to maximize shareholder wealth, the appropriate management variable is the value of equity. The gap model does not consider the change in the market value of assets and liabilities that occur when market interest rates change.

Adjusting the gap with the use of futures

Limitations exist if the firm attempts to hedge interest rate risk by equating rate sensitive assets and rate sensitive liabilities over the relevant time period. Cash market responses to gap may not be a feasible alternative since opportunities to change the maturity or pricing terms of a loan may be limited. Long term, low cost funds may not be available. Futures markets offer an alternative method for gap management.
A futures hedge can be utilized to maintain the margin between the cost of funds and asset rates of return. Table 3.4 illustrates a bank balance sheet with negative gap for the next 90 days. The dollar gap equals $5 million. With this gap position, the institution funds part of fixed rate assets with variable rate liabilities over the next 90 days. If interest rates increase, the net interest margin deteriorates. The appropriate hedging strategy would be to enter the futures market selling short. The specific contract and number of contracts to use should be determined by one of the methods reviewed in the theoretical section on futures.

If interest rates increase the institution faces increases in the cost of liability funds. This increase in costs is offset by the gains in the futures market occurring when the position is closed with the purchase of contracts. As interest rates increase the value of the futures contract falls meaning the sale on day one was for a price greater than that paid to subsequently close out the position.

Hedging the gap requires identifying the net exposure. The use of futures represents part of overall asset/liability management activities. Futures allow the bank to alter the maturity structure of the balance sheet without undertaking a cash market transaction.

Interest rate risk management must be linked to overall liquidity, capital, credit and marketing decisions. Use of
Table 3.4: Example of a balance sheet gap

<table>
<thead>
<tr>
<th>Rate sensitive over</th>
<th>Assets</th>
<th>Liabilities</th>
<th>Dollar Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>90 day period</td>
<td>$10,000,000</td>
<td>$15,000,000</td>
<td>$5,000,000</td>
</tr>
<tr>
<td>Fixed Rate</td>
<td>12,000,000</td>
<td>7,000,000</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$22,000,000</td>
<td>$22,000,000</td>
<td></td>
</tr>
</tbody>
</table>
futures requires careful monitoring and expert management. Although futures offer institutions balance sheet flexibility, their use by the industry has been limited. The accounting treatment of hedging activity offers a possible explanation of the reluctance to use this risk management tool.

Accounting issues

Goodman and Langer (1983) discuss the problems created by the accounting treatment of futures activity at commercial banks. With the deferral method of accounting the gains and losses from a futures hedge are realized at the time the position is closed. Regulators' guidelines allow banks to carry the futures contract at the lower of cost to market or marked-to-market.

Since daily settlement occurs in the futures market gains and losses are credited and debited to trading accounts each day. Since banks are presumably using the futures market to hedge, the changes in the margin account would be offset by changes in cash market values.

Marking to the lower of cost or market results in monthly accounting statements that include losses but not offsetting gains. Gains are included in the statements only when they are actually realized. Accounting earnings will demonstrate increased variability if the cash account being hedged is not marked-to-market at the same time as the futures account.
An example illustrates the problem that exists even if the bank marks-to-market. Assume a bank makes a one year fixed rate loan and finances it with a six-month certificate of deposit. If interest rates increase, the institution pays a higher cost of funds in six months and the spread between costs and returns deteriorate. If interest rates fall, then the institution gains in the liability cash market due to the lower cost of funds.

Now assume the institution decides to hedge the risk using futures markets. If rates increase over the next six months, the bank realizes a gain from the futures position. The offsetting loss for the loan will be realized in months six through twelve.

If rates decline losses are realized prior to the offsetting cash market gain introducing initial accounting losses in earning. Although the hedge is successful in an economic sense because gains and losses are offsetting, the accounting treatment of the hedge makes its use unattractive to management.

Deferral accounting might indicate that an institution was successful by realizing gains on a futures position when, in economic terms, the banks had increased its interest exposure by using the futures markets and was earning speculative profits.
Resolution of the accounting issues may lead to an increased use of futures to hedge interest rate risk. In addition, institutions will develop an understanding and expertise in the market.

Duration Gap Analysis

Limitations of traditional gap analysis and the ability of duration measures to explain changes in the value of financial assets and liabilities led to modifications of the method of measuring institution interest rate risk. The duration gap method explicitly considers the effect of interest rate changes on the market value of the financial intermediary. This section reviews the basic theoretical models.

An example of a simple duration gap

A simple model of a financial intermediary is presented in Table 3.5. The institution is modeled with only one asset and one liability. The duration of assets exceeds the duration of liabilities. An initial position of a zero net worth approximates the highly levered position of financial intermediaries. Changes in the net worth of the firm are calculated for a 100 basis point, a one percent, increase in the interest rate.

The first step is to calculate the new dollar value of the asset side of the balance sheet after the interest rate change. This is analogous to calculating the new price of a
Table 3.5: A simple duration gap

<table>
<thead>
<tr>
<th>Portfolio value</th>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1000</td>
<td>$1000</td>
<td></td>
</tr>
<tr>
<td>Interest rate</td>
<td>10%</td>
<td>8%</td>
</tr>
<tr>
<td>Duration</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

Change in net worth for a 100 basis point increase in interest rates.

Change in the market value of assets

\[ \Delta P_A = -D_A \Delta i_A P_A = -5 \times 0.01 \times 1000 = -5,000 \]

\( \Delta P_A = \) change in the market value of assets

\( D_A = \) duration of assets

\( \Delta i_A = \) change in asset interest rate

\( P_A = \) the initial dollar value of the asset portfolio

Change in the market value of liabilities

\[ \Delta P_L = -D_L \Delta i_L P_L = -2 \times 0.01 \times 1000 = -2,000 \]

\( \Delta P_L = \) change in the market of liabilities

\( D_L = \) duration of liabilities

\( \Delta i_L = \) change in liability interest rate

\( P_L = \) initial dollar value of the liability portfolio

Change in institution net worth

\[ \Delta NW = \Delta A - \Delta L = -5,000 -(-2,000) = -3,000 \]
bond after an interest rate change. The value of assets in the example declines by $5,000 for a one percent increase in the interest rate.

The next step is to calculate the dollar change in the liability side of the balance sheet. Financial liabilities will also decline in value when the interest rate increases. In this example, the liabilities decline in value by $2,000.

Since net worth equals assets minus liabilities, the effect of the interest rate change on capital is calculated by subtracting the change in the value of liabilities from the change in the value of assets. Net worth in the example declines by $3,000. The new net worth of the firm has declined from zero to a negative value. A 100 basis point decrease in the interest rate would have had the opposite effect. The institution is immunized if the duration of assets equals the duration of liabilities.

This example illustrates the fundamental concepts of balance sheet immunization. If the institution is unwilling to accept any interest rate risk, then the balance sheet is structured so that changes in the market value of the assets are exactly offset by changes in the market value of liabilities for any interest rate change. A passive strategy represents the extreme risk averse position. Bierwag and Kaufman (1985) suggest that the institution use the duration gap measure to map risk-return frontiers. Modelling
alternative interest rate changes and various duration gap 
values provides the institution with an opportunity set for 
interest rate risk management. The specific position selected 
would depend on management risk preferences.

If the institution management is confident of their 
forecast of future interest rates, the duration gap offers the 
opportunity to profit from interest rate changes. If the 
duration of assets exceeds the duration of liabilities, 
declining interest rates results in additions to net worth. 
If rate increases were anticipated, the institution willing to 
accept interest rate risk would attempt to set the duration of 
liabilities greater than the duration of assets in order to 
achieve increases in net worth.

The acceptable level of interest rate exposure is 
dependent on the confidence management places in the interest 
rate forecast, the risk preferences of the institution and the 
overall risk position of the firm. The ability to manage 
duration gaps will also be limited by the controllability of 
various balance sheet categories. Loan demand or contractual 
terms may limit short term adjustments.

The Toevs model of duration gaps

Toevis (1983) develops of a model an institution's balance 
sheet in which gap is defined as the difference between the 
duration of rate sensitive assets and liabilities. His model 
provides a measure of institution risk. With a specific
measure of risk, cash or futures market hedging techniques can be used to manage exposure.

The objective of this model is to find a combination of assets and liabilities that balances changes in interest income and interest expense. To eliminate interest rate risk and obtain the target earnings regardless of rate changes over the relevant time period the weighted market value of assets is set equal to liabilities. Mathematically:

$\sum_{j=1}^{n} MVA_j (1-t_j) = \sum_{k=1}^{m} MVL_k (1-t_k)$

The definitions of the variables are as follows:

- $MVA_j$ = the market value of asset that will be repriced during the year
- $MVL_k$ = the market value of the liability that will be repriced during the year
- $t_j, t_k$ = the fraction of the year until this asset or liability is repriced or is first repriced if repricing occurs more than once a year
- $n, m$ = the number of rate sensitive asset or liability payments

The model is based on the same assumptions that (1) the relevant period for the gap model is one year (2) the yield curve is flat (3) the magnitude of the change in asset returns and liability costs is equal (4) no deposit withdrawals or loan prepayments occur and (5) the term structure of interest rates is defined by the unbiased expectations theory.

An item by item match of asset and liability categories is not required. The calculation utilized by management is
the value of the duration gap. The duration gap, based on the
Macaulay duration, is calculated as follows:

\[
(3.50) \quad DG = \text{MVRSA}(1-D_{rsa}) - \text{MVRSL}(1-D_{rsl})
\]

where:

\[DG = \text{the duration gap}\]
\[\text{MVRSA} = \text{the market value of the rate sensitive asset}\]

and:

\[\text{MVRSL} = \text{the market value of the rate sensitive liability}\]
\[D_{rsa} = \text{duration of rate sensitive assets}\]
\[D_{rsl} = \text{duration of rate sensitive liabilities}\]

\[
(3.51) \quad D_{rsl} = \sum_{j=1}^{n} \left( \frac{\text{MVA}_j}{\text{MVRSA}} \right) t_j
\]

\[
(3.52) \quad D_{rsl} = \sum_{k=1}^{m} \left( \frac{\text{MVL}_k}{\text{MVRSL}} \right) t_k
\]

Earnings are immunized if the duration gap equals zero. If the duration gap is greater than zero, the situation is similar to a positive gap in the conventional literature. If interest rates fall realized earnings are less than was anticipated at the beginning of the year. If the duration gap is less than zero a negative gap exists.

This measure can be used to completely hedge interest rate risk or undertake an active strategy. If the duration gap is less than zero then earnings are hedged by the addition of \$X in market value to the net rate sensitive assets with a duration \(Y\) where:
If the duration gap is greater than zero $X$ in market value is added to the value of net rate sensitive liabilities based on the equation above. The institution can alter $X$ in market value of assets or liabilities using federal funds purchases or sales. Other cash market actions might involve changing loan terms or adjusting the investment portfolio.

Long or short positions in the futures market can add flexibility to balance sheet management by replacing or supplementing cash market balance sheet changes. A futures hedging strategy using the duration gap is a macro strategy that hedges the entire bank and not individual assets or liabilities.

Kaufman model of duration gap

Kaufman (1984) approaches the issue of financial intermediary interest rate risk from a different perspective. He concludes that the institution needs to target a particular duration measure for immunization depending on the objectives of management.

The target accounts are capital, the capital to asset ratio and the ratio of net income to total assets. The strict focus on capital represents the perspective of regulators. Shareholders are concerned with the capital to asset ratio and
management focus is on return on assets. Each of the target accounts has a duration gap calculation that represents modification of the measures previously discussed.

The first measure of duration gap uses total capital as the target account. The duration gap for capital is:

\[
\text{GAP}_k = D_A - wD_L
\]

where:

- \( D_A \) = the duration of assets
- \( \text{GAP}_k \) = the duration gap of the capital account
- \( D_L \) = the duration of liabilities

The value of \( w \) is a weight calculated as follows:

\[
w = \frac{L}{L+K} = \frac{L}{A}
\]

\( L \) = total liabilities
\( K \) = total capital
\( A \) = total assets

This measure is the focus of regulators because a negative net worth indicates a condition of insolvency. If the firm continues to operate, cash flows generated from asset returns may not be sufficient to meet repayment obligations. Regulators will need to pay off depositors or arrange for acquisition of the failed bank by another firm in the case of insolvency.

The capital to asset ratio represents another possible target account. The \( \text{GAP}_k \) duration measures only the total dollar change in net worth or capital. A more appropriate measure is the change in capital relative to total assets. The firm may experience large changes in net worth, assets and
liabilities without a change in overall risk exposure. If the
firm is growing or contracting in size the GAP_k measure
duration will not accurately reflect the exposure of the firm.
The GAP_k/a measure solves the problem. Mathematically:

\[(3.56) \quad GAP_{k/a} = D_A - D_L\]

where:

- GAP_{k/a} = the capital to asset duration gap
- D_A = the duration of assets
- D_L = the duration of liabilities

Duration also influences an institution's income. The
third measure of duration standardizes net income in terms of
total assets and represents a net return on asset measure.

This duration calculation is made as follows:

\[(3.57) \quad GAP_{Ni/A} = w_A D_A - w_L D_L - K/NI\]

- GAP_{Ni/A} = net income gap measure
- K = capital
- NI = net income

The values of W_A and w are determined as follows:

\[(3.58) \quad W_A \approx \frac{i_A}{A} - 1\]

- i_A = the asset interest rate
- A = total assets
- NI = net income

\[(3.59) \quad W_L \approx \frac{i_L}{L} - 1\]

- i_L = the liability interest rate
- L = total liabilities
- NI = net income
An example best illustrates the effect on assets, liabilities and ratios when alternative target accounts are immunized. The Tables 3.6 and 3.7 were generated using a duration gap simulation, DGAME, developed by Kaufman (1985). A general trend of falling interest rates was selected from the menu. Six random interest rates were generated by the program with the output indicating changes in asset and liability accounts and selected ratios for each of the interest rate changes.

The gap measure, GAP(NI/A) was initially selected as the immunizing account. Next the GAP(K/A) was selected as the immunizing target account and changes in assets, liabilities and ratios were generated with the new immunizing target under the same pattern of interest rate changes. The results from that alternative are presented in Table 3.7.

The choice of the immunizing duration determines which of the balance sheet categories is targeted for reduced variability. With the net income to assets gap measure, the income cash flows and the net income to asset ratio remain the same for the simulated changes in interest rates. Using the net income measure for duration gap results in changes in the amount of capital. In this example falling interest rates resulted in dollar increases in the capital account.

Selecting the measure of duration gap that immunizes the capital to asset ratio reduces variability in the capital
Table 3.6: Immunizing an institution with GAP(NI/K)\(^a\)

<table>
<thead>
<tr>
<th>TIME</th>
<th>RATE CHANGE</th>
<th>CASH</th>
<th>BUSINESS LOANS</th>
<th>HOME LOANS</th>
<th>TOTAL ASSETS</th>
<th>1-YEAR CDs</th>
<th>5-YEAR CDs</th>
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<tr>
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<td>$100</td>
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<td>100</td>
<td>399</td>
<td>491</td>
<td>990</td>
<td>322</td>
<td>571</td>
</tr>
<tr>
<td>2</td>
<td>-25</td>
<td>100</td>
<td>400</td>
<td>500</td>
<td>1000</td>
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<td>1020</td>
<td>324</td>
<td>592</td>
</tr>
<tr>
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<td>-50</td>
<td>100</td>
<td>405</td>
<td>537</td>
<td>1042</td>
<td>326</td>
<td>607</td>
</tr>
</tbody>
</table>

\(^a\)Kaufman (1985).
<table>
<thead>
<tr>
<th>CAPITAL</th>
<th>TOTAL LIABILITIES</th>
<th>CAPITAL ASSETS</th>
<th>NET INCOME ASSETS</th>
<th>GAP (K)</th>
<th>GAP (K/A)</th>
<th>GAP (NI/K)</th>
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</thead>
<tbody>
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<td>0.99</td>
<td>0.63</td>
<td>0.76</td>
</tr>
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<td>0.99</td>
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<td>1.80</td>
<td>1.15</td>
<td>0.77</td>
<td>1.57</td>
</tr>
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</table>
Table 3.7: Immunizing an institution with GAP(K/A)$^a$

<table>
<thead>
<tr>
<th>TIME</th>
<th>RATE CHANGE</th>
<th>CASH</th>
<th>BUSINESS LOANS</th>
<th>HOME LOANS</th>
<th>TOTAL ASSETS</th>
<th>1-YEAR CDs</th>
<th>5-YEAR CDs</th>
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<td>990</td>
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<td>680</td>
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<td>400</td>
<td>500</td>
<td>1000</td>
<td>211</td>
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<td>706</td>
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</table>

<table>
<thead>
<tr>
<th>CAPITAL</th>
<th>TOTAL LIABILITIES</th>
<th>CAPITAL ASSETS</th>
<th>NET INCOME ASSETS</th>
<th>GAP (K)</th>
<th>GAP (K/A)</th>
<th>GAP (NI/K)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1.75</td>
<td>0.69</td>
<td>0.29</td>
<td>-0.82</td>
</tr>
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</table>
account and results in changes in net income. The simulated institution maintains a target capital to asset ratio while the net income to asset ratio varies.

Once the target duration gap is selected, the other gap measures change in response to the asset/liability management activities designed to immunize the balance sheet. The choice of a target account would depend on the objectives of management. Although one account is selected, earnings, capital and interest rate risk as calculated from the other gap measures should be monitored.

Use of futures in duration gap management

Adjusting loan maturities and terms to match durations using the futures markets represents an alternative to cash market transactions. If the futures market is used, the duration gap must be measured and the type of exposure examined.

Negative duration gaps indicate that interest rate declines will cause a decline in the value of the selected target account. A positive duration gap results in losses if interest rates decline. Once the duration gap is measured a future hedge ratio must be estimated using one of the previously discussed techniques.

For example, if an institution uses the GAP$_{n/A}$ duration with the objective of immunization, this implies that the value for duration gap equals zero. Depending on the size and
sign of the gap, a long or short futures position can achieve immunization. Mathematically:

\[
\text{GAP}_\text{NI/A} = D_A - D_L \pm D_{FP}
\]

\( D_{FP} = \) the duration of the futures position

If the gap equals zero the institution is immunized. As with traditional gap analysis, both cash and futures market hedging possibilities exist. In order to hedge the risk, it must first be quantified. Although the concept of duration gap is theoretically appealing there are some practical limitations to its use.

**Practical problems of duration gap analysis**

The data requirements needed to calculate duration are strenuous. Complete information on expected cash flows along with repricing dates and terms is required. This problem will be solved in the future as the computerized data for financial institutions is made available through changes in currently used software. Other problems remain.

Actual maturities may differ from contractual maturities due to prepayment, due-on-sale clauses or default. In addition, commercial banks have deposit liabilities with no specific maturity date. As banks begin using duration, statistical analysis of these issues will be needed to provide estimates of actual and expected account behavior.

In spite of its limitations, the duration gap measure is a significant improvement over traditionally calculated gaps.
The affect of interest rate changes on the net worth of the firm has important consequences. If book values are used instead of market values the extent of interest rate risk is significantly understated.

Empirical Studies of Futures Hedges

The objective of financial intermediary futures market activity is to hedge either specific asset or liability rates and cash flows or the entire balance sheet. Hedging a specific asset or liability account involves a micro hedge. If overall interest rate exposure is the target, a macro hedge is placed.

Mortgage banking

Bookstaber (1982) examines the use of futures to hedge loan commitments. The loan commitment represents an option contract. At origination, the mortgage banker commits to a price or rate. At the sale of the loan, the bonds are priced according to the prevailing interest rate. If the origination fails or interest rates change, the mortgage banker can decline to deliver bonds. In that case, the loan commitment premium is the only penalty.

The objective of the study is to consider futures and option strategies and compare the resulting trade off between risk and return from hedging activity. The goal of the hedging program is to affect the return distribution by altering the variance or insuring downside risk.
This study employs the portfolio hedge ratio which Edington (1979) applied to futures markets. Bookstabler considers the return distribution when the number of contracts is varied from the value of the minimum variance hedge ratio.

Using a logarithmic utility function, Bookstabler reviews hedging performance. He calculates expected utility for various parameter of risk aversion. His conclusions suggests that the futures strategy yields greater expected utility for a risk averse mortgage banker when compared to the unhedged position.

Kolb, Corgel and Chiang (1982) use duration to calculate a minimum risk hedge ratio. Their objective is to better estimate the relationship between changes in the future and cash instrument. Their general methodology is to determine the price sensitivity of the cash and future to changes in the risk free rate of interest. The goal of the hedge is to equalize price changes in the cash and futures instruments and is represented by the equation:

\[(3.61) \Delta P_C + \Delta P_F(N) = 0\]

\[\Delta P_C = \text{the change in value of the cash instrument}\]
\[\Delta P_F = \text{change in value of the futures instrument}\]
\[N = \text{the number of contracts}\]

The authors solve for \(N\) using the following equation:

\[(3.62) \frac{\delta P_C}{\delta R_i} + \frac{\delta P_F}{\delta R_i} (N) = 0\]

\[R_i = 1 + \text{the risk free rate of interest}\]
Using the Macaulay duration yields the value of $N$:

\[
(3.63) \quad N = \frac{R_c P_c D_c \times \delta R_f / \delta R_i}{R_f P_f D_f \times \delta R_c / \delta R_i}
\]

The study uses monthly data for the cash instrument, commercial mortgage commitments, and the futures contract, GNMA CDRs. The duration strategy is tested against a strategy in which the hedge ratio is calculated by dividing the dollar size of the cash position by the dollar size of the future contract.

Two different assumption concerning the interest rates for the cash and futures instrument are tested. First, the authors assume the futures and cash yields are risk free. Next they estimate a value for the second expression in equation 3.63 above.

The relationship of the cash and futures yields to the risk free rate is estimated with OLS regression. The results of the regression are adjusted for serial correlation.

Numerical examples indicated that the duration strategy results in a lower dollar value of error compared to a naive strategy. The study does not consider the regression hedge ratio nor does it examine more than one hedge for one hypothetical rate change.
Hedging the gap and cost of funds

Koppenhaver (1984) develops a model of a representative banking firm to simulate the effect of a treasury bill futures hedging strategy. The objective of the hedge is to maximize the expected utility of bank profits at the end of a three month planning horizon. Loans are default free, so interest rate risk from a negative gap position is the only source of risk.

This study covers the period from 1976-1981. Since hedges are placed for a three month period, the study has 23 separate hedge periods. Koppenhaver uses the traditional gap measure to institute a macro hedge. The objective of the macro hedge is to use a measure of overall balance sheet interest rate risk to determine the number of contracts to purchase or sell.

A micro hedging strategy to lock in the value of T-bills is also considered. The objective of the micro hedge is to lock in the cost of one liability account. This risk is measured by the variability in the selected account rather than by calculating overall exposure using one of the gap measures.

Two alternative interest rate forecasts are modeled: a perfect forecast using the actual rate at the end of the hedge period and a forecast based on futures prices observed at the beginning of the hedge period.
The dollar value of the gap is hedged with a dollar equivalent hedge ratio. The dollar equivalent hedge ratio is obtained by dividing the dollar amount of the cash instrument by the dollar size of the futures instrument. This is a dollar-for-dollar match of the cash and futures instrument rather than a match based on relative price sensitivities. The variance of unhedged profits for the institution is reduced by 80 percent. The study does not consider the duration gap measure nor are the regression hedge or duration hedge ratio analysis included.

Parker and Diagler (1981) examine a hedge of money market certificates of deposit costs employing the treasury bill futures contract. The gap measure is the dollar difference between fixed-rate loans and money market certificates of deposit. The hypothetical institution has a negative gap. The authors use a dollar equivalent hedge ratio.

An adjustment is required in the number of contracts to reflect the differing maturities of the cash and futures market instrument. Money market certificates of deposit have a six-month maturity and the treasury bill futures contract is based on an underlying security with a 90 day maturity. Dividing the number of days to maturity for the cash instrument by the measure for the futures contract, $180/90$, provides the adjustment. The hedge ratio is then multiplied by the adjustment.
The authors did not statistically evaluate the performance of the hedge. Their analysis does indicate that the use of the futures market locks in the cost of funds over the hedging period. Alternative hedge ratios are not considered.

Maness and Senchack (1986) utilize a continuous hedging program for a savings and loan institution for January 1979 to January 1983. The time period is divided into two subperiods. From January 1979 to April 1981 rates were generally increasing. October 1981 to January 1983 was a period of declining rates.

The study also demonstrates the effect of the hedging program on earnings when each day's gains and losses are marked-to-market. Quarterly earnings when the deferral method of accounting is used form the basis of their comparison. The review of the effect on accounting statements provides information on reported performance for comparison to the economic results of the hedge.

The savings and loan is modelled so that the loan rate for adjustable rate mortgages is hedged. The loans are funded with money market certificates of deposit issued each week with rates tied to the weekly cash yield at the treasury bill auction. The objective of the hedge is to minimize the variance and reduce the effective cost of short term borrowing. The institution shares interest rate risk with
borrowers. The hedging program is an attempt to manage the variance of costs and subsequent loan rates rather than pass all interest rate changes on to borrowers.

The study provides an important comparison of the naive and a minimum variance hedge ratio methodology. The optimal hedge ratio method utilized is the regression technique. The study uses a micro rather than a macro hedge. The authors decided to set a reduced variance of cost as the hedging objective based on the assumption that fixed-rate loans would have a maturity longer than the two year maximum maturity of contracts trading in the futures market.

Adjustable rate mortgages are the only asset and are funded solely with money market certificates of deposit. The study implicitly assumes that mortgage rates are adjusted each month. Other loan repricing terms and their effect on the performance of the hedging strategy are not considered.

The results of the study indicate that a continuous hedge program reduces the variance of cost compared to the unhedged position. The short hedge is successful even when weekly money market rates are declining during the second subperiod in the study. This means that the hedge locks in the cost of funds. When rates are declining the overall cost of funds is higher due to losses in the futures market. The optimal hedge ratio methodology compared to the naive strategy produced a lower variance of cost but had a higher net average cost.
The authors also review the effect of both the deferral and the marked-to-market method of accounting on the reported financial condition of the institution. The net dollar interest expenses reported on the quarterly income statement is graphed for both methods of accounting. Although the hedges are successful in an economic sense, marking gains and losses to market produces more variability in reported earnings.

Bookstabler (1986) examines two methods employed by institutions to manage balance sheet interest rate risk: the use of adjustable rate mortgages compared to fixed-rate lending hedged in the futures market. Semi-annual loan repricing on the adjustable rate mortgages converts the loans to a stack of shorter-term instruments. Contract terms including the index used, the frequency of repricing and caps restrict the adjustment of the loan price and create mismatch of assets and liabilities.

The simulation used in the study begins with yield curve estimation. The adjustable rate mortgage contract allows a two percent adjustment per year and is capped at five percent above the initial rate. Repricing occurs semi-annually based on the treasury bill rates. Three alternative mark-ups to the treasury bill rate are modeled: 150, 200 and 250 basis points.
Bookstabler initially prices the adjustable rate mortgage and the fixed rate loans so a borrower would be indifferent between the two. The objective of the author is to equate the prices of the two alternatives at the start of the simulation period.

The prepayment risk on the fixed rate loan is an option that the borrower holds but its exercise depends not just on interest rates but also demographic factors and refinancing transactions costs. The simulation includes the possibility of prepayment for the fixed rate loan contract.

The institution hedges the duration gap. A liability duration of six months and three years is incorporated into the simulation. The focus is on the comparing the risks of the two alternative asset hedging strategies.

Using the assumption of a six month liability duration, the caps and ceilings on the adjustable rate mortgage fail to track rate changes when interest rates are volatile. More intermediate-term exposure represented by the three year duration assumption is hedged effectively using adjustable rate mortgages. The futures hedge of a conventional fixed-rate mortgage better tracked the interest rate change with the shorter liability duration. The authors contend that adjustable rate mortgages are not necessarily the best tool to reduce interest rate risk. When rates are rapidly changing,
the lag between changes in the cost of funds and loan repricing results in interest rate exposure.

The conclusion of the studies is that futures represent an effective hedging tool on either a micro or a macro level.

Empirical Studies of Duration Gap

There are few empirical studies of the use of duration to manage financial intermediary interest rate exposure. Although the theoretical literature suggests the advantages of the technique, data requirements and computation complexity have resulted in limited theoretical application. The following studies simplify the balance sheet structure of the institution in order to calculate duration gaps and interest exposure.

This first study develops a duration model of a rural commercial bank using accounting data obtained from the institution. The second study considers duration gaps for savings and loan institutions using data from Section H reports prepared for the Federal Home Loan Bank Board. The empirical studies represent approximations of actual institution balance sheets but provide insight in the overall interest exposure at the banks modeled.

Diz and Brake study

Diz and Brake (1986) apply the Toevs model of duration gap to a rural bank. The duration gap is;
\begin{equation}
DG_{ni} = MVRSA \left(1-D_{RSA}\right) - MVRSL \left(1-D_{RSL}\right)
\end{equation}

\begin{itemize}
\item MVRSA = market value or discounted present value of rate sensitive assets
\item MVRSL = market value or discounted present value of rate sensitive liabilities
\item D_{RSA} = Duration of the rate sensitive assets
\item D_{RSL} = Duration of the rate sensitive liabilities
\end{itemize}

A one year planning horizon is assumed. In addition, simplifying assumptions are made eliminating loan prepayment or deposit withdrawals during the year. The Macaulay duration measure with its underlying assumptions is used to calculate the gap.

The authors' objective is to calculate the sign and the absolute dollar value of the institutions interest rate exposure. Taking the asset and liability side on the balance sheet, the cash flow characteristics and the proportions of each asset and liability category are modeled to allow calculation of market values and durations.

The dollar duration gap is \(-20,577,152\). The negative duration indicates that the net worth will decline if rates decrease. The duration gap in years is \(-1.2378\). This calculation is based on the assumption that all asset and liability rates change by the same proportion. Relaxing that assumptions requires calculating an index to indicate the relationship between balance sheet rates and some risk free rate.
Rate changes for all accounts are perfectly correlated but have different magnitudes. The proportionality constant, $p$, is calculated using OLS regression where the benchmark rate is the treasury bill yield.

The difference in market values repriced during the year, the average time to repricing and the magnitude change in account interest rates relative to one another all contribute to interest exposure. The authors multiply the proportionality constant by a term designed to account for the influence of the average time to repricing. The adjusting expression is:

\[(3.65) \quad D = (1-t)p\]

- $D$ = duration
- $t$ = time from repricing to the end of the planning horizon
- $p$ = the proportionality constant

The expression indicates the magnitude and timing pattern of repricing. The new duration is calculated by multiplying the market value of assets and liabilities times $(1-t_i)p_i$ for each category. The new duration gap has the same sign but is smaller in absolute value than the previous calculation.

Finally, the authors examine the issue of calculating duration based on historical relationships between asset and liability rates and the treasury bill benchmark. The exposure of an institution in the future will depend on actual relationships over the relevant time period. This test is
constructed by using actual ex post earnings data to test the assumptions made at the beginning of the planning horizon.

Results calculated in this scenario indicate a duration gap that is larger in absolute value and has a different sign than the first two examples. This means that the institution 'suffers losses if rates fall rather than increase. A large federal funds transaction occurred during the time period under study and after adjusting for that balance sheet change the historical proportionality constants accurately reflect the realized net interest income in the future time period.

The authors methodology involves maintaining the balance sheet proportions in the asset and liability categories at the values in effect at the start of the time period. Differences between the calculated earnings and actual earnings occur because of changes in the institutions balance sheet proportions. The duration gap analysis reflects only the effect of interest rates and not changes in the proportions of assets and liabilities on the balance sheet.

The study does not consider cash or futures market hedges of the duration gap. Further studies could examine the effect of activities designed to immunize the balance sheet. Further research is needed to compare the results for alternative gaps in the target accounts of capital, net income and the capital to asset ratio. The net income measure of gap represents only one dimension of exposures.
Although the study makes several simplifying assumptions, it represents an important application of duration analysis to actual intermediary activities. Further research should modify the assumptions or consider alternative gap measures and management strategies. Examining the mean and variance of target accounts using an immunization strategy would provide better information on the applicability of duration theory and hedging to the problem of institution interest rate exposure.

The next study of saving and loan institutions is designed to determine if reported accounting data for that industry can be used to approximate the cash flow information required to implement a duration strategy. The research also measures the exposure using a sample of 50 saving and loans based on the duration gap.

Bennett, Lundstrom and Simonson

Bennett, Lundstrom and Simonson (1986) calculate durations using actual cash flow data from a single savings and loan and compare their results to estimated durations based on data available in published information from Section H data in the Federal Home Loan Bank Boards Quarterly Reporting System.

The authors use their duration measure to examine changes in the savings and loan net worth under alternative interest rate assumptions and for various prepayment patterns for loans. Since use of Section H data provides adequate
durations and duration gaps are calculated for a sample of 50 large institutions.

In the period 1984-1985, the net worth position of the sample banks as measured on a market value basis increased. The duration of assets was larger than the duration of liabilities and declining interest rates over that time period resulted in an interest rate induced increase in net worth. The net worth is derived as follows:

\[
\Delta NW = [PA_{84}^D_A - PL_{84}^D_L] \Delta i
\]

\(\Delta NW\) = the change in net worth
\(PA_{84}^A\) = the 1984 market value of assets
\(D_A\) = the weighted duration of assets
\(PL_{84}^L\) = the 1984 market value of liabilities
\(D_L\) = the weighted duration of liabilities

The authors use data from the sample banks to fit the above equation using OLS regression. Their results indicate that the durations of assets and liabilities approximate changes in the value of net worth for various term structure changes.

Portfolio net worth is not influenced exclusively by interest rate changes. Changes in portfolio composition influence the yields and cash flows. The study separates the price appreciation effect that results from the duration gap from other factors influencing the balance sheet by using simple variance analysis.
The effect of the gap on net worth is isolated using the duration price risk equations. Mathematically:

\[
(3.67) \quad \frac{PA_{85} - PA_{84}}{PA_{84}} = -D \cdot \Delta i
\]

and;

\[
(3.68) \quad \frac{PL_{85} - PL_{84}}{PL_{84}} = -D \cdot \Delta i
\]

\( PA \) = the present value of assets
\( PL \) = the present value of liabilities
\( DA \) = weighted asset duration
\( DL \) = the liability duration
\( i \) = the term structure for zero coupon bonds as of December 1984
\( \Delta i \) = the change in the zero coupon term structure

This study represents an important step in applying theoretical duration to actual institution management. The data used in available to bank management and offers a solution to the problem of measuring interest rate risk with duration when current accounting systems do not provide data in the exact form required for the analysis.

**Summary**

This chapter has reviewed several alternative measures of interest rate risk along with management techniques designed to reduce that exposure. Interest rate risk is not the only uncertainty faced by financial intermediaries but its management is of primary importance. The extent of interest rate risk acceptable to management will depend on the
volatility of the rate environment, risk preferences and the combined overall exposure of the institution to other risks.

The chapter next begins with a description of the general operation and activities of institutions in the Farm Credit System. Next, studies of debt and interest rate risk management are reviewed. The chapter concludes by integrating the theoretical discussion of portfolio theory and interest rate risk with the present management activities and operations at Federal Intermediate Credit Banks.
CHAPTER IV. LITERATURE REVIEW OF THE FCS

Introduction

This chapter reviews the basic structure and objectives of the Farm Credit System to develop its role as a conduit intermediary. Next is a review of the literature on government agency debt followed by studies specific to the Farm Credit System. Finally the theoretical discussion of the previous chapters is related to FCS institutions.

Institutions and Operation of the FCS

History of the Farm Credit System

The Farm Credit System was established in 1933 to provide the agricultural sector with a reliable source of credit and consolidate existing farm programs. Although the system was initially capitalized by the federal government, that capital has been repaid.

The Farm Credit System consists of 12 Federal Land Banks and 492 Federal Land Bank Associations, 12 Federal Intermediate Credit Banks (FICBs) and 424 Production Credit Associations (PCAs), and 13 Banks for Cooperatives.

Federal Land Banks and Federal Land Bank Associations are authorized to make long-term real estate mortgage loans. They were organized and capitalized by the government under the authority of the Farm Loan Act of 1916. The Federal Land Banks were established in response to the poor terms for real estate loans provided by the private sector. Private sector
loan maturities seldom exceeded 5 years and the loans had high interest rates. In addition, renewals were difficult to obtain.

Federal Intermediate Credit Banks provide short- and intermediate-term credit to Production Credit Associations and other financial institutions. FICBs were established in 1923 to discount agricultural loans made by private sector banks. FICBs were created to provide operating credit to farmers. Production Credit Associations were established in 1933 because private sector financial institutions did not extensively use FICB services.

Present operations

Figure 4.1 indicates the organizational structure of the Farm Credit System. The last of the government's investment in the Farm Credit System was repaid in 1969. The system is now self-sustaining obtaining capital through the purchase of stock by their borrowers. Each borrower is required to purchase stock in an amount proportional to the amount of funds borrowed. The stock provides a major amount of the equity capital financing PCAs and FICBs. The equity capital raised through the stock requirement provides the capital base that allows the Farm Credit system to sell its securities to investors.

The Production Credit System includes both the FICBs and PCAs. Production Credit Associations make short- and
Figure 4.1: Farm Credit System Organizational Structure
intermediate-term loans. Most of their loans are for production or operating purposes and mature within a year. However, loans to farmers, ranchers, rural homeowners and farm-related businesses can be extended for up to ten years. Short-term loans are frequently made for the purchase of feed, seed, fertilizer and chemicals, or for operating or living expenses. Intermediate-term loans are typically made for the purchase of machinery, equipment, vehicles and fishing vessels or for other capital improvements.

Most of the funds lent by Production Credit Associations are obtained from Federal Intermediate Credit Banks. There are 12 of these banks, one in each Farm Credit District. In addition to providing loan funds to PCAs, the banks may participate with the Associations or with each other in making loans. They also discount the notes of agricultural producers or other financial institutions.

Each of the 12 district FICBs is an incorporated cooperative. Voting stock, nonvoting stock, participation certificates, legal reserves, and surplus reserves represent equity capital. Local PCAs hold voting stock that has no stated redemption rights. Local PCAs hold voting stock in proportion to each association's indebtedness.

Farm Credit System banks are not depository institutions. Their source of funds is the national money market. Since January 1979, consolidated discount notes and
bonds have provided funds for the Farm Credit System. Discount notes have maturities ranging from 5 to 270 days. Bonds with six- and nine-month maturities are issued each month. Long-term bonds are issued about eight times a year.

The operation of the Farm Credit System evolves with legislative changes made in response to changing needs and conditions. Significant changes were made by the Farm Credit Act of 1971 which decentralized authority and mandated the role of the system in the provision of credit to the agricultural sector.

Amendments to the Act made in 1980 authorize banks for cooperatives to finance export transactions of U.S. agricultural cooperatives. Other provisions allow Federal Land Banks and Production Credit Associations to more fully finance the processing and marketing activities of agricultural producers and attempt to increase the interaction between institutions of the Farm Credit System.

1985 and 1986 Amendments to the Farm Credit Act

In 1985 and 1986 the Farm Credit Act was amended further in response to the Farm Credit System's financial problems. The 1985 legislation contained three major provisions. First, the Amendment strengthened the regulatory powers of the Farm Credit Administration. Although institutions of the FCS were previously examined on an annual basis, the
regulatory authority lacked the power to enforce changes in management practices.

The second major provision of the law created the Capital Corporation to facilitate the flow of capital and financial assistance between districts and institutions. The Capital Corporation has the power to access sound institutions to provide financial assistance to weaker banks. Additional powers include the provision of technical assistance to institution management and the ability to help restructure or purchase nonperforming loans at individual banks.

The final major provision of the 1985 law provides a mechanism for assistance from the federal government. If congress appropriates the money the treasury can assist the FCS as long as the Farm Credit Administration certifies that assistance is required. The need is based on evidence that the System has committed capital and reserves to solving its problems and that a further drain on resources would impair the ability of the FCS to raise reasonably priced funds in money and capital markets.

The 1986 Amendments gave greater power to individual institutions to set interest rates on loans. Prior approval of the regulatory authority is no longer required. In addition, the legislation allows institutions to prepare financial statements that are not in accordance with
generally accepted accounting principals. The objective is to reduce reported operating losses in the short run to provide additional time for solving financial problems.

**FICB Assets and Liabilities**

The next section presents a description of the assets and liabilities of Federal Intermediate Credit Banks as presented in Moody's Bank and Finance Manual (1986) and financial statements from the Federal Farm Credit Banks Funding Corporation (1987). Although these institutions are financial intermediaries, their balance sheet structure differs from that of depository institutions. FICBs have access to national money and capital markets. Those funds are used primarily to make agricultural loans.

**FCS debt securities**

Both bonds and notes are offered on a regular basis by the fiscal authority, the Federal Farm Credit Funding Corporation, located in New York. The debt is not an obligation of the government. A fairly active secondary market provides investor liquidity.

Systemwide notes represent a short term source of liquid funds. The proceeds are used to finance institution operations between debt issues. Discount notes have a maturity of less than a year and have yields based on a 360 day year.
Consolidated, Systemwide bonds are the joint liability of all FCS institutions. Restrictions exist on the amount of debt that is issued as the primary responsibility of a particular institution. For FICBs, obligations can not exceed twenty times the surplus and paid-in capital.

Denomination sizes are in multiples of $5,000. Bonds are issued in book-entry form. Investors do not need to present coupons in order to receive their interest payments. Payment is made by wire transfer through the Federal Reserve System. Since the 1980 Monetary Control Act, any depository institution can maintain book entry accounts with the Federal Reserve System.

The ownership of the securities is represented by computerized entries. Investors receive a custodial receipt from the bank or dealer rather than a definitive security. Any institution that maintains book entry account with the Federal Reserve System may be selected as the custodial bank by the investor (Martin, 1985).

The interest payments to investors are exempt from state and local taxes. Investors are required to pay federal tax on bond income. In addition, gains from the sale of the securities are taxable. The tax exemption is retained in the 1986 tax reform law.

The interest on short term obligations is paid at maturity. These securities include six- and nine-month bonds.
Securities with a maturity of a year or more at the time of issue pay coupon interest on a semi-annual basis. Interest for these securities is computed based on a 360 day year with 30-day months.

Farm Credit System securities are acceptable as collateral under section 13 of the Federal Reserve Act. Federal Reserve banks accept FCS securities as collateral for 15-day borrowing by financial intermediaries. Any Federal Reserve bank can purchase bonds with six months or less remaining to maturity.

Commercial banks represent a major investor in FCS debt securities. Bank regulations restrict the type of debt securities that can be held by banks for investment purposes. Restrictions limit the kind and proportion of securities held. FCS securities are allowed because of the active secondary market and the perception of low default risk.

Corporate debt containing a call provision allows the firm to retire the debt and refinance if interest rates decline. The corporation is required to pay a penalty, a call premium, if the debt is retired early. The decision to refinance is made by comparing the present value of current debt costs to the costs of refunding. Refunding involves payment of the call premium and transactions costs. The corporation will refinance if the present value of the refunding alternative is positive.
Call provisions introduce flexibility into debt management. A small decline in interest rates will not be sufficient to induce refunding. If rates decline significantly the firm can reduce the cost of funds by calling outstanding debt and reissuing at lower interest rates.

The debt issued by the FCS is not callable. Because the System cannot redeem debt and refinance at lower interest rate, the cost of funds depends on debt decisions in previous time periods. Although call features would require payment of premium to induce investment on the part of the public, lack of the provisions reduces the ability of FCS institutions to refinance if interest rates drop. During the recent decline in interest rates lack of call provisions resulted in higher costs for borrowers.

Method of issuing debt securities

Puglisi and Vignola (1983) provide a comprehensive description of the procedures used to issue and price agency debt. Although their analysis includes all the federal agencies, the FCS follows the same general procedures when issuing debt throughout the year.

FCS securities are not available to the public in the primary or issue market. The securities are sold to the public in the secondary or resale market. Bank and nonbank security dealers make a market for FCS security issues. The coupon is not determined until the day the bonds are priced.
Dealers are provided with information on the maturity, the issue size and denomination size.

The dealers contact investors interested in purchasing the securities. The investors indicate the yield they would require in order to purchase securities from the issue. The dealers supply this information to the fiscal agent who uses it to price the securities. Investors must agree to commit to purchase without the option to cancel the order before the fiscal agent sets the price.

Although this may seem unusual, it is important to recognize that federal agencies are active and frequent participants in the market. Failure to correctly set yields would result in a loss of future liquidity. The objective of the fiscal agent is to price the debt competitively in the absence of a price set by participants in an auction market.

Dealers are not allowed to purchase the issue until after it begins trading in the secondary market. This is to insure that they provide accurate information on investor yield preferences. Dealers receive compensation in the form of concessions or commission. Fiscal agents seek a broad range of investor preferences by contacting a large number of dealers before setting the price.

Order solicitation day represents the point when the investor information is initially sought. Pricing occurs in
the afternoon the day after solicitation day. At noon on the
day after pricing, trading commences in the secondary market.

**Stock requirements and FCS capital**

Institutions of the FCS are borrower-owned cooperatives. The stock purchases are required to obtain loan funds. The specific requirements are determined by the individual institutions in each of the FCS districts. This stock represents borrower ownership. One objective of the stock purchase requirement is to ensure that institution capital adequacy is maintained as borrowing occurs.

The equity investment is subject to the risks of institution operation. When the financial condition of an institution declines, the value of the stock declines. Borrower-owners are not guaranteed that they will receive the price initially paid for the stock.

The stock purchase requirement has important consequences. First, it ensures that patrons also are owners in the firm. The organizational form of a cooperative was used to allow borrowers to exercise control over institution operations. The dual role as borrower and owner creates potential problems.

Although the stock does not trade in the open market, the required purchase represents an investment on the producer's balance sheet. This investment is at risk if the institution has financial problems. Borrowers under financial stress face
loss if the value of their FCS stock declines and further reduces their asset base.

Financially sound borrowers with the ability to borrow from other sources would have an incentive to retire their debt held by FCS institutions and refinance with other institutions. The possibility of declines in the value of stock or institution insolvency provides a motive to exit the Farm Credit System and borrow elsewhere.

The stock purchase is financed out of loan proceeds. The required stock purchase raises the effective cost of the debt to the borrower. If loan rates are not competitive with other sources of funds in the market, the existence of the stock requirement provides an added incentive for financially sound producers to borrow at commercial banks or other institutions outside the FCS.

A final issue is the effect of interest rate changes on the economic or market value of capital. The previous chapter developed the theoretical framework of interest rate risk indicating that interest rate changes affect the value of an institutions net worth.

The direction and magnitude of changes in net worth depends on the size of the duration gap between assets and liabilities and the sign of the gap. A negative gap results in losses in the market value of net worth if interest rates decline while a positive gap has the opposite effect.
The accounting statements of FCS institutions report the condition of capital based on book value. The affect of interest rate changes on the market value of capital have not been considered. If institutions are exposed to asset/liability gaps that subject the balance sheet to interest rate risk, additional impairment of capital can occur.

**Assets of FICBs**

The major FICB asset is agricultural loans. The institutions provide funds for seasonal needs of producers. Borrowers-owners use the funds to finance crop production and marketing. Although a typical commercial bank holds approximately sixty to seventy percent of its assets in a diversified loan portfolio, FICBs lend exclusively to agriculture. Loans represent eighty to ninety percent of total assets.

Loans are priced on the basis of the average cost of funds. Rates change each month as new debt is issued and the average cost of funds changes. The objective of variable rate pricing is to eliminate interest rate risk from the balance sheet of FICBs.

The institution is required to hold collateral equal to the outstanding debt for with it is primary responsible. Loans, marketable securities and cash represent acceptable
collateral. Allowable assets include agency securities, treasury securities and federal funds.

The purpose of the investment portfolio is to provide liquidity. Liquidity is obtained by selling marketable securities or by maturing short term debt. The investment portfolio has not been used to offset risks in other areas of the balance sheet. The primary function of the institution is to make loans rather than investment.

The preceding section provides a brief description of the basic balance sheet activities. The next section presents empirical studies of agency and Farm Credit System debt. The research can be categorized as follows. First there are general studies of the agency security market. These examine market efficiency, risk premiums and yield spreads over treasury securities.

Next is a series of studies examining the specific financing strategies of FCS institutions and FICBs. This research considers the issues of maturity, issue size, the timing of debt issues as well as the mix of maturities in the liability portfolio. The overall objective of these studies is to determine the appropriate activity required to minimize debt cost or the variance of costs.
Studies of Agency Debt Securities

Silber study

Silber (1974) studies agency debt securities to determine if there is an optimal size of issue. The objective is to minimize the yield spread between treasury and agency securities. The analysis compares newly-issued agency securities with outstanding treasury securities.

The research considers two categories of issues that affect yield spreads: the characteristics of the security and the characteristics of the market. The characteristics of the security include the size of issue, the maturity and the minimum denomination. General market conditions that influence yield spreads are credit conditions and the supply of treasury securities in the market.


Silber finds three factors that significantly explain the yield differential between treasury and agency debt. These
factors are the term to maturity of the security, the issue size and the general market conditions.

The size of the issue is related to marketability. The larger the total dollar amount of the issue, the greater the secondary market liquidity. This results in smaller liquidity premiums. This relationship holds over a range of issue sizes. If the size of the issue is too large relative to the total size of the market it would be difficult to place the issue. This would require greater yield spreads.

During the time period studied, the agencies were just beginning to issue longer term debt. Silber suggests the premium required for longer terms to maturity result from investor unfamiliarity with these kinds of issues.

If money conditions are tight, the yield spread increases. The author concludes that there are two reasons for this relationship. First, the agency securities are not backed by the government and the required yield for risk assets increases during tight credit conditions. The second explanation stems from the data the study uses. The treasury securities are outstanding securities but the agency securities are new issues.

This study concludes that the daily price changes of smaller issues show greater variability than larger issues. The author solves for the issue size that minimizes the variance and obtains a solution of $300 million.
The results hold only for the period of study. As trading becomes more active in secondary markets or the securities gain wider acceptance the optimal issue size would drop. The results also imply that shorter maturities involve lower costs but does not explicitly examine an optimal maturity structure.

**Barbade and Hunt study**

The first objective of the study by Barbade and Hunt (1978) is to consider the risk premiums required for federal agency securities. The bid and offer prices of agency and treasury debt are used to calculate the risk premiums. The data consists of Federal Intermediate Credit Bank debt and Bank for Cooperative securities covering the time period from 1965 to 1976. Prices from the first trading day in each month are used resulting in 126 observations for each issue.

The authors construct forward contracts in both treasury and FCS securities in order to compare yields and examine risk premiums. A forward loan contract is constructed in which an investor buys a security with a maturity of \( n + 1 \) and sells short a security with a maturity of \( n \). At the end of period \( n \), the investor must purchase a security to close out the short position and at the end of period \( n + 1 \) the investor receives the maturity value of the security and accrued interest.
If time periods are based on monthly terms, then the difference between \( n + 1 \) and \( n \) is one month. Over that month the investor earns a rate of return. The authors construct portfolios of forward loan contracts which represent the debt securities.

The yield earned during the monthly period consists of pure interest component and a risk premium. The risk premium is determined by the liquidity of the debt and the chance of default. The authors isolate the risk premium by comparing the forward loan rate for the agency security for the monthly period with the treasury security rate. The assumption is that treasury securities are default-free and so offer a riskless rate of return.

After calculating the risk premium for each of the months in the study, the authors find the mean value of the risk premium of the agency debt securities. The authors attribute the risk premium to the illiquidity of the agency secondary market rather than default risk. This is based on the observation that longer maturities have a higher mean value for the risk premium.

The authors develop a regression model to test the sensitivity of the risk premium to the general level of interest rates. During periods of high interest rates the short term debt is less liquid. The authors attribute the
existence of risk premiums to liquidity rather than credit risk.

The results of the study are valid only for the time period covered by the analysis. The recent problems with FCS institutions have widened the yield spread between treasury and FCS institutions. Empirical studies to explain the current yield differentials have not been conducted. Since this study provides information on the liquidity premium, future studies could focus on the default risk premiums.

The second objective of the study is to determine if agency security markets are efficient. The efficiency of the market for agency debt securities is tested by examining the potential rate of return available from security arbitrage. If arbitrage profit opportunities do not exist then expected arbitrage return on an n month forward loan is zero. The authors consider arbitrage between treasury and agency securities.

Using the expected return requires a forecast of the expected future risk premium. The authors develop a regression model to examine the relationship between risk premiums and the general level of interest rates. This model is used to forecast future rates.

The results suggest that the agency market is efficient. After the transaction costs of trading are included in the
model, a review of 125 periods indicates few in which the arbitrage rate of return is not equal to zero.

Puglisi and Vignola

Puglisi and Vignola (1983) examine the pricing efficiency of the agency debt market. The concept of efficiency in this study pertains to price-setting by fiscal agents in the absence of an auction market. Efficient pricing involves accurately estimating the price that would be attained in a competitive equilibrium.

Inefficiency has important consequences. If fiscal agents consistently underestimate coupons, investors will not receive their required rate of return. Future liquidity and marketability could be impaired. From the perspective of the issuing agency, overestimating the required coupon results in higher debt costs over the life of the security. These costs could represent substantial amounts for longer term securities.

Data for the study consists of 59 Federal Home Loan Bank issues, 69 Farm Credit System issues and 69 Federal National Mortgage Association issues covering the time period from 1976 to 1980. The FCS issues include bonds issued to finance operation of Federal Intermediate Credit Banks, Banks for Cooperatives and Federal Land Banks.

Price data collected includes information at the approximate time of day the initial pricing occurred. Data
also cover the point in time when the issue first trades in the secondary market. Because of the elapsed time between the pricing date and the beginning of trading in the secondary market, the market value of the bonds can change due to deviations in the coupon from the market interest rate.

The authors adjust the bond prices using the simple duration measure. The duration accounts for the changes in bond price due to interest rate changes. Each agency security is matched with an equivalent treasury security which is also adjusted for duration. The measure to determine pricing errors is represented mathematically by:

\[ \text{error} = \% \Delta P_A - \left[ \frac{\% \Delta P_T \cdot (D_A / D_T)}{D_T} \right] \]

- \( \% \Delta P_A \) = the percentage change in the price of the agency security
- \( \% \Delta P_T \) = the percentage change in the price of the treasury security
- \( D_A \) = the duration of the agency security which has been divided by \( 1 + i \)
- \( D_T \) = the duration of the treasury security divided by \( 1 + i \)
- \( i \) = the yield to maturity of the security

A value of zero for the error term represents pricing efficiency. The results indicate that the error term is not significantly different from zero for any of the agencies except the Federal National Mortgage Association.

The authors also attempt to determine if any of the characteristics of the issues are related to pricing errors.
Maturity, coupon and issue size represent the tested influences on pricing error. In addition, the authors conduct a test to determine if volatility in general market interest rates influences pricing errors.

Pricing errors are larger when market interest rate are more volatile. Greater pricing errors also occur when the securities have shorter maturities. The coupon size is also significantly correlated to pricing errors. Issue size is significant only for Federal National Mortgage Association pricing errors.

The general conclusions of the study are that fiscal agents correctly approximate the equilibrium that would be attained in a competitive market. The implication is that changes in investor perceptions of agency risk will be reflected in the cost of funds because pricing of issues is efficient. The study does not explicitly examine investor risk premiums.

The efficiency of both the primary and secondary market prices indicates that the cost of funds required by investors will depend on the perceived risks of the issuing entity. Risk consists of default and liquidity risk. Liquidity risk can be reduced by an issue size large enough to result in marketability. Default risk depends on the financial condition of agency. Financial problems translate into higher debt costs.
These studies indicate the factors the market considers when determining the equilibrium security price. If meeting the preferences of the marketplace with respect to issue size and maturity can reduce borrowing costs, the issuing agency should try to tailor debt characteristics to investor preferences.

The next section presents studies of FCS financing strategies. The liability side of the balance sheet is the focus of the research. The implicit assumption is that variable rate pricing eliminates interest rate risk exposure. The overall objective of the following studies is to analyze alternative policies that either minimize the average cost of funds or reduce the variability of debt cost.

**Farm Credit Debt Policy Studies**

**Bildersee studies**


Investor holding periods are defined as one-month intervals. Holding period return indexes are then constructed for thirteen different yield to maturity periods. It is
assumed that investors buy a security at the beginning of the holding period and sell it at the end of the holding period.

The mean and standard deviation for each of the thirteen maturity categories are calculated for two time periods: February 1965 to March 1969 and April 1969 to May 1973. The results suggest that no single funding strategy would minimize costs because neither short or intermediate term debt had a consistently smaller variance through all alternative time periods.

In this study the variables that influence cost include maturity and free reserves. Free reserves is used to represent general market conditions. There is an inverse relationship for free reserves and debt cost suggesting a higher yield spread during tight money conditions. This is consistent with the Silber study.

There is a direct relationship between maturity and the yield spread. This result may occur for the reasons Silber suggests including investor uncertainty concerning new long term debt issues and market liquidity.

Bildersee (1973b) also develops an optimization model to examine the cost minimizing combination of bond issue features. Cost is a function of both market conditions and bond characteristics. General market conditions include interest rates in the economy and the supply of other securities. Based on given market conditions, the agency
selects the optimal term to maturity and issue size. Selection is based only on cost considerations.

The model's focus is the financing decision for the firm. The author assumes that the quantity of funds or the issue of asset acquisition is determined separately from borrowing. The demand for assets is inelastic with respect to the interest rate. The model is not developed for overall balance sheet management. The objective of the study is to determine a cost minimizing liability management strategy.

The author applies the model to review alternative financing strategies. The objective is to determine which funding method minimizes debt costs. Bildersee constructs a cost function based on FCS data and considers various alternative funding levels and maturity combinations.

The results indicate that a range of policies have similar costs. The problem with the financing alternatives modeled is that they ignore the uncertainty associated with interest costs. The market conditions expected over future time periods would have an important impact on financing decisions. The pattern of interest rate changes influences debt costs over the course of the planning horizon.

In addition, the study fails to consider the asset side of the balance sheet. The cost minimizing strategy may expose the institution to interest rate risk because of the mismatch
of maturities or durations. In turn, interest rate exposure could affect the required cost of funds in subsequent periods.

**Percival study**

Percival examines factors affecting the yield spread on agency securities. These include maturity, marketability and supply and demand conditions in capital markets. By determining the influence of each of the factors on debt required yields, agencies could make selections to minimize costs.

The relationship of maturity to cost for the borrower is based on the unbiased expectations model of the yield curve discussed in the previous chapter. The borrower is assumed to match maturities of assets and liabilities. The borrower with the need to fund a three year asset has several alternative funding choices.

First, the borrower could issue debt with a maturity of three years. Alternatively the borrower could issue debt for shorter periods, refinancing throughout the maturity period. A series of one year bonds or a series of two year and a one year bond could be selected. If rate declines are anticipated the cost minimizing strategy would be to issue a series of one year bonds. The author uses dynamic programming to make debt choices.

The major problem with this method is its reliance on a forecast of future interest rates. After generating a
forecast it would have to be compared to that implied by the current term structure of interest rates. Only if the borrowers expectations differed from the markets would the short term strategy be appropriate. The current yield curve or the rate on a three year bond is based on the market's best guess about future rates.

An important problem remains. Even if the borrower makes a forecast that differs from the market, that forecast may be incorrect. The borrower can profit from this strategy only if the forecast is correct. The penalty for incorrect forecasts is an increase in borrowing costs over time. The problem is more acute if debt is not callable and the agency is locked into a high cost of debt over a long period of time.

The author suggests that simple maturity matching is important if reduced variability in operating income is the firm's objective. The short term nature of FCS assets leads Percival to suggest use of short term financing. However, the short term strategy involves fixed issue costs each time the borrower refinances.

The relative supply and demand of securities at the same maturity could also influence yield spreads. An excess supply of a particular type of security requires an increased yield in order to restore equilibrium. Using regression analysis Percival attempts to determine the influence of supply and demand factors on yield spreads.
The explanatory variables are the outstanding supply of both agency and treasury securities and the expected change in supply expected over the next six months. The actual values of six month supplies proxies the expected value.

Outstanding supply is defined as the difference between the supply of treasury and agency securities. This is the only statistically significant variable. The effect on the spread is negative.

The author also re-estimates the regression equation under alternative maturities. These results indicate that short and long term yields are sensitive to changes in relative supply. The intermediate term yields were not related to supply measures.

**Morris study**

In order to examine debt costs, Morris develops a simulation model of FCS debt issues. He considers both a stochastic and historical simulation. The objective is to determine an optimal maturity policy that minimizes cost.

The historical model uses the actual loans made over the period 1955 to 1972. A simulation for the period June 1972 to June 1992 was developed with interest rates entering the model as a random variable.

The simulation is not an optimization model but does allow policy makers the opportunity to review the consequences of their decisions under alternative conditions. The results
of the model indicate that the appropriate strategy depends on the pattern of interest rate changes. The agency would need to forecast interest rates correctly and better than the market in order to undertake a cost minimizing strategy in one time period that minimized costs over the long run.

For the historical period January 1955 to December 1972, the weighted average cost of debt was minimized with the use of longer term debt securities. The upward trend of interest rates over the study period explains the results. The average cost of debt was minimized by locking in borrowing costs over a longer term rather than by refinancing with a series of shorter term securities. The key conclusion is that the cost minimizing strategy depends on the pattern of interest rates over the planning horizon.

In the simulation study, Morris modeled interest rates as a Markov process. The term structure of interest rates in each month was a random variable dependant on the term structure occurring in the previous month. The author used three separate probability matrices to model interest rate condition. The first model reflected rate stability over time. The second added some instability in rates but with no trend either up or down. The third alternative modeled interest rates with more instability and a definite downward trend.
Use of long term debt minimized costs when interest rates were stable. A mix of strategies minimized costs in the simulating using the second model of interest rates. When rates are decreasing, a short term strategy minimized costs. Debt costs were minimized with the use of a long term strategy when rates had an upward trend. Simulations run using the third model of interest rates with a downward trend minimized cost with the use of a short term policy.

The results of the study indicate that the cost minimizing strategy is dependant on the pattern of interest rates over the planning horizon. Uncertainty regarding future interest rates and the lack of accurate forecasts limits management's ability to minimize costs over the entire planning period. The author suggests that the Farm Credit System focus on developing accurate long range forecasts of interest rates prior to implementing a cost minimizing debt maturity strategy.

Tauer study

The focus of the Tauer study differs from the previous FCS research considered. The Percival, Bildersee and Morris studies consider the objective of cost minimization. Tauer considers both risk and cost developing a portfolio model for FICB debt selection. Quadratic programming is the mathematical technique used to generate efficient frontiers.
The objective of this study is to generate an efficient frontier representing the tradeoff between debt cost and the variability of cost. This is an appropriate objective recognizing that debt cost variability translates into lending rate variability due to the FICB method of repricing loans each month on the basis of the average cost of debt.

Debt cost variability for the FICB represents cost variability for borrower-owners. Significant increases in debt costs can result in default if the borrower is unable to make payments out of current cash flows. The method the study uses allows policy makers to select the minimum variance portfolio at the given level of expected cost.

The study includes unconstrained and constrained solutions. Constraints are modeled to match the management restrictions on the proportions of different maturities that are allowed in the debt portfolio. The model assumes a three year planning horizon.

Results for the unconstrained model indicate that the highest cost portfolio or the low variance portfolio finances with a high proportion of term bonds in the first year. Some nine month bonds and discount notes are used in the second year. In the third year only discount notes are used.

The low cost portfolio selects debt activities on the basis of expected cost. The results from this model do not
consider variance and represent solutions that would be obtained using linear rather than quadratic programming.

Moving along the efficient frontier from higher to lower expected cost portfolios causes a shift from the use of term bonds to discount notes supplemented with six month bonds and then discount notes. The nine month bond is not included in the first year portfolios along the unconstrained frontier.

The constraints modeled include restrictions on the amount of debt from various funding sources. The objective of management is to reduce debt cost volatility. Examining the constrained efficient frontier indicates that these restrictions do not necessarily produce the desired results.

The constrained efficient frontier shifts to the right so that at any level of standard deviation there is a higher expected cost. The constraints do not limit volatility and raise the standard deviation of the minimum variance portfolio from that attainable in the unconstrained solution.

The study is important because it indicates that management debt policies that impose restrictions on activities result in an efficient frontier with less desirable risk and cost trade-offs than the unconstrained solution. The major shortcoming of the study is that funding decisions are not integrated with asset activities.
Heffernen and Lee study

Heffernen and Lee (1984) also focus on the variability of debt costs by modelling the Louisville FICB. The objective of their study is to design a futures trading strategy to reduce the variance of debt costs. This objective is selected based on the assumption that borrowers are granted loans determined by their ability to repay at the interest rate in effect at the time the loan is issued.

The Louisville FICB responded to interest rate volatility by shortening the maturity on debt. Reliance on six and nine month bonds increased the proportion of short term debt in the liability portfolio and resulted in an increased volatility in the average cost of funds. In turn, the rate charged borrowers become subject to greater volatility.

The objective of the hedge in the study is to reduce the variability of debt cost. The study covers the period from June to December 1981. A hedge is instituted for one month periods using the contract with an expiration date nearest to the cash market closing date. The T-bill contract is used as the hedging instrument.

An optimal hedge ratio is constructed using the regression technique developed by Edington (1979). This strategy is compared to a naive hedge ratio calculated by the dollar equivalence method. Then the ratio was adjusted by recognizing the maturity differences between six and nine
month bonds and the 90 day treasury bill instrument.

The results of the study indicate that the hedging program reduces the mean cost of funds by 1.59 percent for the naive hedge. The portfolio hedge strategy reduces the cost of funds by 2.61 percent. The general trend in rates over the study period was increasing.

The study results suggest that current regulations that prohibit FICB use of futures markets may limit management's ability to reduce the variability of debt cost. There are two problems with the study. First, theory suggests that the duration or dollar gap between assets and liabilities is the correct measure of interest exposures. This study does not consider asset activities.

Empirical research indicates that a hedge ratio based on duration provides better results than a naive or optimal hedge ratio. The study does not consider a hedging program based on this measure. Results might be improved by using the duration hedge ratio.

Summary of Empirical Research

Several important issues are apparent from a review of previous research. The general studies of agency debt risk premiums and market efficiency indicate that the fiscal agent can not price securities below the rate required to compensate investors for risk. The investors in the study period required a premium for liquidity risk.
The studies do not analyze the time period in which the FCS had widely publicized financial problems. Access to reasonably priced funds depends on the effective management of overall institution risk. Examining the link between FCS financial problems and security risk premiums requires further study.

The studies that specifically consider the debt management policies of the FCS are flawed. There are several problems with studies that attempt to determine the financing strategy that minimizes cost. First, the objective of cost minimization does not recognize the effect of variability on debt and lending rates. Variability in lending rates exposes the borrower to interest rate risk that may lead to default if sufficient cost increases occur.

The studies conclude that the maturity strategy that minimizes cost depends on the pattern of interest rate changes over time. Effective debt management requires that the institution prepare long range forecasts for interest rate changes based on more information than is embodied in the current yield curve. The penalty of an incorrect forecast is higher debt costs.

The studies focus solely on liability management. The asset side of the balance sheet is not explicitly considered. Theory indicates that the risk faced by an institution depends not only on the liability side of the balance sheet but the
interrelationship between assets and liabilities.

The studies by Tauer and Heffernen and Lee incorporate consideration of the variability of debt costs. This represents inclusion of an important issue. Debt cost variability translates into lending rate variability. Minimizing or reducing the variance of cost is an important first step in managing institution risk and reducing borrowers exposure to interest rate risk.

The problem with these studies is that they also focus on liability management. The interest rate risk of an institution is more appropriately measured by analyzing the relationship between both sides of the balance sheet. The theoretical framework introduced in the previous chapter indicates the importance of integrated asset and liability management.

Studies of Farm Credit System institutions and policy that consider lender risks exclude analysis of interest rate risk. Lins, Drabenstott and Brake (1987) indicate that variable rate loan pricing allows FCS institutions to avoid interest rate risk. This is not substantiated with empirical analysis.

The next two chapters develop the empirical models used in this study to consider the issue of institution risk exposure and asset/liability management. Previous studies have not considered both sides of the balance sheet or
considered risk exposure within the framework of traditional or duration gap theory.
CHAPTER V. PORTFOLIO MODEL OF AN FICB

This chapter develops a portfolio model of a Federal Intermediate Credit Bank. A micro-model of a representative institution is presented using data from the Omaha Federal Intermediate Credit Bank. First, a general description of the methodology and bank activities is presented. Then the results of the model are reviewed. The chapter concludes with a summary of the empirical results and their implications.

Modeling FICB Activity

Portfolio analysis

Previous studies of FICB financial activity focus exclusively on the liability side of the balance sheet. The studies reviewed in the previous chapter do not integrate asset and liability management. Using the framework of portfolio analysis, this study considers the risk and return of each asset category simultaneously with the risk and cost of funding sources.

The returns the FICB earns on assets and the costs of liability funds are random variables. In this model, risk is characterized as the variance of returns and costs over the planning horizon. Unlike commercial banks, the FICB is not subject to deposit withdrawal risk because it acquires funds only in the national money market.
Liquidity risk is not explicitly modeled based on the assumption that the institution can raise all the required funds as long as it pays the prevailing market rate. A temporary need for funds between bond issues is met through the use of discount notes. The cost of liquidity is the rate the bank pays to meet funding requirements through the use of discount notes.

**FICB management objectives**

The financial condition of the FICB has an important affect on the cost of funds. Table 5.1 lists the yield spread in basis points occurring with the release of financial information on the condition of the Farm Credit System. Investors require a premium for both liquidity risk and default risk. Treasury securities are used to approximate the riskless rate of return based on the assumption that the government will not default on its debts. Negative information on the financial condition of the System results in a higher default premium or spread above that required for treasury bills.

Chapter IV reviewed empirical studies indicating that the agency market is efficient. Reviewing the yield spreads suggests that the cost of funds varies as the present and future financial condition of the FCS institutions is re-evaluated. Investors use all available information to determine the return required.
Table 5.1: Spreads between the treasury bill and FCS bond^a

<table>
<thead>
<tr>
<th>DATE</th>
<th>INFORMATION</th>
<th>SPREAD IN BASIS POINTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>9/4/85</td>
<td>Wall Street Journal reports FCS will ask Congress for assistance.</td>
<td>65</td>
</tr>
<tr>
<td>10/23/85</td>
<td>FCS announces its first quarterly loss since the depression.</td>
<td>97</td>
</tr>
<tr>
<td>12/24/85</td>
<td>President Reagan approves the 1985 Farm Credit Act Amendments</td>
<td>28</td>
</tr>
<tr>
<td>9/17/86</td>
<td>FCS announces federal assistance is likely.</td>
<td>32</td>
</tr>
<tr>
<td>10/20/86</td>
<td>Congress approves the 1986 Farm Credit Act Amendments.</td>
<td>34</td>
</tr>
<tr>
<td>1/15/87</td>
<td>FCS announces that independent auditors will likely qualify its opinion of the 1986 financial results.</td>
<td>42</td>
</tr>
<tr>
<td>2/18/87</td>
<td>FCS announces 1986 financial results</td>
<td>32</td>
</tr>
</tbody>
</table>

^aFederal Farm Credit Banks Funding Corporation (1987).
This study assumes that the objective of the FICB is overall risk management. The balance sheet structure is based on analysis of overall portfolio risk rather than the risk and return characteristics of individual assets and liabilities.

Interest rate risk is influenced by the variability of asset and liability rates. The FICB attempts to manage interest rate risk by repricing loan assets each month. A high correlation between asset returns and liability costs implies that the institution hedges interest rate risk by matching loan terms with costs. However, a high correlation between costs and returns would reduce the effectiveness of diversification.

The assumption is made that management is risk averse, but the form of the utility function is not specified. Risk aversion is consistent with the FICBs role as a borrower-owned cooperative. The cost of funds is affected by the overall institution risk as is the value of the capital stock. Greater risk requires a higher risk premium, raising the cost of funds.

The study does not select a single optimal portfolio. The solution procedure involves use of quadratic programming to generate an efficient frontier. The management of the FICB could then select a portfolio of assets and liabilities based on their particular risk preferences.
**Quadratic programming model**

The quadratic programming package used in this study is the Rand QPF4 computer program. It is used to generate the efficient frontier. The general mathematical problem is to minimize:

\[(5.1) \quad X'QX \]

subject to two constraints:

\[(5.2) \quad X'R = K \text{ and;} \]
\[(5.3) \quad X'e = 1 \]

where:

- \(X\) = an \(n \times 1\) vector of activity weights
- \(Q\) = an \(n \times n\) variance-covariance matrix
- \(R\) = an \(n \times 1\) vector of expected returns
- \(K\) = the selected return
- \(e\) = an \(n \times 1\) unit vector

The first constraint is the return constraint where \(K\) represents various alternative rates of return. The second constraint is the balance sheet constraint so the sum of the proportions equals one.

The efficient frontier is generated by minimizing risk at selected levels of return. Portfolios consist of combinations of asset and liability activities. The variance-covariance matrix represents the variances and covariances between the asset activity returns and the liability costs.
Mathematical model of an FICB

The Federal Intermediate Credit Bank balance sheet is modeled to solve for the proportions rather than the dollar amounts of assets and liabilities in portfolios along the efficient frontier. The methodology discussed is similar to the Francis (1978) study of commercial banks.

The asset and liability activities of the FICB are restricted by legislation. Loans are made only to the agricultural sector. An investment portfolio provides liquidity and earnings. The institution finances asset acquisitions by issuing bonds and notes. The balance sheet is represented by the equation:

\[(5.4) \quad (\text{Assets}) - (\text{Liabilities}) = \text{Equity}\]

The balance sheet model of the Omaha FICB consists of assets and liability activities as follows:

\[(5.5) \quad \sum_{i=1}^{n} X_i r_i - \sum_{j=1}^{m} X_j r_j = Kr_k\]

- \(X_i\) = the dollar amount in each asset category
- \(r_i\) = asset returns
- \(X_j\) = the dollar amount in each liability category
- \(r_j\) = liability costs
- \(K\) = dollar amount of capital

Dividing equation 5.5 by capital yields the bank's rate of return as a function of the asset returns and liability costs. Mathematically:
\[(5.6) \quad r_k = \sum_{i=1}^{n} w_i r_i - \sum_{j=1}^{m} w_j r_j \]

\( r_k \) = the rate of return

\( w_i, j = x_i, j / K \) the weight of the \( i \)th asset or \( j \)th liability as a percentage of equity capital.

The weights represent the decision variables in the model. The weights of the assets are positive as represented by:

\[(5.7) \quad w_i \geq 0 \]

The liability weights are negative. Mathematically:

\[(5.8) \quad w_j \leq 0 \]

The sum of the assets and liabilities is constrained to equal unity as is the standard convention in Markowitz portfolio analysis. Mathematically:

\[(5.9) \quad \sum_{i=1}^{n} w_i + \sum_{j=1}^{m} w_j = 1 \]

The next section discusses the alternative asset and liability activities included in the quadratic programming model. A brief description of each activity along with the source of the data is presented.

Activities Included in the Empirical Model

The data consist of asset rates of return and liability costs over the five year time period from 1979 to 1983. Review of the data indicates that the study period includes periods of increasing and decreasing rates. T-bills reached a maximum value 14.7 percent and had a minimum of 7.54
percent. The FICB six-month borrowing cost had a maximum of 17.95 percent and a minimum of 8.1 percent. Lending rates also exhibited a wide range of values with a minimum lending rate of 9.1 percent and a maximum of 16.05 percent.

**General asset activities**

FICB asset activities include cash holdings, investments and agricultural loans. Cash represents a nonearning asset and is held primarily as a liquidity reserve. Cash is modeled with a zero variance and a zero rate of return.

For investment and liquidity purposes, the FICBs hold marketable securities and sell federal funds. Rates represent those available on the national money market. The source of the data for all money market instruments is monthly issues of the *Federal Reserve Bulletin* published by the Federal Reserve Board of Governors.

**Description of money market instruments**

The Omaha FICB did not actively invest during the study period in all the money market instruments modeled. The instruments described below have been considered because the institution currently invests in a wider variety of money market securities. Treasury securities and federal funds represent the investments actually utilized by the institution in the period 1979 to 1983.

The Omaha FICB has been purchasing bankers acceptances and certificates of deposit in an effort to earn higher rates
of return since 1983. Two additional money market activities, Eurodollar deposits and commercial paper are modeled in order to consider the effects of investment in additional money market instruments.

**Treasury securities** Treasury securities are issued by the United States government to finance the national debt. The securities included in the model are the 90 day, 180 day and one year securities. The rates collected from the *Federal Reserve Bulletin* represent an average monthly rate. The yield used is the secondary market rate for the securities.

**Federal funds** Federal funds or fed funds are the deposits held by financial institutions at the Federal Reserve district bank. The amount on deposit is based on reserve requirements. Banks with amounts in excess of the minimum can lend to those needing funds to meet reserve requirements or acquire assets.

**Certificates of deposit** Willemse (1986) reviews the characteristics of the certificate of deposit (CD). CDs are issued by commercial banks. The bank accepts a deposit for a fixed period of time and issues a certificate of deposit indicating the amount, maturity and rate.

CDs can be negotiable or non-negotiable. Negotiable CDs represent a liquid short term asset because they can be sold in the secondary market before maturity. The interest on a
CD with a maturity of less than a year is paid at maturity. CDs with maturities longer than a year pay interest on a semiannual basis. This study assumes investment in CDs with a maximum maturity of three months.

CDs pay a spread over the treasury bill yield. The spread exists because of credit risk or the chance the issuing agency may default. Higher interest rates result in an increase in the spread. The tax exempt status of treasury securities requires a larger spread when the general level of interest rates increases.

**Bankers acceptances** Bankers acceptances are best described by example. The following is based on Hill (1986). An American car dealer, Mr. Jones, decides to import Korean automobiles. Mr. Jones has a customer relationship with an American bank. Mr. Jones will purchase the cars from Mr. Sato. Mr. Jones would prefer to pay for the cars after they arrive in the United States.

The problem is that the parties to the transaction do not know one another. The Korean auto manufacturer is not sure of the credit worthiness of the American importer. If the cars are shipped and Mr. Sato does not receive payment, large losses could result.

The bankers acceptance solves the problem. Mr. Jones offers to pay Mr. Sato for the cars a certain number of days after shipment. Mr. Sato collects the discounted present
value of the offered price from the foreign bank. The American bank issues a letter of credit on behalf of Mr. Jones in favor of Mr. Sato.

The letter of credit contains the terms of the transaction: the price and the invoice date. The American bank agrees to back Mr. Jones' promise to pay. The letter of credit informs the exporter that the invoice is eligible for acceptance in 60 days if accompanied by shipping documents.

The American bank sends the letter to a Korean bank. The Korean bank contacts Mr. Sato. The cars are shipped and Mr. Sato presents the shipping documents to the Korean bank transferring the title. The bank pays the discounted present value of the invoice price. The Korean bank takes the documents to the American bank which stamps them "accepted" creating an bankers acceptance.

A bankers acceptance is an unconditional obligation to pay the accepted draft when it matures in 60 days. Mr. Jones arranges to pay the bank in 60 days and receives the shipping documents for the cars.

The bankers acceptance can be held by the Korean banker or the American bank can discount the acceptance at the request of the Korean bank. Discounting the acceptance means it is entered into the books of the American bank as a loan.

A money market instrument is created if the American bank sells the acceptance to an investor. The investor then
finances the import activities of Mr. Jones. At maturity, Mr. Jones pays the American bank and the investor presents the acceptance for redemption.

Commercial paper Rowe (1986) provides a description for the commercial paper money market instrument. Financial and nonfinancial corporations issue commercial paper. This instrument is a substitute for short term borrowing from a commercial bank.

Commercial paper is an unsecured promissory note issued for a specific amount with a stated maturity. The maximum maturity is 270 days. Longer maturities requires registration with the Securities and Exchange Commission. Data on three month commercial paper rates were considered in this study.

Yields represent a spread over the treasury bill rate and most closely track certificates of deposit and bankers acceptances. The paper has carried quality ratings since 1970 when the Penn Central Transportation company defaulted on $82 million in commercial paper. Riskier paper provides a higher yield spread above the treasury bill rate.

Most commercial paper is backed by a bank line of credit. Although maturities are short, there is a chance that the issuer may be unable roll over the debt at maturity.

Eurodollar deposits Eurodollar deposits are dollar denominated deposits in banks not subject to United States
banking regulations. Eurodollars are not just deposits in European banks. The deposit may be with any bank or branch outside the United States or even an International Banking Facility inside the U. S.

Although there are alternative maturities, an instrument with a three month maturity is included in the portfolio of the FICB. Eurodollar deposits are not insured. In addition, there is a substantial penalty if the funds are withdrawn prior to maturity. The deposits pay a rate greater than that offered by domestic accounts because the funds are not subject to reserve requirement regulations.

**Lending activities**

Loans are made to PCAs and other financial intermediaries. The interest rate on loans changes each month based on the new average cost of funds. The loan rate is based on a mark up over the average cost of funds to pay operating expenses. Because loans are not priced on the basis of the marginal cost of funds, lending rates lag changes in debt costs. The maturity structure and cost of previously issued debt determines the extent of the lag.

All borrowers are charged one interest rate regardless of their financial condition. No risk premium is included in the lending rate during the study period. Data on monthly loan rates were provided by the Omaha Federal Intermediate Credit Banks. For the period 1979 to 1983, these rates were
available on unpublished reports titled "Federal Intermediate Credit Bank of Omaha Consolidated Bonds and Farm Credit Investment Bonds Outstanding."

**Liability activities**

The FICB raises funds from four sources. The liability activities include discount notes, six-month bonds, nine-month bonds and intermediate term bonds. The securities are sold through the Farm Credit Funding Corporation on the national money and capital markets. Bonds are issued approximately sixteen times a year. Notes are used to finance short term liquidity.

The characteristics of the FICB debt instruments have been discussed in the previous chapter. Rate data for each of the instruments were collected from the "Consolidated Monthly Bond Outstanding" report. In addition to outstanding issues, the report lists the bonds that have been issued during the course of the month.

**The Empirical Model**

**The variance-covariance matrix**

Monthly data consisting of asset return rates and liability costs were collected for the period 1979 to 1983. The variance covariance matrix was estimated using a routine in SAS, the Statistical Analysis System. The routine also calculated the mean of each asset and liability rate category providing the value for the objective function.
Table 5.2 provides the definition of the asset activity abbreviations. Table 5.3 lists the values of the variance-covariance matrix. The RAND program solves for the minimum risk portfolio of assets and liabilities at each of the alternative rates of return selected. The specific format required to input the matrix into RAND is provided in the appendix.

Model results

Table 5.4 provides the model results. The solution values are graphed in risk return space in Figure 5.1. To earn higher rates of return the institution invests in the Eurodollar money market instrument and loans. The Eurodollar instrument offered the highest return and also exhibited the highest risk as measured by the standard deviation. The asset activities are financed exclusively with the instrument notes payable.

Moving down the efficient frontier to the left results in less use of leverage. The institutions purchase Eurodollar deposits and makes loans in smaller multiples of equity. At a rate of return of 13 percent the portfolio uses both notes and nine-month bonds to finance acquisition of loans and Eurodollar deposits.

At a rate of 12 percent, asset acquisitions are again financed exclusively with one liability instrument. The nine-month bond provides the funds for loan and money market
### Table 5.2: Activity definitions

<table>
<thead>
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<th>Code</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>CASH</td>
<td>Cash</td>
</tr>
<tr>
<td>FUNDS</td>
<td>Federal funds sold</td>
</tr>
<tr>
<td>LOANS</td>
<td>Loans</td>
</tr>
<tr>
<td>ACCPET</td>
<td>Bankers acceptances</td>
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<tr>
<td>CD</td>
<td>Certificates of deposit</td>
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<tr>
<td>PAPER</td>
<td>Commercial paper</td>
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<tr>
<td>EURO</td>
<td>Eurodollar deposits</td>
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<tr>
<td>BOND6</td>
<td>Six-month bonds</td>
</tr>
<tr>
<td>BOND9</td>
<td>Nine-month bonds</td>
</tr>
<tr>
<td>IBOND</td>
<td>Intermediate term bonds</td>
</tr>
<tr>
<td>NOTES</td>
<td>Notes payable</td>
</tr>
</tbody>
</table>
Table 5.3: Variance-covariance matrix

<table>
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<th>ACTIVITY</th>
<th>CASH</th>
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<th>LOANS</th>
<th>ACCEPT</th>
<th>CD</th>
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</thead>
<tbody>
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<td>+.000000</td>
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<td>LOANS</td>
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Figure 5.1: The efficient frontier
investments. Eurodollars are replaced by the assets cash and bankers acceptances.

**Explanation of model results**

The actual asset and liability activities of the Omaha FICB differ from the portfolios included in the efficient frontier. There are several possible explanations for the contrast. The Omaha FICB has a portfolio dominated by lending activity that is financed by a mix of liabilities.

The proportion of Eurodollar assets exceeds loans in that area of the efficient frontier characterized by higher risk and higher return. Notes payable represents the liability with the lowest cost and highest standard deviation explaining its use to fund Euro and loan assets.

Due to the rate data over the study period, profits could be attained by issuing debt and purchasing the money market instrument with the highest spread over the cost of funds. These solutions fail to recognize that the function of the institution is to make loans rather than select from a broad menu of activities to earn profits.

Loans are included in each of the portfolios along the efficient frontier. Loans offer a mean rate of return that is higher than all but the Eurodollar asset. The standard deviation of loans is the lowest of all asset categories. Bankers acceptances have the next lowest standard deviation
and are included in solutions in the lower risk portions of the efficient frontier.

The high return, low risk characteristics of the loan asset seem inconsistent with financial theory that suggest a higher return is required for higher risk assets. Part of the problem is the pricing policy of the FICB. The FICB charged one loan rate to all customers during the study period. The risk statistic does not include credit risk and may understate actual exposure in the loan portfolio.

Lower portions of the efficient frontier fund a three asset portfolio with one liability, nine-month bonds. Cash is represented on the balance sheets along the efficient frontier in the low risk, low return segment. This result occurs because cash has a zero variance and a zero return.

Portfolios with a money market investment, loans and cash more closely approximate the institutions asset diversification but are not consistent with the observed liability diversification in the actual bank portfolio. The correlation coefficients in Table 5.5 provide insight into the problem.

The high correlations between the liability instruments limit the diversification opportunities. The correlation coefficients for the assets also indicate limited opportunities to diversity. As discussed in the section examining portfolio theory, diversification results in risk
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reduction only if there is low or negative correlation between the activities.

The high correlation between the liability rates results from the relationship between yields on money market instruments. Various money market instruments trade with a spread above the treasury bill rate. Although the size of the spread changes over time, the instrument yields are closely related. The FICB liabilities represent borrowing in the money market.

Loans represent the asset with the lowest correlation to other activities. The loan rate is based on the average cost of funds and lags the yields and costs of the other activities. Other asset returns are highly correlated with coefficients above .9. The alternative investment activities are also money market instruments. These instruments also trade at a spread above the treasury bill rate.

Additional Quadratic Programming Models

This section examines alternative models of the FICB portfolio. Constrained and unconstrained models were developed to consider additional activities and management objectives.

A model with restricted investments

The initial model included several money market instruments which were not actively used by the Omaha FICB in the period from 1979 to 1983. During the study period, the
investment portfolio at the Omaha FICB included government agency securities and treasury securities.

The rates for treasury securities were collected from various monthly issues of the Federal Reserve Bulletin. Models including asset yields for 90-day treasury securities did not result in feasible solutions. Two treasury securities activities remained in the model: 180-day treasury bill and one-year treasury bills. A possible explanation of the problem is the stability of the yield spread between the rate on FICB debt and the treasury bill rate over the study period.

After excluding 90-day treasury bills from the variance-covariance matrix, the model was run at rates of return between one percent and twenty percent. In the range from a twenty to a thirteen percent rate of return, the only asset activity on the efficient frontier was loans. Loans were financed with intermediate term bonds.

At a twelve percent rate of return loans and one-year treasury bills were financed with a combination of nine-month bonds and intermediate term bonds. From an eleven percent to a one percent rate of return cash, loans and one-year treasury bills were financed exclusively with the nine-month bond.
A model including a futures instrument

The futures instrument included in the basic model described above was the 90-day treasury bill futures contract. Data on rates were collected from various issues of the Wall Street Journal. Settlement prices were collected for the contract nearest to delivery on the first or second day of the month. The beginning of the month was selected to correspond to the debt issue dates for the FICB.

When included in the basic model, the futures contract did not come into solution at any of the rates of return. The low rate of return relative to the standard deviation suggests that this asset would not be included in the efficient frontier.

Constraining Eurodollar deposits

The initial model included several money market instruments. The Eurodollar asset represented the highest proportion activity at rates of return above thirteen percent. Constraining the proportion of Eurodollar deposits at high rates of return resulted in the substitution of another high risk high return asset, federal funds.

The results of this model were similar to the initial model. At higher rates of return, the high return high risk money market instrument dominated the asset portfolio. Beginning with rates of twelve percent, loans were represented along the efficient frontier in the highest
proportions. Cash and bankers acceptances were also selected. The asset activities were again financed exclusively with the nine-month bonds in the lower portion of the efficient frontier. In the upper portion of the efficient frontier, asset activities were financed with the liability notes payable.

**Constraints on bond activities**

In order to compare the unconstrained solution with the actual activities of the Omaha FICB, models were developed requiring the institution to diversify on the liability side of the balance sheet. Constraining the model to include the six-month bond in the proportions of ten and twenty-five percent resulted in a similar pattern of asset and other liability activities along the efficient frontier.

At rates of return above twelve percent, Eurodollar deposits represented the highest proportion asset on the balance sheet. At a twelve percent rate of return, loans represented the highest proportion asset. The liability activities of nine-month bonds and notes in addition to the constrained proportion for six-month bonds financed asset acquisitions.

Below the twelve percent rate of return assets included loans, bankers acceptance and cash. Loans represented the largest asset activity. Moving down the efficient frontier to lower rates of return resulted in a decline in the size of
each asset category. At rates below twelve percent, six- and nine-month bonds financed assets. At rates above twelve percent notes payable and six-month bonds were the only liabilities included.

Other models were developed requiring the institution to use successively higher proportions of the six-month bond liability. The same general mix of assets and liabilities came into solution along the efficient frontier for these models.

Reviewing total risk for the models requiring use of the six-month bond indicated that the efficient frontier for these models shifted to the right of the initial model. Total risk was greater compared to the initial model at each of the selected rates of return.

Additional models were developed to force the institution to hold a diversified liability portfolio. Constraints requiring the institution to select both the six-month and intermediate term bond activities generated efficient frontiers with both a diversified asset and liability portfolio.

At a thirteen percent rate of return nine-month bonds, six-month bonds intermediate term bonds and notes represented the liability side of the balance sheet. The largest asset category was loans. In addition, the money market instrument of Eurodollars was represented in the asset portfolio.
At a twelve percent rate of return, six-month bonds, intermediate term bonds and nine-month bonds funded an asset portfolio consisting of loans and acceptances. At rates of return below twelve percent the assets of cash, loans and acceptances were financed with six-month, intermediate term and nine-month bonds.

Each of the models that constrained the institution to diversify on the liability side of the balance sheet resulted in an efficient frontier with higher risk at each level of return as compared to the unconstrained model.

Summary and Implications of Results

There are several possible explanations of the model results. First, the time period of the study involved two subperiods: one in which interest rates were increasing and one in which they were declining. The period of rate increases peaked with a cost of borrowing for the FICB in the nine- and six-month bond market at 17.95 percent. The magnitude of interest rate changes and volatility over the period are reflected in the statistics used as inputs into the portfolio model.

The high correlation between activities explains the lack of diversification in the liability portfolio. The high correlations between asset activities also resulted in limited diversification in the investment component of the portfolio. A review of the financial statements for the
institutions over the study period also indicates limited investment diversification.

In all models except those excluding the Eurodollar market instrument, the high return, high risk portions of the efficient frontier involved extensive investment in money market instruments. It is unlikely that bank management would select portfolios for which money market instruments represent the main investment. The FICBs are mandated by law to operate as agricultural lenders. These restrictions limit them to the lower portions of the efficient frontiers where lending represents the major asset activity.

The institution management is likely to make choices from along the lower risk, lower return areas of the efficient frontier. The objective of the institution is to focus on agricultural lending rather than money market investments. Since the institution is owned by borrowers, benefits to stockholders in the form of lower borrowing costs accrue if the bank limits the spread between the cost of funds and the lending rate. Ratio analysis based on data from institution accounting reports indicates that the banks typically operate in the region of a ten percent rate of return.

The results do suggest that the FICB should consider investing in additional money market instruments to earn higher returns. Current bank operations indicate more
aggressive investments in all the additional money market instruments except Eurodollar deposits.

Finally, the model may fail to capture all aspects of debt maturity policy. The close relationship between the cost of debt and the loan rate indicates that the bank is able to effectively manage interest rate risk. Variable rate lending results in a match of the average cost of debt and borrowing rates.

A possible explanation of the observed diversification of liability maturities on the part of the FICB is that it represents an attempt to minimize costs and manage interest rate exposure. Since the study was based on data over a five year historical time period, actual debt costs were available. The bank manager makes decisions without knowledge of the future course of interest rates. Maturity diversification seems to be an attempt to limit reliance on one particular maturity. The implicit assumption seems to be that this strategy will reduce interest rate volatility.

Further Research

The method of FICB loan pricing based on the average cost of funds suggests effective interest rate risk management. The use of variable rate pricing has been utilized by commercial banks with increasing frequency in recent years. The FICB method of loan pricing requires
further analysis. Rates change every month rather than on a quarterly or annual index.

Since the loan rate is based on the average rather than the marginal cost of funds, changes in borrowing costs are not fully reflected in monthly changes in loan rates. Possible exposure to interest rate risk remains on the FICB balance sheet.

The next chapter develops a balance sheet model of the FICB in order to examine further the issue of interest rate risk. The model is based on the concept of duration and is designed to consider more closely the interrelationship between loan and debt maturities.
CHAPTER VI. A DURATION MODEL OF AN FICB

Introduction

This chapter presents a duration model of the FICB balance sheet. The results of the quadratic programming model in the previous chapter suggest that the Omaha FICB effectively manages interest rate risk with the use of variable rate loan pricing. The rate charged borrowers is a markup from the average cost of funds. The bank is assumed to hedge its interest exposure by matching the monthly price changes of debt costs with changes in the lending rate.

Interest rate risk theory indicates that matching price changes does not necessarily insulate the firm from interest rate risk. Unless changes in the market value of assets and liabilities are equalized, the institution is exposed to potential changes in net worth when interest rates change.

The literature and FICB management makes the broad assumption that interest rate risk has been eliminated from the balance sheet with the use of variable rate pricing. No studies exist examining the actual relationship between the asset and liability maturity structures of an FICB.

The objective of this study is to examine the duration of the liabilities and assets of the Omaha FICB in order to determine the extent of institution interest rate exposure. The balance sheet is modeled using the Lotus spreadsheet program.
The next section discusses the format of the model and is followed by a review of the results.

A Duration Model of the Omaha FICB

The focus of the study is the debt liabilities and loan assets of the FICB. The detailed information required in duration analysis is not available for all balance sheet categories. Sufficient information is available to calculate the weighted average duration for debt securities and loans. This section reviews the sources of data for the model, its format and the time period of analysis.

Sources of data for balance sheet categories

The data required to calculate duration includes the coupon interest rate, the maturity date, the time remaining to maturity, the par or maturity value of the security and the yield to maturity. The duration measure utilized in this study is the Macaulay duration. Mathematically duration equals:

\[
D_1 = \frac{\sum_{t=1}^{n} S_t \cdot t \cdot (1+i)^{-t}}{P_0}
\]

- \(D_1\) = duration measured in years
- \(S_t\) = cash flows including both coupon and principal
- \(i\) = the yield to maturity
- \(t\) = time period in which the cash flow occurs
- \(P_0\) = present value price of the bond

This equation was used to calculate the duration of both assets and liabilities for the bank. The duration gap
calculated was based on the difference between the asset and liability duration. This measure is the GAP $K/A$ duration. Mathematically:

\[
(6.2) \quad \text{Gap}_{K/A} = D_A - D_L
\]

$\text{Gap}_{K/A}$ = the capital to asset duration gap
$D_A$ = the duration of assets
$D_L$ = the duration of the liabilities

If the duration of the assets is greater than the duration of the liabilities, the FICB is exposed to decreases in its net worth if interest rates increase. If rates decline, this type of gap position results in additions to the market value of capital. The interest rate is exposed to losses in the market value of net worth if the duration of liabilities exceeds the duration of assets and interest rates decline.

If a financial institution has a duration gap equal to zero then it is immunized. Changes in the interest rate will not affect the market value of capital. An immunized position would be taken if management is uncertain about the future direction of interest rates and is unwilling to accept any interest rate exposure.

**Liability data** The data required to calculate duration came from several sources. The source of information for liabilities was the "Government Agency Issue" financial data published daily in the *Wall Street Journal*. 
Extensive information is published on all agency issues including the debt securities of the Farm Credit System.

Prior to 1979, FICBs and other Farm Credit System institutions issued debt separately. Those rates were listed as FICB debt. After 1979, the FICBs participated in consolidated Systemwide debt issues. That financial information is listed in the Wall Street Journal in the column containing price and rate information for agency issues under the heading "Federal Farm Credit".

The "Government Agency Issue" column lists the daily bid and offer prices for agency debt securities in the secondary market. Also included is the yield to maturity based on that day's bid and offer prices. Individual issues of the FICBs and Farm Credit System are referenced by coupon rate and maturity date.

The Wall Street Journal data were used in conjunction with the unpublished monthly report of the Omaha FICB titled "Consolidated Bonds and Farm Credit Investment Bonds Outstanding". The information in this report includes the date of issue, the maturity date, the coupon interest rate and the par value at maturity.

The list of liabilities from the Omaha unpublished reports was matched with data from the Wall Street Journal. The coupon and maturity date were used to determine the correct yield to maturity for each of the bonds. Each
security on the FICB balance sheet traded each month in the secondary market. There were two exceptions. The yields for two months for one of the securities was estimated by reviewing the yields for other agency securities. Securities with the same coupon and maturity date as the Farm Credit System bond provided the yield estimate.

Bonds with a maturity of a year or more pay interest on a semiannual basis. The coupon payment was based on the recorded coupon interest in the monthly report of outstanding debt. If the original maturity of the bond was less than a year, it was designated as a zero coupon bond. Zero coupon bonds were assigned a value of zero for the coupon interest payment. For the zero coupon bonds, the duration equalled the time remaining to maturity.

**Asset data** Data on the asset side of the balance sheet were approximated based on information contained in the annual reports published by the FICB. Monthly data on loan rates and volume were available on the report listing outstanding bonds. Monthly data were not available on the investment portion of the balance sheet.

Data other than monthly interest rates were unavailable for the loan portfolio. The duration calculation requires information on the exact terms of the loan and the pattern of repayment cash flows. The loan asset was modeled based on the assumption of a one year maturity. Calculations for
alternative terms were not made. Selection of a one year
term may overstate the duration of assets. The consequences
of this assumption are included in the discussion of results.

The annual market value of the investment portfolio was
available from the annual reports of the FICB. Insufficient
data existed to calculate investment duration. The annual
report listed only broad maturity categories for each type of
bond investment. Although insufficient data existed to model
investment, the results section discusses the possible effect
of alternatives.

Methodology of Duration Estimation

Figure 6.1 indicates the method utilized to calculate
the approximate duration of loans and debt securities. A
data input table was constructed. This information was
linked to a model that calculated the duration for each of
the securities. The results from the duration model were
then returned to the last two columns of the data table. The
spreadsheet package used was Lotus 1-2-3. Macro command
language automated the process of duration calculation.

The duration calculation model required data inputs for
the coupon rate, time to maturity and yield to maturity.
Based on that data the model calculated two outputs: the
market value of the asset or liability and the duration. The
source of the code used to calculate the duration was
Cretien, Ball and Brigham (1987). The model contained a
Duration Calculation Module
Input: Yield to Maturity
       Coupon Rate
       Time to Maturity
Output: Duration
       Market Value of Asset of Liability

Data Table of Inputs into the Duration Module

Data Table of Output from the Duration Module

Balance Sheet Totals

Figure 6.1: Spreadsheet Model
subroutine for annual and semiannual compounding. An additional routine was added to consider monthly compounding.

**Time periods of analysis**

The model was constructed on a monthly basis for three separate years: 1979, 1981 and 1983. These three years were selected based on the interest rate environment in those periods. The objective was to examine durations during periods in which interest rates were increasing and decreasing.

The year 1979 was selected because the FICB balance sheet for that period contained several debt securities issued individually by the Omaha FICB. This year also marks the first time debt was issued on a consolidated Systemwide basis. In addition, interest rates started increasing over this time period. Changes in Federal Reserve monetary policy in October of 1979 that targeted a money supply rather than an interest rate variable contributed to the interest rate environment. Deregulation of financial markets and macroeconomic conditions also affected interest rates. Therefore, 1979 was modeled to examine the effect of an increasing rate environment.

The year 1981 was modeled because interest rates and the cost of FICB debt reached their highest levels during this period. Finally, 1983 was modeled because it represented a period in which rate declines occurred. By studying these
years a review alternative interest rate environments is achieved.

Model Results

Duration calculations for 1979

Table 6.1 lists the duration gap on a monthly basis for the year 1979. The duration gap in each month is negative because the duration of the loan assets is less than the duration of the liabilities. This indicates that the institution is exposed to interest rate risk. The interest exposure from a negative duration gap would result in additions to the market value of capital when interest rates were increasing as they were in the 1979 time period.

This position requires further discussion. The asset duration was calculated based on the assumption that the average maturity of loans is one year. If this assumption overstates the actual maturity of the loans, the magnitude of the duration gap would increase.

Other interest bearing assets on the FICB balance sheet include investments in treasury securities, agency securities and federal funds sold. In the model, the effect of these two asset categories was not included.

Although insufficient data exists to model additional investments, analysis of the 1979 financial statements indicates that 70 percent of the investment portfolio
Table 6.1: Duration gaps for the year 1979

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<tr>
<th>Month</th>
<th>Duration gap</th>
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<td>February</td>
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<td>March</td>
<td>-.85</td>
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<td>-.66</td>
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<td>May</td>
<td>-.79</td>
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<tr>
<td>June</td>
<td>-.77</td>
</tr>
<tr>
<td>July</td>
<td>-.88</td>
</tr>
<tr>
<td>August</td>
<td>-.85</td>
</tr>
<tr>
<td>September</td>
<td>-.80</td>
</tr>
<tr>
<td>October</td>
<td>-.75</td>
</tr>
<tr>
<td>November</td>
<td>-.65</td>
</tr>
<tr>
<td>December</td>
<td>-.63</td>
</tr>
</tbody>
</table>
consists of securities with a one to five year maturity. This would tend to increase the duration of assets.

Two factors tend to offset duration of the investment portfolio. First, federal funds sold during 1979 represented approximately half of the nonloan investments. The term of the fed funds was less than a year. This would reduce the duration of nonloan assets. Second, the proportion of nonloan assets on the balance sheet is three percent. The major asset activity is loans representing 91 percent of the asset side of the balance sheet. Loan duration would be the most significant influence on overall asset duration.

FICB management prices loans at variable rates in order to eliminate interest rate risk from the balance sheet. Based on the duration calculations, the debt security and loan portfolio are not matched in terms of duration. This results in interest rate exposure. If rates were declining over the study period rather than increasing the net worth position of the firm would be deteriorating due to the downside of interest rate risk.

Duration calculations for 1981

The period 1981 was characterized by high and volatile interest rates. Borrowing costs ranged from 11.6 to 17.2 percent for six-month bonds and from 11.75 to 17.2 percent for nine-month bonds. This time period was modeled to examine the effect of an increasing rate environment.
The duration gap results are listed in table 6.2. The calculations indicate that the duration gap of the institution for this time period was smaller when compared to the 1979 time period. Management seems to have responded to changes in the interest rate environment by decreasing the maturity and duration of the liability side of the balance sheet.

The duration gap is negative indicating the duration of the liabilities exceeds that of the assets. Although assets other than loans were not modeled, the major nonloan investment category was federal funds. Federal funds represented approximately eight percent of total assets. The short term maturity of fed funds would tend to reduce the duration of the assets.

Duration calculations for 1983

At the beginning of 1983, the cost of six-month bonds and nine-month bonds was 8.65 and 8.8 respectively. Over the course of the year, rates increased reaching a maximum of 10.23 percent for six-month bonds and 10.1 percent for nine-month bonds. Rates increased through August and declined for the last four months of the year.

The values of the duration gap for 1983 are listed in table 6.3. Again the duration gap is negative with the duration of the liabilities greater than assets. The size of
Table 6.2: Duration gaps for the year 1981

<table>
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<tr>
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</tr>
</thead>
<tbody>
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<tr>
<td>February</td>
<td>-.53</td>
</tr>
<tr>
<td>March</td>
<td>-.49</td>
</tr>
<tr>
<td>April</td>
<td>-.45</td>
</tr>
<tr>
<td>May</td>
<td>-.44</td>
</tr>
<tr>
<td>June</td>
<td>-.46</td>
</tr>
<tr>
<td>July</td>
<td>-.46</td>
</tr>
<tr>
<td>August</td>
<td>-.43</td>
</tr>
<tr>
<td>September</td>
<td>-.41</td>
</tr>
<tr>
<td>October</td>
<td>-.41</td>
</tr>
<tr>
<td>November</td>
<td>-.44</td>
</tr>
<tr>
<td>December</td>
<td>-.57</td>
</tr>
</tbody>
</table>
### Table 6.3: Duration gaps for the year 1983

<table>
<thead>
<tr>
<th>Month</th>
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</thead>
<tbody>
<tr>
<td>January</td>
<td>-.68</td>
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<tr>
<td>February</td>
<td>-.72</td>
</tr>
<tr>
<td>March</td>
<td>-.75</td>
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<tr>
<td>April</td>
<td>-.74</td>
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<tr>
<td>May</td>
<td>-.76</td>
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<tr>
<td>June</td>
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<tr>
<td>July</td>
<td>-.73</td>
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<td>August</td>
<td>-.69</td>
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<tr>
<td>September</td>
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</tr>
<tr>
<td>November</td>
<td>-.76</td>
</tr>
<tr>
<td>December</td>
<td>-.78</td>
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</table>
the duration gap is comparable to 1979 and exceeds the values calculated for 1981.

The results have an interpretation similar to previous years. The institution is exposed to interest rate risk because the duration of liabilities exceeds assets. During the periods in which interest rates decline, the market value of the capital of the FICB would be declining.

Summary and Implications of Results

The model results suggest several implications. First, the general assumption that variable rate loan pricing eliminates interest rate risk from the balance sheet is not correct. The institution is still exposed to interest rate risk and will face declines or increases in the market value of net worth depending on the size and sign of the duration gap and the direction and magnitude of changes in the interest rate.

The consistent pattern of a negative duration gap has important consequences during periods of declining rates. A negative gap means that the value of capital declines as interest rates decrease. Currently, FICBs and other Farm Credit System institutions are experiencing declines in capital due to loan defaults. The existence of a negative gap and recent declines in interest rates suggests that analysis of the book value of capital understates deterioration of this balance sheet category. Unless the
duration gap equals zero, the capital account changes in value as interest rates change.

A declining rate environment presents additional problems. Because the FICB prices loans on the basis of the average cost of funds, declines in borrowing rates lag decreases in debt costs. If borrowers can refinance loans at other financial institutions, a declining rate environment would result in loan repayments. This would tend to shorten the duration of loans even further when interest rates fall.

The management of FICBs are in a unique position to utilize the tool of duration. Detailed information is available on the market yield of the liabilities. Management has access to accounting information on investments and loans that could be used to determine the duration gap more exactly than was possible with this study. The structure of the liability side of the balance sheet and the duration gaps calculated in this study suggest opportunities for changes in the terms of loans offered producers. Repricing less frequently than on a monthly basis is suggested.

The investments in nonloan assets also offers an additional method of duration gap management. If the FICB decides to reduce the size of the duration gap, longer term investments with a higher duration could be substituted for the high proportion of short term investments currently on the balance sheet.
It is important for management to more closely examine the interest rate exposure of the FICB balance sheet. The assumption that risk is adequately managed with the use of variable loan rates is not supported by calculation of balance sheet durations. The management may be willing to accept a negative duration gap during periods of rising interest rates but this strategy conflicts with the objective of protecting and maintaining capital adequacy.

At this time, the regulatory authority for Farm Credit institutions does not consider interest rate risk exposure in the examination process. The focus on capital adequacy indicates that a more rigorous examination of interest rate risk should be included in the review of the financial condition of FCS institutions.

The duration gap for FICBs is opposite that of commercial banks. Depository financial intermediaries have shorter maturities on the liability side of the balance sheet. This exposes commercial banks to declines in the market value of capital when interest rates increase.

The final section in this chapter discusses possible areas for further research. The results from this study suggest several important areas for further analysis.

Further Research

Several additional issues merit further research. First, the total balance sheet duration inclusive of all
asset and liabilities categories should be calculated. The detailed data available for liabilities needs to be supplemented with more information on asset terms and cash flows.

A statistical analysis of the prepayment and repayment pattern for loans under alternative interest rate environments would allow duration calculations to incorporate the effect of rate changes on loan paydowns. This information is available for the mortgage loan market and is utilized there to incorporate balance sheet interest rate risk management for mortgage lenders. Calculations in the mortgage loan market are based on the actual experience in loan repayments over time.

An additional area of analysis is the effect of government farm programs on loan repayments. Since these programs affect the cash flows of producers and their subsequent loan repayment activities, a statistical analysis of the relationship would provide greater information on the actual duration of loan assets.

The stated policy objective of the FICB is to eliminate interest rate risk from the balance. The use of variable rate loan pricing does not accomplish this goal. The application of cash market risk management represents one possible solution to the problem. Further research should also examine the use of hedging tools utilized by other
financial intermediaries. These include futures, options and interest rate swaps.

The existence of a negative gap suggests potential for additional research and the application of interest rate risk management tools. Since the duration gap of depository financial intermediaries is the opposite of that for FICBs, the potential use of interest rate swaps should be examined.

In an interest rate swap, institutions with opposite interest rate exposure contract to exchange cash flows thereby eliminating risk for both firms. The FICB and other Farm Credit System institutions are not able to issue callable debt. The use of interest rate swaps could offer an opportunity to reduce interest rate risk.
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I would like to thank Dr. Arne Hallam for his help and guidance in this study. I appreciate the long hours he spent helping me complete this dissertation. I want to thank Dr. Michael Boehlje for his help in this study and during the course of my graduate career. I also want to thank Dr. Jean Adams, Dr. Roger Stover and Dr. Dennis Starleaf for serving on my committee.

Several other individuals also provided me with help and support as I worked to complete this dissertation. I would like to thank Diana McLaughlin for her continued interest in this project and my progress. I also want to thank Sally Chambers for her help in typing parts of the manuscript. Carol Scott also deserves mention for her work on my behalf.

I appreciate the assistance I received from my colleagues at Idaho State University. Dean and Connie Longmore provided invaluable support and encouragement during critical periods when the work was difficult. Corey Schou deserves thanks for his many hours working on the RAND fortran program. I also want to thank Jill Smith and Bill Stratton for their suggestions and support.

My family deserves thanks for a lifetime of support. Their patience and understanding helped motivate me as I finished work on this dissertation.
Table A.1: Input for RAND QPF4 program

```
OBJ
BAL
END
MATRIX
CASH OBJ 0.000000
CASH BAL 1.000000
CASH CASH 0.000000
CASH FUNDS 0.000000
CASH LOANS 0.000000
CASH ACCEPT 0.000000
CASH CD 0.000000
CASH PAPER 0.000000
CASH EURO 0.000000
CASH BOND6 0.000000
CASH BOND9 0.000000
CASH IBOND 0.000000
CASH NOTES 0.000000
FUNDS OBJ 1.124530
FUNDS BAL 1.000000
FUNDS CASH 0.000000
FUNDS FUNDS 0.106540
FUNDS LOANS 0.036420
FUNDS ACCEPT 0.094820
FUNDS CD 0.100190
FUNDS PAPER 0.095470
FUNDS EURO 0.104580
FUNDS BOND6 -0.081490
FUNDS BOND9 -0.080530
FUNDS IBOND -0.067150
FUNDS NOTES -0.094060
LOANS OBJ 1.124790
LOANS BAL 1.000000
LOANS CASH 0.000000
LOANS FUNDS 0.036420
LOANS LOANS 0.037100
LOANS ACCEPT 0.031690
LOANS CD 0.034920
LOANS PAPER 0.032460
LOANS EURO 0.037100
LOANS BOND6 -0.033770
LOANS BOND9 -0.036680
LOANS IBOND -0.028760
LOANS NOTES -0.032610
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ACCEPT BAL 1.000000
ACCEPT CASH 0.000000
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